

The Spacetime of Desire: Emergent Relational Geometry through Spin-Foam and Group-Field-Theoretic Formalisms

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Abstract

This discussion paper develops a revisable account of historically constituted subjective possibility as an emergent relational geometry. The expression *spacetime of desire* names a scale-indexed organization of accessibility, obstruction, detour, repetition, and trajectory support. The paper gives equal weight to a philosophical problem and a formal construction. The philosophical problem concerns a subject whose possible movements are partly constituted through language, embodied history, social mediation, fantasy, and subsequent reinterpretation. The formal construction asks how a family of relational histories can induce effective neighborhoods and distances without treating those structures as pre-given.

Lacan's graph of desire supplies a central conceptual constraint. A close reconstruction distinguishes statement from enunciation, demand from desire, ego from subject, imaginary identification from symbolic determination, and fantasy from drive. The complete graph is reproduced and read as a layered diagram of crossings, returns, and retroactive punctuation. A new typed-graph reconstruction represents selected dependencies while preserving a strict distinction between Lacan's mathemes and the paper's mathematics. A partial realization relation and preservation profile connect selected source structures to relational histories while recording information loss. Spin-foam and group-field-theoretic research supplies a second architectural source through labeled complexes, weighted histories, boundary data, generated combinatorics, and micro-to-effective reconstruction.

The exact results remain confined to a positive weighted-graph toy sector. Schur reduction produces effective adjacency and resistance geometry; Dirichlet-energy minimization characterizes the reduced boundary response; compatible nested eliminations are consistent; and finite examples establish aggregation–reduction order sensitivity and hidden-history divergence under a shared update. Further propositions formalize retrospective semantic dependence without physical backward causation and show how latent upper-layer mediation can alter observable lower-layer relations. These results support a research programme and leave empirical, clinical, continuum, and ontological evaluation open. The manuscript therefore advances a conditional model, an interpretive discipline, and a set of explicit obligations for future work.

Keywords: Lacanian psychoanalysis; emergent relational geometry; spacetime of desire; spin foams; group field theory.

Discussion Paper Note

This paper is a preliminary discussion paper intended to share an evolving idea and invite further dialogue, criticism, revision, and independent development.

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Notices

Status. This is a working draft circulated for discussion. Its concepts, formal assumptions, examples, and statement numbering remain revisable. The exact mathematical claims are confined to the stated finite graph sector. Empirical, clinical, and broader ontological evaluation remains open.

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Statement on the use of language models. Drafting, literature search, formal checking, and argumentative criticism for this paper were conducted in dialogue with ChatGPT (OpenAI). The author selected the research question, accepted or rejected suggestions, determined the manuscript’s position, and assumes responsibility for the definitions, constructions, source use, argument, and remaining errors. Every cited reference received an independent website-verification pass before admission to the bibliography.

Companion papers. This paper develops an emergence problem that arose in two earlier working papers. *Subject Development and Knowledge Emergence* develops a field-theoretic coupled-dynamics framework. *Topology or Metaphor?* states formal obligations for topological models of language and subject development. The present paper concentrates on the intermediate reconstruction from discrete relational histories to a trajectory-relevant effective geometry.

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1 Introduction

This section establishes the paper’s problem, contribution, limits, and sequence. The discussion begins from the distinction between motion within a given state space and historical change in the organization of possible motion. It then introduces the expression *spacetime of desire* as an effective modeling term and states the paper’s formal and interpretive commitments.

Many dynamical descriptions begin with a carrier space and ask how a state moves across it. That strategy is appropriate when the relevant states, neighborhoods, distances, and transition relations can be treated as fixed for the period under study. Subject formation presents a stronger possibility: relational history may alter the organization of what later appears accessible, proximate, costly, articulable, or unreachable. A history may therefore change the geometry in which later trajectories become available.

The proposed shift is from

$$\text{state dynamics on a fixed carrier} \longrightarrow \text{historically generated carrier, geometry, and dynamics.} \quad (1.1)$$

The phrase *spacetime of desire* is reserved here for the right-hand side of Equation (1.1). It denotes an effective geometry of subjective possibility indexed by a declared resolution and history. Its definition and evidence arise within the target-domain construction. The term *spacetime* highlights structured adjacency, distance, boundary, path, and scale. The term *desire* introduces a later psychoanalytic interpretation concerning a divided subject whose orientations are mediated by signifiers, others, demand, and fantasy (Lacan, 2006; Johnston, 2026).

