Reidentifying Collective Entities after Splitting and Merger in Generative Relational Models

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Abstract

Collective splitting and merger distribute members, practices, commitments, resources, and recognized standing among different successors. This paper develops a method for reidentifying organized collectives through such changes within a generative relational framework. It distinguishes source records, normalized appearances, typed transmission, identity relations, and the assignment of subsequent consequences. Positive and negative identity premises determine families of globally admissible equivalences; their secure and possible information remains distinct from a complete historical finding. Conditional derivations examine consistency, conflicting premise repairs, uncertain normalization, temporal aggregation, hidden appearances, and the joint information lost by pairwise summaries. Membership, operation, undertaking, and institutional criteria receive separate justification and criticism. Evidence symmetry constrains unique selection, while adjacent matching and path examples expose the effects of objective, horizon, and score semantics. Rule-changing histories and grounded authority distinguish realized acts from their institutional warrant. Fallible documentary queries separate recovery of a target from recovery of an entire history and make shared-source failures explicit. Constructed merger, dormancy, allocation, and revision dossiers connect the results to complete inquiries. A bounded comparison of official Manchester documents distinguishes formation, heritage, membership carryover, and ceremony. Further analysis addresses historical action, undertaking succession, criterion validation, and exact integration of local findings. The contribution is an integrated evidential procedure with explicit antecedents and inspectable assumptions. Its mathematical results are conditional; broader empirical adequacy and a uniquely privileged numerical-identity criterion remain open.

Keywords: collective continuity; reidentification; organizational splitting; merger; evidence; generative relational models

Research Scope and Method

This inquiry concerns the reidentification of organized collective entities when their members, operations, commitments, and recognized standing follow different historical paths. It develops a proposed method through explicit definitions, conditional mathematical results, constructed cases, and philosophical criticism. Its immediate purpose is to clarify the reasoning needed for a collective continuity finding and the limits of the evidence offered in its support.

The generative relational framework supplies representations of configurations, transformations, appearances, and histories. These representations leave the metaphysical nature of social entities open. The paper treats collective units and their continuity criteria as objects of inquiry whose appropriateness must be defended. A formal construction can demonstrate coherence, incompatibility, or recoverability after its assumptions have been supplied. The interpretation of those assumptions remains part of the philosophical and empirical work.

The numerical examples are constructed illustrations. Their membership counts, transmission relations, and institutional procedures are stipulated to expose distinctions. A separate, bounded comparison of official Manchester documents examines the interpretation of formation, heritage, membership, and ceremonial statements. The documentary study design identifies the further evidence needed to examine practical continuity in actual splitting and merger cases. Each example retains the distinction between stipulated conditions and documented historical claims.

The mathematical arguments use elementary relations, partitions, finite histories, and query functions. Their principal contribution lies in making the commitments of collective reidentification inspectable. Claims of novel mathematics remain outside the present argument. The manuscript distinguishes its proposed interpretation from cited antecedents and limits each comparison to the position actually examined.

The present manuscript develops a conceptual and formal proposal. Review of the new formulations, their philosophical adequacy, and their eventual publication scope remains open. Documentary validation requires the further research described in the study design.

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1 Introduction

Collective organizations often change through processes that distribute their continuities among several successors. A divided association can send most of its members to one branch while another branch continues its central activity, retains its archive, and accepts its outstanding commitments. A merger can bring two established organizations into one administrative structure while preserving different operational units and public affiliations. The question of a continuing collective entity consequently requires more than a record of similarity between earlier and later appearances. It requires an account of which continuation is being investigated, what evidence supports it, and how the proposed identification fits the surrounding history. This paper develops that account within a generative relational modeling framework.

The focal question has a definite scope. It concerns organized collectives whose members, procedures, activities, commitments, and recognition records can be examined across a splitting or merger episode. It begins with candidate appearances whose grouping remains open. An appearance is a represented collective unit at a specified time and resolution; a source record is evidence through which such an appearance is described. This distinction prevents two documents describing one association from being mistaken for two simultaneous associations. It also prevents one administrative name from automatically absorbing several operational units. The first task is therefore to construct the domain of the question with enough care that the later identity relation has an intelligible subject.

The motivating difficulty can be stated without formal notation. Suppose that an earlier organization gives rise to two distinct organizations operating at the same later time. Each inherits something important. If both are declared numerically identical to the predecessor, the transitivity and symmetry of identity identify the two successors with each other. That consequence conflicts with their stipulated distinction. One can preserve the two inheritance paths through a directed relation of continuation, select one successor for a particular reidentification purpose, terminate the earlier identity, or revise the initial account of the units. These are different responses to the case. The evidence and reasons appropriate to one response need not warrant another.

Merger presents the reverse difficulty. A later organization can inherit members, assets, activities, and responsibilities from two predecessors that were distinct at the same earlier time. Treating the later organization as identical to both predecessors would retrospectively identify them. A ledger that records two predecessor links can express inheritance without taking that further step. The conceptual issue is therefore broader than deciding which name survives. It concerns the logical form of a historical claim and the consequences that may legitimately follow from it.

Previous work in the research programme established a distinction between identity on appearances and equivalence between states. Two organizations can display the same behavior under a selected observation scheme while remaining distinct organizations. One organization can also change its behavior while remaining a candidate for reidentification. Likewise, a graph construction can derive connected units at successive configurations without deciding which later unit continues an earlier one. These results create the starting point for the present paper. Their repetition would leave the decisive task unfinished: the independent justification and comparison of continuity criteria.

The proposed answer is an evidentially constrained method. The inquiry first declares its organizational and temporal resolution. It then distinguishes positive identification constraints, required distinctions, and open continuation possibilities. A family of admissible identity partitions retains every grouping compatible with the accepted constraints. Some identities hold throughout the family; others occur only in some candidates. This structure allows an investigator to state a secure conclusion without filling every remaining gap. It also exposes an inconsistent set of premises before their equivalence closure propagates a mistaken merger through the historical record.

The contribution builds on explicit antecedents. Jansen already develops the simultaneous reestablishment restriction [4], and Collins and Lawford-Smith distinguish morally relevant succession from numerical identity [1]. The present work adds an integrated evidential procedure: uncertain record normalization, globally admissible identity families, traceable premise repairs, exact reduction of hidden

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appearances, and a bounded documentary application. Its elementary proofs reconstruct the tools needed for that procedure; the philosophical contribution concerns their warranted use and limits.

A second part of the method concerns criteria. Membership retention, continuing operations, succession to commitments, and authorized registration can matter for different inquiries. Each requires its own interpretation of the evidence. A membership measure needs an eligible population and a rule for dual affiliation. An operational measure needs a specification of the activity and a distinction between inherited practice and independently reproduced practice. A commitment inquiry needs the particular undertaking, its holder, and the grounds for its continuation. Registration requires the relevant system of authority and the effective acts under that system. The method therefore subjects each criterion to a task-specific adequacy argument before combining any of them into a score.

The resulting pluralism has constraints. A researcher cannot choose a criterion solely because it favors a preferred claimant and then treat the outcome as independent evidence. A continuity policy must state the question it answers, apply its rules consistently across relevant cases, expose the evidence that could defeat its conclusion, and account for its practical consequences. Disagreement can then concern evidence, the meaning of the criterion, the legitimacy of the inquiry, or the consequence policy. Keeping these disputes distinct makes criticism more precise. It also prevents a claim of methodological usefulness from silently becoming a metaphysical claim about what organizations ultimately are.

The paper develops several conditional results. A branch constraint identifies the incompatibility between a single identity and two distinct simultaneous successors. A constraint-completion result characterizes when positive and negative identity requirements admit an equivalence. An intersection result distinguishes secure identity from the weaker union of possibilities. A matching construction shows how a coherent sequence of single-successor selections can generate identity paths. A query result characterizes the evidence needed to discriminate historical possibilities with opposing verdicts. Record-level pullback and descent results locate uncertainty in the construction of the appearance domain itself. A conflict analysis identifies the premise withdrawals that restore coherence and the effect of source bundling on repair selection. These are elementary mathematical arguments. Temporal aggregation and exact visible-family reduction further show which identity information a compressed record must preserve. Grounded authority inference locates the accepted premises needed for one explicit discipline of institutional argument. Their role is to expose dependencies in the proposed method and to separate what follows by proof from what still requires interpretive or empirical support.

Two further analyses test the demand for a uniquely selected successor. An evidence-preserving symmetry can exchange the eligible branches while leaving the retained reasons unchanged. A single-valued selection based on those reasons then requires a candidate fixed by the relevant symmetries. A procedural tie-break can resolve a practical choice, with its rationale distinguished from historical warrant. Separately, exact optimization of additive scores over independent boundary matchings does decompose by boundary. An anchored path objective, a changed horizon, or a shared constraint changes that optimization problem. These results make the grounds of selection more precise than a general preference for either local or global methods.

The evidence analysis also permits fallible observations. It asks when the answer patterns compatible with opposing historical verdicts can be distinguished under an explicit error model. The resulting recovery condition concerns the target verdict, which can be recoverable even when the full history remains unresolved. A shared-source model shows how several copied reports can lose the apparent protection suggested by treating their errors separately. The number of records therefore has to be interpreted through their provenance and observation process.

The constructed association used throughout the paper gives these distinctions a common setting. Its members, activities, and commitments divide unevenly. The same historical data support one branch under a membership emphasis and another under an operational or commitment emphasis. Weighted selection produces an explicit threshold at which the preferred successor changes. Additional records can narrow uncertainty about the profile while leaving that evaluative threshold untouched. Later merger and suspension variants show how endpoint resemblance can conceal a different transmission history. The constructed cases establish possibilities and expose specific inferential failures. A separate compar-

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ison of official Manchester records examines the coding of institutional formation, predecessor heritage, membership carryover, and ceremonial recognition. Its bounded documentary findings complement the constructed examples and identify further questions about practical continuity.

The later application chapters bring these parts together. A complete constructed finding passes through source interpretation, appearance normalization, a ten-partition family, boundary refinements, contradiction, two repairs, and a supported correction to a certificate's scope. Another case shows that a correct forecast of programme responses can coexist with failure to recover an earlier identity verdict, even when the complete stipulated policy-response profile is available. These cases distinguish progress in historical knowledge from a successful decision or an accurate prediction.

The inquiry finally considers how findings can be reused across local histories. Exact integration requires agreement on shared appearances and avoidance of global distinctions by the combined closure. Two studies can each be coherent while their combination identifies distinct simultaneous branches through a shared predecessor. The integration result locates that conflict before a common identifier carries it into subsequent research. It also preserves the difference between a least compatible relation and a uniquely warranted historical account.

The exposition moves from the question and its antecedents to domains, branching constraints, criteria, and the divided-association case. It then develops admissible partitions, incomplete evidence, and adjacent selection. Conflicting premise bundles and rule-changing histories receive separate treatment because they change the interpretation of an otherwise coherent identity path. Comparative histories, retrospective revival, and the distinction between historical action and present undertaking succession test the method in more demanding settings. The final sections provide a documentary research design, an integrated finding and revision exercise, sustained philosophical objections, and an analysis of preservation and cross-study integration. This order keeps the formal construction tied to the problem that motivates it. The central objective is a defensible continuity judgment with a visible evidential and conceptual basis.

2 Inquiry and Antecedents

This section places the proposed method within its immediate research setting and its principal antecedents. It first specifies the suspension of ontological judgment, then separates three kinds of justification, and finally identifies the literature relations actually used. The purpose is to make the paper's commitments explicit before formal definitions give them an appearance of inevitability.

2.1 Ontological Suspension and the Object of Inquiry

The paper investigates representations and warranted judgments of collective continuity. It leaves open whether a successful representation identifies a fundamental social entity, a constituted entity, a persisting pattern, or an object whose status needs another account. Suspending this choice allows the inquiry to ask a substantive question that arises within several positions: given the available records and an articulated continuity criterion, what historical identification is supported? The suspension therefore has a positive methodological role. It defines which disagreements the present argument can address and which require further premises.

This choice preserves an insight from the source discussion. An efficacious pattern can have a continuing role in a relational system without being treated at the outset as an independently self-subsisting substance. The insight motivates a representational strategy; it remains distinct from a proof that every collective entity has a particular metaphysical nature. Causal efficacy, temporal persistence, and reidentifiability each need their own evidence. A pattern might affect later activities yet fail to be the same organization. A named organization might persist under a recognized rule while losing the practical capacity that originally made it significant.

Huang's Part I supplies the programme's abstract starting point: a carrier, composable mechanisms acting on it, and a preserved equivalence [3]. The present inquiry adds collective appearances, typed

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histories, evidence constraints, and independently examined continuity criteria. Those additions are model commitments requiring their own warrant.

The relevant carrier in the generative relational framework represents configurations of a situation. A configuration may contain several organizations, participants, resources, and conditions. Collective appearances are therefore indexed within configurations. This separation avoids taking the entire situation to be one individual or treating every trajectory of the situation as the history of one organization. It also permits a collective unit to emerge from relations inside a configuration while leaving its continuity across configurations available for investigation.

The word "entity" in the title consequently names the subject of reidentification, with its philosophical status held open. The paper's definitions are working definitions for a specified inquiry. Their value depends on whether they preserve the distinctions needed for that inquiry and whether the proposed judgment survives objections. The notation is a discipline of explicitness. Its use does not settle the larger ontological discussion that the research programme deliberately suspends.

2.2 Constitutive, Evidential, and Practical Justification

A continuity finding can be justified in several ways. This subsection distinguishes a rule constituting a status, evidence supporting a historical claim, and reasons for allocating a consequence. These forms of justification can cooperate while remaining different.

A constitutive rule can specify which registered organization continues after a permitted reorganization. Under a stipulated registry, an authorized act may transfer or preserve a status according to declared procedures. The rule is part of what makes that status obtain. Determining whether the act occurred and whether its conditions were met remains an evidential task. Furthermore, the constituted status can leave the continuity of daily practice, social affiliation, and historical contribution unsettled. Its authority is located in the domain in which the rule operates.

An evidential argument begins with a proposition about a history. Records of training, transfer, participation, or commitment uptake can support a continuation claim. Their evidential force depends on provenance, relevance, reliability, and competing explanations. A new finding changes the warranted reconstruction of a prior event; the investigator should preserve the difference between the reconstructed event and the later finding. This is especially important when an organization subsequently describes itself as a successor. That description is evidence of its present claim and may also provide testimony about the past. The two uses require different assessments.

A practical justification concerns what should follow. Access to an archive, stewardship of an undertaking, compensation for harm, or recognition of contribution can require a rule that considers continuity along with other facts. A branch might be best able to complete a project even if another branch receives the continuing registration. Historical contribution can deserve preservation even when an organization has ceased to exist. Such judgments require normative premises about the consequence at issue. Identity can be relevant without supplying a complete allocation rule.

The same document can play more than one role. A signed succession agreement may perform an authorized transfer, report earlier negotiations, and give reasons for future cooperation. Its performative, evidential, and normative aspects should be recorded separately. Treating the document as a single unqualified proof of identity loses these distinctions and obscures which aspect a later objection challenges.

2.3 Structural and Agency Antecedents

The paper's use of organization and relational structure has substantial antecedents. Ritchie develops a structural account of groups and explicitly considers membership change and distinct groups with coincident members [10]. This supplies a relevant comparison for the present distinction between a membership set and a collective unit. The current paper's evidential construction leaves the associated ontological account open. Its question concerns how a particular continuity judgment is warranted when records or criteria remain unsettled.

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List and Pettit's book defends group agency and connects its significance to explanation and accountability [5]. This paper separates the continuity of a collective from the further explanatory question of agency: a continuity relation on collective appearances does not by itself establish the organization of beliefs, goals, or capacities required by a particular account of agency.

A recent and especially relevant comparison is Tałasiewicz's model of constituted groups, whose definitions involve statutes, membership, operation, and histories of membership and statutes [11]. That model makes the need for a precise continuity construction particularly clear. The present paper begins earlier in the evidential process, where the complete history and the appropriate grouping of records remain to be established. Its family of admissible partitions preserves that incompleteness rather than assuming a fully specified group history at the outset.

These comparisons establish a boundary on originality. Structure-based group modeling, membership replacement, and the distinction between group and members are established subjects of inquiry. The proposed contribution here is the coordinated treatment of branch constraints, independently argued criteria, partial evidence, and revisable findings in one scoped reidentification method. Whether that combination advances the wider literature requires a fuller comparison than the present access permits.

2.4 Provenance and the Limits of Formal Vocabulary

The W3C provenance specification provides a useful record-design antecedent. It distinguishes entities, activities, derivation, alternate descriptions, specialization, and membership [7]. The paper uses this typed approach to motivate preserving different relations in an identity record. Encoding two descriptions or a derivation link still requires a justified interpretation in the application.

This limitation generalizes. A graph edge labelled "continuation" can accurately store a declared claim while leaving its truth disputed. An equivalence relation can organize a chosen identity policy while leaving its adequacy unsettled. A score can compare measured features while leaving its weights open to criticism. Each representation becomes more useful when it retains the reasoning that made the label, relation, or number appropriate. Formalization should therefore increase the visibility of assumptions and objections. The following sections use it for that purpose.

3 Collective Appearances and Relational Histories

This section constructs the domains on which collective reidentification will operate. It distinguishes records from appearances, appearances from whole configurations, and collective-unit formation from temporal continuity. It then introduces a directed history without assigning identity to its paths. These preparatory distinctions determine the meaning and limits of the subsequent proofs. The later subsections examine mechanism occurrences, organizational context, vacant roles, and the evidential status of configuration descriptions. Their purpose is to connect the formal domains to the particular forms of organization the inquiry seeks to understand.

3.1 Records and Normalized Collective Appearances

An investigation usually encounters documents, interviews, registers, photographs, accounts, or activity traces. These are records. Several records may concern one collective appearance, and one record may concern several appearances. Their relation to the focal organization therefore needs an interpretive mapping. A photograph of a meeting can show participants from two associations. Two newsletters can describe the same association using different names. Counting either photographs or names directly as collective units would incorporate an unwarranted identity judgment into the domain.

Let R be a set of indexed records, $T = \{0, \dots, n\}$ a finite sequence of observation times, and I_t a set of candidate collective appearances at time t. Define the disjoint appearance domain

$$I = \coprod_{t \in T} I_t, \qquad \tau : I \longrightarrow T. \tag{1}$$

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The disjoint union preserves the time of each appearance even when its local description repeats. An element $i \in I_t$ is a represented collective unit at the selected resolution. The map τ supplies its time index. A record-support relation $W \subseteq R \times I$ records which sources bear on which appearance. It may be many-to-many and can itself remain uncertain.

Normalization means that two elements of I_t represent distinct candidate units for the present inquiry, after duplicate descriptions have been examined. It is a modeling and evidential operation, not an assurance of infallible recognition. If two descriptions might denote one unit or two, the investigation can retain alternative normalizations. The initial formal results fix one normalization to state their assumptions cleanly. A later comparison can vary it. The important point is to expose the choice rather than conceal it inside a supposedly neutral list of entities.

The temporal resolution also needs a statement. A slice may represent a day, a meeting, a reporting period, or an episode bounded by a reorganization. Treating an entire year as one slice can hide a sequence of organizations that occupied the same office at different times. Treating every email as a slice can multiply appearances without adding relevant organizational transitions. The resolution should be fine enough to capture the splitting or merger mechanism and coarse enough to make the focal collective unit intelligible. Sensitivity to that resolution belongs to the adequacy analysis.

3.2 Configurations and Internal Collective Units

The generative relational setting represents a whole configuration by $x \in S$. A mechanism b acts through a transformation T_b on the configurations where it is defined. An actual recorded history can be written as a sequence of configurations and indexed mechanism occurrences. For a fixed history, an appearance map

$$p : I \longrightarrow S$$

places each collective appearance in the configuration that describes its setting. Many appearances can share the same configuration because several organizations may coexist. The map need not be injective.

A collective unit may be supplied by institutional records or constructed from the relations represented inside a configuration. For example, a connected-component rule on a graph derives units from elementary positions and edges. Such a rule can make formation and division explicit, but it also embeds a choice about what unity means. A single connecting edge can join two otherwise independent clusters. If that edge records occasional contact, connectedness may be unsuitable for an inquiry into coordinated governance. If it records an indispensable joint procedure, it may have greater relevance.

The present paper keeps the unit-construction rule visible and open to challenge. A proposed rule should state its elementary positions, the kinds of relations it uses, and the organizational question it serves. The rule's mathematical consequence can then be distinguished from its social adequacy. A partition of a graph proves a partition of vertices. It becomes a model of collective organization through an additional interpretation of those vertices, edges, and grouping conditions.

This separation is especially useful under membership change. A role can remain while its occupant changes. An activity can continue through a new arrangement of roles. A registry can preserve an organization while the operations represented in its configuration cease. The choice of the appearance domain should allow these differences to be described before an identity verdict is imposed. Otherwise the formal model answers only the question that its input has already settled.

3.3 Transmission and Directed Lineage

Once candidate appearances have been specified, the inquiry can record relations of transmission between them. Let

$$L \subseteq \{(i, j) \in I^2 : \tau(i) < \tau(j)\}$$

be a directed lineage relation. A pair iLj records a specified kind of continuation from i to j. The edge can carry a type, such as transfer of members, instruction in an activity, transfer of a resource, or uptake of a commitment. Its evidence must support that type. The mere chronological order of two appearances

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supplies a necessary ordering condition for this model, while leaving the existence of a transmission edge unsettled.

Different edge types can yield different lineages. Members may leave an association and form a new one without taking its activities. A manual can be copied by a collective with no shared members. Two branches can jointly continue a project through a federation. Recording these continuations in separate relations permits their histories to overlap without forcing them into a single identity partition. The evidence for one type also remains available when another type is disputed.

The transitive closure L^+ records paths of the selected transmission. Whether an indirect path warrants the same interpretation as a direct edge requires care. A sequence of instructions can support a history of learning, but the final practice may change substantially along the chain. A sequence of member transfers can connect organizations that share no members at the endpoints. The path demonstrates a composed route in the represented history. Any claim that a particular property survives the whole route needs a preservation argument for that property.

Lineage can branch or merge. Its in-degree and out-degree need not be one. These possibilities are among its advantages for the present inquiry. A directed relation can represent several inherited continuities without implying that all their bearers are one entity. A later identity policy may select some of these edges, terminate an earlier identity, or recognize several kinds of successor. It should retain the lineage evidence that its more selective output leaves outside a single identity path.

3.4 Changing Attributes and the Identity Question

Attributes such as membership, location, resources, or procedure belong initially to appearances. Write $a : I \to V$ for an attribute and let E be a proposed equivalence on I. If a is to become an invariant attribute of the quotient I/E, it must be constant on every E-class. That requirement is appropriate for an invariant identifier. A changing attribute should instead be indexed by identity and time, or retained at appearance level.

This elementary distinction prevents a common error in reasoning about organizational change. If membership changes, the membership function fails to descend as a time-independent property of the proposed entity. That fact alone leaves identity open. To infer a different entity, one must add that membership is constitutive of identity for the relevant inquiry. Conversely, a stable name does descend under an identification rule based on that name, but the resulting constancy is partly built into the rule. Its historical adequacy still requires an argument.

The formal domains therefore prepare a disciplined sequence. First establish what the records describe, then identify candidate units, then reconstruct typed transmission, then assess a continuity criterion. Each step can depend on earlier judgments and can be revised by later evidence. Their separation does not require an impossible observer without assumptions. It requires that assumptions remain available for criticism and that their consequences are not mistaken for independent discoveries.

3.5 Coincident Membership and Overlapping Organizations

The appearance domain should be tested against a case in which the same people constitute two organized collectives. Imagine six participants who operate both a reading association and a venue cooperative. The two bodies have different decision procedures, undertakings, and purposes, although their membership sets are identical at the observation time. This is a constructed example of the distinction between membership description and collective unit.

A domain built by identifying collectives with their membership sets would produce one unit. A domain that distinguishes their organized procedures would produce two. The choice matters when the participants later divide: one branch may continue the reading activity while another continues management of the venue. If the original two organizations were collapsed at the outset, the later history could appear to be a split of one entity when it is more appropriately described as different continuities of two already distinct organizations.

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The evidence for the two-unit description should concern the organizational distinction. Separate names can be useful leads, but different procedures, undertakings, and spheres of decision provide a more substantive basis. The investigator should also consider the alternative that the two activities were departments of one organization. The model's purpose and resolution determine which distinctions are relevant, while the records constrain which interpretations are defensible.

Overlapping membership also complicates a physical boundary picture. The same person can act in different roles at the same meeting. A statement can be made on behalf of one collective while the speaker belongs to both. A resource can be used jointly under an agreement that preserves separate ownership or stewardship. An appearance record should therefore include the relevant role or organizational context when the same individuals participate in several units.

3.6 Distributed Organization and Local Manifestations

A distributed organization presents the opposite risk: treating several local manifestations as distinct entities merely because they occupy different places. A collective can coordinate across several venues or maintain specialized units. If the inquiry concerns the whole organization, its appearance may include all those locations. If the inquiry concerns operational subunits, they can receive separate appearances linked to the larger collective.

The temporal separation assumption should be applied at the chosen level. Two simultaneous departments can be parts of one organization without violating slice separation for the organization-level domain. If the departments are treated as candidate entities in their own right, their identity relation belongs to another domain. Mixing levels would create an artificial contradiction: the model would ask one equivalence to represent both being the same department and belonging to the same organization.

The relation between levels can be recorded explicitly through membership, part, delegation, or organizational-subunit relations, with their meanings stated. Such a relation need not have the same formal properties as identity. A department can move from one organization to another while retaining its local activity; an organization can reorganize its departments while continuing its constituted standing. These histories again separate collective formation, internal relation, and temporal identity.

3.7 Changes in the Unit Criterion

An investigation may discover that its original unit criterion was too coarse or too fine. Suppose it initially treats a federation as one operational entity, then finds that its member organizations make the decisions relevant to the target independently. Refining the unit domain may improve the explanation. The change should be recorded as a revision of the representation, with a correspondence between old and new appearances where possible.

Such a refinement is different from discovering that the federation actually split at a historical time. In the first case, the researcher's description changes. In the second, the represented organization changes. Both can occur together, but their distinction is necessary for a coherent history. A new diagram with more nodes should not automatically be interpreted as evidence that more organizations came into existence.

The model should also preserve the earlier criterion's valid uses. A federation-level description can remain appropriate for one institutional question even if a subunit-level description is needed for another. Refinement can add a level of analysis without declaring the earlier representation universally false. The relevant question is whether each description preserves the distinctions required by its stated purpose.

These cases show why normalization deserves philosophical attention rather than treatment as routine data preparation. It specifies what can be identical, distinct, overlapping, or internally related in the subsequent model. The operation therefore carries conceptual commitments whose effects can propagate through every later theorem. Making them explicit is part of the paper's substantive account of reidentification.

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3.8 Mechanism Types and Historical Occurrences

A generative account should distinguish a represented mechanism from its occurrence in a particular history. This subsection clarifies that distinction and the relation between transformations of configurations and lineage among collective appearances.

A transformation describes how a configuration changes under a specified mechanism and its relevant conditions. A recorded occurrence supplies the particular occasion on which that change is represented as taking place. Two occurrences can instantiate the same mechanism type while affecting different collectives, participants, or institutional settings. Repetition of a procedure name therefore leaves the identity of its historical occurrences and their objects to be established.

Consider a rule-governed replacement of a committee member. The mechanism description can specify eligibility, nomination, approval, and transfer of the role. A historical record may establish only that a replacement was announced. To identify the occurrence described by the mechanism, the inquiry must examine which steps occurred, whether their conditions held, and which organization was involved. The formal transformation can organize those questions without making the announcement itself a complete description of the event.

The transformation acts on the configuration, which can contain several collective units and their surroundings. It does not automatically induce one function mapping each earlier collective appearance to a unique later appearance. A division can reorganize one unit into several; a merger can bring several into one configuration; a role replacement can alter a unit without creating a new organizational grouping at the chosen level. The appearance construction and the transmission relations must be examined after the change.

This point protects the distinction between generative modeling and reidentification. A mechanism can explain how later organizational relations arise while leaving open which identity class contains the resulting appearances. Conversely, a criterion can preserve identity through several different mechanisms. Continuity of an office may survive routine replacement, relocation, or a permitted amendment, although those events have different causal structures.

The use of transformation maps is a modeling choice with declared domains and conditions. An incompletely observed event can remain compatible with several transformations or several input configurations. A model that represents an occurrence by a deterministic map does not thereby establish that every social process is deterministic or that the historical record uniquely identifies the map's inputs. The accepted history family can preserve these alternatives while the conditional results operate on each specified history.

3.9 Configuration Context and Collective Boundaries

A collective's represented boundary is a boundary of the focal unit, while the configuration can include relations beyond that unit. This subsection examines how external conditions enter an explanation without automatically becoming members or parts of the collective.

A workshop association may depend on access to a venue, a training institution, and a group of programme partners. Those relationships can help explain its capacity to act. The venue provider need not become a member of the association merely because its permission is indispensable. The configuration should represent the dependency and the unit boundary separately, so that causal relevance is not confused with organizational inclusion.

The boundary can itself depend on institutional relations. An external body may appoint a role holder, recognize a registration, or set a condition for access. Such relations can enter the criterion's account of the collective even when the external body remains a distinct organization. A purely internal membership description would omit part of the relevant history. The question is which relations constitute or support the unit at the chosen level, and which merely influence its circumstances.

Splitting can redistribute these dependencies unevenly. One branch can retain the venue relationship while another inherits trained participants. A third organization can supply new resources after the division. The later branches may therefore differ in capacity for reasons extending beyond what they in-

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herited directly. A continuity criterion focused only on the proportion of transferred members can miss this explanatory difference.

Merger can change the same relations without simply adding two internal structures together. A combined organization may gain access unavailable to either predecessor, lose a partner that supported only one branch, or create a new procedure for coordinating their activities. The configuration history should represent these changes. Treating the merger as the union of predecessor membership sets would preserve one descriptive relation while leaving the new organization unexplained.

The resulting model remains selective. It need not include every causal condition in the social environment. Its explanatory target determines which dependencies require representation, and evidence tests whether the selection is adequate. The appearance boundary should remain open to revision when an omitted relation turns out to govern the very transition under investigation.

3.10 Roles, Vacancies, and Organizational Activity

A collective can change the occupancy and use of its roles without an immediate one-to-one correspondence between role change and entity change. This subsection distinguishes role description, actual coordination, and institutional continuation through a constructed vacancy case.

Suppose an association's procedure specifies a treasurer, a coordinator, and an assembly. At one time the treasurer position is vacant while the assembly continues to authorize expenditure through a stated interim arrangement. At another time all three role names remain printed in a document, but the participants have ceased meeting and no longer regard the procedure as applicable. The two descriptions differ in the relation between a preserved role scheme and ongoing organization.

A model based only on occupied positions could treat the first case as a new structure and thereby force an identity change. A model based only on printed role names could treat the second as fully unchanged. Both constructions would need a reason for making their selected feature decisive. The vacancy case suggests that role occupancy, the maintained procedure, the ability to coordinate, and actual programme activity should initially be represented as distinct descriptions.

The institutional criterion may permit vacancies and specify how they are filled. The occurrence of a vacancy can then be compatible with continuity under that criterion. An operational criterion may ask whether the remaining arrangement can sustain a particular activity. The same vacancy can matter more strongly to that question if the absent role held an irreplaceable practical capacity. The case's interpretation depends on the target and the independently evidenced organization.

The reverse situation is equally important. An appearance can retain participants capable of performing a programme while lacking authority to act under a former institution's name. Capacity and standing are different relations. A programme can continue through an informal team after the institutional association ends, while a constituted association can remain recognized during a period of reduced programme activity. The model should allow each possibility to be evaluated rather than resolve it by defining activity and institutional existence identically.

These distinctions prepare the suspension analysis later in the paper. A blank programme record, a preserved procedure, and a future resumption do not have one automatic implication for the intervening organization. The configuration should identify what is known about coordination and role applicability, leaving the remaining possibilities available for historical inquiry.

3.11 Repeated States and Distinct Historical Positions

The representation can contain the same descriptive state at different historical positions. This subsection explains why those positions remain distinct and what further evidence a continuity judgment requires.

An association may return to an earlier roster after a temporary change, restore a previous procedure, or resume an earlier programme. If the state description contains only those attributes, the earlier and later configurations can appear identical in the model. The time indices and recorded sequence still distinguish the occurrences. Equality of state descriptions is therefore compatible with a history containing important intervening events.

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Suppose a group of participants closes one association and later forms another with the same name, roster, and written rules. Compare a history in which the first association remains in force through an inactive interval. The endpoint descriptions can agree while the institutional transitions differ. A criterion that depends on termination and formation needs evidence of those events. Comparing only the endpoints cannot recover a distinction that the observation omits.

The history can also matter to operations. A practice restored after independent relearning differs in transmission from a practice maintained through continuing instruction. Both can produce a similar final performance. The distinction may matter to provenance or an explanation of how capacity survived, while a narrow assessment of present performance can legitimately focus on the final capacity. The model should retain the historical distinction whenever the query depends on it.

This is why the appearance domain remains indexed even when descriptions repeat. An appearance at one time is a separate argument of the identity question from an appearance at another time. The proposed equivalence can identify them after the criterion and evidence are applied. Building their identity into the equality of a descriptive state would bypass the question the inquiry seeks to answer.

The converse inference also requires care. A change in the represented state supplies a change in its included attributes. It establishes a new entity only under a criterion that makes the relevant change decisive. The configuration history and the identity history therefore remain linked but distinct objects of analysis.

3.12 Evidence for Relations within a Configuration

A relational representation can appear richly explanatory while its edges remain weakly supported. This subsection examines the evidential status of those edges and the distinction between documenting a claim and establishing the relation it describes.

A source can state that one committee supervises another, that a person holds a role, or that a programme belongs to an association. Each statement is evidence requiring interpretation. Its support can depend on who made it, the procedure through which the relation was established, the period to which it applies, and the activities that give it practical effect. An authenticated source identifies a genuine document; its assertions remain available for substantive evaluation.

The same relation can have several evidential dimensions. A governing document can specify formal reporting authority. Meeting records can show whether reports were made. Participant accounts can describe who actually coordinated the work. Disagreement among these sources can reveal a gap between specified and effective organization. The model should preserve that difference when it matters to the continuity criterion, rather than select one relation label to stand for both.

Negative relational claims need comparable discipline. Failure to find a handover record can mean that the handover did not occur, that the record was lost, that it was kept elsewhere, or that the practice used another means of transmission. The evidential force of absence depends on the expected observation process. A model can explicitly stipulate complete observation for a bounded domain, but the application of that stipulation to an archive requires its own warrant.

The record-support relation introduced at the beginning of this section therefore indexes possible evidential relevance. It should not be read as a truth predicate. A source can bear on an appearance by supporting its existence, disputing an attributed role, or revealing ambiguity in its name. The extracted proposition and its assessed status must remain distinguishable from the source’s mere connection to the topic.

A well supported configuration is consequently more than a detailed diagram. It is a representation whose relations have typed meanings, temporal scope, and inspectable grounds. Those grounds give the later identity analysis an evidential subject and make it possible to revise the configuration when a source or interpretation changes.

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3.13 The Explanatory Status of the Relational Construction

The final preparatory question concerns what the construction commits the inquiry to. This subsection locates the model's explanatory ambition while suspending a general ontological adjudication of collective entities.

The model treats collective appearances as objects that can be described, related, and queried within a specified inquiry. It permits an analysis of their formation through participants, procedures, activities, and institutional relations. This representation can be assessed for whether it captures the relevant organization and supports warranted historical distinctions. Its use does not require settling every philosophical question about the ultimate constituents of social reality.

Suspending that broader adjudication leaves substantive demands intact. The inquiry must still distinguish a realized organization from a proposed one, justify its unit boundary, and explain which mechanisms sustain the represented activity. A purely convenient grouping can be useful for one descriptive purpose while failing as an account of the collective object under another. The suspension of ontology should not become permission to treat any mathematically available partition as equally adequate to the evidence.

The same balance applies to causal explanation. A relational configuration can help explain what coordinated participants can accomplish and how changes in their arrangement affect activity. The account needs evidence for those relations and a defensible interpretation of the mechanism. Naming a collective entity or drawing a connected component supplies neither requirement by itself.

The construction therefore serves a disciplined intermediate role. It makes the object of reidentification explicit, preserves the relations that a later identity verdict must respect, and allows competing accounts to be compared on common questions. The subsequent formal results test the coherence and information content of those accounts. Their philosophical use depends on maintaining the connection between the represented relations and the evidence that gives them social meaning.

4 Record Normalization and Organizational Resolution

The appearance domain carries commitments that deserve a formal analysis of their own. This section models the movement from referential record tokens to candidate collective appearances. It derives how identity on appearances becomes a relation on records, characterizes when a record relation can be represented on a normalization, and combines the result with the positive and negative constraints used later in the paper. A four-token example and an exact-transport result then show how apparently routine changes of organizational resolution can change the identity question itself.

4.1 Referential Tokens and Their Evidential Status

A source document can refer to several organizations and several times. To represent its identity questions precisely, the model begins with referential tokens. A token is one indexed use of a reference to a focal collective occurrence, linked to the document and passage that supply it. A meeting record naming both branches yields at least two relevant tokens. A later history describing the predecessor at two different times can yield two more. Tokens are units of reference in the research representation; they are not automatically distinct organizations.

Let R be a finite set of such tokens, and let $\tau_R : R \to T$ record the time of the referred-to occurrence. This time differs from the creation time of the source. A document written after a merger can refer to a predecessor before the merger. Combining the two times would place the historical reference in the wrong observation slice and could create an artificial conflict with another record. The source timestamp and referred-to time therefore need separate fields.

The construction also presupposes a reference interpretation within each candidate model. A document can misidentify an organization, make a false claim, or use an ambiguous name. Assigning a token to a candidate appearance does not establish that the appearance actually occurred. Reference validity,

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source reliability, and existence of the referred-to unit remain evidential questions. An investigator can retain alternative interpretations or leave a token unresolved.

For the following proofs, every token has one supplied focal interpretation within the candidate model. This makes normalization a function. The assumption is useful for a first exact construction and should not be confused with a general claim about language. A many-to-many or partial reference model requires an extension. The present results state what follows once the simpler interpretation has been adopted and its status made explicit.

4.2 Candidate Normalizations

A normalization groups tokens proposed to describe one collective appearance. Let I be its appearance domain and let

$$\nu : R \longrightarrow I \tag{2}$$

be a surjection. Surjectivity means that every appearance in this restricted evidential domain is represented by at least one token. It does not require that every actual organization has left a record. Unrecorded appearances remain outside this particular domain unless introduced through an explicit extension.

The normalization is time-preserving when a map $\tau_I : I \to T$ satisfies $\tau_R = \tau_I \circ \nu$. Tokens grouped into one appearance therefore refer to the same occurrence time at the chosen resolution. If the time interpretation is uncertain, the inquiry can compare several candidate models with different time assignments. It should not silently merge references to different times solely because they use the same name.

The kernel of ν is the relation

$$\ker \nu = \{(r, s) \in R^2 : \nu(r) = \nu(s)\}.$$

It records which tokens the normalization treats as descriptions of one appearance. Since equality is reflexive, symmetric, and transitive, the kernel is an equivalence on the record-token domain. This equivalence concerns duplicate or co-referential descriptions within an appearance, before temporal identity has been assigned.

The distinction is conceptually important. A normalization can already force a same-appearance judgment. If that grouping is mistaken, every identity relation on the normalized domain inherits the mistake when read back onto the tokens. The formal identity stage cannot recover a distinction that normalization has removed without revising the normalization itself. The model therefore keeps the map and its evidence available for review.

4.3 Identity Pulled Back to Records

Let E be an equivalence on the appearance domain I. Define a relation $F_{\nu,E}$ on record tokens by

$$rF_{\nu,E}s \quad \Longleftrightarrow \quad \nu(r)E\nu(s). \tag{3}$$

This relation answers whether the two references belong to one continuing collective according to the normalization and identity criterion. It can connect tokens at different times and can connect several tokens describing the same appearance.

Proposition 4.1 (Identity pullback and normalization commitments)**.** *The relation $F_{\nu,E}$ is an equivalence on R, and* $\ker \nu \subseteq F_{\nu,E}$.

Proof. For every token r, reflexivity of E gives $\nu(r)E\nu(r)$, hence $rF_{\nu,E}r$. If $rF_{\nu,E}s$, symmetry of E gives $\nu(s)E\nu(r)$, hence $sF_{\nu,E}r$. If $rF_{\nu,E}s$ and $sF_{\nu,E}u$, transitivity of E gives $\nu(r)E\nu(u)$, hence $rF_{\nu,E}u$. These are the three equivalence properties.

If $(r, s) \in \ker \nu$, their images are equal. Reflexivity of E then relates those images, and Equation (3) gives $rF_{\nu,E}s$. Thus the kernel is contained in the pulled-back identity. □

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The containment states an unavoidable commitment of the representation. Once two tokens are assigned to one appearance, an identity equivalence on that appearance domain cannot treat them as references to different entities. A source reviewer who needs that distinction must change the normalization or the interpretation of the tokens. Calling the later identity procedure neutral would conceal this dependence on its input.

The result also supplies a common domain for comparing normalizations. Each pair (ν, E) gives an equivalence on the same token set R. The inquiry can compare which record-level identities hold across admitted pairs. Such a comparison still requires a common referential question; different organizational levels may refer to different subjects even when the same source passage is used.

4.4 Descent of a Record-Level Identity

The reverse question asks whether an equivalence on records can be expressed through a proposed normalization. This is useful when some record-level identity judgments have already been warranted and the investigator wants to consolidate their descriptions without losing those distinctions.

Proposition 4.2 (Descent through a normalization)**.** *Let F be an equivalence on R and let $\nu : R \to I$ be surjective. There is a unique equivalence E on I with $F = F_{\nu,E}$ if and only if* $\ker \nu \subseteq F$.

Proof. Necessity follows from Proposition 4.1. For sufficiency, assume $\ker \nu \subseteq F$. For $i, j \in I$, choose representatives r, s with $\nu(r) = i$ and $\nu(s) = j$, using surjectivity, and define iEj when rFs.

To show representative independence, take other representatives r', s' with the same respective images. Kernel containment gives rFr' and sFs'. If rFs, symmetry and transitivity give $r'Fs'$. Reversing the argument gives the converse. The truth of the proposed iEj therefore depends only on i, j.

Reflexivity and symmetry of E follow directly from those of F. For transitivity, suppose iEj and jEk. Represent the two occurrences of j by tokens s, s'. They are F-related because their images agree. The two given relations and that middle relation yield the relation between representatives of i and k by transitivity of F. Hence iEk.

The pullback of E equals F by construction. Any other relation with that pullback must give the same answer to every pair i, j by choosing representatives. Surjectivity supplies representatives for every pair, so the descending equivalence is unique. □

The theorem is an elementary quotient construction. Its research significance lies in the test it provides. A normalization is compatible with an accepted record-level identity exactly when it merges only tokens that this identity already permits to coincide. It may consolidate duplicate descriptions, but it cannot erase a required distinction and still represent the same relation exactly.

This condition should be interpreted with the status of F in view. If F is a provisional criterion output, compatibility preserves that output; it does not establish its truth. If F is itself uncertain, the inquiry can ask which candidates descend and preserve the alternatives that do. The mathematical test keeps those commitments explicit without deciding their philosophical adequacy.

4.5 Slice Separation on the Token Domain

The identity model distinguishes normalized simultaneous units. To express that requirement on record tokens, define

$$N_\nu = \{(r, s) : \tau_R(r) = \tau_R(s),\ \nu(r) \neq \nu(s)\}. \tag{4}$$

These are pairs referring to the same time but assigned to distinct appearances by the normalization. Their required distinction is conditional on that normalization. It differs from an independently warranted negative judgment about the references.

Proposition 4.3 (Record form of slice separation)**.** *Suppose F descends to E through a time-preserving surjection ν. Then E is slice-separated if and only if $F \cap N_\nu = \varnothing$.*

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Proof. Suppose first that E is slice-separated. If rFs and the token occurrence times agree, the images are E-related and have equal times. Slice separation gives $\nu(r) = \nu(s)$, so the pair cannot belong to N_ν.

Conversely, suppose F avoids N_ν. Take E-related appearances i, j at the same time and choose representatives r, s. Time preservation gives equal occurrence times, and the pullback property gives rFs. If $i \neq j$, the pair belongs to N_ν, contradicting avoidance. Therefore $i = j$. □

The result distinguishes duplicate simultaneous descriptions from distinct simultaneous units. Two tokens can share a time and belong to one identity class when normalization assigns them to one appearance. They are excluded only when the normalization treats their images as distinct units. Applying slice separation directly to all raw tokens would therefore be too strong and would falsely separate duplicate descriptions.

The distinction also explains how a revised normalization can alter the negative constraints without changing the documents. If two earlier appearance labels are found to refer to one unit, their corresponding pairs leave N_ν. The revision concerns the interpretation of the records. An independent negative constraint would still require separate review; it cannot disappear merely because the normalization has changed.

4.6 Joint Feasibility of Normalization and Evidence

Let $P, N \subseteq R^2$ contain independently warranted positive and negative identity requirements about the same target. Positive requirements are stronger than similarity or lineage, and negative requirements are stronger than missing evidence. For a candidate normalization, form

$$C_\nu = \mathrm{Eq}(P \cup \ker \nu),$$

where Eq denotes the smallest equivalence containing the displayed relation. It incorporates both the accepted identity evidence and the normalization's same-appearance commitments.

Proposition 4.4 (Feasibility of a candidate normalization)**.** *A slice-separated identity on I compatible with the record requirements P, N exists if and only if*

$$C_\nu \cap (N \cup N_\nu) = \varnothing. \tag{5}$$

Proof. Any compatible identity pulls back to an equivalence F containing P and $\ker \nu$. It therefore contains their equivalence closure C_ν. The independent negative evidence requires F to avoid N, and Proposition 4.3 requires it to avoid N_ν. Its subset C_ν must therefore avoid their union.

Conversely, suppose Equation (5) holds. Set $F = C_\nu$. It is an equivalence containing $\ker \nu$, so Proposition 4.2 gives a unique descending equivalence E. It contains the positive evidence and avoids the independent negative evidence by construction and the assumed condition. Avoidance of N_ν makes E slice-separated. Thus E witnesses feasibility. □

The procedure can filter a family of competing normalizations before a final identity partition is selected. It is constructive: a successful candidate has the explicit witness obtained by descending C_ν. A rejected candidate has a positive closure path that conflicts with either independent negative evidence or its own simultaneous-unit distinctions. That path can be examined in the source record.

The theorem does not rank evidence or prove the normalization family complete. A conflict can reveal a bad normalization, an unwarranted positive link, an incorrect negative claim, or a mismatch between the targets of the constraints. The procedure identifies the incompatible combination and preserves the premises that must be reconsidered.

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4.7 A Four-Token Reconstruction

Take four tokens r_1, r_2, s_1, s_2. The r-tokens refer to time zero and the s-tokens to time one. Suppose independent evidence under one accepted criterion warrants $r_1 F s_1$ and $r_2 F s_2$, while another warranted finding requires s_1 and s_2 to refer to distinct continuing entities. These are premises of a constructed case, not reported findings about actual documents.

Consider first the fine normalization that assigns every token to a different appearance. Positive closure gives two classes,

$$\{r_1, s_1\}, \qquad \{r_2, s_2\}.$$

Each contains one appearance at each time, so the slice condition is satisfied. The independent negative requirement keeps the two classes apart. The identity relation is therefore feasible and unique under these constraints.

Now consider a normalization that groups r_1 with r_2 as descriptions of one initial appearance while keeping s_1, s_2 separate. Its kernel supplies $r_1 F r_2$. Together with the two positive cross-time links, transitivity joins all four tokens. This contradicts both the independent later distinction and the normalization's distinction of the later units. The candidate is infeasible.

A third normalization groups the two initial tokens and also groups the two later tokens. It removes the later pair from its conditional set N_ν, but it does not remove the independently warranted negative requirement in N. Positive closure still joins all four tokens and violates that requirement. The candidate remains infeasible. Changing the representation cannot erase independent evidence without an explicit revision of that evidence.

The example can look like an inference that the initial records must describe two units because their later continuations are distinct. That interpretation is valid only relative to the accepted identity links and criterion. If the cross-time links were merely branching lineage, one initial unit could produce two later units without contradiction. The status of the links is therefore indispensable to the inference.

4.8 Interpretation of a Normalization Conflict

A normalization conflict should lead to a diagnostic discussion rather than an automatic correction. In the four-token case, several repairs are possible. The initial records may refer to two organizations. One cross-time link may be a continuation of practice without identity. The later distinction may be mistaken. The two positive links may use different criteria. Each repair changes a different premise and preserves different parts of the original interpretation.

The researcher should compare these repairs against the source evidence. If a document records two independent decision procedures at time zero, it can support the two-unit normalization. If a handover only concerns a copy of a manual, it may support a lineage interpretation of one link. If the later records describe departments of one larger organization, the normalization of the later slice may need a different level of analysis. The formal contradiction directs attention to these possibilities without settling them.

Preserving the rejected normalization remains useful. It records why the initial grouping was plausible and what evidence or argument defeated it. A later source revision may reopen the alternative. The research record should therefore retain candidate models and their rejection reasons, rather than store only the final map as if it had been obvious from the beginning.

This practice also protects the distinction between uncertainty and error. An unresolved normalization can remain one member of the feasible family. A normalization shown incompatible with accepted premises receives a conditional rejection. If the premises themselves remain disputed, the rejection should state that dependence. The model can be exact about the logic while remaining careful about the evidence.

4.9 Exact Transport across Organizational Resolutions

Suppose two appearance domains are connected by a surjection $\pi : I_f \to I_c$, interpreted as a coarsening from finer units to coarser units. Assume it preserves occurrence time. One might hope to preserve the

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identity question exactly by requiring

$$iE_f j \quad \Longleftrightarrow \quad \pi(i) E_c \pi(j). \tag{6}$$

The following result exposes the limit of that hope when both domains assert distinct simultaneous units.

Proposition 4.5 (Exact transport and unit coarsening)**.** *If E_f, E_c are slice-separated equivalences and Equation* (6) *holds for a time-preserving surjection π, then π is bijective. Conversely, a time-preserving bijection transports a slice-separated equivalence uniquely.*

Proof. Suppose $\pi(i) = \pi(j)$. Reflexivity of E_c relates the equal images, and the right-to-left direction of Equation (6) yields $iE_f j$. Time preservation gives equal times for i, j. Slice separation of E_f therefore gives $i = j$. Thus π is injective; the assumed surjectivity makes it bijective.

Conversely, use the inverse of a time-preserving bijection to transfer the equivalence to the other domain. Its equivalence properties follow under the bijection. Equal-time related images have equal-time related preimages, which are equal by slice separation. The transferred relation is therefore slice-separated and is unique by the required correspondence. □

The theorem concerns exact preservation of this identity relation, not every useful relationship between levels. A department can belong to an organization, and several departments can be mapped to one organization. That map can be useful for aggregation or organizational description. It cannot make being the same department equivalent to belonging to the same organization when distinct departments coexist.

4.10 Descriptions, Units, and Competing Hypotheses

The contrast between record consolidation and unit coarsening is central. Several tokens can describe one simultaneous appearance without violating slice separation, because the tokens are descriptions. Several asserted distinct simultaneous units cannot become one identity while retaining exact slice-separated identity at the finer level. The two operations can look similar in a database but have different conceptual meanings.

There are also two reasons to compare normalizations. They may be competing hypotheses about the same referents: the records describe one organization or two. Or they may represent different levels: local units and a federation. In the first case, evidence can favor one hypothesis about a shared target. In the second, both descriptions may be appropriate for different questions. A model should state which comparison it undertakes.

The exact-transport result helps expose a concealed change of subject. If a researcher merges several asserted local units and then carries every identity judgment over unchanged, the conditions cannot all hold. The researcher must weaken the required correspondence, change the relation, or explain that the target is now the larger organization. This is a constructive clarification of scale, not a prohibition on multilevel social analysis.

4.11 Uncertain Dates and Partial Reference

The model fixes occurrence times and one focal interpretation per token. Actual records can leave either uncertain. A statement about "the association before the merger" may refer to a range of times. A shared name may be ambiguous between branches. A purported event may lack adequate support. These conditions require extensions before the exact results are used.

One option is a family of candidate time and reference assignments. Each candidate carries its own normalization and identity family, and the inquiry compares common record-level questions across them. Another option uses partial maps for unresolved references. The domain of each partial map then becomes part of the model; treating an unresolved token as absent or false would be an additional inference.

Many-to-many reference can also be appropriate when one phrase refers collectively to several organizations. It should not be forced into one token with one image unless the phrase is first decomposed in

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a justified way. The decomposition itself can change what the model represents. These issues belong to the interpretation of records and cannot be settled by an equivalence closure after the fact.

4.12 Research Use and Remaining Obligations

The normalization construction gives the documentary protocol a distinct stage before temporal identity. Researchers can record source references, propose candidate appearance maps, state independent identity requirements, and test their joint feasibility. Rejected maps have traceable conflicts; retained maps preserve unresolved alternatives. The process produces an explicit account of how the domain of the later identity question was constructed.

The strongest practical benefit is the prevention of hidden certainty. A polished list of organizational appearances can look like a set of observed facts even when its grouping depends on disputed interpretation. Retaining normalization hypotheses shows where that uncertainty enters. The later partition analysis can then distinguish a conflict in historical identity from a conflict in reference or unit boundaries.

The formal results remain conditional. They do not establish which source is reliable, whether a collective appearance is actual, or which organizational resolution is best. Their contribution is to make those obligations visible and to derive the consequences of particular answers. That is the appropriate role of mathematics in the present philosophical inquiry: precise dependence and inspectable counterconditions, with the substantive judgment still answerable to evidence and argument.

5 Temporal Aggregation and the Identity of Observed Units

Temporal aggregation changes the domain on which a continuity inquiry is conducted. An annual organizational record can combine several appearances that were distinguished in a finer history. This section derives the conditions under which that combination preserves a proposed identity exactly, examines whether one coarse normalization can preserve competing identity candidates, and separates identity transport from aggregation of membership descriptions. The final discussion concerns hidden splitting, endpoint sampling, and the design of records that preserve uncertainty about which units continued within an observation interval.

5.1 Fine Times, Observation Bins, and Coarse Appearances

Let I be a finite domain of fine appearances, with time map $\tau : I \to T$. Let T and B be finite ordered sets, and let $\beta : T \to B$ be an order-preserving surjection. The elements of B are coarser temporal bins. They may represent months containing several recorded meetings or years containing several organizational transitions. The order assumption makes their temporal interpretation explicit. The algebra below uses the weaker condition of compatibility among the displayed time maps.

Write

$$T_\beta = \{(i, j) \in I^2 : \beta(\tau(i)) = \beta(\tau(j))\} \tag{7}$$

for the equivalence of belonging to the same temporal bin. A proposed coarse domain J has a time map $\tau_J : J \to B$ and a surjection $\pi : I \to J$ satisfying

$$\tau_J \circ \pi = \beta \circ \tau. \tag{8}$$

The surjection assigns every fine appearance a coarse referent. It therefore represents consolidation of appearances, with complete coverage of the supplied fine domain.

The proposed coarse normalization uses one appearance of an entity within each bin. This is the interval counterpart of the earlier normalization discipline. It permits several different entities to be represented in one bin, including entities whose periods of activity only partly overlap. Membership in the same bin consequently supplies less temporal information than exact simultaneity at a fine time. A researcher should preserve that difference when interpreting the coarse units.

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Let E be a candidate identity equivalence on I satisfying the fine slice-separation condition. The question is whether an equivalence E_B on J can preserve every fine identity verdict through exact pullback while satisfying the one-appearance-per-entity condition in each coarse bin. This is a precise transport problem. A deliberately lossy record can remain useful when the condition fails, but its interpretation must state which information it has discarded.

5.2 The Kernel Required for Exact Identity Transport

The two requirements can be written together:

$$iEj \quad \Longleftrightarrow \quad \pi(i)E_B\pi(j), \tag{9}$$

$$xE_By \text{ and } \tau_J(x) = \tau_J(y) \quad \Longrightarrow \quad x = y. \tag{10}$$

The first preserves the fine identity relation. The second specifies the normalization of the coarse appearances. Together they determine exactly which fine appearances the map π must combine.

Proposition 5.1 (Temporal identity kernel)**.** *For a time-compatible surjection $\pi : I \to J$, an equivalence E_B satisfying Equations 9 and 10 exists exactly when*

$$\ker \pi = E \cap T_\beta. \tag{11}$$

When it exists, E_B is unique.

Proof. Suppose such an E_B exists. If $\pi(i) = \pi(j)$, reflexivity of E_B and exact pullback give iEj. Time compatibility also gives $\beta(\tau(i)) = \beta(\tau(j))$. Hence $\ker \pi \subseteq E \cap T_\beta$.

For the reverse inclusion, suppose iEj and the two appearances lie in the same bin. Exact pullback gives $\pi(i)E_B\pi(j)$, and time compatibility gives equal coarse times. Coarse separation therefore gives $\pi(i) = \pi(j)$. Thus $E \cap T_\beta \subseteq \ker \pi$.

Conversely, suppose Equation 11 holds. In particular, $\ker \pi \subseteq E$, so the earlier descent result defines a unique equivalence E_B through representatives. If two coarse appearances are E_B-related and have the same coarse time, their representatives are E-related and belong to the same bin. They therefore lie in $E \cap T_\beta = \ker \pi$, making the coarse appearances equal. The descended equivalence satisfies both required conditions. □

The result gives temporal aggregation a substantive interpretation. Within each bin, appearances belonging to the same proposed entity must be combined. Appearances belonging to different proposed entities must remain separate. The appropriate consolidation therefore depends on the identity relation whose exact preservation is being claimed. Temporal binning alone supplies the bin membership relation; it does not supply the additional identity information in Equation 11.

For a fixed E, the required coarse domain always exists. The intersection $E \cap T_\beta$ is an equivalence, so take its quotient and assign each class the common bin of its members. The resulting projection has exactly the required kernel. This construction proves conditional existence. In an empirical inquiry, the candidate E may remain unsettled, so constructing its quotient cannot be used as evidence that the candidate was the correct identity relation.

5.3 Candidate Families and a Shared Coarse Domain

Suppose the evidence admits a nonempty family $\mathcal{A}$ of fine identity equivalences. An analyst may want one coarse normalized domain that represents every candidate without already choosing among them. The temporal-kernel result identifies when that is possible.

Corollary 5.2 (Shared temporal normalization)**.** *A common time-compatible normalization can support exact coarse-separated descents of every $E \in \mathcal{A}$ exactly when the relations $E \cap T_\beta$ are identical across $\mathcal{A}$.*

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Proof. If one normalization π works for every candidate, the proposition makes every $E \cap T_\beta$ equal to its fixed kernel. Conversely, if all candidates share a relation K of that form, take the quotient I/K. The induced coarse-time map is well-defined, and the proposition supplies an exact coarse-separated descent for each candidate. □

The condition concerns agreement about particular within-bin pairs. Agreement about the number of coarse entities is weaker. Let a occur at one fine time and let distinct b, c occur at the next. Put both fine times in one bin. One candidate identity has classes $\{a, b\} \mid \{c\}$; another has $\{a, c\} \mid \{b\}$. Each requires two coarse appearances. Their required consolidations nevertheless differ: one combines a with b, and the other combines a with c. A common two-row output does not determine which fine records belong to which row.

The example clarifies a practical risk in constructing longitudinal datasets. A system can report a stable number of organizations while its assignment of records to continuing units remains uncertain. If an analyst treats the stable count as confirmation of a particular unit mapping, the uncertainty has been hidden inside the normalization step. The coarse table may then appear to provide independent evidence for a continuity decision that was used to create the table.

One response is to preserve a common record-token domain with candidate-dependent normalization maps. Each candidate can then state how it groups the records and which identities follow at the coarse level. Another response is to choose finer temporal bins whose within-bin identity restrictions are secure. A third is to retain unresolved correspondence explicitly in the coarse record. These responses have different costs, but each preserves the question that a single unexplained normalization would settle without argument.

The earlier same-time coarsening result appears as a special case. If β leaves time unchanged, fine slice separation makes $E \cap T_\beta$ the diagonal relation. Exact normalized transport then requires an injective π. Temporal aggregation can combine successive appearances of one entity because their fine times differ. Merging distinct appearances at one unchanged time is a different operation and faces the earlier restriction.

5.4 Changing Descriptions within a Coarse Identity Class

Exact transport of identity leaves a further descriptive question. Let $\psi : I \to X$ assign a selected state description to each fine appearance. The description might be a roster, a participant count, a procedure, or a recorded activity. An exact coarse description with the same meaning would be a function $\psi_B : J \to X$ satisfying

$$\psi = \psi_B \circ \pi. \tag{12}$$

Proposition 5.3 (Description preservation within temporal bins)**.** *Under exact temporal identity transport, a function satisfying Equation 12 exists exactly when*

$$E \cap T_\beta \subseteq \ker \psi. \tag{13}$$

Proof. If ψ factors through π, every pair with the same π-image receives the same description. Hence $\ker \pi \subseteq \ker \psi$. Conversely, that inclusion makes the assignment $\psi_B(\pi(i)) = \psi(i)$ independent of the chosen representative. Substituting $\ker \pi = E \cap T_\beta$ gives the displayed condition. □

A continuing entity can change its roster within one year. Its fine appearances can therefore be combined for an annual identity record while failing the condition for one unchanged roster description. The apparent tension disappears when identity and state description retain their different roles. The annual unit can have a roster history, an interval union, an average count, or an endpoint roster, each with its own semantics.

The choice among these descriptions should follow the target. An interval union identifies people who participated at some recorded time. An endpoint roster identifies participation at a selected boundary. An average count summarizes a time-indexed quantity under a specified weighting scheme. These

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descriptions need not agree, and none supplies every question answered by the fine history. A data dictionary should preserve the construction used to produce the reported value.

This point also distinguishes a genuine invariance from an artifact of limited measurement. A collective can replace members while maintaining a constant membership count. The count then descends exactly through an interval consolidation, while the roster itself fails exact descent. A study interested in staffing level may retain the needed information; a study interested in participant continuity may lose it. The formal condition is therefore indexed to the description selected for the inquiry.

5.5 Interval Membership and a Threshold Reversal

A constructed membership path shows how aggregation can change a continuity test even when its numerical threshold remains unchanged. At four successive fine times, let the rosters be

$$\{1,2\}, \quad \{2,3\}, \quad \{3,4\}, \quad \{4,5\}. \tag{14}$$

Each adjacent pair shares one participant and has a union of three. Its overlap-to-union ratio is therefore $1/3$. Under a stipulated local transmission criterion with threshold $1/4$, every fine adjacent transition is eligible. A single-continuation selection can consequently preserve the whole path under that criterion.

Put the first two appearances in one bin and the last two in another. If interval membership is defined by union, the two coarse rosters become $\{1,2,3\}$ and $\{3,4,5\}$. They share one participant and have a union of five, giving ratio $1/5$. The same threshold now rejects the comparison, because

$$\frac{1}{3} > \frac{1}{4} > \frac{1}{5}. \tag{15}$$

The calculation reveals a changed measurement variable. A fine roster records membership at a particular represented time. A bin union records membership at some time within an interval. The latter can contain a combination of people that never appeared together in any fine roster. Applying the same threshold to the two variables does not preserve the earlier rule's behavior merely because both variables are sets of people.

The example also distinguishes temporal aggregation from the earlier local-versus-endpoint counterexample. The present coarse comparison uses an interval union on each side of a boundary. It includes people who had already left before the boundary and people who arrived afterward. Their inclusion changes the denominator and therefore changes the meaning of the reported continuity score. The effect can occur even when the fine path's local transmission conditions are completely observed.

A researcher can respond by recalibrating the criterion for the aggregate variable, retaining boundary rosters, or using a criterion defined over the fine history. Each response changes a specified part of the research design. The example supplies a test case for those choices: a revised procedure should explain whether it intends to preserve the fine path's verdict and why. Choosing a convenient threshold after seeing the desired answer would leave the criterion's independent warrant unresolved.

The constant count in this example adds a useful comparison. Every fine roster has two participants, so the count description satisfies the exact-descent condition inside each bin. The interval-union rosters have three participants each. A report of three annual members can be accurate under an interval-union convention while a report of two members at each fine time is also accurate. Treating these as contradictory counts would repeat the same failure to identify the predicate that the aggregation was designed to express.

