

Relational Fields and Criticality Preparation: A Conditional Model of Generative Escape

Wanhong HUANG
huangwanhong@serendip.ngo

Abstract

A perturbation can disappear quickly in one dynamical regime and cross a transition boundary in another. This paper studies the gradual transformation that changes that response. It proposes *criticality preparation* as a typed, route-specific comparison in which a persistent intervention changes local recovery, threshold geometry, potential barrier, finite-time gain, stochastic escape, or viable receiving access over a declared horizon. The concept is developed independently from its oceanographic inspiration. A dimensionless fold model yields exact expressions for the stable branch, unstable threshold, recovery rate, threshold margin, and potential barrier. A negative kick of fixed magnitude h crosses the frozen threshold exactly when $a < h^2/4$. Along a linear slow path, this condition gives a closed-form preparation time. A small-noise escape expression states a separate stochastic route with explicit asymptotic conditions. A triangular two-state extension then preserves both stable eigenvalues while cross-channel transient gain changes with relational coupling. Reproducible simulations verify the deterministic threshold within the toy system. With fixed noise and horizon, the estimated escape frequency rises from zero observed events among 10,000 paths at the two deepest parameter settings to 0.9732 at the shallowest setting. Step-size and noise checks preserve the qualitative parameter ordering. These results establish conditional model consequences and a computational benchmark. Target-domain interpretation requires construct validation, coherent intervention contrasts, longitudinal identification, mechanism discrimination, robustness, replication, and a separate safety profile. Historical explanation, normative classification, and policy endorsement remain open research stages.

Keywords: criticality preparation; relational fields; generative escape; fold bifurcation; basin margin; transient amplification; stochastic escape; viability.

Discussion Paper Note

This paper is a preliminary discussion paper intended to share an evolving idea and invite further dialogue, criticism, revision, and independent development. Its definitions, distinctions, and formal constructions remain provisional. Circulation across scholarly and practical communities is part of the purpose of releasing the manuscript at this stage.

The author treats the viewpoints, concepts, and lines of reasoning presented here as contributions to a shared field of inquiry. Similar or related ideas may have appeared in other intellectual, cultural, and disciplinary traditions. The manuscript therefore states its known antecedents, separates the researcher-origin proposal from later formal reconstruction, and leaves historical priority open pending a systematic originality review.

The arguments should be understood as provisional and historically situated. Readers are encouraged to question, test, revise, extend, reinterpret, or independently develop the ideas presented here. Where appropriate, acknowledgment of this paper as one point of encounter in the development of a related idea is appreciated. Such acknowledgment records an intellectual route; the ideas themselves remain available for criticism, revision, and independent development.

Responsible Use and Rights Reservation

This section separates requested scholarly conduct from the legal permissions stated on the following page. It records an ethical request for responsible use and then defines the narrower scope of retained legal rights.

The author encourages good-faith discussion, criticism, independent inquiry, and responsible use of the material in this work. Separately from the licence's terms, the author asks users to consider foreseeable harms when adapting or applying the proposed framework. This ethical request leaves the licence's permissions and legally authorized uses unchanged.

The author retains the rights preserved under CC BY-NC 4.0 and may pursue remedies to which the author is legally entitled for breach of the licence or violation of the author's independently applicable rights. Reuse remains independent from authorial endorsement. Third-party rights require authorization from their respective holders where applicable. Copyright exceptions and limitations, including applicable forms of fair use or fair dealing, remain fully available.

Notices

This page consolidates the manuscript’s publication status, licence, development disclosure, research-programme relation, and suggested citation.

Status. This working draft records an evolving stage of the author’s position and is circulated for discussion. Definitions, section structure, formal statements, and numbering remain subject to revision. Specialist proof review, target-domain validation, historical application, normative evaluation, legal analysis, and policy design remain outside its present scope.

Licence. Except where otherwise indicated, copyright 2026 Wanhong Huang. This work is made available under the Creative Commons Attribution-NonCommercial 4.0 International License (CC BY-NC 4.0). Subject to its terms, the licence permits sharing and adaptation for noncommercial purposes with appropriate attribution, a link to the licence, an indication of changes, and attribution that preserves the licensor’s independence from the reuse. Reuse is governed solely by that licence; the responsible-use request on the preceding page remains separate from its terms. The licence deed and legal-code link are available at <https://creativecommons.org/licenses/by-nc/4.0/>. The licence governs in case of conflict with this summary. Third-party material remains subject to the rights held by its respective rights holders.

