

Resource Constraints on Maintaining Revisability near Criticality

A Preliminary Discussion

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Abstract

Maintaining the possibility of revision requires practices whose continuation is itself resource dependent. This paper develops a philosophical and formal extension of the Generative Relational investigation of functional resource distribution and distributed optionality. Its primary object is a maintainable revision repertoire: a selected family of later responses supported by one feasible maintenance policy before the response to be exercised is known. This object distinguishes nominal alternatives, individually reachable outcomes, common maintenance, and executable revision. It also makes a distinction within convergence: a shared implementation can economize maintenance, whereas deletion of capacities can reduce expenditure by eliminating the possibility of reopening a decision. The discussion positions these questions against research on irreversibility, organizational learning, adaptive policy pathways, institutional persistence, infrastructure, and international cooperation. History-bearing relational spacetimes provide the primary representational setting. Restricted resource frontiers and resource-filtered history operators clarify compatibility and composition; an operation-matching criterion identifies when compression preserves selected revisions. A finite group-domain field model generates ribbon two-complexes and admits an explicit nonlinear response analysis. Its vanishing restoring mode provides a specific criticality mechanism, while exact recovery times expose the limits of an unrestricted critical-slowness argument. A separate positive history ensemble distinguishes hard feasibility from soft resource penalties. These constructions are several possible models, with explicit interfaces and no presumption of a single social resource scale or literal physical ontology. The resulting account identifies maintenance, authorization, and timely execution as distinct research obligations. It neither establishes the novelty of a new resource programme nor recommends preserving every alternative.

Keywords: revisability; resource maintenance; convergence; criticality; relational spacetime; group fields; operator algebra

Reading Note

This paper extends the argument of *Governance and Survival near Criticality: Functional Resource Distribution and Distributed Optionality* [10]. Heterogeneous resource carriers, dissipative waiting, joint support constraints, and distributed optionality belong to that prior investigation. Their appearance here establishes continuity, not a priority claim. The present reconstruction concentrates on common maintenance of later alternatives, resource-sensitive composition, operational compression, and a specified field-theoretic mechanism.

Generative Relational (GR) language identifies a research orientation, not a settled ontology. An actor may be investigated as a history-bearing relational spacetime; a two-complex can represent aspects of that spacetime; a labelled spinfoam is one possible further instantiation. These descriptions are not interchangeable. A group-domain field construction can generate complexes, but a separate interpretation is required before its amplitudes describe institutional practices.

The mathematical results below are conditional. A budget boundary is not automatically a critical point. The criticality calculation concerns a restoring mode of one nonlinear variational model. Its linear response limit has a diverging relaxation scale, while finite-amplitude recovery can remain finite. Likewise, mathematical reversal is not an authorization to undo an institutional action.

The literature review uses verified sources with recorded reading scopes. Some comparisons rely on publisher abstracts; others use selected primary passages. The bibliography is not a claim to exhaustive coverage. Computational examples are illustrative and reproducible, and provide no empirical calibration. This working draft is intended for discussion and further correction.

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Status. This working draft is prepared for the author’s review. It records an evolving research argument; definitions, section structure, and conclusions remain subject to revision. Completion of the draft does not imply publication, acceptance, or final author approval.

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Statement on the use of language models. The author selected the research problem, working title, and paper queue and provided the exploratory discussion and corrections. OpenAI Codex assisted with source retrieval, conceptual analysis, formal reconstruction, drafting, computational examples, and technical checks. This completed working draft awaits the author’s review; its new formulations and arguments have not yet received final author approval. Authorship credit remains with the human author. This disclosure should be reviewed before external circulation.

1 Maintainable Revision Repertoires

This section identifies the paper’s analytical object and the distinction it clarifies. The question is how a capacity to revise remains practically available while the practices supporting it consume attention, material resources, organizational effort, and time. The argument begins with the constitution of that capacity, then identifies the contribution and its limits.

1.1 Supported Alternatives

A decision can remain formally open after the means of changing it have disappeared. An institution may retain an appeal procedure while losing the personnel able to interpret its records. A negotiating process may leave several proposals on the table while only one retains an executable sequence of commitments. An organization may preserve a technical design while allowing the relevant skills, permissions, and material interfaces to decay. In each case, the alternative survives in one description and fails in another.

The discrepancy concerns the relation between a possible future and the activities that would make it available. A record of an alternative is one component of this relation. Recognition of a reason to revise, authority to act on that reason, and the means of completing the revision are other components. They need not persist together. Their temporal organization matters: resources arriving after a response has lost its point do not necessarily restore the lost opportunity.

The primary object of this paper is a *maintainable revision repertoire*. Informally, it is a selected family of later responses for which one feasible maintenance policy can preserve an executable continuation before the particular response is selected. A repertoire is always relative to participants, a horizon, a history, an interpretation of the responses, and an account of admissible operations. It is not an undifferentiated quantity of freedom.

Consider an institution that can train one specialist before learning which of two kinds of review will be requested. Either review could be supported if its identity were known in advance. It does not follow that the institution can preserve both possibilities while remaining ignorant of that identity. The distinction is a difference in the order of commitments: separate plans may depend on future information that a common maintenance plan does not yet possess. Sections 9 and 10 make this difference explicit.

Preserving a repertoire does not mean executing mutually exclusive futures simultaneously. A maintenance policy can support an eventual choice between alternatives. The resource demand lies in preserving the ability to make that choice later, including the subsequent means of acting on it. Confusing this demand with the sum of all hypothetical future expenditures can exaggerate the costs of revisability. Confusing it with the cheapest individually feasible option can underestimate them.

1.2 Capacity Reproduction

GR directs attention to the historical practices through which an actor, relation, or institution acquires its operative capacities. A review body is not exhausted by its legal description, its members at one instant, or its archive. Its capacity depends on historically organized relations among records, competence, access, authorization, expectations, and material support. The relevant object is therefore a history-bearing configuration rather than an isolated decision point.

Maintenance is part of this configuration’s continued formation. It can reproduce an existing capacity, adapt the means through which the capacity is exercised, or alter the very standards by which a later revision will be recognized. The distinction between maintaining and changing an institution is consequently practical and selective. The question is which relations of continuity matter for the selected purpose, not whether absolutely nothing has changed.

Unused capacity is especially easy to misdescribe. The absence of an appeal does not show that an appeal mechanism is unnecessary; it also does not show that the mechanism remains usable. The relevant evidence concerns executable possibilities: whether an eligible request could be received, interpreted, authorized, and acted upon within the pertinent conditions. This is a counterfactual inquiry grounded in

operations and their supports, rather than a metaphysical claim that unrealized futures already exist as complete objects.

1.3 Convergence Scope

Convergence can economize resources. Shared records, common procedures, and stable expectations can reduce repeated negotiation and duplicated maintenance. Yet the same observable reduction in expenditure can arise from closing channels of revision. A unified record may permit several kinds of later review, or it may erase the distinctions needed to conduct them. The paper therefore evaluates convergence through the operations it preserves and the obligations it changes.

This inquiry extends prior GR work on resources and criticality [10]. It does not rediscover that waiting consumes resources or that multiple functions can compete for common support. The proposed extension is more specific: it relates a common maintenance policy to a selected family of later revisions, and investigates the transformations under which this relation survives.

The formal investigations serve different purposes. Relational spacetimes retain historical and compositional structure. Finite resource frontiers expose an anticipatory quantifier problem. Operator constructions distinguish preservation of a space of histories from the fuel required to execute operations. A group-domain field model investigates generation and nonlinear response. A positive history ensemble distinguishes feasibility from weighting. None supplies a universal representation of trust, political power, or institutional life.

1.4 Argument Structure

Sections 2–7 develop the literature position and philosophical distinctions. Sections 8–12 construct history-sensitive support, resource frontiers, operators, and an operational criterion for compression. Sections 13–15 provide the field-theoretic and ensemble investigations. Sections 16–20 return to institutional interpretation, normative selection, objections, and empirical obligations. Appendix A supplies derivations, Appendix B records computational checks, and Appendix C identifies the model interfaces.

Criticality enters only through a specified mechanism. Much of the resource problem already exists in ordinary uncertainty or institutional change. The title’s qualification becomes analytically warranted to the extent that a loss of restoring capacity changes the time for which enabling conditions must be sustained. The paper examines that possibility without making it a universal explanation of difficult cooperation.

2 Resource Allocation Research

The review organizes existing research by the problem it helps formulate: irreversible commitment, learning, adaptive sequencing, institutional persistence, infrastructural support, and changing cooperation. Its purpose is to identify inherited insights and remaining translation obligations. It does not attribute the entire resource question to GR or assume that conceptual differences establish novelty.

2.1 Irreversible Commitment

Arrow and Fisher investigate environmental preservation when development is irreversible and information may improve later [1]. Their argument supplies a disciplined reason to distinguish immediate returns from the value of retaining a later decision. It does not authorize an unconditional preference for waiting. The comparison depends on a specified decision problem and the consequences of irreversibility.

The present investigation concerns a complementary question: what must continue to exist during the interval in which a decision remains revisable? A retained physical site, an organization capable of reassessment, and a recognized authority to change policy are different supports. Their maintenance may involve different actors and different temporal constraints. Representing the benefit of later information

is therefore insufficient, by itself, to specify the practices through which that information could still matter.

This is a proposed change of analytical emphasis, not a claim that option-value research cannot represent maintenance costs. Such costs can enter a decision model. The GR obligation is to explain when treating them as externally supplied parameters suppresses changes in the actors and relations that perform the later decision.

2.2 Organizational Learning

March's exploration–exploitation framework examines the allocation of organizational resources between exploring possibilities and exploiting established competences [18]. The published abstract emphasizes differences in the timing and distribution of returns and the tendency for exploitation to improve more rapidly, with possible long-run consequences. This is an existing account of how local efficiency can reshape the space of future learning.

A revision repertoire is not identical to an exploration budget. A well-understood alternative may require maintenance without new discovery; exploratory work may generate possibilities that cannot be carried into practice. The distinction matters when a research programme continues to produce proposals while the organization loses the capacity to implement any proposal outside its established routine.

Bourgeois treats organizational slack as a construct requiring attention to its functions and measurement, including financial-data approaches [4]. Slack offers a relevant point of comparison for capacities not committed to current demands. Here, however, a reserve is specified by the operations it can support. Unallocated funding does not establish the availability of interpretation, legal authorization, or a timely material substitute. The paper therefore treats financial slack as one possible support rather than a complete measure of revisability.

2.3 Adaptive Pathways

Dynamic adaptive policy pathways already connect sequences of actions, future options, monitoring, and contingent adjustment. Haasnoot and colleagues combine adaptation pathways with adaptive policy making to construct plans for deeply uncertain conditions [6]. Their framework directly limits any claim that preserving future pathways or preparing for later adjustment is an original contribution of this paper.

The additional question is the compatibility of maintenance obligations before a branch is selected. Two pathways can each have feasible implementations while depending on preparations that cannot be maintained together. This issue can be represented within adaptive planning; the present formalization makes it the primary object rather than treating it as a secondary implementation detail.

An adaptation tipping point in a policy pathway identifies conditions under which a policy ceases to meet its objectives. That criterion should not automatically be read as a dynamical bifurcation. A policy can fail because a legal deadline passes, a resource is exhausted, or a performance requirement changes. Section 7 retains this distinction when introducing a specific loss of a restoring mode.

2.4 Institutional Persistence

Pierson develops increasing returns as an account of political path dependence, highlighting the significance of temporal sequence, contingent developments, and the difficulty of reversing established paths [19]. This provides an existing explanation for why present alternatives depend on institutional history.

The maintenance problem is not simply the opposite of path dependence. Maintaining several alternatives can itself depend on accumulated competence and stable procedures. Conversely, dissolving a path can destroy capacities that would have enabled later correction. The relevant contrast is between different historically supported repertoires, including their switching and renewal conditions.

Stirling distinguishes opening up and closing down in the social appraisal of technology, connecting these orientations to power and pluralism rather than treating participatory and expert processes as uni-

formly distinct [23]. That distinction directs attention to how an apparently inclusive procedure can still narrow the options that receive effective consideration.

The present account adds an operational question to this comparison. An appraisal may recognize several alternatives while the material and institutional preparations support only one. Conversely, a temporarily standardized implementation may retain several credible revision procedures. Neither openness of discussion nor convergence of current action settles the status of the later repertoire.

2.5 Cooperative Infrastructure

Star's methodological account treats infrastructure as relational and ecological, with different significance for different groups, and makes often invisible work an object of investigation [22]. This supports attention to the people and practices through which apparently available capacities remain usable. It does not establish a general quantitative law of institutional maintenance.

International relations already contains substantial accounts of the resources and arrangements required for cooperation. Keohane examines how regimes can assist agreements through information and transaction-cost arrangements [17]. Fearon's distinction between bargaining and enforcement problems prevents an agreement's formation from being treated as equivalent to its continued support [5]. Powell's commitment-problem analysis identifies circumstances in which changing power can undermine agreements [20]. Barnett and Duvall's treatment of multiple forms of power cautions against reducing resource access to a single stock possessed by an already constituted actor [3].

The translation obligation is consequently demanding. A GR account must identify a historically constitutive relation or a compositional constraint that an existing explanation leaves unresolved in the case at hand. Relabelling monitoring as a field, or an institution as a spacetime, does not meet that obligation. Section 16 uses institutional illustrations to specify the additional observations that would be needed.

2.6 Transition Research

Holling's distinction between resilience and stability prevents persistence of a system from being identified with rapid return toward an equilibrium [7]. It does not determine which institutional functions ought to persist. Scheffer and colleagues discuss early-warning signatures associated with critical transitions, including mechanisms related to slowing recovery [21]. Such signatures require the relevant mechanism and observation conditions; they are not universal diagnostics of social fragility.