5.6 Endpoint Sampling and Hidden Intermediate Branches

Temporal consolidation covers the fine domain by assigning every appearance a coarse referent. Endpoint sampling performs a different operation: it retains some appearances and omits others. If only the first and last appearances of a history are observed, the evidence can leave intermediate splitting, suspension, and reunion unresolved. The omitted events may matter even when the endpoint identity query has a secure answer.

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Consider two candidate histories that agree that an initial appearance a and a later appearance d belong to one entity. In one history, the continuing path passes through intermediate branch b; in another, it passes through branch c. The other branch remains distinct in each history. An endpoint sample containing only a, d can preserve the shared answer to their identity query while losing which intermediate branch carried that continuity. The resulting record is adequate for one query and inadequate for another.

This difference matters for downstream attribution. An undertaking entered during the intermediate period may depend on the authority of the branch that entered it. A study of institutional lineage may need to distinguish which practices were transmitted through each branch. A membership inquiry may need the actual route taken by participants. The secure endpoint identity cannot automatically supply these omitted historical relations.

The converse is possible as well. Several candidate histories can display the same endpoint attributes while disagreeing about endpoint identity. A dormant collective and a newly founded successor can share a name and procedure at the later time. The endpoint descriptions then require transmission and appointment evidence to settle the selected identity query. The sampling design should be evaluated against that query before its adequacy is assumed.

A temporal dataset should consequently distinguish an unobserved interval from an interval whose internal history has been consolidated. The first lacks specified observations; the second contains observations whose relations have been summarized. The evidence that would improve each record is different. Recovering an omitted source addresses the first problem. Reopening an aggregation and its normalization assumptions addresses the second.

5.7 Interval Units and the Design of Reusable Findings

The preceding results suggest a concrete design for a reusable coarse continuity finding. The record should identify the fine or source-token domain from which it was constructed, the temporal bins, the proposed unit normalization, and the identity query whose preservation is claimed. It should state whether the normalization is common to every retained historical candidate or depends on a particular candidate assignment.

Descriptions attached to the coarse units should record their aggregation semantics separately. An annual membership union, an end-of-year roster, and a count of distinct participants serve different purposes. Preserving their construction allows later researchers to test whether a continuity criterion can legitimately reuse the value. A stable column name alone is insufficient evidence that two datasets measure the same relation.

The record should also retain the questions that survive aggregation and those that remain unresolved. A coarse identity relation may preserve endpoint reidentification while omitting intermediate branch selection. A membership count may be exactly preserved while a roster history changes. These are specific information boundaries, each supported by the corresponding model or source comparison.

The philosophical implication is that observation resolution can participate in the construction of the collective subject being tracked. Choosing a year instead of a meeting as the observation interval does more than alter the date format when the system also insists on one normalized appearance per entity in that interval. It can require a decision about which fine appearances belong together. The temporal-kernel condition makes that dependency explicit and identifies when uncertainty can be retained within one common coarse domain.

This result strengthens the paper's initial separation of records, appearances, and identity. A coarse organizational record can be a valuable research object while containing interpreted identity assignments alongside observed facts. A defensible continuity inquiry preserves those assignments and their warrants, allowing the record to be reused and criticized without mistaking its construction for independent confirmation of the entity whose history it represents.

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6 Branching Constraints on Reidentification

This section derives the basic constraint on identity through splitting and merger. It states the relevant identity model, proves the obstruction, and examines possible responses. The formal argument is deliberately elementary: its importance lies in identifying the assumptions that an apparently innocent successor label can combine inconsistently. The subsequent analysis examines the force of this constraint for exact identity, the timing of an organizational division, the interpretation of reunion, and the changes of premise concealed by apparent counterexamples.

6.1 A Declared Model of Single-Unit Identity

Fix the normalized appearance domain of Section 3. An equivalence relation E on I represents a proposed reidentification. The class $[i]_E$ collects the appearances treated as one continuing entity under that proposal. Reflexivity includes an appearance in its own class, symmetry permits identification in either direction, and transitivity makes successive identifications cohere. These properties concern the proposed identity relation; a directed transmission relation can have another form.

Assumption 6.1 (Separation within an observation slice)**.** For normalized collective appearances, a proposed identity relation satisfies

$$iEj \text{ and } \tau(i) = \tau(j) \quad \Longrightarrow \quad i = j. \tag{16}$$

The assumption expresses a chosen organizational resolution: two distinct simultaneous units count as distinct entities in this identity model. It should be defended in the domain construction. If the inquiry represents two departments as simultaneous manifestations of one larger organization, the departments are subunit appearances and the larger organization requires its own appearance domain. If two records describe the same unit, normalization should connect them to one appearance. Equation (16) therefore cannot be imposed indiscriminately on raw descriptions or arbitrary spatial parts.

The assumption is compatible with distributed organizations. A collective can operate at several places while having one appearance in a slice if that appearance represents the distributed unit. It is also compatible with overlapping membership between distinct organizations. What matters is the declared distinction between candidate units, not the uniqueness of their members or locations. This is why the scope of a formal identity axiom must be stated before it is interpreted philosophically.

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6.2 Organizational Division and Its Description

Applying the separation assumption requires a reason to distinguish the simultaneous units independently of the desired successor verdict. A constructed translation association illustrates this requirement. The example separates delegated work, a later division of organizational authority, and the records describing those arrangements.

Suppose the Harbour Translation Association initially undertakes one programme under a common assembly. It later opens two translation desks in different venues. The assembly continues to settle their shared undertaking, and its decisions remain effective for both desks. For an inquiry into the whole association, the two desks can be represented as parts of one appearance. Two local reports may describe that appearance from different locations. Their separate headings and the public use of the word *branches* leave the whole-association normalization to be justified by the organizational relations.

Now stipulate a subsequent arrangement in which two associations undertake separate programmes under their respective procedures. The earlier common procedure has ceased to govern either. For the selected inquiry, the case treats these as distinct whole units. This is a substantive premise about the represented organization. The mere existence of two committees would supply insufficient grounds, since a single association can contain several committees with delegated responsibilities. Evidence concerning their authority, dependence, and scope would have to support the chosen unit boundary in an actual investigation.

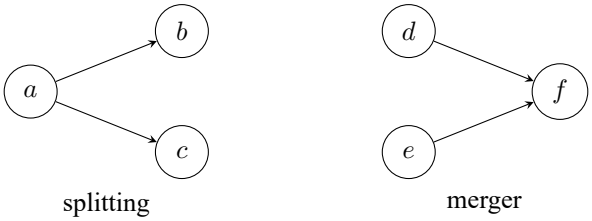

Figure 1: Directed transmission under splitting and merger. Simultaneous branches remain distinct normalized collective appearances.

The contrast makes two kinds of revision intelligible. Discovering that the two local reports described one association corrects a reference or normalization judgment. Discovering that a formerly effective common procedure ceased to coordinate the desks concerns a historical change in the represented relations. An inquiry can encounter both revisions, and their evidence can overlap. Keeping their inferential roles separate prevents the number of labels in the researcher's diagram from becoming the explanation of the number of organizations in the history.

A proposed normalization also needs protection against circular defense. If the sole reason for treating the two later associations as one unit is that both must be identical to the predecessor, the normalization already contains the verdict whose consistency it purports to establish. Independent evidence for a persisting distributed whole would give the revision a different standing. It could show that the earlier domain represented local manifestations at the wrong organizational level. The branching proof therefore gives the investigator a reason to expose the simultaneous-unit judgment, together with the sources supporting it. It supplies a diagnostic pressure on that judgment while leaving room for its warranted correction.

6.3 Splitting and the Successor Obstruction

Consider an appearance a at time t and two appearances b, c at a later time u. Suppose $b \neq c$. A lineage can contain both aLb and aLc. The following result shows the constraint on converting both links into identity.

Jansen gives the corresponding symmetry-and-transitivity argument for distinct simultaneous reestablishments [4, §5.4]. The present reconstruction makes appearance normalization and its slice condition explicit before applying that elementary restriction to an evidential family.

Proposition 6.2 (Identity through a divided successor slice)**.** *An equivalence satisfying Assumption 6.1 cannot contain both aEb and aEc when $b, c \in I_u$ and $b \neq c$.*

Proof. Suppose both identifications hold. Symmetry gives bEa. Combining it with aEc, transitivity gives bEc. Because b and c occur in the same slice, Assumption 6.1 yields $b = c$. This contradicts their stipulated distinction. At least one of the two identification claims must therefore be excluded within this model. □

The result leaves several questions open. It gives no reason to choose b over c. It permits both links to remain lineage links. It also permits the earlier entity to terminate while both successors begin new identities. The proof identifies an incompatibility among premises; a substantive criterion and evidence are needed to choose an interpretation of the case.

Figure 1 displays the two directional patterns. The arrows are transmission possibilities. Their graphical continuity should not be read as an identity assertion without the additional criterion.

6.4 Exact Identity and the Force of a Conditional Result

The proposition concerns an exact equivalence within a declared model. Its philosophical force depends on distinguishing that requirement from both a degree of resemblance and an independently established

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fact of numerical identity. This distinction also explains why the elementary proof can matter without supplying a complete theory of collective persistence.

Within one proposed identity relation, identification licenses the composition of identifications. If an archival inquiry treats an earlier appearance and a branch as appearances of one entity, then treats that earlier appearance and the other branch in the same way, it inherits a commitment concerning the two branches. Treating every pair locally while refusing to examine this inherited commitment would remove the very coherence that makes one continuing subject available across the records. The proof exposes this consequence of exact linkage. It becomes useful where different records, claimant narratives, or institutional decisions each seem persuasive in isolation.

An equivalence alone establishes less than actual numerical identity. Classification by a shared founding year or membership in a historical family can also produce equivalence classes. The interpretation of E therefore requires a reason why its classes answer the particular identity question. Formal consistency is one condition on the proposed answer. Evidence concerning the focal organization and the relevance of the criterion supplies further conditions. A consistent partition can remain historically unwarranted, and several consistent partitions can remain live alternatives.

The phrase *at most one* carries a further limitation. Excluding joint identification with two branches leaves open identification with one branch and also termination of the predecessor's identity. The proof provides no existence premise ensuring that the entity survives. It supplies no comparison that favors the larger branch, the branch retaining a name, or the branch with an apparently closer practice. Those possibilities require their own arguments. Treating the obstruction as a command to select some winner would add an existence and decision requirement absent from its assumptions.

The same point limits a normative interpretation. A branch excluded from one exact identity path can still have transmitted expertise, fulfilled an inherited undertaking, or sustained relationships valued by former participants. Its members can have reasons to recognize their history in that continuation. Exact organizational identity and the grounds of those claims need separate examination. Conversely, a selected branch's place in an identity class gives no automatic exclusive entitlement to every contribution associated with the predecessor. The logical restriction clarifies what one identifier can represent. Further institutional or moral conclusions require a bridge whose premises concern the relevant authority, undertaking, or claim.

6.5 Delayed Division and the Time of Application

A division can be announced before distinct organizational units exist. It can also occur through a sequence of changes whose timing remains uncertain. Returning to the translation association shows how the branching argument accommodates delay while retaining its constraint at the relevant simultaneous slice.

Write a for the initial whole-association appearance and d for its later appearance while the two desks remain within the stipulated common organization. At a subsequent time the normalized domain contains distinct whole associations b and c. The case has thus placed the organizational division after the opening of the desks. An accepted aEd is compatible with their earlier spatial separation. The appearance d already represents the whole unit at that time; its two desks have not been entered as two distinct whole-association appearances in the same domain.

Once b and c appear together as distinct units, the familiar restriction applies. Asserting both aEb and aEc still yields bEc. The intervening appearance d leaves that inference intact. Alternatively, asserting both dEb and dEc reaches the same contradiction directly. Several earlier stages of continuous coordination therefore cannot accumulate into permission for two simultaneous exact identities. The argument concerns the relation among the relevant appearances, irrespective of how long the predecessor persisted before the division.

This analysis separates a logical question from a difficult historical one. The formal obstruction needs distinct simultaneous units under the chosen normalization. Determining when that description becomes warranted can require records of decisions, undertakings, and effective coordination. An announcement might state a future intention; an organizational procedure might continue after participants begin using

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different names. Conversely, a retained common name might outlast the relations on which the selected whole-unit description depended. Each possibility changes the evidence for the appearance domain or the occurrence time, rather than changing the equivalence proof.

Where dates remain unresolved, the researcher can compare candidate histories with different times of division. The claim that a particular dated pair violates slice separation is then conditional on the time assignment and unit interpretation. A coarse reporting interval containing successive units also requires the temporal analysis of Section 5; sharing a database interval need not establish simultaneous coexistence. These qualifications preserve the correct order of dependence. The investigator first specifies what the candidate slice represents, then tests the identity claims made on it. The result remains exact while the historical premises retain their actual degree of uncertainty.

6.6 Merger and Retrospective Collapse

Merger has the same logical structure with time reversed in the identity argument. Let $a, b \in I_t$ be distinct and let $c \in I_u$ for $u > t$. The lineage can contain aLc and bLc. If both aEc and bEc are asserted, symmetry gives cEb, and transitivity gives aEb. Slice separation contradicts $a \neq b$. The proposed common successor therefore cannot be identical to both distinct predecessors in this model.

This consequence is easy to overlook when an institution uses a single later name for the merged body. The later name can designate a new entity that inherits from two predecessors. It can also designate a continuing entity that absorbs another under a particular rule. The existence of one later name leaves those alternatives open. A historical description should state which relation the name is being used to express.

Retrospective collapse has practical costs. If a historical database replaces both predecessor identifiers with one identity key, records that originally referred to distinct organizations may become indistinguishable. A contract made with one predecessor, a contribution by the other, and a dispute between them can then appear to involve a single party. A successor relationship can connect those records while preserving their original subjects. The logical distinction thus has a concrete archival purpose.

6.7 Reunion and the Unity of a Historical Narrative

A later reunion can give the branches a common successor and a shared account of the earlier division. This subsection examines the inference from that later unity to identity during the divided interval. Its target is the preservation of an earlier simultaneous distinction, while the fuller inheritance analysis belongs to the merger section.

Continue the constructed sequence by adding a later joint organization with appearance r. Both b and c transmit participants or practices to this body. For this constructed case, suppose transmission is supported from a to d, from d to each branch, and from each branch to r. A proposed identity class can contain a, d, b, r, leaving c distinct. Another can contain a, d, c, r, leaving b distinct. Each proposal respects slice separation and can be represented by eligible adjacent selections of the kind developed in Section 15. These alternatives establish consistency witnesses. The case has supplied insufficient grounds to prefer either one, and all five transmission edges can remain in the lineage record under either proposal.

Adding the later appearance changes the scope of the historical inquiry without erasing the earlier coexistence of b and c. If both are identified with r, symmetry and transitivity again identify them with each other. If both are identified with a, their later reunion likewise leaves the earlier contradiction untouched. A later agreement about representation can be relevant evidence or can have a specified institutional effect. Its interpretation must still state whether it revises the account of the earlier units, changes the relation being recorded, or supplies a different criterion of continuity.

The phrase *one history* permits another coherent reading. A historical narrative can cover the whole branching and reunion sequence, preserving the contributions of both branches within a shared genealogy. A historical-family equivalence can consequently place all five appearances in one class. Its meaning

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differs from the slice-separated identity under examination, because that class contains the distinct simultaneous units *b* and *c*. The shared history can be informative precisely because it records their interaction and difference.

The distinction matters when a reunion narrative becomes a database operation. Linking both branch archives to the later joint body can preserve the full genealogy. Replacing their original organizational subjects with one unrestricted identity key would additionally change the account of who acted during the divided period. A disagreement between the branches, or an undertaking entered by one alone, needs their distinction to remain expressible. Historical unity can thus justify a common presentation while leaving the exact linkage question open. The desired accessibility of the history supplies a reason to connect records; it leaves the semantic form of that connection to be justified.

6.8 Closure Operations and Their Interpretations

One response to a lineage graph is to forget arrow direction and take connected components. This produces an equivalence relation. It also joins both successors of a split and both predecessors of a merger. The construction is mathematically legitimate as a relation of shared lineage component. It fails the intended slice-separated identity model whenever a component contains distinct simultaneous units. The issue concerns the meaning assigned to the equivalence, not a defect in the graph operation.

Another response is mutual reachability. In a strictly time-directed finite history, an appearance cannot be reached from a later appearance through forward edges. Mutual reachability therefore reduces to equality after reflexive closure. It preserves distinction but fails to connect ordinary forward continuities. This demonstrates that choosing an equivalence closure purely for its algebraic convenience can make the representation either too coarse or too fine for the question.

A more explicit response selects one continuing path under a declared criterion. Such a policy can be suitable for maintaining one registration or one longitudinal analytical subject. Its output is conditional on the criterion, the candidate set, and the evidence. The unselected branch still has whatever transmission relations the evidence supports. Selection for one identity purpose therefore gives no warrant for erasing the other branch's inheritance, contribution, or continuing practice.

6.9 Relation Types and the Composition of Continuities

An apparent conflict can originate in the word *continuity* covering several relations. This subsection examines the bridge required to compose such claims into exact identity. The distinction is relevant both to the interpretation of testimony and to the integration of results produced under different criteria.

Suppose one translation branch carries forward the predecessor's training practice, while the other carries forward an institutional registration. Those claims can both be supported by their respective evidence. An instructor's participation may bear on the first; a competent registration decision may bear on the second. Their common reference to the predecessor gives the inquiry a shared historical subject. It supplies insufficient grounds for treating the two continuities as instances of one identity relation.

The formal reason is specific. Transitivity composes two premises belonging to the same relation. If each criterion is modeled by a separate equivalence, one link in the practice criterion and one link in the registration criterion still fail to supply the two premises of either equivalence's transitivity rule. Integrating them requires an additional claim connecting the criteria. Where that claim asserts that both are sufficient for the same exact identity, the branching obstruction returns. Where the criteria answer different questions, their separate answers can remain simultaneously valid.

This distinction prevents a misleading reconciliation. Saying that each branch is the same predecessor *in its own respect* can either preserve the different questions or conceal their difference. The first reading gives a structured account of what each branch continues. The second reading invites an unrestricted identity inference while withdrawing its transitive consequences when a conflict appears. A research report should make the intended reading available at the point where the claims are combined. The ordinary convenience of one phrase supplies insufficient justification for one exact relation.

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Even a single transmission label can require composition checks. Members can move from one organization to another, and those later members can participate in a further body. The resulting lineage path establishes the represented sequence of transfers. Whether the final body retains the relevant initial participants, training, or undertaking requires evidence appropriate to that property. The same caution applies when an archive combines different edge types into one visible route. A route can show a connected historical process while leaving the preservation of any specified organizational feature unresolved.

The advantage of typed relations is consequently substantive. They allow the inquiry to retain both branches' supported histories without forcing a choice merely to obtain an exact identifier. They also show where an identity argument needs an extra premise: from transmitted practice to continuing organizational unit, from registration to operational continuity, or from a combination of features to one common target. Each bridge can be defended, challenged, or left open. Naming the bridge gives the subsequent criterion discussion a precise task, beyond ranking branches that have already been described through incompatible senses of continuation.

6.10 Premise Audits and Proposed Counterexamples

A useful response to a proposed counterexample identifies the premise it changes and the evidence or argument supporting that change. This subsection collects the resulting diagnostic questions. Its purpose is to distinguish a substantive alternative model from an inconsistent use of the original one.

The first possibility concerns the domain. Two descriptions may refer to one simultaneous appearance, or two local units may be parts of a larger whole relevant to another inquiry. Both responses can be defensible. The first revises normalization; the second changes the organizational target or introduces a part relation. In either case the researcher should state which earlier references retain their meaning. Otherwise a successful answer about the whole can appear to resolve an unanswered question about its distinct parts. The normalization analysis supplies the fuller conditions for examining such changes.

A second possibility concerns time. Two units recorded in the same annual collection may occur successively, or the division date may remain disputed. Revising their simultaneous interpretation changes the slice premise used in the obstruction. Such a revision needs occurrence evidence, rather than an appeal to the inconvenience of the identity conflict. Conversely, evidence that the branches coexisted at the relevant level makes a mere change in reporting format insufficient to resolve the conflict.

A third possibility changes the relation's formal properties. A reflexive, symmetric relation can connect the predecessor to each branch while omitting the relation between the branches, provided transitivity is relinquished. A directed lineage can similarly retain both outgoing links while using different rules of composition. These are coherent alternatives for appropriate questions. Their interpretation must explain which record inferences remain licensed. In particular, a procedure that relies on transitive identity linkage cannot adopt the alternative relation and continue to treat every connected record as concerning one exact subject without an additional argument.

A fourth possibility revises a positive identity claim. Evidence initially treated as identity evidence may warrant only transmission, or a claimed registration may lack the authority required by the selected criterion. This response preserves the domain and the equivalence model while changing the premises supplied to it. It can be a stronger historical explanation than redefining the unit merely to retain every proposed identification. Its adequacy nevertheless depends on the evidence for the revised link, so the formal contradiction alone cannot dictate which claim should be withdrawn.

The distinction between these responses gives the proof a constructive role. It narrows the site of disagreement while keeping several repairs available. A scholar defending a distributed whole, a participant describing inherited practice, and an archivist requiring one registration key may each be making an intelligible claim. Their disagreement becomes assessable when the target, time interpretation, relation type, and warrant are stated. An unresolved case can then preserve the alternative models and their different consequences. Selecting a single database output may remain practically useful, but the grounds of that selection and the surviving historical uncertainty require separate records.

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6.11 The Scope of the Constraint

The proof should not be inflated into a universal metaphysical impossibility of branching beings. Its premises are an equivalence representation and the separation of normalized simultaneous units. A theory can question either premise or choose a different kind of continuant. It then owes a clear account of the replacement relation and the inferences it licenses. The present method makes the cost of that choice visible.

For the organizational inquiry at hand, the premises are often useful because participants distinguish coexisting branches and because identity claims support transitive record linkage. Their usefulness is still case-relative. The investigator must be prepared to revisit the unit boundary if evidence shows that the supposed branches remain one coordinated organization with internal divisions. That would change the domain on which the proof is applied. It would not refute the conditional proof itself.

The next task is consequently substantive. The branch obstruction tells the investigator where a choice or a suspension of judgment is required. It does not supply the reasons for that choice. Those reasons must be developed from the organizational inquiry and the evidence of continuity.

7 Grounds for Continuity Criteria

This section develops the substantive criteria that may support a collective continuity judgment. It treats membership, operations, commitments, and institutional recognition separately before considering their combination. For each criterion, the inquiry must identify the relevant object, the independent evidence, the reason that the continuity matters, and the counterconditions that would defeat the proposed use.

7.1 Membership and Continuing Association

Membership continuity can be relevant when the focal entity is an association of participants engaged in a shared organizational life. A branch retaining most active participants may preserve patterns of interaction, accumulated experience, and the practical relationships through which the association operates. These are reasons to investigate membership, not a proof that majority retention establishes identity. The majority can leave the activity behind, exclude a minority that maintains it, or reorganize around another purpose.

The membership population must therefore be defined before retention is measured. A formal roster may include inactive members; participation records may omit members who contribute remotely or intermittently. Eligibility can change during the disputed episode. A branch can claim supporters who never accepted membership, and a person can belong to both branches. The numerator and denominator of a retention fraction are consequently substantive choices. An apparently precise percentage can conceal incompatible populations.

Let M_a be the eligible predecessor membership set and M_b the eligible successor set under a fixed rule. A directional retention measure is

$$m(a,b) = \frac{|M_a \cap M_b|}{|M_a|}, \qquad |M_a| > 0.$$

It answers how much of the predecessor population appears in the successor. It differs from a measure of how much of the successor comes from the predecessor. The distinction matters in a merger: a small association can transfer every member to a much larger body, giving complete predecessor retention while constituting a small share of the successor. Neither direction settles identity by itself.

Independent evidence should distinguish the fact of affiliation from the reason it matters. Membership applications, attendance histories, accepted roles, and participants' statements can support affiliation claims. Evidence of continuing coordination can support the significance of those affiliations for the chosen inquiry. When these sources disagree, the method should retain the disagreement and test whether the verdict depends on one contested classification. That is more informative than allowing a roster definition to silently determine the result.

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7.2 Operations and Transmitted Practical Organization

Operational continuity concerns the collective activities through which an organization does something. A successor may continue a programme, a method of coordination, a schedule of work, or a capacity to respond to particular conditions. The inquiry must specify which operation is relevant. A general description such as "community service" can cover activities with very different participants, resources, and purposes. A narrow description such as one weekly event can overlook the organizational system that sustains it.

Similarity of outputs leaves transmission open. Two collectives can perform the same task independently. One can imitate another after observing public materials. A branch can inherit trained participants and routines but change the stated purpose of the activity. These histories can produce comparable output measures while supporting different continuity claims. Operational evidence therefore needs a path: training, handover, shared routines, transferred equipment, continuing coordination, or another documented mechanism connecting the earlier and later practice.

The inherited operation may also transform. A method can survive through adaptation to a new venue or population, whereas literal repetition can fail to preserve its practical function under changed conditions. The criterion should state which changes it permits and why. Describing every permitted change as identity preserving would revive the original circularity. The justification must refer to independently characterized features of the activity and to the explanatory question, then examine whether their preservation warrants the proposed collective identification.

Operational continuity is particularly useful when an inquiry tracks a collective capacity. It can explain why a small branch remains able to complete a project after losing most members. It can also reveal that a registered successor has inherited little practical organization. These findings are valuable even when numerical identity remains disputed. The method should preserve their explanatory content without demanding that every operational result become an identity verdict.

7.3 Commitments and Continuing Undertakings

A commitment criterion focuses on a particular undertaking: a promise to complete a project, maintain an archive, provide an activity, or meet an agreed condition. Such undertakings can give participants a reason to regard a branch as continuing something they entrusted to the predecessor. The relevant question can be more specific than the identity of the organization in every respect. It concerns who continues the undertaking and on what grounds.

The commitment itself needs a record. Its content, addressee, authorized maker, conditions, and temporal scope should be identified. A general mission statement differs from a specific undertaking to a partner. A later branch may endorse a mission while declining a particular obligation. A successor may voluntarily accept an undertaking without receiving the predecessor's registration. The evidence should preserve these differences before a continuity criterion is applied.

Uptake and implementation also differ. A branch can announce that it will continue a project yet lack the resources to do so. Another can perform the activity while disputing that it has assumed the earlier obligation. A commitment inquiry may need both the authorized acceptance and the practical capacity for discharge. Which condition controls a particular consequence depends on the nature of that consequence. The formal model can record both; it cannot derive their normative priority from a scalar score.

This criterion therefore provides a demanding test of the relation between identity and responsibility. If responsibility is assigned solely through an identity label, a reorganization can appear to remove an undertaking when the label terminates. If responsibility follows any causal lineage, it can spread indiscriminately to distant successors. A defensible account needs the particular undertaking and the grounds of its transfer or persistence. The collective identity finding may inform that account while leaving the normative argument explicit.

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7.4 Recognition and Constituted Standing

Institutional recognition can supply a relatively definite continuity rule. A registry, federation, governing body, or internal procedure can identify a successor under its own standards. The inquiry should specify the authority, the domain of its competence, the relevant act, and the current status of that act. The resulting standing can be an important social fact with practical effects.

Its meaning remains domain-specific. Recognition by a sporting federation can govern participation in its competitions. Recognition by participants can concern their continuing association. Recognition by an archive can concern the organization under whose name records are described. These judgments can interact without being interchangeable. The authority relevant to one domain requires further argument before it governs another.

Recognition can also be disputed or divided. A numerical majority of statements should not automatically be treated as independent confirmation. Statements may repeat one announcement, reflect unequal visibility, or come from constituencies with different relationships to the organization. A researcher needs to record who recognizes what, when, through which source, and with which authority or interest. This allows a recognition finding to be examined both as evidence and as an event that can shape later continuity.

7.5 Combining Criteria and Exposing Tradeoffs

A combined rule can be useful when a decision requires one successor. Its construction should follow the separate analysis of the criteria. A weighted sum assumes that the component measurements have meaningful scales and that compensation between them is permissible. If a missing authorization is a disqualifying condition, a sufficiently high membership score should not automatically compensate for it. Eligibility constraints and comparative scores need distinct roles.

A proposed policy can first exclude candidates that fail justified requirements, then compare the remaining profiles. It should preserve ties, incomparability, and sensitivity to disputed values. An investigator can also decline scalar aggregation and report a profile of continuities. This is a substantive output when the question concerns historical inheritance rather than a compulsory single registration.

The criterion's justification finally depends on a public argument about its purpose. A rule suitable for longitudinal analysis of a membership association may be unsuitable for stewardship of a specific project. The researcher should state that difference directly and examine the consequences of each choice. A transparent criterion is more than a declared preference: it identifies what evidence matters, what would defeat the finding, and why the resulting identification serves the inquiry without erasing relevant continuities.

7.6 A Procedure for Justifying a Criterion

The preceding criteria identify relevant considerations. A further procedure is needed to distinguish an independently justified criterion from a list of features selected after the desired successor is known. The procedure proposed here has five connected tasks: formulate the question, identify the explanatory or practical role of continuity, characterize the evidence in terms independent of the verdict, specify counterconditions, and compare the rule across discriminating cases. Each task constrains the next.

First, formulate the question in terms that allow an answer other than a preferred branch's success. "Which branch preserves our organization?" can express a participant's concern, but its research use requires an account of what preservation concerns. A question about continuing participation differs from a question about completing an undertaking. The inquiry should identify the object that would be lost by using an inappropriate criterion. This makes the criterion answerable to a concrete problem.

Second, state why continuity matters to that problem. A longitudinal study may need to track one organized decision procedure so that changes in its behavior can be interpreted. An archive may need to distinguish the provenance of records from the current custodian. A community may need to arrange continued access to an activity after a split. These purposes create different requirements. A criterion

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that selects the same branch in every case can still fail if it fails to preserve the relation relevant to the inquiry.

Third, characterize the evidence without defining it through the identity verdict. A transfer of a roster can be described as an event, with named participants and a documented date. Continuing training can be reconstructed through instruction and performance records. An undertaking can be specified through its content and authorized uptake. These descriptions may use institutional concepts, but they need not assume that the whole predecessor and successor are one entity. The criterion must explain the further inference from those facts to reidentification.

Fourth, specify counterconditions. A membership rule should say what happens when the majority abandons the relevant activity. An operational rule should distinguish inherited practice from independent imitation. A commitment rule should address an announced uptake that lacks authority or implementation. A registration rule should specify its domain and effective procedure. Counterconditions give the rule an inferential boundary. Without them, almost any continuity can be redescribed as satisfying a vague criterion.

Fifth, compare the criterion across cases where the relevant dimensions separate. Agreement in an easy case provides little evidence about the rule's treatment of division. The constructed association is useful because membership, operation, and commitment point in different directions. A merger that reverses their relative sizes supplies another test. A criterion that changes its interpretation whenever its favored branch loses needs a principled explanation or revision.

7.7 Constitutive Criteria and Empirical Indicators

Some criteria define a status within a practice. Others use observable features as indicators of a continuity whose meaning is independently specified. These uses require different standards of assessment. A rule that constitutes registry continuity can be evaluated for clarity, coherence, authority, and legitimacy. An empirical indicator can be evaluated for reliability, sensitivity to relevant change, and failure under counterexamples. Treating a constitutive rule as an empirical discovery can make its authority seem natural; treating an indicator as constitutive can protect it from evidence of failure.

Suppose a register defines its continuing member through a valid succession decision. The correct application of that rule determines registry standing. The number of retained participants may be evidence relevant to the decision, but its relation to standing is mediated by the rule. By contrast, a study may use retained participants as an indicator of continuing coordination. Its adequacy depends on whether the participants actually sustain the relevant relations, which can vary even when the roster is unchanged.

A single organizational practice can combine the two. A constitution may define eligible membership, while actual participation varies among eligible members. A continuity criterion can therefore use both a constituted population and empirical evidence of its activity. The model should record these different dependencies rather than seek a single category called "objective data." The data acquire their meaning through the rule and observation process that generated them.

This distinction also clarifies validation. Reproducing the registry's decision validates a reconstruction of the registry rule only if that is the target. It does not independently validate the rule as a criterion of practical continuity. Predicting future activity can validate an indicator for that target under appropriate conditions, while leaving numerical identity open. A criterion should be assessed against the kind of claim it makes.

7.8 Multiple Purposes and a Shared Historical Record

An organization can be relevant to several inquiries at once. A historian may study a movement of participants, a project partner may track an undertaking, and a governing body may administer a registration. The method permits their identity criteria to differ while requiring them to share the historical evidence where their factual premises overlap. A disagreement about the purpose should not license contradictory claims about who attended a meeting or which document was signed.

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The shared record can therefore support a common factual layer and several explicitly indexed interpretive outputs. This does not require every source claim to be settled. It requires each output to retain the historical alternatives on which it depends. A criterion cannot treat a disputed handover as established solely because that interpretation favors its consequence policy.

Cross-purpose criticism remains possible. A registry procedure may ignore an operational continuity that matters to the people whose access it controls. A historical account may use a membership criterion that erases an excluded minority's continuing work. These criticisms concern the justification and consequences of the inquiry, not merely whether a rule was applied consistently. The framework should provide a place for them in its objection record.

An integrated conclusion can then say, for example, that branch B continues the registered association under a stated procedure, branch C continues the specified programme through documented transmission, and both inherit parts of the predecessor's social history. Such a conclusion can guide a later allocation decision without pretending that the allocation has already been derived. The remaining normative task is to explain which continuity should govern which consequence.

7.9 The Risk of Retrospective Criterion Fitting

A final methodological risk is retrospective fitting. After observing which branch prospered, an investigator may define continuity through the features that explain its success and then claim that success confirms its identity. The reasoning can become self-supporting if the criterion, the evidence selection, and the validation target all come from the same outcome.

A stronger study records the criterion before examining the decisive outcome or distinguishes exploratory development from later evaluation. Exploration is valuable: a difficult case can reveal that a criterion misses an important feature. The revision should then be described as a response to that case and tested elsewhere where feasible. It should not be presented as an independently confirmed rule on the same evidence that generated it.

The criterion also needs stable counterfactual interpretation. If a branch had retained the undertaking but lost a popular name, would the rule still recognize the relevant continuity? If the majority had moved to the other branch, would the membership rule follow them? These questions examine consistency of application. Their answers remain arguments until actual cases provide evidence, but they can expose an unacknowledged preference embedded in the proposed rule.

8 Institutional Rules and the Warrant for Continuity Criteria

Institutional criteria require a closer account of the rules through which an organized collective can change. This section examines the distinction between a specification of identity over complete histories and the reconstruction of a disputed transition from incomplete records. It then analyzes rule contents, enacted rules, amendment, effective operation, and external authority. The final discussion develops a procedure for defending an institutional criterion without defining the decisive event as identity-preserving in advance. The inquiry remains directed toward organized collectives whose activities and standing depend partly on identifiable institutional procedures.

8.1 Complete-History Conditions and Historical Reconstruction

A complete-history account can specify when two descriptions represent one group by comparing their members and organizing rules throughout time. Tałasiewicz's model supplies a relevant formulation: its identity condition compares membership and statutes over time, while its definitions distinguish permitted changes from the enactment of a new statute [11, §1.1]. This provides a focused point of confrontation for the present evidential inquiry. The question is how an investigator can establish the historical inputs and interpret a disputed transition before the complete group histories have been recovered.

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There are two logically different uses of a history. In a theoretical specification, the history can be an input whose membership and rule assignments are already fixed. An identity condition can then compare those inputs consistently. In a historical investigation, the assignment of a person or rule to the same continuing group can be part of what is disputed. A succession of rosters labeled with one organization name cannot automatically supply the input history, because the continuity of the label may be one of the contested observations.

The distinction does not undermine the value of a complete-history condition. It identifies the work required to apply it. The researcher needs an appearance-level description that can be assembled without presupposing the final identity partition. Sources can establish who participated in particular meetings, which procedures were applied, what appointments were issued, and which successor claims were made. A criterion can then evaluate possible ways of connecting those appearances through time. The historical reconstruction becomes an argument with visible inputs rather than a restatement of the desired group history.

This difference is especially important after a division. A single pre-division roster can be followed by two successor rosters, each accompanied by a claim to carry the earlier statute. If the investigator first assigns the earlier organization to one successor and then compares its complete history with the competing successor, the result may reflect the initial assignment. An appearance-level reconstruction preserves both candidates while asking which documentary and procedural facts support each link. The global partition constraints then test whether the proposed links can coexist under the stated identity policy.

The contribution of a criterion therefore has several layers. It states which historical features are relevant, explains why their continuity bears on the chosen target, and supplies a rule for the cases in which those features diverge. The evidence establishes which relevant features were present. A completed model comparison can then show which verdicts follow. Keeping these layers distinct allows a strong theoretical condition to guide inquiry without treating its unavailable inputs as already observed.

8.2 Rule Contents, Enactments, and Organizational Appointments

Two organizations can use documents with the same wording. The interpretation of this similarity depends on whether the inquiry concerns rule content, a particular enacted rule, or the organizational appointment through which that rule operates. These distinctions can be relevant even when the membership rosters also coincide. The same persons can participate under two separate appointments, with different reporting relations or mandates supplied by the surrounding institution.

A constructed example illustrates the issue. A governing body appoints one committee to review a particular proposal and later appoints another committee under the same standard procedural text. The same five persons serve on both. The second appointment can concern a new task under a fresh authorization even though the copied procedure and participant set remain unchanged. A model whose rule coordinate records only the document's wording may therefore omit the feature that distinguishes the appointments under the selected institutional criterion.

The example is a test of representation, not an attribution that a particular structural theory must ignore appointments. A richer rule description can include the enacted token, its issuing authority, its scope, and the task to which it applies. Under that representation, two identical textual documents can express distinct institutional rules. The important methodological step is to state what equality of rules means in the model. Equality of strings, equality of procedural content, and identity of an enactment have different implications.

The same distinction operates through time. A revised document can record an authorized amendment to an existing appointment. An unchanged document can be reenacted under a new appointment after an earlier body has ceased to operate. A comparison based only on textual change can reverse the significance of these events. It may mark an amended continuing procedure as discontinuous and an identical newly enacted procedure as continuous. The relevant transition is supplied by the enactment relation and its institutional context.

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An evidence record should consequently identify the rule's content and the event through which it acquired the asserted role. It should preserve the distinction between a draft, a proposal, an adopted procedure, and an operative instruction. Copies and later consolidations can be useful evidence, while their relation to the enactment needs to be established. This structure helps a reviewer determine whether the continuity finding depends on preserved content, an authorized transformation, or a newly created appointment that reuses familiar language.

8.3 Amendment and Replacement through Independently Described Acts

An institutional continuity criterion often distinguishes amendment from replacement. That distinction becomes circular if an amendment is defined solely as a change that preserves the organization and the organization is then said to persist because its changes were amendments. A more informative account describes the relevant acts through their participants, procedures, authority, and scope before drawing the continuity conclusion.

The first component is the rule under which change is proposed. It may specify who can initiate the process, which participants are eligible, what notice or deliberation is required, and how approval is registered. The second component is the event that allegedly satisfies the rule. The third is the content and intended scope of the resulting change. These descriptions can be examined without inserting a verdict about numerical identity into the event's label.

A criterion can then argue that a specified class of authorized changes continues an institutional appointment. The argument depends on the practice's own rules and on the inquiry's target. For an inquiry into authority to act under that appointment, a correctly enacted amendment can provide direct evidence of continued authorization. For an inquiry into the continuity of a social practice, the same amendment may be relevant alongside evidence that participants actually implemented the changed procedure. For an inquiry into personal affiliation, the amendment's significance may depend partly on recognition by those whose relationship is being studied.

The procedure also needs a boundary for changes that exceed the accepted amendment power. A rule may authorize changing membership procedures while reserving the collective's task to an external appointing body. A group could then correctly amend the former and purport to change the latter without the required authority. The criterion should identify which transition conditions apply to each kind of change. The mere presence of a signed document or a majority vote leaves that scope question open.

These are general analytical requirements, and the detailed rules must be established in each actual setting. The benefit of the decomposition is that a disputed transition becomes susceptible to specific evidence. A missing notice, an ineligible participant, an unauthorized subject matter, and a disagreement over the identity significance of a valid act are different possible explanations. The inquiry can separate them and preserve the strongest finding supported at each stage.

8.4 Effective Operation and Institutional Validity

A collective can continue to operate through a procedure whose institutional validity is disputed. Conversely, an authorized change can have limited immediate effect on actual practice. This distinction creates two histories worth reconstructing: the history of the rules claimed or applied in activity, and the history of the acts that confer standing under a selected institutional framework. Their relation is an empirical and interpretive question.

Consider a constructed association whose members continue using an earlier admission rule after an authorized amendment. The current institutional procedure and the enacted practice temporarily diverge. A study of participation may find continuity through the old practice; a study of authorized admission may find a changed rule and a period of noncompliant conduct. Either account loses precision if it assumes that the institution's formal record directly describes everything the participants actually did.

The reverse situation is equally informative. Participants may begin using a new procedure after a meeting that failed the earlier approval requirements. The new procedure can shape actual coordination

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and produce stable outcomes. Its practical effectiveness supplies evidence for one history. Its authorization under the prior framework remains a different question. Subsequent recognition may alter its standing, but the relevant recognition event and its temporal scope must be represented explicitly.

The distinction helps avoid an equivocation in the phrase "the rule of the group." The phrase can refer to a formally applicable procedure, an informally accepted practice, a publicly claimed rule, or the procedure actually used. An organized collective can exhibit several of these at once. A model may represent their agreement in simple cases, while difficult transitions require separate coordinates or typed records. The richer description is warranted when the difference changes a continuity judgment.

This analysis also explains why causal effectiveness cannot by itself certify institutional validity. An unauthorized procedure may be highly effective, and an authorized procedure may be poorly implemented. The difference matters for criticism as well as description. A researcher can investigate the effects of a procedure and evaluate the grounds of its standing without forcing the two findings to coincide. The continuity criterion should state which of those targets it serves and how the other history enters the explanation.

8.5 External Appointment and Distributed Authority

The authority relevant to a collective's continuity may lie partly outside the collective. A committee can depend on an appointing body, a project can depend on a partnership agreement, and a local branch can depend on rules of a wider association. An internal history of members and activity can therefore be insufficient for an inquiry whose target is continuing institutional standing.

A constructed pair of histories makes the informational issue clear. In both, the same people continue meeting under the same procedure and produce the same kinds of outputs. In one history, their external appointment is renewed for the next period. In the other, it expires and the participants continue meeting informally. Observations limited to the meetings can be identical while the answer to the appointment question differs. The missing external event supplies the difference under that criterion.

The example does not imply that external recognition determines every form of collective continuity. The informally continuing participants may preserve their social practice, mutual relationships, and practical capacity. The example shows that an internally complete activity description can remain incomplete for an externally constituted status. The study must include the relations that its target makes relevant, even when those relations lie outside the initially drawn organizational boundary.

Authority can also be distributed among several actors whose powers concern different subjects. One body may appoint members, another approve expenditure, and a third recognize professional standing. A merger can transfer some powers while leaving others tied to a predecessor framework. A single field recording whether the organization is "recognized" can conceal this distribution. The criterion needs to identify the actor, the competence, the decision, and the consequence to which that recognition applies.

This analysis suggests a disciplined stopping point for authority regress. An inquiry can state which initial appointments and competence rules it provisionally accepts, then reconstruct the continuity of authorization from that starting basis. A separate normative inquiry can challenge the legitimacy of that basis. The historical reconstruction remains conditional on its anchor, and the normative challenge remains visible. Treating every local continuity question as requiring a completed theory of legitimate authority would obscure the particular evidence the study can establish.

8.6 A Procedure for Defending the Institutional Criterion

The preceding distinctions support a positive procedure for selecting and defending an institutional continuity criterion. The procedure begins with the claim that the inquiry needs to answer. For example, a study might ask which successor is authorized to complete an undertaking entered through a specified appointment. The target already identifies a relevant institutional relation, while the actual successor and the applicable transition facts remain to be established.

The next step identifies the acts and conditions that the relevant practice treats as changing that relation. These conditions should be described through independently observable or documentable features.

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The researcher records the issuing authority, the scope of the appointment, the procedure for change, the transition event, and any subsequent recognition with a stated effect. This supplies the proposed evidential bridge from historical records to a continuity judgment.

The criterion then faces discriminating cases. An authorized change of procedure with sustained appointment should be compared with an identical procedure under a new appointment. Continued activity after expiry should be compared with renewed authority during inactivity. A divided successor set should be compared with a single successor whose membership has changed substantially. These comparisons reveal whether the criterion tracks the institutional relation named in the target or merely follows a familiar label, a majority roster, or visible activity.

A further step examines the criterion's consequence sensitivity. If its result will govern access, representation, or attribution, the study should state why the chosen institutional relation is relevant to that consequence. The fact that a procedure is authoritative for one purpose does not automatically give it authority over another. This test permits criticism from affected participants without requiring the historical evidence to change merely because the outcome is disputed.

Finally, the study records failure conditions. A criterion may fail because the relevant authority is unidentified, the transition records support several interpretations, the procedure permits several successors, or the target concerns a practice whose continuity extends beyond the institutional appointment. Each failure identifies a different next step. Some call for more evidence; others call for a narrower conclusion or a different criterion. The result is a defended and revisable method whose application can be audited without accepting an identity label at the outset.

9 Evidence Symmetry and the Grounds of Unique Selection

This section examines a specific demand placed on a continuity criterion: selecting one historical successor when the retained evidence treats several successors alike. It first constructs a divided association with balanced continuity profiles. It then states a conditional symmetry requirement, derives its implication for a unique selection, and examines the additional evidence or practical authority that could justify a more determinate result. The analysis concerns what a declared body of reasons supports. It leaves open whether a fuller history would reveal a relevant difference.

9.1 Balanced Evidence in a Divided Association

Consider a constructed association with twelve active participants, six exclusive equipment sets, and an established programme conducted through two teams. The teams have equal membership and equipment. Each contains three participants trained in the association's central method, and each has performed the same number of programme sessions during the period under study. The association divides into two simultaneous bodies, here called North and South. Each body receives the corresponding team, three equipment sets, a copy of the programme manual, and records sufficient to continue its work. The two bodies subsequently perform the same number and kind of sessions under comparable conditions.

The equality is stipulated for the example. A real inquiry would need to establish the populations, observation interval, relevant kinds of skill, condition of equipment, and meaning of comparable activity. Equal totals could conceal differences in practical organization. One team might contain the only participant able to repair equipment, or maintain a relationship with a venue that the other branch lacks. The construction excludes those differences from its accepted history in order to examine a precise inferential limit. It should not encourage an investigator to declare two actual branches equivalent after comparing a short spreadsheet.

Both branches accept the same continuing programme undertaking. The undertaking permits performance by several providers, so this shared acceptance is coherent. It supplies no exclusive office whose holder is already determined by the undertaking itself. Both branches also claim to continue the earlier association. The retained documents contain no independently warranted decision designating either branch as the sole successor, and both claims have the same evidential standing. Distinct simultaneous

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organization is accepted: North and South make their own decisions, have separate participants in this construction, and can alter their programmes independently.

Suppose an inquiry additionally accepts that the predecessor continues in exactly one of these branches under its chosen identity criterion. This acceptance narrows the eligible domain to two histories. Its warrant needs to be supplied separately; the present example treats it as a premise whose consequences can be examined. In the first history, North is the same collective entity as the predecessor and South is a new entity. In the second, South continues the predecessor and North is new. The retained membership, equipment, practice, undertaking, and recognition evidence is balanced between those alternatives.

The difference between the two histories is therefore a difference the inquiry seeks to justify, rather than a distinguishing observation already contained in its evidence. An output that simply names North would resolve the requested format of the answer. It would still require an explanation of why the retained reasons favor North. Selecting the alphabetically first name, the first row in a database, or the branch initially visited would supply a reproducible procedure. Reproducibility would establish how that procedure operates; its historical significance would remain to be argued.

9.2 Relabeling and Substantive Distinctions

The relevant symmetry concerns an exchange that preserves the inquiry's reasons. This subsection distinguishes such an exchange from the removal of an inconvenient difference. A symmetry test has force only after the dossier specifies which features carry evidence and why.

In the constructed association, interchange the labels North and South, together with the corresponding team, equipment, and record labels. The typed relations remain the same: each branch receives six participants, three equipment sets, three trained participants, a manual copy, and the same undertaking. Each occupies the same temporal position relative to the predecessor. Each has an equally supported continuation claim. The exchange changes names while preserving the structure to which the proposed criterion appeals. An investigator who regards those names as arbitrary should obtain an answer that changes consistently with the exchange.

A genuinely evidenced asymmetry changes this analysis. Suppose North holds a documented appointment to a continuing office, while South lacks such an appointment. If the criterion gives that office a justified role, exchanging the branch labels must also transport the appointment relation. An exchange that leaves the relation attached to its former textual name fails to preserve the complete dossier. The original balanced case has been replaced by a different evidential situation. Symmetry cannot be maintained by hiding the appointment in an omitted column.

Conversely, the mere existence of different personal names need not supply a criterion-relevant asymmetry. Every participant can have a distinct name without the alphabetical order of those names carrying historical warrant. The model must distinguish referential labels from attributed properties. An identity-sensitive source statement about a particular participant may be relevant because that participant held an independently evidenced role. The relevant distinction then lies in the role and its history, even though the name helps identify its bearer. Treating all strings as arbitrary would erase evidence; treating every difference between strings as a reason for collective continuity would manufacture it.

The same discipline applies to source order and format. A record copied earlier into an archive is earlier in that archive's acquisition sequence. That fact can matter when the target concerns archival priority. Its bearing on an earlier organizational transition requires a connection between acquisition and the transition. Page position alone supplies a display asymmetry. A criterion using that asymmetry should describe the practical rule it is adopting and explain why such a rule is appropriate to the decision. The existence of a difference is weaker than a reason to make the difference decisive.

Identifying permitted relabelings is consequently part of philosophical and documentary judgment. The formal result below begins after that judgment has specified the invariant structure. Its value is to expose a compatibility condition between the declared reasons and the demanded answer. It cannot discover the relevant reasons by inspecting anonymous symbols alone.

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9.3 A Conditional Fixed-Candidate Result

The formal construction makes the requirement of consistent relabeling explicit. Its proof is elementary. The substantive application lies in deciding when this requirement expresses a fair account of the evidence on which a continuity judgment claims to rest.

Let G be a finite group of permitted relabelings acting on dossiers and on a set X of candidate outputs. A dossier d includes the target, appearance domain, times, typed evidence, and accepted criterion or criterion family. Let $C(d)$ be its nonempty set of eligible outputs. Require eligibility to respect the action: $C(gd) = gC(d)$. A selector s on the orbit Gd is *eligible* when $s(d') \in C(d')$ and *equivariant* when $s(gd') = gs(d')$. The latter condition says that relabeling the reasons and then selecting has the same effect as selecting and then relabeling the selected output.

Define the dossier stabilizer and its fixed outputs by

$$G_d = \{g \in G : gd = d\}, \qquad \mathrm{Fix}_X(G_d) = \{x \in X : gx = x \text{ for every } g \in G_d\}.$$

A stabilizing relabeling preserves the complete retained dossier. It can still exchange candidate outputs, because the issue under investigation is which candidate those reasons support.

Proposition 9.1 (Eligible selection under evidence-preserving relabeling)**.** *An eligible equivariant single-valued selector on Gd exists if and only if*

$$C(d) \cap \mathrm{Fix}_X(G_d) \neq \varnothing.$$

Proof. Suppose such a selector exists and put $c = s(d)$. Eligibility gives $c \in C(d)$. For every $g \in G_d$, equivariance gives $gc = gs(d) = s(gd) = s(d) = c$, so c is fixed by the stabilizer. Conversely, choose an eligible fixed c and define $s(gd) = gc$. If $gd = hd$, then $h^{-1}g \in G_d$, whence $h^{-1}gc = c$ and $gc = hc$. Thus the definition is independent of the representation of a dossier in the orbit. Eligibility follows from $gc \in gC(d) = C(gd)$. Finally, $s(k(gd)) = (kg)c = ks(gd)$ for every $k, g \in G$, proving equivariance. □

For the divided association, write a for the predecessor and b, c for the two branches. The eligible identity partitions are

$$E_b = \{\{a, b\}, \{c\}\}, \qquad E_c = \{\{a, c\}, \{b\}\}.$$

The branch exchange preserves the stipulated dossier and interchanges these partitions. Neither partition is fixed. Proposition 9.1 therefore excludes an eligible unique selection satisfying the stated relabeling requirement. The two-element family is preserved as a set and supplies a coherent representation of the remaining alternatives.

This conclusion concerns the combination of a target, eligible domain, and source of warrant. It does not derive a general thesis that historical identity is indeterminate whenever evidence is balanced. A fuller record could break the symmetry. A different criterion could make another feature relevant. A decision institution could intentionally introduce a rule that selects one branch. Each response changes a stated element of the inquiry and can be evaluated on its own grounds.

9.4 Candidate Domains and the Termination Alternative

The eligible domain has an indispensable role in the preceding result. This subsection examines a tempting response to the symmetrical split: choosing termination because it is the only branch-neutral identity partition.

Add the partition

$$E_0 = \{\{a\}, \{b\}, \{c\}\}$$

to the eligible domain. It represents a predecessor that ends and two new collective entities. The branch exchange fixes this partition. An eligible equivariant selector now exists; it can choose E_0 throughout

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the relabeling orbit. The formal obstruction has disappeared even though the membership, programme, and documentary evidence has remained unchanged.

This result should be read with care. Invariance is a requirement of the proposed selection discipline, and satisfying it removes one defect. It supplies no general reason to prefer every invariant candidate over every candidate that belongs to a symmetric pair. The predecessor may in fact continue through one branch while the retained evidence fails to disclose which. A report that treats this uncertainty as positive evidence of termination would add a historical conclusion unsupported by the symmetry argument.

The distinction is visible in the semantics of a family. The expanded family contains three possible histories. A set-valued report can retain all three without selecting one. Its invariance reflects the balance of the evidence. The partition E_0, by contrast, states a specific historical answer: every appearance belongs to a different entity. Its mathematical position as a fixed point is a property of that answer under the allowed relabelings. Historical acceptance requires reasons to exclude the continuation alternatives, or a declared policy that chooses among them for a practical purpose.

The reverse error arises when the eligible domain is restricted to exactly one continuing branch solely to force a dilemma. If the evidence permits termination, that candidate belongs in the historical family unless an independently defended criterion excludes it. If the evidence permits an additional collective unit that received the relevant authority or operation, that unit must also be considered. The two-branch construction is valuable because it states its restriction openly. It becomes misleading when the restriction is treated as an automatic implication of the word “split.”

Candidate-domain revision can therefore change the mathematical status of a selection problem without adding evidence about the original pair. The change may correct an omission, adopt a different organizational resolution, or replace the inquiry’s purpose. A useful finding record identifies which of these occurred. Otherwise, the disappearance of an impossibility result can be mistaken for the discovery of a uniquely supported history.

9.5 Independent Evidence that Breaks a Symmetry

A symmetry analysis can guide evidence acquisition by identifying which distinctions would alter the accepted family. The relevant task is to seek an independently warranted difference that bears on the chosen criterion. The mere accumulation of further descriptions of the balanced features would leave the initial uncertainty intact.

Suppose the criterion tracks a specified continuing administrative office. The initial dossier contains equal branch claims to that office. A newly recovered appointment record names a person who later participates in North. The document may establish that someone wrote an appointment statement at a particular time. To establish office continuity, the inquiry must also investigate the appointing body’s competence, the procedure’s applicability, the appointment’s effective conditions, and any later events that changed the office. These questions correspond to the authority analysis developed in Sections 8 and 24. Their answers can break the original symmetry without using the desired collective-identity verdict as a premise.

If the appointment was effective and the criterion independently treats its continuation as decisive, the new dossier can favor North. If both branches possess authenticated appointment statements issued under rival procedures, the appearance of an asymmetry in document count may conceal continuing uncertainty about competence. Counting one document for North and two for South also leaves open whether the two South documents are copies of a single source. Source dependence and procedural applicability can be more important than the volume of the archive.

An operational inquiry would seek different evidence. A previously omitted training sequence could show that North preserved a practical capacity which South reconstructed independently. That finding would distinguish transmission from similarity. Its significance for numerical identity would still depend on the criterion, while its significance for the history of the practice could be direct. A distinction can settle one target without settling every target associated with the predecessor.

The timing of the evidence matters. A document newly discovered today can describe an earlier event and thereby change present knowledge of the past. A decision newly made today can establish a

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prospective office or allocation and thereby change the institutional situation. Both events can reduce practical uncertainty, but they do so through different relations. Discovery changes the accepted history family by supplying evidence. A constitutive act changes what happens in that history from the act's effective time, subject to the institution's rules. Retroactive effects require their own typed account, as the revival discussion explains.

The symmetry result thus supports a selective research strategy. Before collecting another source, specify which retained exchange it might defeat and through which independently described property. A source that could favor either branch is often useful precisely because its result is open. A source chosen solely because it is expected to repeat the preferred branch's self-description supplies less discrimination. The criterion should remain stable long enough to test whether the proposed evidence actually bears on it.

9.6 Procedural Tie-Breaking and Historical Judgment

Practical institutions sometimes need a unique recipient even when the historical inquiry supports several possibilities. This subsection examines how a deliberate tie-break can be justified while preserving the content of the unresolved historical finding.

Suppose a foundation can fund one programme site during the coming year. North and South are equally eligible under its published programme criteria. The foundation might use a rotation, a public lottery, or a previously announced priority rule. Each procedure supplies reasons of a practical kind: distributing opportunities, avoiding favoritism, reducing decision costs, or coordinating expectations. The decision can be fully determinate even when the identity of the predecessor's continuing collective remains unresolved.

The foundation should state the object it is selecting. A recipient for one grant is an output different from the historical identity partition. The choice can influence future practice by giving one branch more resources. It can also become evidence of the foundation's recognition of that branch. Neither consequence automatically converts the grant rule into a discovery procedure for the earlier collective identity. A history can contain a fair allocation to North and a continuing predecessor in South under the adopted historical criterion, if that criterion permits such a combination.

A registration procedure may have stronger constitutive effects than a grant choice. If a competent institution can assign a particular registration to one claimant, its rule can establish who holds that registration from a specified point. The authority and scope of that assignment must be stated. An exclusive registration may settle the registration query while preserving shared causal lineage and divided undertakings. A broader identity claim needs a bridge between the registration and the collective object under investigation.

The practical choice should also preserve the reason for its determinacy. If the decision relies on a lottery, the record should identify the result as a lottery outcome. The probability assigned by the lottery concerns the selection mechanism. Equal chances for North and South express a procedural policy; they provide no automatic posterior probability that either branch is historically the predecessor. A probability over histories would require a separately justified model of evidence and underlying possibilities.

This separation permits decisive action alongside an accurate account of knowledge. The institution can say which branch receives the grant, which office has been assigned, or which timetable will govern the next event. The historical report can retain its alternatives. A demand that every practical choice settle numerical identity would burden action with a question it may have no authority or evidence to answer.

9.7 Symmetry, Determinacy, and the Scope of the Object

The philosophical significance of the result depends on the relation between the retained dossier and the object of inquiry. Three interpretations deserve separate consideration: incomplete evidence about a determinate history, incomplete specification of a criterion, and a practical object partly constituted by an authorized selection.

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Under the first interpretation, the collective has a determinate continuation under a fixed criterion, while the dossier omits a relevant event. The balanced evidence is then epistemically insufficient. The symmetry result states that an answer claiming to depend only on that evidence cannot choose one branch consistently with the declared relabeling discipline. It offers no reason to deny the existence of the omitted event. Historical inquiry may continue by searching for it or by stating the remaining uncertainty.

Under the second interpretation, the retained history is comparatively rich, but the criterion has left unresolved how competing continuities should matter. North might preserve one practice and South another, with both recognized as central in different parts of the association's life. A symmetry may arise after these features are assigned equal significance. Additional historical detail can illuminate their development while leaving the evaluative choice open. In that situation, asking for a better source will help only if the source bears on a reason that the criterion has agreed to treat as discriminating.

Under the third interpretation, an accepted institution has authority to constitute a particular status through selection. Before its decision, the status may remain unassigned or multiply claimed. Afterward, a unique holder may exist under the relevant rules. The decision is part of the history that the model must represent. The equivariance issue then concerns the decision procedure's justification and the status it creates, alongside a distinct question about earlier continuity. Treating the decision as an event keeps the analysis sensitive to its timing and competence.

These interpretations can coexist within one case. The association's programme transmission may be a historical matter with incomplete records; the importance of that transmission for identity may remain disputed; and a later registrar may assign a prospective status. The word "successor" can travel across all three relations. A careful report identifies the relation each use expresses, preserving agreement where it exists and locating the actual disagreement.

The object restriction matters equally. The model concerns organized collectives represented through normalized appearances, roles, practices, and institutional relations. Other social categories can be imposed from outside, sustained through diffuse classification, or shared without a central procedure. Applying the same selection discipline to such categories would require a fresh account of the candidates and relevant symmetries. The formal theorem is general; the reasons for construing a particular social dossier in its terms remain domain-specific.

9.8 Criterion Revision after an Unresolved Selection

An unresolved symmetric case can reveal a weakness in a criterion, but different weaknesses call for different repairs. This subsection distinguishes an incomplete evidence requirement, an unjustified demand for uniqueness, and a criterion whose declared purpose gives a legitimate reason for further selection.

If a criterion claims to reconstruct a historical transmission mechanism, the balanced case may show that its measurement omits the mechanism. Equal membership totals and output counts might be poor substitutes for records of coordination, training, or authorization. The appropriate revision is to specify those relations and seek their evidence. Arbitrarily perturbing weights until one branch wins would improve the criterion's output determinacy while leaving its connection to the proposed mechanism unclear.

If the criterion's real purpose is to describe multiple inheritances, the demand for one winner may be misplaced. A typed lineage account can record both branches' transmitted equipment, practices, and undertakings. It can then pose a narrower identity question only where the research requires one. The value of a criterion should be assessed partly by whether it preserves the phenomena that motivated the inquiry. A single number can be convenient while concealing the relations the investigation sought to understand.

If the purpose requires a unique administrative decision, the criterion can be supplemented by an explicit tie-break with an independent rationale. The resulting procedure should distinguish evidential comparison from the tie-break stage. This structure permits later review to identify whether a different outcome would follow from corrected evidence, a changed priority, or another lottery result. It also prevents the decision's finality from being attributed entirely to historical evidence that remained balanced.

A criterion can therefore improve by becoming more discriminating, by accepting a set-valued answer, or by separating a practical selection from a historical finding. These are substantive alternatives.

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Their adequacy depends on the inquiry's purpose and the object it has defined. The symmetry test contributes a precise question to that assessment: which retained reason supports the asymmetry of the proposed output? An answer to that question can be examined, challenged, and revised without presupposing a universal theory of collective persistence.

10 A Divided Association

This section develops one constructed case in enough detail to test the criteria and connect the later formal results. The association, its records, and every numerical value are stipulated. The example compares continuities, derives a selection threshold, and examines how uncertainty about the measurements differs from disagreement about their importance.

10.1 The Predecessor and Its Two Branches

Consider an association A with one hundred eligible active members. It maintains a programme of collective activities and has undertaken a set of specified commitments. Following a dispute over its direction, it separates into two organizations, B and C, that operate simultaneously and recognize their organizational separation. Eighty of the original members join B; twenty join C. For this first version of the example, membership is exclusive and every original member joins exactly one branch. These assumptions simplify the counts and will be relaxed below.

The operational history follows a different distribution. Branch B continues a small part of the predecessor's programme and begins new activities. Branch C receives most of the trained organizers, operating materials, and relationships required to continue the earlier programme. Both have a historical connection to A, but the inherited operation is concentrated in C. The example stipulates independently verified transfer paths; the operational score therefore summarizes a known transmission history, rather than inferring one from output resemblance alone.

The commitments also divide unevenly. Branch B accepts and implements a limited part of the earlier undertakings. Branch C accepts and implements all undertakings included in the selected commitment inquiry. This does not describe a legal transfer. The case stipulates voluntary uptake under the relevant organizational arrangements. Any legal or moral obligation beyond that uptake would require a separate argument.

We normalize these observations into the profiles shown in Table 1. The operational and commitment dimensions require declared denominators and comparison rules. Their numbers are illustrative summaries under such rules, not naturally given measurements of organizational essence.

The table supports several descriptions. Most original members continue in B. Most of the specified operation continues in C. The selected commitments receive more complete uptake in C. These findings can all be true together. The disagreement begins when a policy must identify one branch as the continuing organization for a purpose that combines those dimensions. The choice cannot be read directly from the table without a rule for their relative importance.