Statement on the use of language models. The exploratory discussions and preparation of this paper involved OpenAI’s Codex. Codex supported exploratory dialogue, formal reconstruction, source discovery followed by website verification, simulation coding, argumentative criticism, and drafting in L^AT_EX. The author selected the research questions, directed and approved the theoretical commitments and epistemic status of the claims, and bears sole responsibility for the manuscript, including its definitions, formal constructions, taxonomy, numerical design, arguments, conclusions, and errors. Authorship credit remains with the human author. The access level and claim limit for every cited source are recorded in the accompanying literature audit.

Related research programme. This paper is project P019 and the fifth promoted paper in the research-stage *Generative Injustice* programme. P015 develops a typed lifecycle diagnostic, P016 develops transaction ontology, P017 supplies the ocean-source and analogy-transfer firewall, and P018 develops recursive power. The present paper takes responsibility for the target criticality-preparation model, toy derivations, simulation, sensitivity check, and safety firewall. Later projects retain responsibility for historical liberation, dynamic consent, effective exit, portability, and governance.

Suggested citation. Huang, Wanhong. “Relational Fields and Criticality Preparation: A Conditional Model of Generative Escape.” Working discussion paper, 2026.

1 Introduction

This section establishes the motivating distinction, the paper’s formal question, its limited position, and the sequence of analysis. It begins from the researcher-origin staged hypothesis, defines criticality preparation, and states the exact and computational contributions developed below.

Some systems absorb an ordinary disturbance and return rapidly to a familiar regime. The same disturbance can later persist, combine with other changes, or cross a threshold. The difference can arise from a larger disturbance. It can also arise from a transformed background: recovery has weakened, a basin boundary has moved, a barrier has fallen, a coupling route has gained transient amplification, or a receiving pathway has opened. The second configuration motivates the present paper.

Wanhong Huang proposed this distinction during a wider discussion of relational generativity, domination, and ocean-inspired transport. The discussion placed structured sparks and stochastic turbulence after a prior stage of relational reconfiguration. Its central insight was that gradual transformation of the relational field can make an ordinary perturbation dynamically consequential. The oceanic material supplied a source of questions concerning transport, background gradients, boundaries, and approach to sensitive regimes. A companion project, P017, reconstructs that source domain and establishes an analogy-transfer firewall. The present paper begins wholly within a target toy model.

Definition 1.1 (Criticality preparation). Fix a target system, reference regime, coherent intervention and comparison arrangements, perturbation class, transition region, horizon, and viability conditions. *Criticality preparation* occurs over a declared route when the intervention changes a transition-accessibility diagnostic so that the same perturbation class has a different finite-horizon response.

The term *criticality* functions broadly in Definition 1.1. A local eigenvalue approaching zero supplies one route. Basin-boundary movement, finite-amplitude crossing, non-normal gain, noise-induced escape, rate-induced tipping, and topology change supply other routes. Each application must select and justify its mechanism.

Claim 1.2 (Conditional preparation position). Relational criticality preparation should be represented as a vector of route-specific dynamical contrasts. A transition claim requires a declared perturbation and target. An empirical mechanism claim additionally requires construct and causal evidence. A generative, liberatory, just, or safe classification requires independent premises and observations.

The paper supports Claim 1.2 through four contributions. First, it gives an adaptive comparison architecture and a nonaggregated route profile. Second, a fold normal form yields exact recovery, margin, barrier, and fixed-kick consequences. Third, a triangular extension demonstrates changing transient gain under a fixed stable spectrum. Fourth, deterministic and stochastic simulations publish their parameters, seeds, raw output, and sensitivity grid.

Section 2 positions the model within bifurcation, critical-transition, basin, transient, stochastic, adaptive-network, simulation, and viability traditions. Section 3 defines the typed comparison and route profile. Section 4 derives the fold results. Section 5 develops the stable-spectrum extension. Section 6 reports numerical results. Section 7 states target-domain and safety obligations. Section 8 consolidates the present position.