Ashwin and colleagues distinguish bifurcation-, noise-, and rate-induced tipping [2]. Their distinction is important here because rapid forcing can create difficulty without a vanishing restoring eigenvalue. A resource shortage, a missed response window, and a critical transition may interact, but their analytical identities should be preserved.

2.7 GR Inheritance

Prior GR work already develops resource carriers, functional support, dissipative delay, distributed optionality, and the criticality-governance programme [10, 9, 13]. The present paper accepts those as inherited investigations and treats their application as requiring case-specific support. The spacetime and field constructions likewise continue existing representational work rather than introducing those mathematical families into GR [8, 14].

Table 1 records the main boundary between inheritance and the present obligations. The right column describes work attempted here, not an established list of novel results relative to every relevant literature.

Table 1: Inherited structures and present analytical obligations.

Research object	Inherited contribution	Present obligation
Resource support	Heterogeneous carriers and joint allocation constraints	Support a selected repertoire through one anticipatory policy
Relational history	Elements, couplings, events, and composable histories	Record duration, renewal, authorization, and consumption
Generative fields	Interaction kernels generate complex combinatorics	Derive a tractable nonlinear maintenance-related response
Institutional change	Learning, path dependence, and adaptive pathways	Distinguish retained operations from cheaper closure
Criticality	Multiple transition mechanisms and conditional warning signals	Separate a restoring-mode loss from ordinary feasibility failure

3 Historical Resource Capacities

This section develops the GR account of resources needed by the later models. It proceeds from resource carriers to historically formed capacities and then examines substitution. The objective is to avoid treating money, competence, authority, and recognition as interchangeable entries in a universal stock.

3.1 Functional Resource Support

A thing functions as a resource in relation to an operation and the conditions of its use. A server can carry an archive but fail to support review if its formats are unreadable. An expert can possess relevant knowledge while lacking access or standing. A budget can purchase equipment while remaining unable to procure a legally valid authorization. Resource availability is therefore a relation among carriers, functions, and situated possibilities of activation.

Following the prior resource investigation, let $\mathcal{R}$ denote a declared domain of resource-carrier configurations and $\mathcal{F}$ a declared family of functions [10]. Neither domain is assumed to be numerical. For $f \in \mathcal{F}$, a readout $\kappa_f : \mathcal{R} \rightarrow D_f$ has a function-specific codomain D_f . A readout may classify whether a record is recoverable, identify an authorized office, or report a quantity of fuel. Different codomains are retained unless a justified translation is supplied.

The relevant configuration includes relations among carriers. Two individually competent people may lack a common language; two organizations may each possess information while being prohibited from sharing it. A list of carrier attributes can therefore overstate joint support. Conversely, a relation can create a capacity absent from either isolated carrier, as when independently held records permit mutual verification.

A budget B will initially name a constraint specification, not a scalar amount. Its feasible allocation domain $\mathcal{D}(B)$ includes permitted uses of the relevant carriers. Constraints can concern quantity, timing, exclusivity, access, or authority. The finite vector budget introduced later is an explicitly restricted instance of this construction.

3.2 Constituted Possession

Describing an actor as possessing a resource is often a useful abbreviation. It can also conceal the practices that make possession effective. An institution's competence depends on recruitment, apprenticeship, routines, records, and relations with those who recognize its decisions. A change in these relations can alter competence without changing the institution's legal name or its nominal budget.

GR's history-bearing object permits these practices to enter the representation of the actor itself. The actor's boundary is an analytical selection within a wider relational history. This does not require

dissolving every actor into the entire world. It requires recording which excluded relations must remain effective for the selected actor's capacities to continue.

This perspective changes the interpretation of maintenance. Maintaining a capacity need not mean preserving its original material carrier. Personnel can change, archives can migrate, and procedures can be revised while selected operations remain executable. Conversely, retaining the original carrier can fail to maintain the operation. The identity relevant to this inquiry is an operationally tested continuity, not mere material sameness.

There is also a limit to that operational description. Two institutions may perform the same selected task while having different histories, responsibilities, or relations of legitimacy. Prior GR work distinguishes historically formed identity from a record sufficient for a particular prediction [11]. This paper uses operation equivalence only relative to a declared repertoire; it does not infer complete relational identity.

3.3 Resource Substitution

Resource conversion is a practice with its own requirements. Purchasing additional interpretation time requires available interpreters, access to the record, a contracting mechanism, and sufficient lead time. An exchange rate expresses a possible transaction only within these conditions. It cannot, by itself, establish the transaction's availability.

A noncompensable resource is correspondingly relative to a repertoire and a horizon. If no admissible operation can replace a required authorization before the deadline, additional equipment cannot compensate within that problem. This does not imply that authorizations are metaphysically irreplaceable. A longer horizon or a different institutional process may alter the result.

The distinction also prevents an unwarranted moral conclusion. A formally authorized operation can be unjust; a prohibited operation can be ethically defensible. Authorization is included first to describe executable institutional possibilities, while normative evaluation remains a separate obligation. An analyst should not convert present administrative restrictions into an unquestionable limit of political imagination.

3.4 Analytical Consequences

Three consequences organize the remaining investigation. Resource readouts are selective descriptions of historically formed supports. Conversion claims require executable intermediate operations. Continuity claims require a declared family of tasks rather than an undifferentiated assertion that the institution remains the same.

The finite models below deliberately hold much of this background fixed. Their proofs identify consequences of that restriction. Their philosophical usefulness lies partly in making the omitted relations visible: a result about two consumable resource types cannot be directly generalized to legitimacy, trust, or interpretive competence. Those require their own carriers, readouts, and transition rules.

4 Practical Revisability

This section distinguishes the elements of an effective capacity to revise. It examines recognition, authorization, execution, and continuity, then specifies the temporal character of meaningful alternatives. These distinctions supply the interpretation of the support predicates introduced later.

4.1 Practical Recognition

A revision requires some difference to become practically relevant: a changed circumstance, an error, a contested interpretation, or a newly articulated demand. Detecting a signal is not yet recognizing that difference. A monitoring system may register an anomaly without possessing a category through which the anomaly can challenge the established arrangement.

Recognition can itself depend on a relation that is changing. An objection previously treated as procedural noise may acquire standing, or an established warning may cease to identify the same process. The interpretive conditions of the revision are therefore part of the history to be investigated. A model with a fixed menu of requests is one tractable case; it does not exhaust changes in what can count as a request.

This matters for epistemic lag. The practical difficulty is not always that an actor knows too little. It can be that the actor continues to interpret new manifestations through relations that no longer organize the situation. Additional observations may then enlarge an outdated classification without supplying a usable account of the change. The present paper retains this as a qualitative mechanism unless a case-specific measurement model is available.

4.2 Executable Authority

A recognized correction may lack an authorized route to action. Different participants can agree that a record is mistaken while disagreeing over who can amend it or what obligations follow. Authority can also be distributed across several bodies whose decisions must occur in a particular order. Available resources support revisability only if they can enter this sequence.

Execution adds a further demand. A response can be legally possible and materially impossible, or materially possible after its relevant interval has ended. A review that establishes an error after an irreversible operation is complete may still support accountability, compensation, or future learning. It does not thereby restore the earlier alternative.

The phrase *effective correction* consequently names a relation to a specified objective. Correcting a public record, preventing an action, compensating an injury, and rebuilding a relationship are different operations. A finding can succeed in one respect while failing in another. Treating all of them as a single binary variable would conceal the mechanism that the resource analysis is intended to explain.

4.3 Revisable Continuity

Revision presupposes enough continuity for a later response to concern the earlier arrangement. It can also require transformation of that continuity. A successor institution may inherit a responsibility while rejecting the old interpretive categories. Participants may continue cooperating while revising the basis on which cooperation is understood.

A useful distinction is therefore between preserving an implementation and preserving a revisable relation. The former retains a current procedure; the latter retains an executable route by which the procedure itself can be reconsidered. These can conflict when the procedure consumes the resources needed for its own review.

The relation can remain meaningful even when the original outcome cannot be restored. What matters is the response family selected for investigation. If the family concerns material reversal, compensation is not an equivalent substitute. If it concerns public accountability, a reasoned acknowledgment may remain relevant after material reversal becomes impossible. The model must declare the family before counting responses as interchangeable.

4.4 Revision Horizons

A horizon is an analytical restriction on the interval over which support is assessed. It can represent a deadline, a replenishment cycle, a period of legal competence, or the end of an agreed review process. These have different structures. Some intervals are fixed externally; others are altered by the very practices being studied.

An alternative's meaningfulness can also be discontinuous. It may be executable now, unavailable during a transition, and available again after reconstruction. A single shrinking scalar window would conceal this pattern. Prior resource work already permits changes in the configurations and support

available through time [10]; the present finite models state more restrictive horizon assumptions when needed.

A claim that a repertoire is maintained should accordingly answer four questions in its substantive account: what responses remain available, for whom, through which executable operations, and until when. These are not administrative additions to a fully specified option. They help determine the option's practical identity.

5 Maintenance Obligations

This section examines how continued support consumes and reproduces resources. It distinguishes retention, exercise, renewal, and coordination, then considers the visibility of their costs. The objective is to give the later budget models an interpretation without assuming that every relevant burden is measurable on a common scale.

5.1 Capability Costs

Retention costs concern keeping a capability available before it is selected. Exercise costs concern carrying out the selected response. Training reviewers, maintaining interpretable records, renewing credentials, and testing access can support retention. Conducting a particular review and implementing its result can require additional resources.

The distinction is functional rather than permanent. A training exercise can simultaneously maintain a capability and carry out a real correction. A completed review can create records that reduce later retention costs. A good model should allow these overlaps where they matter, rather than assigning every activity to an exclusive accounting box.

Nevertheless, the distinction exposes an important error. A reserve sufficient to pay for a correction does not establish that the conditions for exercising it will survive until selection. An institution may hold money for a future review while losing its relevant competences. Conversely, maintaining a capability without any credible means of exercising it preserves an incomplete repertoire.

5.2 Renewal Sequences

Waiting can be an activity. It may require keeping participants engaged, renewing access, updating records, or maintaining equipment. The same elapsed duration can involve very different burdens depending on the activities through which the relation persists. A categorical identity operation does not capture these burdens unless duration and resource use are explicitly represented.

Renewal complicates a simple depletion account. A cooperative practice can replenish skills, relationships, or funding while consuming them. Maintenance may therefore depend on the timing of inflows rather than an initial stock alone. A process can be sustainable over a cycle but fail before the first replenishment arrives.

This temporal issue also limits retrospective accounting. A sequence with a nonnegative final balance may have required an impossible negative balance at an intermediate step. In institutional terms, a reimbursement available after a review does not necessarily fund the preparation required to begin it. Section 10 formalizes this prefix constraint in a deliberately simple consumable model.

5.3 Joint Support

Several revisions may share an archive, a legal channel, or a trained team. Joint maintenance can then be cheaper than separate maintenance. It can also create congestion or expose the alternatives to a common failure. Sharing and resilience are different properties.

A repertoire requires support for a later choice, not the simultaneous exercise of every response. Some shared capacity can therefore be reused across mutually exclusive branches. Other preparations

cannot be postponed until selection. The relevant cost structure depends on which activities must occur before the uncertainty resolves.

This distinction makes the policy of maintenance analytically prior to an aggregate cost. The cost belongs to a sequence of commitments, observations, and contingent actions. Summing separate pathway costs ignores possible sharing; taking their minimum ignores the requirement to remain prepared for more than one pathway. A common policy resolves the comparison only after these timing relations are specified.

5.4 Invisible Burdens

Maintenance burdens may be displaced rather than removed. Standardizing an interface can reduce one institution's interpretation work while requiring others to translate their records into its categories. A faster decision procedure can save administrative time while increasing the time participants spend demonstrating eligibility. The present paper records such distributions when they change the repertoire being evaluated.

The full strategic problem of preserving one actor's optionality by burdening another is reserved for a separate investigation. Here the narrower requirement is accounting honesty: a claim of resource economy must identify its boundary and the activities excluded from it.

Archival retention illustrates the same issue. Keeping more records can support later interpretation but impose privacy, access, curation, and contestation burdens. Prior GR work on power and historical records cautions against treating retention as an unqualified good [12]. The appropriate repertoire may require selective preservation, controlled access, or eventual deletion. An indefinitely expanding archive is not a general solution to revisability.

5.5 Maintenance Evidence

The presence of a procedure is evidence of a nominal possibility. Evidence of maintained capacity is stronger when it concerns the components required for an actual response: access attempts, interpretation exercises, valid delegations, replenishment schedules, and completion times. Such evidence remains selective. Testing one kind of appeal does not establish every kind of appeal.

This difference supports a practical research design. Instead of asking only how many alternatives an institution lists, investigate the activities whose interruption would make a selected alternative unavailable. The object is a conditional support structure with traceable failure points. Its reconstruction can reveal dependencies without pretending that all of them are numerical.

6 Convergence Economies

This section examines the conditions under which convergence can reduce maintenance demands while retaining revision. It distinguishes shared implementation, compressed description, and closure. The argument is conditional: neither diversity nor convergence has a uniform resource advantage.

6.1 Shared Implementations

A common procedure can eliminate repeated translation, duplicate record keeping, and incompatible authorization chains. If several later responses use the same preliminary operations, a shared maintenance arrangement may support them at lower cost. This is a substantive economy rather than an illusion created by counting fewer alternatives.

The relevant comparison is between implementations of a selected repertoire. Suppose two departments maintain separate ways to review the same classes of requests. A shared archive and training programme may reduce routine upkeep while preserving each department's authority to initiate and complete review. The economy depends on the shared arrangement's continued accessibility and on the compatibility of its procedures with both request types.

Such an arrangement can also create a new dependence. If the shared archive becomes inaccessible, several responses fail together. A cost comparison for normal operation should therefore be distinguished from a comparison across failure scenarios. The same convergence can economize one class of obligations and increase another.