10.2 A Disclosed Weighted Decision Rule

Let $v_j \in [0,1]^3$ be the profile of candidate j. For nonnegative weights w_1, w_2, w_3 summing to one, define

$$s_w(j) = w_1 v_{j1} + w_2 v_{j2} + w_3 v_{j3}. \tag{17}$$

The score is a proposed decision construction. It presupposes that each dimension can be compared on the stated scale and that the tradeoffs are admissible. Its derivation clarifies the consequence of adopting those premises; it does not provide their independent justification.

For a one-parameter comparison, let membership receive weight λ and let the two other dimensions divide the remaining weight equally. Thus $w = (\lambda, (1-\lambda)/2, (1-\lambda)/2)$, with $0 \le \lambda \le 1$. Subtracting

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Table 1: Stipulated continuity profiles after the association divides

Candidate	Membership retention	Operational continuity	Commitment uptake
B	0.8	0.2	0.2
C	0.2	0.9	1.0

the two scores gives

$$\begin{aligned} s_w(B) - s_w(C) &= 0.6\lambda - 0.7\frac{1-\lambda}{2} - 0.8\frac{1-\lambda}{2} \\ &= 0.6\lambda - 0.75(1-\lambda) \\ &= 1.35\lambda - 0.75. \end{aligned} \tag{18}$$

The difference is positive exactly when $\lambda > 5/9$, zero at $5/9$, and negative below it. A policy assigning more than that share to membership selects B; a policy assigning less selects C, provided the candidates are otherwise eligible and the rule chooses a strict maximum.

The threshold identifies where the practical disagreement lies. If all participants accept the data, asking for more precise counts alone will not settle whether membership should receive a weight above or below $5/9$. That question concerns the purpose and justification of the selection rule. Conversely, disagreement about the operational measurement can shift the threshold even when the weights are accepted. The two issues can interact without becoming one issue.

10.3 Robustness across Weight Choices

One might hope to find a successor that wins independently of the weights. The following elementary result states exactly when that hope is justified under the displayed score family.

Proposition 10.1 (Universal weak ranking over simplex weights)**.** *For two profiles $v_j, v_k \in \mathbb{R}^d$, the inequality $w \cdot v_j \geq w \cdot v_k$ holds for every nonnegative weight vector whose coordinates sum to one if and only if $v_{jr} \geq v_{kr}$ for every coordinate r.*

Proof. If every coordinate difference is nonnegative, multiplying each by a nonnegative weight and adding preserves nonnegativity. Conversely, if the difference at coordinate r is negative, choose the weight vector whose rth coordinate is one and whose other coordinates are zero. The weighted difference is then precisely that negative coordinate. Universal weak ranking fails. These two directions prove the equivalence. □

Neither branch in Table 1 dominates the other coordinatewise. The membership dimension favors B while both remaining dimensions favor C. Universal ranking therefore fails. A restriction on the allowed weights could produce a robust selection within that restricted family, but the restriction would become another premise requiring reasons. The result helps locate the open judgment instead of disguising it as an inevitable mathematical answer.

10.4 Measurement Uncertainty and Dependence

Suppose the profile differences are uncertain. Let the difference in each dimension lie in an interval $[\ell_r, u_r]$. For fixed nonnegative weights, the weighted difference lies in

$$\left[\sum_r w_r \ell_r,\ \sum_r w_r u_r\right]. \tag{19}$$

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The lower bound follows by multiplying each lower inequality by its weight and summing; the upper bound follows similarly. If every combination in the coordinate box is feasible, the simultaneous endpoints attain the bounds. If the dimensions are dependent, the interval can be conservative because the extreme combination may be impossible.

Dependence is plausible in an interpreted organizational case. The same organizers may supply both operational continuity and commitment implementation. A disputed classification of their work can affect both scores. Counting the dimensions as independent confirmations would then overstate the evidential diversity. The uncertainty record should preserve joint scenarios when the same source or causal condition controls several measurements.

The interval can still support a useful conclusion. If its lower endpoint is strictly positive, every admitted measurement scenario favors B under the fixed weights. If its upper endpoint is strictly negative, every scenario favors C. If the interval crosses zero, the score alone leaves the choice unresolved. A secure ranking within the measurement model remains conditional on the weighting policy and the eligibility rules.

10.5 Eligibility and Alternative Case Variants

Some inquiries need constraints before ranking. Suppose the task is to name the steward of a particular archive and the policy requires demonstrated capacity to maintain access. A branch failing that requirement can be ineligible even if it retains more members. Such an exclusion should be stated and justified directly. Hiding it in a very large operational weight makes the policy harder to interpret and can permit unintended compensation in other cases.

Dual membership changes the simple population story. If some participants join both branches, the two retention fractions can sum to more than one. That is compatible with overlapping affiliation. It would contradict only the earlier exclusive-allocation assumption. Membership continuity should therefore not be treated as a conserved resource without an explicit exclusivity condition. The same person can carry experience into several collectives, and a historical account can preserve both transmissions.

A third branch can also change a selection without changing either original profile. If it preserves the relevant operation and receives an accepted authorization that the first two lack, it can become the preferred candidate under a previously fixed rule. The earlier claim of a unique successor was therefore indexed to the candidate universe. The example will return to this issue when coherent histories are constructed across several times.

The divided association provides a common test for the rest of the paper. It demonstrates that a rich description of continuity can precede a unique identity verdict, that numerical precision leaves some philosophical questions open, and that a good method can report the location of those questions without discarding the insights already supported by the evidence.

10.6 Measurement Scales and an Ordinal Recoding Test

The weighted comparison deserves a further audit because a normalized number can appear more commensurable than its interpretation warrants. Placing three dimensions in the interval $[0, 1]$ fixes convenient endpoints. It does not establish that a change of 0.1 has comparable significance across membership, operation, and commitment. The following calculation shows how much the decision can depend on that additional assumption.

At weights $(0.6, 0.2, 0.2)$, the original profiles give

$$s(B) = 0.6(0.8) + 0.2(0.2) + 0.2(0.2) = 0.56, \qquad s(C) = 0.6(0.2) + 0.2(0.9) + 0.2(1) = 0.50.$$

Now suppose the membership dimension has only an ordinal interpretation: higher values mean greater continuity, but numerical intervals have no independently established meaning. A strictly increasing transformation preserves that ordering. Apply $f(x) = x^4$ to the membership coordinate alone and keep the same numerical weights. The endpoints remain zero and one, and B still has greater membership continuity than C.

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The transformed scores are

$$s'(B) = 0.6(0.8)^4 + 0.2(0.2) + 0.2(0.2) = 0.32576,$$
$$s'(C) = 0.6(0.2)^4 + 0.2(0.9) + 0.2(1) = 0.38096.$$

The selected candidate reverses. The calculation uses $(0.8)^4 = 0.4096$ and $(0.2)^4 = 0.0016$, followed by the same multiplication and addition rule. Every coordinate's ordering of the candidates remains unchanged. What changes is the tradeoff encoded by numerical distances and fixed weights.

This counterexample does not show that the original membership fraction is merely ordinal. When the ratio genuinely measures the fraction of a specified population, it has a stronger interpretation, and replacing it with its fourth power changes the measured object. That is precisely the point: the investigator must say which numerical interpretation is warranted. If the original scale is a count ratio, that fact constrains legitimate transformations. If it is a qualitative judgment translated into numbers, the justification may be much weaker.

The operational and commitment dimensions need the same scrutiny. Counting activities assumes that the selected units are comparable for the question, or else requires justified weights among them. Counting commitments can give a trivial undertaking the same contribution as an indispensable one. Assigning expert scores can incorporate judgments about importance, similarity, and evidence in one number. The arithmetic can be correct while its interpretation remains unclear.

A more defensible procedure separates measurement from valuation. It can report how many eligible members transferred, which operations continued, and which undertakings were accepted, then state a decision rule that uses those facts. If a scalar is still required, its construction should identify the compensation it permits. The recoding test becomes a diagnostic: a verdict that depends on transformations the measurement theory treats as equivalent needs a different aggregation rule or a stronger interpretation of the scale.

10.7 Baseline Choice and the Meaning of Retention

The predecessor is itself a history. Selecting one time as the baseline can influence the retention profile. Suppose A had eighty members a year before the division, grew to one hundred during the dispute, and then split. A branch that retains most recent entrants can have a strong final-roster score and a weaker long-term participation score. Neither description is automatically mistaken. They concern different predecessor populations.

The study should state why the baseline belongs to the inquiry. A question about a long-standing association may need a membership history. A question about authority at the moment of division may need the eligible voters under the rule then in force. A question about continuing operation may need the team that actually carried the activity during the relevant period. One baseline cannot be assumed to serve all three.

Baseline choice can also be strategic. If participants know that a particular roster date will control recognition, they may alter membership before that date. A historical study should examine such timing where records support it. The possibility alone is not evidence of manipulation in a case. It identifies a mechanism through which the measurement rule can influence the phenomenon measured, making the rule's institutional role part of the analysis.

Operational baselines have analogous problems. A programme can expand, contract, or change purpose before division. Comparing the successor with one selected week can overstate or understate continuity. A range of baseline periods may provide a more informative sensitivity analysis. The researcher should report whether the conclusion is stable across defensible choices and, when it changes, which historical transformation produces the difference.

These audits deepen the divided-association example. The first calculation located a disagreement over weights. Scale and baseline analysis reveal earlier choices that determine what those weights act on. A serious formalism should expose that sequence: define the population and activity, choose the

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reference interval, establish the measurement meaning, state eligibility, justify tradeoffs, and then derive the selection. Reversing that order would allow a preferred verdict to govern every preceding choice.

11 Admissible Identity Partitions

This section supplies a positive procedure for constructing and assessing identity judgments. It begins with required identifications and distinctions, derives a consistency test, and classifies each pair of appearances as securely identical, securely distinct, or unresolved. It then explains why possible identity must be represented differently from secure identity.

11.1 Positive and Negative Identity Constraints

Fix a finite normalized appearance domain I. Let $P \subseteq I^2$ contain positive constraints: pairs that the accepted evidence and criterion require to be identified. Let $N \subseteq I^2$ contain negative constraints: pairs required to remain distinct. For a slice-separated identity model, N includes every pair of distinct appearances in the same slice. Further negative constraints can arise from independently established concurrent histories, incompatible constitutive records, or other case-specific grounds. Their interpretation must be recorded.

A positive constraint is stronger than a similarity observation. The finding that two appearances share most members belongs initially to a profile. To enter the pair in P, the investigator needs a criterion under which that evidence warrants identity. Likewise, an absence of observed transmission should not automatically enter N. Missing evidence and evidence of distinction have different inferential roles. The method depends on this discipline because every accepted constraint can have consequences for other pairs through transitivity.

Define the admissible family

$$\mathcal{A}(P, N) = \{E \subseteq I^2 : E \text{ is an equivalence, } P \subseteq E,\ E \cap N = \varnothing\}. \tag{20}$$

The displayed definition imposes no upper bound on the number of entities. It also omits additional path or agency requirements. These omissions specify the domain of the following theorem. A richer application can add constraints, but it must then examine whether the theorem still applies.

11.2 Closure and the Consistency Test

Let P^* be the smallest equivalence containing P. Concretely, join two appearances when an undirected finite path of positive links connects them, and include every appearance's diagonal pair. The resulting connected components form the classes of P^*. Reversing a positive link uses symmetry; following several links uses transitivity. The construction records exactly the identities forced by those two operations and reflexivity.

Proposition 11.1 (Feasibility of identity constraints)**.** *The family $\mathcal{A}(P, N)$ is nonempty if and only if*

$$P^* \cap N = \varnothing. \tag{21}$$

When the family is nonempty, P^* is its least member under inclusion.

Proof. Every equivalence containing P contains all pairs connected by a positive path. It therefore contains P^*. If a negative pair lies in P^*, every equivalence containing P includes that forbidden pair, and the admissible family is empty.

Conversely, suppose $P^* \cap N = \varnothing$. By construction P^* is an equivalence and contains P. The assumed disjointness makes it avoid N. Thus P^* itself belongs to $\mathcal{A}(P, N)$ and witnesses nonemptiness. Its inclusion in every other member, established in the first paragraph, proves leastness. □

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The theorem gives a practical audit. Form the positive components, then check whether any required distinction falls within one component. A conflict identifies an incompatible set of premises. The method should preserve the links and their sources so that the conflict can be investigated. Removing the last entered link automatically would confuse order of data entry with evidential weakness. The mathematical test detects the inconsistency; it does not determine which premise should be revised.

11.3 Exact Classification of Pairwise Questions

The same construction gives more than a consistency test. It supplies an exact three-way answer for a pair of appearances under the displayed constraint family. Assume the family is nonempty and let C, D be positive components.

Proposition 11.2 (Pair classification under unrestricted class count)**.** *Appearances within one positive component are identified by every admissible equivalence. For two distinct positive components, every admissible equivalence keeps them separate exactly when some negative constraint joins a member of one to a member of the other. If no such negative constraint exists, identification and distinction are both possible in the family.*

Proof. The first statement follows because every admissible equivalence contains P^*. For distinct components C, D, suppose $(c, d) \in N$ with $c \in C$ and $d \in D$. Identifying any member of C with any member of D would, through their positive within-component identities and transitivity, identify c with d. Every admissible equivalence must therefore keep the components separate.

Now suppose there is no negative pair between C and D. Merge exactly these two positive components and leave every other component separate. The resulting partition still contains P. Its merged class contains no negative pair: internal pairs within either original component are safe by consistency, and cross pairs are safe by the present supposition. Other negative pairs remain between separate classes. This partition is admissible and witnesses possible identity between C and D.

The unmerged partition P^* is also admissible and keeps C, D separate. It witnesses possible distinction. The two possibilities prove the final statement and the converse direction of the characterization. □

The procedure is constructive. It can state a secure identification, cite a negative constraint that forces distinction, or exhibit two admissible partitions that disagree. An unresolved pair thus has an inspectable basis. The output can be used to plan further inquiry: the missing information is whatever would justify a new positive or negative constraint between the relevant components under the selected criterion.

11.4 Secure Identity and Possible Identity

For any nonempty family $\mathcal{A}$ of admissible equivalences, define

$$E_{\text{sure}} = \bigcap_{E \in \mathcal{A}} E, \qquad E_{\text{poss}} = \bigcup_{E \in \mathcal{A}} E. \tag{22}$$

The secure relation contains pairs identified under every admitted account. The possible relation contains pairs identified under at least one account. Their quantifiers differ, and that difference changes their algebraic form.

Proposition 11.3 (Closure of secure identification)**.** *The relation E_{sure} is an equivalence. The relation E_{poss} is reflexive and symmetric and can fail transitivity.*

Proof. Every candidate contains each diagonal pair, so their intersection does. If a pair lies in every candidate, its reverse lies in every candidate by symmetry. If aEb and bEc hold in every candidate, transitivity in each same candidate gives aEc there. Hence the intersection is transitive.

The union inherits reflexivity and symmetry from its candidates. For failure of transitivity, take three appearances a, b, c at different times and the two partitions $\{\{a, b\}, \{c\}\}$ and $\{\{a\}, \{b, c\}\}$. The union contains ab and bc and omits ac. It is therefore nontransitive. □

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The example explains why possible links cannot simply be chained. The first link may be supported only by one historical interpretation and the second only by an incompatible interpretation. Combining their conclusions without their premises creates a third account that the evidence never admitted. A database should therefore preserve the candidate or scenario supporting each possibility. Secure links can support ordinary equivalence closure; possible links require their modal qualification.

11.5 Additional Constraints and the Scope of the Procedure

A numerical cap on entity classes changes the problem. If an investigator requires at most two entities among three positively separate components, the unmerged positive closure may cease to be admissible. Negative constraints can then interact globally through the cap. The exact pair classification proved above relies on the freedom to leave every positive component separate. Its conclusion should be re-established after any such global restriction is added.

Similarly, a requirement that identities follow specified lineage paths can remove the simple merged witness. Two components may lack a negative constraint yet be impossible to join under the additional path rule. Admissibility then has both evidential and structural conditions. The general intersection and union construction remains available for a nonempty family, but the simpler local classification need not remain exact.

Independent finite verification enumerates all four-way positive, negative, both, or unmarked assignments on the six unordered pairs of four appearances. Comparing the displayed procedure with full partition enumeration checks the theorem's implementation in 4,096 constraint systems. The proofs above establish the general conditional reasoning; the finite check helps detect mistakes in the representation and examples. Neither form of verification establishes that an actual organizational source deserves a positive or negative identity interpretation. That remains the central evidential obligation.

11.6 A Complete Split–Merger Dossier

A six-appearance constructed dossier shows how the procedure develops a positive answer without presuming that every continuity question is already settled. There is one appearance a at time zero, two appearances b, c at time one, two appearances d, e at time two, and one appearance m at time three. The middle appearances are normalized as distinct simultaneous organizations. The final appearance is a body formed after a merger episode. These statements describe the candidate units and chronology; their identity relations remain to be assessed.

The dossier contains accepted continuity evidence within each established branch: b is identified with d, and c with e, under one stated criterion for continuing local organization. For example, the stipulated criterion can permit ordinary role replacement while requiring an uninterrupted procedure of collective coordination. The hypothetical records are assumed to establish those conditions for the two intervals. The example does not infer the two positive links merely from matching names or membership percentages.

The required constraints are therefore

$$P = \{(b, d), (c, e)\}, \qquad N = \{(b, c), (d, e)\},$$

with reverses understood where the pairs are read symmetrically. Reflexive pairs and the symmetric, transitive consequences of P enter through closure. The two negative pairs record the accepted distinction of simultaneous units. All other pairwise identity questions remain open. No bound on entity count, no gap prohibition, and no extra path constraint is imposed in this version.

The positive components are

$$A = \{a\}, \quad B = \{b, d\}, \quad C = \{c, e\}, \quad M = \{m\}.$$

The capital letters here abbreviate blocks of appearances. They are not additional entities discovered by the construction. The negative constraints prohibit merging B with C. Neither negative pair lies inside a positive component, so the consistency test passes.

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Table 2: Complete admissible identity family for the constructed dossier

Case	Partition of positive components	Additional identifications
1	$A \mid B \mid C \mid M$	none
2	$AB \mid C \mid M$	A with B
3	$AC \mid B \mid M$	A with C
4	$AM \mid B \mid C$	A with M
5	$BM \mid A \mid C$	B with M
6	$CM \mid A \mid B$	C with M
7	$ABM \mid C$	A, B, M together
8	$ACM \mid B$	A, C, M together
9	$AB \mid CM$	two selected continuities
10	$AC \mid BM$	the alternative paired selection

11.7 Enumeration of the Ten Admissible Partitions

The remaining problem is to partition the four positive components while keeping B and C separate. There are fifteen partitions of four labelled objects. Five identify B and C: contracting those two into one object gives a bijection with the five partitions of three labelled objects. Subtracting leaves ten admissible partitions. The subtraction is an exact count because contraction neither loses nor duplicates a partition containing the prohibited identification.

Table 2 lists every admissible partition. A vertical bar separates identity classes. Juxtaposition within a class means that the corresponding positive components are merged. The table therefore provides complete witnesses, not only a count.

The list can be checked by class sizes. There is one partition into four singletons. There are five allowed partitions with exactly one merged pair: every pair except BC. There are two allowed partitions with a triple and a singleton, namely $ABM \mid C$ and $ACM \mid B$. There are two allowed partitions into two pairs, namely $AB \mid CM$ and $AC \mid BM$. A four-object class is forbidden because it contains B and C. The total is $1 + 5 + 2 + 2 = 10$.

The secure findings are already substantive. The two branch continuities bEd and cEe hold in every case. Every cross pair between B and C is securely distinct. Thus b is distinct from e, and d is distinct from c, even though these latter pairs were not explicitly listed in the original negative set. Their distinction follows through the accepted within-branch identities. Every other pair of positive components is ambiguous under this limited family, with both merged and separate witnesses in the table.

The appearance a can be identified with m while remaining separate from both intermediate branches in case four. That possibility is deliberate. The constraint family has not required continuous observed appearance through every slice. If the inquiry rejects such a gap identity, it must add a reason and a corresponding restriction. The ten-case count should not be transferred unchanged to the adjacent-path model of Section 15.

11.8 Sequential Evidence and a Unique Conditional Answer

Suppose a further record, interpreted under the same accepted criterion, warrants identifying a with b. The new positive component is $AB = \{a, b, d\}$, while $C = \{c, e\}$ and $M = \{m\}$ remain separate. The negative relation keeps AB and C apart. Exactly three partitions remain:

$$AB \mid C \mid M, \qquad ABM \mid C, \qquad AB \mid CM.$$

The first terminates both earlier identities before the final body; the second continues the AB identity into m; the third continues the C identity into m. These are interpretations of the identity partition, with the fuller lineage retained separately.

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Now suppose another accepted record warrants identifying e with m. Transitivity identifies c with m as well, because cEe was already required. The sole remaining partition is

$$\{\{a, b, d\}, \{c, e, m\}\}.$$

It gives a unique answer under the accumulated constraints. The uniqueness is earned by explicit premises and can be traced to them. It does not mean that the final body lacks historical inheritance from d; it means that the present identity criterion selects the c, e continuation while the lineage can preserve both inputs to the merger.

The new records are deliberately described by the role they must play, not fabricated as actual documents. In a real application, the investigator must produce the record, assess its provenance and content, and justify the inference to the positive link. A proposed handover or a later self-description may be insufficient. The worked calculation shows exactly what follows if that evidential obligation is met.

11.9 Conflict, Minimal Explanations, and Revision

Suppose instead that the dossier accepts both aEb and aEc. The positive path b, a, c identifies b with c, contradicting the required distinction at time one. The family becomes empty. A compact explanation of the conflict contains the two new positive links and the negative pair bc. Removing irrelevant records does not affect this logical incompatibility.

The explanation can guide review. One link may have been inferred from a criterion suitable only for lineage. The two appearances may have been incorrectly normalized as distinct units. The evidence for one handover may be unreliable. Or two institutional domains may have been combined without preserving their different identity questions. The procedure does not choose among these diagnoses, but it shows which premises need attention.

Withdrawing a premise should preserve its historical record and the reason for withdrawal. If aEc is reclassified as an operational lineage link, its evidential insight remains valuable even though it leaves the positive identity set. The revised family can then support a coherent identity answer without erasing the continuity that generated the conflict. This illustrates the broader preservation principle of the paper: formal repair should separate meanings before discarding observations.

Independent enumeration of all 203 partitions of the six raw appearances confirms the counts ten, three, one, and zero for the successive constraint variants. That audit was performed before reducing to the four positive components, giving an independent check of the block construction. The general proofs explain why the procedure works; the complete dossier demonstrates how it can turn a partially understood organizational episode into a precise set of conditional findings.

12 Conflicting Evidence and Revisable Continuity Premises

An empty admissible family reveals a conflict among the supplied conditions. The next task is to locate that conflict and identify the kinds of revision that could resolve it. This section develops a finite model of revisable premise bundles, derives the relation between conflicts and withdrawals, and applies the construction to a divided-collective dossier. The discussion then examines source dependence, repair costs, background assumptions, and the status of a continuity verdict after revision. The objective is to make a failed reconstruction informative while preserving the historical record and the distinction between consistency and truth.

12.1 Premise Bundles and Fixed Background Conditions

The analysis begins after a candidate appearance domain and a continuity question have been specified. Let I be a finite domain of appearances and let $\mathcal{B}$ be a nonempty family of equivalence relations on I that satisfy the conditions held fixed for the present analysis. These conditions may include slice separation

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and a set of independently retained positive or negative constraints. Their fixed status is local to this analysis. A later review may challenge them through a different model comparison.

Let A be a finite set of labels for revisable premise bundles. A label $r \in A$ supplies a predicate $\varphi_r(E)$ on candidate equivalences. The predicate can be a single identity requirement, a single difference requirement, or a conjunction whose premises are being reviewed together. The term "bundle" specifies the unit of hypothetical withdrawal. It does not imply that every statement in a document must be accepted or suspended together. The bundling choice needs its own reason, such as a common interpretive assumption or a shared extraction process.

For a retained set $J \subseteq A$, define

$$\mathcal{M}(J) = \{E \in \mathcal{B} : \varphi_r(E) \text{ for every } r \in J\}. \tag{23}$$

The notation preserves a distinction between the archival source and the premise used in the model. A document can remain fully preserved while the researcher suspends one interpretation of its content. A photograph can remain in the archive while the proposed identification of the meeting it depicts is questioned. An official statement can remain an observation of authorized recognition while its use as evidence for practical continuity is withdrawn.

This distinction also preserves duplicate support. Two labels may supply the same identity constraint because two sources, or two separately defensible interpretations, support it. If one label is withdrawn, the constraint remains active whenever another retained label supplies it. An implementation that deletes an edge merely because one supporting label has been removed would misrepresent the premise set. The record should therefore retain a many-to-many relation between sources, interpretive premises, and formal constraints.

The fixed-domain assumption isolates a useful stage of inquiry. It permits a precise comparison of alternative treatments of a disputed premise. Questions about false referents, duplicate appearances, or changed organizational resolution remain available through the normalization model. They should be reopened explicitly when the evidence points toward them, rather than simulated by silently withdrawing an arbitrary identity edge.

12.2 Minimal Conflicts and Hypothetical Withdrawal

A conflict is a set of premise bundles whose conjunction has an empty model family relative to the fixed background. An inclusion-minimal conflict loses that property whenever any of its premises is removed. Minimality concerns the set inclusion relation. Several minimal conflicts can have different sizes, and a small conflict need not identify a more serious historical error than a larger one.

Definition 12.1 (Minimal premise conflict)**.** The minimal-conflict family is

$$\mathcal{C} = \{C \subseteq A : \mathcal{M}(C) = \varnothing \text{ and } \mathcal{M}(C') \neq \varnothing \text{ for every } C' \subsetneq C\}. \tag{24}$$

A withdrawal $D \subseteq A$ is a consistency repair when $\mathcal{M}(A \setminus D) \neq \varnothing$.

The definition supplies a deliberately limited meaning of repair. It identifies a hypothetical premise set under which a coherent interpretation exists. It leaves open which retained premises are true, whether the fixed background is appropriate, and whether the withdrawal is evidentially justified. A proposed repair may be useful for locating a disputed interpretation even if further investigation eventually rejects it.

The finite structure has an established diagnostic antecedent. Reiter's theory of diagnosis characterizes minimal diagnoses through minimal hitting sets of conflicts [9, §4.1]. The present application concerns revisable identity premises and supplies a direct proof for that restricted setting. The role of the diagnosis analogy is combinatorial. A source withdrawal remains an inquiry operation, with its historical and interpretive warrant separately assessed.

Proposition 12.2 (Conflict coverage by withdrawals)**.** *For finite A and the model families in Equation 23, a set $D \subseteq A$ repairs consistency exactly when it intersects every member of $\mathcal{C}$.*

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Proof. Suppose first that $\mathcal{M}(A \setminus D)$ is nonempty. If a minimal conflict C were disjoint from D, then $C \subseteq A \setminus D$. Every model of the retained premises would consequently satisfy all premises of C, contradicting $\mathcal{M}(C) = \varnothing$.

Conversely, suppose D intersects every minimal conflict and that $\mathcal{M}(A \setminus D)$ is empty. The finite inconsistent set $A \setminus D$ contains an inclusion-minimal inconsistent subset C. That subset belongs to $\mathcal{C}$ and is disjoint from D, which contradicts the assumed intersection property. Therefore the retained premise set has a model. □

The proposition immediately identifies inclusion-minimal repairs with inclusion-minimal sets that intersect all minimal conflicts. It also explains the importance of the background condition. If $\mathcal{B} = \varnothing$, the empty premise set is already inconsistent. The resulting empty conflict can be intersected by no withdrawal. If the full premise set is compatible, the conflict family is empty and the empty withdrawal is the unique inclusion-minimal repair. These boundary cases separate a need to revise optional premises from a need to reconsider the model that made every candidate inadmissible.

12.3 Positive Paths and Inspectable Conflict Certificates

The abstract bundle model allows many predicates. The positive and negative identity constraints used earlier admit a particularly transparent conflict certificate. Suppose the background contains fixed positive requirements P_0 and fixed negative requirements N_0. Each optional bundle r supplies positive requirements P_r and negative requirements N_r. For retained J, write

$$P_J = P_0 \cup \bigcup_{r \in J} P_r, \qquad N_J = N_0 \cup \bigcup_{r \in J} N_r. \tag{25}$$

The earlier closure result gives

$$\mathcal{M}(J) = \varnothing \quad \Longleftrightarrow \quad P_J^* \cap N_J \neq \varnothing. \tag{26}$$

An inconsistent pair therefore has two inspectable components. A chain of positive requirements connects its endpoints, and a negative requirement requires the endpoints to differ. For distinct endpoints, the positive chain can be reduced to a simple undirected path by removing repetitions. For a negative requirement on an appearance and itself, the positive connection is the reflexive zero-length path. The certificate contains the actual inferential route through which the contradiction arises.

For example, a series of succession decisions may require that an initial appearance be identified with an intermediate appearance and that the intermediate appearance be identified with a later one. A separate criterion may require the initial and later appearances to differ because their total membership overlap falls below a threshold. The certificate then reveals a conflict between a transitive chain of local decisions and a global endpoint exclusion. It directs attention toward the semantics of the two criteria, rather than treating the final contradiction as an unexplained failure of the data.

The provenance labels on the path matter. Each positive edge and the final negative pair should carry a supporting bundle or an identified background condition. The optional labels used in a certificate form an inconsistent subset of the retained premises. If the retained set is already an inclusion-minimal conflict, the labels of any complete certificate must include that entire set; a smaller inconsistent label set would contradict minimality.

A short path and a small premise conflict are nevertheless different objects. One source bundle can support several edges, while another path uses one independently supported edge from each of several sources. Minimizing edge count can therefore retain more premise bundles than a longer path. Moreover, a bundle may contain an additional constraint that makes it inconsistent with the background by itself, even though the chosen path certificate also uses another bundle. The displayed certificate reveals a conflict; a separate minimality check establishes whether every listed premise is needed.

The certificate's completeness follows from the specialized equivalence model. If the background also imposes an upper bound on the number of entities, a conflict may arise because the separation graph

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requires more classes than the bound permits. That conflict can occur without a positive path ending at a forbidden pair. The representation should then use a certificate appropriate to the additional condition, or return to the general predicate formulation of Equation 23.

12.4 A Divided-Collective Dossier with Competing Repairs

A three-appearance dossier makes the difference between locating a conflict and selecting a historical account explicit. Let a occur before a division, and let b and c be distinct simultaneous candidate successors. The background therefore requires $b \not E c$. Three revisable premise bundles are supplied:

$$u:\ aEb, \qquad v:\ aEc, \qquad w:\ a \not E b. \tag{27}$$

The labels can represent differently interpreted records. Their formal contents are stipulated here; the example supplies no actual source testimony or institutional decision.

The pair $\{u, v\}$ is inconsistent because transitivity would identify b and c. The pair $\{u, w\}$ is inconsistent because its two premises require opposite verdicts on the same pair. Each singleton is compatible, and $\{v, w\}$ is compatible with the partition $\{a, c\} \mid \{b\}$. The complete minimal-conflict family is therefore

$$\mathcal{C} = \big\{\{u, v\}, \{u, w\}\big\}. \tag{28}$$

A withdrawal can intersect both conflicts by removing u. It can also do so by retaining u and removing both v and w. These are the two inclusion-minimal repairs. Under the first, the retained premises uniquely yield $\{a, c\} \mid \{b\}$. Under the second, they uniquely yield $\{a, b\} \mid \{c\}$. Thus each minimal repair supports a complete and coherent answer, while the two answers disagree about the continuing successor.

The conflict analysis has still produced a useful result. It shows exactly which packages of premises can coexist and identifies the premise choices on which the rival accounts depend. A researcher can now ask why u was introduced, whether v and w have independent support, and whether the background distinction between the successors is warranted at the stated resolution. Those questions are more specific than a request for additional information about the collective in general.

The example also clarifies what a repair says about a withdrawn premise. Removing u means that its requirement is suspended in that hypothetical reconstruction. The retained premises happen to imply its contrary in this particular dossier. In a less constrained case, withdrawing a positive identity premise can leave the pair unresolved. Suspension and rejection are therefore distinct operations even though they coincide in some examples. A finding record should state whether the remaining evidence supports the contrary claim or merely ceases to establish the original one.

Finally, the uniqueness of the partition under each repair is a feature of this small dossier. A larger repair can retain several admissible identity partitions. Selecting one repair then leaves additional uncertainty inside its model family. The analyst must evaluate the query across that whole family rather than choose an arbitrary partition and report it as the repair's unique consequence.

12.5 Inclusion Minimality, Cardinality, and Repair Costs

The two repairs invite a practical preference: removing one premise may appear preferable to removing two. That preference introduces a new rule. Inclusion minimality only says that a repair contains no unnecessary withdrawal relative to its own retained premises. It does not compare two repairs when neither is a subset of the other. Minimum cardinality supplies such a comparison by assigning equal unit cost to every bundle.

More generally, let each bundle r have a nonnegative withdrawal cost c_r. A minimum-cost repair minimizes

$$\operatorname{cost}(D) = \sum_{r \in D} c_r \tag{29}$$

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over consistency repairs. The cost might encode a review policy, the strength of an independently justified presumption, or the effort required to reopen a source interpretation. These meanings differ. The numerical values should retain the meaning used to justify them.

In the three-appearance dossier, unit costs assign cost 1 to withdrawing u and cost 2 to withdrawing v, w. The resulting choice identifies a with c. If the costs are $c_u = 3$ and $c_v = c_w = 1$, the costs become 3 and 2, so the selected repair identifies a with b. Both calculations are correct under their supplied costs. The change in verdict is produced by the ranking of withdrawals, not by a new historical observation.

Strictly positive costs have a useful formal consequence. Every minimum-cost repair is inclusion-minimal, because removing a redundant withdrawal would strictly lower cost. With nonnegative costs, an inclusion-minimal optimum still exists: repeatedly remove redundant withdrawals until none remain. Its cost cannot increase, and optimality prevents it from falling below the minimum. Other optima may retain redundant withdrawals of zero cost. A report that equates every cost optimum with an inclusion-minimal repair must therefore preserve the strict-positivity assumption.

The choice of costs requires more than formal convenience. A high cost can protect an authoritative institutional record from withdrawal even when that record was used beyond its appropriate subject. A low cost can make an informal participant account easy to suspend even when it provides the most direct observation of a disputed practice. These possibilities show why source genre, competence, interpretive scope, and actual support should be reviewed before their combined effect is compressed into a number. The optimization can organize a defended policy; it cannot supply the defense by returning a unique solution.

12.6 Shared Sources and the Unit of Revision

The dossier changes again when the provenance of the premises becomes visible. Suppose v and w were generated by two interpretations of one report, and suppose the present inquiry has a reason to review those interpretations together. They can be represented by one bundle z whose predicate is their conjunction. The optional label set then becomes $\{u, z\}$, with one minimal conflict $\{u, z\}$. The two singleton withdrawals yield the same two rival partitions as before, but unit bundle costs now assign them equal cost.

The disappearance of the earlier preference illustrates a representation effect. The underlying pair of possible reconstructions has remained available. The preferred answer changed because the unit counted by the repair rule changed. A cardinality policy therefore needs a justified account of what counts as one independently revisable premise. Counting sentences, documents, witnesses, or interpretive decisions can produce different rankings even where the formal constraints ultimately express the same alternatives.

Common provenance alone is insufficient to determine the correct bundle. A document can contain several observations made through different processes. An error in its date may leave its participant list usable. Conversely, several documents can repeat a single mistaken identification from a shared source. A faithful model should represent the dependencies that matter to the alleged error mechanism. A simplistic rule that groups everything by document can hide internal independence; a rule that treats every sentence as independent can create artificial corroboration.

The distinction between evidence and interpretation adds another layer. A source can accurately report that two successor bodies claimed the same name. The analyst can then incorrectly translate each naming claim into a positive identity constraint. Revising those two formal premises need not withdraw either source observation. The error lies in the inference rule from recognized naming to numerical identity. A bundle that captures the shared interpretive rule may therefore be the appropriate unit of revision.

This layered provenance is particularly important in a contested split. Several accounts may share a legal assumption, a membership definition, or a temporal segmentation inherited from the same administrative practice. Their agreement on a continuity verdict can be less independent than their distinct authorship suggests. The conflict model becomes useful when it directs inquiry toward that shared de-

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pendency. It becomes misleading when its labels are treated as self-evident units whose number already measures evidential strength.

12.7 Background Revision and the Scope of a Conflict

The fixed background permits a clear local diagnosis, but its choice also shapes the repair space. In the three-appearance dossier, the separation of b and c is held fixed. If they were duplicate descriptions of one successor, the conflict between u and v would disappear after a justified normalization. If they were distinct simultaneous units, identifying them to save the positive premises would sacrifice the meaning of the unit distinction. The appropriate response depends on the referential evidence, not on which alteration most easily restores a nonempty model family.

One can therefore distinguish several levels of revision. A source-level revision questions an observation or its reliability. An interpretive revision changes the proposition extracted from a source. A criterion revision changes the rule that makes a feature count toward identity. A normalization revision changes which appearances the records describe. A background revision changes an assumption such as slice separation, contiguity, or an entity-count bound. The formal effect of each revision may be a changed admissible family, while its intellectual meaning is different.

These levels should be compared through reasons appropriate to them. A photograph that reveals one meeting described twice bears directly on normalization. A governing procedure that distinguishes transfer from continuation bears on criterion interpretation. Evidence of a transcription error bears on source extraction. The fact that one revision produces a larger or more convenient model family provides a coherence observation; it supplies limited evidence about which of these causal explanations of the conflict is correct.

A disciplined investigation can retain several background models in a higher-level uncertainty family. Each model has its own appearances, fixed assumptions, premise bundles, and repairs. Comparisons across models then require a common target or an explicit correspondence between their targets. The earlier normalization results show why an unrestricted merger of distinct simultaneous units cannot preserve every fine-level identity judgment through exact pullback. That limitation prevents a background revision from being presented as a harmless change of notation.

The point of holding a background fixed is therefore methodological focus. It isolates the consequences of reviewing one class of premises. The result should always state that focus. A conditional repair analysis can be rigorous and useful while leaving the larger choice of model open to further evidence and philosophical criticism.

12.8 Query Security after Repair Selection

A researcher may retain several repairs because the available evidence does not justify choosing among them. Let $\mathcal{S}$ be a nonempty family of consistency repairs and define

$$\mathcal{U}_{\mathcal{S}} = \bigcup_{D \in \mathcal{S}} \mathcal{M}(A \setminus D). \tag{30}$$

The nonempty selected-repair family and the consistency of each repair ensure that $\mathcal{U}_{\mathcal{S}}$ is nonempty. A query is secure under this policy exactly when all its models agree on the answer.

This construction preserves two sources of uncertainty. Different repairs can support different retained premise sets. Within a single repair, the retained premises can still permit several identity partitions. The universal query test must range over both. Agreement among one arbitrary representative from each repair would omit unresolved alternatives and could produce a false appearance of security.

In the three-appearance dossier, retaining both minimal repairs leaves the identity of a with either successor unresolved, while the separation of b and c remains secure. A minimum-cardinality policy selects one repair and thereby makes an additional identity query secure under that policy. The strengthened verdict is correctly described as policy-dependent. Its emergence is not evidence that the historical record itself has become more informative.

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A subsequent observation can discriminate the repaired histories. Its value depends on whether the alternatives predict different observations at the target. A new document that merely repeats a premise already supported through the same source chain may add little discrimination. An independently established handover event, a corrected reference, or an applicable procedural text can change the admissible set more directly. The query framework developed below provides the formal vocabulary for this next stage.

The resulting report can be precise without forcing a single conclusion. It can state the minimal conflicts, the alternative repairs, the reasons for retaining them, the queries secure across all of them, and the observations that could distinguish their remaining answers. This is a positive research product: it transforms a contradictory dossier into a structured map of inferential dependencies while retaining the unresolved historical and philosophical issues that the evidence still supports.

12.9 Revision Records and the Integrity of the Archive

The final methodological requirement concerns preservation. A repair changes which premises participate in a particular inference. The source record, earlier interpretation, and reason for revision should remain traceable. Otherwise a later reader sees only the repaired conclusion and loses the conflict that motivated the improvement.

A useful revision record identifies the query and appearance domain, the fixed background, the conflicting premise labels, and the certificate or other argument establishing inconsistency. It then describes each considered withdrawal and the surviving model family. If a repair is selected, the record states the additional evidential or practical reason for that selection. It distinguishes this reason from the fact that the repair restores coherence.

The record should also preserve the status of the withdrawn material. A false date, a misidentified referent, an overextended interpretation, and a still-credible but unresolved observation deserve different descriptions. Collapsing them into a generic rejected-source category can distort the archive and damage subsequent research. A later inquiry with a different target may legitimately use a source whose earlier identity inference was withdrawn.

This preservation requirement supports cumulative criticism. A future researcher can test whether a correction resolved the actual conflict, whether duplicate support was retained, and whether the adopted repair policy privileged a preferred successor without adequate reason. The formal proof makes the logical consequences reviewable. The revision record makes the intellectual choices that supplied those consequences reviewable as well.

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13 Incomplete Evidence and Criterion Disagreement

This section distinguishes uncertainty about a history from disagreement about the criterion applied to that history. It constructs a joint family of possibilities, derives a refinement property, and states the condition for recovering a fixed identity verdict from an observation. The analysis prevents additional records from being credited with resolving a question that their content leaves untouched.

13.1 Histories and Interpretive Criteria

Let H be a set of candidate histories consistent with the domain of inquiry, and let Θ be a set of articulated continuity criteria. For each admitted pair (h, θ), assume a proposed equivalence $E_{h,\theta}$ on a common normalized appearance domain. The common domain is a simplifying condition: it permits pairwise questions to retain their subjects across possibilities. When candidate histories differ in their appearance domains, the inquiry needs correspondence maps or a larger common record domain.

The feasible set is

$$\Omega \subseteq H \times \Theta, \qquad \Omega \neq \varnothing.$$

It contains the history/criterion combinations that the inquiry currently admits. Writing $\Omega = K \times \Theta_0$ assumes that every admitted history can be combined with every admitted criterion. This factorization can be convenient but requires justification. A criterion may be meaningful only when a particular kind of organization existed, or a historical hypothesis may include an institutional rule that restricts which criterion applies. The joint set preserves these dependencies.

For a focal pair i, j, define

$$q_{h,\theta}(i, j) = \mathbf{1}\{iE_{h,\theta}j\}.$$

This is a Boolean verdict within one fully specified possibility. Its meaning remains conditional on the criterion and history. It is not a probability or confidence score. The family of values over Ω records which answers remain compatible with the current inquiry.

13.2 Secure Verdicts and Valid Refinement

The verdict is securely true when every member of Ω gives one, securely false when every member gives zero, and unresolved when both values occur. These categories are exact relative to the feasible family. Their usefulness depends on whether that family includes the relevant alternatives and whether the assumptions defining it are defensible.

Proposition 13.1 (Persistence under nonempty refinement)**.** *If $\varnothing \neq \Omega' \subseteq \Omega$, every secure verdict over Ω remains secure with the same value over Ω'.*

Proof. Suppose every member of Ω gives value $v \in \{0, 1\}$. Every member of Ω' is a member of Ω, so each gives the same value v. Nonemptiness ensures that this universal agreement represents an admitted possibility rather than an empty family. The conclusion follows. □

The proposition describes valid restriction of a fixed possibility space. Actual research can also expand or revise that space. A newly discovered branch, an alternative interpretation of a source, or a defect in the normalization can introduce possibilities omitted earlier. A formerly secure verdict may then become unresolved. This is a revision of the model or its evidential basis, not a counterexample to the subset result.

An empty family needs a separate status. If new evidence excludes every candidate, the investigator has an inconsistency among the admitted histories, criteria, and evidence constraints. Treating universal statements over the empty set as substantive findings would make every identity securely true and securely false by vacuity. The research procedure should diagnose the conflict and reopen the relevant assumptions.

13.3 Historical Disagreement with an Accepted Criterion

Return to the divided association and suppose a commitment criterion has been accepted. Under one candidate history, branch C received an effective handover and accepted the predecessor's undertaking. Under another, the document described as a handover was a proposal that never took effect. The criterion can yield opposite verdicts because the historical facts differ. Additional records about acceptance, authority, or implementation could discriminate the histories.

The evidential task should be directed at those differing premises. A larger collection of photographs showing C performing similar activities may add little if the open question concerns acceptance of a specific undertaking. Repeated statements copied from the same announcement may also leave the historical alternatives intact. The inquiry needs evidence whose content changes the relevant feasibility set, with its reliability separately assessed.

This targeted approach also clarifies negative findings. If a complete and reliable archive of the relevant procedure contains a required act's rejection, that can support a negative historical constraint. The absence of a document in an incomplete archive has a different meaning. A continuity model should record the difference between a counter-record and a missing record. Their effects on Ω depend on explicit assumptions about the archive's completeness and the procedure's evidential traces.

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13.4 Criterion Disagreement with a Shared History

Now suppose all participants accept the complete history in Table 1. A membership criterion identifies B as the continuing association, while an operational criterion identifies C for the continuity of its programme. The disagreement persists even when H contains one history. The remaining variation lies in Θ.

Further evidence can still matter indirectly. It can clarify what the inquiry is for, expose a consequence of a criterion, or show that a measurement fails to represent the intended activity. But a new membership count that confirms the already accepted value does not by itself determine whether membership should control the verdict. The argument must address the purpose and justification of the criterion.

This distinction prevents a false opposition between facts and values. A criterion can contain constitutive definitions, explanatory priorities, and normative reasons. Its application can depend on detailed facts. The useful separation concerns the role of each premise: which assumptions describe the history, and which specify how that history answers the reidentification question? Locating the dispute at that level makes further inquiry more productive.

13.5 Observation and Exact Verdict Recovery

For a fixed criterion, let $q : H \to \{0, 1\}$ be a focal identity verdict and let $\rho : H \to Z$ be the observation record available to an investigator. The record can be a dossier, a coded table, or a reduced historical description. Exact recovery means there is a function $\bar{q}$ on $\rho(H)$ with $q = \bar{q} \circ \rho$.

Proposition 13.2 (Verdict recovery from a record)**.** *Exact recovery exists if and only if*

$$\rho(h) = \rho(h') \quad \Longrightarrow \quad q(h) = q(h') \qquad (h, h' \in H).$$

Proof. If $q = \bar{q} \circ \rho$, equal records give equal arguments to $\bar{q}$ and hence equal verdicts. Conversely, assume the displayed condition. For a record value $z \in \rho(H)$, define $\bar{q}(z)$ to be the common verdict of histories in its nonempty fibre. The condition makes this value independent of the chosen history. The resulting function satisfies the required factorization. □

The result supplies a precise form of an evidential limitation. Two histories with the same available record and opposite verdicts demonstrate failure of identification for that criterion and record scheme. More elaborate calculation on the same record cannot remove the ambiguity without another assumption. A probabilistic model can assign different plausibilities if justified probabilities are supplied, but a probability ranking is a different output from exact recovery. The distinction should survive the transition from a philosophical argument to a data system.

13.6 Model Counts and the Meaning of Uncertainty

The ten admissible partitions in the constructed dossier are ten possibilities, not ten equally probable histories. The constraint model contains no probability distribution. Counting how many partitions contain a pair can describe the combinatorics of that representation, but it does not establish a confidence level for the corresponding identity. A posterior probability would require a justified probability model on histories, criteria, evidence, or another explicitly defined domain.

Uniform counting is sensitive to representation. Suppose one historical possibility is described through two variants that differ only in an irrelevant record detail, while another is left undivided. Assigning equal probability to each described variant changes the total weight attached to the first possibility without adding evidence. A change of normalization can have a similar effect on the number of admissible partitions. The choice of counting units therefore needs justification before it becomes a probability model.

This matters for institutional presentation. A system might display that seven of ten generated models support a successor and call that "seventy percent confidence." The percentage would measure only the frequency within the generated model list unless the generation process has a defensible probabilistic

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interpretation. The model family can support secure, impossible, and unresolved verdicts without that additional assumption. Those are meaningful outputs in their own right.

Probabilities can still be useful when their interpretation is clear. A historian may use a structured assessment of source reliability; an institution may adopt an explicit decision model under uncertainty. The model should state which uncertainties receive probabilities and which remain unresolved criterion choices. A precise posterior for a fixed criterion cannot settle whether that criterion should govern the consequence at issue.

13.7 Uncertain Normalization and Correspondence

So far the candidate histories share one appearance domain. In difficult cases, the number and boundaries of collective units can themselves be uncertain. Two local chapters may be one organization's subunits or two organizations in a federation. Two names may describe one unit or two coexisting units with overlapping members. Fixing one normalization too early can settle the identity problem by construction.

A cautious inquiry can preserve a record-level domain and several normalizations. Each normalization specifies how records support candidate appearances and which simultaneous units are distinct. The identity analysis then runs within each candidate domain. Comparing the outputs requires a stated correspondence between their appearances or a shared proposition expressed directly about the records.

For example, the shared proposition can ask whether two specified recorded actions belong to the same continuing collective under the criterion. One normalization may assign the actions to one appearance; another may assign them to distinct appearances. The disagreement is then located before temporal identity. Additional evidence about governance, coordination, or institutional boundaries may resolve it. The researcher should not report the later partition calculation as independent support for the earlier normalization that made it possible.

This extension also qualifies slice separation. The mathematical assumption applies within a normalized domain. If a later review shows that two simultaneous appearances were duplicate descriptions, the domain changes and their earlier negative constraint may be withdrawn. That revision is compatible with the original theorem's conditional validity. The explanation should state the changed premise rather than portray the theorem as having discovered that one entity was simultaneously two distinct units.

13.8 Evidence Dependence and Apparently Convergent Sources

Multiple records can converge on one identity claim because they share a common source. A register may be updated from a committee decision; a website may copy the register; a later interview may rely on the website. Their agreement can establish the dissemination of a recognition claim while adding limited independent support for the historical premises behind it. A provenance analysis should distinguish these two forms of convergence.

An evidential model can represent the shared source explicitly. Candidate histories then specify whether the original decision was correctly recorded and whether its historical assertions were warranted. The copied records can be compatible with both a reliable and an unreliable original claim. If so, collecting more copies does not distinguish those histories. The decisive query concerns the original evidence or a genuinely independent source.

Agreement can nevertheless carry information about recognition. If the question is whether one branch was publicly treated as a successor during a period, the spread of the claim matters. The same records can thus be weak evidence for one historical identity criterion and strong evidence for the circulation of a social designation. Their evidential value is indexed to the proposition, not a fixed property that applies equally to every inquiry.

This point reinforces the distinction between records and evidence. An archive contains objects that become evidence through an interpretation directed at a question. The interpretation should state the observation process and competing explanations. A continuity method gains rigor by exposing those relations, rather than treating every available document as an independent vote on the final identity verdict.

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13.9 The Limits of a Fixed Possibility Space

The refinement theorem presumes that the relevant true or adequate account remains inside the feasible family. An omitted mechanism can invalidate that presumption. The investigator may have modeled only formal branches and missed an informal continuation, or assumed exclusive membership when dual affiliation occurred. New evidence then calls for expansion or reconstruction of the family rather than further restriction within it.

A research report should distinguish this model uncertainty from uncertainty inside a fixed model. Secure agreement over the current family is conditional on its coverage. The report can state the alternative mechanisms considered, the searches undertaken, and the reasons for treating others as outside scope. This gives the reader grounds to assess the practical strength of the conclusion.

The method's openness to new alternatives should also have discipline. A bare imaginable story does not deserve the same status as a history supported by records. Candidate histories need coherence with known facts and a reason for their relevance. Preserving uncertainty means retaining warranted alternatives and their status, not indefinitely multiplying unsupported possibilities. The study design should make those admission standards explicit.

13.10 Common Targets and Comparison across Questions

The criterion coordinate needs a final clarification. Two criteria can be competing answers to one question, or they can define different questions that use similar identity language. The mathematics can compare their equivalences in either case, but the interpretation of agreement changes. A family of alternative measurements of one accepted target differs from a family containing registry continuity, operational continuity, and membership-association continuity as separate targets.

When the criteria compete over one target, a secure verdict means that every admitted criterion and history gives the same answer to that target. The warrant remains conditional on the family, but its logical subject is shared. For example, several operational measures can be proposed as indicators of one explicitly characterized continuing programme. The inquiry can ask whether the verdict survives their differences and whether the indicators are adequate to the target.

When the criteria concern different targets, their intersection records a cross-question agreement. If registry and operation criteria both identify a with b, the researcher can report that agreement. It does not establish that the two criteria mean the same thing or that their common answer supplies an unrestricted identity fact. The agreement may fail in another case precisely because the questions remain different. Its significance needs the descriptions of the targets alongside the common pair.

The difference is consequential for negative results. If one criterion identifies a pair and another distinguishes it, the researcher should first ask whether they answer the same question. An apparent contradiction can dissolve when one finding concerns a registration and the other a practical operation. Conversely, two sources claiming to answer the same registration question can genuinely conflict about an effective act. A record that stores only two opposing Boolean values would obscure this distinction.

The feasible set Ω should therefore include enough metadata to identify the target of each criterion. For a single-target analysis, the family should admit only criteria proposed for that target. For a cross-question comparison, the report should state that its secure relation records agreement among specified reidentification practices. This keeps the formal intersection useful while preventing a shift from plural inquiry to an unargued universal identity claim.

A similar issue arises when the appearance domains differ. A membership association and its operational programme may not present the same units, even if the same names appear in their records. Comparing their equivalences requires a correspondence between the focal appearances. That correspondence can be an explicit relation between records or organized units; it cannot be assumed merely because the investigator wants one common matrix. The shared-domain assumption in the formal model is therefore another substantive condition to check.

The point also limits the phrase "robust identity." Robustness is always relative to specified variations. A finding can be robust across measurement intervals, source interpretations, or eligible criteria

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while remaining sensitive to a changed question. The report should state the varied inputs and the fixed target. An unqualified robustness claim can conceal which philosophical assumptions were held constant throughout the exercise.

The framework consequently supports two disciplined outputs. One is a conditional answer to a fixed continuity question, with evidence and criterion uncertainty explicitly represented. The other is a comparison of different continuity questions, showing where their answers align or separate. Both can advance the inquiry. The first improves the warrant of a finding; the second clarifies the conceptual structure of collective persistence and the reasons that different stakeholders can speak past one another.

This distinction should guide later expansion of the model. Adding another criterion to the family is not automatically adding another source of uncertainty about the same fact. It may introduce a new research object. The project should record that change and decide whether the new object belongs in the current comparison or requires a separate analysis. Preserving this boundary keeps the model comprehensive without making its central question so broad that every disagreement becomes one undifferentiated uncertainty.

14 Joint Historical Questions and Partial Identity Records

A table of secure, possible, and excluded identity pairs can be a clear summary of a study. Its clarity creates a further obligation: the table's scope must match the questions later asked of it. This section derives a limitation of pairwise summaries, shows how it affects subsequent evidence updates, and specifies what a research record must retain for joint historical questions. The distinction is especially important when global constraints or alternative criteria restrict the admissible family beyond positive and negative pairs.

14.1 Pair Statuses and Candidate Histories

Let $\mathcal{A}$ be a nonempty family of equivalences on a finite appearance domain. Its secure relation and possible relation are

$$S(\mathcal{A}) = \bigcap_{E \in \mathcal{A}} E, \qquad U(\mathcal{A}) = \bigcup_{E \in \mathcal{A}} E. \tag{31}$$

These relations determine each pair's status. A pair in S is identified throughout the family. A pair outside U is separated throughout it. A pair in $U \setminus S$ has both interpretations among the candidates. For a report concerned only with those current pair questions, the two relations provide all the required verdicts.

A candidate history carries more information. It specifies which identifications hold together in one account. The possible relation records that each of several links holds somewhere, while leaving open whether they hold in the same candidate. The earlier nontransitive-union counterexample already exposes one consequence. The limitation persists even when the possible relation happens to be transitive and therefore looks like a coherent partition.

The secure relation has a related interpretation. Its equivalence classes collect appearances already known to belong together under every admitted account. Separate secure classes indicate a lack of secure identification between them. Depending on the family, they may still have to merge in each complete candidate, with uncertainty about which merger occurs. Reading the secure quotient as an exact count of historical entities can therefore go beyond the knowledge it represents.

These issues concern the difference between a partial description of what is settled and a possible complete history. They remain relevant when every admitted history is precise. The uncertainty can concern which complete arrangement obtains rather than the logical status of the arrangements themselves.

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14.2 A Three-Appearance Comparison

Take three appearances a, b, c at distinct times. Because each time contains one appearance, every equivalence on this domain satisfies slice separation. Write E_{ab} for the partition identifying a, b and separating c; define E_{ac} and E_{bc} in the corresponding way. Let E_{top} identify all three, and let D separate all three.

Consider the two families

$$\mathcal{F} = \{E_{ab}, E_{ac}, E_{bc}\}, \qquad \mathcal{G} = \mathcal{F} \cup \{E_{\text{top}}\}. \tag{32}$$

The first admits exactly the three two-identity arrangements. The second also admits one common identity for all appearances. These are stipulated possibility sets. An actual inquiry would need reasons for their global restrictions.

Proposition 14.1 (Equal pair summaries with different joint possibilities)**.** *The two families have the same secure relation D and the same possible relation $I \times I$. Nevertheless, the joint query that a, b, c belong to one identity is false throughout $\mathcal{F}$ and possible within $\mathcal{G}$. The secure partition D belongs to neither family.*

Proof. Each off-diagonal pair is identified in one of the three two-class partitions and separated in the other two. Their intersection is therefore the diagonal, while their union contains every pair. Adding the universal equivalence changes neither relation. Identifying both a with b and a with c forces all three into one class by transitivity, so the joint query fails in every two-class partition and holds in E_{top}. Finally, D has three classes, while both families contain only partitions with one or two classes. □

The example gives the possible relation its strongest apparent form. It is universal, hence reflexive, symmetric, and transitive. Its coherence as an abstract equivalence still fails to establish that it belongs to the first family. Transitivity of a summary supplies a formal property of the summary; admissibility remains a property relative to the full historical constraints.

The secure relation illustrates the converse temptation. Its three classes might be displayed as three identified organizational histories. Under $\mathcal{F}$, however, every candidate contains exactly two histories. The record knows that some pair continues one identity while leaving the pair's identity unresolved. Three separate secure classes express that unsettled allocation of appearances. They do not establish three separate historical entities.

14.3 Global Restrictions and the Pair-Constraint Model

The families in the example deliberately go beyond the unrestricted positive-and-negative model. If a nonempty family consists of every equivalence containing P and avoiding N, the intersection of its members is itself admissible. It remains an equivalence, contains every positive premise, and respects every negative one. The secure relation can therefore serve as a complete candidate within that particular family.

Neither $\mathcal{F}$ nor $\mathcal{G}$ has that form. Every off-diagonal pair is separated somewhere, so none can be a required positive premise. Every pair is identified somewhere, so none can be a required negative premise. Those pair constraints would admit all five partitions of the three-element domain, including D. The global restriction excluding D needs another part of the model.

One possible interpretation of $\mathcal{F}$ stipulates that exactly two collective identities occur in the dossier. An interpretation of $\mathcal{G}$ permits at most two. The earlier discussion of entity caps shows why such assumptions need their own evidential justification. Here they provide a compact way to exhibit the missing joint information. A different historical constraint could produce a similarly restricted family without using a numerical cap.

This distinction prevents the general family method from inheriting every property of its simplest constraint instance. A family generated by histories, criteria, or path conditions can preserve correlations that pairwise closure omits. The method should state which construction generated the alternatives before using an intersection or summary as a complete candidate history.

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The exact visible reduction in Proposition 19.1 has the same boundary. It preserves the full visible family for the specified pair-constraint model. Its proof gives hidden components fresh classes, which a global cap may forbid. The present example explains a broader consequence of that boundary: a pair summary may cease to be a complete language for the reduced possibilities.

14.4 A Common New Record with Different Refined Findings

Lost joint information affects later pairwise questions, even when the report never asks a joint query directly. Suppose a new accepted record establishes aEb. Apply that same positive refinement to the two families in Equation (32).

In $\mathcal{F}$, only E_{ab} remains. The refined dossier securely separates a from c and b from c. In $\mathcal{G}$, both E_{ab} and E_{top} remain. The same two pairs therefore stay ambiguous. The initial summaries and the new pair constraint were identical, yet the correct updated summaries differ.

An update procedure retaining only the initial S, U pair cannot distinguish these inputs. It must produce the same output for both unless it retrieves additional information. At least one output will then fail to represent the full refinement correctly. The counterexample identifies the lost dependency: the original global restriction on which pairs could hold together.

This matters when a research finding becomes a maintained historical record. A display can summarize the current pair verdicts while the underlying system retains the complete constraints or candidate family. New evidence should refine that underlying representation before the display is regenerated. Updating the display directly can convert a presentation shortcut into an inferential error.

The example also clarifies the meaning of preserving uncertainty. Keeping a mark of ambiguity on each pair can preserve less than the original study knew. The study may know that one of several identities must hold, or that certain possible links are mutually exclusive. Those positive constraints on the alternatives are part of its result and deserve a durable representation.

14.5 Joint Questions in a Merger Narrative

A merger history naturally produces questions with several appearances. An investigator may ask whether one organization persisted through the whole episode, whether an intermediate branch was distinct from both endpoints, or whether an undertaking passed through one uninterrupted institutional route. Answers to isolated pairs can leave these questions unresolved.

Consider an initial association, an intermediate claimant, and a final organization. A report may mark each pair as possibly identical. That report leaves open whether there is a candidate in which all three belong to one identity. If the evidence allows only pair-specific alternatives, the narrative of one continuous organization exceeds what the pair table establishes.

The reverse error is to treat unsettled pair links as a history containing breaks at every interval. A family may establish that some continuity exists while leaving its location unresolved. Selecting the secure intersection can make the output appear more fragmented than every admitted history. That appearance can mislead a reader about whether the study established actual termination or only failed to identify a unique path.

The practical solution is to state the joint target directly. A finding about one continuous organization through three stages should be evaluated in each complete candidate. A finding about an undertaking's uninterrupted route should use its typed transmission relation and the required intermediate conditions. The target's logical structure determines the needed information.

A constructed review exercise can make the mistake concrete. Let the three appearances be an association before a reorganization, a committee during the transition, and a later association. Stipulate that an independently accepted register criterion requires exactly two identities across the three appearances, while three retained readings of the appointment records support the alternatives in $\mathcal{F}$. One reading links the earlier association and committee; another links the two endpoint associations; the third links the committee and later body. Each reading separates the remaining appearance.

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A reviewer extracts a possible pair from each reading and joins the three into a single continuous-organization narrative. The resulting narrative violates the register condition and belongs to none of the retained readings. The error concerns the combination of warrants: each possibility has a witness, but the witnesses are different complete accounts. A corrected finding preserves the three alternatives and reports that the all-three identity is excluded under the stipulated criterion. If the register's two-identity condition is later challenged, that premise can be revised explicitly; the original combination remains unsupported by the earlier family.

This approach avoids turning one summary device into a universal historical language. Pair tables are valuable for locating disagreement and reviewing individual links. Narrative claims can require combinations of links, exclusions, and events whose coexistence must be checked within one candidate account.

14.6 Preserving a Sufficient Research Record

A research record can preserve joint information in several ways. A small constructed family can be listed in full. A larger family can be specified by complete constraints, with its global conditions stated alongside the positive and negative pairs. A history model can retain the events and criterion applications that generate the alternatives. The appropriate choice depends on which queries the record is expected to answer.

Each choice needs a sufficiency argument. If a compressed record is intended to support only current pair verdicts, S, U suffice by definition. If it must support arbitrary later refinements, the example shows that more information can be required. If it must support a specified joint query, the recovery criterion asks whether the compression places opposing query answers in one observational fibre.

The record should also distinguish a retained witness from a complete enumeration. Showing one candidate that supports a possible identity establishes possibility. It leaves open whether another candidate supports a competing link or whether the two links can coexist. Showing a counterexample to a universal claim establishes failure of that claim, while a positive universal finding needs an argument covering the whole admitted family.

This distinction can guide human-readable reporting. A table may list the pair verdicts, a short constraint statement may preserve the joint restriction, and linked witnesses may show the decisive alternatives. The reader can then understand both what remains open and what the openness is allowed to contain.

14.7 Partial Knowledge and the Meaning of a Complete Finding

A complete research finding need not select one complete historical identity. It can give a complete account of the evidence's current implications, including a structured set of alternatives. The distinction matters because pressure to produce one partition can destroy knowledge about the relations among unresolved options.