Table 1: Antecedent traditions and contribution boundaries

Tradition	Licensed role	P019 boundary
Bifurcation and fast-slow systems	Fold normal forms, parameter variation, and multiscale transition analysis [11, 10]	Relational interpretation and the fixed-kick preparation statement belong to the present construction.
Early-warning analysis	Candidate slowing, variance, and autocorrelation indicators, together with quantified detection limits [15, 3]	Indicator movement receives route-specific interpretation and error analysis.
Tipping-route plurality	Bifurcation-induced, noise-induced, and rate-induced transition mechanisms [1]	The current exact toy result concerns a frozen fold and fixed kick; a full rate-induced model remains an obligation.
Basin and transient stability	Global basin measures and stable-spectrum transient amplification [12, 16]	The scalar margin and triangular response are elementary toy witnesses.
Stochastic escape	Barrier crossing, metastability, exit problems, and quasipotential methods [9, 5]	The reported expression is a small-noise frozen-potential specialization.
Adaptive networks	Coupled evolution of node states and network topology [6]	Every target edge requires independent semantics and measurement.
Numerical and sensitivity practice	Euler-Maruyama simulation and systematic sensitivity workflow [7, 13]	The present grid is a reproducible demonstration with limited convergence scope.
Viability and resilience	State constraints, viability kernels, stability domains, and resilience [2, 8]	P019 reports a safety vector and leaves kernel calculation to applications.
Model-world and causal relations	Target-directed modeling and longitudinal treatment-confounder feedback [17, 14]	Simulation adequacy and target causal identification form separate gates.
Generative justice	Circular value flow and generative-justice context [4]	Criticality preparation and all equations are present-project constructions.

2 Antecedent Structure and Model Boundary

This section locates the proposed construction within established mathematical, modeling, and safety traditions. Its objective is to identify which results support the toy model and which inferential steps remain project-specific. Table 1 organizes the traditions by their licensed role and transfer boundary.

Bifurcation theory supplies a precise local language for changes in qualitative dynamics under parameter variation. Critical-transition research embeds such changes in fast-slow and stochastic settings [11, 10]. The resulting vocabulary supports the fold example below. It also imposes discipline: a fold normal form represents one model class, and time-scale separation governs the relation between frozen calculations and a moving parameter path.

Local recovery tells only part of the transition story. Basin stability measures global attraction under a distribution of perturbations [12]. Non-normal systems can amplify inputs over finite intervals while every eigenmode decays asymptotically [16]. Stochastic systems can cross finite barriers, and a sufficiently rapid parameter path can generate a transition through a rate mechanism [9, 1]. These antecedents support a route profile with several coordinates.

Early-warning studies motivate measurement of recovery and fluctuation statistics [15]. Detection performance depends on record length, noise, model class, prior information, and decision rule;

false-alarm and missed-detection rates therefore belong in the report [3]. P019 uses exact toy quantities and direct simulated first passage. Its warning claim remains limited to the declared model class and diagnostics.

The model’s relational adjective refers to change in coupling, channels, barriers, or other explicitly typed conditions. Adaptive-network theory shows how topology and node dynamics can coevolve [6]. This formal possibility leaves every application responsible for defining what an edge is, who can change it, and how that change is observed. Target-directed modeling similarly connects a model’s adequacy to its purpose and intended target [17].

Finally, transition accessibility carries an open evaluative sign. Ecological resilience distinguishes persistence properties across stability domains [8]; viability theory studies state-constrained evolution and capturability [2]. These traditions support a separate safety profile. They supply neither a social welfare ordering nor a classification of escape as liberation.

3 Typed Preparation Architecture

This section defines the general comparison system used to interpret the toy results. Its objective is to separate evolving state, relational configuration, background parameters, interventions, perturbations, transition targets, and safety conditions. The route profile then prevents a scalar criticality score from concealing distinct mechanisms.

Let X_t denote a target state, R_t a typed relational configuration, θ_t background parameters, q_t a coherent intervention, and W_t a declared stochastic process. A general adaptive system is

$$dX_t = f(X_t; R_t, \theta_t) dt + B(X_t; R_t, \theta_t) q_t dt + \Sigma_t dW_t, \quad (1)$$

$$\dot{R}_t = g_R(R_t, X_t, q_t), \quad \dot{\theta}_t = g_\theta(\theta_t, R_t, X_t, q_t). \quad (2)$$

Equations (1)–(2) permit feedback between state and structure. They provide an architecture; empirical fitting remains a separate stage.

Assumption 3.1 (Coherent comparison). Each intervention arm specifies a feasible complete arrangement, initial distribution, observation law, perturbation process, horizon H , transition region $\mathcal{O}$, viability set K , and all co-interventions required for implementation.

The preparation profile is

$$\Pi_{\text{CP}} = \langle \alpha, b, \Delta V, \mathcal{G}_H, p_{\text{exit}}(H), p_{\mathcal{O}}(H) \rangle. \quad (3)$$

Here α is local spectral abscissa, b is a declared threshold or basin margin, ΔV is a potential or quasipotential barrier when defined, $\mathcal{G}_H$ is finite-time gain, and the final coordinates are hitting probabilities. Table 2 records their meanings and counterconditions.