6.2 Compressed Descriptions

Convergence may occur in descriptions rather than practices. Several detailed histories can be represented by a common summary. This can reduce the effort of interpretation, but the summary's adequacy depends on the future operations it must support.

A current decision rule may require only a final status. A later appeal may require the sequence of authorizations through which that status arose. A summary sufficient for current processing can therefore be insufficient for revision. The failure is not simply a lack of detail; it is the loss of a distinction that an admissible future operation needs.

This observation connects to prior GR analysis of historical identity and operational records [11]. Adequacy is indexed to a declared task family. It is possible to compress without preserving every historical feature, but the preserved features must be justified through the operations being claimed. Section 12 gives a precise sufficient condition for a restricted model.

6.3 Administrative Closure

Closing a revision channel can reduce expenditure. If an institution no longer accepts appeals, it may no longer need appeal personnel. That is a genuine reduction in one budget, but it is not an economy in maintaining the same repertoire. The comparison has changed its object.

Closure can still be justified. A procedure can be redundant, harmful, or incompatible with other obligations. The point is to state that justification directly rather than presenting the deletion as if it preserved the original possibilities more efficiently. A resource argument cannot settle the value of the deleted responses without a normative premise.

A similar ambiguity appears in convergence of interpretation. Shared categories can make cooperation possible by reducing persistent misunderstanding. They can also make some experiences harder to articulate as relevant. The maintenance of a revision repertoire may therefore require procedures through which categories themselves can be contested. Whether such procedures are feasible is an institutional question, not a consequence of affirming pluralism in the abstract.

6.4 Reopening Capacities

A converged arrangement remains practically revisable when there are supported operations for challenging and altering it. These operations need not reproduce the full diversity that preceded convergence. They must preserve the selected forms of reopening at the relevant horizon.

Reopening can require a different competence from routine execution. A highly efficient routine may have no internal resources for diagnosing its own failure. The institution can then depend on an external review body, a retained alternative practice, or a periodically renewed testing process. These are additional support relations whose costs belong in the comparison.

The resulting claim is modest but important. Convergence may economize the maintenance of revisability when it changes the implementation while preserving relevant operations. It may also economize by narrowing the repertoire. The former claim requires an operation-preservation argument; the latter requires an argument for the narrowed object. A single scalar measure of convergence cannot distinguish them.

6.5 Evaluation Boundary

An adequate comparison declares the participating actors, maintenance horizon, response family, failure scenarios, and resource readouts. It can then ask whether the new arrangement supports the same family

with fewer required allocations, a different distribution of burdens, or a changed exposure to failure.

No universal ordering follows. One implementation can require less funding but more interpretive work, while another has a larger fixed cost and a cheaper exercise phase. The resource frontier in Section 10 retains such incomparability. This permits a precise resource claim without embedding an unargued social welfare function.

7 Criticality Qualification

The resource argument so far does not require criticality. This section specifies the additional mechanism that would justify a near-critical investigation. It separates feasibility boundaries, changing environments, and dynamical restoring structure, then states the limited claim tested by the field model.

7.1 Distinct Sources Of Difficulty

An arrangement may become unrevisable because its budget is exhausted. It may fail because a request arrives after a deadline, because a relevant category changes, or because the environment moves faster than a response can be completed. None of these descriptions alone identifies a critical point.

A criticality claim requires a specified structure and a change in that structure. In the later model, the structure is the restoring response of a field configuration to a perturbation. A parameter controls a mode whose linear restoring rate approaches zero. At its critical value, the qualitative organization of stationary configurations changes.

This is one mechanism among several. It is not a universal definition of social criticality. Prior criticality research distinguishes different transition mechanisms [2], and the GR programme retains those distinctions when interpreting survival and preparation [9, 13]. The present model deliberately selects a tractable mode-loss case.

7.2 Holding Time

The resource connection concerns the time over which a capability must be sustained. If a response process takes longer to resolve, the supports that must remain active during that process may incur additional maintenance. A slowing response can therefore transform a dynamically defined difficulty into a resource obligation.

Several bridge assumptions are necessary. The response mode must matter to the selected revision; the institutional process must be unable to bypass it; relevant supports must incur a positive holding burden; and replenishment or adaptation must not fully offset that burden. Without these assumptions, a slow field mode may have no effect on the revision repertoire.

Recognition time and dynamical relaxation time are also different. An actor may recognize a problem before the represented field relaxes, or remain unable to interpret it after relaxation. The formal calculation below supplies a possible duration within a maintenance problem. It does not establish a general equation between epistemic lag and a physical-looking relaxation constant.

7.3 Nonlinear Qualification

Linearization describes sufficiently small perturbations near a specified configuration. A divergent linear relaxation scale does not automatically imply that every finite disturbance takes arbitrarily long to decay by a fixed proportion. Nonlinear restoring terms can matter precisely when the linear coefficient becomes small.

The field construction retains those terms and derives an exact recovery time in an invariant sector. It thereby distinguishes two limits: small-amplitude response at fixed restoring coefficient, and approach to the critical coefficient at a fixed nonzero amplitude. The limits give different answers. This distinction prevents an attractive but invalid universal claim that maintenance costs must diverge whenever criticality is approached.

The calculation also separates a dynamical bifurcation from a thermodynamic singularity. The model's field space is finite dimensional and its quartic term makes the partition integral finite. Its variational response can undergo a bifurcation while the finite integral remains smooth. These are distinct mathematical objects.

7.4 Research Obligation

The resulting near-critical claim is conditional. Where a selected revision depends on a mode with weakening restoration, the duration and organization of its enabling support can change. Whether that change destroys practical revisability depends on the nonlinear response, the horizon, the available carriers, and the maintenance policy.

A case study must therefore demonstrate more than rapid change or anxiety. It must identify the relevant response process, show why a restoring-mode account is appropriate, and compare that account with ordinary depletion, delay, rate effects, and changing interpretation. If those simpler accounts suffice, the resource analysis remains useful but its criticality qualification should be withdrawn for that case.

8 Relational Maintenance Spacetimes

This section constructs the history-bearing representation used by the support and operator models. It follows the prior GR distinction between elements, couplings, and coupling events. The construction is representational: a two-complex records selected features of a relational spacetime, and its incidence data do not supply a social dynamics.

8.1 Boundary Data

Let P be a finite set of element occurrences and C a finite set of coupling occurrences. An incidence assignment $\iota : C \rightarrow \mathcal{P}(P)$ gives each coupling the set of elements it joins, where $\mathcal{P}(P)$ is the power set of P . Every $\iota(c)$ contains at least two elements, and each element belongs to at least one and at most two such sets in the graphical sector considered here. Elements with one incident coupling are boundary half-edges. A typed boundary is denoted $\Gamma = (P, C, \iota)$, with additional labels for the kinds of elements and couplings.

This convention follows the relevant GR construction: graph nodes represent couplings, while graph edges or half-edges represent elements [8]. It is deliberately not the familiar convention in which nodes are already constituted actors. An actor can occupy an extended, historically connected subconfiguration involving several elements and couplings.

Labels can record a material carrier, a competence, or a relation of authorization, but such readings require a separate interpretation. Incidence alone establishes neither possession nor trust. In particular, a graph edge between two represented practices does not establish that information can pass between them or that either may authorize the other.

8.2 History Composition

A finite history complex $F = (V, E, \Phi)$ has vertices V for coupling events, edges E for histories of couplings, and faces Φ for histories of elements. Selected initial and final boundaries have the preceding form. Temporal incidence is required to admit an acyclic order in this restricted directed sector. Face attachments and boundary types are part of the data.

Two histories can be glued when a chosen type-preserving isomorphism identifies the final boundary of the first with the initial boundary of the second. Interiors are renamed to be disjoint before gluing, and incidences must remain compatible. If F_1 and F_2 satisfy these conditions, write $F_2 \circ F_1$ for the composite. The notation does not make every pair composable.

A concrete history record h includes such a complex, its labels, the selected boundary correspondence, and provenance for the operations applied. The record can be refined when an interpretation

requires additional distinctions. Restricting attention to a record is a modelling choice; it is not a claim that relational reality is exhausted by what has been recorded.

A categorical identity history represents persistence without a coupling event in the selected description. It does not establish a costless interval. A duration assignment, a resource-use record, or explicit maintenance events must be added to represent what persistence requires. Otherwise an identity morphism can inadvertently erase the very maintenance problem under investigation.

8.3 Support Annotations

An operation label a specifies a permitted history extension together with its boundary type. Its implementation can depend on the available resource carriers and on an authorization relation. These are not reducible to the incidence pattern. A staff member can be incident to a record-handling practice without possessing permission to change the record.

Duration marks can be attached to events, edges, or intervals, depending on the intended interpretation. They need not define a single universal clock. In the finite scheduling models below, a common discrete clock is introduced explicitly. In the later field flow, the evolution parameter belongs to an auxiliary variational model. Neither is silently identified with every participant's lived or institutional time.

Maintenance can preserve a selected boundary capability while changing the interior history that supports it. Replacing an archive medium, renewing an authorization, or training a successor can all extend the represented spacetime. The resulting histories need not be identical, even when their available revision operations match.

8.4 History Representation Layers

A two-complex supplies combinatorial organization. Adding group or representation labels and amplitudes can produce a spinfoam-type instantiation in a suitably specified model. A group field can generate complexes through its interactions. These are different layers, and none follows merely from calling an institution a spacetime [14].

The operator model uses recorded extensions of these histories. The finite resource model forgets most internal incidence while retaining the operations and their support conditions. The field model specifies a separate generating construction and then investigates one response sector. These translations are intentionally partial. They demonstrate several ways to investigate the object; they do not assemble an unproved universal formalism.

The gain from the spacetime representation is a disciplined place for history, concurrency, boundary compatibility, and renewal. Its cost is a substantial interpretation burden. A case study must identify what its elements, couplings, events, and resource marks actually describe, and what distinctions are lost in the chosen representation.

9 Anticipatory Support

The objective of this section is to formalize the difference between individual feasibility and common maintenance. It first uses a deterministic finite-horizon setting, then explains the additional requirement imposed by uncertain observations. No probabilities or scalar measures of trust are introduced.

9.1 Support Primitives

Fix an initial history record h , a horizon, and a resource constraint specification B . Let U be a finite nonempty set of selected revision requests. Each request names a response requirement, not merely a target label. Its interpretation includes the relevant claimant, authorization, completion conditions, and deadline.

Let $\mathcal{M}(h, B)$ be the set of admissible maintenance implementations from h to the declared selection stage. Membership requires feasible use of carriers throughout the maintenance interval. A maintenance implementation χ determines a resulting history h_χ and residual support data B_χ . Let $\Pi(u \mid h_\chi, B_\chi)$ be the set of permitted continuation implementations for request u . Define $E(u, \pi, h_\chi, B_\chi)$ to mean that continuation π executes the required response, with all support and timing conditions satisfied.

All these objects depend on the chosen interpretation. In particular, E is not established by a formal path alone. In a concrete application its clauses must be supported through evidence about execution and authority.

9.2 Quantifier Order

Individual feasibility allows preparation to depend on which request will later be made. Common support requires one preparation that works before that selection. The two predicates are defined as follows.

$$\begin{aligned} I(U; h, B) &\iff \forall u \in U \exists \chi_u \in \mathcal{M}(h, B) \exists \pi \in \Pi(u \mid h_{\chi_u}, B_{\chi_u}) E(u, \pi, h_{\chi_u}, B_{\chi_u}), \\ M(U; h, B) &\iff \exists \chi \in \mathcal{M}(h, B) \forall u \in U \exists \pi \in \Pi(u \mid h_\chi, B_\chi) E(u, \pi, h_\chi, B_\chi). \end{aligned} \quad (1)$$

Equation (1) defines the precise sense of a maintainable repertoire used in this restricted setting. The same χ must serve every request, while the continuation π may depend on the request after it has been selected. There is no demand to execute all continuations.

Proposition 9.1 (Common support). *For the fixed primitives above, $M(U; h, B)$ implies $I(U; h, B)$. The converse need not hold.*

Proof. If M holds, choose its witness χ for every χ_u . The corresponding continuation witnesses establish I . To disprove the converse, take $U = \{a, b\}$ and a maintenance domain containing precisely two implementations, χ_a and χ_b . Let the first support only a and the second only b , with one feasible continuation for each supported request and none for the other. Then each request has a witness for I , while neither maintenance implementation witnesses M . $\square$

Proposition 9.1 is an elementary quantifier result, not a novel theorem of planning. Its role is to expose what a claim of retained alternatives must demonstrate. A list of separately feasible plans can satisfy the weaker predicate while being presented as if it satisfied the stronger one.

9.3 Observation-Dependent Policies

A maintenance policy can respond to observations during the interval. Let Ω be a declared set of disturbance histories, and let $o_t(\omega)$ be the observation record available by stage t in disturbance history ω . A policy is nonanticipative when its action at t depends only on the available observation record and previous actions. Two disturbance histories with the same such record must receive the same current action.

Let $\mathfrak{M}(h, B)$ contain the policies satisfying this condition and feasible for the disturbance histories being considered. Each $\chi \in \mathfrak{M}(h, B)$ induces a history $h_{\chi, \omega}$ and residual support $B_{\chi, \omega}$. The robust version of common support has the following form, where $U(\omega)$ is the request family pertinent to that disturbance history.

$$\exists \chi \in \mathfrak{M}(h, B) \forall \omega \in \Omega \forall u \in U(\omega) \exists \pi \in \Pi(u \mid h_{\chi, \omega}, B_{\chi, \omega}) E(u, \pi, h_{\chi, \omega}, B_{\chi, \omega}). \quad (2)$$

Equation (2) is a worst-case requirement over the declared family, not a claim about every imaginable future. Continuations must obey the same observation restrictions if uncertainty persists during their execution; the definition of Π must then contain policies rather than clairvoyant paths.