The method proposed here therefore separates three products. The accepted dossier records premises and their warrants. The admissible family records the complete possibilities under those premises. The displayed summary answers selected questions about that family. Each product has a useful role, and each requires a different interpretation when it is revised.

This separation preserves a positive conception of uncertainty. An unresolved identity question can contain exact information about which histories remain possible, which combinations are excluded, and what a new record would settle. The three- appearance example shows that such information can exceed a list of uncertain pairs. A well-maintained collective history should retain that additional structure whenever it matters to the inquiry.

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15 Selection across Adjacent Times

This section constructs a coherent single-continuation policy from directed lineage possibilities. It derives the relation between adjacent partial matchings and identity paths, then examines the limitations of independent pair scores and candidate-dependent selection. The purpose is to supply a usable construction while preserving the distinction between a selected identity and the fuller history of inheritance.

The later worked cases distinguish three meanings of selection: establishing which edges are eligible, choosing a feasible matching, and optimizing a declared objective over feasible selections. Exact boundarywise optimization is sufficient under one separable policy. Greedy edge selection and a policy valuing an anchored complete path raise different questions. The distinction prevents a claim about global coherence from becoming an unsupported demand for global optimization in every application.

15.1 Admissible Edges and Local Selection

Let $I_0, \dots, I_n$ be normalized appearance slices and let $L_t \subseteq I_t \times I_{t+1}$ contain the lineage edges eligible for selection. Eligibility is a prior judgment. It can require a documented transmission, an effective succession act, or another justified condition. The present construction begins after those conditions have been stated.

Select $K_t \subseteq L_t$ subject to

$$\#\{j : (i,j) \in K_t\} \le 1, \qquad \#\{i : (i,j) \in K_t\} \le 1. \tag{33}$$

Each earlier appearance has at most one selected successor, and each later appearance has at most one selected predecessor. The set K_t is a partial matching between the slices. "Partial" permits termination and new formation. A requirement that every appearance receive a match would force continuity even when the evidence supports a beginning, ending, or suspension of judgment.

The selected edges need not exhaust the lineage. If a transmits members to b and operations to c, the lineage may contain both links while the identity policy selects at most one. A record should retain the unselected edge with its type and supporting evidence. Selection concerns the single-continuation policy; it does not retroactively remove a transmission.

15.2 Identity Paths and Their Derivation

Let $K = \bigcup_{t=0}^{n-1} K_t$. Form connected components after forgetting the directions of the selected edges. The following proposition gives the condition under which these components are appropriate identity classes.

Proposition 15.1 (Path components from adjacent partial matchings)**.** *For finite slices and selections satisfying Equation* (33)*, each connected component of K is a directed path or an isolated appearance. It contains at most one appearance in each slice. Its component equivalence therefore satisfies Assumption 6.1.*

Proof. Every selected edge increases time by exactly one, so a directed cycle is impossible. The matching bounds give every appearance at most one incoming and at most one outgoing edge. An undirected cycle would require a vertex of minimum time on that cycle to have two outgoing edges, contradicting the bound. Thus every undirected component is acyclic.

Every vertex has undirected degree at most two, so a finite connected acyclic component is a path or a singleton. At an internal path vertex, the degree bounds require one incoming and one outgoing edge. Direction is therefore consistent along the path. Time strictly increases in that direction, and the component contains at most one appearance at each time. Component membership consequently defines a slice-separated equivalence. □

The converse needs another assumption. Suppose an identity equivalence is slice-separated, each class occupies a contiguous interval of slices, and each pair of its consecutive appearances belongs to the

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eligible lineage. Connecting those consecutive appearances gives partial matchings: the class has only one appearance in each slice, and different classes are disjoint. The resulting paths recover the identity classes. Contiguity is essential. An entity recognized as returning after an unrepresented interval requires gap edges or a different representation.

15.3 Pairwise Scores and Global Inconsistency

A pairwise similarity rule can fail before any difficult long history is considered. Let one predecessor a have two distinct simultaneous successors b, c. Suppose each pair exceeds an accepted similarity threshold. If both pairwise judgments enter the positive constraint set, their equivalence closure identifies b and c. The result contradicts their required distinction. High scores on individual pairs therefore do not guarantee a coherent identity partition.

The same issue can arise over a longer interval through indirect links. An investigator may accept aEb from one set of records and aEc from another, with the simultaneous conflict visible only after the records are integrated. This is why local decisions need a global constraint audit. The problem is structural even when every score was calculated correctly. The score's threshold was never sufficient to enforce the intended relation.

Partial matching is one repair when the purpose requires single continuation. It prevents both outgoing links from being selected together. It still leaves the choice between them open. A tie-breaking convention can make a software system deterministic, but it should not be presented as an evidentially established identity. A defensible procedure can retain both eligible matchings until additional evidence or a justified policy resolves the choice.

15.4 Longitudinal Criteria and Path Dependence

Even a sequence of coherent local matchings can be inadequate for a longitudinal criterion. Suppose every adjacent transition retains a majority of members, but repeated replacement eventually removes every original member. A local majority rule can maintain one path. An endpoint rule based on original membership can deny the final identification. The difference concerns what kind of continuity the criterion requires, rather than an arithmetic inconsistency.

A path criterion can require intermediate transmission instead of endpoint overlap. It can also impose a cumulative loss bound, continuity of a particular procedure, or preservation of an undertaking. Such conditions couple selections across times. Optimizing each adjacent score independently may then fail to optimize or even satisfy the longitudinal policy. The appropriate object becomes a family of complete admissible paths or partitions, with the path-level constraints stated explicitly.

This distinction matters for interpretation. Gradual replacement can sustain an organizational practice through continuous training, while an abrupt replacement with the same final roster may leave that transmission absent. A model retaining only endpoints cannot distinguish those histories. A model retaining every step still needs a criterion specifying which transmissions matter. More detailed data increase the range of questions that can be asked; they do not automatically choose the question.

15.5 Candidate Universes and Provisional Uniqueness

Comparative selection depends on the set of candidates. For a fixed score s and finite nonempty set C, write

$$F(C) = \operatorname{argmax}_{j \in C} s(j).$$

If $C = \{b\}$ and $s(b) = 0.7$, b is the unique maximizer. Add c with $s(c) = 0.9$ and the maximizer changes even though the evidence and score of b remain unchanged. A finding of uniqueness must therefore state its candidate universe.

In an organizational dispute, the missing candidate may be a less visible branch, an informal continuation, or an operational unit omitted by a registry. The issue is not simply a larger sample of the same objects. It can change the alternatives among which a successor is selected. A research design

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should search for plausible candidates before treating a unique observed claimant as a unique warranted successor.

Absolute eligibility and comparative selection remain different. A branch can continue to satisfy every eligibility condition after a stronger candidate appears. Its inheritance has not vanished. The comparative policy has changed its selection. Preserving both facts helps avoid a winner-takes-all historical narrative in which later discovery erases earlier continuities.

15.6 Local Overlap and an Incompatible Global Rule

The difference between local transmission and global similarity can be derived in a complete three-time example. Let the membership sets be

$$A_0 = \{1, 2, 3, 4, 5\}, \quad A_1 = \{2, 3, 4, 5, 6\}, \quad A_2 = \{3, 4, 5, 6, 7\}.$$

Each transition replaces one member. Define the overlap ratio for nonempty finite sets by

$$r(A, B) = \frac{|A \cap B|}{|A \cup B|}.$$

This displayed formula is a descriptive set comparison. Its use as an identity rule requires the additional argument under examination.

Adjacent intersections have four members and adjacent unions have six. Both adjacent ratios therefore equal $2/3$. The endpoint intersection is $\{3, 4, 5\}$ and the endpoint union is $\{1, \ldots, 7\}$, giving $r(A_0, A_2) = 3/7$. Choose threshold $3/5$. Both adjacent ratios exceed the threshold, while the endpoint ratio lies below it.

Now combine two proposed rules. The first says that adjacent appearances above threshold must be identified. The second says that any two appearances below threshold must remain distinct. The first supplies $i_0 E i_1$ and $i_1 E i_2$; transitivity supplies $i_0 E i_2$. The second supplies the required distinction of i_0 and i_2. The joint constraint family is empty. Every individual calculation was correct; the rules were incompatible when applied to one history.

One coherent response treats the threshold as a local transmission condition only. The identity then follows the connected sequence, and endpoint similarity need not exceed the same threshold. Another treats the threshold as a necessary condition for every pair in an identity class. It must then reject at least one adjacent identification in this example. The two policies answer different continuity questions. Their distinction should be settled before a researcher combines local links into a longitudinal record.

15.7 Gradual Replacement and the Meaning of a Chain

The example can be extended through repeated replacement until no original member remains. Each local overlap can remain high while endpoint overlap becomes zero. This is a structural possibility of membership turnover. Whether the chain preserves an organization depends on the criterion and the processes sustaining it.

A transmission account can give a positive explanation. Continuing members may train newcomers, maintain procedures, and carry undertakings across the replacements. The eventual absence of the original roster is compatible with that chain. The relevant continuity lies in the organized transfer, not an invariant set of people. A criterion focused on the original association of those particular people may reach another conclusion. The two accounts should state their different subjects.

The chain itself is also insufficient evidence if its links record only overlap. Participants can remain members while abandoning the shared practice. They can preserve a name while changing the organization's governing purpose. A local transmission criterion must therefore identify what is transmitted and why its repeated transmission warrants the intended continuity. The overlap calculation locates the logical issue; it leaves the substantive criterion open.

This analysis shows why an equivalence closure can be both correct and misleading. It correctly derives the implications of accepted identity links. It becomes misleading when those links were merely

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similarity judgments whose nontransitivity was ignored. The research record should retain their original status and the criterion used to promote them into identity premises.

15.8 Selection Objectives and the Value of Abstention

A practical system may seek a complete set of identity paths because it needs one longitudinal identifier for every appearance. Completion can improve administrative convenience while adding unsupported claims. The partial matching construction permits unmatched appearances precisely to avoid making completeness an implicit evidential requirement.

Abstention can have several meanings. It can mark a known new formation, a known termination, or an unresolved link. These statuses should be distinguished. An unmatched successor with strong evidence of new formation is different from an unmatched successor whose predecessor remains unknown. Treating both through one blank field would lose information even though the selected graph is formally the same.

An objective function that rewards the number of matched pairs can favor weak links simply because they connect more appearances. A rule rewarding high similarity can favor a path that omits a historically important undertaking. A rule penalizing every new identity can conceal genuine collective formation. Such objectives are proposed policies with consequences, not neutral completions of the evidence.

A defensible selection design can require minimum evidential support, permit abstention, and publish competing optimal or admissible histories. If one path must be chosen for current operations, the choice can be labelled provisional and tied to a review procedure. The resulting practical decision is concrete while its epistemic qualification remains intact.

15.9 Continuity through a Later Merger

The six-appearance dossier in Section 11 illustrates how local paths interact with a final merger. After accepting the links aEb, bEd, cEe, and eEm, the two identity classes are $\{a, b, d\}$ and $\{c, e, m\}$. The historical transmission from d to m can still be recorded as lineage. It is excluded from the selected identity path because joining it would merge the two simultaneous branches through transitivity.

This outcome can initially seem asymmetric: one branch appears to continue into the merger while the other terminates. The asymmetry belongs to the stated identity criterion and its supporting records. It should be explained rather than hidden. The terminated path may contribute most resources or members to the merged body, and those contributions remain part of its history.

A different criterion could select the other path or treat the merged body as newly formed. The common lineage can support all three interpretations of identity while preserving the same transmission facts. Comparing them requires the reasons developed in Section 7, with the consequence policy made explicit. This is the point at which mathematical coherence and philosophical justification cooperate without replacing one another.

Figure 2 makes the selected identity paths and the additional lineage visible together. Solid arrows show the selected continuities in the unique conditional partition. Dashed arrows retain other stipulated transmission routes. The two arrow styles encode different claims; they are not confidence levels for one relation.

15.10 Evidence Eligibility and the Status of a Selected Edge

An eligible edge is a candidate for selection under a declared rule. Its eligibility should be distinguished from a required identification and from the eventual output of a matching procedure. This subsection specifies those statuses and explains how each can enter an evidential record.

Suppose records support a transmission from an earlier association to two later bodies. One receives equipment and experienced participants; the other receives an undertaking and the procedure for carrying it out. A lineage description can retain both transmissions. If the selected identity policy permits each

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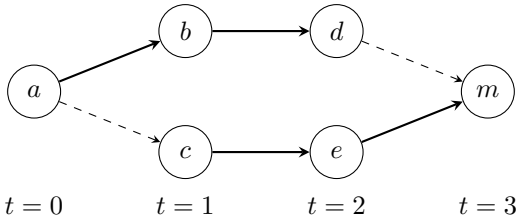

Figure 2: Selected identity paths and additional lineage in the constructed split–merger dossier. Solid arrows are selected identity continuities; dashed arrows record further transmission.

type of transmission to support candidacy, both edges can enter the eligible set without already being accepted as identity premises.

This interpretation matters to the relationship with positive constraints. A pair in the required-positive set must occur in every admissible identity under that constraint system. A pair in the eligible lineage can be selected or left unselected. Treating all eligible pairs as required positives would restore the branching conflict that partial matching was introduced to manage.

The evidential basis of eligibility can also vary by type. A transfer document can establish the movement of a specified asset under suitable source assumptions. Its use as an eligibility certificate for practical continuation depends on the criterion. The document's authenticity and the historical accuracy of its content remain separate questions, as they do elsewhere in the paper.

An edge can therefore possess substantial historical support while remaining insufficient for unique identity. The support may establish a transmission that several coherent identity policies retain in different ways. Selection cannot increase that evidential force merely by assigning the edge to a unique path.

Exclusion has similarly different meanings. An edge can fail an eligibility condition because the required evidence is absent, because contrary evidence defeats the proposed link, or because the criterion deliberately ignores that type of transmission. These reasons should remain distinct from exclusion caused by competition with another eligible edge.

For example, a matching can leave a well-supported link unselected because its target already receives another selected predecessor. That decision expresses the single-continuation policy. It supplies no new evidence that the omitted transmission failed to occur. A narrative that describes the omitted branch as historically unrelated would exceed the result.

The same distinction applies to unmatched appearances. A singleton in one selected partition is different from a pair shown distinct in every admissible partition. The former may be an operational labeling decision under uncertainty. The latter is a secure consequence relative to an explicit nonempty family and its constraints.

A practical record can preserve these statuses without requiring one universal label. It can state the transmission asserted, the source and its limits, the criterion under which the edge is eligible, whether it is required or optional, and the reason for its inclusion or exclusion in the selected output. This structure keeps the route from evidence to decision available for criticism.

An objection is that such a record is burdensome for routine administration. The required detail should be proportionate to the claim's consequences. Where the output merely organizes files, a compact provisional label may suffice. Where it is used to allocate historical credit or address a continuing undertaking, the reason for selection becomes substantively important.

The constructive point is that eligibility can be a disciplined intermediate stage. It excludes unsupported candidates under a stated rule while retaining plural supported routes. Matching then addresses coherence among those routes. Neither stage needs to pretend that it has already completed the historical argument supplied by the other.

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Eligible edge	Stipulated policy score
$a \to c$	9
$a \to d$	8
$b \to c$	8

Table 3: Eligible edges in the constructed boundary-choice dossier. The scores evaluate selections under a disclosed policy; they are not probabilities or independently established identity verdicts.

15.11 Competition within a Single Boundary

A global difficulty can arise inside one adjacent-time matching, before any longitudinal objective is introduced. This subsection gives a complete small case distinguishing sourcewise preference, greedy edge selection and exact matching optimization. The scores are stipulated policy values, rather than measured probabilities of identity.

Let the earlier slice be $I_0 = \{a, b\}$ and the later slice $I_1 = \{c, d\}$. The eligible edges and their scores appear in Table 3. The absence of $b \to d$ means that the dossier supplies no eligible edge of that form under the declared rule. It does not by itself establish every possible historical statement about the pair.

An independent choice by each earlier appearance selects $a \to c$ for a and $b \to c$ for b. The result has two selected incoming edges at c. It therefore fails the matching condition even though each source has selected its own highest-scoring eligible successor.

A greedy procedure that first selects the largest edge avoids that immediate inconsistency but can still give an inferior score. It selects $a \to c$ with value 9. Both remaining edges then conflict with the selected one: $a \to d$ repeats the source, and $b \to c$ repeats the target. The completed greedy matching has score 9.

The exact comparison is small enough to enumerate. The feasible partial matchings are the empty set, each of the three singleton edges, and the two-edge matching $\{a \to d, b \to c\}$. Their scores are respectively $0, 9, 8, 8, 16$. The two-edge matching is the unique maximizer of the stipulated sum.

The example shows why an exact boundary decision concerns the eligible graph as a whole. The value of choosing one edge depends on which other edges the choice prevents. Independent sourcewise maximization and a descending edge-by-edge procedure ignore different parts of that interaction.

The superiority of score 16 is conditional on the objective. If the policy instead gives priority to one specified appearance or treats an edge as required, the optimization problem changes. The numerical example establishes an error in a proposed algorithm relative to a declared objective, not the universal correctness of adding scores.

The example also distinguishes score maximization from a rule maximizing the number of selected edges. Both happen to choose the same matching here. Their agreement in this dossier leaves them different policies. Another distribution of scores could make a one-edge matching preferable under the sum while a cardinality rule still favored two edges.

No historical conclusion follows from the score loss alone. The omitted $a \to c$ edge remains the individually highest-valued candidate in this construction. Its omission reflects the consequences of the joint decision. Calling it a false continuity would confuse a comparative policy judgment with the evidential status of the underlying transmission.

A further objection concerns the empty matching. Because the stated scores are positive, the empty set loses under the sum. Its presence in the feasible domain nevertheless matters. It makes the option to abstain explicit and shows that the optimization's preference for selecting an edge follows from the scoring policy, rather than a hidden requirement that every appearance must continue.

If an investigator wants weak evidence to support abstention, that preference should enter eligibility, the objective or a separate decision constraint with its meaning explained. Treating every positive numerical value as a reason for historical identification would otherwise make the sign convention carry an unargued epistemic threshold.

The case provides a useful positive method. Establish the eligible graph, state the matching condi-

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tions and compare feasible matchings under the declared policy. Preserve the reasons for omitted edges. The procedure produces a coherent selection while leaving its historical warrant tied to the independently assessed evidence and criterion.

15.12 Exact Optimization across Independent Boundaries

The previous case concerns coordination within one boundary. A separate question is whether exact solutions at different boundaries can be combined. Under the basic adjacent-path conditions and an additive all-edge objective, they can. This subsection derives that result and states the assumptions that make it possible.

Let $\mathcal{M}_t$ be the set of all partial matchings contained in L_t. The empty matching belongs to every $\mathcal{M}_t$. Because the eligible graphs are finite, these sets are finite and nonempty. Suppose the joint feasible domain is exactly

$$\mathcal{F} = \prod_{t=0}^{n-1} \mathcal{M}_t.$$

This product assumption says that the basic matching conditions are the complete constraints on the selection. It excludes an additional shared budget, endpoint condition, prescribed class or cumulative path restriction.

For a fixed score $w_t : L_t \to \mathbb{R}$, define the score of a boundary matching by

$$v_t(K_t) = \sum_{e \in K_t} w_t(e), \qquad S(K) = \sum_{t=0}^{n-1} v_t(K_t).$$

The objective counts the scores of every selected edge in the whole collection of paths. Its meaning differs from valuing only the path associated with one initial appearance.

Proposition 15.2 (Boundarywise optimization of a separable selection)**.** *Under the finite product-domain and additive-score assumptions just stated,*

$$\max_{K \in \mathcal{F}} S(K) = \sum_{t=0}^{n-1} \max_{K_t \in \mathcal{M}_t} v_t(K_t).$$

The family of maximizing selections is the Cartesian product of the boundarywise maximizing families.

Proof. Write m_t for the maximum of v_t on $\mathcal{M}_t$. Each maximum exists because its domain is finite and nonempty. Every feasible K satisfies $v_t(K_t) \le m_t$ at every boundary. Summation gives the upper bound $S(K) \le \sum_t m_t$.

Choose a maximizing matching at each boundary. Their tuple belongs to $\mathcal{F}$ by the product-domain assumption, and its score is $\sum_t m_t$. It attains the bound. Every tuple of boundarywise maximizers therefore maximizes the whole score.

Conversely, suppose a globally maximizing tuple contains a nonmaximizing boundary choice. Replace that coordinate by a maximizing matching while holding every other coordinate fixed. Product feasibility is preserved and the total score strictly increases, a contradiction. Thus every coordinate of a global maximizer is boundarywise maximizing. □

The middle appearances create no extra coupling under these assumptions. An appearance may receive one selected incoming edge at one boundary and send one selected outgoing edge at the next. Those choices are jointly allowed by Equation (33). Proposition 15.1 then gives coherent directed path components.

It would therefore be incorrect to say that a long history always requires one inseparable optimization merely because its paths cross several times. Under the stated policy, exact optimization of complete

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Boundary	Eligible edge	Stipulated policy score
0	$a \to b$	6
0	$a \to c$	5
1	$b \to d$	1
1	$c \to e$	10

Table 4: One eligible graph evaluated under two different objectives. The whole-path-collection policy counts every selected edge; the anchored policy counts the complete trajectory beginning at a.

boundary matchings is sufficient. The two uses of global should remain distinct: a boundary matching already coordinates all competing edges at that boundary, while the cross-time objective can still decompose.

Additional constraints can change the result. A requirement that an initial appearance reach a specified terminal slice couples choices along its path. A limit on total changes across the whole history can link otherwise independent boundaries. A required or forbidden endpoint identification can constrain several adjacent selections through transitivity.

The objective can also create coupling without changing the eligible edges. If it values only the trajectory from a particular initial appearance, the score of a later edge matters only when the earlier selections place that edge on the trajectory. The all-edge additive objective and that anchored objective are different constructions.

The theorem's philosophical importance is a discipline of specification. An investigator should identify what makes a longitudinal problem genuinely coupled. The phrase whole history supplies a description of temporal extent, while the constraints and objective determine the optimization's structure.

The theorem also separates computation from warrant. Even exact optimization under a perfectly specified objective leaves open why that objective should govern identity. A correct decomposition can make a policy easier to apply without supplying evidence that its unique maximizing path is the historically correct one.

15.13 An Entire Path Collection and One Anchored Trajectory

The difference between a separable policy and a longitudinal objective can be shown without changing the eligible graph. This subsection supplies a complete three-slice example. It holds the evidence and edge scores fixed while changing what the objective evaluates.

Let $I_0 = \{a\}$, $I_1 = \{b, c\}$ and $I_2 = \{d, e\}$. The only eligible edges are listed in Table 4. They create two possible complete trajectories from a, together with paths that can begin independently in the middle slice.

First evaluate the sum over every selected edge. At the first boundary, $a \to b$ is the unique best matching, with score 6 rather than 5. At the second boundary, both $b \to d$ and $c \to e$ can be selected, since they have different sources and targets. Their joint score is 11.

Proposition 15.2 therefore gives the whole selection

$$\{a \to b,\ b \to d,\ c \to e\}$$

with total score 17. Its path components are $\{a, b, d\}$ and $\{c, e\}$. Replacing the first edge by $a \to c$ preserves feasibility but gives total score 16.

Now change the objective. Require a path beginning at a and reaching slice 2, and evaluate only the scores on that anchored complete path. The eligible paths are $a \to b \to d$ and $a \to c \to e$. Their scores are $6 + 1 = 7$ and $5 + 10 = 15$. The second path is the unique maximizer.

There is no contradiction between the answers. The high-valued edge $c \to e$ contributes to the whole-collection objective whether or not a is linked to c. Under the anchored objective, its value counts only when the earlier choice places it on the path from a. This conditional membership couples the consecutive choices.

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The example identifies the exact change in the problem. The eligible transmissions, their numerical values and the basic matching bounds remain fixed. The objective's subject changes from every selected edge to the trajectory of one designated appearance, together with the requirement that this trajectory reach the last slice.

That subject needs a practical justification. A longitudinal identifier assigned to every observed unit may motivate an all-collection problem. An inquiry into a particular predecessor's continuation may motivate the anchored comparison. These purposes can be reasonable while yielding different outputs. The output should state which purpose it serves.

Neither purpose establishes historical truth through optimization. The number 15 is larger under the stipulated anchored score. It is not a probability that a and e are one entity, and the comparison does not convert the weak first edge into stronger documentary evidence. It changes the policy's ranking of complete alternatives.

The requirement of a complete path also deserves attention. Partial matching permits termination, but the anchored domain here admits only paths reaching slice 2. That restriction is an additional assumption. If an ending at b or a refusal to identify any successor were relevant possibilities, the policy would need to include and evaluate them.

A critic may prefer to require minimum support at every transition. That condition can be added to eligibility before optimization. Another may prefer a cumulative criterion that permits one weaker link to be offset by other features. These are different evidential or decision policies. The additive example cannot decide which one is warranted.

The constructive method is to make the subject, endpoint requirement and aggregation rule explicit. A researcher can then explain why the selected path answers the intended question, compare it with alternatives and retain the historical transmissions outside that path. The method supplies a coherent analysis without treating a score as a complete theory of collective persistence.

15.14 Forward and Backward Evaluation of One Problem

The direction in which an algorithm examines a history can affect a heuristic result. Exact optimization of the same finite problem has a more constrained interpretation. This subsection uses the anchored dossier to separate an algorithmic difference from a change of target or evidence.

A forward greedy procedure begins at a and selects the higher-valued first edge, $a \to b$. Its only eligible continuation is $b \to d$. The resulting complete path has score 7. The procedure has failed to maximize the declared anchored objective because it ignored the later consequence of the first choice.

An exact forward comparison can retain both first-step possibilities, extend them through their eligible continuations and compare the complete scores 7 and 15. It selects $a \to c \to e$. The improvement comes from retaining information relevant to the whole objective, rather than from reversing the direction of historical explanation.

An exact backward evaluation gives the same result. At b, the value of the only remaining continuation is 1. At c, it is 10. At a, the complete alternatives therefore have values $6 + 1$ and $5 + 10$. The larger is 15, with the same path through c.

Backward evaluation is appropriate here because the terminal condition and remaining subproblems are specified. It does not establish that later events cause earlier ones or that historical identity is determined by the endpoint. It is a method for evaluating the consequences of present choices under a fixed mathematical objective.

If two exact implementations of the same finite optimization report different objective values, at least one has failed to solve the stated problem or they are using different inputs. If they return different paths with the same maximal value, the difference may reflect multiple optima. An arbitrary tie-break should then be distinguished from a uniquely supported selection.

The evidential interpretation requires similar discipline. A researcher starting from a present institution may search backward for predecessors. Another starting from an earlier association may search forward for successors. Their observed candidate sets can differ because the searches have different

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coverage. That difference concerns the dossier supplied to the method, rather than an inherent temporal asymmetry of equivalence.

Forward and backward searches can usefully expose omissions in one another. A present-centered search may overlook a terminated branch. An origin-centered search may overlook an independently formed body that later receives resources. These are constructed possibilities motivating a search discipline, not empirical findings about every organizational history.

The appropriate response is to reconcile the appearance domain and evidence before comparing final selections. If the two inquiries genuinely ask different questions, they should retain that difference instead of forcing their outputs into one alleged contradiction.

The account therefore treats direction as a feature of investigation or computation whose consequences must be identified. Under fixed inputs, domain and objective, exact procedures agree on the optimal value and maximizing family. Where the outputs differ, the inquiry should locate a changed assumption, an incomplete search, a tie or an algorithmic error before drawing a philosophical conclusion.

15.15 Endpoints, Horizons and Retrospective Selection

An endpoint condition can change the admissible alternatives, and a longer horizon can change the objective's comparison. This subsection distinguishes those effects from new evidence about an earlier event. The distinction is important when a later successful institution is used to organize a story about its past.

In the anchored dossier, requiring the terminal appearance to be d leaves only $a \to b \to d$. Requiring the terminal appearance to be e leaves only $a \to c \to e$. Each answer is unique relative to its terminal restriction. Their uniqueness provides no independent reason for choosing one terminal restriction over the other.

A present-centered inquiry can legitimately ask how a specified current institution is connected to earlier appearances. The restriction then belongs to the question. It becomes misleading when the answer is presented as establishing that every other branch lacked continuity in any relevant respect.

The horizon produces a different effect. If the anchored comparison ends at slice 1, the values 6 and 5 favor b. If it ends at slice 2, the complete path values 7 and 15 favor the route through c. The score and evidential description of $a \to b$ can remain unchanged.

This change need not be a correction of a mistaken historical fact. The later inquiry evaluates a longer trajectory under a policy that values its subsequent edges. The earlier conclusion should therefore have been stated as a selection for its declared horizon, rather than an unqualified finding of identity.

The all-collection policy gives a useful contrast. Under its additive objective, adding the second boundary does not change the preferred first boundary in this dossier. Both second-boundary edges contribute regardless of the first choice, so the first link remains $a \to b$. The effect of horizon depends on the objective's structure.

Later information can, of course, also bear evidentially on an earlier history. A newly discovered contemporary document may establish that an alleged transfer was unauthorized or that two appearances were separately organized. That is a revision of the evidential dossier. The numerical horizon example holds such evidence fixed and therefore demonstrates a different mechanism.

Keeping those mechanisms separate improves retrospective explanation. An earlier selection can be revised because its facts were wrong, because its candidate set was incomplete, because the criterion changed, or because the same criterion was applied to a longer target. The record should identify which transition occurred.

An endpoint-sensitive policy also risks selecting its own validation material. If an investigator first chooses the institution whose continuation is to be explained and then treats the resulting unique backward path as independent confirmation of that institution's privileged status, the endpoint assumption is doing hidden work.

A stronger design can state the endpoint in advance for a question that requires it, or compare several terminal conditions when the endpoint itself is contested. It can preserve all supported lineage outside the chosen path and explain the practical use of the selected identifier.

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The resulting conclusion remains concrete. A path can be the unique solution to a clearly specified predecessor or successor inquiry. The qualification concerns what that inquiry establishes. A selected endpoint and a successful optimization do not jointly remove the need to justify the historical and criterial premises.

15.16 Gap Permissions and Combined Path Constraints

Allowing a selected identity path to cross an unrepresented interval changes the adjacency assumption. This subsection develops the precise graph condition needed for that extension. It concentrates on feasibility and selection, leaving the distinct historical questions of dormancy and reconstitution to their later treatment.

Let selected edges be permitted to move from an earlier slice to any strictly later slice. Adjacent edges remain a special case; a gap edge skips at least one represented boundary. The one-incoming and one-outgoing bounds must now apply to the union of all selected edges.

Under those combined bounds, the path argument continues to hold. A directed cycle is impossible because time strictly increases along every edge. An undirected cycle would contain a minimum-time vertex with two outgoing edges on the cycle, contradicting the outgoing bound.

Every finite connected component is therefore acyclic and has undirected degree at most two. It is a path or an isolated appearance. At an internal vertex, one edge enters and one leaves, so direction is consistent. Strictly increasing time prevents two appearances from the same slice from belonging to the path.

What changes is contiguity. A path can contain an appearance in slice 0 and another in slice 2 while containing none in slice 1. Its slice-separated identity interpretation remains formally coherent under the declared gap policy. The path alone supplies no account of activity, absence or an unobserved appearance during the interval.

The combined-bound requirement is easy to miss if different edge types are processed separately. Let $I_0 = \{a\}$, $I_1 = \{b\}$ and $I_2 = \{c\}$. Suppose $a \to b$ is an eligible adjacent edge and $a \to c$ an eligible gap edge. Each type considered alone contains one edge and satisfies its own bound.

Selecting both gives two outgoing edges at a. Their component also contains the branch structure the single-path policy was designed to exclude. The gap permission therefore requires a joint audit across edge types, rather than an independent matching routine for each type.

Grouping edges by source slice is also insufficient in general. Replace the adjacent edge in this small example by $b \to c$, retaining the gap $a \to c$. Independent selections at the two source slices can then give c two incoming edges. The constraint couples the selections across their starting boundaries.

This observation marks a limit of Proposition 15.2. Its product domain was justified for adjacent partial matchings. Gap permissions can create shared constraints across groups that previously operated independently. An additive score remains additive, but feasibility may cease to factor in the required way.

The extension also needs a rule for eligibility. A gap can be permitted only under specified evidence, a maximum interval, a maintained undertaking or another criterion appropriate to the question. Those conditions are substantive assumptions. The graph proof begins after they have been assessed.

Selecting only $a \to c$ in the original gap case leaves b outside the selected path. That output should not be read as proving that the earlier eligible transmission to b was fictitious. Nor does the gap edge prove a hidden intermediate unit. The selected identity, the transmission record and the possible missing appearances retain distinct statuses.

The positive construction is therefore available but qualified. Strictly forward gap edges with combined degree bounds preserve path components and slice separation. They require a new feasibility analysis and an explicit account of what warrants crossing the interval. The extension relaxes contiguity without making a graph edge a substitute for historical inquiry.

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15.17 Score Origins and the Number of Selected Links

A policy summing edge scores needs more than an ordering of individual edges. This subsection examines the role of the score's origin when partial matchings contain different numbers of links. The example keeps eligibility and feasibility fixed while changing the numerical representation of the same edge ordering.

Return to Table 3. Subtract 8 from every edge score. The new values are 1 for $a \to c$ and 0 for each other eligible edge. The individually highest edge remains highest, and the other two remain tied.

The scores of the five feasible matchings become $0, 1, 0, 0, 0$ in the previous enumeration. The singleton $\{a \to c\}$ is now the unique maximizer. The two-edge matching that previously had score 16 now has score 0. The numerical change preserves every comparison between individual edges while reversing the selected matching.

The mechanism can be stated exactly. Apply one positive affine recoding $w'_t(e) = \alpha w_t(e) + \beta$, with $\alpha > 0$, to all edge scores. Write

$$n_{\text{sel}}(K) = \sum_t |K_t|$$

for the total number of selected edges. Distributing the sum gives

$$S'(K) = \alpha S(K) + \beta n_{\text{sel}}(K).$$

The offset contributes once for every selected edge. A positive offset therefore introduces a reward for additional links; a negative offset introduces a penalty.

If every feasible selection has the same total edge count, the offset is common to every alternative. Positive scaling then preserves the maximizing family. The reversal arises because the partial-matching policy permits different counts. Its interpretation depends on how links and abstentions are valued.

This result qualifies a possible defense of the original scores. It is insufficient to say that 9 represents a stronger continuation than 8. That ordinal statement leaves both numerical codings available. A sum over matchings of different sizes requires a reason for the units, origin and aggregation rule.

The empty matching makes the origin especially visible. Under the stated sum its score is zero. If every eligible edge receives a positive value, some selection is favored whenever an edge can be added feasibly. If a negative recoding makes the same eligible edge costly, abstention can become favored. The sign convention therefore affects continuity decisions.

A defensible policy can deliberately include a cost for unsupported extension or a benefit for maintaining a usable longitudinal record. Those values should be stated as policy considerations. They should not be hidden inside a numerical normalization presented as harmless.

The example leaves open whether a sum is the right construction for the inquiry. A lexicographic rule, a minimum-support condition, a constrained cardinality rule or retention of several admissible selections would answer different decision questions. Each needs an interpretation tied to the task.

The practical lesson concerns the role of arithmetic. A correct optimizer faithfully implements the numerical policy it receives. When the score origin changes the balance between continuation and abstention, that change belongs in the philosophical explanation of the policy. It cannot be treated as a detail with no effect on the identity output.

15.18 Policy Optimality and Historical Warrant

A uniquely optimal selection can be useful for current operations while remaining insufficiently warranted as a historical finding. This subsection states the difference in the terms of the worked dossiers. It focuses on what optimization establishes and the further premises an identity claim would require.

In the boundary-choice dossier, exact optimization establishes that the two-edge matching maximizes the original stipulated sum. The conclusion depends on the eligible edges, the matching rules and the numerical objective. Its proof is complete under those inputs.

To conclude that this matching is the historically correct identity partition, an investigator needs an additional argument connecting the objective to the intended historical criterion. The calculation

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alone cannot supply that connection. It shows which alternative the policy favors, not why the policy's preference tracks the historical target.

The distinction remains even when the optimum is unique by a large margin. A large difference in stipulated score measures separation within that objective. Its epistemic interpretation requires evidence concerning the score's meaning and the candidate domain. A confidently chosen convention can remain a convention.

The anchored-path dossier makes the issue particularly clear. The same graph and scores yield one first link under the all-collection objective and another under the anchored objective. Both calculations can be exact. Their disagreement reflects a choice about the question and policy rather than a disagreement about arithmetic.

An institution may have a legitimate reason to choose one policy for a limited administrative purpose. It might need a current identifier to coordinate records or a provisional route through an unresolved history. The authority to make that assignment should be distinguished from a claim that the assignment independently discovers every historical fact it represents.

The selected output can subsequently influence the record. Later participants may repeat the assigned identifier, organize materials under it and describe their activity through the resulting history. Those consequences can be real. Their existence does not automatically supply independent confirmation of the original selection.

Conversely, optimization can contribute to historical inquiry without deciding it. It can expose incompatible local claims, reveal which alternatives depend on one weak edge and identify where additional evidence would change the feasible or maximizing family. These are useful consequences of making the policy explicit.

A historical finding becomes stronger when the needed premises receive independent support. A document may establish the relevant succession act, a source comparison may rule out a competing appearance interpretation, or a fixed criterion may have a defended connection to the question. The matching calculation then organizes the consequences of those premises rather than replacing their warrant.

The relation to the secure and possible identity families remains important. An optimizer can select one member of a wider admissible family. Its selection does not make every pair in that member secure across the family. A report should distinguish the selected operational partition from the identities that all admissible alternatives share.

The constructive conclusion is a division of explanatory labor. Evidence and criterion arguments establish which claims are eligible, required or excluded. Matching establishes coherence under the declared path policy. Optimization ranks feasible selections under an explicit objective. Historical warrant concerns whether these premises and transitions justify the final claim being made.

15.19 A Reviewable Selection Record

The completed examples suggest a compact but substantive record for a continuity selection. This final subsection specifies its components and their relation. The purpose is to make a practical output reviewable without requiring the investigator to reproduce every calculation in every use.

The record should first identify the normalized appearance domain and the temporal resolution. A finding about one collective appearance per slice presupposes that duplicate records and distributed components have already been handled under a stated interpretation. A later normalization change can require recomputing the selection.

It should next state the eligible edges and the basis of their eligibility. Required identity constraints and required distinctions should be marked separately. This prevents an optional transmission candidate from entering the analysis as an unquestionable identity premise.

The feasibility conditions should identify whether paths must be adjacent, whether gaps are permitted, and whether endpoint or whole-history constraints apply. If a decomposition is used, the record should explain why the feasible domain has the product form needed by Proposition 15.2.

The objective should identify what is valued: every selected edge, one anchored trajectory, a fixed number of links, a minimum support condition or another declared target. Score units, offsets and the

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treatment of abstention matter when they can change the maximizing family.

The output should preserve its multiplicity. A unique optimum, several equal optima and a wider unresolved admissible family are different results. If one alternative is chosen for operational reasons, the tie-break or assignment authority should be stated independently from the evidential status of the history.

Unselected lineage should remain available. It can explain inherited resources, shared participants and continuing undertakings that the single-path identity policy leaves outside its chosen classes. The policy's structural constraint supplies no reason to erase those transmissions from the historical account.

Finally, the record should state what would warrant revision. New appearances, corrected sources, a changed endpoint question, revised gap permissions or a better justified score interpretation can each alter the selection through a different route. Preserving that route makes the change intelligible instead of treating every new output as proof that the earlier computation was defective.

The adjacent construction thus supplies a positive method with clear limits. It can produce coherent paths, identify exact conditional optima and expose the consequences of policy choices. Its historical use remains answerable to the evidence and criteria that give those paths their meaning.

16 Rule-Changing Histories and Procedural Continuity

The preceding account of adjacent selection assumes that the relevant continuity conditions can be evaluated at each transition. Institutional changes introduce a further complication: an accepted procedure can alter the procedure that governs later changes. This section separates realized updates from their authorization, derives a cumulative test for a declared procedural lineage, and examines an amendment example with changing approval thresholds. The final analysis concerns historical observation, ratification, branching, and the limits of using procedural validity to justify numerical identity.

16.1 State, Event, and Validation Data

Let S be a space of states containing a currently recorded or applied procedure together with the contextual information that the model requires. Let $\mathcal{E}$ be a set of event records. A partial function

$$U : S \times \mathcal{E} \rightharpoonup S \tag{34}$$

represents a realized update. Its value records the state that follows an event in the described history. A separate predicate

$$V : S \times \mathcal{E} \longrightarrow \{0, 1\} \tag{35}$$

evaluates the event under a declared prior-rule authorization test.

The separation of U and V is essential. A procedure can be subsequently applied even when the event introducing it failed the earlier authorization requirements. Representing that change through U records a realized or claimed procedural history. Assigning $V = 1$ makes an additional statement under the selected institutional test. If the update function were defined only for already authorized events, a history of effective but disputed rule change could disappear before its status was examined.

The state and event descriptions must also retain the information on which V depends. An amendment rule can depend on eligibility, attendance, affirmative votes, notice, subject matter, and the competence of an issuing body. A simplified model can hold some of these conditions fixed, provided their role is stated. The numerical example below focuses on affirmative votes and therefore uses an approval threshold. Attendance requirements would be a distinct part of the procedure.

The representation leaves the interpretation of the current procedure visible. A procedure appearing in a state may be the rule that participants apply or the rule they claim is operative. Its authorization through the entire preceding history is a cumulative question. A locally described state cannot acquire that historical status merely because its rule coordinate has been populated. This distinction allows the model to represent disputed authority without converting every effective update into a certified continuation.

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16.2 Anchored Procedural Validity and Composition

A finite realized history has the form

$$H = (s_0, e_0, s_1, \ldots, e_{n-1}, s_n), \qquad U(s_t, e_t) = s_{t+1}. \tag{36}$$

Fix the interpretation of V and provisionally accept the initial authority represented at s_0. The following definition tests continuity of authorization from that anchor.

Definition 16.1 (Anchored procedural validity)**.** The history's anchored procedural validity is

$$C(H) = \bigwedge_{t=0}^{n-1} V(s_t, e_t). \tag{37}$$

For an empty history, the empty conjunction has value 1.

The definition expresses a particular target: every transition in the supplied history meets the selected prior-rule test. It can be evaluated incrementally. Set $c_0 = 1$ and update

$$c_{t+1} = c_t \wedge V(s_t, e_t). \tag{38}$$

At the first step, the flag records the first validation result. If the flag after t steps equals their conjunction, conjoining the next result gives the conjunction after $t + 1$ steps. Finite induction therefore establishes $c_n = C(H)$.

This cumulative flag preserves an earlier failure even when later events meet the procedures then being applied. A later locally acceptable vote can be significant evidence about present coordination. Under the anchored criterion, it does not erase the earlier step that failed the selected authorization chain. A different institutional rule may provide for ratification or fresh founding; that rule requires a separate description of its operation and effect.

Proposition 16.2 (Composition of anchored histories)**.** *Let H and K be realized histories with matching final and initial states. Under the same validation predicate and state interpretation,*

$$C(H \cdot K) = C(H) \wedge C(K). \tag{39}$$

Proof. The event tests in the concatenation are exactly the tests in H followed by those in K. Associativity of conjunction permits the first segment and the second segment to be grouped separately. Their values are $C(H)$ and $C(K)$ by definition. The empty-segment cases follow from the identity value 1 of the empty conjunction. □

The proposition explains what is preserved by the formal composition. It joins histories under one stated criterion. If the second segment uses a different authority, a different meaning of its state variables, or a different validation test, the boundary requires an additional interpretive argument. The fact that two records display the same procedure name supplies insufficient information to establish that correspondence.

16.3 An Amendment Sequence with Evolving Approval Thresholds

Consider a constructed association with five eligible participants. Its currently applied amendment approval threshold is $q \in \{2, 3, 4\}$. An event (k, q') records k eligible affirmative votes and a proposal to apply threshold q' afterward, where $0 \le k \le 5$ and $q' \in \{2, 3, 4\}$. The simplified scheme permits these proposed threshold values and holds other procedural conditions fixed. Define

$$U(q, (k, q')) = q', \qquad V(q, (k, q')) = 1 \iff k \ge q. \tag{40}$$

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Event	Prior q	Votes k	Proposed q'	Prior test	Context
First amendment	4	4	3	1	Valid sequence
Second amendment	3	3	2	1	Valid sequence
Direct disputed change	4	3	2	0	Separate history

Table 5: Constructed amendment events under a prior-rule approval test. The first two rows form one history. The third row is an alternative direct change from the initial threshold. Attendance and other procedural conditions are held fixed.

Begin at threshold 4. An amendment with four affirmative votes changes the applied threshold to 3. A later amendment with three affirmative votes changes it to 2. The first event passes because $4 \geq 4$. The second passes because its prior threshold is now 3 and $3 \geq 3$. The history consequently has anchored validity $1 \wedge 1 = 1$ under the evolving prior-rule test.

Table 5 separates the two-step history from a different direct change. The distinction matters because a frozen criterion that demands four affirmative votes at every stage rejects the valid sequence's second event. That criterion evaluates every transition against the initial procedure, even though the stipulated amendment power permits the procedure to change. The frozen and evolving criteria therefore answer different questions.

The example provides a precise sense in which continuity can involve change in the conditions governing subsequent change. The procedural lineage is preserved through correctly evaluated transitions, even though the approval threshold at the end differs from the one at the beginning. Comparing only the thresholds would reveal difference in state. Requiring every later act to meet the initial threshold would impose a stronger invariant than the chosen procedure supplies.

The construction also has a clear limit. It assumes that the initial procedure permits changes to the approval threshold through the specified rule. If the threshold were reserved to an external authority, the same vote counts would be insufficient. If different subject matters required different approval conditions, the event record would need to identify the subject. The arithmetic expresses the consequences of the stated scheme; the institutional interpretation determines whether that scheme fits the inquiry.

16.4 Retrospective Evaluation and the Direction of Authorization

Now consider the direct disputed change in the third table row. Three affirmative votes propose a new threshold of 2 from a prior threshold of 4. The prior-rule test fails because $3 < 4$. Evaluating the event against the proposed successor rule would instead produce $3 \geq 2$ and accept it. The disagreement identifies the temporal direction through which authorization is being assessed.

Under the anchored model, a proposed rule obtains its place in the certified sequence through an event evaluated by the prior accepted procedure. Allowing the proposed rule to certify the event that introduces it changes that model. Such a change may be a deliberate institutional design in a different setting, but it must be described as a different authorization rule. It cannot be treated as evidence that the original prior-rule condition was satisfied.

This distinction has a practical evidential counterpart. A later consolidated rule document can accurately record the procedure then being used while omitting the circumstances of its adoption. Reading the current rule backward into the earlier event can supply an apparent authorization that the record never established. The study needs the prior procedure and the relevant adoption evidence to evaluate the historical claim under the anchored target.

A similar error can arise in the opposite direction. An investigator may apply a superseded rule to a later event and reject a transition that was properly governed by an authorized amendment. The remedy is a time-indexed procedure history, with evidence for the sequence through which each rule became applicable. The formal distinction between frozen and evolving tests clarifies the issue without presupposing that change itself favors continuity or discontinuity.

The direction of authorization also concerns responsibility for interpretation. A criterion should state

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whether it reconstructs what participants actually accepted, what an institution currently recognizes, or what follows from a specified earlier rule. These targets can differ after a contested transition. Their comparison can illuminate the history, while an unmarked substitution can make a preferred successor appear certified by a procedure selected only after the event.

16.5 Endpoint Observations and Lost Procedural Information

The example supplies an exact failure of recovery from endpoints. Let

$$H_+ = (4, (4, 2), 2), \qquad H_- = (4, (3, 2), 2). \tag{41}$$

Both histories begin with the applied threshold 4 and end with the applied threshold 2. Their event records differ in affirmative votes. The selected test gives $C(H_+) = 1$ and $C(H_-) = 0$.

Suppose an observation map ρ retains only the initial and final thresholds. Then $\rho(H_+) = \rho(H_-) = (4, 2)$. If a function f recovered procedural validity from this observation, it would have to assign both 1 and 0 to the same pair $(4, 2)$. Hence the validity query fails to factor through ρ on this two-history domain.

The result gives a concrete instance of the earlier historical-query condition. It also locates the omitted information. The endpoints describe the procedure before and after the change; they omit the event property that the authorization test evaluates. Adding the affirmative vote count would distinguish these two histories under the simplified assumptions. In a richer case, the count could remain insufficient because eligibility, notice, or the event's subject matter is also disputed.

An augmented state carrying a correctly established validity flag would formally restore recovery. That construction can be useful for an information system that preserves an audited authorization history. It leaves the evidential task of establishing the flag intact. A field labeled "valid continuation" provides information only when its value comes from an adequately supported process. Treating the field as an unexplained primitive would relocate the original uncertainty into the data entry.

This result bears directly on institutional continuity criteria. An unchanged present procedure can follow different histories of appointment, amendment, imposition, or replacement. A changed procedure can follow a valid chain of authorized modifications. The historical target determines which distinctions the representation must preserve. A state description adequate for predicting the next meeting's voting behavior can consequently be inadequate for reconstructing the authority through which that procedure came to be applied.

16.6 A Reconstructed Amendment Dossier with Disputed Eligibility

A richer constructed dossier shows how the event variables acquire their meaning. Five people attend an amendment meeting. The minutes record four affirmative ballots and one negative ballot. The prior procedure requires four eligible affirmative votes. A subsequent rule sheet displays the proposed new threshold. On this initial evidence, a simple comparison of the recorded ballot total with the threshold appears sufficient to approve the transition.

The appearance of sufficiency depends on the meaning of the count. Suppose three affirmative ballots are uncontroversially eligible and the fourth was cast by participant p, whose membership status at the meeting is disputed. A resignation notice exists, but the date at which it became effective remains unresolved. In one candidate history, p was still eligible and the four affirmative ballots satisfy the approval threshold. In another, the resignation was already effective and only three affirmative ballots count under the prior procedure. The same minute entry can therefore support two different authorization verdicts.

The analyst should preserve the recorded raw total in both histories. The disagreement concerns the rule-governed classification of one ballot, not necessarily an error in counting the physical ballots. Replacing the minutes' four with a corrected three would blur that distinction. A more precise record contains the raw tally, the eligibility rule, the status evidence for p, and the derived eligible tally under each supported interpretation.

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The next evidential query can then be specified. A source establishing the effective date of the resignation may discriminate the histories. A current roster issued long after the meeting may establish that p eventually left while leaving the earlier eligibility question open. A letter written before the meeting may express an intention to leave without establishing that the relevant institutional procedure had already taken effect. The value of each record depends on the event and temporal relation that the eligibility rule actually evaluates.

Now add a second independent uncertainty. The association's procedure may permit ordinary amendments while reserving changes to the approval threshold to an external appointing body. A complete copy of the applicable procedure has yet to be found. One interpretation permits the members to make the proposed change; another requires the external body's authorization. Even if p was eligible, the vote total alone then leaves the authority to amend this particular subject unresolved.

Under the deliberately restricted dossier, there are four combinations of the two open conditions: eligible or ineligible ballot, and internal competence or externally reserved competence. If all other conditions are satisfied, only the combination of eligibility and internal competence certifies this meeting's amendment through the specified route. The other combinations fail at least one required condition. The result shows why a positive authorization finding can require several distinct evidential bridges. A securely established failure of one necessary condition can settle the negative verdict for that route while leaving other facts unresolved.

This asymmetry matters for research design. An investigator seeking a complete procedural history may still need to resolve both conditions. An investigator asking only whether this meeting validly enacted this amendment can stop once a decisive failure is established, provided the inquiry's scope and remaining uncertainty are reported. A later authorization through another route remains a separate query. The study should avoid allowing the route-specific negative result to become an unrestricted conclusion about every possible source of standing.

The dossier also exposes a possible conflict between evidence bundles. One interpretation of the minutes may supply the premise that the amendment was approved. An independently established eligibility record may supply a contrary premise under the same rule. A conflict analysis can locate the inconsistent combination, while the richer reconstruction shows that the minutes' factual content can remain intact. The revisable element may be the inference from a raw affirmative total to an eligible affirmative total. This is a concrete example of revising an interpretive bridge while preserving the source observation.

The same pattern applies to the successor question. Suppose the amended procedure later governs the selection of a successor after division. A finding about that successor may depend on the amendment's earlier authorization, even if the final selection meeting is fully documented. The decisive uncertainty lies upstream of the apparently disputed identity event. A source search focused only on the split can therefore miss the historical condition on which the continuity rule depends. The cumulative validity model identifies the dependency and directs attention toward the earlier record.

This constructed dossier extends the simple threshold arithmetic in a specific way. It separates physical ballots from eligible votes, ordinary amendment power from competence over the proposed subject, and a route-specific finding from broader present standing. Those distinctions determine which additional records could change the verdict. They also show how a serious institutional criterion can be empirically demanding even when its final mathematical test is elementary.

16.7 Ratification, Fresh Authority, and Target Revision

The anchored conjunction deliberately preserves every earlier failure. Some institutional inquiries need to represent later ratification or a fresh source of authority. The model can be extended to do so, but the extension should identify the question being changed. A later act can establish present standing under a new rule while leaving the historical proposition that every earlier step met the old rule false.

Consider a constructed external authority that recognizes a successor body from a specified later date. The recognition may answer a present authorization question. It need not certify that the successor's earlier formation complied with the predecessor's procedure. An inquiry could therefore return a secure

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present standing result alongside a negative or unresolved anchored-history result. These conclusions concern different temporal claims.

A retroactive ratification rule requires still more explicit semantics. The researcher must identify which act is ratified, which defects the ratifying authority can address, the period covered, and the consequences the practice assigns to the ratification. A generic instruction to reset the cumulative flag to 1 would erase these distinctions. The appropriate model may need several queries: original compliance, later recognized effect, present competence, and the status of intervening acts.

Fresh founding produces a different transition. A successor can receive a new appointment that supplies its present authority without continuing the earlier chain. The predecessor's history can remain relevant through lineage, transferred undertakings, or membership carryover. A new authority anchor therefore need not remove historical relations, and a preserved historical relation need not make the new anchor an extension of the old authorization sequence.

These distinctions make model revision constructive. The researcher can preserve the original anchored result, add a separately specified recognition or founding relation, and report the claims each supports. The revised account becomes richer without presenting a changed question as a correction of a previous arithmetic calculation. The evidential and normative grounds of the new authority remain available for their own review.

16.8 Authorized Branching and Numerical Reidentification

Procedural validity does further work when a collective's rules permit division. Suppose an accepted procedure authorizes two successor bodies, each assigned a defined task and part of the earlier activity. Both resulting procedural branches can satisfy the applicable transition rules. The validity of their histories then supports a directed account of authorized succession.

Under the paper's normalized-unit assumptions, the two simultaneous successors remain distinct appearances. Identifying both numerically with the predecessor would identify them with each other by equivalence transitivity. The branch constraint therefore still applies. The authorization of several branches does not by itself select one of them as the continuing numerical entity.

A procedure can contain an additional selection rule. It may reserve one specified institutional appointment to one successor while authorizing a new appointment for another. A criterion concerned with that appointment can use the rule as evidence, subject to the ordinary questions of applicability and implementation. The selected identity relation should remain separate from the broader authorized-lineage graph, which preserves the legitimate emergence of both branches.

The same structure appears in merger. Two organizations can authorize contributions to a new body. Their valid procedural participation establishes relations to the merger event and its outcome. It leaves the numerical status of the outcome dependent on the relevant formation and continuity rules. Calling the merger authorized settles one important question while leaving the identity relation to the target-specific analysis.

This boundary prevents an attractive procedural criterion from becoming circular or overgeneralized. The criterion's independent features establish authorization. A further argument explains why a particular authorization relation bears on a reidentification claim and whether it has the single-continuation structure that an equivalence partition requires. The formal framework retains both achievements: a certified procedural history and a globally coherent account of the identity conclusions that can follow.

17 Merger and Unequal Inheritance

This section examines a merger in which distinct predecessor organizations contribute different members, practices, resources, and undertakings. It distinguishes symmetric successor language from asymmetric transmission, derives a limited accounting condition for conserved resources, and explains why identity alone supplies an incomplete rule for responsibility or credit. An integrated merger-and-division dossier then carries these distinctions through one complete sequence. Its numerical ledgers, alternative identity

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partitions, and documentary questions make the consequences of a claimed return to the predecessors explicit.

17.1 Asymmetry within a Common Successor

Consider two organizations A and D that form a later organization M. Suppose A supplies most members and the administrative office, while D supplies the trained team and procedure that sustain the main activity. Both contribute resources. A public announcement describes the result as a merger. This label records an important institutional event, but the underlying inheritance remains asymmetric and multidimensional.

A membership inquiry may emphasize A. An operational inquiry may emphasize D. A newly constituted registration may identify M as a new body. An absorption rule may preserve one predecessor's registration. These answers can be jointly coherent because they concern different relations. The historical record becomes misleading when "successor" is used without specifying which of them is intended.

The branch constraint of Section 6 still applies. If A and D were distinct simultaneous units, a slice-separated equivalence cannot identify M with both. A lineage relation can connect both predecessors to M and preserve their different contributions. The choice of a single continuing identity is therefore a further policy or philosophical judgment. Its adoption should not flatten the transmission history into an undifferentiated origin.

17.2 Transfer Ledgers and Conserved Quantities

Some inherited objects obey an accounting constraint. Suppose a predecessor holds a divisible conserved resource, measured in fixed units, and a reorganization allocates it among successors. For predecessor a, let α_{ab} be the share assigned to successor b. A stipulated conservation ledger requires

$$\alpha_{ab} \geq 0, \qquad \sum_{b} \alpha_{ab} \leq 1. \tag{42}$$

The unallocated residual is $1 - \sum_b \alpha_{ab}$ and must receive an explicit holding or disposal status if the ledger is to be complete.

The derivation follows the meaning of a share in this model. Each successor receives a nonnegative part of an original unit, and the total assigned amount cannot exceed that unit under the conservation premise. Summing the allocations gives the transferred total; subtraction gives the residual. The result is bookkeeping under stated units and boundaries. It is not a theorem about every kind of social inheritance.

Copyable information illustrates the limit. Two branches can each receive a complete copy of a manual. Assigning each a "share" of one would violate Equation (42) only if the manual were incorrectly modeled as an exclusive conserved quantity. The relevant relation is duplication or access, with potentially distinct rights and responsibilities. Membership can also overlap, and practical knowledge can be transmitted to several collectives. Their lineages should not be forced into a conservation law borrowed from a resource ledger.

Identity and allocation have different domains. An organization treated as a new entity can receive the entire resource. A continuing entity can transfer most of its resource to another body. A positive allocation does not imply identity, and identity does not imply a positive allocation. The two relations can inform one another in a governance arrangement, but any such connection must be stated as an additional rule.

17.3 Undertakings and the Grounds of Responsibility

An outstanding undertaking differs from a conserved resource. Its content may require performance, remedy, explanation, or stewardship. Several parties can have overlapping responsibilities concerning it. Dividing a resource into percentages does not establish a corresponding division of responsibility. A

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model that treats both through the same allocation matrix would need an independent normative argument for that identification.

The investigator should begin with the undertaking's history. Who made it, through which authority, to whom, under what conditions, and with what subsequent uptake? A merger can include an explicit assumption of an undertaking. It can also transfer the practical capacity required to fulfil it without an accepted assumption. These facts have different relevance to possible moral, organizational, or legal claims. Their evaluation requires the relevant normative framework and, for actual legal questions, verified jurisdiction-specific sources.

An identity verdict can matter because a rule may attach an undertaking to the continuing organization. Yet the validity and scope of that attachment are additional premises. If the predecessor terminates, another rule may still assign responsibility to a successor, a participant, or a custodian. The mathematical termination of an identity path therefore supplies no general warrant for treating earlier undertakings as extinguished.

This observation also limits causal attribution. Receiving some resources from a predecessor does not make a successor responsible for every earlier action. Conversely, changing a name or membership roster does not independently settle whether a responsibility persists. The relevant historical and normative relations require separate reconstruction. The purpose of a careful identity model is to prevent those questions from being answered accidentally by a record-linking convention.

17.4 Historical Contribution and Public Credit

A merger can alter the visibility of contributions. The common successor's name may become the heading under which earlier work is described, making one predecessor's role disappear from public summaries. A lineage record can preserve who contributed which practice, resource, or idea before the merger. Such preservation does not require choosing a single organization that owns the whole history.

Credit is also not automatically a conserved scalar. Several participants can make indispensable contributions, and later reinterpretation can change the understanding of their significance. A percentage allocation may be appropriate for a specific institutional purpose, but historical description can require a richer account of roles and dependencies. The identity of the current organization is one fact within that account.

The asymmetry of inheritance should therefore remain visible after administrative consolidation. A merged entity can have one current governance structure and several distinct historical sources. Describing those sources precisely supports accountability, scholarship, and future revision. It also preserves the generative insight that a later collective can arise through several relations whose importance cannot be reduced to one predecessor label.

17.5 Merger Reversal and Subsequent Division

Suppose M later divides into B and C. A temptation is to identify the new branches with the original predecessors because their names or activities resemble them. The intervening history can complicate that claim. Training, shared work, new commitments, and changed membership during the merger may have transformed both branches. The apparent reversal of a diagram need not reverse the historical process.

The inquiry should therefore reconstruct the complete path. One branch may recover an old registration while inheriting a mixed operation. Another may continue a practice created jointly during the merged period. Endpoint similarity alone cannot determine which earlier entity, if any, is reidentified. A preserved lineage permits these possibilities to be examined without forcing a return to the original partition.

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Table 6: Constructed membership and inventory transmission through merger and division

Appearance	Members from A	Members from D	Kits from A	Kits from D
A at time 0	60	0	12	0
D at time 0	0	40	0	8
M at time 1	60	40	12	8
B at time 2	50	20	10	5
C at time 2	10	20	2	3

17.6 A Complete Merger and Division Dossier

A sustained case allows the different inheritance questions to be tested against the same history. The following dossier fixes memberships, inventory, and a sequence of collective appearances, then leaves reidentification open. Every quantity and event is stipulated for analysis. The case represents a possible arrangement within the model and supplies no empirical finding about a particular association.

At the initial time, A and D are distinct organizations with disjoint memberships. A has sixty members and twelve individually tagged recording kits; D has forty members and eight kits of the same standardized kind. At the middle time, they have formed M, represented as one organized collective at the declared resolution. At the final time, M has divided into distinct organizations B and C. Each letter denotes an appearance at its stated time, so the domain contains five appearances in three slices.

The interval contains no entry or departure of members. Each person belongs to exactly one represented collective at each slice. Every kit is also accounted for: there is no purchase, disposal, loss, or duplication during the interval. These restrictive assumptions make the ledger in Table 6 exact. Later applications would need to record the additional routes rather than infer their absence from missing entries.

A also contributes three experienced administrators, while D contributes four trained facilitators. During the merged period, members develop a joint procedure for preparing and interpreting recordings. At the later division, two administrators and two facilitators join B; one administrator and two facilitators join C. Both organizations receive copies of the procedure manual. Their ability to use it successfully remains a further question, since possession of the document and the presence of named roles leave the organization of their activity incompletely specified.

The dossier distinguishes three kinds of persistence from the outset. Individually tagged objects can be traced through physical transfers. Members can be linked across rosters under the stipulated personal-reference assumption. Collective identity concerns a partition of the five organizational appearances. Accurate links in the first two domains give evidence relevant to the third while leaving its criterion to be defended. This prevents the precision of the numerical ledger from deciding a philosophical question through an unnoticed change of subject.

17.7 Membership Continuity and the Meaning of Integration

The membership ledger permits several exact comparisons, each with a different denominator. Examining them together shows why a statement about the dominant predecessor needs an explicit question. The same counts can support distinct descriptions of retention, composition, and participation in the merged organization.

B receives fifty of A's sixty original members and twenty of D's forty. It therefore retains five sixths of the former A membership and one half of the former D membership. Within B itself, five sevenths of the seventy members formerly belonged to A. These fractions answer different questions. The first two concern the destination of a predecessor's members; the third concerns the source composition of a successor. Their numerical precision makes the difference easier to state, while a phrase such as substantial membership continuity can conceal it.

C has ten former members of A and twenty of D. Its majority comes from D, yet B and C receive

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equal numbers of former D members. A rule that selects the destination containing the greatest number of a predecessor's members therefore ties for D. A rule that selects the successor whose membership is most concentrated in former D members favors C. Neither calculation is erroneous. Their disagreement exposes two different meanings of a membership-based continuation criterion.