Definition 3.2 (Route-specific preparation contrast). For coherent arms $q = 1$ and $q = 0$, a preparation contrast is the full coordinate report

$$\Delta \Pi_{\text{CP}}(q) = \Pi_{\text{CP}}^{\text{do}(q=1)} - \Pi_{\text{CP}}^{\text{do}(q=0)}$$

with coordinate-specific units, uncertainty, horizon, and direction.

Definition 3.2 leaves a universal sign pattern unsupported. A smaller barrier can accompany reduced transient gain. A larger accessible target can accompany a more dangerous path. A larger

Table 2: Criticality-preparation routes and diagnostic boundaries

Route	Diagnostic	Model meaning	Principal countercondition
Local recovery	spectral abscissa α	asymptotic response near a reference equilibrium	finite-amplitude or rate transition with bounded local margin
Basin geometry	margin b or basin measure	distance or probability of remaining in a reference regime	metric, initial distribution, and boundary dependence
Potential barrier	ΔV or quasipotential	small-noise action cost under an admitted stochastic model	nongradient drift, colored noise, or changing noise law
Transient routing	gain $\mathcal{G}_H$	finite-time input-output response	nonlinear saturation and signed adverse amplification
Stochastic access	$p_{\text{exit}}(H)$	finite-horizon first passage	nonstationarity, endogenous noise, and boundary error
Target access	$p_{\mathcal{O}}(H)$	entry into a declared target region	harmful destination and nominal access
Rate and topology	path speed or structural event	loss of tracking or changed reachability	mechanism confusion with fold or noise routes

stochastic escape probability can arise from stronger noise while the deterministic landscape remains fixed. Aggregation would require a common scale and defended weighting rule.

The target transition region also requires independent content. For a party i , a viable target may be written

$$\mathcal{O}_i^K = \{x \in \mathcal{O}_i : \exists \text{ an admissible path to } x \text{ that remains in } K\}. \quad (4)$$

This construction separates nominal boundary crossing from a path that preserves declared viability constraints. Its empirical meaning depends on participant-relevant states, feasible controls, and the governance of K .

4 Fold Toy Model and Exact Consequences

This section supplies a solvable model for one preparation route. Its objective is to derive the stable and unstable branches, local recovery, threshold margin, potential barrier, fixed-kick condition, and slow-path preparation time in continuous order. Table 3 fixes the symbols before the derivation.

Assumption 4.1 (Fold toy regime). The preparation coordinate satisfies

$$a(r) = a_0 - \beta r, \quad \dot{r} = \varepsilon u(t), \quad a(r) > 0, \quad (5)$$

with $a_0, \beta, \varepsilon > 0$. The fast state follows

$$dX_t = (a(r_t) - X_t^2 + s_t)dt + \sigma dW_t. \quad (6)$$

Frozen calculations hold a constant over the response interval.

The scalar a is a toy compression with dimensionless model-internal units. An application may associate a vector of resources, barriers, relations, and institutional conditions with a lower-dimensional model only after measurement and model-comparison work.

Table 3: Symbols in the fold preparation toy model

Symbol	Type	Role
r_t	dimensionless slow coordinate	declared relational preparation coordinate within the toy system
$a(r)$	positive scalar parameter	controls the frozen fold geometry
X_t	real fast state	position relative to the stable branch and threshold
s_t	structured input	declared deterministic perturbation channel
σdW_t	stochastic input	additive Gaussian white-noise channel
x_s, x_u	equilibrium states	stable branch and unstable threshold
h	positive magnitude	instantaneous negative state kick
H, N, dt	numerical settings	horizon, path count, and time step

Proposition 4.2 (Frozen fold geometry). *Under Assumption 4.1, set $s = \sigma = 0$ and freeze $a > 0$. The drift $f(x; a) = a - x^2$ has stable equilibrium $x_s = \sqrt{a}$, unstable equilibrium $x_u = -\sqrt{a}$, local eigenvalue $\lambda_s = -2\sqrt{a}$, threshold margin $b = 2\sqrt{a}$, and gradient potential barrier $\Delta V = 4a^{3/2}/3$.*