Robustness can be demanding or empty when the scenario set is poorly chosen. A probabilistic or graded criterion is another possible model, but requires justified probabilities or preferences. The present paper does not infer such a criterion from the mere fact that some futures are uncertain.

9.4 Changing Request Languages

Both formulas presuppose that the analyst can specify the response family. Near-critical interpretive change can undermine this presupposition. A new form of objection may emerge that the initial menu did not contain. Preserving $M(U; h, B)$ then establishes support only for the old menu.

One response is to include operations for revising the request language itself: accepting a new category, authorizing a novel review, or preserving records needed for reinterpretation. This extends the repertoire rather than escaping the problem. Those operations have their own maintenance requirements and limits.

The formalism therefore cannot certify complete openness. It can certify a declared relation between a maintenance policy and selected later responses. The philosophical investigation remains responsible for explaining why that family is relevant and what forms of transformation it excludes.

10 Resource Frontiers

This section constructs a finite consumable-resource instance of anticipatory support. Its purpose is to derive resource requirements without summing mutually exclusive exercises or treating distinct resource types as interchangeable. A second calculation shows why replenishment requires prefix feasibility.

10.1 Restricted Accounting

Assume m independently measured consumable resource types and an initial stock $b \in \mathbb{R}_{\geq 0}^m$. The componentwise order is written $\leq$. Fix a finite set J of maintenance modes. Mode j has cost $k_j \in \mathbb{R}_{\geq 0}^m$ and supports a subset U_j of the request family. If $u \in U_j$, its selected exercise has cost $e_{j,u} \in \mathbb{R}_{\geq 0}^m$.

For this subsection, costs are deterministic, there is no replenishment, exercise alternatives are mutually exclusive, and the component stocks can be retained without loss. Timing, authority, and incidence compatibility have already been checked in defining J and U_j . These assumptions make the following accounting sufficient; they are not facts about institutions generally.

For a requested repertoire U , mode j is eligible only if $U \subseteq U_j$. Write $\bigvee$ for the componentwise maximum of a finite nonempty family of vectors. The required stock $r_j(U)$ and the feasible budget region $\mathcal{B}(U)$ are defined as follows.

$$r_j(U) = k_j + \bigvee_{u \in U} e_{j,u}, \quad \mathcal{B}(U) = \bigcup_{j: U \subseteq U_j} (r_j(U) + \mathbb{R}_{\geq 0}^m). \quad (3)$$

Equation (3) reserves enough of each resource for whichever response is selected. It does not add the resource demands of branches that will never be executed together.

Proposition 10.1 (Finite stock criterion). *Under the stated accounting assumptions, a common maintenance mode supports U from stock b if and only if $b \in \mathcal{B}(U)$. The undominated vectors among the eligible $r_j(U)$ form its finite minimal frontier.*

Proof. Mode j is feasible for every selected request precisely when $b - k_j \geq e_{j,u}$ for every $u \in U$. Coordinate by coordinate, these inequalities are equivalent to $b \geq k_j + \bigvee_{u \in U} e_{j,u}$. Taking the union over eligible modes yields the budget region in Equation (3). A dominated vector adds no point to this upward-closed union beyond the points supplied by its dominator. Every undominated vector is necessary as a minimal generator, after duplicate vectors are identified. $\square$

The free-disposal assumption is essential to Proposition 10.1. More nominal resources do not always expand feasible action when they create new duties, political dependencies, storage burdens, or restrictions. The proposition applies to fixed transition rules and usable stocks whose surplus can be left unused.

Table 2: An illustrative maintenance menu. Exercise costs apply only to the selected response.

Mode	Maintenance cost	Supported requests	Exercise cost
j_a	(1, 1)	$\{a\}$	(1, 1)
j_b	(1, 1)	$\{b\}$	(1, 1)
j_{ab}	(2, 2)	$\{a, b\}$	(1, 1) for either
j'_{ab}	(4, 0)	$\{a, b\}$	(1, 1) for either

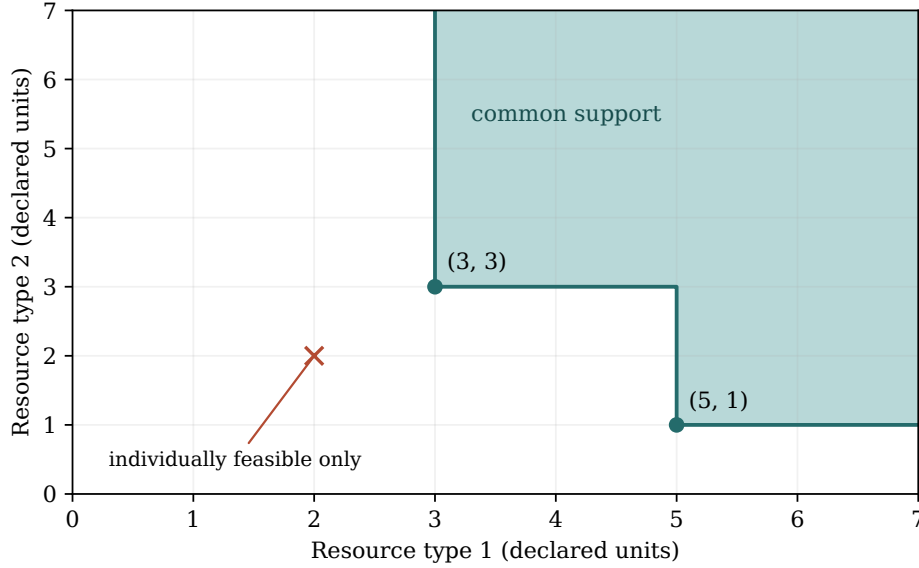


Figure 1: Illustrative stock frontier for the two-request repertoire. The shaded region supports one common maintenance mode. The marked stock (2, 2) supports either request separately but fails common support.

10.2 Two-Resource Example

Take two resource types measured in declared units, and two requests a, b . Four possible maintenance modes and their exercise costs are shown in Table 2. The numbers are illustrative, with no conversion rate between the coordinates.

At $b = (2, 2)$, each request is individually feasible through its dedicated mode. Neither joint mode is feasible. The example thus realizes the converse failure in Proposition 9.1 with explicit resources. The common-support frontier is $\{(3, 3), (5, 1)\}$. Neither point dominates the other: one uses less of the first resource and more of the second.

No universal scalar cost follows from this frontier. A weighted sum could select one point after weights are justified, but would add a preference or exchange model. The componentwise comparison retains the underlying incomparability.

Figure 1 shows the feasible common-support region and the individually feasible budget used in the counterexample. It is a depiction of the declared menu, not an empirical estimate of institutional flexibility.

10.3 Replenishment Timing

Now let a scheduled implementation have N steps, consumption vectors $c_i \geq 0$, and inflows $a_i \geq 0$ credited after consumption at step i . Define $r_0 = b$ and the post-step stock r_i . Execution requires the

following conditions.

$$r_{i-1} \geq c_i, \quad r_i = r_{i-1} - c_i + a_i, \quad i = 1, \dots, N. \quad (4)$$

Equation (4) makes the ordering explicit. The final condition $r_N \geq 0$ is insufficient. With one resource, $b = 1$, $c_1 = 2$, and $a_1 = 2$, the final accounting balance would be 1, but the first action cannot be funded.

If inflows arrive before consumption, the execution condition changes. If borrowing is permitted, its terms become additional operations and constraints. If resources decay, the update changes again. These are legitimate extensions, but none is supplied by a net-resource calculation alone.

10.4 Resource Frontier Interpretation

The frontier is a restricted representation of resource carriers, not their replacement. It applies when units, retention, and compatibility have already been justified. Its contribution is to identify the resource requirement of a shared preparatory commitment and to preserve alternatives among incomparable implementations.

The model also identifies a diagnostic for administrative claims. An institution claiming that several response pathways remain available should be able to exhibit an eligible common mode or a nonanticipative policy, rather than only a set of separately affordable plans. A case can fail this diagnostic even when its total reported budget appears adequate.

11 Resource-Filtered History Operators

This section develops an operator representation of composable history extensions. Its purpose is to separate algebraic availability, preservation of a selected history domain, and resource-feasible execution. The Hilbert space is a representation of records; no quantum ontology is inferred.

11.1 Recorded Extension Operators

Choose a countable alphabet $\mathcal{A}$ of typed, deterministic partial extension operations on history records. A record retains its initial configuration and the complete ordered word of applied operations, including their boundary identifications. Let $\mathcal{H}$ be the resulting countable set of valid recorded histories, and let $\mathcal{H} = \ell^2(\mathcal{H})$ have orthonormal basis $\{|h\rangle : h \in \mathcal{H}\}$.

For $a \in \mathcal{A}$, write ha for the extended record when the operation is admissible. Retaining the full record makes $h \mapsto ha$ injective on its domain. Define an append operator L_a by its action on the basis.

$$L_a|h\rangle = \begin{cases} |ha\rangle, & \text{if } a \text{ is admissible at } h, \\ 0, & \text{otherwise.} \end{cases} \quad (5)$$

Equation (5) extends linearly to a bounded partial isometry of norm at most one. Injectivity ensures that distinct nonzero images remain orthogonal. For a word $w = a_1 \cdots a_n$, define $L_w = L_{a_n} \cdots L_{a_1}$, and let the empty word act as the identity.

The use of complete records is important. If distinct histories were silently merged into the same output, the operator could fail to be a partial isometry. A compression requires its own normalization or quotient construction; it cannot be substituted into Equation (5) without checking the change.

11.2 Resource Filtration

Use $\mathbb{N} = \{0, 1, 2, \dots\}$ for resource indices. Assign each generator a history-independent consumable grade $c(a) \in \mathbb{N}^m$, with no replenishment. For a word, set $c(w) = \sum_i c(a_i)$. This assumption restricts the operator sector to resource types with additive nonnegative costs. History-dependent costs can be represented by refining labels, but may require a larger alphabet and additional data.

Let $\mathfrak{F}_b$ be the norm-closed linear span of append operators whose grade does not exceed $b \in \mathbb{N}^m$. These spaces satisfy the following filtration rule.

$$\mathfrak{F}_b = \overline{\text{span}}\{L_w : c(w) \leq b\}, \quad \mathfrak{F}_b \subseteq \mathfrak{F}_{b'} \ (b \leq b'), \quad \mathfrak{F}_a \mathfrak{F}_b \subseteq \mathfrak{F}_{a+b}. \quad (6)$$

The last inclusion in Equation (6) means that each product of an element of the first space with an element of the second belongs to the stated larger space. It follows first for finite linear combinations by word concatenation and grade addition, then for norm limits by continuity of multiplication.

A fixed-budget space need not be an algebra. Consider one always admissible generator x with cost one, acting by the unilateral append shift on words $1, x, x^2, \dots$. Then $\mathfrak{F}_1 = \text{span}\{I, L_x\}$, while $L_x^2 \notin \mathfrak{F}_1$: its action on the empty history reaches the orthogonal word x^2 . Thus two individually allowed operations can have a composition outside the same budget.

Linear combinations here belong to a mathematical operator space. They do not represent an institution literally executing coherent superpositions of decisions. Claims about a feasible implemented operation concern the selected generators or explicitly interpreted combinations, not every element of their linear span.

11.3 Stock-Gated Execution

Resource stocks can be represented explicitly. Let $\mathcal{K}_R = \ell^2(\mathcal{H} \times \mathbb{N}^m)$, with basis $|h, r\rangle$ for a history and a stock. The stock-gated extension for generator a is defined as follows.

$$\widehat{L}_a |h, r\rangle = \begin{cases} |ha, r - c(a)\rangle, & a \text{ admissible at } h \text{ and } r \geq c(a), \\ 0, & \text{otherwise.} \end{cases} \quad (7)$$

Equation (7) is again a partial isometry. The complete output history identifies the removed suffix, and the stock update is injective on the admitted domain.

Proposition 11.1 (Consumptive execution). *Consider the additive nonnegative cost sector. The operator product $\widehat{L}_{a_n} \cdots \widehat{L}_{a_1} |h, r\rangle$ is nonzero exactly when every history extension is admissible and $r \geq \sum_{i=1}^n c(a_i)$. Its final stock is $r - \sum_i c(a_i)$.*

Proof. At the first step, Equation (7) checks admissibility and subtracts $c(a_1)$. Inductively, the stock before step k is $r - \sum_{i < k} c(a_i)$ whenever earlier steps succeeded. The next step succeeds exactly when this stock dominates $c(a_k)$. Since all costs are nonnegative, the final total inequality implies every prefix inequality. Conversely, success of the last step implies the total inequality. The stock expression follows by repeated subtraction. $\square$

Proposition 11.1 ceases to reduce execution to a final inequality when replenishment or decay is included. Those processes require separate operators with the temporal conditions of Equation (4).

The adjoint $\widehat{L}_a^*$ removes a suffix and restores the recorded stock on its mathematical range. It is not automatically an admissible institutional reversal. Spent time, violated confidentiality, or expired authority need not be restored by any real operation. The distinction between an adjoint and an executable undo operation is essential.

11.4 Preservation Algebras

Let $S \subseteq \mathcal{H}$ be a declared family of acceptable history records, and let P_S be its orthogonal projection in $\mathcal{H}$. Set $Q_S = I - P_S$. Within the norm-closed unital algebra $\mathfrak{A}$ generated by the append operators, define the preservation algebra.

$$\mathfrak{A}_S = \{T \in \mathfrak{A} : Q_S T P_S = 0\}. \quad (8)$$

Equation (8) expresses invariance of the selected history subspace. It does not establish that S is normatively acceptable, empirically accurate, or affordable.