The inquiry must explain which meaning serves its object. A study of where former participants continued their association may care about destination counts. A study of the social composition of an organization may care about within-successor proportions. A procedure conferring a particular institutional status may specify one measure for its own purpose. The appropriateness of that procedure and the historical use of its output remain separate questions. Calling one proportion the natural measure would avoid the very argument that the case requires.

Membership provenance also leaves the quality of integration unresolved. During the merged period, the original memberships might continue to work largely in separate teams. They might form mixed teams around a jointly revised practice. They might share an administrative office while retaining independent deliberation. The fixed roster totals are compatible with these contrasting organizational patterns. An account of integration therefore needs evidence about actual activity and governance in addition to a common membership heading.

This observation has a consequence for the appearance domain. The dossier stipulates a single collective M at the chosen middle resolution. A real inquiry would have to justify that representation. Evidence of separately organized subunits could motivate a more detailed domain, especially if the continuity question concerns those subunits. Two rosters preserved under one office would be relevant evidence, but the rosters alone would still leave their organizing relations to be examined. The normalization problem in Section 4 precedes the identity consequences of choosing either description.

The dossier thus supports a positive use of membership evidence. It can establish exact transmission statements, discriminate between measures, and direct inquiry toward the form of integration. Its role becomes weaker when one silently converts those statements into a complete identity criterion. The additional burden is to explain why the selected membership relation should determine the collective reidentification at issue.

17.8 Inventory Lineage and Claims of Restoration

The conserved inventory provides a simpler domain in which to examine the language of return. Individual tags make the transfer path inspectable, and the common unit makes the arithmetic meaningful. The analysis then asks which further historical claims that exact ledger can support.

Of A's twelve kits, ten reach B and two reach C. Of D's eight, five reach B and three reach C. The final stocks are consequently fifteen kits in B and five in C. Relative to each predecessor's stock, the shares are $5/6$ and $1/6$ for A, and $5/8$ and $3/8$ for D. Each row of the predecessor allocation sums to one, satisfying Equation (42) with zero residual under the stipulated complete transfer boundary.

Suppose B later adopts A's former name and C adopts D's. Calling the division a restoration of the inventory would then be inaccurate if restoration means returning each earlier stock to the correspondingly named body. B holds several former D kits, and C holds several former A kits. The current names leave those source relations unchanged. A later exchange could restore the original distribution of tagged objects, but that additional action would need its own record.

Even an exact return of every original kit would establish a limited result. The equipment might have been used in the intervening joint practice; the members might have developed new working relations; and the current organizations might have accepted new undertakings. Restoration in the inventory domain therefore coexists with change in other domains. The researcher can state the exact material return without expanding it into a return of the entire earlier social configuration.

An inquiry into control or ownership would require further premises. A tag shows which physical object is being tracked. A custody record shows who held it at a time. An authorized transfer can establish a distinct institutional relation under its applicable rules. The quantitative ledger does not itself decide whether the current holder acquired an entitlement to keep the object or whether an earlier condition requires its return. Those questions remain open even when the object's path is completely known.

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Copyable material gives another useful contrast. Both successors can possess the full manual developed during the merged period. The two copies need no division into complementary fractions, because their possession is compatible with the continued possession of the original. A ledger suited to exclusive kits and a record suited to copied information therefore preserve different relations. Treating the second through the first would create an artificial conflict; treating the first through the second would conceal exclusive control and possible competing claims.

The practical benefit of separate ledgers is cumulative. The inventory record can answer where tagged resources went. The information record can answer which versions were available to whom. An institutional record can address permissions and accepted conditions. Their combination supports a richer reconstruction, provided the conclusion retains the contribution and limitations of each source.

17.9 Inherited Practices and Work Produced Jointly

The merged organization can transmit older practices while producing a new practice through their combination. This subsection examines that difference using the dossier's administrators, facilitators, and shared procedure. It asks what evidence would distinguish preservation, adoption, joint development, and a later reconstruction.

Suppose D's facilitators already used a method for preparing interviews, while A's administrators organized access and scheduling. In M, these activities become connected through a new review process in which interview preparation, access decisions, and interpretation inform one another. The result can depend on contributions from both predecessors. Describing it as the continuation of D's interview method captures one relation while leaving the jointly organized review process incompletely described.

The case requires a sufficiently specific account of the practice. If the investigator defines the activity simply as producing recordings, both the earlier and later arrangements may satisfy the description. A description of how recordings are prepared, interpreted, and made available can reveal consequential changes. The appropriate resolution depends on the question. Broad functional similarity may be adequate for counting one kind of output; transmission of a particular procedure requires attention to its steps and their organization.

The source of the later practice also remains an evidential question. Attendance at a training meeting can show an opportunity for transmission. Possession of a manual can show access to a specified version. A report of using the method can supply testimony about uptake. These observations have different strengths, and their conjunction still requires an argument about actual activity. In the constructed dossier, the joint development is stipulated. A documentary application would need to reconstruct it through records capable of distinguishing the relevant alternatives.

One alternative is independent reproduction. A successor might produce similar outputs through a procedure learned elsewhere. Another is partial adoption: it might retain the vocabulary of the manual while changing a step important to the original practice. A third is active adaptation, where participants understand the earlier method and deliberately revise it for a new setting. Similarity at the endpoint leaves these routes open. The route matters when the claim concerns inherited knowledge, historical contribution, or the grounds for treating a practice as continuous.

The later division creates a further distinction. B and C each receive two facilitators and a copy of the manual, but their administrative teams have different sizes. Equal facilitator counts establish neither equal capacity nor equivalent organization. The relevant work might depend on coordination, available time, complementary skills, and access to material. An operational comparison should identify those conditions before projecting the merged organization's performance onto either branch.

This point also constrains the interpretation of failure. If one branch struggles after division, the result may concern missing complementary conditions rather than a complete loss of inherited knowledge. If it performs well, the success may depend on a new arrangement rather than faithful reproduction of every inherited procedure. An explanation can therefore preserve both continuity and transformation by identifying which parts were transmitted and which relations were reorganized.

The jointly developed procedure introduces a historical object whose origin is the merged period itself. Even if a later identity policy treats M as continuing A, the procedure's production can involve

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both former memberships. The identity classification leaves the actual contribution history intact. A public account that attributes the whole procedure to A merely because that identity was selected would need an additional argument connecting institutional authorship to the finer history of work.

The generative relational description earns its place here by directing attention to that organization of contributions. The later practice is examined through the relations that produce and sustain it, including new relations formed during the merger. The account's explanatory success depends on how specifically it identifies those mechanisms. Attaching the word generative to a mixed roster would leave the important part of the explanation unfinished.

17.10 Predecessor Identity through the Merged Interval

The five-appearance domain permits a direct test of the claim that the later division restores both predecessors. The test separates a gap-permitting reidentification from a continuous trace through every intermediate slice. Its conclusion follows from the previously established identity constraints under an additional stated condition.

Let the slices be $I_0 = \{A, D\}$, $I_1 = \{M\}$, and $I_2 = \{B, C\}$. An admissible identity is an equivalence E respecting the distinction between different appearances in the same slice. The endpoint return claim is AEB together with DEC. Consider first the model that permits gaps between appearances of an entity. The partition $\{A, B\}; \{D, C\}; \{M\}$ satisfies that endpoint claim and slice separation. It describes two identities reidentified across the middle period and a separately identified M.

Other partitions support the same endpoint claims while assigning M differently. The partition $\{A, M, B\}; \{D, C\}$ places M on the first identity trace. The partition $\{A, B\}; \{D, M, C\}$ places it on the second. Each requires at least one predecessor trace to have a gap at time 1. The examples are candidate descriptions, with no probability assigned to them and no claim that the list exhausts every admissible interpretation.

Now add the contiguity requirement used in the converse part of Section 15: whenever an identity class meets two slices, it meets each intermediate slice. Under this requirement, the two endpoint returns become jointly impossible in the fixed domain. The class containing A and B must contain an appearance at time 1. Since M is the only appearance there, it must contain M. The same argument puts M in the class containing D and C. Symmetry and transitivity then identify A with D, contradicting their stipulated simultaneous distinction.

The argument exposes the exact incompatible conjunction. It combines both endpoint returns, a single middle appearance, slice-separated identity, and contiguous traces. Removing the contiguity condition admits the previously displayed partition. Revising the domain to include independently organized middle subunits can change the available paths. Weakening the endpoint claims to directed lineage can preserve both predecessor connections without identifying either with M. Each response changes a specific commitment and should be recorded accordingly.

The single-middle-appearance premise deserves special attention in a documentary study. A common registration, office, or publication heading can provide evidence for one administrative description while leaving operational subunits distinguishable. The researcher should establish the relevant organizational resolution before applying the obstruction. Adding two records that merely describe M differently would leave the unit domain unchanged; identifying two actual organized subunits would raise a different normalization question.

The contiguity premise also has a substantive interpretation. Some inquiries allow an entity to remain identifiable through dormant periods in which its ordinary activities or appearances are absent. Others restrict a trace to connected represented activity at each slice. The present case gives conditional consequences of those choices. It supplies no general verdict about whether dormancy preserves a collective or whether reconstitution restores it. Those philosophical and institutional questions require the criteria discussed elsewhere in the paper.

An endpoint name match therefore understates the content of a return claim. The researcher needs to say whether the claim concerns a historical affiliation, a restored institutional status, a gap-permitting identity, or a contiguous organizational trace. The same phrase can be defensible under one interpretation

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and inconsistent under another. Specifying the claim protects the valuable historical relation from being either inflated or discarded because a stronger interpretation fails.

17.11 Registration, Representation and Separate Consequences

Institutional decisions can settle particular statuses within the merger history. Their practical force depends on the authority and scope of the decision. This subsection examines how such decisions enter the dossier without becoming a universal description of its past.

Suppose a competent registering body recognizes M under a new registration, then later authorizes B to use A's former designation. Under the stipulated procedure, the second act can settle the current use of that designation. The act's historical interpretation depends on what the procedure actually confers. It may permit a name, restore an office, or assign a specified continuity status. Those are distinct possibilities, and the research record should preserve the operative wording and conditions.

If the rule expressly confers continuity for its registration system, the result has a defined institutional target. It can guide dealings conducted under that system. Transferring it to an operational or historical identity inquiry requires an argument about the relation between the targets. Such transfer may be justified in a particular case, but its justification cannot consist solely in repeating that the decision was official.

The same distinction protects the authority of the registering body. A historian can retain uncertainty about the organization of activity while accepting that the stated registration act took effect. The historical uncertainty leaves the act's specified institutional consequence intact. Conversely, a source can document a group's effective activity while leaving the competence of a claimed registering authority unresolved. Evidence for operation and evidence for authorized status can therefore develop independently.

Representation raises another question. B may claim to speak for the entire former A membership because it receives most of those members. The dossier shows that ten former A members belong to C. A majority destination supplies a numerical basis for describing B's composition; it supplies a representative mandate only if an independently justified procedure connects the two. Former members' participation, consent, and the scope of an appointed role can matter to that further question.

A shared commemorative event could also include both B and C without settling their disputed identities. Participants can recognize different routes from the earlier work and cooperate in preserving its record. Whether one body has a particular institutional mandate remains open to the procedure governing that mandate. Cooperative remembrance and exclusive representation need different warrants, even when they appear in the same public announcement.

The researcher's task is to locate each consequence at the level where its grounds operate. A name authorization belongs in the status record. A roster belongs in the membership record. A representative appointment belongs in the authority record. Linking these records allows the connections to be assessed while preserving which additional rule supports each move from evidence to conclusion.

17.12 Undertaking Histories through a Common Office

The merged office can hold commitments originating at different times and through different actors. A precise undertaking history distinguishes predecessor acceptance, transmission during the merger, a new joint undertaking, and a later successor's current response. The dossier provides a setting in which these distinctions change what an investigator should seek.

Suppose A accepted material for a limited archive, while D promised participants a review session before a specified use of their recordings. During the merger, M publicly announces that it will carry those arrangements forward. Whether that announcement validly assumes the undertakings depends on the actors' authority, the terms of acceptance, and the position of the parties to whom the arrangements matter. The announcement's occurrence can be established separately from that normative and institutional assessment.

The common office can make performance easier by coordinating resources previously held separately. It can also obscure differences in the undertakings' scope. An access condition attached to one

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contribution need not govern every item in the merged archive. A promise concerning one review session need not create an unlimited duty to maintain the whole programme. A reliable record should preserve the object, parties, content, and applicable conditions of each undertaking through the administrative consolidation.

Now suppose the joint procedure created in M produces a further undertaking to return a set of annotated recordings. Its formation belongs to the merged period. After division, B holds some of the materials, C holds others, and each has members involved in the earlier work. This arrangement creates practical questions about performance. It also shows why assigning every commitment to a single predecessor by identity would erase part of the formation history.

The analysis should first establish which actor accepted the new undertaking under the stipulated organizational procedure. It should then identify the material and capacities needed for the promised response. Finally, it should state the grounds on which a current party is expected or required to act. An accepted transfer, a continuing role, present custody, or an independently defended normative principle can supply different possible grounds. Their actual force requires argument at that level.

The later account of historical action and current claimants in Section 26 provides the relevant separation. An identity choice can change which current claimant is associated with M's historical action. It leaves the historical actor attribution and the practical distribution of materials as separately recorded facts. The additional response relation needs its own warrant. Consequently, disagreement about which branch continues M need not prevent every bounded practical arrangement concerning the material.

For example, the parties might agree that each present custodian will prepare the material it holds for an authorized transfer, while leaving the broader identity dispute unresolved. The justification would depend on legitimate custody, agreed access conditions, feasible work, and competent acceptance of the arrangement. It would support that specified response. The agreement need not express a judgment that the custodians share responsibility for every act in M's history.

This possibility also has limits. Material held by one branch may require knowledge available only in the other. A proposed division of tasks may burden someone who never accepted the new role. A beneficiary may reasonably contest a transfer that changes the promised access conditions. The investigator should preserve these counterconditions rather than treat an administratively convenient allocation as automatically justified. The ability to coordinate a response remains different from the authority to impose it.

The undertaking record therefore contributes to the identity inquiry without becoming its substitute. It can reveal continuing relationships that a termination label would otherwise conceal. It can also expose new relationships that a continuity label would incorrectly attribute to a predecessor. These findings make the history more exact and the current practical question more tractable.

17.13 Evidence for a Claimed Return

A later claim of restoration should generate a discriminating evidence plan. This subsection identifies what observations could change the interpretation of the dossier and which questions would remain open even after those observations were secured. The plan begins with the particular meaning of return under investigation.

For a membership claim, the relevant evidence includes the initial and later rosters, a justified method of linking individuals, and the conditions of affiliation. The constructed counts assume disjoint membership and complete retention. An actual finding of dual affiliation, departure, or new recruitment would require a revised ledger. It could change both the numerical comparison and the interpretation of integration. The investigator should preserve the original denominator and record why the revised one is appropriate.

For a material-return claim, tags and transfer records can establish which objects moved. Missing tags create a reference problem; missing transfer records create a historical-route problem. These can require different inquiries. A current inventory may locate an object while leaving its intermediate custodian unknown. Conversely, a transfer receipt may document an earlier movement while leaving the present

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location unresolved. A complete restoration claim should identify which of these questions its evidence answers.

For a practice claim, versions of manuals, records of training, descriptions of actual procedures, and accounts of particular work episodes can discriminate between transmission and resemblance. They should be compared with serious rival routes, including learning from another source or independently developing a similar method. A claim about the organization's routine practice requires more than one specially selected demonstration. The sampling and interpretation of episodes therefore become part of the research design.

For a status claim, the evidence should identify the operative decision, its issuer, the authority invoked, the time of effect, and the specified consequence. A commemorative statement can be evidence of public affiliation while leaving the operative decision elsewhere. A later certificate can report an earlier status or create a current one. The document's date alone does not determine which role it has. The temporal analysis of authority supplies the needed distinctions.

The most consequential evidence may concern the middle slice. If the accepted domain contains only M, the contiguous-return obstruction applies. If records establish two independently organized subunits relevant to the target, the normalization should be reconsidered. The additional appearances need to be justified through their organization, rather than introduced solely to obtain a preferred identity verdict. A useful study should state what evidence would support that change and what evidence would leave the single-unit description preferable.

Even complete reconstruction can leave criterion disagreement intact. Two investigators may agree on the rosters, transfers, procedures, and authorized acts while disagreeing about whether dormant identity is admissible or which institutional status should guide the historical classification. Further archive work would then have a different value from further philosophical argument. A report should locate the remaining disagreement so that effort addresses the unresolved premise.

The evidence plan consequently avoids a total-information ideal. It seeks information capable of changing the declared conclusion. If the question concerns the current location of an entrusted kit, the entire collective identity dispute may be unnecessary. If it concerns two continuous predecessor traces, the middle organizational domain becomes central. Selecting evidence in this way connects the formal query analysis with a proportionate documentary practice.

17.14 A Revised Historical Account after Division

The completed dossier supports a historical account with several coordinated findings. This final discussion shows how those findings can remain readable without compressing their different grounds into a single successor label. It also specifies what a later revision would change.

The membership finding records that both B and C contain people from both predecessors, with different concentrations and retention fractions. The inventory finding records an exact mixed allocation of exclusive objects. The practice account distinguishes inherited procedures from work developed during the merged period. The undertaking history preserves the formation and possible transfer of particular commitments. Each finding has a specified object and an evidential route that can be reviewed independently.

The identity finding can then state the criterion and its consequence. If the study accepts contiguous traces over the fixed five-appearance domain, it must reject the conjunction of both predecessor returns. If it allows identity across gaps, the endpoint claims remain mathematically compatible, with further criteria needed to select among candidates. If the appearance domain is revised, the finding must be reevaluated on that revised domain. These are explicit conditional conclusions rather than competing unqualified slogans.

Suppose new records later show that D retained a separately organized deliberative body during the merged period. That discovery could materially change the unit analysis if the body falls within the inquiry's chosen scope. It would leave the stipulated physical transfer paths unchanged unless those records also challenge them. The revision should therefore identify which domain and inference changed. A change in collective normalization need not require rewriting every other part of the historical account.

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Alternatively, suppose the new evidence establishes only that former D members met informally from time to time. Whether those meetings support a distinct collective appearance requires an argument about the organizing relations at issue. The presence of a familiar name or social connection can be relevant without settling that question. Preserving the weaker observation protects it from either premature promotion or dismissal.

The same discipline applies to public narratives after division. A branch can accurately acknowledge that its members and practices descend partly from an earlier organization while also recognizing what was created jointly in M. It can report an authorized current designation while preserving another branch's contributions. Such an account need not choose between a completely discontinuous history and exclusive ownership of a predecessor's past. Its precision lies in the named relations and supported consequences.

The merger case therefore makes a bounded contribution to collective reidentification. It demonstrates how exact ledgers, explicit unit construction, conditional identity constraints, and independently warranted practical arrangements can cooperate in one inquiry. The later division changes the organization again; its appearance of return becomes a question to investigate through the intervening history. The resulting account preserves the transmission and new formation that the simpler reversal diagram leaves unresolved.

18 Suspension and Reconstitution

This section examines continuities that include a period without observed activity. It distinguishes dormant organization, missing observation, termination, and later reconstitution. It then considers temporal resolution and the information lost when a history is reduced to matching endpoints. These cases test the contiguous-path construction and reveal where a richer record is needed. A common-archive dossier then separates a justified narrow absence finding from unresolved questions about organization and private activity. The final discussion follows maintained procedures, changing capacity, and the first returning event through that evidential distinction.

18.1 Different Meanings of an Inactive Interval

An interval without recorded activity can have several explanations. The organization may remain constituted while suspending its programme. Its members may continue informal coordination that the available records omit. It may cease to operate and later be reconstituted by some of the same participants. The archive may simply be incomplete. These possibilities can produce the same visible gap while supporting different continuity judgments.

The initial observation should therefore be stated narrowly: no activity of a specified type was found in a specified record source over an interval. To infer inactivity, the investigator needs reasons to expect that the source would record the activity if it occurred. To infer termination, a further criterion is required. The absence of one kind of operation may be compatible with continuing roles, commitments, or authority. Each inference adds a premise that can be tested or challenged.

The meaning of dormancy also depends on the organization. A collective created for an annual event can have long intervals between performances. A standing committee may continue through intervals without a meeting. An informal project group may dissolve when coordination ceases. These are possible organizational arrangements, not empirical generalizations about particular institutions. They show why a universal inactivity duration would need an argument that the present model does not supply.

18.2 Latent Appearances and Gap Relations

The adjacent matching construction connects appearances in consecutive slices. A continuity claim across an empty interval can be represented in two principal ways. One can introduce latent appearances supported by evidence of continuing organization, or one can allow a typed gap relation between the last observed and first returning appearances. The choice depends on what the evidence supports.

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A latent appearance should not be inserted merely to make a desired path continuous. Its role is to represent a warranted claim that the collective continued during the interval under the selected criterion. Evidence could concern maintained authority, ongoing obligations, retained coordination, or another specified condition. The record should label the appearance’s evidential basis and distinguish it from a directly observed activity.

A gap relation can express a more cautious judgment. It links two observed appearances under an asserted continuity while leaving the intervening manifestation unresolved. Its semantics should state whether the claim is one of continuing registration, uninterrupted organization, historical reconstitution, or another relation. A generic edge marked “same” would conceal precisely the distinction the gap makes important.

Allowing edges across several times changes the simple proof in Section 15. Degree bounds can still prevent branching in the selected graph, but the resulting class may have non-contiguous support. The converse characterization through adjacent pairs no longer applies. This is a controlled extension of the model: the assumptions are changed explicitly, and the properties required by the new interpretation must be checked again.

18.3 Reappearance and Reconstitution

Suppose an association stops meeting, then several former members form a body with the same name and activity five years later. Membership overlap, name continuity, and practical resemblance support different observations. They leave open whether the body persisted through the interval or was reconstituted. A record of a decision to dissolve, a maintained governing procedure, or an explicit later foundation can discriminate some of these possibilities, depending on the criterion.

Reconstitution can itself be a significant continuity relation. The later body may deliberately recover an earlier practice, use its archive, or accept its unfinished project. Treating it as a new identity under a particular model need not diminish these connections. The historical question can ask which earlier resources and commitments made the reconstitution possible. That question often preserves more insight than an exclusive contest between complete identity and complete discontinuity.

An identical membership set at the endpoints is also insufficient to settle the intervening history. The same people can disband one organization and later create another. Alternatively, they can maintain a dormant organization throughout. If the criterion distinguishes these histories, endpoint membership is an insufficient record in the precise sense of Proposition 13.2. An observation scheme that retains only the two rosters places opposing verdicts in one fibre.

18.4 Temporal Coarsening and Hidden Branches

Temporal coarsening can conceal more than inactivity. Suppose a collective splits in March and merges again in November. Annual records taken in January can show one organization before and after the episode. A yearly path appears unbranched, while a monthly reconstruction contains distinct simultaneous branches. An identity policy that uses only annual slices may therefore omit a conflict that becomes visible at a finer resolution.

This observation does not imply that every inquiry should record every instant. The appropriate resolution depends on the question. An annual financial series may legitimately describe one reporting entity under a specified accounting rule. A history of internal commitments during the split needs finer distinctions. The model should state which events its resolution can represent and which it intentionally leaves outside the current analysis.

A useful sensitivity check reconstructs the same episode under at least two defensible resolutions. If the identity verdict changes, the investigator should locate the decisive hidden appearance or transition. The difference may reveal an inadequacy in the coarser record or a legitimate difference between inquiries. The result should be reported with that explanation, rather than averaged into one apparently resolution-independent identity.

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Table 7: Constructed histories compatible with a common blank public-event register

History	Organization during the interval	Programme activity	Later transition
Dormancy	Coordinating roles and procedure remain	Suspended	Programme resumes
Termination	Earlier organization ends	Absent	A later body is formed
Private activity	Coordination continues	Private events occur	Programme becomes public

18.5 Persistence, Capacity, and the Possibility of Return

A dormant organization can retain the capacity to resume activity. That capacity can depend on members, authority, materials, access, and shared expectations. Retaining a feasible route to future activity may be relevant to a continuity criterion. It remains distinct from actually exercising the capacity and from being the same organization under every criterion.

The availability of a return route is also conditional. A group may be able to resume only if a venue remains accessible or a custodian releases an archive. A former practice may be reproducible by several distinct groups. Such possibilities can support an analysis of latent collective capacity without selecting a unique bearer of identity. The inquiry should preserve the conditions and alternative routes through which the practice could reappear.

These cases deepen the original continuity insight. Causal efficacy can persist through records, training, expectations, and resources after an organization’s identity has ended under a chosen criterion. Conversely, an identity can remain recognized while the route to renewed operation becomes unavailable. A serious account of collective continuity needs room for both possibilities. The appearance, lineage, identity, and capacity records give them different places in the model.

18.6 A Shared Archive with Three Different Histories

A single observation record can support a definite conclusion about one kind of inactivity while leaving broader continuity questions open. The following constructed dossier makes that difference explicit. It fixes the retained archive and compares three histories under one declared organizational criterion.

The archive contains an initial membership roster, an earlier procedure document, a register of publicly advertised programme events covering a five-year interval, and a later roster accompanied by a public description of renewed activity. The two endpoint memberships are identical. The event register has no entries during the interval. Assume, favorably, that it completely records publicly advertised programme events in that interval. Its scope excludes internal coordination, private programme events, termination acts, and the continuing applicability of the procedure document.

Table 7 gives three histories compatible with those observations. In the dormant history, accepted coordinating roles and the relevant procedure remain in force while the programme pauses. In the termination history, the organization ends and a later body recovers the name and activity. In the privately active history, the organization continues to conduct programme work that it leaves outside the public event channel. At the final time, each history produces the same retained roster and public description.

The fixed continuity criterion in this comparison concerns maintenance of the relevant coordinating organization. Under the stipulated facts, it classifies the first and third histories as continuous and the second as discontinuous. Its philosophical adequacy is an independent premise of the comparison. The dossier tests recovery of the answer under that criterion; it leaves other persistence criteria available for separate assessment.

The same archive therefore leaves two queries unresolved. Organization continues in some compatible histories and ends in another. Programme activity occurs in one and is absent in the others. Both answers vary within one observation fibre, so the recovery result of Proposition 13.2 applies. By contrast,

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publicly advertised programme activity is absent in every history under the completeness assumption. That narrower answer is constant on the fibre.

This positive result matters. Evidential caution can preserve a well supported absence finding while resisting an enlarged conclusion. The blank register accurately describes its specified public domain. The mistake would be to treat that domain as exhausting the organization’s activity or conditions of persistence. The case thus locates the inferential boundary without requiring the investigator to abandon every conclusion drawn from an incomplete archive.

18.7 The Warrant of a Positive Absence Finding

An absence finding depends on what the observation process could have detected. This subsection examines that warrant by varying the constructed register’s scope and completeness. It separates missing entries, missing events, and missing organizational relations.

The favorable assumption in the dossier states that every relevant advertised programme event would appear in the retained register. Combined with an empty register, that premise excludes such events within the specified interval. The premise must concern the actual recording process. A document’s title, such as complete programme, would supply a claim about completeness that requires evaluation alongside its production and preservation history.

A register may cover one venue, one organizing team, or one channel of public communication. An event elsewhere could then be absent without violating the register’s actual function. Equally, a record could have been complete when produced and become incomplete through later loss. These possibilities concern different stages of the observation route. A useful archive description identifies the intended coverage, the recording practice, and the retained material.

Suppose a missing folder is discovered to have contained event announcements from part of the interval. That discovery weakens the completeness premise. Additional histories with omitted public events may become admissible, and the earlier narrow absence finding may require revision. The change concerns evidence about the observation map. It leaves the conditional reasoning from a complete empty register intact while changing whether its premise is warranted in the actual inquiry.

The distinction between an announcement and a completed event introduces another target. A programme register might list planned activities that were later cancelled. Its positive entries would then require a completion bridge before supporting a count of performed activities. Conversely, the absence of announcements could coexist with a private event. A study should name whether it is reconstructing plans, public offers, completed programmes, or internal coordination. Their relations can be investigated without defining them as identical.

Time boundaries also matter. A decision made before the interval can authorize activity within it, while a record produced afterward can describe an earlier event. Sorting records solely by publication date can therefore move relevant evidence outside the apparent window. The event time, the time of recording, and the period of applicability should be represented according to the query. This care is particularly useful when a later account describes a long period in one retrospective paragraph.

The resulting absence statement can be exact and readable: within the justified coverage of the specified record, the defined public programme is absent during the interval. Whether the organization continued privately, remained dormant, or ended is then assessed through additional evidence and a declared criterion. Such a statement preserves what the archive establishes and identifies the work required for a stronger conclusion.

18.8 Preserved Procedures and Their Continuing Application

A retained constitution, manual, or appointment record can outlast the organization that once used it. Its presence is therefore relevant to continuity while leaving continuing application open. This subsection identifies the additional relations that would make the preserved procedure informative about the dormant period.

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In the common-archive dossier, all three histories contain the same earlier procedure document. In the dormant and privately active histories, it remains connected to accepted coordinating roles. In the termination history, it survives as a record of an earlier arrangement. Reading the words of the document can establish what procedure was described. Establishing that the procedure remained applicable requires evidence concerning the organization and the relevant acts during the interval.

The additional evidence can take several forms. Participants may continue to recognize an appointed coordinator within a specified scope. A competent act may renew an appointment. A retained process may govern the admission of members or the handling of entrusted material. A record of these activities can support a claim of continuing organization, provided its accuracy and institutional interpretation are warranted. An authentic document's existence remains distinct from the truth and applicability of its content.

The procedure can also remain operative without producing the public programme it was designed to organize. A group may reserve a venue, maintain a contact route, or continue limited decisions about future work. These actions can preserve some organizing relations during a pause. Whether they suffice for the selected collective criterion depends on what that criterion requires. The investigator should identify the role of each relation rather than treat every surviving administrative trace as sufficient.

A competing interpretation may describe the same actions as stewardship after termination. A custodian can preserve an archive and answer inquiries because of a limited undertaking, even when the earlier collective has ended. That work can be meaningful and well warranted. Its importance supplies a reason to record it carefully, while the claim that the collective itself persisted requires a further link between stewardship and organizational continuity.

The contrast helps discriminate the histories. Evidence that participants continued to make decisions through the applicable collective procedure differs from evidence that one former member kept the papers. A later account of the archive may conflate them because both involve preservation. The research record should retain who acted, the role invoked, the decision's object, and its scope. Those details make a continuity inference open to criticism.

This inquiry can remain bounded. It need not derive the authority of every background institution from an ultimate beginning. It can state which procedures are accepted for the present question and examine whether they continued to operate in the relevant way. The fuller analysis of authority grounding addresses disputes about those accepted premises. Here the task is to distinguish a surviving description from the organizational relation it would need to describe during the gap.

18.9 Maintained Capacity and the Conditions of Resumption

The ability to resume a programme depends on several conditions that can change independently of recognized identity. This subsection follows those conditions through a dormant interval and explains how their assessment differs from a prediction that the programme will return.

Suppose participants retain the manual and sufficient knowledge of the activity. The venue may nevertheless be unavailable, an essential piece of equipment may require repair, or a person with a necessary coordinating role may be unable to participate. These are distinct possible barriers. A finding that the method survives in the archive addresses the informational condition; it leaves the remaining practical conditions to be examined.

The capacity question should specify the proposed resumption. A one-time gathering can require less coordination than a recurring programme. A small event for former members can be feasible when an open public programme is beyond the group's resources. The proposition that the association can return is therefore incomplete until the intended activity, participants, time, and supporting arrangements are identified.

The assessment should also distinguish an available resource from authorized access to it. An archive may exist in a location that participants cannot presently use. Equipment may be held under an arrangement that limits its transfer. A volunteer may possess a relevant skill while declining the proposed role. A feasible plan must respect those conditions. Treating every physically available resource as freely deployable would build an unsupported authority assumption into the capacity finding.

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Support can change the feasible route. A current custodian may agree to provide limited access, a new participant may accept coordination, or a different venue may support a revised version of the activity. These changes can make resumption possible while changing its organization. The resulting route should be described as the arrangement actually available, with its own conditions and consequences for the continuity question.

Capacity and current intention also differ. Participants can possess the means to resume while deciding that the earlier programme has completed its purpose. They can strongly wish to resume while lacking an indispensable condition. Their reports are relevant evidence about intention and perceived barriers; they become a capacity finding through a further assessment of the proposed activity and available means.

The capacity analysis supplies a practical complement to reidentification. It can show what a dormant organization's recognized standing currently enables, or what a later reconstitution would require after termination. It can also identify a continuing practice whose means are available to several distinct groups. In that case, explaining the possible return of the practice leaves selection of a unique collective identity open.

18.10 Change within a Dormant Organization

A pause in programme activity need not freeze the organization. This subsection examines changes that can occur during dormancy and their consequences for the interpretation of the returning appearance. The aim is to preserve temporal development within an interval that the public record describes as inactive.

Members can enter or leave, roles can be renewed, materials can be transferred, and the meaning of an earlier objective can be reconsidered. Some such changes may occur through the continuing procedure; others may challenge its application. The fact that the public programme remains paused leaves these possibilities open. An endpoint account should therefore avoid treating every later difference as a change that occurred only at resumption.

Consider an association that spends part of the interval reconsidering its method. Its next public event uses a revised format developed through that work. The event may be the first visible programme activity in several years, while its formation has an extended internal history. A narrative of sudden reconstitution would miss that history if it inferred the beginning of organization from the first public performance.

The opposite case is also possible. A later group may retrieve the old manual and reproduce the familiar public format with little of the former coordinating structure. Strong endpoint similarity can then coexist with a substantial organizational break. The same observed format should be evaluated through its route of production and the criterion being applied. Neither novelty of output nor familiarity of output settles collective identity on its own.

Dormancy can change the distribution of influence. A person who maintains the contact list or archive may acquire practical control over the route to resumption. That control can support valuable coordination while leaving its authority and scope open to assessment. The inquiry should identify which decisions the role permits and how other participants can contribute to or contest them. Possession of the route back is a consequential relation that deserves its own description.

Such changes also affect the value of older evidence. An early statement of purpose may remain relevant while leaving current uptake unresolved. An old roster may identify former members without establishing who now participates. A retained procedure may have been validly amended. The historical record should preserve the earlier state and the intervening revisions so that the returning appearance can be understood through its actual development.

The result is a more discriminating temporal description. Programme suspension names a relation to one type of activity. Organizational maintenance and internal transformation name other processes that may occur over the same period. Representing them separately allows a continuity claim to accommodate change without making change irrelevant to the criterion.

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18.11 The First Returning Event and the Identity Question

The first visible event after a gap often becomes the focal point of a continuity narrative. Its evidential role is narrower than the entire history it may be used to summarize. This subsection distinguishes what the event establishes from what its description invites an investigator to infer.

A successfully held event establishes activity at that occasion under the adopted event definition. It can also reveal current participants, procedures, and available resources. A description of the event as a resumption records a claim about its relation to earlier work. A description as a foundation records a different claim or institutional act. The occurrence of the event and the success of those descriptions require their own evidential assessment.

The event may be organized by a temporary team that intends to create a continuing association afterward. It may be produced by an already maintained body whose programme was suspended. It may bring together several groups that share an earlier history. These possibilities can produce a similar public occasion. The investigator should establish which organized unit the appearance represents before using it as the endpoint of an identity trace.

A current event can also fail to establish the capacity for ongoing activity. Exceptional effort or temporary access may make one occasion possible while leaving a recurring programme unsecured. That observation concerns the conditions of future operation. It should not erase the event that actually occurred or automatically determine whether the organizing collective has a continuing identity under the selected criterion.

Claims made at the event can create new practical relations. Participants may accept roles, undertake to preserve material, or agree to meet again. Those acts belong to the present formation history and should be recorded with their content and uptake. Their significance can coexist with unresolved questions about the gap. A later programme can be well organized without the investigator having settled every retrospective identity claim.

The first returning event therefore supplies a new observation and potentially new acts. Its narrative connection to the earlier association remains a target of inquiry. Placing both roles in the record allows the present activity to retain its practical significance while the historical interpretation continues to be assessed.

18.12 A Proportionate Account of the Unobserved Interval

The final report should connect each conclusion about the gap to the evidence and criterion that support it. This subsection shows how to do so without requiring an exhaustive reconstruction of every intervening event. The relevant depth follows the question the report is intended to answer.

For the common-archive dossier, the report can establish the absence of the defined advertised programme under its stated coverage assumption. It can describe the identical endpoint memberships and the surviving procedure document. It should then state that these observations leave both organizational continuity and private programme activity unresolved in the accepted history family. The unresolved findings have exact objects; they are more informative than a general declaration that the past is uncertain.

Further inquiry can be directed toward the decisive alternatives. A reliable record of private programme activity would challenge the inactivity description without by itself establishing every condition of the continuity criterion. An applicable termination act could discriminate the organizational histories. A record of continuing coordination could support a maintained-body interpretation while leaving a different criterion open to argument. Each finding should be integrated at the level it actually addresses.

The report should also preserve the cost and feasibility of further recovery. Some missing records may be unavailable, and some surviving testimony may support only a limited description. Where the unresolved distinction would not change the practical task, a bounded conclusion can be sufficient. Where it would determine a significant attribution or undertaking, the limitation may need to remain prominent. The degree of historical detail is justified through the consequence of the claim rather than through an ideal of collecting everything.

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New evidence can refine the history, while a new criterion can change the question. A clear report preserves both possibilities. It records the previous conclusion with its assumptions, identifies the material revision, and explains which later findings depend on it. This structure supports continued inquiry without retrospectively making the earlier archive appear more complete than it was.

The suspension case consequently preserves several forms of continuity across a gap: maintained organization, transmitted practice, retained capacity, and deliberate reconstitution. Their evidential and philosophical conditions remain distinct. A careful account can recognize the relation that survives while leaving a stronger identity judgment unresolved, and it can accept a termination finding while preserving the history through which later work becomes possible.

19 Comparative Histories and Discriminating Evidence

A continuity model becomes more informative when it can distinguish the mechanisms that produce superficially similar disputes. This section develops constructed comparisons in which the decisive difference lies in replacement processes, omitted appearances, dormant organization, concurrent collective arrangements, delegation, or asymmetric merger. Each comparison states the target and identifies evidence that could change the finding. A formal reduction result explains how accepted intermediate appearances can be removed from a report while preserving their implications for visible identity questions. The comparisons are analytic instruments for designing a study; their stipulated histories remain separate from the documentary illustration in Section 23.

19.1 Comparative Design and the Location of a Difference

A useful comparison changes an assumption that performs inferential work. Replacing an association's name while reproducing the same numerical profile adds little to an argument. Changing the mechanism through which participants, authority, or activities continue can expose a different limitation in the model. The present comparisons therefore hold selected observations fixed while varying a historically relevant relation.

The first step is to name the target at a level that the comparison can test. A target concerning a continuing decision procedure differs from one concerning the reproduction of a skilled practice. Their evidence can overlap because the same people may carry both. A comparison becomes discriminating when it identifies an event that preserves one relation and changes the other. It then becomes possible to explain a disagreement without treating every continuity criterion as an arbitrary preference.

The second step is to state which facts are held fixed. Two endpoint rosters can be identical even when the histories between them differ. Two organizations can issue indistinguishable public descriptions while their internal authorization relations differ. A controlled construction allows the analyst to isolate that difference. In an actual study, holding facts fixed is an evidential achievement and may remain only approximate. The constructed case establishes the logical possibility that a given record omits a decisive distinction.

The third step identifies a discriminating observation. Such an observation must concern the varied relation. Collecting another endpoint roster cannot reveal an omitted dissolution if the two candidate histories already agree on that roster. A meeting record can be decisive if the criterion gives the recorded act a specified role and the record warrants that it occurred. The observation's value comes from this connection between a historical difference and the target verdict.

The fourth step considers the cost of the distinction. A richer model requires additional records, interpretive work, and decisions about resolution. The appropriate response to a difficult distinction may be a more limited finding, such as established practice transmission with unresolved organizational persistence. The comparison should identify that attainable result. It should also state when a proposed extra distinction contributes little to the research question.

These four steps supply the structure of the cases below. Their common purpose is to turn a broad identity puzzle into a specific account of what the accepted observations establish, what they leave open,

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and which additional premise would change the answer.

19.2 Gradual Substitution and an Identical Final Roster

Consider an association initially coordinated by five participants. Over five intervals, each participant leaves and a replacement enters. Every departure and appointment occurs through an accepted procedure. During the transition, continuing participants train the newcomer, pass on unfinished tasks, and maintain the association's decision records. At the final time, every original participant has left. The roster has changed completely while a connected sequence of authorized and practical handover events links the initial and final appearances.

Now construct a second history with the same initial and final rosters. The initial participants dissolve their association. At a later date, the five final participants independently create a body with the same name and a similar programme. They may have read the earlier association's public material, but the construction stipulates a fresh procedure of constitution and a gap in organized coordination. The two histories can have identical endpoint counts, names, activities, and stated ambitions. Their intermediate organization differs.

A criterion based on an authorized succession of coordinating roles can distinguish these histories. In the first, each replacement belongs to a sequence that the criterion accepts. In the second, the relevant sequence ends and another begins. A practice-oriented inquiry may produce a more complex result. It can recognize trained transmission in the first history and documented reconstruction from public material in the second. Both may be causally connected to the earlier practice, although the mechanisms and the warranted organizational identity differ.

The comparison clarifies the role of gradualness. Small changes make it possible for particular relations to be maintained through each interval. Their small size alone supplies a weak reason. A sequence can consist entirely of individually modest changes while one change violates the criterion's required authorization or terminates the relevant procedure. Conversely, a large, explicitly authorized replacement can preserve the relation that a role-based criterion treats as decisive. The relevant question concerns what each transition carries, not merely how many participants it replaces.

The overlap calculation in Section 7 already shows that adjacent similarity can decline into endpoint dissimilarity under repeated replacement. The present comparison supplies a philosophical reason to examine the underlying path. If a criterion defines persistence through the continued exercise of a procedure, endpoint roster distance is an indicator with limited scope. It can describe the magnitude of member replacement while leaving procedural persistence intact. Treating that distance as a conclusive identity test would change the target.

The evidence design follows the distinction. An endpoint-only study needs appointment records, contemporaneous coordination records, and evidence about the status of the procedure during the intervening intervals. The most useful records concern the purported handovers. A complete list of the later participants' names adds less than a reliable account of how the first replacement entered and what authority remained for the next. The study should also test whether records described as appointments actually record completed acts, proposals, or retrospective narratives.

Several findings are possible. The records may warrant the full handover sequence. They may establish training transmission while leaving one appointment's status uncertain. They may reveal that participants continued the programme after the association's procedure had ended. Each finding separates a different continuity from a different interruption. The comparison therefore resists an all-or-nothing description of what the association passed on.

19.3 Intermediate Appearances in a Reduced Report

An investigator often knows more appearances than a reader needs to see in a particular report. A study of two current claimants may omit several intermediate committees from its main narrative while retaining their records in the archive. Reducing the displayed domain can be legitimate if the report preserves the

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implications of those intermediate appearances. A naive crop of the pair list can lose exactly the evidence that supports the visible conclusion.

Let I be the finite appearance domain and let P, N be the accepted positive and negative constraints. Required co-presence exclusions are included in N. Assume the family $\mathcal{A}(P, N)$ is nonempty and impose no further limit on the number of identities or the temporal support of their classes. Write $C = \mathrm{Eq}(P)$ for positive closure and let $V \subseteq I$ be the appearances retained in the report. Here "hidden" means omitted from this output. The underlying dossier still contains the appearances and the premises involving them.

Define the visible positive relation and the transported negative relation by

$$P_V = C \cap (V \times V), \tag{43}$$

$$N_V = \{(v, w) \in V^2 : \exists x \in [v]_C \; \exists y \in [w]_C, \; (x, y) \in N\}. \tag{44}$$

The first relation retains every accepted positive connection whose endpoints remain visible. The second retains every exclusion between the full positive components represented by those endpoints. An edge of N_V can therefore be supported by a negative observation involving two omitted appearances.

Proposition 19.1 (Exact reduction of the visible identity family)**.** *Under the stated assumptions, the restrictions of full admissible identities to V are exactly the equivalences on V that contain P_V and avoid N_V:*

$$\{E \cap (V \times V) : E \in \mathcal{A}(P, N)\} = \mathcal{A}_V(P_V, N_V). \tag{45}$$

Proof. The restriction of an admissible E is an equivalence and contains P_V, since $C \subseteq E$. If $(v, w) \in N_V$ and vEw, choose witnesses xCv, yCw, and xNy. Transitivity would imply xEy, contradicting admissibility. Every full restriction therefore belongs to the right-hand family.

For the converse, take an equivalence Q on V containing P_V and avoiding N_V. Every full C-component that meets V intersects a single Q-class, because its visible members are already related by P_V. Assign that component the label of its Q-class. Give each component disjoint from V a fresh label, distinct from the visible labels and from every other fresh label. Equality of component labels defines an equivalence E on I containing C, and its restriction to V is Q.

An N-edge within one C-component is excluded by compatibility of the full dossier. An N-edge between components with distinct labels is respected by construction. If an N-edge joined two distinct components with the same label, both components would meet V. Their visible representatives would then be Q-related while belonging to N_V, contradicting the choice of Q. Thus E avoids N and extends Q admissibly. This proves the reverse inclusion. □

The fresh labels in the extension proof perform substantive work. They permit hidden-only components to remain distinct without disturbing the visible choice. A cap on the number of identities removes that freedom. Likewise, a requirement that every identity class form an eligible contiguous path could reject one of the extensions constructed here. Equation (45) concerns the declared positive-and-negative constraint model.

19.4 Mediated Identity and Mediated Difference

Two small dossiers show the distinct losses caused by cropping a report. The first loses a positive path. Suppose appearances a, h, b occur at successive times and the accepted positive constraints identify a with h and h with b. A report showing only a and b may contain no original direct pair joining them. Positive closure nevertheless forces their identity. The summary P_V preserves this implication and can link it to the two recorded handovers that justify it.

The second dossier loses an exclusion. Suppose a continues to h, b continues from k, and h and k are warranted distinct simultaneous units. A report showing only a and b omits the moment at which their difference became visible. Their positive components still contain the exclusion hNk, so the transported

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relation N_V requires a and b to remain different. The later or earlier endpoints need not be contemporaneous for the full history to establish their distinction.

These results give a precise sense in which intermediate organization can matter after it disappears from a concise narrative. The summary's visible positive and negative pairs are derived claims. They should retain the path or component witness that supports them. A reader who challenges the normalization of h and k can then locate the premise on which the endpoint difference rests. Without that provenance, the summary risks presenting a historically mediated conclusion as a direct observation.

The same distinction matters when transferring findings into a database. A field linking two visible identifiers can record the final pair, but the field alone leaves the derivation opaque. A more informative record stores the criterion, the relevant intermediate appearances, and the constraint chain. Updating one accepted handover then becomes a reasoned revision of a dependent finding. It becomes possible to ask which published links survive the change instead of rebuilding the whole history from memory.

Successive exact reductions also have a useful property. If a report first retains V and a later display retains $W \subseteq V$, the family obtained by restricting twice equals the family obtained by restricting directly from I to W. This follows because set restriction composes. The property belongs to the complete preserved family. Selecting one representative partition or one highest score before the second reduction can discard alternatives and yield a different result.

The distinction between omitted output and absent evidence is essential to this use. If the original study never observed or warranted h, the reduction theorem supplies no premise involving it. An analyst cannot insert an intermediate committee simply because doing so would connect two endpoints. That proposal belongs to the candidate-history stage and requires its own evidence. Exact reduction conserves implications from an accepted dossier; it supplies no historical existence theorem.

19.5 A Class Limit and the Failure of a Local Summary

A numerical limit on identities changes the reduction problem. Consider three appearances a, h, b at distinct times, with negative constraints aNh and hNb and no positive links. The unrestricted model permits three different identities, and its visible summary on $\{a, b\}$ leaves their relation open. If the model also permits at most two identities in the whole dossier, a and b must belong to the same identity. The hidden appearance already requires the other class.

The example isolates the role of a global resource constraint. The two negative links impose no direct exclusion between the visible endpoints. Their implication changes because a third class is unavailable. A summary containing only transported pair constraints from positive components therefore becomes incomplete under the cap. The extension proof fails at the step where the hidden component receives a fresh label.

A study may encounter such a restriction when it has independent reasons to believe that only a fixed number of organizations could have existed in the relevant domain. That belief requires a scope argument. A registry containing two names may enumerate only recognized bodies while omitting informal ones. A funding programme may allow two grants without limiting the number of applicant organizations. Converting those observations into a universal entity cap can create forced identities that the source never established.

The practical consequence is to record numerical domain restrictions separately from pairwise continuity evidence. If a conclusion depends on the cap, the report should state that dependency. A later discovery of an additional eligible organization can then reopen the conclusion without contradicting any of the original pair observations. The revision changes the domain assumption that made the earlier identification necessary.

This case also distinguishes global consistency from simple local agreement. The visible endpoints have no local conflict. Their permitted relation is constrained by an omitted third appearance and a global limit. A method designed only to compare the visible pair's features cannot represent that dependence. The relevant information concerns the whole admissible arrangement.

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19.6 Dormancy, Missing Activity, and a Retained Decision Procedure

Return to a collective that disappears from activity records for several years and then resumes its programme. Construct two histories with the same observed performances before and after the gap. In one, appointed custodians retain the governing procedure, maintain the unfinished undertakings, and use that procedure to authorize the return. In the other, the earlier procedure terminates and later participants establish a new body that adopts the programme. An activity log alone can leave both histories admissible.

The comparison introduces a target that ordinary event frequency cannot settle. The inquiry concerns continuity of a coordinating arrangement during an interval in which its main public activity was absent. The relevant records can include appointments, retained decision authority, communications about pending undertakings, and the act authorizing the return. Their importance depends on the criterion. A practice-history question may instead focus on how instructions and skills survived the gap.

Now vary the observation process while keeping the first history fixed. Suppose the investigator initially consults only public event notices. Those notices contain the visible gap. An internal register later reveals periodic decisions by the custodians. The new record changes the study's description of the interval without changing the organization that existed there. The difference is evidential. It should be distinguished from a case in which participants actually establish a new authority after the investigator begins asking about continuity.

That latter event is a possible consequence of inquiry. An interview can prompt former members to clarify who may act for the dormant body. They may then meet and adopt a fresh procedure. The resulting record is evidence of the new act. Its relevance to the earlier interval requires an argument about whether the act recognizes, ratifies, or creates the authority at issue. Treating the newly produced coordination as evidence that the same arrangement continuously existed would collapse the time of inquiry into the time of the target.

The case therefore needs two timelines. One contains the purported organizational events, including any continuing authority and later return. The other contains the acquisition and production of evidence. The timelines can interact, but their events have different roles. A study that records only the order in which documents enter its archive can misplace a retrospective account or an investigator-induced clarification within the organizational history.

A defensible finding might establish that the public programme ceased, that some custodial authority persisted, and that the return occurred through an accepted decision. Another might establish only preservation of the archive and later reconstitution. The difference concerns the relations carried through the gap. The number of inactive years becomes meaningful in connection with those relations and the record's capacity to observe them.

19.7 Concurrent Organizations with a Shared Roster

Membership overlap can be complete while organizational arrangements remain distinct. Consider five people who coordinate a neighborhood reading association and, during the same period, organize a separate temporary exhibition. Stipulate that each arrangement has its own decision procedure, commitments, and records, and that the participants can authorize an act for one without thereby authorizing it for the other. Their participant sets coincide. The two arrangements nevertheless supply distinct candidate appearances at the selected organizational resolution.

The evidential problem arises when public records describe the same people using a collective label that omits the arrangement in which they act. A photograph, attendance list, or contact address can support several interpretations. One interpretation treats the exhibition as a programme of the reading association. Another treats it as the activity of an independently coordinated temporary collective. The investigator must reconstruct the authorization and undertaking relations before normalizing the references into one or two appearances.

The case differs from ordinary duplicate records. Two documents can describe the same meeting of one organization. Here the candidate histories disagree about whether two organized arrangements

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coexist. The normalization decision changes the available identity question. If the researcher merges the arrangements at the outset because their rosters coincide, subsequent slice separation cannot recover the lost distinction. The formal model will faithfully enforce a unit choice that may already be mistaken.

A discriminating record should concern the relation between the arrangements. A minute authorizing the exhibition through the association's ordinary procedure can support the programme interpretation. A separately accepted exhibition undertaking, with a distinct decision arrangement and a limited mandate, can support the independent-collective interpretation. The exact combination needed depends on the working definition of a collective unit. Shared funding or materials may connect the arrangements while leaving their decision relation unresolved.

The temporary arrangement can later become permanent. If that happens, the event should be described through the change in its procedure and commitments. A later separate name is useful evidence, but the time at which the name appears may differ from the time at which independent coordination began. A name-based dataset can therefore date the separation later than a procedure-based reconstruction. The disagreement can be traced to the indicator and the record schedule.

This comparison identifies a reason to preserve actor roles alongside rosters. The same person can occupy a role in each arrangement, and an action performed in one capacity need not be an action for the other. Historical action attribution in Section 26 depends on this distinction. A later merger of the two arrangements cannot make the earlier role difference disappear from the event record.

19.8 Delegated Activity and Emerging Independent Coordination

A collective can transfer the performance of an activity without transferring the authority that governs it. Consider an association that delegates its educational programme to a team. The team schedules the sessions and teaches the participants, while the association retains authority to define the programme, accept undertakings, replace the team, and end the delegation. Activity records may make the team appear to carry the entire practical operation. The stipulated authority relation still places that operation within a delegated arrangement.

Now vary the history. The team acquires authority to accept its own undertakings, determine the programme independently, and continue even if the original association ends the delegation. These changes can support normalization of an independent collective. The transition can occur gradually through several decisions. A study needs to identify which change the adopted unit definition treats as decisive and why. Simply observing that the same team continues teaching under both arrangements leaves the difference unresolved.

The case separates operation transmission from organization formation. The teaching practice can pass continuously through the transition. An independently coordinated body can emerge while the activity's schedule, personnel, and techniques remain stable. Conversely, a new team can replace the performers while the association's authority remains continuous. An operation-only record treats both changes as variations in practical production and can miss their different organizational implications.

The evidence needed for this distinction concerns powers and their exercise. Who can accept a new undertaking, alter the programme, end the arrangement, or appoint replacements under the applicable practice? The study should extract the relevant acts and provisions. It should also examine whether the purported powers were recognized and effective in the episode. A written delegation and an independently operating team can diverge, generating competing histories of formal authority and effective coordination.

One possible result is layered. The association continues under an institutional criterion, an independent team emerges at a later point, and the educational practice has a continuous transmission history through both. These findings occupy different relations in the model. They permit a later split dispute to be described with greater precision: the team may claim a practice and a new coordinating arrangement while the association claims continued institutional identity.

The comparison also prevents every successful delegation from being classified as a split. Branching in a lineage graph can represent the distribution of work among teams inside one collective. A split in

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the normalized entity domain requires a warranted change in the unit structure. Keeping those events distinct preserves the organizational resolution on which the branch constraint depends.

19.9 Asymmetric Merger and Crossed Continuities

Consider two simultaneous organizations, u and v, that combine their activities in a later appearance m. Stipulate that u supplies the continuing appointment and administrative procedure, while v supplies the specialist programme, most of the relevant skilled participants, and the archive required to reproduce that programme. The merger has crossed continuities: the different relations converge on m from different predecessors.

An institutional criterion can identify m with u if its specified conditions are satisfied. A programme inquiry can trace the new body's specialist operation to v. The normalized identity relation cannot identify m with both distinct simultaneous predecessors while retaining equivalence and slice separation. The typed lineage can retain both contributions without imposing that impossible identity pattern.

The asymmetry matters for explanation. Suppose the later body succeeds because the administrative procedure secures access to resources while the specialist team supplies the capacity to use them. An explanation that follows only the selected numerical predecessor can omit an indispensable contribution. A record of both transmission relations can show how the merger's capacities depend on their combination. The identity criterion determines one continuity judgment; the causal explanation requires the relevant input relations.

The asymmetry also matters for an undertaking. The historical party that accepted the undertaking may be u, while the practical capacity to perform it comes chiefly from v. An accepted merger arrangement can connect that undertaking and capacity at m. The warrant for present performance then concerns both the undertaking's succession and the acquired means of fulfillment. A roster-based allocation of past responsibility would answer a different question and may track neither relation adequately.

Further evidence should be chosen accordingly. Appointment and constitution records bear on institutional persistence. Programme handover records, training relations, and access to the archive bear on the origin of the specialist capacity. Accepted undertaking records bear on the present commitment. These sources can converge on a coherent account without converging on a single undifferentiated notion of continuity.

The comparison preserves a particularly valuable insight about merger. A later collective can depend on several histories even when a particular identity policy selects one. The policy's exclusivity should be confined to the relation it defines. Historical contribution, transmitted competence, and undertaking succession retain their own evidence and can remain plural.

19.10 Comparative Findings and the Next Evidence Task

Table 8 gathers the cases by their decisive difference. The table is a research-design summary. Its entries name evidence tasks rather than report observations from actual organizations. The purpose is to help a reader choose the next inquiry that bears on the remaining alternative.

The cases support a common procedure without requiring a universal criterion. First state the accepted observations. Then identify the historical relation on which the competing interpretations differ. Next determine how the selected criterion uses that relation. Finally ask which available record could distinguish the alternatives. This sequence gives evidence acquisition a target and prevents repeated collection of observations that every candidate already accepts.

The exact reduction result adds a second discipline. When a report omits parts of a warranted history, it should preserve the consequences that those parts carry for the remaining questions. A shorter report can retain a richer argument through derived constraints and linked provenance. Its economy should concern presentation, while the underlying evidential dependency remains available for review.

The comparisons also show where more data reaches a limit. If two criteria evaluate the same fully specified history differently, additional records of that history may leave their disagreement untouched. The next task concerns the criterion's suitability for the question. If two histories differ on an unrecorded

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Comparison	Decisive relation	Relevant additional record	Finding retained across alternatives
Gradual substitution and fresh foundation	Sequence of accepted handovers	Appointment and termination acts	Endpoint roster change
Visible endpoints and omitted appearances	Mediated positive or negative constraint	Intermediate component and witness path	Accepted source-level constraints
Dormancy and reconstitution	Continuing authority during the gap	Custodial decisions and return authorization	Absence of recorded public activity
Shared roster and concurrent organization	Independent coordination of undertakings	Role-specific decision and commitment records	Participant overlap
Delegation and independent formation	Distribution of decision powers	Delegation, appointment, and uptake acts	Transmission of practical activity
Asymmetric merger	Distinct institutional and practical inheritances	Constitution, programme, and undertaking records	Multiple predecessor contributions

Table 8: Discriminating relations and evidence tasks in the constructed comparative histories.

handover, a new document can matter directly. If the unit normalization is unresolved, the inquiry must first establish which organized arrangements the records describe. Locating the disagreement determines the appropriate form of further work.

20 Evidence Acquisition and Discriminating Queries

This section develops a query-specific account of evidence acquisition. It characterizes which sets of observations can distinguish histories with opposing identity verdicts, illustrates complementary evidence, and examines the limits of applying the result to organizational research. The objective is to connect an unresolved finding to a concrete evidential task.

20.1 A Finite Family of Historical Questions

Fix a finite candidate history set H and a criterion yielding a Boolean target $q : H \to \{0, 1\}$. A query $f_r : H \to Y_r$ represents the answer that would be obtained from a specified evidence source under each history. Examples include whether a handover was accepted, whether a procedure remained in force, or whether a particular training relation occurred. The formal function assumes that the query and its interpretation are already specified. Reliability and accessibility require separate modeling.

For a selected set of queries J, the joint observation is $f_J(h) = (f_r(h))_{r \in J}$. Exact identification of q from these queries means that histories sharing the joint answer also share the target verdict. This is an application of the recovery criterion, now used to choose a record design rather than merely evaluate an existing one.

For histories with opposite verdicts, define their distinguishing set

$$D(h, h') = \{r : f_r(h) \neq f_r(h')\}. \tag{46}$$

It lists the available queries capable of separating that particular pair. The set can be empty. In that case the supplied query family leaves those opposing histories observationally indistinguishable.

20.2 The Separation Criterion

Proposition 20.1 (Queries sufficient for a fixed verdict)**.** *A selected query set J exactly determines q on H if and only if*

$$J \cap D(h, h') \neq \varnothing \quad \textit{for every } h, h' \textit{ with } q(h) \neq q(h'). \tag{47}$$

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Proof. Suppose the condition fails for an opposite-verdict pair. Every selected query gives the same answer on that pair, so their joint observations are equal while their target values differ. Exact recovery is impossible.

Conversely, suppose the condition holds. If two histories share every selected query answer, no selected query distinguishes them. They cannot be an opposite-verdict pair, because the condition requires a selected distinguishing query for every such pair. They therefore have the same target value. The recovery criterion supplies a well-defined function from joint answers to the verdict. □

The result is a finite separation argument. It does not claim a new algorithmic theorem or guarantee that an optimal query set is easy to find. Its conceptual use is direct: an evidence plan should cover the pairs of histories that matter to the target question. A large number of records that leave one decisive pair indistinguishable cannot establish exact identification under the model.

20.3 Complementary Evidence

Evidence can be useful jointly even when each item alone leaves the verdict unresolved. Take $H = \{00, 01, 10, 11\}$, where the two bits represent answers to two historical questions. Let the target be one exactly when the bits differ. The first query reports the first bit, and the second reports the second bit.

If only the first bit is known, either target value remains possible: 00 and 01 share the first answer and differ on the target, as do 10 and 11. The same holds for the second query. Knowing both bits determines the target completely. Thus the two queries are complementary for this stipulated target. Neither has a secure verdict by itself, yet their joint record does.

An organizational analogue could involve two conditions whose relation matters, such as whether an undertaking was transferred and whether it was subsequently superseded under an accepted rule. The analogy requires a carefully stated target; the binary example alone does not show that an actual succession rule has this form. Its lesson is methodological: the value of a record can depend on the other records with which it is combined. Evaluating sources only by their isolated ability to settle the whole question can miss such complementarity.

20.4 Costs, Access, and Reliability

Suppose each query has a cost $c_r \geq 0$. A research design can seek a set J satisfying Equation (47) within a budget or with minimum total cost. This is an optional design problem after the history set, target, query functions, and cost model have been fixed. The cost can represent time, money, archival access, or burden on participants. Combining these different burdens into one number requires a further policy.

Actual sources can be unreliable, unavailable, or ambiguous. A witness may misremember; a document may have uncertain provenance; a required archive may be closed. In such cases the query is better represented by a set of possible answers or a justified observation model. The deterministic separation theorem then describes an idealized benchmark. The investigator must examine whether the source can deliver the assumed discrimination.

Repeated dependent sources deserve particular attention. Ten records that copy one announcement may produce the same query answer under every candidate history. Their number alone adds no distinguishing power in the formal model. Independent sources can still agree, but their independence must concern the relevant evidential mechanism, not merely different file names or publishers. A provenance record helps locate dependence without automatically deciding reliability.

20.5 Adaptive Inquiry and Changed Targets

An investigator can choose later queries after seeing earlier answers. Such adaptive inquiry may save effort by excluding histories early and asking only questions relevant to the remaining alternatives. Its advantages depend on the candidate set and query structure. A fixed query set and an adaptive strategy

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answer different design problems and should be compared under the same costs and reliability assumptions.

Inquiry can also change the organization. Asking branches to clarify an undertaking can induce them to accept, reject, or renegotiate it. An adjudicative request can change affiliation or public recognition. The resulting record then concerns both a prior history and a new event. A post-inquiry commitment should not be silently treated as evidence that the same commitment already existed before the question was asked.

The research design should therefore separate targets by time. A query about a prior act seeks evidence of that act. A request for a current commitment can help construct a future arrangement. Both may be useful, but they have different relations to identity reconstruction. Preserving the distinction allows evidence acquisition to contribute to practical resolution without rewriting the historical uncertainty that preceded it.

20.6 Discriminating the Initial and Final Identity

The six-appearance dossier permits a more organizationally specific illustration of complementary evidence. Restrict attention to four histories in which the initial collective continues into exactly one middle branch and the final body continues exactly one middle branch. Let X name the branch selected at the initial division and Y the branch selected at the final merger. Their possible values are B and C. The target asks whether a and m belong to one identity class under the fixed criterion.

The four admitted histories are BB, BC, CB, and CC, where the first letter gives X and the second gives Y. The corresponding partitions are $ABM \mid C$, $AB \mid CM$, $AC \mid BM$, and $ACM \mid B$. This restricted family is explicitly smaller than the ten-partition family. It incorporates the additional premise that both reorganization boundaries select one of the established branches. It therefore excludes new formation and gap-only continuity from this particular query illustration.