Proof. Solving $a - x^2 = 0$ gives $x_s = \sqrt{a}$ and $x_u = -\sqrt{a}$. Since $\partial_x f = -2x$, the derivative is negative at x_s and positive at x_u . The one-dimensional margin is

$$b = x_s - x_u = 2\sqrt{a}.$$

Choose $V(x; a) = x^3/3 - ax$, so $-\partial_x V = a - x^2$. Direct evaluation gives

$$V(x_s; a) = -\frac{2}{3}a^{3/2}, \quad V(x_u; a) = \frac{2}{3}a^{3/2},$$

and their difference is $4a^{3/2}/3$. □

Proposition 4.2 couples three diagnostics because of the selected normal form:

$$|\lambda_s(a)| = 2\sqrt{a}, \quad b(a) = 2\sqrt{a}, \quad \Delta V(a) = \frac{4}{3}a^{3/2}. \quad (7)$$

Broader systems can vary these quantities independently. The fold therefore provides a transparent case and a computational benchmark.

Proposition 4.3 (Fixed-kick preparation threshold). *Apply an instantaneous negative kick of magnitude $h > 0$ at x_s , then freeze a . The post-kick state lies beyond x_u exactly when*

$$h > 2\sqrt{a} \iff a < \frac{h^2}{4}. \quad (8)$$

Proof. The post-kick state is $x^+ = \sqrt{a} - h$. The crossing condition $\sqrt{a} - h < -\sqrt{a}$ is equivalent to $h > 2\sqrt{a}$. Positivity of a and h makes this equivalent to $a < h^2/4$. □

Proposition 4.3 holds perturbation magnitude fixed. Its changing effect comes entirely from the frozen background parameter. This supplies the paper’s simplest formal version of gradual preparation.

Corollary 4.4 (Linear-path preparation time). *Let $a(t) = a_0 - \nu t$ with $\nu > 0$, $a_0 > h^2/4$, and an admissible path through the equality. The fixed kick first reaches the threshold at*

$$t_h = \frac{a_0 - h^2/4}{\nu}. \quad (9)$$

The strict crossing condition holds immediately after t_h under the frozen-response approximation.

The deterministic result describes a finite-amplitude route. A stochastic route uses the same potential with distinct assumptions. For fixed $a > 0$ and small σ , the classical overdamped escape asymptotic specializes to

$$\mathbb{E}\tau_a \asymp \frac{\pi}{\sqrt{a}} \exp\left(\frac{8a^{3/2}}{3\sigma^2}\right). \quad (10)$$

The prefactor follows from the curvatures $|V''(x_u)| = V''(x_s) = 2\sqrt{a}$; the exponent follows from $2\Delta V/\sigma^2$ [9, 5]. Equation (10) requires a frozen potential, small noise, metastable separation, and a compatible time horizon. Its near-fold use therefore demands care.

5 Stable-Spectrum Transient Extension

This section establishes a second exact route. Its objective is to show that a relational cross-channel coefficient can change finite-time response while both eigenvalues remain fixed and stable. The construction prevents the fold's joint movement of recovery and margin from becoming a general definition.

Linearize a two-state extension around a declared stable reference state:

$$\dot{\mathbf{u}} = A\mathbf{u}, \quad A = \begin{pmatrix} -p & -\kappa \\ 0 & -\gamma \end{pmatrix}, \quad p, \gamma > 0. \quad (11)$$

The first coordinate may represent a transition-relevant state displacement, the second an input channel, and κ their signed routing coefficient. These meanings remain toy meanings until a target supplies observables.

Proposition 5.1 (Stable-spectrum cross-channel response). *For every real κ , the eigenvalues of A are $-p$ and $-\gamma$. If $\mathbf{u}(0) = (0, h)^\top$, then*

$$u_2(t) = he^{-\gamma t}, \quad (12)$$

$$u_1(t) = -\kappa h \frac{e^{-\gamma t} - e^{-pt}}{p - \gamma}, \quad p \neq \gamma, \quad (13)$$

with $u_1(t) = -\kappa h t e^{-pt}$ when $p = \gamma$.

Proof. Triangularity gives the two diagonal eigenvalues. The second equation solves directly. Variation of constants for the first equation gives

$$u_1(t) = -\kappa h \int_0^t e^{-p(t-s)} e^{-\gamma s} ds,$$

which evaluates to Equations (13) and its continuous $p = \gamma$ limit. □

Define the peak cross-channel gain

$$G_{2 \rightarrow 1}(p, \gamma, \kappa) = |\kappa| \sup_{t \geq 0} \left| \frac{e^{-\gamma t} - e^{-pt}}{p - \gamma} \right|. \quad (14)$$

For fixed p and γ , this gain scales with $|\kappa|$ while the spectrum remains unchanged. Trefethen and colleagues establish the broader importance of non-normal transient amplification in hydrodynamic stability [16]; Proposition 5.1 supplies an elementary target-independent witness.