Proposition 11.2 (Preservation closure). *The set $\mathfrak{A}_S$ is a norm-closed unital subalgebra of $\mathfrak{A}$. It need not be closed under adjoints, even when the ambient operators admit adjoints.*

Proof. The defining condition is linear and norm closed, and the identity satisfies it. If T, U preserve the range of P_S , then $UP_S = P_SUP_S$ and $Q_S TUP_S = Q_S TP_SUP_S = 0$, so their product also preserves it. For the adjoint claim, take a two-dimensional example with S spanned by the first basis vector and $T = |1\rangle\langle 2|$. Then T preserves S , but $T^*|1\rangle = |2\rangle$ leaves it. $\square$

Proposition 11.2 clarifies a limitation of purely algebraic preservation. Repeated maintenance can remain inside S at every step while exhausting a stock. Preservation of a relationally admissible domain and feasible continuation through that domain are different requirements. The resource filtration and the stock representation add the latter question.

This use of accessible operators is consistent with the representational restraint of prior GR algebraic work [15, 16]. The algebra records allowed transformations at a declared level of description. Its existence does not establish the ultimate structure of relational reality.

12 Operational Compression

The objective of this section is to specify when a reduced representation preserves the selected revision operations. It gives a sufficient condition in a deterministic finite-operation model and then explains why resource economy requires an additional comparison. The criterion is deliberately stronger than agreement on present observations.

12.1 Summary Maps

Let H be a set of detailed history records and $\bar{H}$ a set of summaries. A surjection $q : H \rightarrow \bar{H}$ defines the compression. For each operation label a from a fixed alphabet, let δ_a be a partial map on H , and let $\bar{\delta}_a$ be a partial map on $\bar{H}$. Write $A(h)$ and $\bar{A}(\bar{h})$ for the labels available at each record.

For this model, availability includes authority and any nonresource preconditions. Let $c_a(h) \in \mathbb{R}_{\geq 0}^m$ and $\bar{c}_a(\bar{h}) \in \mathbb{R}_{\geq 0}^m$ be the respective consumable costs. Stocks update by subtracting these costs, with no replenishment or decay. Let $G_u \subseteq H$ and $\bar{G}_u \subseteq \bar{H}$ be the completion sets for each selected revision request u .

A present observation map can factor through q even when these operational structures do not. If a summary records only that a decision was approved, it may reproduce the current status perfectly while omitting the signatory needed to reopen it. Present observational adequacy is therefore weaker than the following requirement.

12.2 Operation Matching

Assume that for every $h \in H$ and every available a , the following conditions hold.

$$\begin{aligned} A(h) &= \bar{A}(qh), & q(\delta_a h) &= \bar{\delta}_a(qh), \\ c_a(h) &= \bar{c}_a(qh), & h \in G_u &\iff qh \in \bar{G}_u \quad \text{for every selected } u. \end{aligned} \tag{9}$$

The equality of available labels in Equation (9) supplies both projection and lifting. It prevents the summary from inventing a response that some represented detailed record cannot execute. The cost equality is a substantive assumption, not a result of the commutative transition equation.

Proposition 12.1 (Finite operational preservation). *Under Equation (9), a finite operation word is executable from h with a given initial stock if and only if it is executable from qh with that stock. Final records correspond under q , total costs agree, and completion of every selected request is preserved. Consequently, existence of a finite common maintenance word followed by a request-dependent finite continuation is preserved.*

Proof. For the empty word, records correspond and costs are zero. Suppose the statement holds after a prefix. Equality of available labels gives the same admissibility for its next label. Equality of costs gives the same stock test and residual stock, while the transition equation gives corresponding successor records. Induction proves the statement for any finite word. The completion equivalence gives the same response result. Applying the argument to the common maintenance word and then to each selected continuation preserves the corresponding existential witnesses and their shared prefix. $\square$

Proposition 12.1 concerns a deterministic word model. A policy using observations discarded by q is not covered merely because transition diagrams commute. Uncertain environments require matched observation structures and disturbance transitions, with lifting conditions appropriate to the chosen robust or probabilistic semantics. Those additional conditions have not been proved here.

The criterion is sufficient, not necessary. A compression may preserve a particular repertoire even if it fails to preserve irrelevant operations. One can restrict the alphabet and completion sets to the declared task, provided that the restriction does not conceal an operation needed by its maintenance.

12.3 Implementation Economies

Compression of a record does not automatically reduce material maintenance. A short summary may require an expensive process to update, and retaining the ability to regenerate its details may cost more than storing them directly. An economy claim therefore compares implementations, not just description lengths.

Suppose two implementations satisfy the operation-matching criterion for the selected repertoire. Let their fixed preselection upkeep vectors be k and $\bar{k}$, and let corresponding exercise words have identical costs. If $\bar{k} \leq k$, with strict inequality in at least one coordinate, the compressed implementation has a lower declared upkeep requirement without losing the matched operations. This conclusion follows from the componentwise stock criterion, once the upkeep and operation assumptions have been independently established.

This is the precise form of the convergence economy sought here. The theorem does not manufacture the lower upkeep; it identifies the operational obligation that must accompany a reported saving. A new common dependency or a changed failure scenario can invalidate the comparison and must be added to the model when pertinent.

12.4 A Failure Example

Let two detailed records h_A and h_B have the same present output, but let only h_A authorize response a and only h_B authorize response b . Merge them into one summary $\bar{h}$. If the summary declares both responses available, it invents an operation at each detailed record. If it declares only their intersection available, it loses a valid response at both.

The summary can accurately reproduce the current output in either case. What fails is operation matching across its fibre $q^{-1}(\bar{h})$. Keeping a recoverable authorization mark, or adding an admissible process for obtaining the missing authority, changes the example. The additional process has its own costs and horizon.

The example illustrates a GR concern: histories can be equivalent for one manifestation and different in their future relational effects. It does not prove that every historical distinction must be retained. The correct question is which distinctions support the selected operations and whether their continued accessibility is itself feasible.

13 Generative Maintenance Fields

This section introduces a finite group-domain field model with an explicit interaction, a convergent field integral, and a generating interpretation. Its purpose is to provide a genuinely field-theoretic investigation

rather than place a field label on a scalar trust variable. The resource interpretation is attached separately to supported operations and response durations.

13.1 Field Domain

Choose a finite group G of order n . The group is a mathematical index structure, with no automatic interpretation as a set of social actors. Let $\mathfrak{h}$ be the complex vector space of functions $\varphi : G \times G \rightarrow \mathbb{C}$ invariant under simultaneous right multiplication. Equip it with the normalized inner product below.

$$\begin{aligned} \mathfrak{h} &= \{\varphi : \varphi(ih, jh) = \varphi(i, j) \text{ for all } i, j, h \in G\}, \\ \langle \varphi, \psi \rangle &= \frac{1}{n^2} \sum_{i, j \in G} \overline{\varphi(i, j)} \psi(i, j), \quad \|\varphi\|^2 = \langle \varphi, \varphi \rangle. \end{aligned} \quad (10)$$

The invariance in Equation (10) reduces the field to n complex degrees of freedom, because its values depend on ij^{-1} . It is a declared redundancy of this model, not an empirical symmetry inferred from institutions.

Define a normalized composition and an adjoint on this space.

$$(\varphi \star \psi)(i, k) = \frac{1}{n} \sum_{j \in G} \varphi(i, j) \psi(j, k), \quad \varphi^\dagger(i, j) = \overline{\varphi(j, i)}. \quad (11)$$

Both operations in Equation (11) preserve simultaneous right invariance. For composition, substitute $j = \ell h$ in the transformed sum; for the adjoint, use the defining invariance directly. Composition is associative because finite sums can be regrouped. Its matrix representation also gives $\|\varphi \star \psi\| \leq \|\varphi\| \|\psi\|$.

Let $e(i, j) = 1$, so $\|e\| = 1$. Define $P_0\varphi = \langle e, \varphi \rangle e$, let $z = \langle e, \varphi \rangle \in \mathbb{C}$, and set $\chi = (I - P_0)\varphi$. Thus $\varphi = ze + \chi$ with $\langle e, \chi \rangle = 0$. The scalar z is one mode of a defined field. It is not an amount of trust, recognition, or political legitimacy.

13.2 Finite Field Measure

Choose real parameters $\mu, \kappa > 0$, $\lambda > 0$, and $\eta > 0$, and let $K = \mu P_0 + \kappa(I - P_0)$. The model action is the following real functional on $\mathfrak{h}$.

$$S(\varphi) = \langle \varphi, K\varphi \rangle + \lambda \|\varphi \star \varphi\|^2 + \eta \|\varphi\|^4. \quad (12)$$

Equation (12) contains a quadratic kinetic term, a quartic composition interaction, and a positive norm-quartic stabilizer. The action is a modelling functional, not a measured social utility or a universal energy of relations.

Let $d\varphi$ be Lebesgue measure in real and imaginary coordinates of an orthonormal basis of $\mathfrak{h}$. Define the finite field integral as follows.

$$Z_{\text{fld}}(\mu, \kappa, \lambda, \eta) = \int_{\mathfrak{h}} \exp[-S(\varphi)] d\varphi. \quad (13)$$

Equation (13) defines a positive finite-dimensional measure after normalization. It does not identify the terms of a perturbative graph expansion with positive probabilities.

Proposition 13.1 (Finite field integrability). *For every real μ and positive κ, λ, η , the integral Z_{fld} is finite and positive. At fixed positive κ, λ, η , it is a smooth function of μ .*

Proof. The interaction norms are nonnegative and $S(\varphi) \geq -|\mu| \|\varphi\|^2 + \eta \|\varphi\|^4$. Outside a sufficiently large ball, the right side is at least $(\eta/2) \|\varphi\|^4$ minus a finite constant. Its exponential is integrable in the finite real dimension $2n$. The integrand is strictly positive, giving positivity. On a compact interval of μ , every μ derivative introduces a power of $|z|^2$, bounded by a corresponding power of $\|\varphi\|^2$. A polynomial times a uniform quartic-decay bound is integrable. Differentiation under the integral is therefore valid to every finite order. $\square$

Proposition 13.1 permits a bifurcation of an auxiliary response flow without a singular finite partition integral. No thermodynamic limit or empirical phase transition has been established.

13.3 Generated Complexes

The composition interaction has an explicit index contraction.

$$\|\varphi \star \varphi\|^2 = \frac{1}{n^4} \sum_{i,k,j,\ell \in G} \varphi(i,j) \varphi(j,k) \overline{\varphi(i,\ell)} \overline{\varphi(\ell,k)}. \quad (14)$$

Equation (14) defines a four-field vertex with two-index strands. In a Gaussian expansion with positive K , pairings connect field slots through the covariance K^{-1} on the invariant field space. Following the paired index strands gives edges and closed faces of ribbon diagrams. Filling those faces yields combinatorial two-complexes.

The quadratic kernel is K , while the propagator is its inverse where that inverse defines the Gaussian covariance. This convention follows the prior GR group-field construction [14]. For $\mu > 0$, the eigenvalues of K are μ on the constant mode and κ on its orthogonal complement, so this Gaussian reference is available.

For $\mu \leq 0$, K is not a positive Gaussian reference. One can choose a different positive quadratic form and include the quadratic difference among the interactions. The convergent integral remains defined by Proposition 13.1; the perturbative series is a formal generating expansion and is not claimed to converge to it.

The η term is a double-trace quartic interaction in the matrix description. Its vertices can have disconnected local ribbon boundaries. The generated objects therefore include a broader class than a single prescribed triangulation family. Gauge projection can introduce strand identifications or group averages. The construction is rank two and finite, and makes no claim to generate all relational spacetimes.

13.4 Complex Realization

A generated diagram does not yet name a maintenance practice. A realization would assign its open strands, interactions, and boundaries to selected elements, couplings, and events, then check the typing and authorization conditions of Section 8. Acyclic temporal admissibility is an additional selection; the raw diagram expansion does not guarantee it.

Likewise, resource use can be attached to realized operations without equating a coupling constant with money or legitimacy. A graph weight is a formal amplitude from the generating model. A carrier allocation is a claim about executable support. Connecting them requires a justified map and an interpretation of what is being weighted.

A representation-basis expansion on G can introduce representation labels, providing one route toward a spinfoam-type instantiation. That further construction is optional here. The field model's present contribution is more limited and explicit: it generates combinatorial histories through interactions and supplies a nonlinear response structure that can be analyzed exactly in one sector.

14 Critical Response Modes

This section derives the response dynamics associated with the field action and identifies its critical mode. It then calculates the duration of a finite recovery and the conditional resource demand of sustaining support during that duration. The calculation preserves the nonlinear terms and distinguishes the order of limiting operations.

14.1 Variational Response

Use the complex gradient convention $dS_\varphi[v] = 2 \operatorname{Re}\langle v, \nabla_{\bar{\varphi}} S \rangle$. The action in Equation (12) has the following derivative.

$$\nabla_{\bar{\varphi}} S = K\varphi + \lambda[(\varphi \star \varphi) \star \varphi^\dagger + \varphi^\dagger \star (\varphi \star \varphi)] + 2\eta\|\varphi\|^2\varphi. \quad (15)$$

Appendix A derives Equation (15), including the normalized composition factors. The gradient remains in $\mathfrak{h}$, so no further projection is needed.

Choose an auxiliary parameter t and define the variational response flow. Along it, the action decreases.

$$\dot{\varphi} = -\nabla_{\bar{\varphi}} S, \quad \frac{d}{dt} S(\varphi(t)) = -2\|\nabla_{\bar{\varphi}} S\|^2 \leq 0. \quad (16)$$

Equation (16) is an additional dynamical choice, not a law derived from GR ontology. Other kinetic operators, stochastic dynamics, or nonvariational terms would give different response structures. Its parameter becomes institutional time only through an independently justified interpretation.