The target is true in BB and CC and false in BC and CB. If $X = Y$, the initial and final appearances connect through the same middle branch and transitivity identifies them. If $X \neq Y$, the required distinction of the branches prevents that identification. The target therefore depends on the relation between the two boundary selections, not on either boundary alone.

A query about the initial division can reveal X. Knowing $X = B$ leaves histories BB and BC, which disagree on the target. Knowing $X = C$ leaves CB and CC, again with opposing verdicts. A query revealing Y has the same limitation. The two queries together determine the history within this restricted family and therefore determine whether the initial and final identities coincide.

The example shows why evidence at a distant point in a history can matter to an endpoint identity question. Establishing the initial succession is insufficient if the final merger's continuity remains open. Conversely, a precise final registration can leave the initial branch connection unsettled. A research plan focused only on one dramatic event may therefore miss an indispensable part of the target's history.

20.7 A Query Plan with Explicit Evidential Assumptions

An actual query about X cannot simply ask a document to reveal the true successor. It must identify the historical conditions that the fixed criterion uses at the division. If the criterion concerns a continuing procedure of collective coordination, the query must examine that procedure's transmission. If it concerns an authorized registration, the query must examine the effective act under the relevant rule. The target function is meaningful only after those conditions have been specified.

The corresponding query about Y requires evidence at the merger. It should distinguish an effective continuation act from a proposal, a transfer of a practice from its independent reproduction, and a claim of succession from the conditions warranting the claim. A single record may answer part of the query while leaving another part uncertain. The formal query function then needs to represent the partial answer or retain several historical possibilities.

The study can prepare a query worksheet that lists the target premise, the source sought, the expected discrimination, the reliability assumptions, and the consequences of an ambiguous answer. This makes

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evidence collection directed and reviewable. If the sought source cannot discriminate the remaining histories, the plan should change rather than collect it solely because it is easy to obtain.

The idealized two-query result is thus a benchmark, not a promise that two documents will resolve an actual case. One historical query can require many records and a sustained interpretive argument. Conversely, one well-supported document can answer several queries. The formal units are propositions and observation functions, while the research units are sources, passages, interviews, and acts of interpretation. Keeping those levels separate prevents an elegant finite model from understating the work of documentary inquiry.

20.8 Redundancy and the Preservation of Unused Records

A record can become unnecessary for one current verdict without losing its research value. Once the two decisive queries are answered, additional records may be redundant for identifying the endpoint pair within the fixed four-history family. They can still support source verification, reveal omitted histories, answer another question, or become important after the criterion changes. Logical sufficiency for one target is not a general instruction to discard the archive.

This distinction suggests two coordinated records. A working evidence set contains the sources and inferences currently needed for a finding. A preservation record retains the broader authorized material and its provenance. The first can be concise enough for review; the second supports later reinterpretation. Their relation should be explicit so that compression in the working analysis does not erase historical material.

The method should also preserve failed queries. If a source was sought because it could have distinguished two histories but proved unavailable or ambiguous, that outcome belongs in the research record. It explains why the verdict remains unresolved and prevents later investigators from repeating the same inquiry without knowing its limitation. A failed evidential attempt can therefore carry substantive information about the conditions of knowledge.

20.9 Evidence Sufficiency and Decision Sufficiency

An institution may need to decide before the available evidence identifies a unique history. It can adopt a rule for acting under uncertainty, such as a provisional appointment or a shared stewardship arrangement. The evidence sufficient to justify that decision can be weaker than the evidence needed for an exact historical identity verdict. The decision's justification should state the uncertainty it accommodates and the consequences it seeks to manage.

Conversely, exact identification of a historical relation may be insufficient for a present decision. Knowing which organization continued an undertaking does not alone establish which actor currently has the capacity, legitimacy, or resources to perform it. A decision can require additional facts and normative reasons. The query plan should therefore distinguish the historical target from the practical target instead of assuming that one answer automatically supplies the other.

The distinction supports a constructive response to difficult cases. Research can preserve an unresolved historical finding while helping an institution design a transparent provisional arrangement. Later evidence can revise the finding and inform the arrangement's review. This maintains the integrity of the historical record while allowing collective activities and undertakings to continue under explicit conditions.

21 Fallible Records and Recovery of a Continuity Verdict

The earlier query construction assumes that an acquired answer has the value assigned to its candidate history by the observation map. Real documentary inquiry can include inaccurate testimony, transcription mistakes, incomplete extracts, and source-dependent repetitions. This section extends the model

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to explicitly fallible answers. It derives the condition for recovering a fixed continuity verdict, specializes that condition to bounded coordinate errors, and then examines failures that affect several records through one source. A constructed authority dossier makes the difference between target recovery and complete historical reconstruction precise.

21.1 A Fixed Target under an Explicit Observation Model

An error model needs a stable question. If the investigator changes the continuity criterion while collecting records, a difference in answers can reflect a changed target rather than an inaccurate observation. The present construction therefore fixes the candidate history family, appearance normalization, and target verdict before describing the permitted errors. Later revisions to those inputs remain possible, but they produce a new model comparison.

Let H be a nonempty finite family of accepted candidate histories and let $q : H \to \{0, 1\}$ be the target. For each history h, let C_h be the nonempty set of observed answer patterns permitted by the proposed observation model. All C_h lie in a common answer domain Y. They can represent an ideal report together with specified alternatives caused by recording or interpretation failures. They can also directly list the range of answers that the investigator regards as compatible with the source under that history.

The word "error" names a difference relative to the stipulated ideal report. It need not imply dishonesty. A participant may misremember an appointment date, a transcriber may misread a name, or an extract may omit a qualification. A source can report its speaker's belief accurately while that belief misdescribes the event. The observation model should identify which level it treats as fallible: the event report, the transcription, the extraction, or the institutional interpretation.

A recovery function, or decoder, assigns the correct target to every admitted answer. Its natural domain is

$$Y_C = \bigcup_{h \in H} C_h. \tag{48}$$

An answer outside this domain conflicts with the combined history and observation model. The decoder can remain undefined there. Treating such an answer as automatically supporting one verdict would replace a model failure with an unsupported conclusion.

The model is deliberately explicit about where its substantive assumptions enter. The history family must include the possibilities relevant to the inquiry. The criterion must make q intelligible. The sets C_h must describe errors the sources can plausibly produce or a declared sensitivity scenario. A proof about these sets cannot establish their empirical adequacy.

21.2 Separation of Opposing Answer Possibilities

The general recovery condition depends on opposing verdicts, rather than on every distinction between histories. Two histories can remain indistinguishable when they give the same answer to the research question. This permits a study to settle a continuity verdict while leaving other historical details open.

Proposition 21.1 (Recovery from fallible answer sets)**.** *A function $d : Y_C \to \{0, 1\}$ satisfying $d(y) = q(h)$ for every $h \in H$ and $y \in C_h$ exists exactly when*

$$q(h) \neq q(h') \quad \Longrightarrow \quad C_h \cap C_{h'} = \varnothing. \tag{49}$$

Proof. If an answer y belongs to both sets for histories with opposing verdicts, recovery would require $d(y)$ to equal two different bits. Thus the separation condition is necessary. Conversely, take any $y \in Y_C$. At least one compatible history exists, and every compatible history has the same verdict because opposing histories have disjoint answer sets. Define $d(y)$ to be that common verdict. The definition is well-defined and recovers every admitted answer. □

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For a particular observed answer y, define

$$H_y = \{h \in H : y \in C_h\}. \tag{50}$$

When H_y is nonempty, the target is secure exactly when q is constant on it. The global proposition requires this condition for every admitted answer, while a particular investigation can obtain a secure result for its observed y even if other possible answers would leave the target open.

This distinction matters for research planning. A set of queries may fail to guarantee a verdict in advance while still being useful in the episode being studied. Conversely, a guarantee under the stated model can fail in practice if the actual error lies outside the admitted sets. The study should distinguish a successful observation from a uniform guarantee and should name the conditions under which either conclusion holds.

The proposition also gives a direct counterexample method. To show that a proposed record system cannot recover the verdict under its own assumptions, find two opposing histories and one answer pattern that both permit. Additional confidence in a decoder's implementation cannot remove that overlap. A successful repair must change the available observations, justify a narrower error model, refine the history family, or revise the target.

21.3 Bounded Errors in Binary Documentary Answers

A binary specialization gives the separation condition a familiar geometric form. Let $f : H \to \{0,1\}^m$ assign the ideal answers to m fixed queries. A bit can record whether a specified act occurred, whether a named role was occupied, or whether a stipulated criterion condition was satisfied. The operational definition of each query remains part of the model.

For binary vectors x, z, define their Hamming distance by

$$d_H(x,z) = |\{i : x_i \neq z_i\}|. \tag{51}$$

It counts the coordinates on which the two patterns differ. Hamming's geometric account connects this distance with bounded error detection and correction [2, §5]. The present application reconstructs that established reasoning for a fixed documentary target.

Suppose the admitted errors change at most k coordinates, where k is a nonnegative integer. Then the permitted answers under h form the set

$$C_h = \{y \in \{0,1\}^m : d_H(y, f(h)) \leq k\}. \tag{52}$$

The assumption constrains the total number of altered answers. It introduces no probabilities and requires no probabilistic independence among their errors. Any selection of at most k coordinates can be altered.

Proposition 21.2 (Cross-verdict distance under a coordinate budget)**.** *Under Equation* (52)*, uniform exact recovery of q is possible exactly when*

$$q(h) \neq q(h') \quad \Longrightarrow \quad d_H(f(h), f(h')) > 2k. \tag{53}$$

Proof. If a vector y is within k coordinates of both x and z, every coordinate where x, z differ contributes to at least one of the comparisons with y. Hence $d_H(x,z) \leq d_H(x,y) + d_H(y,z) \leq 2k$.

Conversely, let $d = d_H(x,z) \leq 2k$. Choose an integer r with $\max(0, d-k) \leq r \leq \min(k,d)$; such an integer exists under the assumed inequality. On r of the differing coordinates let y agree with z, and everywhere else let it agree with x. Then $d_H(x,y) = r \leq k$ and $d_H(y,z) = d - r \leq k$. The two permitted-answer sets intersect exactly when $d \leq 2k$. Applying Proposition 21.1 to opposing histories gives the result. □

The restriction to opposing histories is essential. Recovering the whole history would require separating every distinct candidate that the investigation treats as a different answer. A continuity question can

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History	Authority ground	Target q	Ideal answers	Distance from 1100
h_0	Neither accepted ground	0	0000	2
h_1	Earlier appointment	1	1110	1
h_2	Delegation	1	1101	1

Table 9: Constructed authority histories and one observed answer pattern under a fixed institutional criterion.

be less demanding. It can leave two authorized routes unresolved while establishing that an authorized route existed.

The result also exposes the role of the budget. Increasing k enlarges each permitted-answer set and can destroy separation. Adding well-chosen queries can increase the distance between opposing histories. Repeating a displayed answer may increase the vector distance while also changing the plausible error model. The next comparisons examine that dependency.

21.4 A Four-Query Authority Dossier

Consider a constructed inquiry into a historical appointment. The institutional criterion is fixed. The accepted background allows three candidate histories: the appointment lacked an accepted authority ground, an earlier appointment supplied the ground, or a distinct delegation supplied it. The two positive routes are mutually exclusive within this stipulated candidate universe. Its exhaustiveness is an assumption of the example.

The first two query coordinates are ideal correct reports of the appointment result through two documentary channels. The third concerns the earlier-appointment route; the fourth concerns the delegation route. Calling the first two ideal reports correct defines the observation map for the construction. An actual certificate or minute would need its own interpretation and authority evidence.

Table 9 states the entire candidate family. The target is whether an accepted authority ground existed. The route by which it existed is a different historical question.

The distance from h_0 to either positive history is three. Their own distance is two. Under a one-coordinate error budget, the target is recoverable for every admitted answer, although the complete history need not be recoverable. The cross-verdict separation condition holds even where the two positive neighborhoods overlap.

Suppose the observed answer is 1100. The first two reports affirm the appointment result while the two route queries are negative. This pattern lies one coordinate from each positive history and two from the negative history. Under the stated budget, the target is securely positive. The earlier appointment route and delegation route remain unresolved.

The finding has a precise conditional form. Given the accepted three-history universe, the fixed criterion, and at most one altered coordinate, the appointment had an authority ground. The investigation has yet to establish which of the two grounds supplied it. It should preserve that residual uncertainty because later questions about the appointing actor or the route's scope may depend on it.

The example also shows that a contradictory-looking set of reports need not make the whole model empty. The observation model can explicitly permit the discrepancy. The compatible family then describes its possible explanations. A finding that one record differs from its ideal value should retain which alternative history makes that assessment and why.

21.5 Copied Reports and a Shared Failure

Now keep the three histories and their ideal answer vectors fixed, but change the source model. Suppose the first two report channels copy one underlying source. A mistake in that source can alter both coordinates together. The third and fourth coordinates have separate source origins.

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Assume at most one of the three source bundles fails. The affected coordinate sets are $\{1, 2\}$, $\{3\}$, and $\{4\}$. Within a failed bundle, permit any alteration of its affected answers. This is a different deterministic budget from at most one altered coordinate.

Under the bundle model, observed 1100 is compatible with all three histories. Starting from 0000, failure of the first bundle changes the first two coordinates. Starting from 1110, failure of the third-coordinate bundle produces the observation. Starting from 1101, failure of the fourth-coordinate bundle does so. The target is therefore ambiguous.

The change in finding does not contradict the distance theorem. The first model excluded a pattern with two altered coordinates. The second permits that pattern when one shared source causes both. The theorem remains correct, while the source-dependence evidence determines which observation model is appropriate.

A simpler repetition example makes the same point. Ideal answers 000 and 111 have distance three and permit one-coordinate correction. If all three records copy one fallible source, a single source failure can replace one whole pattern with the other. Three displayed records then supply one source-dependent error opportunity under the bundle interpretation.

Copying can still have value. It can preserve a record after an archive is lost, make a source accessible in several places, or reveal changes across versions. Those benefits concern access and provenance. They should be distinguished from the discriminating protection supplied by a genuinely different source route under an appropriate failure model.

21.6 Affected Coordinates and Source-Bundle Covering

The bundle comparison has an exact finite formulation. Let R be a finite set of source labels, and let $B_r \subseteq \{1, \dots, m\}$ contain the coordinates that a failure of source bundle r can affect. Different bundles may overlap. Under at most k failed bundles, define

$$C_h^{\mathcal{B},k} = \left\{ y : \exists J \subseteq R,\ |J| \le k,\ \{i : y_i \neq f(h)_i\} \subseteq \bigcup_{r \in J} B_r \right\}. \tag{54}$$

The definition permits arbitrary changes inside the affected union and preserves coordinates outside it. A more restricted causal error pattern needs its actual permitted answer set in Proposition 21.1.

For a set of coordinates D, let $\mathrm{cover}_{\mathcal{B}}(D)$ be the smallest number of bundle sets whose union contains D. Its value is infinite if the sets cannot cover D, and zero if D is empty. This is a combinatorial measure of the admitted corruption structure, not a probability or a measure of a source's moral reliability.

Proposition 21.3 (Source-bundle separation)**.** *The two corruption sets around ideal vectors x, z intersect exactly when*

$$\mathrm{cover}_{\mathcal{B}}(\{i : x_i \neq z_i\}) \le 2k. \tag{55}$$

Consequently, exact target recovery requires the strict opposite inequality for every pair of histories with opposing verdicts.

Proof. If y is an admitted corruption of both x and z, choose failed label sets J_x, J_z, each with at most k members, that cover the changed coordinates in the respective comparisons. Every coordinate where x, z differ must lie in the union of their affected sets. At most $2k$ labels therefore cover the disagreement coordinates.

Conversely, suppose a set J of at most $2k$ labels covers the disagreement set D. Split J into J_x, J_z, each with at most k labels, and write U_x, U_z for their affected coordinate unions. Outside D, keep the common value. On $D \setminus U_z$, let $y = z$; on $D \setminus U_x$, let $y = x$; where both unions cover a differing coordinate, choose either value. These instructions are consistent because $D \subseteq U_x \cup U_z$. Every change from x lies in U_x and every change from z lies in U_z. Thus y is admitted under both histories. The general separation proposition gives the recovery consequence. □

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A coordinate affected by no source set acts as protected evidence within this model. If opposing ideal answers differ there, the covering number is infinite and these errors cannot make the answers coincide. An actual inquiry needs a strong reason for such protection. Treating an official field as unalterable because it is official would simply place a disputed reliability claim inside the model.

Overlapping source sets introduce another useful distinction. One document can depend on a witness for its factual content and on a common transcriber for its wording. Either failure can affect the same coordinate. The covering model can represent both routes if the dependencies are specified. Counting each document as one independent unit would omit that structure.

21.7 Evidence for an Error Budget

The strongest practical obligation concerns the budget itself. A researcher rarely knows simply from the number of documents that at most one contains a relevant error. The claim requires an account of how the records were produced, checked, and selected, and which mistakes the model permits. A numerical bound without that account is a sensitivity assumption.

Sensitivity analysis can still be informative. A report may state that its verdict survives every single-coordinate error but fails under a two-coordinate budget, then identify the opposing histories that create the failure. This locates the conclusion's dependence on the error assumption. It also guides additional collection toward a source that distinguishes those histories.

The source-bundle version improves that inquiry when dependencies are known. A copied announcement and a retrospective history based on it may belong to one affected set. An independently maintained appointment register can provide another route. Independence here should be described through the documented production relation and the permitted failures, rather than inferred from different publication venues.

An audit can also reveal that a hard budget is an unsuitable description. The errors may be systematic, the source universe may be selectively preserved, or the same ambiguous institutional term may affect every extract. The general answer-set model can retain such possibilities without forcing them into a small coordinate count. Its result may then remain ambiguous, which is the appropriate finding under the admitted possibilities.

The mathematical framework supplies an exact account of the assumption's consequences. Its historical use requires a separate argument that the assumption describes the documentary case well enough for the intended purpose.

21.8 Target Recovery and Continued Historical Inquiry

Recovering a verdict can leave important work unfinished. In the four-query case, a secure positive result under the coordinate budget leaves the authority route unresolved. A later inquiry about the officer's scope of power or the validity of a further appointment may need precisely that distinction. The retained family should therefore remain available after the current target is settled.

A change in the continuity criterion can also make a previously irrelevant historical difference decisive. Two routes may both satisfy an institutional standing question while only one carries a specified undertaking. The earlier decoder was sufficient for its fixed target. Reusing its output for the new question requires a new sufficiency argument.

This is the practical counterpart of the joint- information result. A summary can answer its intended question correctly while discarding distinctions needed by another. The appropriate research product connects the verdict to its candidate histories, observation assumptions, and criterion version. It can then support a reasoned extension of the inquiry.

Fallibility consequently deepens the paper's evidential method. It adds an explicit account of how records can disagree with their ideal meanings, what combinations of discrepancies remain admissible, and which verdicts survive them. The result preserves both a positive recovery procedure and a precise description of the evidence still needed when recovery fails.

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22 Institutional Recognition and Authority

This section examines the relation between a continuity finding and an institution empowered to recognize a successor. It uses expressly stipulated procedures rather than claims about any jurisdiction. The analysis separates constituted standing, historical evidence, public acceptance, and the effects of the recognition act itself.

22.1 A Stipulated Successor Register

Imagine a federation whose members agree that one registered identifier may continue after a division only through a recorded vote of a specified committee. The rule further permits the committee to close the predecessor identifier and register both branches under new identifiers. This procedure is a hypothetical institutional arrangement. It supplies a clear example of a domain in which a successor status depends partly on an authorized act.

To determine the registered successor, the investigator must establish the rule in force, the committee's competence, the occurrence and content of the vote, and any effective review of that decision. These are evidential questions about a constituted status. If the rule is accepted and the conditions are met, the register can give a definite answer for federation participation even when operational continuity remains divided.

The same finding does not automatically settle which branch maintains the predecessor's programme, which branch its former members regard as their association, or which branch should hold a specific archive. Those questions may be affected by the federation's decision, but the connection needs reasons. The domain of the recognition rule should travel with the finding whenever it is cited elsewhere.

22.2 Authority, Correct Application, and Legitimacy

Three questions arise around the committee's decision. First, does the institution's rule give it the power to make the decision? Second, did it apply the rule correctly in this case? Third, is the rule or exercise of power justified? These questions can receive different answers. A formally effective decision can remain open to criticism on evidential or normative grounds. A persuasive historical judgment may lack authority to alter the register.

The method should preserve these distinctions in the record. The decision is an event with a date, issuer, scope, and review status. Its reasons are arguments that can be examined. Its historical assertions are claims that can be supported or challenged by sources. Its practical effects are changes in access, eligibility, or recognition that can be studied. Treating all four aspects as one fact of "official identity" conceals the structure of the dispute.

This analysis also applies when the institution is internal to the organization. A constitution can specify amendment or succession procedures, while rival branches dispute which meeting or office was authorized. The rule cannot settle its own historical application merely by appearing in a document. Evidence is needed about who acted, which version governed, and how the action related to the procedure.

22.3 Public Recognition and Unequal Visibility

Public recognition can differ from formal registration. Participants, partners, audiences, and other organizations may identify different successors. Their judgments can reveal important relationships: trust in a continuing team, attachment to a name, concern for an undertaking, or acceptance of an official decision. A study should record the content of these judgments and the constituency to which they belong.

Counting statements requires care. One branch may control the old website and mailing list, giving its narrative greater visibility. Another may continue the programme among participants with little public documentation. These are possible mechanisms of unequal visibility, to be investigated in a real case. A simple count of public mentions can then reflect control over communication infrastructure as well as independent recognition.

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Recognition is therefore both an observation target and a possible causal factor. Publicly treating one branch as the successor can direct resources, members, and opportunities toward it. The branch's later continuity can become stronger partly because of the recognition. A retrospective argument that uses those later advantages to prove the original recognition correct needs to account for this feedback.

22.4 Recognition Acts and Historical Targets

Let an organization be disputed at time t_0 and let a recognition decision occur at t_1. The decision can constitute a status from t_1 onward under the relevant rule. It can also state a conclusion about the organization's history before t_1. The first effect depends on authority and procedure; the second depends on evidence and the historical criterion. A record should separate these temporal targets.

Suppose a committee asks branch C to accept a previously disputed undertaking and then recognizes it after acceptance. The new acceptance can supply a reason for the future arrangement. It does not establish that C had already accepted the undertaking before the request. The earlier uncertainty can remain relevant to responsibility for actions during the intervening period. Preserving it is compatible with a practical resolution going forward.

This distinction gives a constructive way to handle disputes that resist a single historical verdict. An institution can establish a future coordination arrangement while retaining a qualified account of the past. It can state that the arrangement is a settlement or constitutive choice, identify the reasons supporting it, and preserve competing historical claims for further inquiry. The resulting clarity is stronger than a fiction that one official act has answered every identity question.

22.5 Review, Contestation, and the Record of Reasons

A review procedure should identify what can change a finding. New evidence may challenge the occurrence of an authorized act. An interpretation may show that the rule was applied beyond its scope. A normative objection may question the rule itself. A later event may justify a new arrangement without undermining the earlier finding. These are different revision paths and should receive different explanations.

The record should preserve the earlier decision, the challenge, the relevant sources, and the revised conclusion. Updating a current status field is useful for present operations; preserving the version history is necessary for understanding past actions that relied on it. A user of the record should be able to determine which finding was available at a given time and why it later changed.

Institutional recognition thus belongs inside the continuity model as a typed, historically situated relation. It can provide authority, evidence, and causal influence in different respects. A rigorous account keeps those respects visible so that an institution's practical need to decide does not erase the philosophical and evidential questions that remain open.

23 Institutional Formation and Heritage in Manchester Merger Records

The analysis now turns to a bounded documentary illustration involving the University of Manchester. The objective is to examine how differently typed historical statements can coexist in an institutional record and how a researcher might introduce an identity conflict through inappropriate coding. The section first establishes the limited documentary basis. It then separates formation, predecessor inclusion, membership, and ceremonial recognition; develops alternative inquiries that could be addressed to those records; and identifies the additional evidence required for a stronger continuity finding. The illustration concerns the interpretation of public documents. A complete history of the merger would require a substantially wider archive and a different empirical undertaking.

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23.1 Documentary Basis and Access Scope

Four official sources supply the narrowly defined observations. The University's foundations page describes the current corporation as beginning on 1 October 2004 after the dissolution of the Victoria University of Manchester and UMIST. It identifies a charter and statutory transfer from the former institutions [14]. A bicentenary account acknowledges the 2004 merger while tracing the UMIST branch to the Manchester Mechanics' Institution in 1824 [12]. These statements already provide different temporal reference points within one institution's public presentation.

The current online charter gives the corresponding formation and transfer recitals. Article III includes graduates of the former universities within the specified membership category of the University [13]. The July 2011 degree-congregation programme distinguishes the institutional transition at the end of September 2004 from the formal inauguration on 22 October 2004 [15]. The sources therefore provide observations about several relations whose historical dates and participants need to remain distinct.

The substantive foundations and bicentenary pages, the charter's printed pages 1–3, and the congregation programme's cover and unnumbered inauguration page form the consulted material. The current charter copy has blank witness-date fields. Its presence on the official site establishes the provenance of the accessed text; the present comparison leaves authentication of an executed original and the full amendment genealogy open. The operative provisions of the University of Manchester Act 2004 require a separate primary reading before detailed statutory conclusions could be supported. Individual graduate entries, subsequent programme pages, and the bicentenary video's contents fall outside the consulted basis.

This access description has an argumentative function. A public institutional summary may provide evidence of the institution's description of a transition, while a legal inquiry into a particular entitlement requires the relevant operative text and its interpretation. A commemorative narrative may identify the predecessors included in a heritage account, while an inquiry into the continuity of laboratory work requires records of personnel, methods, equipment, and activity. The present comparison derives its value from specifying these differences before asking the documents to answer a single question about organizational sameness.

The following interpretations are the paper's proposed analysis of the documentary distinctions. They are not additional historical findings about unexamined departments, participants, decisions, or legal relations. Where an example extends beyond the verified observations, it is introduced as a possible research question or a constructed alternative. This separation allows the case to constrain the method without requiring the limited documentary set to carry the evidential weight of a completed field study.

23.2 Formation Events and the Choice of Collective Unit

An institutional formation account identifies a particular kind of unit and an event relevant to its establishment. A researcher can therefore begin with a question about the corporate organization described by that account. At that resolution, a newly constituted post-merger organization is a plausible candidate unit. The description supplies a reason to distinguish formation from the mere replacement of a name in a list of enduring organizations. The remaining evidential task is to establish the applicable rule and its operation if the research requires a legal conclusion, rather than a report of official institutional self-description.

The same formation event can be studied at other resolutions. A department may be represented as a continuing organized unit within a changed parent institution. A teaching programme may be represented through successive authorizations and deliveries. An informal scholarly community may be represented through collaborative practices whose participants cross formal boundaries. These are possible analytical units, each requiring evidence appropriate to its organization. Their admissibility cannot be inferred from the fact that a parent institution has either a new corporate beginning or a long predecessor history.

The normalization issue becomes concrete here. A record naming a predecessor university and a record naming one of its departments may concern different levels of organization. Treating the records as competing names for a single appearance would collapse a parent relation into co-reference. Treating

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every change in institutional naming as the appearance of a new department would make the reverse mistake. A valid normalization requires an explicit focal unit, a time of reference, and an account of the relation between the unit and the names used to describe it.

The consequence for comparison across a merger is substantial. Suppose a study seeks to measure continuity of a research group and uses corporate registration changes as its only segmentation rule. Its time series can mark a complete organizational break while every measured practice, member role, and equipment relation remains continuous. A different study can preserve the research group's label across the transition while overlooking changes in the authority under which it operates. Either representation can be useful for a specified question. Their findings become misleading when the chosen unit disappears from the reported conclusion and the word "organization" carries both meanings without qualification.

This is an analytic possibility, not a claim about any Manchester research group. Its role is to identify an evidence gate created by the documentary example. Corporate formation records help fix one level of institutional description. They leave the continuity of subsidiary practices to a study that can identify those practices independently. The method accordingly keeps the corporate formation predicate available without turning it into a universal discontinuity predicate for everything associated with the former institutions.

23.3 Predecessor Inclusion and Historical Narrative

A heritage narrative can include several predecessor branches because its organizing relation is historical contribution. The formation of a new institution can therefore be situated in a longer account of inherited work, sites, associations, or educational aims. The chronological reach of such an account tells a reader which earlier histories have been included. A further argument is required before that inclusion can be translated into numerical identity between every included predecessor and the current organization.

Consider a researcher who codes the sentence that an institution has roots in an earlier organization as a positive identity edge. If the researcher also codes two distinct predecessor roots in this way, transitivity can identify the predecessors with each other. Under a resolution in which those predecessors are distinct simultaneous units, the resulting partition violates the slice-separation condition. The contradiction arises from the coding rule that converted ancestry into identity. The public historical statement need never have asserted the equivalence relation that produces the conflict.

The directed-lineage representation provides a less committal and more informative first description. It can preserve the contribution of each predecessor, the times at which they operated, and the relations through which their histories entered the later institution. A separate identity relation can then address the chosen unit under an explicitly defended criterion. This division permits a new institutional beginning and a long historical lineage to appear together without demanding that one of the descriptions be removed. Their compatibility depends on their different predicates, which should be visible in the representation.

Historical inclusion is itself a selective activity. A lineage account may follow a chain of organizational predecessors, a tradition of teaching, the history of a site, or the development of a discipline. Those selection rules can yield different beginning points even when their factual statements are mutually consistent. A serious inquiry should therefore ask what makes an earlier episode relevant to the narrative, what other branches could have been included, and whether the selected scope suits the question being asked. These are questions about historical interpretation and evidence. They need not presuppose a dispute about the numerical identity of the current institution.

The present illustration supplies no basis for alleging misleading commemoration or for ranking the value of different predecessors. Such judgments would require an account of the audience, the actual communicative claim, the omitted material, and the relevant standard of fairness or accuracy. The modest positive lesson is that a researcher should preserve the difference between a statement about institutional roots and a statement about the identity of a corporate unit. Treating both statements faithfully can produce a richer historical account than forcing them into one unqualified starting date.

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23.4 Membership Categories and Carryover Relations

The charter provision about former graduates creates a particularly useful analytical distinction. The organizational subject and a specified relation to persons can change together according to different rules. A person may qualify for a category of membership in a later organization by virtue of a relationship to an earlier one. That rule can establish a continuing institutional connection while the organization itself is described through a new formation event.

The general form of the distinction concerns relational carryover. Let an earlier collective stand in a defined relation to a class of persons. A transition rule may establish a corresponding relation between the later collective and persons meeting the earlier qualification. This mechanism links the histories of the two organizations through a specific predicate. It leaves open whether the organizations are numerically identical, whether the persons exercise the same activities, and whether their own understanding of affiliation changes. Each additional proposition introduces a further target and a further evidential burden.

The distinction also guards against an overly narrow view of continuity. If a study records only the creation of a new organization, it may miss explicitly preserved relationships to people whose histories precede that creation. If it records only the preserved membership category, it may overlook the rule through which the later relation was established. The transition itself can be the mechanism of continuity for a particular relationship. Continuity then involves an institutional act with a history, even where the resulting membership language resembles earlier language.

The practical significance depends on the precise category. Membership can refer to a constitutional status, participation in governance, employment, enrollment, alumni affiliation, or an informal sense of belonging. These relations can overlap without coinciding. A source that establishes one category should be coded at that level. Inferring current participation, particular entitlements, or subjective attachment from the category alone would extend the claim beyond the consulted provision.

This attention to the predicate improves the earlier membership criterion. A raw head count can conceal the difference between persons who actively continue an organization and persons whose inherited affiliation is recognized through a rule. Both relations may matter, but their significance for persistence requires separate reasoning. A preserved category can be important evidence for institutional acknowledgment of predecessor histories. Active role continuity may be more directly relevant to the maintenance of an operation. Neither should disappear into a generic measure called “membership retention” without an explanation of which relation is being counted and why its continuation bears on the target inquiry.

23.5 Institutional Transition and Ceremonial Recognition

The distinction between institutional transition and formal inauguration demonstrates a second form of temporal differentiation. Several events can mark the beginning of an organization for different purposes. A legal or administrative transition, the commencement of practical operations, and a public ceremony can occur at distinct times. A historical account becomes more precise when it identifies the event type associated with each date.

This matters for the construction of time slices. Suppose an analyst records an organization as existing only after the date of its public inauguration. Activities between the institutional transition and the ceremony may then be assigned to a predecessor or left without an organizational subject. Suppose a different analyst uses the earliest preparatory meeting as the beginning of the completed institution. Activities of a planning body may then be assigned retrospectively to the later organization. Both errors arise when a convenient date is used without specifying the relation it indexes.

A more careful representation can preserve a sequence of events and their different targets. Formation may concern the establishment of a corporate unit. Operational commencement may concern the performance of a particular function. Recognition may concern the public acknowledgment of the institution. Preparatory activity may concern a committee or agreement whose purpose is to bring the institution about. Their connections can be represented through directed generative relations without

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identifying every participant in that process with the institution that eventually results.

Ceremonies can also have constitutive force in some institutional settings. The present point therefore concerns the evidential interpretation of an event, rather than a universal rule that ceremonies are merely symbolic. A criterion that treats a ceremony as identity-relevant must state the practice or rule that gives it that role. A criterion that treats the ceremony as subsequent recognition must preserve the earlier event that establishes the target relation. The historical question is settled through the rule and its implementation, not through a general preference for administrative or ceremonial records.

For the present documentary comparison, the explicit distinction between two events helps prevent accidental substitution. It also shows why a database field labeled “founded” is often too compressed for the research question. A single field can be useful for a directory whose conventions are known. A study of collective continuity needs the event description, the referent, the temporal precision, and the source supporting the date. The additional structure serves interpretation rather than ornamenting a simple fact with unnecessary notation.

23.6 Alternative Queries over the Same Documents

The consulted documents support the formulation of several different queries. One asks how the current institution describes its corporate formation. Another asks which predecessor histories appear in its public heritage account. A third asks whether a defined constitutional category includes persons connected with former institutions. A fourth asks how the account distinguishes institutional transition from formal inauguration. These queries overlap historically while requiring different predicates in the model.

A fifth possible query asks whether a specified teaching practice continued through the merger. The present document set leaves that inquiry open. Its answer would require an independently specified practice and evidence of its enactment across the relevant transition. A sixth asks whether a group of participants experienced the later institution as the continuation of their earlier collective. That inquiry requires participant evidence and an account of how such recognition bears on the target claim. A seventh asks which organization should receive historical credit for a discovery. That question adds a standard of credit allocation to the reconstruction of organizational history.

The distinction among queries prevents a common inferential shortcut. An analyst may have a highly authoritative answer to the formation question and then use it to settle the practice, affiliation, or credit question. The authority of the source for its own subject remains intact; the transfer of the answer to a different subject requires justification. The same caution applies in the other direction. Evidence that a practice or personal affiliation continued can be highly significant while leaving the corporate formation question to the appropriate institutional records.

The admissible-family method can accommodate these differences by fixing the target before collecting positive and negative constraints. A proposed corporate-unit inquiry will have one appearance domain and one relevant constraint vocabulary. A practice-continuity inquiry may have a different domain or a relation that permits several successors. A credit inquiry may produce an allocation over contributions. Combining these outputs can enrich a historical explanation. Intersecting them as if they were rival answers to one binary identity query can conceal the distinct work each output performs.

Table 10 records the resulting coding distinctions. Its final column identifies the next inquiry generated by each supported description, rather than treating every document as an answer to one unqualified identity question.

23.7 Source Genre, Institutional Authority, and Evidence Dependence

The four sources occupy different documentary roles. A governance page provides an accessible institutional account. A charter text specifies a constitutional structure in its stated form. A ceremonial programme locates an event within a public occasion. A heritage story selects a historical through-line for commemoration. These differences guide the questions for which each document is initially relevant. They also guide the further checks needed before one document is used to corroborate another.

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Record	Target predicate	Narrow finding	Further inquiry
Foundations account	Institutional formation	A stated beginning for the current corporate unit	Continuity of a specified activity
Heritage story	Predecessor inclusion	Earlier histories enter the institutional narrative	Selection and interpretation of predecessor branches
Charter category	Membership qualification	A stated relation to former graduates is carried forward	Particular affiliations, participation, and effects
Inauguration account	Event classification	Transition and ceremony are separately dated	Role of recognition in the chosen continuity question

Table 10: Proposed coding of the consulted Manchester documentary statements. Source descriptions and their access limits are given at the beginning of Section 23. The additional inquiries require their own evidence and criteria.

Official provenance has several possible implications. It can establish that a description is issued through an institution's own channels. It can make a document relevant evidence of authorized self-presentation. It can also provide a route to primary constitutive texts. These roles should be distinguished from the claim that every sentence is independently verified by its repetition across official pages. Shared authorship, common source material, or editorial reuse can make several documents dependent witnesses to one account.

This dependence matters if the study later compares conflicting evidence. Four institutional documents repeating a formation date do not automatically provide four independent observations of the event. Their agreement can still be significant for the stability of the institution's public account. A different inference, concerning the accuracy of that account against a contemporaneous archive, requires investigation of the documents' sources and production histories. Counting citations cannot replace that investigation.

An evidence record should therefore preserve both the document's content and the reason for assigning it a particular role. It should record whether the researcher treats the source as constitutive, descriptive, retrospective, or evidential of recognition. A source may occupy several roles, with different claims attached to each. This typed treatment makes criticism possible: a reviewer can challenge the relevance or competence assignment without needing to deny that the document exists or that it says what the record reports.

23.8 A Bounded Coding Result and Its Research Extensions

The positive result of the illustration is a differentiated coding of the consulted descriptions. A formation claim concerns a specified institutional unit. A heritage claim relates that unit to selected predecessor histories. A membership provision carries a specified relation to persons across the organizational transition. A ceremonial account locates recognition in a distinct event. Keeping these predicates separate permits the documents to be represented together without manufacturing an unrestricted equivalence between every institution, activity, and affiliation they mention.

A fuller investigation could select one organizational activity whose continuity matters to a specified question. The selection should precede the search for a favorable story. The study could then identify the roles, authorizations, resources, and practices that constitute the activity at the chosen level, inspect records before and after the transition, and examine how participants describe their connection to it. Competing continuity criteria could be applied to the same evidence with their assumptions kept visible. The expected result could be agreement, a criterion-specific finding, or a persistent difference among inquiries.

Another extension could examine the production of heritage narratives. That project would ask how predecessor branches are selected, how audiences understand institutional dates, and how public narratives change over time. Its evidence would include successive versions of documents and reception

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material. It would address recognition and historical representation rather than using narrative chronology as a direct measurement of numerical identity. The present analysis suggests the distinction that makes such a project coherent; it supplies none of the uncollected reception evidence.

The next comparative cases should include explicit disagreement about a shared target. The Manchester illustration primarily tests faithful predicate coding; the constructed dossiers test the consequences of competing identity constraints. A research programme can combine those roles while preserving the distinct evidence each supplies.

24 Retrospective Revival and the Grounds of Authority

Reviving a collective introduces a difficult connection between an account of the past and an act performed in the present. This section examines that connection through the authority claimed by returning participants, the evidential grounds for their act, and the identity consequences of competing revivals. It develops a finite rule model of grounded inference and then tests its philosophical limits. The model clarifies one defensible discipline for a bounded inquiry while keeping alternative accounts of mutually supporting institutional practices open.

24.1 Decisions about Continuity and Their Historical Objects

Jansen treats some group persistence judgments as decision-sensitive, including re-establishment after a gap, while recognizing internal, external, and formal constraints [4, §§5.2–5.4]. The present inquiry develops an evidential question within that setting: which independently warranted acts and authority relations support a particular retrospective identification?

Consider a former association whose programme ended several years ago. Some former participants now announce its revival. The announcement can perform several different acts. It can invite people into a new coordinating arrangement, declare that the arrangement adopts an earlier undertaking, claim a continuing institutional standing, or interpret the earlier association's work as part of a shared history. Those acts can occur together. Their combination in one announcement does not give them identical conditions of success.

The first act may depend on present agreement among the participants. The second may depend on the content of the undertaking, the position of its beneficiaries, and competent acceptance of a new commitment. The third may depend on a procedure extending through the inactive period or on a recognized power of reconstitution. The fourth raises questions about the kind of historical association being asserted. An inquiry should identify which of these acts its evidence supports.

The historical object also needs care. The returning participants may refer to a legal body, an informal association, a programme, a tradition of practice, or a network of relationships. Their earlier coexistence can make a familiar name appear to denote one object without ambiguity. The inactive period can separate them. An archive may survive while the appointment procedure ends; a programme may be transmitted while the original membership disperses. A revival claim can reconnect some of those relations without restoring every earlier arrangement.

The appropriate first record is therefore a typed description of the present act. Who made the announcement, under what procedure, with what content, and with what uptake? What earlier appearance or relation did the act identify? Those questions give the retrospective claim an inspectable subject. The later identity judgment can then evaluate the act under the chosen criterion instead of treating the word "revival" as a completed derivation.

24.2 An Officer and a Branch with Reciprocal Claims

A constructed dispute makes the risk of circular authority concrete. An officer claims power to certify which branch continues the former association. Branch b recognizes that officer. The officer then certifies b. When asked for the officer's authority, the branch points to its power to appoint officers as

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the continuing association. When asked for its continuing standing, the branch points to the officer's certificate.

The two documents establish that the appointment claim and the certificate were issued. Their contents can be authentic even when their authority remains unresolved. The investigator must therefore distinguish the occurrence of each act from the validity attributed to it. Treating authentic issuance as sufficient proof of competent issuance would silently supply the missing premise.

The argument has two conditional steps. If the officer has the relevant authority, the certificate can confer or establish the claimed standing under the specified procedure. If the branch has the required standing, its appointment can authorize the officer. Taken alone, the steps leave open how the chain acquires an accepted starting point. They are compatible with a history in which an independent earlier appointment gives the officer authority. They are also compatible with a history in which both claims begin without that support.

A discriminating inquiry therefore looks for the alleged anchor. It may concern an appointment made before the division, a retained power in an accepted constitution, an external authority's competent act, or a new founding procedure with its own basis. Each possibility changes the proof structure. A previously authorized officer can provide an independent route to the branch's standing. A new founding act can establish present organization while leaving the continuity with the predecessor to a separate criterion.

The distinction matters for the language of the conclusion. The researcher may find that the reciprocal claims lack support in the accepted dossier. That finding is narrower than declaring that the officer or branch has no legitimate authority under any possible account. Another theory may treat a sufficiently established practice of mutual recognition as constitutive. Such a theory needs an account of the practice's participants, stability, scope, and normative relevance. The two reciprocal documents alone should not be allowed to supply all those conditions invisibly.

24.3 Typed Premises and Finite Grounding

The logical construction is a finite restricted application of positive-rule semantics. Van Emden and Kowalski connect model intersection, consequence closure, and iterative derivation [16, §§5–8]. The institutional interpretation and its warrant conditions remain the separate subject here.

The following construction gives one precise interpretation of the accepted-anchor approach. Let $\mathcal{B}$ be a finite set of typed propositions relevant to the dossier. Examples concern the occurrence of an appointment, the holder of a role at a given time, the applicability of a rule, and the assignment of a named status. An assertion of present legitimacy can be included as another proposition when the inquiry supplies appropriate normative premises. Its separate type prevents institutional efficacy from becoming a moral conclusion through a change of vocabulary.

Let $S \subseteq \mathcal{B}$ contain the premises accepted at the declared boundary of the inquiry. Let $\mathcal{R}$ be a finite collection of positive rules (B, h), where $B \subseteq \mathcal{B}$ is a finite set of premises and $h \in \mathcal{B}$ is the conclusion. The rule licenses h when every member of B is accepted. Its interpretation needs independent justification. A rule with an empty premise set functions as an unconditional anchor and requires the same scrutiny as a member of S.

Define successive grounded sets by

$$K_0 = S, \qquad K_{n+1} = K_n \cup \{h : (B, h) \in \mathcal{R},\ B \subseteq K_n\}. \tag{56}$$

The construction adds conclusions supported by premises already available at the previous stage. Its stages are logical stages of derivation. They need not coincide with historical time or the order in which the researcher discovered the evidence.

Proposition 24.1 (Least finite grounded conclusion set)**.** *The sequence in Equation* (56) *stabilizes after at most $|\mathcal{B} \setminus S|$ strict growth steps. Its eventual set K^* is the least set containing S and closed under $\mathcal{R}$. An atom belongs to K^* exactly when it has a finite derivation tree from seed premises or unconditional rules.*

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Proof. The sequence increases and remains inside the finite set $\mathcal{B}$. Every strict growth step adds at least one previously absent atom, giving the stated bound. Once an update adds nothing, the same rule application adds nothing at every later stage. The eventual set is therefore closed and contains S.

Let M be another closed set containing S. Induction gives $K_n \subseteq M$: the claim holds for K_0, and every newly added head has all its premises in M, so closure puts the head in M. Hence $K^* \subseteq M$. Since K^* itself is closed, it is the least such set. Equivalently, it is the intersection of all closed sets containing S; that family is nonempty because $\mathcal{B}$ belongs to it.

For finite derivability, give every seed a leaf. A head added at the next stage has a rule whose finitely many premises already have derivation trees; attach those trees beneath the rule's head. An unconditional rule supplies a licensed leaf. This constructs a finite tree for every grounded atom. Conversely, induction on the height of any finite derivation tree places its conclusion in the closed set K^*. The two descriptions therefore agree. □

The result is an elementary closure argument. Its value here lies in making the proposed warrant discipline explicit. It distinguishes an asserted conclusion, a conclusion compatible with the rules, and a conclusion connected to accepted premises through a finite argument. The mathematical existence of K^* supplies no guarantee that its seed facts or inference rules are well warranted.

24.4 Compatibility, Grounding, and the Reciprocal Certificate

Represent the officer's authority by atom o and the branch's standing by atom b. Suppose the retained rules are $o \to b$ and $b \to o$, with an empty seed set. The first update remains empty, so the least grounded set is empty. The larger set $\{o, b\}$ is also closed: whenever it contains a premise, it contains the corresponding conclusion. Thus rule compatibility and grounded derivability yield different assessments of the reciprocal claim.

Adding an accepted seed o changes the result. The next update adds b, and both conclusions become grounded. The formal difference identifies the substantive question precisely: what warrants the officer's authority independently of the disputed branch standing? The investigator can now examine the claimed earlier appointment and the procedure under which it occurred. The case has become an evidence task with a named dependency.

The same reasoning applies if b has an independent basis. A competent external act may establish its standing and thereby authorize the appointment. Alternatively, the participants may establish a new collective through an accepted founding practice. That founding can warrant a present standing atom while leaving the historical identity atom outside the grounded set. The rule connecting present standing to predecessor identity would require an additional criterion.

This separation is especially useful when one word covers several statuses. A branch may have the practical authority to coordinate its consenting participants, recognized standing in a federation, and a contested claim to the predecessor's registration. The first status can be grounded in one practice and the second in another while the third remains unresolved. A single atom called "legitimate successor" would combine the questions before any argument had been supplied.

Absence from the least set also needs careful interpretation. The dossier may omit a true earlier appointment or lack a rule that an adequate institutional interpretation would accept. Nonderivability reports the limits of the supplied argument. It should trigger either further evidence, a review of the rule, or an appropriately qualified finding. It supplies a proof of falsity only if a further completeness principle has been justified, and the present construction contains none.

24.5 Alternative Foundations and Preserved Support Routes

Authority can have several independently warranted routes. Suppose an officer's standing is supported both by an earlier appointment and by a later competent confirmation. Either route may suffice under the accepted institutional rule. A displayed proof tree can show one route clearly, but the research record should retain the other. Revising the first premise then need not remove the conclusion.

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The positive-rule model captures this possibility by allowing several rules with the same head. Their bodies identify different sufficient premises. The derivation of one head can therefore have several trees. A certificate of derivability should be understood as one witness, while the provenance record preserves the relevant alternatives. The distinction parallels duplicate support in the premise-repair model.

Source independence is a separate question. Two documents may support the same historical appointment because one copied the other. They can provide two physical records of one testimony without establishing two independent foundations. The logical head may be the same, but the evidential routes share a source dependency. A critical reader needs that dependency to assess how much of the argument survives a challenge to the original record.

An inquiry can therefore maintain two connected representations. The rule graph shows which propositions support which institutional conclusions. The source graph shows which records warrant the propositions and how those records depend on one another. A challenge to an inference rule differs from a challenge to a document's authenticity. Keeping the graphs linked allows either challenge to be traced to the conclusions it affects.

This approach also permits local inquiry without pretending to derive every institution from an ultimate origin. A study may take a federation's accepted appointment practice as a declared background because the dispute concerns a later branch decision. If the objection later concerns that background, the study can enlarge its boundary. The original finding remains conditional on the earlier acceptance. The boundary is an explicit limit of investigation, not a claim that the institutional foundation is beyond criticism.

24.6 Mutual Support and the Case for a Different Warrant Discipline

A critic can object that institutional authority often depends on an ongoing system of mutual recognition. Participants recognize roles, role holders maintain procedures, and the procedures organize the participants whose recognition matters. Such a practice may have a coherent structure even when an investigator cannot identify a single historically independent founding act. A purely linear picture of authorization can therefore miss the organization of the practice.

The finite-grounding construction need not deny that possibility. Its inference stages are acyclic in their justificatory use, but their premises can describe a reciprocally organized practice. For example, an accepted seed can concern an observed pattern of mutual recognition, with a separately defended rule specifying when that pattern constitutes the relevant standing. The construction then asks whether the seed and rule are warranted. It avoids treating the words "mutual recognition" as a complete institutional theory.

There is also a stronger alternative. A theory may permit mutually supporting conclusions without any accepted description of the supporting practice at the boundary. That theory would use a warrant relation different from finite derivability. Its defense should explain how it discriminates an effective, appropriately constituted institution from an unsupported pair of reciprocal assertions. The empty and full closed models in the officer example show the formal choice it faces. Choosing the fuller model requires a reason beyond closure under the two conditionals.

The philosophical issue is therefore the adequacy of a discrimination standard. A foundation may be too demanding if it requires records that historically workable institutions never produced. A coherence standard may be too permissive if any internally consistent claim can authorize itself. The research design should expose the evidence and institutional conditions that separate those possibilities. The present paper recommends finite grounding for the bounded dossiers in which accepted anchors and positive rules can be specified. Other cases remain candidates for a richer account.

This qualified recommendation preserves the purpose of ontological suspension. The paper can examine whether a particular argument is grounded in its stated premises without claiming that every form of social existence must have a corresponding finite derivation. A formal warrant discipline and a complete metaphysics of institutional dependence remain different projects.

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24.7 Later Recognition and Earlier Institutional Time

An authority finding has at least two relevant times: the time of the act being evaluated and the time at which the finding is made. A later record can reveal a valid earlier appointment. In that case, the researcher's warranted belief changes while the alleged appointment remains located in the earlier history. A later institutional act can also grant authority prospectively. That act changes the organization from its effective time under the applicable rule.

Ratification introduces another possibility. A later body may possess a specified power to approve an earlier act or assign it a retrospective institutional effect. The analysis then needs the ratifying body's authority, the scope of the ratification rule, the act it addresses, and the consequence it produces. A rule that makes an earlier decision effective from a stated date differs from a claim that its original issuer already had authority at the moment of issuance.

The distinction can be represented with typed atoms. One atom records original competence at the earlier time. Another records later ratification. A third records the resulting effective status under the ratification rule. A justified implication can connect the second and third without implying the first. Where original incompetence is independently established, this structure preserves that finding while allowing a later institutional consequence. Ratification alone leaves original competence to its separate evidential inquiry.

The same care applies to revival. A newly recognized body can receive permission to use an earlier name, administer an archive, or complete an undertaking. Those permissions may give the body a compelling practical relation to the past. Whether they also establish numerical identity depends on the criterion and the recognizing authority's domain. A prospective stewardship assignment can succeed without a finding of uninterrupted persistence.

An inquiry should therefore avoid ordering its argument solely by the dates on which documents were published. A retrospective certificate, a newly discovered old minute, and a prospective recognition act can enter the archive on the same day while affecting different time-indexed propositions. Their separate roles determine which earlier findings require revision.

24.8 Two Competent Revivals and the Identity Constraint

Suppose two groups of former participants each establish a competent present collective through an accepted founding practice. Each adopts part of the predecessor's programme and claims to revive its association. A positive-rule dossier can ground the competent formation of both. It can also establish genuine transmissions of skills, materials, or undertakings to both. These results are compatible with distinct simultaneous units.

The conflict appears only when an additional rule converts each competent formation into identity with the same predecessor. Combined with equivalence and slice separation, those two conversions are inconsistent. The branching argument identifies the location of the conflict. Competent formation alone can be retained; the two identity bridges require review under the chosen target.

Several coherent resolutions remain possible. One criterion may privilege a particular earlier appointment and select one branch. Another may treat both present collectives as new organizations carrying different parts of the predecessor's history. A larger collective unit may include both branches if the evidence warrants that organizational normalization, although this changes the appearance domain. The analysis should identify which assumption each resolution uses.

Recognition by different institutions can produce another layered result. A federation can recognize one branch for membership while an archive custodian recognizes another for stewardship. The institutions may be acting competently within distinct domains. Their decisions conflict only if they are interpreted as answers to one exclusive identity question under a common criterion. The record should preserve their actual scopes before declaring an institutional contradiction.

The case gives a positive role to the formal model. It allows multiple successful present formations and plural historical connections, then isolates the additional claim that produces a logical obstruction.

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The investigator can challenge that claim without discrediting every act performed by the competing participants.

24.9 Historical Belonging and the Standing of Other Participants

A revival can be meaningful to participants because it places earlier work within a history they now share. That meaning is itself a social fact worth recording. Its significance can extend beyond the formal institution: practices, memories, and relationships may connect people whose present organizations remain distinct. A precise identity model should make room for these connections.

The claim to a shared history also has other subjects. Former participants who joined another branch may have contributed to the same earlier work. Beneficiaries may have relied on an undertaking that the revival selectively acknowledges. An archive may contain records entrusted under conditions that constrain its use. The present group's decision can express its relation to that history without exhausting the claims of everyone affected by it.

This supplies a reason to separate historical affiliation from exclusive appropriation. A group can celebrate its connection to an earlier programme while acknowledging the other routes through which the programme continues. A claim of sole numerical identity requires a further argument. A claim of sole ownership or exclusive moral credit requires yet another, whose premises concern the relevant rights and contributions.

The research record should preserve the voices and relations needed to evaluate those arguments. An official certificate can settle a particular standing while leaving former members' contributions intact. A participant's rejection of the certificate can reveal a disagreement about authority or affiliation without automatically invalidating its institutional effect. The point is to identify each claim's object and grounds, then allow its criticism to proceed on those terms.

The section's proposal consequently has three connected parts. Present acts receive typed descriptions and separate authority warrants. Their conclusions are traced through an explicit inference discipline, with alternative support and objections retained. Any conversion into a retrospective identity enters the global constraint audit. This sequence preserves the practical force of revival while making the historical and philosophical commitments of a particular reidentification reviewable.

25 Explanatory Adequacy and Group Agency

This section examines what a continuity model contributes to explanation. It separates identity tracking from the causal efficacy of a collective pattern, compares behavioral similarity with historical transmission, and states the additional questions required for group agency. The purpose is to preserve the explanatory ambitions of the relational approach while keeping the reach of its formal results precise. A complete constructed allocation case then tests the relation between successful prediction and a historical identity verdict. The final discussion develops a proposed validation design that separates source recovery, criterion application, explanatory mechanism, and the effects of using the resulting classification.

25.1 The Explanatory Work of a Collective Unit

A collective unit can organize an explanation by locating coordinated activities, resources, decisions, and constraints within a common pattern. The usefulness of that unit depends on what the explanation seeks to understand. A committee structure may explain how proposals become decisions. A trained team may explain how an activity continues despite changes in individual participation. A registration may explain access to an institutional opportunity. These explanatory roles can justify different aspects of a collective representation.

The entity label alone does little explanatory work. Saying that an association persisted because it was the same association restates the identity judgment. An explanation should identify the mechanisms through which relevant relations continued: recruitment, training, rule-governed replacement, resource

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maintenance, commitment renewal, or another documented process. The identity path can then organize the resulting history without substituting for those mechanisms.

This is the constructive sense in which the relational approach supports entity modeling. A collective appearance is connected to the configuration and the relations that sustain its activity. Changes in those relations can explain formation, persistence, division, or loss of capacity. The continuity criterion selects which changes matter to the focal entity question. The explanatory account should make that selection answerable to the mechanisms and evidence.

25.2 Behavioral Similarity and Historical Connection

Two organizations can have the same observable response profile under a specified set of tests. They may perform the same activity, make the same decisions, or use the same procedures. That similarity supports a claim about behavior under those tests. It leaves numerical identity open because distinct organizations can instantiate the same profile.

Conversely, a candidate continuing organization can change its behavior. It can adopt a new procedure, replace a programme, or respond differently to a changed environment. If the identity criterion permits such change, behavioral difference is compatible with continuity. An invariant response profile would then be an overly restrictive criterion. The appropriateness of behavioral evidence depends on the question and the permissible changes.

Historical transmission adds another dimension. An operation inherited through trained members and coordinated handover differs from an independently developed operation with the same observable outputs. The difference can matter to an inquiry into provenance or continuing practice. It may have little relevance to a narrowly defined prediction task. A good model states which question requires the historical distinction and which can legitimately ignore it.

This separation also explains why successful prediction cannot establish identity by itself. A model trained to predict activity may perform well with a coarse behavioral class containing several organizations. Its success shows adequacy for the prediction target under the evaluation conditions. Recovering a particular entity from that class requires additional discriminating information. The earlier recovery theorem states the formal structure of that requirement.

25.3 Causal Efficacy and Evidence of Mechanisms

The relational proposal connects collective representation to causal efficacy. An organization matters because its arrangement affects what can happen. The present paper retains that insight while separating a causal claim from a descriptive association. A pattern that precedes an outcome may be a cause, a correlated indicator, or an effect of another condition. Its causal interpretation requires a model of the relevant alternatives and evidence appropriate to that model.

For a collective operation, useful evidence can include the sequence of training and execution, the role of a coordination procedure, or the consequences of a documented loss of access. Such evidence can support a mechanistic explanation without requiring destructive interventions on an organization. The analysis should nevertheless distinguish observation, counterfactual reasoning, and an actual intervention. Each provides a different kind of warrant.

A hypothetical removal of an edge in a relational model can show that the represented unit depends on that relation under the model's assumptions. It does not establish that removing the corresponding social relation would have the same effect in an actual case. Participants may adapt, replace the relation, or change the activity. The interpreted mechanism must therefore include the relevant possibilities of response. This limitation is especially important when a model is used to assess the resilience of a continuing organization.

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25.4 Agency and Additional Organizational Conditions

Group agency raises a further question about the organization of collective action. A lineage can connect organizations through membership or resource transfer even when none satisfies an accepted criterion of agency. An identity partition can be coherent even when its classes contain units whose activities lack coordinated decision procedures. The formal results therefore leave the agency question open.

An account of agency may require organized goals, representation of relevant conditions, procedures for action, responsiveness to reasons, or other features. The present paper does not choose among detailed theories of agency. It asks the investigator to state the adopted conditions and examine their relation to the continuity criterion. A collective may retain its recognized identity while gaining or losing some agency-related capacity. A reorganization can also create a new arrangement of agency without transferring every earlier identity relation.

The distinction matters for responsibility. Even if a group is an agent, the grounds for attributing a particular act or undertaking to it need historical evidence and a normative account. Identifying an appearance across time is one part of that argument. It should not carry the entire burden of explaining intention, control, knowledge, and accountability. Those questions need the organizational structure relevant to the act.

25.5 Criteria of Explanatory Adequacy

An adequate continuity model should preserve distinctions needed for its explanatory target, support coherent judgments across the relevant history, and identify evidence that could challenge its conclusion. It should also avoid treating irrelevant detail as compulsory. A membership inquiry may not need every internal communication; a commitment inquiry may need one specific authorization record that a large activity dataset omits.

Adequacy also requires comparison with alternatives. If two criteria make the same predictions in every available case, their empirical comparison remains limited. If they disagree only on a philosophical commitment about what counts as the entity, prediction alone may not resolve that dispute. The study should state which part of the criterion is being tested and which part is being defended through argument.

The resulting method can retain a plural account of one history. It can identify a continuing registration, a branching operation, and a shared lineage of knowledge, then explain how these relations interact. Such an account is often more informative than forcing every continuity into one unqualified entity label. Its philosophical value lies in the disciplined connection between questions, representations, and warrants.

25.6 Separate Targets in the Assessment of a Criterion

Criterion validation needs a specified object of assessment. This subsection distinguishes four targets that can otherwise be compressed into the claim that a continuity model works: recovering a past event, applying a declared rule, predicting activity, and supporting a practical intervention.

Recovery concerns the relation between retained evidence and the historical query. If the question asks whether a valid appointment occurred before a division, the assessment should concern that appointment, its effective conditions, and the observation process. A later pattern of activity can be relevant when a separately defended connection makes it informative. Its temporal position alone supplies no guarantee of such a connection.

Rule application concerns whether the procedure follows its declared criterion on the accepted facts. A coding system can apply a membership threshold accurately even when the threshold lacks a persuasive connection to the intended collective object. Conversely, a defensible criterion can be applied incorrectly because a roster contains duplicates or an amendment date has been misread. These failures call for different corrections. A validation report should preserve whether the disagreement concerns the rule’s rationale or its execution.

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Table 11: Constructed historical verdicts and programme responses under two allocation policies.

History	Earlier continuation of a in N	Sessions under p_N, ordered (N, S)	Sessions under p_S, ordered (N, S)
h_N	Affirmed by the fixed criterion	$(8, 2)$	$(2, 8)$
h_S	Denied by the fixed criterion	$(8, 2)$	$(2, 8)$

Prediction concerns an outcome under stated conditions. A model can forecast programme attendance, continued access to a venue, or the next recorded registration entry. Accuracy on that target is a substantive achievement. Its relation to numerical identity depends on what the forecasted outcome reveals about the relevant history. A predictor may succeed by using stable features shared by several distinct entities. That success can make it useful for planning while leaving the identity query unresolved.

Practical intervention concerns the consequences of acting on the model's output. A funder may use a continuity finding to allocate resources; an archive may use it to arrange collections; a collective may use it to coordinate a future programme. The action can alter later observations. Assessment then needs to distinguish whether the model described an earlier condition accurately and whether the action it guided achieved an accepted purpose. The second achievement cannot automatically certify the first.

These targets can support one another when their connections are explicit. A well reconstructed appointment history can improve a forecast of who will be invited to an institutionally restricted meeting. An accurate forecast can then support a planning decision. The sequence contains several links, each with its own assumptions. Keeping them visible allows an investigator to identify where a successful prediction supplies indirect historical evidence and where it merely tracks another feature of the case.

25.7 A Constructed Allocation and Performance Dossier

The following dossier provides a counterexample to an unrestricted inference from predictive success to historical reidentification. It fixes the target and specifies the two candidate histories, then gives identical programme responses under each of two possible allocation policies.

Let a predecessor a divide into distinct simultaneous appearances N and S. An independently accepted criterion in this example identifies a with the branch that received an effective appointment to a continuing programme office before a funding decision. The criterion's adequacy is a premise of the comparison. The example evaluates the recovery of its answer; it leaves the criterion available for philosophical challenge.

The accepted historical family contains two possibilities. In h_N, a competent appointment placed the continuing office in N. In h_S, the corresponding effective appointment placed it in S. The events differ in the underlying histories, while their documentary records are absent from the retained observation. Both histories contain equal practical capacities in the branches before funding. The office difference has no effect on the programme counts specified below. These are stipulated features of a constructed model, chosen to expose a logical relation between assessment targets.

A policy p_N gives N resources sufficient for eight sessions and S resources sufficient for two. A policy p_S reverses those allocations. Assume that the branches conduct the corresponding number of sessions under each policy in either history. The quality and other retained descriptions of the sessions are also identical across histories for a fixed policy. Table 11 states the complete response comparison. The numbers are illustrative; the argument depends on equality of the observations across histories with opposite identity verdicts.

Suppose p_N is implemented and the observed counts are eight sessions in N and two in S. A forecast of those counts is correct in both candidate histories. The observation therefore demonstrates a successful programme prediction within the construction. It supplies the same record whether the predecessor continues in N or in S under the fixed historical criterion.