The sign of u_1 matters. A larger absolute gain can assist threshold crossing in one direction and amplify harmful deviation in another. The relevant input distribution, direction, saturation, nonlinear return, and target boundary therefore belong in any empirical use.

6 Numerical Demonstration and Sensitivity

This section reports the computational benchmark and its uncertainty. Its objective is to compare analytic predictions with deterministic integration, estimate stochastic first passage under published settings, and disclose step-size and noise sensitivity. The code and raw CSV accompany the manuscript.

The deterministic model uses fourth-order Runge-Kutta integration after an instantaneous kick. The stochastic model uses Euler-Maruyama [7]. For N independent paths, the reported finite-horizon estimate and Monte Carlo standard error are

$$\hat{p}_H = \frac{1}{N} \sum_{n=1}^N I_n(H), \quad \widehat{\text{se}}(\hat{p}_H) = \sqrt{\frac{\hat{p}_H(1 - \hat{p}_H)}{N}}. \quad (15)$$

This standard error covers simulated path sampling under the toy model. Discretization, model form, construct validity, and external validity require separate analysis.

Table 4 uses horizon 12 and time step 0.001. Every row agrees with Proposition 4.3. At $a = 0.0625$, the margin 0.5 exceeds the kick. At $a = 0.0225$, the margin 0.3 lies below it. The two cases make changed perturbation efficacy visible with fixed h .

Table 5 uses $\sigma = 0.30$, $H = 20$, $dt = 0.005$, and $N = 10,000$. Escape is first crossing of $-\sqrt{a}$. Seeds begin at 20260824 and increment by row. The two zero entries mean zero observed crossings in those finite samples. They represent run-specific observations; exact zero probabilities remain unsupported.

The estimated first-passage frequency increases sharply as the toy barrier falls. This pattern is compatible with the barrier mechanism. It also reflects the joint movement of curvature, margin, and barrier in the selected normal form. The simulation therefore demonstrates internal coherence. Mechanism identification remains confined to the toy equations.

Each cell in Table 6 uses $N = 8,000$ paths and $H = 20$, with seeds 4101 through 4109. The grid preserves the qualitative ordering by noise level. Differences across step sizes reach several Monte Carlo standard errors in some comparisons. The result supports qualitative sensitivity reporting and leaves higher-precision claims for a fuller convergence study. A global analysis would expand parameter ranges, interactions, sampling designs, and output metrics [13].

For the transient extension, fix $a = 0.36$, hence $p = 1.2$, and set $\gamma = 1$. Equation (14) gives peak gains 0, 0.167449, 0.334898, 0.669796 for $\kappa = 0, 0.5, 1, 2$, respectively, while both eigenvalues remain -1.2 and -1 . The common positive- κ peak time is approximately 0.911608. This numerical evaluation confirms the exact linear scaling in $|\kappa|$.

Table 4: Deterministic fixed-kick benchmark

a	Exact margin	Kick h	Final reported x	Classification
1.0000	2.0000	0.4	1.00000000	return
0.2500	1.0000	0.4	0.49999590	return
0.0625	0.5000	0.4	0.24509117	return
0.0225	0.3000	0.4	-0.25000000	escape at kick

Table 5: Stochastic first-passage benchmark

a	Barrier $4a^{3/2}/3$	Estimated escape probability	Monte Carlo standard error
1.00	1.333333	0.0000	0.000000
0.64	0.682667	0.0000	0.000000
0.36	0.288000	0.0088	0.000934
0.16	0.085333	0.4394	0.004963
0.09	0.036000	0.8159	0.003876
0.04	0.010667	0.9732	0.001615

Table 6: Step-size and noise sensitivity at $a = 0.16$

Step size	$\sigma = 0.25$	$\sigma = 0.30$	$\sigma = 0.35$
0.0100	0.214250	0.430125	0.636500
0.0050	0.210875	0.437500	0.646750
0.0025	0.219000	0.429000	0.639000

7 Target-Domain Promotion and Safety

This section converts the toy results into research obligations. Its objective is to state the evidence required for target-domain interpretation and to keep dynamical accessibility separate from safety, generativity, liberation, justice, and policy endorsement. Table 7 provides the promotion sequence.