14.2 Homogeneous Sector

For invariant zero-mean χ , all row and column sums vanish. Consequently $e \star \chi = \chi \star e = 0$, $e \star e = e$, and $\chi \star \chi$ is orthogonal to e . These identities yield an exact decomposition of the action.

$$S(ze + \chi) = \mu|z|^2 + \kappa\|\chi\|^2 + \lambda(|z|^4 + \|\chi \star \chi\|^2) + \eta(|z|^2 + \|\chi\|^2)^2. \quad (17)$$

Equation (17) shows that $\chi = 0$ minimizes the action at fixed z . It also makes the homogeneous sector invariant under the gradient flow. Define $u = \lambda + \eta > 0$. The restricted action and flow take the following form.

$$S(ze) = \mu|z|^2 + u|z|^4, \quad \dot{z} = -\mu z - 2u|z|^2 z. \quad (18)$$

Equation (18) is derived from the field model, not postulated as a scalar trust equation. The restriction discards transverse history-pattern information and is used only for a response calculation.

Proposition 14.1 (Homogeneous critical mode). *At $\mu > 0$, the zero field is the unique global minimum and has restoring rates μ in the constant-mode plane and κ in transverse directions. At $\mu = 0$, the constant-mode linear restoring rate vanishes. At $\mu < 0$, the global minima are the homogeneous fields satisfying $|z|^2 = -\mu/(2u)$. Their phase direction is neutral.*

Proof. For any fixed z , every term added by nonzero χ in Equation (17) is nonnegative, and $\kappa\|\chi\|^2$ is strictly positive. It suffices to minimize $\mu r^2 + ur^4$ for $r = |z| \geq 0$. For $\mu \geq 0$ its unique minimum is $r = 0$; for $\mu < 0$ differentiation gives $r^2 = -\mu/(2u)$ as the minimizing radius. Linearization of Equation (15) at zero gives $-\mu$ and $-\kappa$ on the respective flow modes. The homogeneous action depends only on $|z|$, so changing phase along the circle of minima has no restoring force. $\square$

Proposition 14.1 identifies a bifurcation with a specific mode loss. The nonzero minima form a circle, not an isolated asymptotically stable point. Calling each point on that circle a stable attractor without this qualification would be incorrect. At the critical value, nonlinear damping remains even though the constant-mode linear term vanishes.

14.3 Exact Recovery Time

Consider $\mu \geq 0$ and an initial nonzero homogeneous amplitude with squared magnitude $y_0 = |z(0)|^2$. Fix an amplitude ratio $0 < \rho < 1$, and let $y_1 = \rho^2 y_0$. Define $y(t) = |z(t)|^2$. The squared amplitude satisfies the following equation.

$$\dot{y} = -2\mu y - 4uy^2. \quad (19)$$

Separating variables in Equation (19) gives the time T to reach y_1 from y_0 .

$$T(\mu; y_0, \rho) = \begin{cases} \frac{1}{2\mu} \log \left(\frac{y_0(\mu + 2uy_1)}{y_1(\mu + 2uy_0)} \right), & \mu > 0, \\ \frac{1}{4u} \left(\frac{1}{y_1} - \frac{1}{y_0} \right), & \mu = 0. \end{cases} \quad (20)$$

Equation (20) is positive and finite for the declared positive endpoints. Appendix A provides the integral and solution.

Two limits differ. At fixed $\mu > 0$, sending y_0 to zero while keeping ρ fixed yields the linear response time. At fixed $y_0 > 0$, sending μ to zero yields a finite nonlinear value.

$$\lim_{y_0 \downarrow 0} T(\mu; y_0, \rho) = \frac{\log(1/\rho)}{\mu}, \quad \lim_{\mu \downarrow 0} T(\mu; y_0, \rho) = \frac{\rho^{-2} - 1}{4uy_0}. \quad (21)$$

Equation (21) prevents an unrestricted divergence claim. The linear approximation is appropriate when $2uy_0 \ll \mu$. At fixed amplitude that inequality eventually fails as μ decreases. Recovery all the way to zero remains asymptotic; the calculation concerns a fixed fractional reduction.

14.4 Holding Resources

Suppose a selected revision must keep a supporting configuration active until this response reaches its declared completion amplitude. Let $v \in \mathbb{R}_{\geq 0}^m$ be a constant vector of consumable holding rates per unit of the model's interpreted time, and let $e_R \in \mathbb{R}_{\geq 0}^m$ be an exercise reserve. Assume no replenishment, no alternative bypass, and independently verified correspondence between this response and the institutional task. The initial stock must meet the following requirement.

$$b \geq v T(\mu; y_0, \rho) + e_R. \quad (22)$$

Equation (22) is a conditional bridge, not a consequence of the field action alone. Coordinates retain their separate units. If some holding rate is zero, that coordinate has no duration-related burden; if support replenishes, the update must instead respect its actual time sequence.

In the small-amplitude regime, the holding demand associated with a positive rate grows as $1/\mu$. At fixed nonzero amplitude it approaches the finite value implied by Equation (21). A finite budget can still be exceeded before this saturation, but such failure must be calculated, not inferred from the word criticality.

Figure 2 displays the exact and linear times for illustrative dimensionless parameters. It also shows the stationary homogeneous radius across the critical value. The figure is not a fit to social data.

14.5 Criticality Scope

The field calculation identifies a circumstance in which criticality adds something beyond ordinary scarcity: a changing restoring structure alters the duration for which support must be sustained. Scarcity determines whether the resulting obligation can be met. The two mechanisms can interact without being identical.

The calculation does not explain why an institution has this field action, how its parameters are measured, or why a particular revision must wait for its relaxation. Those remain empirical and interpretive obligations. Nor does it establish that conserving the represented configuration is desirable. A critical transition can remove a harmful arrangement as well as a valuable one. The resource account identifies conditional capacities, leaving their evaluation explicit.

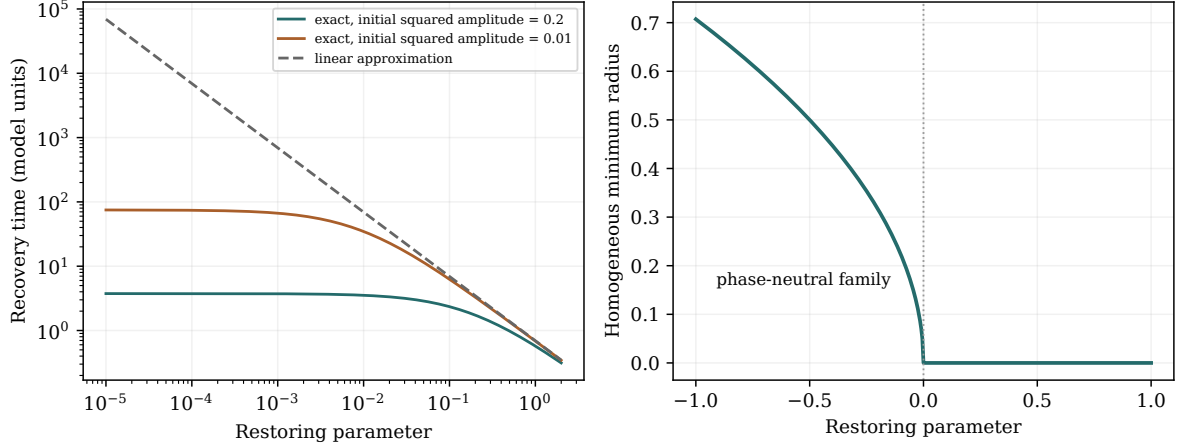


Figure 2: Illustrative field response with $u = 1$ and $\rho = 1/2$. Left: exact recovery times for two initial squared amplitudes and the linear response approximation. The finite-amplitude curves approach finite limits as μ decreases. Right: the radius of the homogeneous global-minimum family; nonzero phases remain neutral.

15 Resource-Weighted History Ensembles

This section examines a second way to represent resource constraints on histories. It distinguishes a hard feasibility restriction from a soft weighting and derives the latter's response identities. The positive ensemble is separate from the field model's generally signed perturbative amplitudes.

15.1 Admitted Histories

Choose a finite set $\mathcal{H}_N$ of realized histories, where N denotes a declared finite bound on the selected construction. Each history has passed the typing, authorization, and temporal-admissibility checks appropriate to the application. In a consumptive sector without replenishment, assign a resource-cost vector $C(h) \in \mathbb{R}_{\geq 0}^m$ and a positive base weight $w(h)$.

The weights can be illustrative or empirically estimated, but their status must be stated. They are not automatically probabilities supplied by the diagram expansion of Section 13. The present finite positive construction supplies its own normalization.

For a stock b , hard feasibility defines a truncated partition sum and, if that sum is positive, a probability distribution.

$$Z_b = \sum_{\substack{h \in \mathcal{H}_N \\ C(h) \leq b}} w(h), \quad p_b(h) = \frac{w(h) \mathbf{1}_{\{C(h) \leq b\}}}{Z_b}. \quad (23)$$

Equation (23) removes unaffordable histories. If none are feasible, $Z_b = 0$ and no distribution is defined. An analyst must retain this infeasibility result rather than normalize an empty support.

15.2 Soft Resource Weights

Let $\theta \in \mathbb{R}_{\geq 0}^m$ be a declared vector of resource-penalty parameters, each with inverse units appropriate to its resource coordinate. Define the soft ensemble as follows.

$$Z(\theta) = \sum_{h \in \mathcal{H}_N} w(h) e^{-\theta \cdot C(h)}, \quad p_\theta(h) = \frac{w(h) e^{-\theta \cdot C(h)}}{Z(\theta)}. \quad (24)$$

Equation (24) uses a scalar exponent only after the model supplies weights that make it dimensionless. It does not establish an objective conversion between the social values of different resources.

For every finite θ , all histories with positive base weight retain positive probability. An unaffordable history can therefore remain in the soft ensemble. A penalty is not a hard budget gate. Even very large penalties need not reproduce a specified componentwise budget region; they favor directions of low weighted cost rather than implement all coordinate bounds.

15.3 Response Identities

Let $\mathbb{E}_\theta$ denote expectation under p_θ . Direct differentiation of the finite sum gives the following identities for coordinates i, j .

$$\frac{\partial \log Z}{\partial \theta_i} = -\mathbb{E}_\theta[C_i], \quad \frac{\partial^2 \log Z}{\partial \theta_i \partial \theta_j} = \mathbb{E}_\theta[C_i C_j] - \mathbb{E}_\theta[C_i] \mathbb{E}_\theta[C_j]. \quad (25)$$

To verify Equation (25), first differentiate each exponential, obtaining $-\sum_h C_i(h) w(h) e^{-\theta \cdot C(h)}$. Dividing by Z gives the first identity. Differentiating that ratio introduces a numerator with $C_i C_j$ and a product of first moments, yielding the covariance.

The Hessian is positive semidefinite because its quadratic form in any real vector a is the variance of $a \cdot C$. This is a finite exponential-family identity. It does not establish a physical fluctuation law for political resources, and cross-coordinate covariances need not be positive.

15.4 Hidden Support

A coarse history record may identify the same observable outcome for histories with different costs. Replacing those costs by their conditional average can hide an infeasible member of the class. Hard support requires checking the relevant member, or a correctly specified worst-case or probabilistic condition, rather than only the average.

The problem becomes sharper with replenishment. Final net cost does not encode the prefix constraints of Equation (4). A history-level indicator must then test the full resource trajectory, including the timing of inflows and consumption. Equation (23) applies directly only where total nonnegative cost is sufficient for execution.

This section therefore provides a second safeguard against an apparently sophisticated but inadequate resource model. Weighting a history by resource use and establishing its executable feasibility are different achievements. A generated ensemble can help investigate distributions and sensitivities, but it cannot substitute for the support relation that defines a maintainable repertoire.

16 Institutional Illustrations

This section translates the formal distinctions into three hypothetical institutional settings. Each illustration identifies an operative history, a selected repertoire, and evidence that would distinguish maintenance from nominal availability. None is presented as an empirical finding.

16.1 Cooperative Review Arrangements

Consider two states that cooperate through a standing review arrangement. The arrangement receives reports, interprets discrepancies, and can authorize either a technical adjustment or a formal dispute procedure. A common secretariat maintains the records before the nature of a future discrepancy is known.

The individually feasible plans might be straightforward. If the discrepancy is technical, a small expert team can resolve it. If it concerns authorization, a legal team can prepare the dispute. Yet the plans may require incompatible preparation: both teams need the same small group of bilingual record specialists during the preselection interval. Separate feasibility studies could overlook that conflict because each assumes the specialists are available for its own pathway.

A maintainable repertoire requires a policy for that common interval. Cross-training, duplicated access, a shared preliminary record, or a sequencing agreement might supply one. Each proposal changes the history-bearing configuration rather than merely increasing an abstract quantity of trust. The relevant question is whether the resulting arrangement still permits both kinds of revision when the discrepancy becomes interpretable.

The formal resource model would represent the specialists' time only if sufficiently meaningful units can be justified. Their interpretive competence may not be divisible into interchangeable hours. A carrier-based account can instead record that a particular review requires a qualified participant available during a specified interval. This constraint need not become a real-valued cost to be analytically effective.

Existing IR accounts already ask how institutions assist agreements, how bargaining differs from enforcement, and how commitment can become problematic [17, 5, 20]. The proposed GR contribution would have to appear in the reconstruction of a particular dependence: for example, the historical loss of an interpretive bridge that changes what either party can recognize as a valid request. Merely observing that a secretariat is expensive would not establish an additional explanation.

A criticality interpretation would require further evidence. One would need an identified restoring process whose response weakens, rather than only a delayed meeting or an exhausted budget. If each dispute takes longer because the same fixed staff faces more cases, congestion may be sufficient. If the interpretive relation itself loses a stabilizing mode, a different model may be appropriate. The distinction is an empirical obligation.

16.2 Infrastructure Adaptation

Consider an infrastructure authority that intends to preserve two future adaptation pathways. Both require a common monitoring programme, but one also requires retaining access to a construction corridor while the other requires maintaining a specialized operating competence. The authority does not yet know which pathway will be selected.