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25.8 Derivation of the Validation Limit

The recovery analysis follows directly from the observation-fibre condition developed earlier. This subsection states the steps because the example's force depends on keeping the historical target fixed throughout the comparison.

Let q ask whether a continues in N at the relevant post-division time under the appointment-based criterion. By construction, $q(h_N) = 1$ and $q(h_S) = 0$. The actual-policy observation places both histories in the same fibre because each produces the pair $(8, 2)$ under p_N. A function of that observation would have to return one value for the shared input. It cannot return both of the values required by q. Exact recovery of the historical query from that retained performance observation is therefore impossible within the accepted family.

Now strengthen the predictive information. Suppose the investigator knows the complete stipulated response profile for both policies, rather than only the realized counts. Each history still has the profile consisting of $(8, 2)$ under p_N and $(2, 8)$ under p_S. The richer observation leaves the two histories in one fibre. Perfect prediction over the entire stated policy domain remains compatible with failure to recover the historical identity verdict.

The result identifies the missing inferential connection. Programme response is controlled by the allocated resources and the equal practical capacities in this construction. The earlier appointment varies without changing those responses. A claim that successful programme prediction validates the appointment-based identity verdict would need an additional premise linking the responses to that appointment. The stipulated histories show that such a link is substantive and cannot be supplied by the word "success."

A reliably observed appointment record could separate the two histories. The investigator would need grounds for its content, effective conditions, and relevance to the criterion. Once those grounds refine the accepted family to one history, the target becomes recoverable. The programme forecast could remain unchanged throughout that evidential improvement. Historical knowledge would have increased even though predictive accuracy stayed constant.

This argument leaves open cases in which activity does reveal continuity. A specified programme may require a capacity transmitted only through one organizational route, with independent evidence excluding alternative acquisition. Under those stronger assumptions, later performance can support an inference about transmission. Its support comes through the defended mechanism and excluded alternatives. The general validation lesson is to identify that mechanism before allowing a successful outcome to settle a historical question.

25.9 Feedback from Classification to Later Activity

The allocation dossier also isolates a possible feedback route. A published continuity classification can influence resources, and those resources can influence the later activity used to assess the classification. This subsection examines that route as a conditional possibility, with the historical and practical questions kept separate.

Suppose an investigator initially names N as the continuing association. A foundation relies on the published finding when implementing p_N. The greater subsequent activity of N is then cited as confirmation that the investigator identified the branch that truly continued the programme. Within the constructed histories, the increased activity follows the allocation in either case. Its occurrence can confirm the resource-response model while leaving the earlier appointment unresolved.

The timing of the argument matters. If the criterion had originally been defined to select the branch that would conduct the most sessions after the foundation's own award, the classification and allocation procedure could jointly create the feature used by that criterion. Such a rule might serve a practical coordination purpose, but its target differs from reconstructing which branch inherited an earlier office. Retaining the same word "continuation" for both targets would conceal the change.

The investigator's influence can be less direct. An archive's cataloguing choice can affect which records later researchers discover. Public recognition can influence who applies for membership. A

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programme partner can redirect referrals to a selected branch. These are possible mechanisms requiring case evidence, not universal effects attributed to every classification. A study that depends on later observations should examine whether its own output participated in producing them.

That examination can be constructive. If the research purpose includes helping a collective maintain an undertaking, the effects of a classification may be relevant to evaluating the intervention. The analysis should then state the accepted undertaking, the action taken, the observed consequence, and the causal alternatives. Success in maintaining the undertaking would be reported as an intervention result. The earlier identity finding would retain its own evidential basis and uncertainty.

The distinction also protects criticism from an opposite excess. A classification's influence on events does not by itself make its historical content false. An accurately identified successor can receive funding because of that accurate identification. The point is that the later performance alone cannot adjudicate the historical truth when the classification itself helped produce the performance. Independent historical evidence can support the truth, while a separate account explains the classification's consequences.

25.10 Evidence Reserved for a Historical Assessment

A proposed validation design should reserve evidence that bears directly on the target and whose production or interpretation is sufficiently distinct from the procedure being assessed. This subsection develops the proposal without assuming that documentary independence is easy to achieve.

For the appointment case, one possible design would first reconstruct the branches and apply the criterion using a specified set of records. A separately held appointment record could then be examined through an independent authority inquiry. The comparison would ask whether the initial procedure recovered the result supported by that later inquiry. The appointment record would function as a further source of evidence, with its own fallibility and scope, rather than an automatically infallible answer key.

The held record must be independently informative in the relevant sense. Keeping a copy in another folder does not change its source dependence. A later certificate may simply repeat the initial report. An official archive may preserve a branch's submitted statement without investigating its accuracy. The study should trace these relations before treating agreement as corroboration. The source-bundle model in Section 21 shows one precise way in which apparently separate observations can share a failure route.

Independence of interpretation deserves similar attention. Two researchers can read different records while relying on the same contested criterion bridge. Their agreement may show consistency in applying that bridge, while the bridge's philosophical warrant remains open. A validation design should therefore record which propositions are independently evidenced and which are common assumptions. Otherwise, repeated agreement can mask dependence at the level of interpretation even when the documents have separate origins.

Reserved evidence can also be selected badly. If the investigator chooses only records expected to confirm the initial finding, the procedure's ability to discriminate opposing histories remains untested. A useful selection identifies possible outcomes that would support different verdicts and states in advance how each outcome would affect the accepted family. This is a proposal for disciplined comparison, not a requirement that every historical inquiry follow one experimental format.

Historical research often encounters a decisive source only after a substantial reconstruction. The source's late discovery can still improve knowledge. The report should distinguish discovery during exploratory work from a previously specified assessment. The difference affects how one evaluates the procedure's demonstrated reliability; it leaves the source's substantive content available for examination in either setting.

25.11 Mechanism Comparison through Contrasting Histories

Explanatory adequacy requires more than testing whether an entity label matches another label. This subsection proposes comparing mechanisms that could sustain the same observed programme while differing in their relation to collective continuity.

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One history may contain direct transmission of a practical routine through experienced participants. Another may contain independent reconstruction from a publicly available manual. A third may contain continuing control by a predecessor organization that delegates performance to a newly formed local team. Equal output counts can fit all three. The explanatory task is to identify what trained the participants, who coordinated the work, which relations maintained access, and how the practice responded to change.

These mechanisms generate different documentary expectations. Direct transmission can leave a sequence of joint work and handover. Independent reconstruction can leave learning records and adaptation without predecessor supervision. Delegation can leave instructions, reporting relations, and retained decision authority. Such expectations are proposals for evidence search. Their absence is informative only when the observation process could reasonably have preserved them, as the dormancy analysis illustrates.

A criterion centered on operational transmission may distinguish the first history from the second while treating the delegated team's independent collective identity as a separate issue. An office-centered criterion may track the supervising body through the third history even when most visible activity occurs elsewhere. Comparing the accounts can reveal that their apparent disagreement concerns different explanatory objects. It can also reveal a substantive conflict when both claim to identify the same continuing organization through incompatible mechanisms.

The useful comparison identifies what each mechanism explains that its alternative leaves unexplained. A continuing coordination procedure may explain orderly replacement of participants. A local training route may explain why a new branch preserves a skill despite institutional separation. A resource allocation may explain increased output without historical continuity. These explanations can coexist in a plural account of the episode. A collective identity partition organizes selected relations within that account; its adequacy depends on preserving the distinctions required by the chosen explanatory question.

Mechanistic comparison also creates a disciplined role for counterfactual reasoning. The investigator can ask which represented activity would remain available if a particular coordination relation failed, while stating the permitted adaptations. A model in which participants cannot replace a missing resource may predict termination; a model permitting substitution may predict reorganization. The different results reveal assumptions about adaptation. Actual organizational resilience requires evidence concerning which responses participants can undertake.

25.12 A Validation Record for Competing Criteria

The preceding distinctions can be assembled into a reviewable assessment record. Its purpose is to make competing criteria answerable to evidence and argument without pretending that every disagreement has the same kind of resolution.

For each criterion, the record should state the collective object, the temporal target, permitted changes, and the events or relations that make a continuation eligible. It should identify the records used to establish those events, the normalization through which they refer to appearances, and the global constraints imposed on the resulting identity history. The criterion's output can then be compared with alternative outputs under clearly identified changes of evidence or assumptions.

The assessment should separately report extraction accuracy, application consistency, predictive performance where prediction is claimed, and the historical discrimination supplied by additional evidence. A correction to a source reading can change the result without undermining the criterion's rationale. A failure under a new case can expose an unjustified criterion restriction even when the source reading was accurate. A successful forecast can coexist with historical ambiguity, as the allocation dossier demonstrates.

The record should also identify the consequences of using the finding. If the output guides funding, registration, or public affiliation, subsequent activity belongs partly to the history of that use. The study can examine those effects through a separately stated purpose. Keeping the assessment targets distinct makes it possible to say that a procedure is useful for one task, unreliable for another, and unresolved with respect to a third.

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Finally, the comparison should retain the reasons for preferring a criterion within its scope. A preference can be grounded in its ability to preserve the relevant historical mechanism, discriminate a documented transition, avoid an unsupported identity bridge, or serve an explicitly authorized decision. The preference should be revisable when those grounds fail. This form of validation connects the mathematical construction to substantive research while preserving the philosophical question of which collective object the inquiry ought to track.

26 Historical Actions and Responsibility after Reorganization

A continuity finding often matters because someone wants to attribute an earlier action, preserve an undertaking, or identify a present party capable of responding to its effects. This section separates those questions and examines the inferential role of reidentification in each. It first distinguishes historical actor records from current claimant associations, then confronts opposing accounts of corporate responsibility, and finally develops a divided-undertaking case. The aim is to preserve the reasons that make collective continuity practically important while preventing one identity decision from silently determining every subsequent attribution.

26.1 Historical Actor Records and Present Identity Histories

An action record has a historical subject. It can identify the committee that issued a decision, the association that adopted an undertaking, or the participants who performed a coordinated activity. The warrant for that attribution depends on the relevant action concept and evidence. A current claimant's connection to the historical subject introduces a second relation: the proposed continuity between the earlier actor and the claimant now being considered.

Let appearances and events take their times in a common linearly ordered domain T. Let $\mathcal{L}$ be a set of events with time map $\tau_{\mathcal{L}} : \mathcal{L} \to T$, and let $\mathrm{Act} \subseteq I \times \mathcal{L}$ be a separately interpreted attribution relation. An attributed appearance and event share a time:

$$\mathrm{Act}(a, e) \implies \tau(a) = \tau_{\mathcal{L}}(e). \tag{57}$$

The relation may associate several actors with one joint event. It may also distinguish an institutional attribution from the actions of particular people who brought the event about. The choice of those predicates belongs in the action record.

For a historical appearance a, identity candidate E, and current slice t, define

$$S_E(a, t) = \{j \in I_t : aEj\}. \tag{58}$$

This set has at most one member under slice separation. If j and k both belong to it, symmetry and transitivity give jEk, and their common time gives $j = k$. The set can be empty when the historical identity has ended or has no represented appearance at the current slice.

For current appearance $j \in I_t$, define its associated past-event history by

$$H_{E,t}(j) = \{e \in \mathcal{L} : \tau_{\mathcal{L}}(e) \leq t,\ \exists a\,[\mathrm{Act}(a, e) \wedge aEj]\}. \tag{59}$$

The definition connects a current claimant to a recorded historical actor under a proposed identity. It preserves the original actor and event time. Saying that the claimant's identity history contains the event has this relational meaning; the present time-indexed appearance is not being placed physically in an earlier event.

Changing E can change this association while the historical action record remains fixed. If a divided predecessor performed an event, one identity candidate can connect that actor to branch b and another to branch c. Their disagreement concerns current reidentification, even when they agree completely about the earlier action. A research record should preserve that agreement rather than allow the disputed successor to make the original event attribution appear uncertain.

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The uniqueness result applies per historical actor. A joint event can involve two distinct organizations, each with its own current successor. Their present histories can both contain an association with that event. Treating the event itself as requiring one exclusive current claimant would add a restriction absent from the actor-level identity argument.

26.2 Corporate Responsibility and Individual Control

The relation between organizational action and moral responsibility remains contested. Pettit distinguishes causal attribution, accountability, and fitness for blame or approval. His account makes significant choice, relevant evaluative judgment, and responsive control central to responsibility, and permits corporate and individual responsibility to coexist [8, §§I,V]. Moen challenges the corporate-responsibility defense by emphasizing individual agenda control and strategic voting, including the capacity to contribute to an outcome one did not sincerely favor [6, §§4–5].

The disagreement matters to a continuity inquiry because an organizational record can omit precisely the action and control relations on which these arguments turn. A record of the final collective decision can conceal which people set the agenda, which alternatives were available, what participants knew about the procedure, and how the decision was implemented. A roster can identify formal membership while supplying little information about those roles. Reidentifying the organization across a later merger leaves the omitted evidence to be reconstructed.

The paper's proposed response is to preserve the relevant relations at several levels. The record should retain the procedure through which an organizational outcome was produced, the individuals who occupied decisive roles, their available information, and the actions or omissions attributed to them. A defended theory of corporate responsibility can then use the organizational history without erasing individual agency. A defended individualist account can use the participant history without discarding the procedure and coordination that made their contributions effective.

This response does more than postpone a theoretical choice. It identifies a common evidential requirement that the identity partition cannot replace. The relevant responsibility theory must connect its normative conditions to particular historical facts. An identity criterion that follows the largest successor membership provides little information about the agenda setter's control during an earlier decision. A criterion that follows an institutional appointment can preserve standing while leaving the participants' knowledge unresolved. The evidential bridge to responsibility requires those additional facts.

The continuity inquiry should also preserve the difference between actual control and a merely imaginable alternative. An analyst may demonstrate that a different vote could have changed an outcome. Whether that alternative was available to the person, understood by them, institutionally admissible, or morally permissible requires further argument. These qualifications can affect the attribution of responsibility while the mathematical description of the voting procedure remains unchanged.

26.3 Available Alternatives and the Meaning of Control

Control claims become more precise when their counterfactual domain is stated. One possible domain varies an individual's action while holding everyone else's action fixed. Another permits changes in communication or coordination among several participants. A third permits redesign of the decision procedure. These comparisons can identify different opportunities to influence an outcome, and they can attach to different actors at different historical times.

Consider a constructed committee whose chair can choose between two authorized agenda orders. The order affects the final decision because an early item constrains the later options. The chair's control concerns the agenda-setting event. Other participants may control their own votes within the chosen order while lacking authority to alter it. A later organizational successor that inherits the committee's records is connected to this history through reidentification, but the earlier distribution of powers remains a separate fact about the decision process.

Now suppose that one voter could have blocked the final outcome by submitting a statement they believed false. A formal manipulation possibility has been established if the procedure would respond

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as described. A responsibility argument still needs to consider the voter's understanding, the norms governing truthful testimony, and any other available response. The existence of a causally effective false statement does not itself establish an obligation to make it. The analyst should distinguish capacity, permissibility, and duty even when one event brings all three into view.

The same issue arises with resignation or refusal to participate. A member may be able to leave a collective without being able to prevent its action. Leaving can change their association with the event, but its effect on the event itself depends on the procedure and the availability of replacement participants. A study should therefore avoid treating every exit option as equivalent to effective control over the collective outcome.

These examples sharpen the information needed for a continuity-based attribution. The relevant event may predate the split by years and involve actors who join different successors or leave all successors. Their control relations do not migrate automatically with a later majority roster. Preserving them separately permits the historical attribution to remain stable while the current organizational identity is disputed.

The broader philosophical lesson concerns the grain of the explanatory claim. A group-level procedure can explain how a class of outcomes is generated, while an individual action explains why a particular instance occurred through one participant's contribution. Whether both explanations support moral responsibility depends on the adopted account of agency and control. Their coexistence in the record is already useful because it prevents a choice of explanatory level from erasing historical facts relevant to the competing argument.

26.4 Obligation Kinds and Particular Succession Claims

Collins and Lawford-Smith examine promising and rectification through their required continuity relations, while allowing morally relevant succession to differ from numerical identity [1, §II.A]. The present analysis develops the narrower task of reconstructing a particular undertaking after division. That task requires several judgments which an argument about the kinds of obligations a group can bear leaves open.

The first judgment identifies the original undertaking. A public aspiration, an invitation to negotiate, and an accepted commitment can use similar language. A study needs the act, its content, its addressee, and the practice under which it acquired its force. The historical identity of the issuing organization helps identify the speaker, but it cannot determine what that speaker undertook. A reidentification method should therefore connect to a separately supported commitment record.

The second judgment identifies the conditions of fulfillment. An undertaking to provide a service may specify a result, a particular mode of delivery, or a named party's performance. Those conditions matter after reorganization. If the beneficiary agreed to a qualified team's instruction, transferring the organization's name to a branch without the relevant qualifications can preserve one institutional status while leaving performance unavailable. If the undertaking permits equivalent delivery by another qualified party, a change in organizational identity may have a smaller effect on fulfillment.

The third judgment concerns later acts. A successor can expressly accept an undertaking, a beneficiary can agree to a revised performance arrangement, or an institution can assign a defined obligation under an applicable rule. These are distinct possible grounds for the present position. The investigator should record which ground actually applies. The availability of one ground does not establish that every other ground applies as well.

The fourth judgment concerns present feasibility. A party can occupy the relevant institutional standing while lacking the capacity to perform by the deadline. Another party can possess the capacity while lacking an accepted undertaking. A complete practical argument must connect authority, commitment, and capacity under a defended policy. Reidentification supplies one relation among them.

This division of the task clarifies the logical distance between a general possibility and a particular verdict. Showing that some organized collectives can undertake obligations leaves open whether this association made this undertaking. Showing that a kind of succession can be morally relevant leaves open which transmission in the actual split has that relevance. Showing that a present branch can perform

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leaves open why it should bear the performance burden. The needed premises become more concrete at each step.

A critic may regard this structure as overly cautious when ordinary participants already know what the promise means. In an uncomplicated case, their shared understanding may permit a short finding. Reorganization makes the distinctions important when participants disagree or when the relevant roles have separated. The method expands the argument only where an inference is disputed, incomplete, or practically consequential.

26.5 Performance by Members and Organizational Continuity

An organizational undertaking is often performed through people who occupy roles. After a split, some of those people can remain willing to perform while their organizational affiliations change. A useful record should preserve the connection between the original undertaking, the performance roles, and the persons who now occupy or could occupy them. It should also preserve the terms under which those persons accepted the role.

Consider an instructional programme with a coordinator, several instructors, and a custodian of teaching materials. The old association's decision procedure assigns their roles. After division, the coordinator joins one branch, the instructors another, and the custodian remains unaffiliated. The original procedure's role distribution does not translate automatically into a complete present performance plan. The plan needs renewed coordination, access to the materials, and an account of the commitments that remain effective for each participant.

This case shows why an organizational capacity should be described relationally. Possessing the instructors without access to the necessary material can leave a branch unable to deliver. Possessing the archive without qualified instructors can have the same effect. A reidentification criterion that selects the branch with more members can fail to identify the coalition of roles needed for performance. The practical question concerns the available coordinated route.

The case also raises a burden question. Individual participants may already have personal commitments, limited time, or conditions on their voluntary service. An argument that an organization retains an undertaking does not settle how its performance costs should be distributed among them. The institutional arrangement and the applicable normative principles must specify that connection. A claim about collective obligation and a claim about each member's contribution can therefore require different reasons.

Overlapping membership makes careless counting particularly misleading. A person who belongs to both successor branches has one schedule and a limited set of capacities. Adding each branch's nominal capacity can count that person's available hours twice. A feasible joint plan needs a resource description that identifies shared participants and incompatible uses. The issue is practical overlap, even when the organizations are securely distinct.

Suppose each branch reports that it can provide sixty hours because both count the same instructor's twenty hours. If the other available hours are disjoint, their combined capacity is one hundred hours, not one hundred and twenty. The arithmetic requires the stipulated overlap and a common time window. If the instructor can teach the same session jointly for both branches and the beneficiary accepts that session once, the performance record should still count the delivered hours according to the undertaking's unit of fulfillment.

The example has a narrow lesson. Distinct organizational identities do not imply disjoint practical resources. Shared members, materials, and access rights can create dependencies between otherwise distinct claimants. A model that preserves their relations can assess a joint plan more accurately than one that treats each identity class as a self-contained capacity container.

26.6 An Undertaking Divided between Two Successors

A constructed undertaking case separates the practical question from the choice of one numerical successor. An association agrees to provide one hundred hours of instruction to a named beneficiary group by

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an agreed deadline. Before completion, the association divides into branches b and c. Branch b retains most members and the registered appointment; branch c retains the instructional programme and most of the people qualified to deliver it.

For the example, suppose b can supply thirty hours and c seventy hours within the deadline. The hours are divisible, equivalent for the specified undertaking, and compatible in schedule. The beneficiary accepts delivery by both branches, and each branch expressly accepts its stated part under the stipulated practice. These assumptions support a complete performance plan of thirty and seventy hours. They are chosen to make the practical arrangement explicit, rather than inferred from the branches' names or membership shares.

A registration criterion may identify b with the earlier association. An operational criterion may identify c with it. Under either criterion, requiring the selected numerical successor alone to perform all one hundred hours exceeds its stated capacity. The jointly accepted plan can complete the undertaking while the two identity criteria continue to disagree. The case therefore gives an example of practical coordination whose feasibility does not require a universal identity verdict.

Several premises make that conclusion possible. The performance can be divided without changing its value to the beneficiary. Both branches have the requisite qualifications for their portions. Their schedules permit completion. The beneficiary's acceptance matters under the stipulated practice, and the branches' uptake supplies a ground for their present commitments. If any of these conditions changes, the same numerical sum can fail to establish an acceptable performance plan.

For example, the undertaking may require one indivisible certification that only one branch can issue. Thirty plus seventy hours would then be an inadequate description of the obligation. Or the instructional relationship may depend on continuity with a particular educator whom neither branch can replace. In that variant, substituting equivalent hour counts can conceal a loss important to the beneficiary. The appropriate allocation model follows the content of the undertaking, including its nondivisible and relational features.

The case also distinguishes uptake from historical blame. A branch can accept a present share of performance without admitting that it committed an earlier wrong. A branch can be connected to the historical actor through identity while lacking the practical capacity to deliver the whole remedy alone. These findings can coexist. Their coordination requires an explicit account of the present undertaking and the reasons for assigning its performance.

26.7 Beneficiaries, Consent, and the Content of Continuity

The beneficiary's role prevents the undertaking from being reduced to a dispute among successors. A collective may reorganize its internal structure, but the significance of a commitment can depend on relationships with people outside that structure. Their expectations, reliance, and accepted terms form part of the question about continued performance.

In the instruction example, consent to divided delivery is one stipulated ground of the arrangement. It should be distinguished from an inference that all obligations depend on present consent. Duties can have other grounds, which require their own normative account. The example uses an explicit agreement to show how a present obligation can be stabilized without allowing numerical identity to do every part of the justificatory work.

Suppose the two branches prefer a division that is convenient for them but burdens the beneficiary with incompatible schedules. Their combined nominal capacity still reaches one hundred hours. The proposed performance fails to preserve a condition of the undertaking. A criterion that tracks only resource transfer or staff retention would miss the failure unless the beneficiary's requirements are included in the target.

Suppose instead that a third organization offers to complete the instruction under terms the beneficiary accepts. That organization can acquire a present undertaking while remaining distinct from the original association and both successors. Its role supplies a counterexample to the idea that every appropriate performance duty must follow numerical identity. The arrangement's justification comes from the newly established relation and the content of the accepted task.

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These possibilities explain why practical reasons for tracking a collective should be recorded before the continuity criterion is selected. If the actual concern is delivery of a service, the study needs the service's conditions and the capacities of possible performers. If the concern is authority to represent a community, the relevant relation may depend on a different set of participants and procedures. A common organizational history can support both inquiries, while their consequences follow different justificatory paths.

26.8 Membership Replacement and the Temporal Distribution of Responsibility

Membership replacement adds a temporal dimension to the distinction between historical attribution and present response. A current collective can contain people who played no role in an earlier action. Earlier participants can remain available outside the current organization. The continuity of a corporate unit therefore cannot be treated as evidence that the current roster reproduces the historical distribution of knowledge, control, or contribution.

A study can preserve a role history indexed to the event. It can identify who designed the relevant procedure, who supplied information, who made the decision, and who implemented it. It can also preserve opposition, absence, and constrained participation where the evidence supports those distinctions. This event-level record remains useful after the organizational roster has changed.

The normative significance of present membership then becomes an explicit question. A person may acquire duties through a current office, through an accepted undertaking, or through their capacity to help remedy an ongoing condition. Those grounds differ from blame for an earlier action. A defensible account can recognize present responsibilities without inventing past participation, and can preserve past responsibility without assuming continued membership.

The distinction is particularly important when successors compete to claim historical achievements while distancing themselves from historical failures. Such selective claims invite criticism of the continuity criterion and the consequence policy. The critic should identify the inconsistency precisely: whether the successor applies different identity rules, whether credit and blame legitimately require different premises, or whether a relevant factual history has been omitted.

Symmetric treatment of every consequence is not automatically required. Historical credit for inherited knowledge and a present duty to repair a harm may have different grounds. The appropriate demand is an explicit and defensible explanation of the difference. The method makes that explanation reviewable by retaining the identity criterion, the event record, and the additional normative premises used in each attribution.

26.9 Termination, Reconstitution, and Continuing Effects

The identity of a historical actor can end while its effects persist. Records, practices, damaged relationships, and unfinished undertakings can outlast the organization that produced them. An empty current-successor set in Equation 58 therefore leaves substantial historical and practical questions available.

This is one reason to preserve lineage alongside identity. A later collective may inherit resources, continue a programme, or occupy a role created by the earlier actor. Those relations can bear on present duties or historical understanding under a defended normative account. The lineage record identifies the factual connection without declaring that every connected successor is the same numerical agent.

Reconstitution supplies a related case. A new organization can revive an earlier collective's activity and accept its remaining undertakings. Its present role can be substantial even when the selected criterion treats the earlier identity as terminated. Calling the revival a new organization should therefore preserve the specific continuities through which it acquires its practical significance.

Conversely, a recognized continuing organization can lose the capacity that made a particular remedy possible. Its identity history remains relevant to attribution, while current performance requires an account of available resources and alternative actors. A response policy may involve several parties, with different reasons for their participation. Its justification cannot be reduced to the existence of one surviving identity class.

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These cases make termination an analytical finding with limited implications. It can mark the end of a particular numerical identity under a specified criterion. It does not erase the historical action, the individual actors, or the later relations through which its effects continue. Preserving those distinctions prevents a formal endpoint from becoming an unexplained closure of the practical inquiry.

26.10 A Structured Responsibility Finding after a Split

A reviewable finding can now be organized around the actual dependencies. It first states the historical event and the actor attribution supported by the evidence. It identifies the relevant procedures and individual roles. It then reports which current claimants are connected to that actor under each retained identity criterion.

The finding next states the normative question. Is the inquiry about blame for the past event, fulfillment of an undertaking, present authority to act, or an appropriate response to continuing effects? The answer specifies which additional facts and principles must be supplied. A role in one question should be carried into another only through an explicit argument.

For the instruction dossier, the report can retain the disagreement between the registration and operational identity criteria while recording the accepted thirty/seventy performance plan. It can state that the plan's adequacy depends on divisibility, qualifications, schedule compatibility, and the stipulated acceptance conditions. A later dispute about those conditions can be examined without reopening every fact about the split.

For a past harmful decision, the report can distinguish a secure historical actor record from uncertain current reidentification. It can preserve the evidence of individual control and organizational procedure for evaluation under competing responsibility theories. This retains the relevant information while the philosophical disagreement remains active.

The resulting method gives collective reidentification a definite practical role. It connects historical subjects to current claimants under inspectable criteria. It also shows where that connection reaches the limit of its justificatory force. Historical action, current identity, and present responsibility become parts of one reviewable argument, with each inferential step supported by the kind of reason it requires.

27 Documentary Study Design

This section specifies a proposed empirical study capable of testing the method on actual collective splitting and merger. It defines the case boundary, record collection, coding, comparison of criteria, and validation obligations. The bounded Manchester comparison in Section 23 already examines primary institutional documents and establishes distinctions among their supported public claims. The fuller study proposed here would reconstruct organizational practices, disputed transitions, and competing criteria across a complete episode. Its empirical findings remain open.

27.1 Case Selection and Inquiry Boundaries

A suitable first case should contain a documented reorganization, at least two plausible continuity criteria, and records sufficient to reconstruct the episode before and after the change. The case should be chosen because it can discriminate among the method's alternatives, not merely because its public narrative resembles the constructed association. A relatively bounded organization with identifiable procedures and activities can make the initial test more inspectable.

The research question should be fixed at the outset. Possible questions include continuity of a membership association, stewardship of a specific programme, or persistence of a constituted registration. The investigator can compare several questions, but each needs its own target and criterion. A broad question about "the real successor" risks allowing the intended object to change during analysis.

The temporal window should begin early enough to establish the predecessor's relevant organization and extend far enough to observe the successors' distinct activities. The organizational boundary

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should include plausible branches, informal continuations, and institutions whose decisions affected the reorganization. Exclusion rules should be recorded. The candidate universe is a material part of every uniqueness claim.

27.2 A Record Inventory with Provenance

The initial inventory should preserve each record's identifier, origin, date, custodian, access conditions, version, and claimed relation to the episode. Records can include governing documents, meeting records, membership lists, activity schedules, financial or resource-transfer records, correspondence, public statements, and interviews. Inclusion in the inventory indicates relevance for examination, not endorsement of the record's claims.

Source dependence should be explicit. A newspaper item may repeat a branch announcement; a later history may rely on a former officer's account; a registry entry may reflect a contested decision. The inventory should preserve these dependencies so that repeated claims are not counted as independent confirmations. A source can still be valuable when dependent, provided its role is correctly interpreted.

The original record and the researcher's extracted claim should remain separate. An extract should identify the proposition it supports, the relevant passage or field, and the interpretive step. If the record is ambiguous, the ambiguity should be retained. Corrections to an extraction should preserve the earlier version and the reason for revision. This practice makes later audits possible without treating the researcher's coding as a substitute for the source.

27.3 Constructing Appearances and Candidate Histories

The next stage groups records into candidate appearances at a declared resolution. The researcher should distinguish duplicate descriptions from claims about distinct units and record the evidence for the grouping. If two normalizations remain plausible, both should be retained through the initial analysis. This prevents the most difficult identity decision from being hidden in preprocessing.

Candidate histories should then state the events and transmissions that connect appearances. Each lineage edge needs a type and supporting record. The analysis should preserve missing intervals, uncertain event ordering, and alternative interpretations. A history is a structured hypothesis about the episode, not simply the order in which documents were found.

The researcher should identify decisive historical disagreements. These are pairs of candidate histories that agree on much of the record but differ on a premise relevant to the target verdict. The query method can then guide further collection. An additional source is especially useful when it can distinguish a remaining opposite-verdict pair under a credible observation model.

27.4 Operationalizing Continuity Criteria

Each criterion should have a written operational definition before it is used to select a successor. For membership continuity, specify eligibility, activity requirements, dual affiliation, and the treatment of missing rosters. For operation continuity, specify the activity, transmission mechanism, permissible adaptations, and the relation between inputs, procedures, and outputs. For commitments, identify the undertaking, authority, uptake, and implementation separately.

Institutional criteria require the applicable rule system and the scope of the decision. Actual legal questions require current jurisdiction-specific verification and appropriate expertise. The study should not infer legal succession from generic organizational similarity. A verified registry finding can be reported within its domain while other continuity questions remain open.

Measurements should preserve denominators and missingness. A percentage of retained members means little without the predecessor population and the rule used to count them. A score for continuing activity needs the activities included and their relative importance. If the score combines ordinal judgments, arithmetic operations need a justification or should be replaced by an ordered or set-valued

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comparison. The method should permit qualitative evidence where quantification would obscure the reasoning.

27.5 Independent Coding and Disagreement Analysis

Where resources permit, more than one researcher should independently code selected records and apply the written criteria. Disagreement is informative when its source is analyzed. Coders may differ about what a document says, how it relates to an appearance, whether a transition occurred, or which criterion governs the target. A single agreement percentage would combine these different issues.

The study should therefore classify disagreements by inferential stage. Factual extraction disagreements can prompt source review. Criterion application disagreements can reveal ambiguous definitions. Normative disagreements can reveal a substantive dispute that more coding practice will not eliminate. The objective is to improve the clarity and adequacy of the method, while preserving disagreements that carry conceptual content.

Consensus coding can produce an operational dataset, but the consensus record should retain the reasons for changes. Agreement among researchers is evidence about the reproducibility of a procedure under their shared training. It is not sufficient proof that the resulting identity verdict is true. The study should compare its findings with independent records and alternative explanations wherever possible.

27.6 Constraint Analysis and Comparative Outputs

For each accepted criterion, the investigator can construct positive and negative constraints with explicit warrants. The consistency test should be applied before publishing a transitive identity record. Conflicts should be returned to the relevant evidence and criteria. A resolution must name the revised premise; a silent deletion would destroy the explanation of the result.

The output should include the secure relation, possible links with their supporting scenarios, and required distinctions. It should also preserve the typed lineage. If a single successor is selected for an institutional purpose, the selection rule and candidate universe should be stated. The remaining historical continuities should be reported alongside it.

Comparing criteria can reveal robust and sensitive findings. Some pairs may be securely identified under every admitted criterion. Others may change only when a disputed historical premise changes. Still others may remain divided under a fully accepted history. The study should identify which pattern occurs and explain its significance for the target inquiry.

27.7 Validation, Error, and Researcher Influence

Validation should use outcomes appropriate to the claim. A prediction of continued activity can be compared with later activity, under a stated forecasting design. A claim about a past handover should be checked against independent historical evidence. An identity criterion's normative appropriateness requires argument and stakeholder criticism. Success on one target should not be reported as validation of all three.

Error analysis should examine false merges, false separations, omitted candidates, incorrect event ordering, source dependence, and changes of criterion during coding. A false merge can propagate widely through equivalence closure. A false separation can fragment a continuing history and hide responsibilities or contributions. Their relative costs depend on the research and institutional use, so the procedure should expose the chosen balance.

Researcher involvement can also shape the case. Interviews may prompt participants to clarify or revise commitments. Sharing provisional findings may affect public recognition. These interactions should be recorded as events in the research process, with prior and subsequent targets distinguished. The investigator should avoid presenting induced changes as independent confirmation of the earlier reconstruction.

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The completed study would provide a test of whether the proposed distinctions improve a real continuity inquiry. It could also show that some are too coarse, too demanding, or misaligned with the case. Such findings would refine the method. The formal framework is designed to make those revisions traceable, rather than protect an initial criterion from evidence that challenges it.

27.8 A Worked Coding Protocol for the Constructed Dossier

The six-appearance example can specify the coding operations that an actual study would need to perform. The example remains constructed: the following record classes describe evidential roles and do not claim that corresponding documents have been collected. Their purpose is to make the transition from a formal constraint to a research task concrete.

Begin with records describing the initial collective at time zero, the two branches at time one, their continuing activity at time two, and the merged body at time three. The first coding pass extracts dates, names, roles, activities, and statements of authority. It leaves identity conclusions in a separate field. A statement such as "we continue the association" is coded as a claim by its speaker, with time and context, before it is assessed as evidence for the claim's content.

The second pass constructs candidate appearances. To normalize b and c as distinct simultaneous units, the study needs evidence appropriate to its organizational resolution. Separate coordinating procedures, independent decisions, distinct undertakings, or an accepted separation act may be relevant. The researcher should record the argument combining those observations. Separate names alone may be insufficient if they describe two activities within one organization.

The third pass reconstructs the two within-branch continuities. For b and d, the hypothetical criterion requires a continuing procedure of collective coordination through the interval. Records could establish who occupied the relevant roles, how replacement occurred, and how decisions and activities continued. The same analysis applies to c and e. The investigator should evaluate the branches under the same criterion, while allowing their different evidence to receive different assessments.

Only after those steps would the accepted positive links enter P. A low-quality record can remain in the inventory without supporting a constraint. A contested link can remain in some candidate histories and be absent from others. A secure negative constraint requires its own warrant. The formal family should reflect these statuses rather than force every coded source into a binary decision immediately.

27.9 Research Decisions at the Three Refinement Stages

At the ten-partition stage, the study has established the two within-branch continuities and the branches' distinction. It has not selected which branch, if either, continues the initial collective or the final merged body. A useful report would state these positive findings and identify the open links. The research question for further collection becomes narrower than the original general question of organizational identity.

To move from ten to three partitions by accepting aEb, the study needs evidence that satisfies the same criterion across the initial division. A document titled "succession agreement" is a lead, not an automatic answer. The investigator must determine its authenticity, whether it records a proposal or effective act, which participants had authority under the relevant arrangement, and whether the criterion treats that act as sufficient.

If the document establishes only a transfer of resources, it may support a lineage edge without supporting identity. If it constitutes registry succession under a different rule from the criterion used within the branches, the researcher must either keep the registry question separate or justify the connection. The example's numerical reduction is valid only after the positive identity premise has been warranted within the intended question.

To move from three partitions to one by accepting eEm, the study faces the parallel task at the merger. It should examine whether the final body continues the relevant organizational procedure, receives a constituted status, or begins a new organization that inherits a practice. These alternatives may require

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different kinds of records. The proposed method prevents the mere presence of a merger announcement from selecting one answer silently.

At the unique-partition stage, the report should still preserve the original lineage from both d and e into m where the evidence supports it. The identity finding narrows one relation; it should not narrow every historical relation by contagion. An outside reader can then distinguish the selected continuing organization from the multiple sources of the merger's operation.

27.10 A Structured Disagreement Exercise

Before applying the method to an actual dispute, researchers can use the constructed dossier for a structured disagreement exercise. Provide the same hypothetical observations to several readers and ask them to state the identity question they believe is being answered, the criterion they would use, and which observations they would promote into constraints. The exercise tests interpretability of the method, not empirical truth about an organization.

The comparison should distinguish agreement about the observations from agreement about their implications. Readers may all accept the branch rosters while disagreeing about membership's role. They may agree on a succession rule while interpreting one document differently. They may use the same words for different objects, such as an association of participants and the programme those participants once maintained. The exercise can reveal which distinctions need clearer explanation in the protocol.

The researchers should then supply explicit definitions and repeat the application on a different constructed case. Improved agreement can show that the procedure became clearer. Persistent disagreement can identify a philosophical or normative question that the procedure should preserve. Neither result warrants claiming that the criterion has been validated for actual collective identity.

Such preparation can reduce avoidable ambiguity before a documentary study begins. It also gives a disciplined role to constructed examples: they test the internal intelligibility and consequences of a method, while actual records test its capacity to support warranted historical findings.

27.11 Reporting a Negative or Mixed Result

The method should be evaluated in cases where it yields a mixed or negative outcome. Suppose the records establish lineage but fail to justify any cross-division identity link. The study can report the continuity of members, practices, and undertakings while leaving the entity verdict unresolved. This is a substantive result if it shows why a widely repeated single successor narrative exceeds its evidence.

Suppose instead that the criterion itself proves unstable under reasonable operational definitions. A membership rule may change its selection whenever inactive members are included or excluded, and the case may offer no defensible way to decide that issue. The result can identify the rule's practical limitation. It should not conceal the sensitivity by selecting the denominator that gives the clearest answer.

A third result may show that the proposed formal distinctions add little to an existing documentary practice that already handles the case well. That finding would narrow the contribution. The model might remain useful for teaching, cross-case comparison, or auditing automated record linkage, while a stronger empirical claim about improved historical inquiry would remain unsupported. The research programme should preserve this possibility as an explicit test of its own value.

28 An Integrated Documentary Finding and Its Revision

This section develops a complete constructed inquiry through the stages that the preceding analysis has separated. It begins with records and candidate units, fixes a continuity target, interprets institutional and transmission evidence, derives a sequence of identity findings, and examines a conflict followed by a supported correction. The purpose is to show how the method's parts constrain one another within one episode. The source descriptions are stipulations of the exercise; the bounded Manchester comparison remains the paper's actual primary-document illustration.

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28.1 The Collective Episode and the Requested Finding

The constructed episode concerns a workshop association whose activities divide between two organized branches and later contribute to an integrated body. The inquiry asks whether the initial association and that later body are the same collective entity under a specified institutional criterion. It also preserves separate questions about transmitted programmes and resources, so that the answer to the identity query can be interpreted without losing the episode's other continuities.

The initial appearance is a at time 0. The division produces b and c at time 1. Subsequent changes yield d and e at time 2, followed by an integrated appearance m at time 3. These symbols denote normalized collective appearances. They are not names for documents, persons, assets, or identity classes. Establishing the appearance domain will require its own evidential account before the partition calculation begins.

The institutional criterion, denoted here by β, tracks an organized association through specified effective continuations of its governing procedure. An amendment can qualify when it is made by a competent body, follows the applicable procedure, and preserves the relevant organizational arrangement under the criterion's permitted changes. These conditions need descriptions independent of the assertion that the association remains the same. The example stipulates the criterion and the relevant act descriptions; its philosophical adequacy remains an explicit premise.

This target is narrower than identifying the single authentic inheritor of everything associated with the predecessor. Programme materials can pass to several recipients. Participants can carry skills into different bodies. A later institution can acknowledge multiple predecessors. The criterion asks a specified question about the organized association. The remaining relations can explain why a body excluded from its identity class still has substantial historical connections to the predecessor.

The temporal window is also fixed. It includes the initial association, the two simultaneous branch stages, and the integration. The base identity family imposes no requirement that every class appear in every intervening slice, and it places no numerical cap on the number of entities. Those choices preserve the assumptions of the six-appearance construction in Section 11. A study imposing continuous appearance paths would need to recompute its family under the additional restriction.

28.2 Referential Records before Collective Appearances

The initial source review concerns what the records refer to. This subsection examines duplicate descriptions and distinct simultaneous units before treating any document pair as evidence of persistence.

Suppose the archive contains a roster headed with the name of a branch assembly and meeting notes headed with the name of a working circle at the same time. Shared participants create a reason to investigate whether the headings describe one collective unit. They leave open the possibility of two coincident organizations formed by the same people. The normalization cannot be justified by matching names or rosters alone.

In the construction, additional records explicitly connect the two headings to one governing meeting, one decision procedure, and one organized body using different descriptions. The inquiry accepts this interpretation after reviewing the records' referents and scope. The corresponding record tokens are grouped into one appearance. This decision concerns the unit at one time; it supplies no successor relation to a unit at another time.

The evidence for distinct branches has a different form. At time 1, the records describe separate governing meetings for b and c, independently taken participant decisions, and each body's capacity to alter its programme without the other's approval. At time 2, corresponding records support distinct organization of d and e. Within the chosen resolution, the inquiry accepts two units in each of those slices. Distinct page headings would have supplied a weaker basis than these organizational relations.

The exercise accepts the referential validity of the selected record tokens. An actual inquiry may encounter a fictitious body, a proposed committee that never assembled, or a statement whose focal referent cannot be resolved. Such cases require a change in the referential model before they can be represented as uncertainty among identities on a fixed domain. Adding a singleton identity class for a

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Table 12: Constructed source roles, supported predicates, and conditional identity uses.

Record role	Narrow extracted proposition	Use in the identity inquiry
Separate branch meetings	Distinct organization of b, c and later d, e	Simultaneous exclusions
Branch amendment records	Effective permitted transitions from b to d and c to e	Positive constraints bd, ce under β
Integration transfers	Programmes, copies, and specified resources pass into m	Typed lineage; transfer alone supplies no identity edge
Earlier continuation record u	Effective continuation of the governing procedure from a to b	Positive ab under β
Later continuation record w	Effective continuation of e's governing procedure in m	Positive em under β
Rival certificate v	Initially interpreted as institutional continuation from a to c	Provisional positive ac
Certificate attachment	Specified programme succession from a to c	Corrected predicate; the identity use of v is withdrawn

nonexistent appearance would conceal the problem by treating reference failure as an ordinary continuity verdict.

The result of this initial stage is therefore conditional and precise. The inquiry supplies six appearances at the chosen times and resolution, with accepted simultaneous distinctions. It retains the records supporting that normalization and the reasons for grouping duplicate descriptions. If later evidence changes the domain, the identity calculation must be revisited. The current calculation begins after this provisional domain commitment, with its warrant available for review.

28.3 An Inventory of Claims and Their Inferential Roles

The record inventory separates source content from the proposition extracted and from the use of that proposition under the criterion. Table 12 summarizes the operative roles in the constructed case. A row describes a source role, not a claim that every row has an independent origin.

A branch amendment record can establish that a meeting adopted particular words. Its identity use depends on further propositions: the meeting's competence, the procedure then applicable, the effect of the vote, and the criterion's treatment of the resulting change. The inquiry preserves those steps even where the exercise stipulates them as accepted. This makes clear what a future objection would have to challenge.

The integration records have another role. They document movement of programme materials, manual copies, and specified resources from d and e into m. These facts can support directed transmission from both predecessors. They do not by themselves identify m with both predecessors, which would conflict with the accepted simultaneous distinction between d and e. The record retains the two transmissions while leaving the institutional identity question to its own criterion.

The inventory also preserves source dependence. A branch announcement may quote the rival certificate, and a later catalogue may repeat that announcement. These are additional records that can help trace reception and circulation. They share the certificate's source route for the substantive claim under review. Their number cannot silently multiply the independent support for its interpretation.

The scope attachment has a specific evidential function. It identifies the object of the rival certificate's succession statement. Its later appearance in the inquiry changes the justified extraction from that certificate. The source remains in the record, along with the former reading and the reason for correction. This distinction will matter when the numerical conflict is resolved: the repair concerns an identity use of a statement, while its programme-succession content survives.

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28.4 The Initial Conditional Identity Family

The first partition stage uses the normalized appearances and the two accepted branch continuations. This subsection derives the initial family and states the limited findings it supplies before either disputed boundary is resolved.

The positive constraints are bd and ce. The negative constraints include bc and de, expressing the accepted simultaneous distinctions. Positive closure therefore has four components: the initial appearance a, the branch pair $\{b, d\}$, the branch pair $\{c, e\}$, and the integrated appearance m. The two branch components are incompatible. The record supplies no further cross-component exclusion at this stage.

There are ten compatible partitions under the stated unrestricted-class model. Section 11 derives this count by examining the possible placement of the initial and final components relative to the two incompatible branch components. Here the count serves a documentary role: it describes the alternatives left by the accepted interpretations, rather than ranking them or assigning probabilities.

The initial finding can securely identify b with d and c with e. It can securely distinguish the two branch histories. It leaves the predecessor's identification with either branch open, and it leaves the integrated body's placement open. The source record also contains directed transfers into m, which the identity uncertainty does not erase.

One possible initial identity class can connect a with m while appearing in neither intervening slice. That possibility follows from the absence of a temporal-contiguity requirement in this family. It should be reported with that condition visible. An investigator who believes the relevant collective must have an accepted appearance in every intervening slice can impose that additional requirement and examine its consequences. The initial ten-case result should not be carried over unchanged.

The finding is already informative despite its breadth. It identifies which relations follow from the currently accepted institutional histories and which depend on missing boundary evidence. It also gives the inquiry a focused next step: evidence concerning the initial division and the later integration could reduce the remaining alternatives. Collecting another copy of the already accepted branch amendment would have a different value, such as strengthening preservation, while adding little discrimination between these particular possibilities.

28.5 Refinement through the Earlier Continuation Record

The next stage adds the record labeled u. Its accepted interpretation connects the initial governing procedure to b. This subsection traces the change in the family and distinguishes a boundary finding from a complete history.

The exercise stipulates that the division settlement was made by a competent body under the applicable procedure and effectively continued the relevant governing arrangement in b. Under β, the resulting positive constraint is ab. The criterion bridge remains visible: the document's institutional content and the identity consequence are connected through the adopted account of the collective object.

Positive closure now joins a to b and d. The other branch component contains c and e. Those two components are distinct because they contain the incompatible simultaneous appearances. The integrated appearance m can form its own class, join the first component, or join the second. Exactly three partitions remain.

The requested initial-to-final query is still unresolved. In one of these partitions a and m are the same entity; in the others they are distinct. The earlier settlement has resolved the initial branch assignment without settling the later integration. A report that names b the successor and then carries that designation automatically into every later organization receiving its programme would skip the still-open boundary.

The stage also illustrates the importance of expressing what the evidence has positively achieved. The inquiry can now state that the initial association continues through b to d under the accepted criterion and record. Its uncertainty concerns the later relation to m. An unqualified statement that identity remains uncertain would conceal the resolved portion of the history and make the next evidence task harder to identify.

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The support record should therefore link the new boundary finding to u and its interpreted conditions. If u is later challenged, the inquiry can locate the affected part of the history. The established branch continuations bd and ce need not be discarded merely because another boundary becomes disputed. The model's modular constraints support a correspondingly discriminating revision.

28.6 The Integration Boundary and a Unique Conditional Finding

The later continuation record w supplies evidence concerning e and m. Its accepted interpretation completes the initial inquiry under the fixed criterion. This subsection examines both the resulting uniqueness and the substantive continuities that remain outside its identity classes.

Suppose the integration arrangement effectively carries e's governing procedure into m under the conditions recognized by β. The organizational changes and transferred authority are described independently of the identity verdict, and their competence and applicability are accepted within the construction. The identity constraint is em.

The remaining components are now $\{a, b, d\}$ and $\{c, e, m\}$. They cannot be joined because doing so would identify the distinct simultaneous branches. The family therefore contains exactly one partition. Under the accepted premises, the initial association a and integrated body m are distinct. The inquiry has obtained a determinate answer to its specified question without identifying every historical inheritance with numerical identity.

The programme and resource records remain relevant. Materials from d can enter m even though d and m belong to different identity classes. Manuals can be copied; participants can teach routines; an integrated programme can combine practices from both branch histories. These facts explain why a public history might acknowledge the initial association among the integrated body's predecessors. The acknowledgement can have a well supported lineage meaning while the institutional identity finding remains distinct.

The finding's uniqueness is conditional on more than the last document. It depends on the normalized domain, simultaneous exclusions, accepted branch continuations, earlier settlement, later arrangement, and criterion. A useful report identifies those dependencies. Doing so does not weaken the derivation; it specifies the circumstances in which the derivation applies and the propositions whose revision would change the answer.

The unique partition should also be kept separate from confidence in its premises. A singleton compatible family means that every accepted model agrees. It supplies no numerical measure of how likely those accepted premises are to capture the actual episode. Fallible observation models and expanded historical alternatives can be added when warranted, but their probability or error assumptions require a separate account.

28.7 An Opposing Certificate and the Location of Conflict

The rival certificate v is initially read as assigning the association's continuation from a to c. This subsection shows how the formal conflict can reveal an interpretive problem without deciding which source is wrong.

Suppose that reading is provisionally accepted under the same institutional criterion. It supplies ac. Together with ab, it identifies the initial appearance with both simultaneous branches. Transitivity then identifies b and c, contrary to the accepted slice exclusion. The compatible family is empty.

The empty result reports inconsistency in the accepted package. It supplies no secure identity finding by universal quantification over an empty family. In particular, the inquiry cannot retain the former unique partition and claim that the new document merely adds another harmless description. The new interpretation, if used as a positive identity constraint, conflicts with the existing interpretation of the episode.

The conflict certificate can be stated briefly: the positive path from b through a to c meets the accepted negative pair bc. This path locates the issue without requiring the entire later merger history. The later

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relations remain part of the complete case, but the immediate contradiction arises at the initial split. That localization helps focus the source review.

Several responses remain possible before further evidence is examined. The earlier settlement may have been misread. The rival certificate may concern another object. A claimed act may lack competence or applicability. The simultaneous units may have been normalized incorrectly. The criterion bridge may be too strong. Mathematical inconsistency reveals that the accepted package cannot all hold under the model; it does not choose among these substantive diagnoses.

The source inventory should retain every record during this review. A contradiction between extracted constraints is a reason to examine content, scope, and warrant. It gives no basis for deleting an inconvenient source or treating the earlier finding as immune to challenge. Preserving the opposing certificate is especially useful because its later clarified meaning will contribute a valid programme-succession claim.

28.8 A Bounded Comparison of Two Repairs

To make the revision alternatives exact, temporarily hold a specified background fixed and treat only the two rival initial-boundary premises as revisable. This subsection derives the repair family and states the scope restriction that makes it manageable.

The accepted hard background contains bd, ce, and em, together with the simultaneous exclusions. It is compatible. The revisable bundles are u, supplying ab, and v, supplying ac. Calling the other premises hard means that this particular review accepts them as its background. It does not claim that they are permanently beyond evidential challenge.

Either bundle alone is compatible with the background. Retaining u gives the unique partition $\{a, b, d\} \mid \{c, e, m\}$. Retaining v gives the unique partition $\{a, c, e, m\} \mid \{b, d\}$. Retaining both produces the contradiction already identified. Since the two singletons are compatible, the pair $\{u, v\}$ is the sole inclusion-minimal conflict within this bounded premise universe.

Its inclusion-minimal withdrawal repairs are therefore to withdraw u or to withdraw v. These alternatives are applications of the conflict/repair result from Section 12. They state the smallest premise withdrawals that restore compatibility under the chosen background. They leave open which withdrawal has a better documentary warrant.

If the investigator admits both repaired accounts, the initial-to-final query becomes ambiguous. In the repair retaining v, a and m belong to one class. In the repair retaining u, they are distinct. The two repairs preserve the later branch and integration history while disagreeing about which initial branch continues the predecessor.

Expanding the revisable universe changes the repair problem. If the simultaneous distinction or the later continuation record is also open to withdrawal, additional compatible packages may arise. The two-repair comparison is useful precisely because its boundary is stated. It cannot be described as every possible explanation of the archival conflict unless the broader premise choices have been independently excluded.

28.9 Secure Information across the Repaired Accounts

The two repaired partitions support a partial finding even before one interpretation is preferred. This subsection distinguishes the secure summary from a complete possible history and identifies the joint alternatives that the summary should preserve.

The intersection of the repaired equivalences has classes $\{a\}$, $\{b, d\}$, and $\{c, e, m\}$. Thus the later branch continuations and the e-to-m integration are secure across the admitted repairs. The initial appearance's relation to those later classes remains unresolved. The two branch classes are securely distinct.

The singleton $\{a\}$ in this intersection expresses limited knowledge. In each repaired history, a actually joins one of the later classes. The secure summary itself is therefore not one of the two complete repaired partitions. Interpreting its singleton as a positive finding that the predecessor terminated before both branches would substitute a new history for the information common to the retained histories.

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The possible links also need joint interpretation. It is possible, in one repair, that a is identical with b. It is possible, in the other, that a is identical with c. Both links cannot be accepted together because the branches remain distinct. The report should retain the two supporting scenarios, or another representation with the same joint information. Listing each pair as possible and then combining all possible pairs would recreate the contradiction.

These findings supply a focused interim report. The inquiry can state the later continuities that survive both repairs, the unresolved initial assignment, and the documentary issue that distinguishes the two accounts. The report can also preserve programme transmission into m, whose existence does not depend on choosing between the rival initial identity readings.

No probability follows from the presence of two repairs. Their symmetry in the formal listing does not make them equally likely, and a larger number of copied records supporting one interpretation does not automatically break the balance. A probabilistic assessment would need a justified model of source reliability, dependence, and candidate histories. The present interim finding states logical compatibility under explicit premises.

28.10 Correction through the Certificate's Stated Object

The later scope clarification supplies a reason to revise one identity extraction. This subsection examines that reason and the historical content that remains after the correction.

Suppose the certificate's attachment specifies that the continuation statement concerns a particular workshop programme. The programme can be transmitted to c while the organized association continues through b under β. The attachment's wording, relation to the certificate, and applicability to the relevant event are accepted within the constructed case. It therefore provides a substantive basis for interpreting the certificate's object more narrowly.

The correction removes ac from the identity constraints. It retains a typed programme-succession relation from a to c, together with the certificate and attachment that support it. The reason for removal is the revised predicate, not a general judgment that the source is unreliable. The certificate can accurately describe programme succession even though its former use as an identity premise was mistaken.

With u and the accepted background retained, the unique partition again has classes $\{a, b, d\}$ and $\{c, e, m\}$. The numerical answer is the same as before the rival interpretation entered the inquiry. Its warrant has changed: the revised finding now includes an explanation of why the opposing certificate supports another relation. The intervening conflict has performed useful work by forcing the scope distinction into view.

This return to the former partition should not be described as proof that the first report was correct in every respect. The first report may have omitted a relevant programme lineage or failed to anticipate the ambiguity of succession language. The later report can improve its account even when its identity classes are unchanged. Intellectual revision concerns reasons and represented relations as well as final labels.

The correction also leaves u's warrant open to its own challenges. The attachment removes one opposing identity interpretation; it does not independently verify the earlier settlement's competence or the criterion's adequacy. A reliable research record can explain why one conflict has been resolved without presenting the surviving account as immune to future evidence.

28.11 A Reader-Facing Finding with Explicit Scope

The final report should give the substantive result in language that a reader can assess without reconstructing every calculation. This subsection describes the content of such a report and the supporting material that keeps it reviewable.

The report would identify the collective object and period, state the institutional criterion, and explain that the initial association continues through the first branch while the integrated body continues through the second branch's governing procedure. It would report the two resulting identity classes and

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the evidence supporting the boundary transitions. It would also preserve the programme and resource transmissions connecting both branches to the integrated body.

The scope clarification deserves a direct account. The rival certificate was initially interpreted as a statement about the association's numerical continuation. Its attachment supports a programme-succession reading. The revised finding therefore retains the certificate's substantive contribution while withdrawing its former identity use. A reader can examine that interpretive move independently of the partition calculation.

The report should state the major conditions that could change the result: different unit normalization, a failed authority premise, a broader revisable background, an alternative criterion, or evidence of another eligible organizational appearance. It need not repeat every technical assumption in the main conclusion. A linked finding record can preserve the complete dependencies, alternatives, and former versions.

The final identity verdict should be accompanied by the continuities it does not subsume. Programme inheritance, transmitted skills, resource movement, and commemorative affiliation can remain well supported across an institutional separation. Explaining these relations helps a reader understand how a negative initial-to-final identity finding can coexist with a substantial historical connection. The account answers a scoped question while retaining the phenomena that made the question important.

This presentation is more informative than publishing only a persistent identifier for each appearance. The identifier can assist retrieval once the finding is accepted. The report provides the reasons for that assignment and the conditions under which it should be revised. A later researcher can therefore use the finding without treating the identifier itself as evidence of the past.

28.12 The Contribution of the Integrated Exercise

The exercise adds an inferential sequence to the separate mathematical results. This subsection identifies what the integration demonstrates and what remains to be established through a full documentary study.

Unit normalization constrains the meaning of simultaneous distinction. The institutional criterion determines which independently described acts supply positive identity premises. Typed transmission preserves historical connections that identity cannot simultaneously absorb. Closure and partition analysis show which complete histories fit the accepted package. Conflict analysis identifies incompatible interpretations, and premise repair exposes the consequences of different withdrawals. A source-scope clarification then supplies a reason to prefer a correction. Each stage depends on the preceding interpretation while retaining its own revision conditions.

The sequence also shows that a method can make positive progress at several levels. It can establish branch continuity before settling the integration, localize a conflict without deciding which source fails, preserve secure later history across competing repairs, and correct an extraction while retaining the source's valid claim. Progress is therefore richer than the transition from an unknown label to a unique label.

The exercise's limitations are equally specific. Its records and effective acts are stipulated. The accepted institutional criterion has been exposed to argument elsewhere in the paper, but the example cannot validate it by choosing facts that fit its conditions. A full study would need to examine actual sources, interpret contested authority, test normalization choices, and compare alternative criteria. The protocol in Section 27 specifies that work.

The integrated contribution is a proposal for preserving warranted distinctions through a difficult inquiry. Its elementary calculations make the consequences of declared assumptions inspectable. Its documentary structure keeps the reasons for those assumptions visible. Together they support a collective history that can be determinate where the evidence and criterion warrant determinacy, plural where several accounts remain, and revisable when the source interpretation changes.

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29 Philosophical Objections and Alternative Interpretations

This section tests the proposal against objections concerning numerical identity, relativism, circularity, vagueness, power, and explanatory reduction. The responses distinguish what the present method can establish from questions that remain open. Their purpose is to strengthen the argument through its most demanding alternatives. The extended cases distinguish independence of historical truth from choice of inquiry, uncertainty of reference from uncertain identity, and descriptive precision from the warrant for a sharp criterion.

29.1 Numerical Identity and Purpose-Indexed Tracking

The strongest objection is that the paper substitutes a useful tracking policy for numerical identity. Numerical identity, the objection maintains, concerns whether one entity is the very same entity, while an investigator's purpose determines only which evidence is convenient. If this is correct, the proposed criteria can organize inquiry without constituting an account of identity itself.

The objection identifies a real limit. The formal construction represents identity judgments and their coherence under explicit criteria. It does not prove that every admissible criterion captures numerical identity. The distinction should remain in the paper's conclusions. A tracking policy can be useful and well justified for a task while a stronger metaphysical claim requires additional argument.

The method nevertheless contributes to that stronger inquiry. Any proposed criterion of numerical collective identity must explain how it applies to the records, why it handles replacement and branching appropriately, and which observations could reveal a mistaken application. The appearance domain, constraint audit, and alternative histories make those obligations explicit. A theory claiming one uniquely correct criterion can enter the comparison and defend its privilege. The framework does not decide the contest in advance.

The title's reidentification problem therefore has an epistemic emphasis. It asks how collective entities are identified again across change and which grounds warrant that act. The results concern the organization of that judgment. Where the manuscript uses an identity partition, it means the relation represented by the stated criterion. Its metaphysical adequacy remains available for further argument.

29.2 Relativism and the Discipline of Reasons

A second objection holds that purpose-indexed criteria allow any answer to be defended by choosing a convenient purpose. A branch could select membership when it retains members, operations when it retains operations, and registration when it controls the register. The resulting flexibility could protect self-serving claims from criticism.

The proposed method requires more than declaring a purpose. The question must be intelligible and relevant to the consequence or explanation at issue. The criterion must be stated before its favorable result is used as evidence of its adequacy, applied consistently across relevant cases, and examined against counterexamples. Its component measurements must have independent meanings, and its exclusions must be justified. These conditions constrain opportunistic selection even when they leave room for legitimate disagreement.

An investigator should also compare the outputs of plausible alternatives. If a preferred criterion changes a verdict only because a disfavored branch is omitted, that dependence should be visible. If a normative consequence requires attention to an undertaking that a membership criterion ignores, the criterion can be criticized for missing the relevant object. A purpose can itself be objectionable or too vague to justify the use. Purpose sensitivity therefore creates additional responsibilities of argument, not an exemption from them.

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29.3 Circularity in Domain Construction and Evidence

The method can be challenged for relocating circularity. Instead of naming identity-preserving mechanisms, it begins with normalized appearances and positive identity constraints. These inputs can already contain the judgment that the construction appears to derive. If so, the formalism may only rearrange an earlier assumption.

The response is to distinguish two uses. When identity is already established for some pairs, the construction organizes their consequences and tests consistency. Its contribution is conditional, and it should be described that way. When identity is under investigation, the researcher should begin with records and independent descriptive evidence, preserve alternative normalizations, and justify the move from a profile to a positive constraint under a declared criterion.

Complete freedom from background interpretation is neither promised nor required. The relevant standard is whether the premises can be examined without merely repeating the target conclusion. A roster, a transfer act, and a record of coordinated practice can be described before deciding whether the whole organization continues. Their relevance to identity then becomes an explicit argument. The method is circular when that argument consists only of calling them the records of the same entity.

29.4 Vagueness and the Pressure of Sharp Boundaries

Collective boundaries and continuity can be vague. Membership may be informal, coordination gradual, and public recognition divided. A partition assigns each appearance to exactly one class, which can seem too sharp for the phenomenon. The admissible-family construction responds to some uncertainty, but it may still assume that one sharp partition is the hidden truth.

The method does not require that assumption. A family can represent evidential uncertainty, unresolved conceptual choice, or several defensible precisifications. These interpretations should be distinguished in the project record. The intersection and union results remain formal properties of the family; their philosophical meaning depends on why the alternatives are admitted.

If the inquiry requires a graded or overlapping identity-like relation, its inferential rules need separate development. A similarity relation can be useful without being transitive. A graded continuation can preserve degrees of inheritance without yielding numerical identity. The danger lies in using the familiar word “identity” while retaining the inferences of equivalence only when convenient. A clear alternative relation can improve the model; an unmarked mixture weakens it.