Construct validation comes first. A target study must define whose state is modeled, which relations change, which actor controls the intervention, which events count as perturbations, and which region counts as a transition. Accessibility may be actor-relative and domain-specific. A credential, resource, organization, exit route, or relationship can create distinct state spaces and boundaries.

Causal interpretation then requires a coherent contrast. If an intervention changes both the landscape and the size of the final perturbation, the fixed perturbation claim loses its intended meaning. Longitudinal settings can contain exposure-confounder feedback: earlier relation changes affect later confounders that also influence subsequent intervention and outcome. Approaches developed for time-varying treatments make that problem explicit [14]. Their use still requires a target estimand, positivity, measurement, and interference assumptions.

Mechanism discrimination requires perturbation-response data at several stages of the background path. Local recovery may be estimated from controlled, ethically bounded perturbations. Threshold geometry may require a distribution of initial states or interventions. Transient routing requires signed input-output observations. Stochastic claims require a noise model, repeated trajectories, and a boundary. Rate-induced claims require parameter speed and tracking evidence [1]. A single observed transition supports few of these distinctions.

The safety profile is

$$\mathbf{S} = \langle \text{viability, destination, plurality, participation,} \\ \text{reversibility, distribution, cascade, new dependency} \rangle. \quad (16)$$

Viability theory motivates explicit state constraints and admissible paths [2]. The destination coordinate asks which regime becomes reachable. Plurality records the range of viable futures. Participation records affected-party authority. Reversibility concerns route-back and revision. Distribution records benefits, burdens, and third-party effects. Cascade records propagation risk. New dependency records whether an enabling channel becomes a new center of control.

Research Obligation 7.1 (Safe transition evidence). Any applied claim of generative escape must report the dynamic contrast and the safety profile separately, preserve adverse entries, and state the participant-relative and institutional basis of each evaluative threshold.

Obligation 7.1 also governs interpretation of persistence. Resilience can preserve valued function or maintain a harmful regime [8]. Transition can expand options or destroy viable conditions. A preparation intervention can support autonomy or create dependence on the new receiving node. These possibilities require observations beyond the transition event.

Table 7: Target-domain promotion and safety requirements

Stage	Required record	Failure signal
Construct	typed state, relation, parameter, perturbation, boundary, target, horizon, and viability measures	variables derive meaning mainly from ocean or landscape vocabulary
Comparison	feasible complete arms and co-interventions	the proposed background change also changes perturbation size or measurement
Identification	longitudinal estimand, timing, interference, feedback, attrition, and confounding assumptions	prior intervention changes later confounders without suitable adjustment
Mechanism	spectral, basin, transient, stochastic, rate, and topology rivals	several routes reproduce the same observations
Numerics	code, seed, discretization, convergence, uncertainty, and sensitivity	qualitative result changes across admissible numerical settings
External validity	target cases, negative cases, scale limits, and replication	parameter interpretation changes across cases
Safety	viability, destination, plurality, participation, reversibility, distribution, cascade, and new dependency	access rises while severe adverse coordinates deteriorate

8 Research Obligations and Conclusion

This section consolidates the paper’s mathematical result, computational evidence, inferential boundary, and future programme. Its objective is to state the current position in a form that can be revised through proof review, alternative models, empirical testing, and participant-centered safety work.

The core exact result is narrow. In the frozen fold model, local recovery magnitude, threshold margin, and potential barrier are explicit functions of a . A fixed negative kick crosses the unstable threshold exactly when $a < h^2/4$. Along a linear slow path, the equality gives the preparation time in Equation (9). The triangular extension supplies an independent witness: cross-channel transient gain changes with κ while the stable spectrum remains fixed.

The computational benchmark adds transparent numerical evidence within the toy family. Deterministic results match the exact kick threshold. Stochastic first-passage estimates change sharply across the declared parameter grid. The step-size and noise check preserves the qualitative ordering and identifies limits on precision. Code, seeds, settings, and raw output accompany the manuscript.

Several obligations remain.

Research Obligation 8.1 (Mathematical extension). Future work should compare fold, finite-amplitude, rate-induced, noise-induced, non-normal, and topology-changing toy families; analyze slow passage and bifurcation delay; test colored and endogenous noise; and add nonlinear saturation to the transient extension.

Research Obligation 8.2 (Numerical validation). Future computation should perform strong and weak convergence checks, seed ensembles, broader sensitivity analysis, alternative integrators, boundary error analysis, and reproducible tests against additional analytic cases.