The maintenance object is not the pathway diagram alone. It includes the corridor's legal and physical availability, the reproduction of competence, the interpretability of monitoring records, and the authority to activate either response. Dynamic adaptive planning already provides a framework for sequencing options and monitoring conditions [6]. The present contribution would be to make the compatibility of these particular preselection supports explicit.

Suppose a new standardized design reduces routine operating costs. It may preserve both pathways through compatible interfaces, or it may remove the physical connection needed by one. In the first case, convergence can lower the upkeep of the selected repertoire. In the second, its lower cost partly reflects a narrower repertoire.

This difference is not settled by whether the standardized design is technologically advanced. It is settled by the operations retained, their resource footprints, and the conditions of their execution. A reversible-looking modular interface can still depend on an unavailable supplier or a lost certification. A seemingly rigid arrangement can have an effective replacement pathway supported by retained records and authorizations.

A resource frontier can help compare two implementations without assuming a common currency. One may require greater material reserves and less specialist maintenance; another may require ongoing expertise and fewer stored components. A decision among these points requires judgments about availability, risk, distribution, and value. The frontier clarifies that judgment rather than replacing it.

16.3 Public Correction Procedures

Consider an institution that retains a right to request correction of an administrative classification. The procedure requires a record, an intelligible account of the classification rule, an authorized reviewer, and an executable remedy. The selected repertoire includes correction before a consequential decision and acknowledgment after the decision has become irreversible.

These responses have different completion conditions. An institution may become better at acknowledging errors while losing its ability to prevent their consequences. Aggregating both as successful corrections would conceal that change. A GR investigation would trace the histories through which records, classifications, and remedies became coupled or separated.

A compressed database can illustrate the operational criterion. If it retains only a final category, current processing may become cheaper while later review loses the evidence needed to identify an exception. If the summary preserves a verifiable route to the relevant history, the same compression may remain adequate. The retrieval route then belongs to the maintenance obligation: permissions, formats, retention periods, and knowledgeable staff must remain available.

Maintaining every record indefinitely is not the automatic answer. Some records expose participants to harm, and access itself can reproduce unequal power. The relevant repertoire might combine correction rights with selective retention, restricted access, and a process for contesting the categories. The design problem is to maintain those selected operations together.

This example also shows why meaningful alternatives are relational. A remedy may exist for an institution that can hire a specialist while remaining practically unavailable to an affected participant who cannot interpret the procedure. The analysis must specify whose executable repertoire is being described. It should not infer general revisability from the institution's ability to revise its own records.

16.4 Comparative Diagnostic

Across these illustrations, three empirical patterns would be especially informative. First, an institution may list several alternatives while its preparatory commitments support only one. Second, a reduction in routine costs may coincide either with preserved reopening operations or with their disappearance. Third, apparently adequate final resources may arrive too late to support the relevant sequence.

These patterns can occur without criticality. The near-critical extension concerns an additional change in the response structure that alters maintenance duration or the interpretation of feasible continuation. The illustrations identify where that mechanism would need to be demonstrated; they do not establish it by analogy.

17 Normative Allocation

This section examines how the selected repertoire acquires normative significance. The formal models describe conditional support; they do not determine which responses deserve preservation or who should bear their maintenance. The argument distinguishes feasibility, selection, distribution, and justified closure.

17.1 Repertoire Selection

Preserving all imaginable alternatives is neither a coherent finite objective nor a generally desirable one. Some alternatives conflict, some require incompatible preparations, and some would reproduce serious harms. A maintainable repertoire is therefore a selected object.

Selection is not merely an analyst's convenience. Choosing to preserve a right of appeal, a capacity to withdraw, or a procedure for reinterpretation expresses judgments about agency, responsibility, and the kinds of error an institution should remain able to address. The resource model can reveal whether that commitment is feasible, but cannot supply its justification.

The distinction is particularly important when the current arrangement is effective. Stable performance may reduce the apparent need for revision while increasing the consequences of losing it. Conversely, a costly revision channel may serve little defensible purpose. Neither conclusion follows from stability or expense alone.

A defensible selection process should identify the affected participants, the kinds of change that merit response, and the grounds for excluding other possibilities. It should also remain open to challenges that

reveal a failure in the initial request language. This is a normative orientation, not a theorem guaranteeing unlimited institutional openness.

17.2 Effective Standing

A response can be maintained for one participant and unavailable to another. Access may depend on language, legal status, time, expertise, or the ability to withstand delay. A repertoire should therefore be indexed to its holders and claimants, rather than attributed without qualification to the institution as a whole.

The distinction changes the interpretation of efficiency. A procedure that saves the institution an hour by requiring every claimant to undertake a day of preparation may reduce its internal cost while increasing the total burden of exercising revision. Whether the change is justified depends on more than the institution's budget.

Power can also shape what appears as a resource constraint. An authority may withhold access, establish an unusually demanding evidentiary standard, or define the relevant problem so that some claims never reach the maintenance menu. Barnett and Duvall's differentiated account of power provides a reason to investigate these constitutive and institutional relations rather than locate power only in a stock of capabilities [3].

The present paper does not develop a full theory of strategic burden transfer. It requires only that a maintenance claim disclose the relevant distribution. A later investigation can ask when one actor deliberately preserves its own options by imposing uncertainty, waiting, and support work on others.

17.3 Responsibility For Maintenance

Responsibility for an operation need not coincide with control over its supports. A review body can be legally required to hear an appeal while depending on another body for records or funding. If that dependence is excluded from the analysis, failure can be attributed to the reviewer alone even when the operative constraint lies elsewhere.

A history-bearing representation helps trace these relations. It can identify when a capacity was transferred without the resources needed to reproduce it, or when a nominal authorization persisted after its practical supports were withdrawn. The representation does not itself settle responsibility, but it supplies a more adequate object for assigning it.

There is a further temporal issue. An institution can consume a capacity whose loss will become visible only to its successor. Deferred maintenance can therefore redistribute burdens across generations of participants. The future claimant cannot always participate in the present allocation, making the selection of preserved operations a matter of public justification rather than current demand alone.

17.4 Justified Closure

Closure is sometimes required. Ending a harmful practice, deleting dangerous records, or withdrawing an abusive permission can improve the relational configuration while reducing one repertoire. The appropriate evaluation concerns the affected responses and participants, not an abstract maximization of optionality.

A justified closure should distinguish its purpose from an alleged economy in maintaining the same alternatives. It should also consider whether some other response remains necessary: accountability after deletion, compensation after irreversible action, or a route to contest the closure itself. These are different operations and may require different supports.

Convergence can therefore be normatively valuable for reasons beyond cost. A shared procedure can establish predictability or equal standing. Those benefits should be argued directly, alongside any loss of interpretive or procedural diversity. The formalism provides a way to make the tradeoff explicit without declaring one side universally superior.

17.5 Limits Of Allocation Models

A resource frontier can show that a declared set of commitments exceeds the available supports. It cannot determine whether the commitments should be reduced, resources redistributed, or the institutions governing access transformed. Treating present constraints as immutable would turn a diagnostic into an apology for the status quo.

The philosophical role of the model is thus double. It resists costless promises of revisability, and it makes the constitution of scarcity available for examination. Both are needed. Feasibility matters to an honest commitment, while the conditions of feasibility can themselves become legitimate objects of revision.

18 Philosophical Objections

This section tests the argument against objections concerning redescription, excessive representation, endogeneity, and institutional conservatism. The responses delimit what the paper can establish and identify cases where a simpler account should be preferred.

18.1 Maintenance Vocabulary

One objection is that a maintainable revision repertoire merely renames ordinary planning with resource constraints. The objection has force. Nonanticipative policies, shared preparation, and resource feasibility are established mathematical ideas. The paper makes no claim to have invented them.

The proposed philosophical contribution lies in making the continued formation of revision capacity the primary object. This connects the compatibility of preparation with the historical constitution of actors, interpretive competence, and authority. Whether that connection yields an explanation unavailable through an existing framework must be demonstrated in a substantive comparison.

A case in which a standard scheduling model fully captures the difficulty does not require a GR reconstruction. A case in which the identity of the relevant operation changes through relational history may benefit from one. This is a criterion for additional work, not a presumption that every complicated institution needs the entire formal apparatus.

18.2 Excessive Formalism

A second objection is that spacetimes, field theory, and operator algebras overcomplicate a practical question. Their use is justified only if they expose different constraints. Here, the spacetime representation records historical composition, the operator construction distinguishes preservation from executable repetition, and the field model supplies a specific nonlinear critical mechanism.

These purposes do not require every application to instantiate every model. A qualitative carrier analysis can be sufficient when the relevant failure is a missing authorization. A finite frontier can be sufficient when measurable consumables and a fixed menu capture the issue. A field-theoretic account becomes relevant only when a justified generative or collective-response structure is at stake.

The pluralism is methodological rather than decorative. Each translation states what it retains and omits. If a formal construction contributes no discriminating inference in a case, it should be omitted from that case's explanation. Mathematical sophistication does not compensate for an ungrounded interpretation.

18.3 Fixed Repertoires

A third objection concerns the use of a predeclared request family. Near-critical relational change may produce possibilities that no participant could specify in advance. A finite menu appears to miss the most important kind of revision.

The objection marks a real limitation. The support predicates establish results relative to selected requests. They cannot guarantee the availability of every future reinterpretation. Including procedures for admitting new requests expands the object but does not provide a view from outside all possible languages.

The appropriate response is to distinguish completeness from corrigibility. An institution can preserve ways to challenge its categories without predicting every challenge. Such capacities still require resources, standing, and interpretive practices. Their maintenance is investigable even when the content of future claims remains open. The present formulas offer a starting representation, not a complete theory of conceptual emergence.

18.4 Self-Referential Maintenance

A fourth objection asks what maintains the maintenance system. If review procedures require their own reviews and every reserve requires a reserve for its upkeep, an infinite regress seems possible.

The analysis does not require an infinite hierarchy of separately supported systems. Maintenance activities can be mutually supporting, share carriers, or operate cyclically. A finite institution can reproduce a set of capacities through recurrent practices without containing a complete duplicate of itself at every level.

Nevertheless, a proposed cycle must be executable. Invoking mutual support cannot conceal a first-step impossibility, an unfinanced interval, or an unavailable authorization. The prefix conditions provide one restricted way to test such claims. More general self-maintaining configurations require an explicit account of their recurrent operations and boundary dependencies.

The regress objection therefore becomes a useful demand for a stopping rule. The analyst should declare which dependencies are represented, which are treated as boundary conditions, and why that boundary is adequate for the question. Ontological suspension permits such a partial account without pretending that the unrepresented world is irrelevant.

18.5 Institutional Transformation

A fifth objection is that maintaining revisability can become a rationale for indefinite delay or institutional self-preservation. An actor may demand continued openness while others bear the costs of waiting. A harmful arrangement may use its capacity for internal revision to postpone a necessary break.

Nothing in the formal support predicate makes the selected repertoire valuable. An operation can be well supported and unjust. The normative argument must therefore consider the consequences of maintenance and the legitimacy of continued delay. Some revisions should be exercised promptly, and some capacities should be dismantled.

The GR perspective can also support transformation. A history-bearing object need not preserve its original organization to retain responsibility or enable meaningful response. New practices can inherit, redistribute, or contest prior relations. The question is which continuities matter and what resources make the transformation executable.

18.6 Criticality Rhetoric

Finally, the near-critical qualification may appear unnecessary because the main support problem already exists under ordinary uncertainty. That objection is correct for much of the paper. The common-maintenance distinction, resource filtration, and compression criterion do not depend on criticality.

The field calculation supplies a narrower addition: a specified loss of restoration can change a required support duration. Its nonlinear qualification prevents a universal divergence claim. If a case cannot establish this or another independently specified critical mechanism, its account should use the ordinary resource and revision vocabulary. The programme benefits from that discipline more than from extending the criticality label to every difficult transition.

19 Empirical Investigation

This section translates the conceptual distinctions into a research design. Its purpose is to identify observations that could support or undermine a maintenance claim. It proposes no universal index of revisability and no empirical validation of the preceding models.

19.1 History Reconstruction

An investigation should begin with a selected revision episode or a declared family of possible requests. It should reconstruct the operations required for recognition, authorization, and execution, then identify the carriers and relations supporting each operation. The unit of analysis can be an extended institutional history rather than a snapshot of organizational attributes.

Relevant material can include procedure changes, staffing continuity, record migrations, access failures, review exercises, authorization chains, and the timing of resource commitments. The evidence should distinguish an absent capacity from a capacity that existed but was not selected. A failed response alone does not identify which condition failed.

The reconstruction should also record alternative interpretations. A delay attributed to missing resources may result from strategic obstruction, contested standing, or a change in the meaning of the request. A history-bearing account is valuable only if it helps discriminate among these possibilities rather than redescribes all of them as relational complexity.

19.2 Common-Support Tests

To examine the quantifier distinction, identify at least two responses claimed to remain available before selection. Reconstruct their preparatory requirements during the shared interval. Test whether one admissible policy could have maintained both, given the observations available at that time.

This test should avoid hindsight. A plan that works only because it knows the later request does not establish anticipatory support. Conversely, a policy can legitimately adapt to an observation that was available before selection. The timing and accessibility of evidence are therefore part of the data.

For a small finite menu, the test can enumerate feasible common modes. In a richer setting it may require process tracing, policy simulation, or a partial proof of feasibility. The method should match the available evidence. A numerical optimization with unjustified costs can be less informative than a carefully documented incompatibility.

19.3 Convergence Comparisons

A before–after cost comparison should identify which operations survived the change. Lower upkeep accompanied by a lost appeal channel supports a claim of closure, not preservation of the same repertoire. Lower upkeep with matched operations supports a convergence economy within the declared scope.

Testing operation matching requires more than comparing average outputs. Researchers should examine difficult requests, authorization exceptions, recovery of archived distinctions, and the execution of reopening procedures. A summary that works for routine processing may fail precisely in the cases that make revisability valuable.