29.5 Power and the Production of Continuity

Another objection concerns power. The evidence available to an investigator is produced within institutions. A branch controlling archives, names, resources, or communication channels can shape the record on which its continuity is judged. A formally careful procedure could then reproduce an unequal distribution of visibility.

The concern extends beyond bias in a dataset. Control over infrastructure can affect which collective practices remain possible. A recognized branch may gain resources that enable future continuity, while another loses access and declines. Later observations can then reflect the effects of the earlier designation. The study must examine that historical mechanism instead of treating the later pattern as an independent verdict on the past.

The proposed remedy is evidential and institutional. Preserve source dependence, search for omitted candidates, distinguish lack of evidence from evidence of absence, and record the effects of recognition acts. Make the criterion and its consequence policy available for criticism. These measures cannot eliminate every power asymmetry, but they identify specific ways in which an identity finding can fail and provide grounds for revision.

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29.6 Reduction to Individuals and the Level of Explanation

A critic may argue that all relevant facts concern individuals, their actions, and documents, making collective identity an unnecessary layer. Another may argue that the proposed model gives too little autonomy to the collective by grounding its description in relations and records. These objections pull in different directions and show the importance of the paper's ontological suspension.

The present construction requires neither a reduction of every collective fact nor a claim of metaphysical independence. It asks whether a collective unit and continuity record perform useful explanatory and evidential work for the chosen question. If the unit conceals decisive individual differences, the description needs refinement. If eliminating the unit obscures organized procedures, commitments, or coordinated capacities, the collective level can retain a justified role.

This methodological answer leaves a deeper account of dependence open. The model's explicit domains and mechanisms can contribute to that later argument by showing which collective claims depend on which records, relations, and organizational conditions. It should not present a useful representation as a completed solution to every dispute about social ontology.

29.7 Mathematical Simplicity and Philosophical Contribution

The mathematical propositions are elementary. An objection may therefore ask whether they add anything beyond familiar logical facts. Their contribution lies in the placement of those facts within an inquiry that otherwise permits unnoticed shifts between similarity, lineage, identity, and authority. The branch proof identifies a conflict; the constraint theorem supplies a constructive audit; the query theorem exposes an evidential limitation.

Mathematical simplicity is compatible with conceptual usefulness, but it cannot excuse an underdeveloped philosophical argument. The paper must show why its domains and criteria fit collective reidentification and must confront existing work that addresses related problems. The current literature comparison establishes several antecedents and identifies further work. A fuller originality assessment remains necessary before strong claims about the paper's place in the field are justified.

29.8 Necessary Features and Historically Maintained Organization

A further objection asks whether the paper avoids identifying the features truly necessary for a collective entity to persist. If every feature can change under some purpose-indexed account, the notion of a continuing organization may become empty. A substantive theory, the objection argues, should name what must remain through the change.

The concern should be separated into two questions. One asks whether a particular criterion requires an invariant feature. The other asks whether collective persistence in general has such a feature. The present method can examine the first by stating the criterion and testing its implications. It leaves the universal question open. A history can preserve an organized transmission process while every member, venue, or individual procedure changes, depending on the rule governing those replacements.

A process of authorized replacement can itself be a candidate continuity condition. Its adequacy needs an account of what authorizes each change and why the chain remains relevant to the same organization. If the authorization derives solely from an already assumed identity, circularity returns. If it is grounded in an independently described practice of collective decision and uptake, it provides a more substantive candidate. The philosophical task is to evaluate whether that practice warrants identity or only a kind of succession.

The method encourages explicit necessary and sufficient conditions within a criterion, together with counterexamples. A condition may be necessary for registry continuity and unnecessary for continuing practice. A set of conditions may be jointly sufficient within a stipulated institution while remaining insufficient for a broader metaphysical claim. Stating those qualifications gives the conditions content without presenting a local success as universal necessity.

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29.9 Intrinsic Similarity and External Historical Relations

Another objection favors criteria based on the organization's current internal features. Historical and external relations can seem to make identity depend on facts outside the organization itself. Yet the present cases show why those relations can be indispensable to the question. Two collectives can have matching current structures while only one inherits the predecessor's practice through a documented path.

A continuity criterion that distinguishes those collectives must retain some historical relation. An internal similarity description cannot recover a distinction it omits. This is an evidential and representational point before it becomes a metaphysical claim. The critic can reject the historical criterion, but then needs to explain why the difference in transmission is irrelevant to the intended identity question.

External authorization supplies another example. A constituted standing can depend on acts of a federation or another institution. Its persistence is partly relational in that sense. The adequacy of such a rule can be criticized, but an inquiry into the standing must represent the relation that constitutes it. Omitting the institution in pursuit of a purely internal description would change the target.

The broader lesson is that the boundary of an entity and the boundary of the facts needed to explain its continuity need not coincide. A collective unit can be the focal subject while its history depends on relations extending beyond it. The generative relational representation is useful because it can retain those relations without turning every influencing organization into the same entity.

29.10 Termination and the Value of a Continuing Practice

A final objection concerns the emotional and normative force of declaring an organization ended. Participants may hear termination as a denial of their work or a judgment that their continuing activity has lost authenticity. The formal model should not let those implications enter unnoticed. Termination under a criterion states the end of one identity path; the value and provenance of the continuing practice need their own assessment.

A reconstituted collective can sustain a valuable undertaking, preserve memories, and develop new capacities. A continuing registered body can abandon a valued practice. Identity therefore supplies an incomplete measure of success or loss. The research should describe what continues and what changes in concrete terms before evaluating their significance.

This preserves an important insight from the original continuity question. The space between numerical identity and complete discontinuity contains substantial historical, practical, and normative relations. A rigorous model should make that space intelligible. Its use of sharp identity constraints is valuable when it helps preserve those relations with their proper meanings, rather than forcing their value into a single verdict of survival or disappearance.

29.11 Historical Independence and the Choice of an Inquiry

The objection concerning numerical identity gains force when its demand for historical independence is stated precisely. A researcher can choose which question to investigate, yet the answer to that question may depend on events that occurred before the investigation. This subsection develops that distinction and examines the more demanding claim that collective continuity has one privileged interpretation across every legitimate inquiry. The two issues require separate arguments because acceptance of historical independence alone leaves the scope of the predicate open.

Consider a question about whether a particular appointment was made under the procedure in force on a specified evening. The question contains a selected institution, office, procedure, and time. Selecting those parameters gives the inquiry its object. Once they are fixed, an investigator's preference for one branch supplies an insufficient basis for changing who attended, which motion was adopted, or whether the required officers participated. The appointment can have occurred even when the investigator lacks its records. Conversely, a persuasive account can misdescribe the event. Purpose dependence in the

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selection of a question is therefore compatible with independence of its historical answer from the present investigator's wishes.

A criterion connecting that appointment to continuity adds a further premise. One institution might constitute a continuing office through a specified appointment procedure. A research question about the continuity of the office would then require the relevant constitutive rule and its temporal application. A question about the continuation of a practical programme could require different evidence, including coordination and transmission of competence. Recognizing these different questions gives each a determinate place in the inquiry. It also permits criticism of a report that answers the programme question and presents its conclusion under the office question. Such criticism depends on maintaining the distinction between the two targets.

The stronger objector can accept every word of that account and still maintain that the whole organization has one numerical identity beyond either office continuity or programme transmission. The present method leaves that proposal available. Its defender would need to explain what the whole-organization question adds, which facts determine its answer, and how the proposed criterion relates to the narrower predicates. The framework can represent the proposed criterion as a candidate and examine its consequences. Calling the criterion metaphysically privileged would express the defender's further commitment. Establishing that privilege requires the substantive argument which the label announces.

This distinction also clarifies disagreement between researchers. Two investigators can disagree about the facts under one accepted criterion. They can agree about all relevant events while disagreeing about whether the criterion captures the intended question. They can also answer different questions while treating their answers as competitors. The appropriate response differs in each case. Documentary evidence can address the first disagreement. Argument about the object and grounds of the inquiry addresses the second. Explicit reformulation of the questions can dissolve the third or reveal a consequential dispute beneath it. A single score of inter-investigator agreement would combine these distinct successes and failures.

The possibility of a constituted fact deserves equal care. An institution can give a competent act the power to establish a standing. The act's effects then belong to the history being investigated. An investigator who discovers the act acquires evidence about that history. An investigator who also occupies the competent office may perform a new act with its own effective time and scope. These roles can belong to one person while retaining different inferential functions. A new designation can settle the institution's future treatment of a branch under an accepted procedure. Whether that designation also establishes a fact about an earlier period depends on the institution's relevant rules and the defensibility of the retrospective interpretation.

The proposed suspension of deeper metaphysical claims therefore has a positive methodological content. It keeps the researcher's choice of question, the historical determinants of an answer, and the authority to change an institutional condition separately examinable. This separation protects a limited finding from both inflation and premature dismissal. A finding about an effective appointment can be well warranted even while the identity of the whole organization remains disputed. An unresolved whole-organization question can remain philosophically important even when a registry has settled its own practical classification. Each finding carries the scope supplied by its evidence and criterion.

There remains a serious limitation. A method that only checks internal coherence could accommodate a criterion with a completely mistaken understanding of collective life. A coherent answer under an arbitrary rule would then have little philosophical value. The method consequently requires an argument about the relevance of the rule, independently interpreted cases, and exposure to objections that the rule's advocates have reason to take seriously. Formal audit identifies what follows from a criterion and where its application breaks down. Substantive assessment asks why that criterion deserves to organize the inquiry. The combination permits disciplined disagreement while leaving a stronger theory of collective identity open to development.

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29.12 Admissible Histories and the Warrant for Exclusion

The admissible-family construction places an important burden on the meaning of admissibility. A model can call a verdict secure because it holds throughout its retained family, yet an omitted history could reverse the verdict. This subsection examines the relation between that conditional result and a justified historical assertion. It distinguishes logical exclusion, evidential exclusion, and the limitations of the researcher's search for alternatives.

In the formal construction, an admissible history satisfies the stated conditions. The word identifies membership in a specified family. Its use becomes evidentially stronger only when those conditions and their application have adequate grounds. For example, a family may exclude every history in which a meeting occurred after the deadline because an authenticated, correctly interpreted record establishes the earlier date. Alternatively, the researcher may have excluded those histories because the database contains a field labeled "appointment date" and the field was assumed to record the effective act. The same formal restriction can arise through these different routes. The warrant for the resulting finding depends on which route the inquiry actually took.

A secure verdict has two separable achievements. First, every retained history gives the stated answer under the fixed criterion. Second, the researcher has reasons to regard the retained family as adequate for the question. The mathematical result establishes the first achievement from the represented premises. The second achievement draws on source assessment, domain construction, interpretive adequacy, and a search for relevant omissions. A report should make the transition between them visible. Its conclusion can then state both the finding and the grounds on which the alternatives were limited.

Consider an archive containing a signed approval certificate but lacking its attachment. The certificate might concern continuation of the organization, transfer of a programme, or permission to use a venue. Treating its approval language as an identity assertion immediately restricts the family. A later attachment may reveal that the document concerned programme succession. The correction changes the interpretation of the premise. It leaves intact the existence of the certificate and can preserve an important historical relation, while withdrawing the identity constraint extracted from it. The earlier algebra may have been correct for the supplied constraint. The revised inquiry locates the failure in the bridge from document to constraint.

This example motivates an exclusion record. For a consequential alternative, the investigator should be able to identify the premise which rules it out, the evidence supporting that premise, and the interpretation connecting the evidence to the modeled condition. A record of this kind makes an objection tractable. A critic can challenge the certificate's authenticity, its scope, its effective time, or the criterion assigning identity significance to its content. Each challenge has a different object. The response can revise the relevant premise without discarding every other established fact in the dossier.

The demand for an exclusion record should be proportionate to the inquiry. Histories can vary in indefinitely many irrelevant details. A missing description of the meeting room's wall color need have no effect on the accepted appointment criterion. Requiring exhaustive enumeration of every conceivable history would prevent finite inquiry. The useful standard is target relevance: retain differences that can change the answer or affect the warrant for a premise on which it depends. A compressed representation can therefore be appropriate when the researcher explains why its omitted variation leaves the target unchanged. The evidence-recovery construction gives that claim an exact form within a specified family, while the family's adequacy remains a substantive obligation.

An adversarial search can improve the family without claiming final exhaustiveness. One route begins with the preferred answer and asks what historical change would reverse it. The investigator then checks whether the retained evidence excludes that change. Another route begins with each decisive premise and varies its interpretation while holding the other premises fixed. A third checks whether the unit domain omits an organization that could alter the meaning of an apparent split or merger. These are proposals for disciplined inquiry. Their value lies in the concrete alternatives they produce and test, rather than in a general assurance that every conceivable error has been eliminated.

The empty-family case imposes a separate limit. When the retained constraints conflict, universal statements over the family can acquire vacuous logical truth. The research procedure treats that state as

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History	Reference at τ	Domain condition	Identity inquiry
h_{two}	Alpha refers to a; Beta to distinct simultaneous b	Two realized normalized units	Different under slice separation
h_{alias}	Both cards refer to a	One unit, two documentary descriptions	Same appearance
h_{plan}	Alpha refers to a; Beta's proposed operating unit is unrealized	Second collective reference fails at τ	Required second argument absent

Table 13: Constructed alternatives for the referential premises of a two-card identity inquiry. The retained cards are identical across the three histories.

a failed joint specification requiring diagnosis. It cannot support a historical finding merely by appealing to the absence of an admitted counterexample. A repair should identify what was withdrawn or retyped and explain why that change deserves acceptance. Different defensible repairs may produce different findings. Their disagreement preserves information about the dossier's unresolved warrant.

Admissibility thus occupies a middle position between unconstrained imagination and an assertion of historical truth. It records the alternatives that survive a declared assessment at a particular stage. The finding derived from them can be precise, informative, and revisable. Its strength grows when the investigator can defend the exclusions that make it possible. The philosophical contribution of the family representation is partly this accountability: it exposes where a claim of certainty depends on the architecture of inquiry and makes changes to that architecture available for explicit criticism.

29.13 Referential Success and Uncertain Identity

An identity query ordinarily presupposes that its expressions successfully pick out the objects being compared. Archives can leave that presupposition unsettled. This subsection develops a small record case to distinguish uncertain identity between supplied collective units from uncertainty about whether the relevant units have been supplied. The distinction matters because adding a partition over records can conceal the very question which should precede the identity calculation.

At a fixed time τ, suppose an archive supplies two unverified cards, r_A labeled Alpha and r_B labeled Beta. Their retained content leaves open whether these labels name two operating collectives, two names of one collective, or an operating collective and a merely proposed second unit. The three stipulated histories in Table 13 represent these possibilities. The table concerns the operating units claimed at τ. A proposal document and the people developing a proposal can themselves be actual objects of another inquiry, while the proposed operating unit remains unrealized at the specified time.

In the first history, the normalized appearance domain contains a and b at the same time. Their distinctness at the declared resolution supports the simultaneous separation condition. Each card has an actual collective referent, and the identity verdict under that condition is different. In the second history, both documentary tokens refer to a. The total record-to-appearance normalization remains available, although its fibers differ from those in the first history. Two documentary tokens then yield one normalized appearance. Their distinctness as pieces of an archive places no pressure on reflexivity of the appearance's identity.

The third history has a different status. The second card's claimed operating collective lacks an appearance at τ. The total-reference premise of the normalization construction is therefore unavailable. Applying that construction to both cards would require the investigator to add a referent which the history fails to supply. A formal system could introduce a special marker for failed reference, provided its semantics were declared. That marker would record failure of the premise. It would have a different role from a second realized collective with an ordinary identity class.

The same distinction constrains the interpretation of a negative answer. Saying that two realized units are different presupposes that both enter the comparison. Reporting that a proposed unit had yet to be realized answers a prior occurrence question. Combining both reports under one binary "different" code would discard information needed for later reasoning. A rule for transfer between two existing

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collectives, for example, could apply to the first history while lacking the required recipient in the third. A common negative identity code would conceal that difference in applicability.

All three histories return the same retained cards. The evidence therefore leaves the reference condition unresolved before it leaves an identity relation unresolved. Conditional on the first two histories, the ordinary identity question has opposing answers. Across the full comparison, the investigator must first distinguish successful reference to two comparison terms from the failure of one term. A report can express the result in that order: the archive has yet to settle whether Beta was a realized operating unit; conditional on actual reference, the retained descriptions permit either an alias or a separate unit. That statement preserves more of the evidential situation than a bare declaration of ambiguous identity.

The case also clarifies why source authenticity and referential adequacy require separate attention. An authentic card can faithfully document a proposal. Its author may have had every reason to create it before the proposed organization existed. Authenticating the card would then strengthen confidence in the occurrence of the proposal while leaving its realization open. The investigator would need further evidence of organization at the relevant time, interpreted under the chosen unit definition. Conversely, a later organization can actually exist even when one purported early card is inauthentic. The truth of the occurrence claim and the quality of that particular evidential route can come apart.

Temporal precision adds another layer. Beta may be merely planned at τ and operating at a later time. A retrospective account might legitimately trace its development through the proposal, provided it distinguishes the project history from the later operating-unit history. Backdating the operating appearance to the first proposal changes the unit criterion or introduces a substantive continuity bridge. A heading that combines proposal and operation into one narrative period leaves that bridge implicit. An explicit temporal account can retain the full development while showing when the conditions for the focal appearance were satisfied.

The philosophical issue extends beyond careless records. A collective description can refer at different organizational levels: a campaign, its administrative committee, a registered association, or the recurring workshop it arranges. Whether two cards describe the same unit may depend on which of these objects the inquiry intends. Evidence about the cards' use can help identify that intention, while evidence about organization helps determine whether the intended unit was instantiated. Treating those tasks separately prevents a convenient domain choice from silently settling a contested question about the object under discussion.

The model's domain consequently deserves a substantive defense alongside its identity criterion. Each can fail independently. A well-motivated continuity criterion may be applied to records which supply an inappropriate unit. A carefully normalized domain may support a criterion whose relation to the intended identity question remains weak. The reference case gives the inquiry a disciplined point of return when such failures appear: re-examine the occurrence and level of the unit, retain the documentary history, and then rebuild the conditional identity comparison with its premises visible.

29.14 Membership Evidence and Criterion Boundaries

Sharp boundaries can enter collective reidentification through the measurement of a feature or through the rule assigning that feature identity significance. These routes create different philosophical obligations. This subsection examines a membership example in which uncertainty about a count can be resolved while the justification of the chosen threshold remains open. It then considers why a transparent threshold can still be insufficient for a globally coherent identity account.

Suppose the predecessor has sixty distinct members at a declared baseline. At a later cutoff, twenty-nine are verified continuing members of a candidate successor, twenty-nine have verified departures, and the status of two remains unresolved. The membership definition and dates are fixed throughout this constructed comparison. Each unresolved person can be a continuing member or a departed member independently of the other. The possible continuing counts are therefore twenty-nine, thirty, and thirty-one. Expressing retention as a fraction of the baseline roster gives $29/60$, $1/2$, and $31/60$.

Now stipulate an eligibility rule requiring at least thirty continuing members. Under that rule, the first count fails and the other two pass. The relevant evidence would concern the two unresolved statuses at the

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cutoff. If both departures are established, the candidate fails the stated rule. If at least one continuation is established, the candidate passes even while the exact count could remain uncertain. The precision needed to answer the eligibility question can thus be lower than the precision needed to reconstruct the complete membership history. The criterion identifies which uncertainty is consequential for its own application.

That calculation leaves the threshold's philosophical warrant untouched. The rule could express a negotiated administrative convention, an organizational constitution, or an analyst's proposed proxy for continuity. Its justification differs across these roles. In an inquiry into a standing constituted by a valid rule, the threshold can be part of the object whose application is being reconstructed. In an inquiry into a broader historical identity, the analyst needs an argument for why crossing this particular boundary carries the asserted significance. An exact count supplies evidence about whether the boundary is crossed. It supplies little by itself about why this boundary deserves the role.

The difference becomes visible at the cutoff. If a researcher regards twenty-nine continuers as insufficient and thirty as sufficient, the additional person's presence changes the rule's verdict. That sharp change may be entirely appropriate for a constituted voting entitlement. Its appropriateness for continuity of organized activity depends on further features. The additional person might hold a unique coordinating role, or might have little involvement in the ongoing practice. A membership-only rule deliberately abstracts from that difference. The researcher must explain why the abstraction fits the inquiry, or expand the criterion so that the relevant organization enters its premises.

Requiring such an explanation differs from requiring every social distinction to vary smoothly. Some institutional acts take effect at a specified moment and can change standing sharply. A gradual process can also support a practical boundary chosen for reasons of coordination or administrability. The question is which role the boundary serves and which grounds support it. A criterion can be precise because a competent procedure constitutes a status, because a measured feature has a defensible threshold for a stated use, or because an analyst imposes a convenient division. These forms of precision deserve distinct accounts of their authority and limits.

Even an independently justified eligibility threshold leaves the branch constraint active. Consider a separate fully specified division of the sixty members into two distinct simultaneous successor units with thirty members each. Both satisfy the weak-majority threshold. If qualification were treated as sufficient for identity with the predecessor, transitivity would identify the two simultaneous units. The normalized domain and slice-separation premise exclude that result. The rule can still serve as an eligibility condition, while a further account determines whether either eligible branch carries the relevant identity. The formal conflict locates a missing part of the criterion's role.

A strict-majority rule avoids that particular double qualification by requiring more than thirty members. In the evenly divided case, it qualifies neither branch. This restores consistency with termination of the predecessor's identity path, but the consistency has a limited meaning. It establishes that this output fits the formal constraints. Whether termination is the correct historical interpretation still requires the substantive continuity argument. Selecting a threshold because it produces a coherent partition can be useful during model construction, provided the researcher retains the further obligation to justify the resulting verdict.

The two-member uncertainty also shows why empirical and conceptual vagueness deserve separate descriptions. In the stipulated case, every person has a definite status under an agreed definition, and records are incomplete. A different case might involve a disputed membership category: occasional participants could count under one understanding of the collective and fall outside another. Additional attendance records could clarify what those people did while leaving the classification dispute open. The admissible family can represent both kinds of variation, yet the inquiry should record which coordinate varies. Evidence acquisition and conceptual revision then receive appropriately different tasks.

A similar caution applies to apparent sensitivity. A verdict that changes between twenty-nine and thirty continuers may reveal a brittle proxy, or it may correctly reflect a sharp institutional rule. Reporting the change is informative; interpreting it as a defect requires an argument about the intended criterion. Sensitivity analysis becomes philosophically useful when it identifies the premise responsible for the

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change and asks whether that premise has the claimed role. The mere fact that a classification has a boundary is compatible with both a justified institutional distinction and an inadequately motivated model choice.

The membership case therefore supports a layered account of precision. The researcher can establish the count, clarify the classification of members, state the eligibility rule, test its compatibility with the identity construction, and defend its relation to the focal continuity question. Success at one layer contributes a specific result. The subsequent layer requires its own reasons. Preserving this sequence lets the mathematical account improve precision while keeping the substantive philosophical commitments available for criticism.

29.15 Participant Testimony and the Scope of Collective Reference

Participant testimony can carry knowledge that formal records omit. It can also combine several continuity relations in ordinary first-person plural language. This subsection examines the interpretive work needed to preserve that knowledge without silently converting every report of continued activity into an institutional identity premise. The proposal concerns the scope and grounds of testimony, with the speaker's own understanding retained as part of the inquiry.

Suppose a participant in a successor collective says, "We continued the association's work." This is a constructed statement for the present analysis. The speaker might mean that a recurring programme continued, that several former participants kept cooperating, that the successor retained an institutional standing, or that the whole association remained the same organization. These meanings can overlap in a particular history while supporting different inferences. The investigator's first task is to understand which claim the speaker intends and which experience or record supplies its grounds.

The participant may have direct knowledge of how a workshop was organized, which skills were transmitted, and what difficulties arose after the division. An administrative document recording termination of a registered unit can coexist with that account of continuing practice. If the inquiry records only the administrative identity verdict, it can lose the very history which the participant describes. If it treats the participant's practical account as establishing registered continuity, it gives the testimony a different object. A careful account can retain both findings with their respective grounds and explain the relation between them.

The pronoun also requires attention. Its referent might be the speaker and a few colleagues, the current group, or the predecessor interpreted through its present participants. A change in the referent of "we" across an interview can be intelligible in ordinary conversation. In a formal argument, the same change can silently join units which the record otherwise distinguishes. Clarifying the pronoun serves the speaker's account when it makes visible which relationships the speaker is describing. It becomes a distortion if the investigator forces every use into a predetermined unit and then treats the resulting formulation as the participant's own settled theory.

Different participants can therefore supply complementary accounts through apparently conflicting identity language. One may say that the association ended because its governing assembly dissolved. Another may say that it continued because the cooperative practice remained active. Before assessing inconsistency, the investigator should ask whether the two statements answer the same question. Their accounts might jointly support a history of institutional termination and practical continuation. They might also express a substantive disagreement about which relation constitutes the association's identity. Preserving the initial formulations and clarifying their intended objects makes that difference assessable.

The position of a participant supplies particular access and particular limitations. A person closely involved in routine work may understand its organization more accurately than a remote official. A person responsible for maintaining a register may have stronger evidence of its procedural history than a programme participant. Neither role automatically settles every continuity question. The evidential assessment should connect each assertion to the speaker's access, interpretive competence, records and opportunities for correction. Authority to constitute a standing requires a further account of the relevant office or procedure.

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This approach also gives excluded participants a determinate place in the inquiry. Their testimony can reveal omitted activity, disputed representation, or a different history of the division. Assessing those claims requires attention to their content and evidential grounds. The fact that a participant lacks control over the official archive is compatible with knowledge relevant to the focal question. Equally, the political importance of hearing an account leaves its specific historical assertions open to examination. Inclusion and critical assessment can support one another when the investigator states the basis of each finding.

The proposed record should therefore preserve the participant's wording, the circumstances of the statement, its clarified scope, and the grounds for any formal extraction. Where clarification remains unavailable, the inquiry can retain alternative interpretations instead of imposing an identity constraint. This practice protects the richness of collective experience while keeping the formal construction accountable to the particular claims from which its premises were drawn.

30 Preservation and Revision of Identity Findings

This section specifies how a continuity finding should be recorded and revised. It distinguishes the current answer from its historical versions, identifies the dependencies that require downstream correction, and explains why an uncertain result can remain a valuable research artifact. The purpose is to preserve the reasoning as well as the final label.

30.1 The Content of a Reviewable Finding

A reviewable finding should name the collective appearances, time interval, organizational resolution, and continuity question. It should state the criterion, the relevant evidence, and the status of the verdict. Where a family of possibilities remains, the record should identify the secure conclusions and the alternatives responsible for unresolved pairs. The finding's scope should remain visible when it is cited elsewhere.

The supporting record should distinguish descriptive observations from identity constraints. A membership overlap is an observation under a counting rule. A positive identity pair is a conclusion warranted by a criterion and evidence. A required distinction can follow from normalized co-presence or another source. Storing these items in separate fields allows later reviewers to identify which inferential step has changed.

The record should also retain the candidate universe. A unique selected successor among two branches is a different claim from a unique successor among every plausible continuation. If the search was limited to registered bodies, that limit belongs with the finding. A later discovery can then be understood as an expansion of the candidate set rather than an inexplicable reversal of an unchanged comparison.

30.2 Versioned Criteria and Evidence

Criteria can evolve during inquiry. An operational definition may be refined after a counterexample reveals that it conflates activity similarity with transmitted practice. A membership rule may change when dual affiliation becomes visible. Such revisions can be intellectually valuable, provided the earlier formulation and the reason for change remain available.

Each version should state whether it changes the target question, the measurement, the admissibility constraints, or the consequence policy. These are different revisions. A change in the question may make earlier and later answers compatible. A correction to a measurement may undermine an earlier answer to the same question. A new consequence policy may alter a decision while leaving the historical finding intact.

Evidence revisions need similar care. A source may be newly discovered, reinterpreted, corrected, or withdrawn. The record should identify which historical possibilities the revision admits or excludes

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and why. The original source should remain preserved where authorized and feasible; the current interpretation should link to it. This allows the researcher to reconstruct the basis of an earlier finding without treating it as the current conclusion.

30.3 Propagation through Identity Dependencies

An identity link can support many later associations. If a is identified with b and b with c, records attached to a can become associated with the entity represented by c. When one premise is withdrawn, the derived associations need review. A system that stores only the final equivalence class can make that review difficult because the source of the transitive connection has disappeared.

The positive-link graph should therefore retain the basis of each edge. After a revision, connected components can be recomputed from the remaining warranted links. A class may divide, or an earlier conflict may disappear. The revision record should explain which links changed and which derived identities were affected. This is a logical dependency analysis rather than a new claim that the past itself changed.

Some downstream claims require more than recomputing a partition. A responsibility finding may depend on a particular undertaking as well as identity. A historical attribution may have independent provenance even after an identity link is revised. The correction process should examine the actual premises of each claim. Automatically removing every association with a revised identity would risk erasing information that still has a separate warrant.

30.4 Uncertain Findings and Research Value

An unresolved verdict can carry substantial knowledge. The investigator may have established the complete membership transfer, identified the operational lineage, verified the recognition act, and narrowed the remaining disagreement to a criterion choice. Reporting only "identity uncertain" would compress away these achievements. A good finding states what is known and where the uncertainty remains.

The admissible-family representation supports this richer output. Secure pairs remain usable, possible links retain their supporting scenarios, and required distinctions prevent unwarranted collapse. The family can also guide future inquiry by showing which records would discriminate the remaining alternatives. Uncertainty becomes structured research state rather than an empty placeholder.

Preserving disagreement can also improve later philosophical work. Two criteria may repeatedly separate along the same distinction, such as membership association versus continuing undertaking. That pattern can motivate a refined question or a theory of their relationship. A single forced label would hide the very insight that the cases generate.

30.5 Publication and Responsible Reuse

Published findings should state their scope in ordinary language before presenting identifiers or formulas. A reader should be able to determine whether the conclusion concerns registration, practice, membership, historical lineage, or numerical identity under a defended criterion. The formal details should support that understanding rather than require the reader to reconstruct the entire data model.

Reuse should preserve qualifications. A finding of operational continuity should not become an unrestricted statement of legal succession in a secondary summary. A possible link should not become a secure identity because a system lacks an uncertainty field. A current status should not erase the historical record of a prior decision. These are concrete requirements for faithful transfer of a research result.

The method's preservation principle is consequently simple and demanding: retain the question, criterion, evidence, alternatives, and inferential path that give a finding its meaning. This makes future revision possible without sacrificing the insights already established. It also connects the philosophical concern for continuity to the practical continuity of the research record itself.

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30.6 Historical Time and the Time of a Finding

A revision record should distinguish the time of an organizational event from the time at which evidence of it becomes available and the time at which an interpretation is accepted. This subsection examines the distinction through the completed documentary exercise.

The division settlement in Section 28 concerns an event at the initial split. Its discovery can occur much later in the investigation. Accepting its interpretation changes the current finding about the earlier boundary. The discovery itself is a new research event, while the represented institutional event retains its historical position. A chronology that substitutes discovery order for event order would misdescribe the association's history.

The scope attachment creates another temporal distinction. The attachment may have existed with the certificate even though the investigator first consulted the certificate alone. Its later examination can correct the current account of what the certificate meant from the outset. That correction differs from a later institution issuing a new statement that changes the certificate's effective scope. The latter would require an account of a new act and its authority, whereas the former concerns improved interpretation of an earlier source.

These differences should remain visible in the finding's provenance. A source date, an event date, an access date, and an interpretation date perform different functions. The record can preserve them without requiring the reader to treat every administrative timestamp as part of the substantive organization. The relevant question is which date supports the historical ordering or evidential claim being made.

An uncertain event date should remain uncertain. A record known to describe an act sometime between two meetings supplies an interval, not necessarily an exact boundary. If the continuity verdict depends on whether the act preceded another transition, that ordering becomes an evidence query. Assigning a convenient exact date can settle the formal order while concealing the historical uncertainty the model ought to preserve.

The temporal discipline also clarifies the interpretation of a changed identifier. A new identifier can record an actual organizational formation, a correction to a former conflation, or a new criterion for the same historical material. Those changes have different meanings. The research record should identify the event or reasoning that made the revision appropriate, so that later users can reconstruct what changed in the world and what changed in the account.

30.7 Support Routes after a Premise Correction

Withdrawing one premise can remove some derived findings while leaving others supported through independent routes. This subsection develops the practical interpretation of the closure and premise-bundle results for a revised collective history.

An identity pair can be supported by a direct institutional record and also by a chain of independently warranted transitions. If the direct record is reinterpreted, the pair may remain justified through the chain. The current finding should identify that surviving support. A simple rule that deletes every conclusion once any cited source changes would lose the difference between a premise's participation in an argument and its indispensability to that argument.

The converse can be equally consequential. Several apparent support routes can depend on one source or one criterion assumption. A direct certificate, a later catalogue entry, and a public history may all derive their identity claim from the same earlier statement. Correcting that statement can affect the entire support family. The number of stored references alone therefore cannot determine whether a finding survives a revision.

The useful revision question is which accepted routes remain after the changed premise is removed or retyped. For the finite positive constraint model, recomputing closure answers the formal portion of that question. The source inventory and criterion record answer the interpretive portion: which links still have warranted premises, and which were merely repetitions of the corrected claim.

This distinction also applies to negative findings. A required difference can be supported by distinct simultaneous organization, an incompatible institutional status, or another criterion-specific reason.

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Revising one source of exclusion leaves the other grounds to be examined. Conversely, a negative conclusion inferred solely from the absence of a recorded continuation can fail when the observation model is weakened. The revision should identify the particular reason for difference that changed.

Preserving support routes makes correction more discriminating. It allows an inquiry to revise the affected identity while retaining independently established transmission, activity, and institutional facts. The result is a current account with a traceable basis, rather than a choice between defending every earlier conclusion and discarding the entire reconstruction.

30.8 Consequences of a Finding and Grounds of a Decision

A published identity finding can inform a later decision whose validity depends on additional grounds. This subsection explains how revision of the finding should enter the review of that decision without silently determining its outcome.

Suppose a programme partner chose a branch to perform a remaining undertaking. The decision considered the branch's documented capacity, its explicit acceptance of the task, and a historical continuity finding. If the continuity finding is later revised, the decision's original justification has changed. The capacity and undertaking premises may remain supported. Their role in the decision must be examined under the partner's actual commitments and decision rule.

A different decision may have depended exclusively on a specified institutional identity. Revising that premise can then have a stronger effect. The relevant question is what relation the decision required, what authority governed it, and what consequences the applicable arrangement assigns to corrected evidence. A generic identity model does not supply those institutional or normative rules.

Historical interpretation and present response can also have different temporal targets. An earlier action can remain attributed to the appearance that performed it even when the current claimant connected to that appearance changes. A present body may have accepted an undertaking independently of its identity claim. The action and responsibility analysis in Section 26 provides the distinctions needed to assess those possibilities.

The revised report should therefore identify the consequences that logically depend on the changed finding and the separate grounds that require further review. This practice supports correction without pretending that one equivalence relation settles every question of responsibility, performance, or institutional standing. It also helps the user of the research understand what the revised finding actually requires them to reconsider.

The same principle protects valid historical content from being discarded because its practical use was contested. A finding can have sound documentary support while a particular allocation based on it was poorly justified. Criticism of the allocation and criticism of the historical finding should each identify the premise they challenge. The research record can preserve that distinction even when public discussion combines the two.

30.9 Comparison of Earlier and Later Answers

A version history becomes intellectually useful when it explains the relation between successive answers. This subsection identifies several possible relations and the evidence needed to distinguish them.

An earlier and later answer can concern the same target under the same criterion but differ because a factual premise was corrected. The revised finding should explain which observation or interpretation changed and how the resulting family differs. The initial-to-final query in the integrated case provides one example: accepting the rival certificate's identity reading creates conflict, while its clarified programme scope removes that particular premise.

The answers can also differ because the inquiry changes its object. A study may first ask who continues a registration and later ask who maintains a practice. The two answers can both be supported without one correcting the other. A version record that labels the later answer simply more accurate would conceal the change in question. The valid use of the earlier result should remain available within its original scope.

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A third possibility is a refinement of measurement under a stable criterion. Correcting duplicate roster entries, resolving a disputed membership date, or distinguishing active participants from nominal members can change a retention calculation. The record should state which population the earlier and later measures represent. Where the population definition changes, the comparison must separate the descriptive correction from a revised criterion of relevance.

A fourth possibility is a changed decision policy. A practical body may replace a weighted preference with a threshold or tie-break rule. The same historical evidence can then lead to another selected recipient. The revision concerns the policy and its rationale; it need not imply that the earlier historical reconstruction was false. The score-origin and horizon cases in the selection analysis show how even modest changes in a policy's semantics can alter its output.

These comparisons make version history part of the argument. They identify where knowledge increased, where interpretation changed, where a question was replaced, and where a practical choice was revised. Preserving that trajectory allows later research to retain the insights of an earlier formulation while improving its limits.

31 Integration of Findings across Local Histories

This section examines the reuse of collective-identity findings produced over different portions of a history. It first identifies the semantic and referential conditions of comparison, then derives a criterion for exact integration of two complete local identity relations. A divided-predecessor case shows how locally coherent findings can conflict only when their domains are combined. The final discussion extends the interpretation to uncertain findings, source dependence, and revisions of a shared history.

31.1 The Object of a Combined Inquiry

Two studies can use the same organizational name while answering different questions. Before their findings are combined, the investigator must establish what each study has identified and which relation its output expresses. This subsection treats that comparison as a substantive part of integration.

One study may track a constituted association through amendments to its governing procedure. Another may track a programme transmitted among several organizations. A third may catalogue records under an institutional name for reasons of custody. The resulting links can all be useful. Their shared vocabulary of continuity supplies no warrant to place them in one equivalence relation.

A combined account can preserve the studies through typed relations. It can record the association's identity path, the programme's branching transmission, and the archive's custody history separately, then examine their interactions. This arrangement retains substantive agreements that would otherwise become apparent contradictions. The programme can move between distinct associations, and the records can enter an archive independently of either association's identity.

The exact integration result below addresses a narrower situation. Both local studies claim to identify the same kind of collective object under compatible criterion semantics. Their appearance domains use a common interpretation of time and organizational resolution. The inquiry seeks a global identity relation that preserves both local findings exactly. These conditions make integration a well-defined question while leaving their evidential justification available for examination.

Compatibility of criterion names is insufficient. Two researchers may both use institutional continuity while differing about effective amendment, termination, or the role of external recognition. Their operational definitions and permitted changes should be compared. A disagreement at that stage is a dispute about the rule by which a historical finding is made. Applying equivalence closure to the outputs would hide the dispute inside a calculation.

The combined inquiry should therefore state its target before attempting to reconcile identifiers. It can adopt one criterion for a specified purpose, compare several criteria in parallel, or preserve a disagreement for later philosophical analysis. Each option produces a different interpretive object. An exact merger of identity relations is appropriate only within a declared common target.

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31.2 Shared Appearances and Referential Correspondence

Local studies seldom begin with identical appearance symbols. This subsection examines the evidence required to establish their overlap and the consequences of treating a correspondence as already settled.

Suppose one study calls an appearance the Northern Workshop Assembly and another calls it Committee Seven. A common date and partially matching roster can support further investigation. They leave open distinct simultaneous bodies, a governing committee and a wider association, or two descriptions of one unit. The correspondence should be grounded in the source references and organizational relations relevant to the chosen resolution.

Once a correspondence is accepted, the formal construction represents the common appearance by one element in both domains. This is a deliberate normalization of the studies' reference systems. The resulting shared element should retain links to the local records and the reasons for their alignment. Otherwise, later users may treat a common identifier as independent evidence that the source descriptions referred to one unit.

Different temporal resolutions can complicate the correspondence. An annual institutional entry may summarize several appearances that a monthly study separates. Equating the annual entry with one monthly appearance would require an account of what the annual unit represents. The temporal aggregation result in Section 5 shows that exact transport of identity through a coarsened domain has strong conditions. Matching dates by approximation cannot supply those conditions.

The same issue arises across organizational levels. A national association and a local branch may share officers and documents while remaining distinct units in the focal inquiry. A source can speak for one while mentioning the other. The mapping between local studies should preserve which unit each claim concerns. A combined history can represent their relationship without collapsing the two appearances.

The theorem assumes that this preparatory work has supplied fixed domains and their actual overlap. If several correspondences remain admissible, the inquiry should retain the alternatives and evaluate integration under each. A conflict appearing under one correspondence can be evidence of a mapping problem, although the mathematical conflict alone cannot establish that diagnosis. Reference and identity remain distinct stages even when their uncertainties interact.

31.3 Exact Local Findings and a Common Domain

The formal setting uses complete local equivalence relations. This subsection defines what completeness means here and why it matters to the interpretation of the integration result.

Let I_A and I_B be finite appearance domains. Put $K = I_A \cap I_B$ and $I = I_A \cup I_B$, with a common time map $\tau : I \to T$. Let E_A and E_B be equivalence relations on the local domains. Each represents a complete local identity finding: appearances in one class are identified, and appearances in different classes are treated as different entities within that finding.

This meaning differs from the secure intersection of an uncertain family. Two appearances can occupy different classes in that intersection because their identity is unresolved. They need not be securely different. A local report that supplies only a secure summary therefore cannot be inserted into the present construction as though all its separate classes were determinate historical distinctions.

Let N_g be a symmetric global negative relation on I. It contains the required distinction between distinct simultaneous normalized appearances, including pairs whose members were previously studied in separate domains. It may also contain other explicitly warranted exclusions. The current model imposes no additional class-count, path, or historical compatibility condition beyond the stated relations.

An exact integration is an equivalence E on I whose restriction to each local domain is exactly the corresponding local equivalence and which avoids N_g. Exact restriction preserves local differences as well as local identities. A global relation that joins two previously distinct classes within I_A would revise study A's finding, rather than integrate it without change.

The question is consequently precise: under which conditions can the two complete findings coexist in one globally constrained identity history? The answer concerns compatibility of the stated findings.

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Their empirical truth and the adequacy of the common criterion remain separate matters.

31.4 Derivation of the Exact Integration Criterion

The combined positive closure provides both a test and a candidate integration. This subsection derives the result by examining how a path through one local domain can affect identity within the other.

Define

$$C = \mathrm{Eq}_I(E_A \cup E_B).$$

Every global equivalence extending the local identities must contain C. A necessary condition for an integration is therefore that this forced closure avoid the global exclusions. Another necessary condition is that the local findings agree on their overlap.

Proposition 31.1 (Exact integration of two local identity findings)**.** *An exact integration exists if and only if*

$$E_A|_{K^2} = E_B|_{K^2} \quad \textit{and} \quad C \cap N_g = \varnothing.$$

Under these conditions, C itself is an exact integration and is contained in every other exact integration.

Proof. An exact integration restricts on K to both local relations, so their overlap restrictions agree. It contains C, which must therefore avoid N_g. Conversely, assume overlap agreement. Consider a path in $E_A \cup E_B$ whose endpoints lie in I_A. Every excursion through $I_B \setminus I_A$ enters and leaves at elements of K. Its boundary endpoints are E_B-related by transitivity, hence E_A-related by agreement on K. Replace each such excursion, and each E_B link with both endpoints in K, by its E_A link. The resulting E_A path shows that the original endpoints are E_A-related. Thus C restricts exactly to E_A. The symmetric argument establishes its exact restriction to E_B. If C avoids N_g, it is an exact integration. Its minimality follows from the defining closure property. □

The proof performs a specific task. Overlap agreement prevents a detour through one study from introducing a new identity within the other's domain. The global exclusion test then checks the pairs that become relevant only when the domains are combined. These are distinct requirements, and the next case shows why the second cannot be omitted.

The complete family of integrations can also be written using the positive/negative construction already established. Use positives $E_A \cup E_B$ and negatives

$$N_* = N_g \cup (I_A^2 \setminus E_A) \cup (I_B^2 \setminus E_B).$$

The local nonidentity pairs enforce exact restriction. Every equivalence in the resulting admissible family is an integration, and every integration belongs to that family. The construction therefore preserves possible global completions beyond the least closure where the local evidence leaves them open.

31.5 A Shared Predecessor and Separately Studied Branches

Consider two local histories that share an earlier appearance while observing different branches after a division. This subsection shows how the original branching constraint can emerge only at the integration stage of a research programme.

Study A contains a at time 0 and b at time 1, with one identity class $\{a, b\}$. Study B contains the same a and a different appearance c at time 1, with one class $\{a, c\}$. Each local domain contains at most one appearance at each time, so each finding respects its local slice separation. Their overlap contains only a, on which the equivalences necessarily agree.

The combined domain contains distinct simultaneous appearances b and c. Suppose their distinct organization has been independently warranted at the common resolution. Global slice separation therefore excludes their identity. Yet the combined positive closure relates b to a and a to c, hence b to c. Proposition 31.1 rejects an exact integration.

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The local studies can each have been internally coherent because each omitted the competing branch. The integration exposes an incompatibility that neither local domain could express. This result has an immediate research implication: a uniqueness claim made within a restricted candidate universe should retain that restriction when reused. Enlarging the domain can introduce relevant simultaneous distinctions and rival continuations.

The global conflict still leaves its substantive diagnosis open. One study may have tracked programme transmission while using identity language. The branch correspondence may be wrong. The criterion may permit only one of the proposed institutional transitions. A source may have overstated its scope. Alternatively, the supposed simultaneous distinction may need revision because the records describe aliases of one body. The test identifies the incompatible package; evidence and argument determine its repair.

The combined account can preserve the local studies' valid contributions during that review. Their source histories, transmitted practices, and participant relationships remain available. If both local links are retained as directed lineage while one identity use is revised, the result can become richer than either original account. The failure of exact integration need not imply that one entire study should be discarded.

31.6 A Compatible Chain and the Limits of Least Integration

A simple compatible case clarifies what successful integration establishes. This subsection then considers a case in which the least integration is only one of several possible completions.

Let study A contain a at time 0 and b at time 1, identifying them. Let study B contain that same b and c at time 2, also identifying them. The overlap is $\{b\}$, and the combined closure has one class $\{a, b, c\}$. If the global exclusions introduce no conflict, this closure is an exact integration. It supplies the initial-to-final identity that neither local domain could state by itself.

The derived connection has an explicit support path. It depends on the first local identity, the second local identity, and the warranted correspondence of their shared appearance b. A revision of any of those premises can change the integrated connection. The fact that the global relation is transitive does not remove its dependence on the local findings from which that relation was constructed.

Now consider two disjoint local domains, each containing one appearance at a different time. Each local study has a singleton equivalence class. Without cross-domain exclusions, the least combined closure keeps the appearances separate. A larger equivalence joining them also preserves both local findings, because neither study contains a pair whose distinction would be violated. Exact integration therefore permits two global completions.

Selecting the least completion would introduce a substantive difference claim about the cross-domain pair. Its mathematical minimality states that it adds the fewest identities beyond those forced locally. That property can be useful for a conservative information representation. It is not, by itself, historical evidence that the two appearances belong to different entities.

The appropriate output depends on the task. An information system can retain separate provisional identifiers while recording that their historical relation is unresolved. A historical finding should preserve both compatible completions unless further evidence distinguishes them. A practical integration policy can choose a representation for a declared purpose, with that choice separated from the epistemic status of the cross-domain identity query.

31.7 Uncertain Local Families and Joint Compatibility

Many local studies produce families of possible histories rather than complete equivalences. This subsection explains how the exact integration result can be used within such an inquiry and why marginal possibility alone is insufficient for combining the alternatives.

Suppose study A retains several candidate identity relations and study B does the same. The combined inquiry must specify which pairs of local histories can jointly describe the episode under its accepted evidence and criteria. If the two studies share a source, a disputed appointment, or a normalization

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assumption, their alternatives can be dependent. Treating every candidate from one family as compatible with every candidate from the other introduces a substantive independence assumption.

For each admitted pair, the exact integration criterion can be applied. Pairs that disagree on shared appearances or force a global exclusion have no exact completion. Compatible pairs contribute their families of global completions. Retaining the union of those families preserves the global histories admitted through at least one jointly supported local pair. The support pair should remain associated with the completion so that later evidence can revise the relevant route.

A global finding can then be secure across the admitted completions even when the local studies remain partly uncertain. Conversely, each local study can securely identify a pair while their global combination fails because their criterion interpretations conflict. The status of a link within one family does not alone establish its compatibility with another family's assumptions.

The pairwise-summary limitation from Section 14 becomes especially important here. A local secure relation and possible relation can omit joint restrictions among candidate identities. Integrating only those summaries can therefore admit combinations absent from every local history. A combined finding should retain enough of the local family or its constraints to preserve the queries it intends to answer.

This requirement need not force publication of every internal data structure. A compact set of compatible scenarios, a constraint description, or a target-specific certificate can sometimes preserve the needed joint information. The adequacy of the representation depends on the future queries. The earlier recovery condition provides a precise criterion: the retained representation must distinguish every pair of admitted histories that require different answers to those queries.

31.8 Shared Sources and the Appearance of Corroboration

Integration can make a claim appear more strongly supported because it occurs in several studies. This subsection examines the source relations that determine whether repetition supplies new evidence or primarily documents the circulation of an earlier interpretation.

Suppose study A derives an identity link from a branch announcement. Study B cites a later institutional history that repeats that announcement. Their agreement can show that the claim circulated through several documentary settings. Its independence as corroboration of the underlying event depends on whether the later history used another evidential route. Separate publications do not automatically provide separate access to the event.

The same issue arises when one study reuses another's normalized appearance identifiers. The shared identifier can make the overlap easy to represent, while the actual correspondence still depends on the first study's source interpretation. A second use of the identifier supplies evidence of reuse. It does not independently confirm the original referential alignment.

The combined record should therefore preserve source routes alongside the formal support graph. A transitive identity path can depend on two claims that originate in one statement. Correcting that statement may affect both edges. The premise-bundle and fallible-query analyses provide different models for such dependence: one examines hypothetical withdrawal of accepted claims, while the other specifies permitted observation failures. Their interpretations should remain distinct.

Repeated sources can still have substantial value. A quotation preserved in another archive can recover wording from a lost document. Differences between copies can reveal an amendment or a later alteration. A study's explicit reasoning can expose an assumption that its source left tacit. These contributions should be recorded for what they establish. The method concerns the type of evidential gain, rather than discounting every dependent source.

The practical aim is a combined history whose apparent density reflects its actual support. Several independent routes can justify stronger confidence when a defensible observation model explains their relation. Several copies of one route can improve access and preservation. The distinction helps both historical interpretation and later revision.

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31.9 Revision of an Integrated Finding

A combined identity relation should remain revisable through the premises that support it. This subsection traces the effects of a local correction and explains why downstream revision must follow the actual dependency structure.

In the compatible chain a to b to c, suppose the second study discovers that its source describes a programme transfer rather than collective identity. Correcting the bc identity use removes the path that supported the integrated ac identity. The programme-transfer relation remains available. The first local finding ab can remain unchanged if its source and criterion warrant survive the correction.

An archive that stored only one combined identifier for all three appearances would need to recover the reason for the earlier merger of records. A preserved support path makes the revision inspectable. It shows which association depended on the corrected premise and which claims have independent routes. The integrated finding can then be recomputed without erasing the historical records attached to each appearance.

The same correction may have different effects on different downstream claims. A claim about a resource's provenance can remain supported by the transfer record. A claim about the current bearer of an earlier institutional office may lose its identity bridge. A moral undertaking can remain relevant under a separately argued succession relation. Revision should examine those premises individually, rather than removing every connection associated with the former identity class.

A source correction also differs from a later organizational event. A new division changes the history from its event time. Discovery that an earlier record was misread changes the current account of an earlier history. Both can alter a database's present output, but their temporal and evidential meanings differ. A combined finding should preserve the event described, the evidence used, and the time at which its interpretation was revised.

These distinctions support accountable reuse. A subsequent researcher can determine whether an identifier changed because the organization reorganized, the reference mapping was corrected, the criterion changed, or a source's scope was reinterpreted. That knowledge is part of the substantive history of the finding, not merely an administrative record of edits.

31.10 The Scope of a Shared Collective History

The integration analysis identifies a constructive limit on shared historical infrastructure. A common account can support coordinated research while preserving plural relations and unresolved questions. This subsection states what the formal result contributes to that aim.

Exact integration provides a test for combining complete local identity findings under a common target. It protects each local finding's identities and differences, makes newly relevant simultaneous constraints explicit, and preserves alternative global completions where local evidence leaves them open. The test can expose a conflict before a shared identifier silently spreads it through later work.

Its application still depends on the interpretation of the studies. Referential alignment, criterion compatibility, source dependence, and the status of local uncertainty require substantive judgment. These obligations are precisely why a research infrastructure should preserve the reasons for its links. A well structured system can make those reasons available; it cannot replace them with the fact that a link has been stored successfully.

A shared history can consequently contain several coordinated layers: collective identity under a defended criterion, branching transmission, institutional recognition, resource provenance, and the succession of particular undertakings. These relations can cross the same appearances in different ways. Their coexistence expresses the historical complexity of splitting and merger, while the integration conditions regulate which claims can jointly function as numerical identity.

The resulting ambition is a history that remains usable without becoming dogmatic. Secure findings can support further inquiry. Possible connections can retain their scenarios. Conflicts can identify premises for review. New evidence can alter the account through traceable steps. This form of continuity

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in research depends on preserving the relations that gave earlier findings their meaning while allowing those findings to change when their warrant changes.

32 Research Obligations

This section identifies the work required to strengthen or revise the proposal. It separates literature confrontation, documentary validation, formal extension, and philosophical adjudication. Each obligation concerns a substantive uncertainty left by the present argument and specifies the kind of result that would count as progress.

32.1 Literature Confrontation and Originality

The present source review establishes several relevant antecedents but remains selective. A fuller comparison should examine organizational identity across reorganization, collective responsibility, theories of group persistence, and methods for linking historical entities under uncertain evidence. Sources must be read at the level required by the claims made about them. Bibliographic discovery alone cannot support a detailed interpretation or an originality judgment.

The comparison should ask which parts of the proposed method are already available in existing accounts and where the integration adds explanatory or practical value. The elementary constraint and query results should be treated as mathematical tools. A stronger contribution would consist in showing that the combined method resolves a persistent ambiguity or improves a documented inquiry beyond an established alternative.

The project should also consider whether the present scope needs to narrow. If one organizational domain supplies coherent criteria and evidence while others require fundamentally different constructions, a focused paper may be more defensible than a general account of collective reidentification. The current title already limits the problem to splitting and merger in a specified modeling framework; the empirical work can refine that boundary further.

32.2 Documentary Validation and Discriminating Cases

The proposed study in Section 27 should be carried out on a case whose records can genuinely discriminate between criteria or histories. A successful application would demonstrate that the method produces findings with clearer warrants, detects a consequential false merge or separation, or identifies an evidential gap that another procedure overlooks. Merely reproducing a familiar public conclusion would supply a weaker test.

Failure is equally informative. The method may require records that are rarely available, produce too many unresolved alternatives for its intended use, or rely on a normalization that participants cannot defend. Such findings should prompt revision of the criterion, evidence model, or practical ambition. A method can remain valuable for archival inquiry while being unsuitable for time-sensitive institutional decisions, or the reverse.

Comparative cases should vary the relation among criteria. One can examine a case where membership and operation align, another where they divide, and another where authority preserves a status despite a major practical discontinuity. The aim is to test the method's distinctions, not to collect a larger number of illustrations that all support the same initial intuition.

32.3 Formal Extensions and Their Conditions

The elementary pair-classification theorem assumes a fixed finite appearance domain and an unrestricted number of identity classes. The paper has developed uncertain normalization, temporal aggregation, gap-permitting path interpretations, and exact integration of local findings under specified conditions. Further combinations require fresh derivations. In particular, class-count constraints, shared historical budgets,

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or defeasible institutional rules can change the family in ways that the unrestricted positive/negative criterion does not capture.

The fallible-query analysis represents observation uncertainty through explicit sets of admissible answers and distinguishes coordinate corruption from shared-source failure. A justified probabilistic extension remains open. It would need a defensible probability model for histories, source dependence, and observations, together with a clear target. Possibility counts or numbers of repairs alone supply none of those probabilities. The resulting numerical assessment should preserve uncertainty about the criterion itself.

Large histories introduce computational questions. Enumerating every partition can become impractical even when the conceptual definition is clear. The positive-closure procedure is efficient for its limited constraint family, but richer global conditions may require other methods. Any computational shortcut should be checked against the mathematical admissibility condition and preserve the meaning of unresolved outcomes.

32.4 Philosophical Adjudication and Consequence Policies

The central philosophical obligation is to determine whether a particular domain warrants one privileged identity criterion or a structured plurality of continuity questions. This requires engagement with numerical identity, the constitution of organizations, and the reasons for tracking them. The current method provides a setting for that argument and leaves its strongest conclusion open.

The relation to responsibility also needs fuller development. Specific undertakings, historical contribution, authority, control, and capacity can matter independently of identity. A consequence policy should state how these factors interact and why. The resource ledger in Section 17 deliberately supplies only a limited accounting model; it cannot replace a normative theory of obligation or credit.

Finally, the research should examine how identity findings affect the collectives they describe. Recognition, resource access, and public narratives can change later continuity. An adequate method should record those effects and distinguish them from evidence about the earlier history. This obligation connects the epistemology of reidentification with the governance of the conditions under which collective practices can continue.

33 Conclusion

Collective splitting and merger distribute continuity across members, operations, commitments, resources, and recognition. A later organization can inherit several of these relations while another inherits the rest. The central task is therefore to establish what a proposed identity finding means, what evidence warrants it, and which consequences can legitimately follow. This paper has developed a method for answering that task within a generative relational representation of collective histories.

The method begins with records and normalized appearances. It distinguishes the formation of a collective unit at a time from its reidentification across time. Directed lineage preserves transmission through branching and merger. A proposed identity equivalence adds a stronger relation, with explicit conditions on the distinction of simultaneous units. Under those conditions, one predecessor cannot be identical to both distinct simultaneous successors, and one successor cannot be identical to both distinct simultaneous predecessors. The result locates a required choice without deciding which continuity should govern it.

The branch restriction has an explicit antecedent in Jansen [4]; the separation of relevant succession from numerical identity also precedes this paper [1]. The present contribution lies in the connected evidential construction and its application, including the point at which each accepted premise enters a reviewable continuity judgment.

The substantive argument therefore examines membership, operation, commitment, and institutional criteria separately. Each needs an independent interpretation of the evidence and a reason tied to the inquiry. The divided-association case demonstrates their possible disagreement. Its weighted comparison

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derives an explicit threshold at which the selected successor changes, while the robustness result states when a ranking survives every admitted weight. These calculations expose the consequences of a policy; the justification of the policy remains a philosophical and practical task.

Evidence symmetry sharpens that task. When the retained reasons permit an exchange of two eligible successors, a unique selection respecting those reasons requires an eligible candidate fixed by the exchange. The absence of such a candidate identifies a limit of the retained warrant. It leaves open a fuller history with a discriminating fact, a revised criterion, or an explicitly justified practical tie-break. Adding termination as a fixed candidate can remove the formal obstruction without providing historical evidence that termination occurred.

Positive and negative identity constraints provide a constructive procedure. Their closure admits an identity partition exactly when it avoids every required distinction. Under the stated constraint family, the positive components and negative cross-links give an exact classification of secure identity, secure difference, and ambiguity. The intersection of admitted equivalences records secure identification; their union records possibilities and can fail transitivity. Preserving that distinction prevents incompatible possible links from being chained into a finding that no admitted account supports.

The construction also locates uncertainty on either side of a compatible partition family. Before partitioning, candidate normalizations determine which record tokens refer to one appearance. Pullback and descent conditions characterize the identities compatible with those choices. When the accepted premises conflict, a bundle-based repair analysis identifies the withdrawals that could restore coherence. Source grouping and withdrawal costs can change the preferred repair, so an optimized answer retains its evidential and policy dependencies.

Temporal aggregation supplies an additional test. Exact transport of identity into a coarser normalized domain requires agreement about the identities being collapsed within each interval. Distinct admissible identities can therefore prevent a common faithful normalization. Eliminating intermediate appearances from an output has a different structure: under the unrestricted constraint model, transported positive and negative relations preserve the exact visible family. This reduction retains the implications of warranted hidden records while leaving their provenance available for review.

Further results connect coherence to evidence. Adjacent partial matchings generate slice-separated identity paths under a contiguous-history model. Independent pairwise scores can fail to provide that coherence. Historical uncertainty and criterion disagreement occupy different coordinates, and valid nonempty refinement preserves secure verdicts within a fixed model. A record identifies a verdict exactly when its observational fibres contain no opposing answers. The corresponding query criterion specifies which evidence sets can discriminate every decisive pair of histories.

The selection analysis also distinguishes two senses of local choice. Greedy selection of individual edges can fail within a boundary because eligible edges compete for the same appearance. Exact optimization of whole boundary matchings does recover the additive global optimum when the feasible family is a product and the objective is separable. An anchored path objective, a terminal restriction, or a changed horizon requires its own comparison. The score-origin example further shows that ordinal rankings of edges can leave an additive policy underdetermined when selected matchings have different cardinalities.

Fallible queries extend the evidential result under explicit observation assumptions. Recovery requires separation between the admitted answer sets of opposing-verdict histories. The binary coordinate model and source-bundle model expose different failure budgets. Copied reports can share one source failure even when their displayed answer vector looks resistant to a small number of coordinate changes. A verdict can also be recoverable while the route by which it became true remains unresolved. These distinctions support a targeted account of what further evidence is needed.

Rule-changing histories give the institutional criteria a temporal structure. A sequence can preserve its declared authorization chain while altering the approval conditions governing later changes. Mechanically realized updates and prior-rule validation occupy separate roles. The constructed threshold example demonstrates both disagreement with a frozen-rule test and the loss of authorization information when only endpoint procedures are observed. Authorized branching still requires a separate numerical-identity

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analysis.

The retrospective revival analysis develops the authority question further. Finite positive-rule closure distinguishes conclusions grounded in accepted premises from conclusions merely compatible with reciprocal claims. Its scope is a declared warrant discipline, open to criticism and alternatives. Typed propositions preserve the differences among an act's occurrence, its institutional competence, its later recognition, and its present normative legitimacy.

Merger, suspension, and recognition show why the richer history should remain available after a single successor is selected. A common later entity can have asymmetric inheritances from distinct predecessors. Reconstitution can preserve a practice without establishing uninterrupted identity. A recognition act can constitute future standing while leaving some claims about the prior history unsettled. Responsibility and credit require their own premises, even when identity forms part of the argument.

The Manchester documentary comparison provides a bounded empirical check on the vocabulary of these distinctions. Formation, predecessor inclusion, specified membership relations, and ceremonial recognition can be coded separately. Converting every historical-root statement into identity would introduce a transitivity claim beyond that documentary content. Practical continuity and disputed shared-target verdicts remain subjects for the more extensive study design.

The performance-validation dossier supplies another boundary. Its two histories have opposite earlier identity verdicts and identical programme responses under every stipulated allocation policy. A correct forecast therefore leaves the historical query unresolved. This counterexample directs validation toward the actual target: historical recovery, criterion application, outcome prediction, and practical effect require distinct assessments. A classification that influences later resources can participate in producing an outcome without that outcome independently verifying its earlier historical content.

The proposed documentary study and revision protocol carry these distinctions into research practice. They preserve source provenance, criterion versions, candidate universes, conflicting evidence, and the dependencies of derived findings. An unresolved identity question can then retain substantial knowledge about membership, operation, lineage, and authority. Future evidence or argument can refine the result without erasing the insights that made the question worth investigating.

The integrated documentary exercise demonstrates that preservation through an entire inquiry. Its unique identity finding becomes inconsistent after a rival certificate receives an identity reading. Two explicit repairs retain different initial continuations. A later scope clarification warrants treating the certificate as evidence of programme succession, restoring the earlier partition through a more complete interpretation. The source's valid contribution survives the correction, and the intervening conflict remains part of the finding's evidential history.

Integration across studies extends the same discipline. Exact local findings can be combined only when they agree on their shared domain and their combined closure respects the global exclusions. Local coherence can conceal a conflict between separately studied branches. Where integration succeeds, the least relation need not exhaust the compatible histories. Uncertain local families require their joint alternatives and shared assumptions to remain available, so that a summary of secure information is not mistaken for a complete history.

The paper's achieved results are conditional and methodological. They provide explicit tests of coherence and evidential sufficiency and a structured account of the reasons that continuity criteria require. Empirical validation, a fuller literature confrontation, and the status of a uniquely privileged numerical-identity criterion remain open. These obligations define the next development of the inquiry and preserve the distinction between a rigorous model of reidentification and a completed philosophy of collective entities.

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