Research Obligation 8.3 (Target-domain identification). A substantive application should preregister typed constructs, intervention arms, perturbation class, transition region, horizon, competing routes, longitudinal identification assumptions, negative cases, and external-validity limits.

Research Obligation 8.4 (Normative and institutional evaluation). Evaluation should specify standing, participant authority, viability, destination quality, plurality, reversibility, burden distribution, cascade risk, and new dependency before assigning generative, liberatory, just, or policy status.

The present position can therefore be stated concisely. Gradual relational reconfiguration can make a fixed perturbation consequential within a declared model. Several dynamical routes can produce that change. Exact toy results and reproducible simulation clarify the proposition’s logical content. Target explanation and ethical endorsement depend on further evidence. This layered structure preserves the motivating insight while keeping its mathematical, empirical, and normative statuses visible.

Acknowledgments

The author thanks Ron Eglash for dialogue that helped inspire the wider inquiry into generative justice and value circulation within which this paper was developed. The present definitions, criticality-preparation construction, formal model, taxonomy, simulations, arguments, conclusions, and errors remain the author’s responsibility. This acknowledgment records intellectual inspiration from dialogue.

References

- [1] Peter Ashwin, Sebastian Wieczorek, Renato Vitolo, and Peter Cox. Tipping points in open systems: Bifurcation, noise-induced and rate-dependent examples in the climate system. *Philosophical Transactions of the Royal Society A*, 370(1962):1166–1184, 2012.
- [2] Jean-Pierre Aubin, Alexandre M. Bayen, and Patrick Saint-Pierre. *Viability Theory: New Directions*. Springer, Berlin, 2 edition, 2011.
- [3] Carl Boettiger and Alan Hastings. Quantifying limits to detection of early warning for critical transitions. *Journal of the Royal Society Interface*, 9(75):2527–2539, 2012.
- [4] Ron Eglash, Kwame P. Robinson, Audrey Bennett, Lionel Robert, and Mathew Garvin. Computational reparations as generative justice: Decolonial transitions to unalienated circular value flow. *Big Data & Society*, 11(1), 2024.
- [5] Mark I. Freidlin and Alexander D. Wentzell. *Random Perturbations of Dynamical Systems*. Springer, Berlin, 3 edition, 2012.
- [6] Thilo Gross and Bernd Blasius. Adaptive coevolutionary networks: A review. *Journal of the Royal Society Interface*, 5(20):259–271, 2008.
- [7] Desmond J. Higham. An algorithmic introduction to numerical simulation of stochastic differential equations. *SIAM Review*, 43(3):525–546, 2001.
- [8] C. S. Holling. Resilience and stability of ecological systems. *Annual Review of Ecology and Systematics*, 4:1–23, 1973.
- [9] H. A. Kramers. Brownian motion in a field of force and the diffusion model of chemical reactions. *Physica*, 7(4):284–304, 1940.

- [10] Christian Kuehn. A mathematical framework for critical transitions: Bifurcations, fast–slow systems and stochastic dynamics. *Physica D: Nonlinear Phenomena*, 240(12):1020–1035, 2011.
- [11] Yuri A. Kuznetsov. *Elements of Applied Bifurcation Theory*. Springer, New York, 2 edition, 1998.
- [12] Peter J. Menck, Jobst Heitzig, Norbert Marwan, and Jürgen Kurths. How basin stability complements the linear-stability paradigm. *Nature Physics*, 9:89–92, 2013.
- [13] Francesca Pianosi, Keith Beven, Jim Freer, Jim W. Hall, Jonathan Rougier, David B. Stephenson, and Thorsten Wagener. Sensitivity analysis of environmental models: A systematic review with practical workflow. *Environmental Modelling & Software*, 79:214–232, 2016.
- [14] James M. Robins, Miguel A. Hernán, and Babette Brumback. Marginal structural models and causal inference in epidemiology. *Epidemiology*, 11(5):550–560, 2000.
- [15] Marten Scheffer, Jordi Bascompte, William A. Brock, Victor Brovkin, Stephen R. Carpenter, Vasilis Dakos, Hermann Held, Egbert H. van Nes, Max Rietkerk, and George Sugihara. Early-warning signals for critical transitions. *Nature*, 461:53–59, 2009.
- [16] Lloyd N. Trefethen, Anne E. Trefethen, Satish C. Reddy, and Tobin A. Driscoll. Hydrodynamic stability without eigenvalues. *Science*, 261(5121):578–584, 1993.
- [17] Michael Weisberg. *Simulation and Similarity: Using Models to Understand the World*. Oxford University Press, 2013.