Distribution also matters. Changes in participant preparation time or translation burdens should be included when they fall within the chosen comparison boundary. Otherwise apparent economies may reflect a transfer of maintenance outside the measured institution.

19.4 Criticality Discrimination

A criticality study needs a specified response process and evidence of changing restoring behavior. It should compare a mode-loss explanation with depletion, congestion, rate-induced failure, and changing interpretation. Observed slowing alone is insufficient to identify the field model.

The nonlinear calculation suggests a further diagnostic obligation: distinguish small-amplitude response from finite disturbances. A recovery-time trend can depend on the amplitude and completion criterion. Reporting those conditions prevents an apparent contradiction between a growing linear response scale and bounded finite-amplitude recovery.

Parameter estimation would require an independently defended map from observations to the field or its selected mode. This paper provides no such map. Until one exists, the model should remain an illustrative mechanism rather than an empirical description of trust, cooperation, or institutional criticality.

20 Research Limits

This section states the results the investigation supports and the work it leaves open. The paper is an extension of existing GR resource research, with several conditional models organized around a maintainable revision repertoire.

The philosophical distinction concerns the continued support of later response. Nominal alternatives, individually feasible plans, one common maintenance policy, and executable revision are different achievements. A repertoire can fail between these stages even when a procedure remains formally open.

The finite results specify part of that difference. The common-support predicate imposes an anticipatory commitment. The resource frontier retains incomparable implementations and distinguishes retention from mutually exclusive exercises. Prefix constraints prevent final accounting from concealing an impossible intermediate step. The operator construction separates closure of an admissible transformation domain from resource-feasible repetition. The compression criterion identifies sufficient conditions for preserving selected operations through a reduced record.

The field investigation adds a specific dynamical mechanism. An explicitly defined group-domain action generates ribbon two-complexes in a formal expansion and supports a nonlinear response analysis. Its homogeneous restoring mode vanishes at a declared parameter value. The exact recovery time shows both a divergent linear response scale and a finite fixed-amplitude critical limit. A maintenance burden follows only when the selected institutional operation depends on that duration and its supporting resources satisfy the stated assumptions.

The positive ensemble supplies a separate distinction between feasibility and weighting. Soft resource penalties do not eliminate unaffordable histories. Neither their scalar parameters nor the field couplings provide a universal measure of social resources.

Several limitations remain substantive. The resource carriers, request families, and authorization relations require case-specific identification. The operation-matching theorem covers a deterministic finite-word setting; uncertain observation and changing request languages need further work. The field construction is finite and rank two, with no demonstrated social calibration or thermodynamic singularity. The literature comparison is selective, and no priority claim has been established for the coordinated account.

These limits leave a concrete research object. A claim that a relation remains revisable should identify the practices that keep its selected responses recognizable, authorized, and executable through the relevant interval. A claim that convergence economizes those practices should show which operations remain. A claim that criticality changes their resource demands should demonstrate the additional mechanism. The resulting inquiry concerns the historically supported conditions of meaningful revision, including the possibility that those conditions themselves must change.

A Field Derivations

This appendix gives the derivative, decomposition, and recovery calculations used in Sections 13 and 14. All sums are finite, and the inner product and composition are those of Equations (10) and (11).

A.1 Composition Adjoints

Write Φ and Ψ for the matrices of two fields. Normalized composition is $\Phi\Psi/n$, and the squared field norm is the squared Frobenius matrix norm divided by n^2 . The Frobenius product inequality therefore gives $\|\varphi \star \psi\| \leq \|\varphi\| \|\psi\|$ with no extra factor of n .

For fixed φ , let $B(\varphi) = \varphi \star \varphi$ be the quadratic composition map. Its derivative in direction v takes the following form.

$$DB_\varphi[v] = v \star \varphi + \varphi \star v. \quad (26)$$

Equation (26) follows by expanding $(\varphi + \epsilon v) \star (\varphi + \epsilon v)$ and taking the coefficient of the real parameter ϵ .

Cyclicity of the finite matrix trace gives the adjoint identities for arbitrary invariant fields v, φ, b .

$$\langle v \star \varphi, b \rangle = \langle v, b \star \varphi^\dagger \rangle, \quad \langle \varphi \star v, b \rangle = \langle v, \varphi^\dagger \star b \rangle. \quad (27)$$

Write V and B for the matrices of v and b in this trace calculation. The first left side is $n^{-3} \text{Tr}(\Phi^\dagger V^\dagger B)$, while its right side is $n^{-3} \text{Tr}(V^\dagger B \Phi^\dagger)$. The traces agree. Equation (27) therefore has the same normalization as the field inner product.

The derivative of the quartic composition norm follows by applying the derivative of a squared norm to Equation (26).

$$d\|B(\varphi)\|^2[v] = 2 \text{Re} \left\langle v, B(\varphi) \star \varphi^\dagger + \varphi^\dagger \star B(\varphi) \right\rangle. \quad (28)$$

Equation (28), the self-adjointness of K , and $d\|\varphi\|^4[v] = 4\|\varphi\|^2 \text{Re}\langle v, \varphi \rangle$ give Equation (15).

Substitution of $v = -\nabla_{\tilde{\varphi}} S$ into the defining gradient identity gives the dissipation formula in Equation (16). The polynomial vector field is locally Lipschitz. Its trajectories remain in a bounded action sublevel set by quartic coercivity and dissipation, so finite-time escape is excluded. The auxiliary flow consequently exists for all forward times for finite initial data.

A.2 Constant-Mode Decomposition

For an invariant field, every row sum has the same value: replacing j by ij^{-1} runs through the group. Every column sum likewise has the same value. For zero-mean χ , both sums vanish. Therefore $e \star \chi$ and $\chi \star e$ vanish entrywise.

The product $\chi \star \chi$ again has zero row and column sums. Its inner product with e is zero. The expansion of $\varphi = ze + \chi$ therefore has the following orthogonal decomposition.

$$\varphi \star \varphi = z^2 e + \chi \star \chi, \quad \|\varphi \star \varphi\|^2 = |z|^4 + \|\chi \star \chi\|^2. \quad (29)$$

Equation (29), together with orthogonality in the quadratic norm, proves Equation (17). In particular, there are no omitted linear cross terms.

At $\chi = 0$, the full gradient is proportional to e . The constant sector is therefore invariant, establishing Equation (18). At a nonzero homogeneous minimum, the radial derivative of the amplitude flow is $2\mu < 0$, while the phase derivative is zero. The transverse linear coefficient is $-(\kappa + 2\eta|z|^2)$, which is negative. These facts explain radial and transverse restoration together with phase neutrality; they do not turn an individual phase into an isolated attractor.

A.3 Recovery Integration

The homogeneous equation implies $\dot{y} = \dot{z}\bar{z} + z\dot{\bar{z}}$, yielding Equation (19). For $y_0 > y_1 > 0$ and $\mu > 0$, separation gives the following integral.

$$\begin{aligned} T &= \int_{y_1}^{y_0} \frac{dy}{2y(\mu + 2uy)} \\ &= \frac{1}{2\mu} [\log y - \log(\mu + 2uy)]_{y_1}^{y_0}. \end{aligned} \quad (30)$$

The antiderivative in Equation (30) follows from $1/[y(\mu+2uy)] = \mu^{-1}(1/y - 2u/(\mu+2uy))$. Evaluating its endpoints gives Equation (20). For $\mu = 0$, direct integration of $1/(4uy^2)$ gives the second case.

The full squared-amplitude solution supplies an independent check.

$$y(t) = \begin{cases} \frac{y_0 e^{-2\mu t}}{1 + (2uy_0/\mu)(1 - e^{-2\mu t})}, & \mu > 0, \\ \frac{y_0}{1 + 4uy_0 t}, & \mu = 0. \end{cases} \quad (31)$$

Substitution of Equation (31) into the differential equation verifies it directly. Setting $y(t) = y_1$ recovers the same time.

For the fixed-amplitude critical limit, the logarithm in Equation (20) can be written as $\log(1 + \mu/(2uy_1)) - \log(1 + \mu/(2uy_0))$. Dividing by 2μ and using the first derivative of $\log(1+x)$ at zero gives the finite limit in Equation (21). For the small-amplitude limit at fixed μ , the ratio inside the original logarithm tends to $y_0/y_1 = \rho^{-2}$, giving the linear time.

A.4 Scope Of The Critical Calculation

The existence of a zero restoring coefficient in this chosen response flow is mathematically established. Its interpretation as a maintenance problem requires the bridge assumptions preceding Equation (22). A different response rule, a different completion criterion, or a resource process with replenishment can change the resulting burden.

The finite partition integral remains smooth by Proposition 13.1. No exchange of an infinite-volume limit and a parameter limit has been performed. The bifurcation concerns stationary configurations and local response of the auxiliary flow, not a demonstrated nonanalyticity of the finite statistical model.

B Computational Checks

This appendix records the reproducible illustrative calculations. The accompanying source packet contains `simulations/check_models.py`, the generated figures, and a machine-readable result file. The script uses a fixed random seed where a sample field is required. No data fitting or empirical calibration is performed.

B.1 Finite Support

The maintenance example enumerates the four modes in Table 2. It verifies that stock $(2, 2)$ separately supports each request and fails common support. It calculates the exact frontier $\{(3, 3), (5, 1)\}$ and checks the resulting feasibility description at every integer budget with both coordinates from zero through seven.

The enumeration is a check of the stated finite example. Proposition 10.1 supplies the general finite-menu argument under its assumptions. The script also verifies the one-step replenishment counterexample: a nonnegative final balance can coexist with an unaffordable first consumption.

B.2 Field Identities

The field checks use the cyclic group with five elements and invariant matrices formed from a complex function of the group difference. This is one instance of the finite-group construction; the algebraic derivations do not depend on commutativity.

The script verifies vanishing cross products for a zero-mean invariant field, the exact action decomposition, preservation of the homogeneous sector, and the explicit four-index contraction in Equation (14). A central finite difference checks the full directional derivative against Equation (15). The relative discrepancy is below 2×10^{-10} in the recorded run. A small step along the negative gradient decreases the action.

These numerical checks can detect normalization and implementation errors. They do not replace the derivation in Appendix A, and one random test field would not establish an identity by itself.

B.3 Numerical Response Checks

The exact recovery time is checked in two independent ways: substitution into the closed-form trajectory and direct composite-Simpson quadrature of the separated integral. The sample includes the critical value, small positive restoring coefficients, several initial amplitudes, and ordinary positive coefficients. The maximum endpoint error is below 2×10^{-17} and the maximum relative quadrature discrepancy below 7×10^{-13} in the recorded run.

The script also verifies the fixed-amplitude critical limit and the small-amplitude linear limit. The plots in Figures 1 and 2 are generated from the declared finite menu and exact formulas, rather than from unreported smoothing or fitted curves.

Finally, a three-history positive ensemble checks the covariance identity in Equation (25) against finite differences of $\log Z$. It includes an over-budget history that retains positive probability under finite soft penalties. This is a counterexample to identifying a penalty with a hard feasibility gate.

B.4 Reproducibility Boundary

All eighteen implemented checks pass in the recorded run. Their assertions concern the finite constructions and numerical tolerances stated in the script. They do not test the validity of the institutional illustrations, the adequacy of a resource measurement, or the novelty of the philosophical argument.

Reproducing the paper requires Python with NumPy and Matplotlib for the figures, together with XeLaTeX and BibTeX for the manuscript. The build scripts also use a PDF parser for diagnostics. The source packet excludes downloaded third-party full texts; bibliographic identity and bounded claim support remain documented in the verification ledger.

C Model Interfaces

This appendix records the formal objects and the limits of their translations. Its purpose is to prevent notation shared across models from creating an unwarranted impression of a single derived social dynamics.

Table 3 identifies the primary objects, their status, and the bridge required for application. Each model retains only part of the maintenance problem.

The boundary and history conventions come from prior GR work. The finite resource and operator models can be attached to interpreted histories, but their additive costs are additional assumptions. The field model generates a class of combinatorial objects; it does not guarantee their admissibility as temporally ordered institutional histories. The positive ensemble requires its own nonnegative weights and is not obtained by treating every field-diagram amplitude as a probability.

The most important interfaces are therefore research obligations. A proposed application must establish how its historical records realize the required operations, how those operations use actual carriers, and how any chosen response dynamics affects the duration of support. Failure at an interface limits the application without invalidating the conditional mathematics on either side.

This modular treatment preserves methodological pluralism. It permits a case to use the carrier account without the field model, or a field investigation without claiming an institutional measurement. The common philosophical object is the supported relation between present maintenance and later revision. The models provide different, explicitly partial investigations of that object.

Table 3: Formal objects and interpretation obligations.

Object	Formal role	Interpretation obligation
$\Gamma = (P, C, \iota)$	Typed boundary incidence	Identify elements and couplings without treating nodes automatically as actors
$F = (V, E, \Phi)$	History two-complex	Identify events, coupling histories, element histories, and boundary gluing
$\mathcal{R}, \kappa_f$	Resource carriers and heterogeneous readouts	Establish functional support, access, and conversion conditions
$M(U; h, B)$	Common maintenance predicate	Specify responses, horizon, authority, and executable continuation
$\mathcal{B}(U)$	Restricted consumable stock region	Justify units, free disposal, retained stocks, and fixed costs
$L_a, \mathfrak{F}_b$	History extensions and cost filtration	Distinguish operator representation from implemented action
$q : H \rightarrow \bar{H}$	Record compression	Verify available operations, costs, and response completion
$\varphi \in \mathfrak{h}$	Finite invariant group-domain field	Supply a semantic realization for generated complexes
S, K, Z_{fld}	Action, kinetic kernel, and finite integral	Separate generating weights from social resource measures
z, μ, T	Constant mode, restoring parameter, and recovery time	Establish relevance to a selected response and its support interval
p_b, p_θ	Hard and soft positive history ensembles	Justify base weights and retain trajectory feasibility

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