Spin-foam and group-field-theoretic formalisms are relevant at the level of research architecture. Spin-foam models organize labeled discrete complexes and covariant histories (Perez, 2013). Group field theory can generate such complexes through field-theoretic expansions and supports programmes that seek macroscopic effective dynamics from collective microscopic states (Freidel, 2005; Oriti, 2016; Gielen et al., 2013). These achievements belong to quantum-gravity research. The present proposal transfers selected structural roles under explicit semantic restrictions.

The paper contributes a typed architecture and one complete finite calculation. The finite sector projects a relational history to a positive weighted graph, eliminates internal nodes through Kron reduction, and equips retained nodes with effective-resistance distance. The calculation establishes five bounded points. Internal equilibration induces the reduced boundary operator. Mediated paths can appear as effective adjacency at a coarser scale. Compatible nested eliminations preserve the final retained operator. Coarse-graining and microscopic averaging can fail to commute. Hidden histories with equal current endpoint geometry can respond differently to one shared update.

The remainder of the paper proceeds in five parts. Sections 2–4 establish the literature corpus, philosophical problem, and rival conceptions of desire. Sections 5–7 define the target construct and reconstruct Lacan’s complete graph. Sections 8–9 state the source-domain transfer and build a typed mathematical descendant of the desire graph. Sections 10–11 connect that reconstruction to a relational-field generator and derive effective geometry, scale consistency, order sensitivity, and hidden-history differentiation. Sections 12–15 return to philosophical interpretation, objections, comparative identification, empirical design, and failure conditions.

2 Literature Corpus and Interpretive Method

This section establishes the evidential and interpretive basis of the manuscript. Its role is methodological: it identifies the literature clusters, distinguishes primary texts from commentary and

comparison, and states how cross-disciplinary sources enter the argument. The discussion proceeds through corpus construction, source hierarchy, interpretive disagreements, and the limits of the present review.

2.1 Corpus construction and source hierarchy

This subsection defines the review procedure so that later citations carry a determinate burden. The review is structured and problem-led. It is neither a bibliometric survey nor a claim of exhaustive coverage. Sources were admitted only after an online verification of bibliographic identity and an accessible publisher, journal, library, or institutional record. Primary Lacanian texts govern claims about Lacan’s formulations. Specialist commentaries assist reconstruction, comparison, and the identification of controversy. Philosophical and scientific sources supply neighboring problems whose relevance is stated separately.

The corpus has five clusters. The first contains Lacan’s 1960 essay “The Subversion of the Subject and the Dialectic of Desire in the Freudian Unconscious,” together with Seminars V and VI, where the graph is constructed and elaborated (Lacan, 2006, 2017, 2019). The second contains sustained readings of the essay and graph by Van Haute, Fink, Eidelsztein, and Chiesa (Van Haute, 2002; Fink, 2004; Eidelsztein, 2009; Chiesa, 2007). The third places Lacanian desire among linguistic, phenomenological, ecological, field-theoretic, and schizoanalytic alternatives (de Saussure, 2011; Jakobson, 1971; Lewin, 1936; Merleau-Ponty, 2012; Gibson, 1979; Deleuze and Guattari, 1983). The fourth concerns discrete quantum-gravity architectures and spacetime emergence, including recent philosophical analysis of recovery from non-spatiotemporal structures (Huggett and Wuthrich, 2025). The fifth concerns scientific representation, analogy, and inferential use (Hesse, 1966; Suarez, 2004; Frigg and Hartmann, 2025).

Table 1 prevents a recurrent collapse of roles. A source may verify that a formal operation is mathematically established while remaining silent about its psychoanalytic meaning. A primary psychoanalytic passage may motivate a target distinction while leaving the proposed numerical representation unsupported. The manuscript therefore cites the source appropriate to each local claim and records interpretive steps as authorial constructions.

2.2 Primary sequence of the graph

This subsection locates the graph within Lacan’s work and states the sequence used for interpretation. Seminar V develops the graph while analyzing formations of the unconscious, including the joke and the mechanisms of signification. Seminar VI continues the construction around desire and its interpretation and contains a part explicitly devoted to constructing the graph (Lacan, 2017, 2019). The 1960 essay presents a compressed four-stage exposition and a complete graph in the *Écrits* (Lacan, 2006). The manuscript treats the stages as pedagogical decompositions of a structure. It avoids reading them as a chronological theory of biological maturation.

The complete graph gains significance through its internal relations. Its two horizontal chains distinguish levels commonly read as statement and enunciation. Its upward and downward paths organize a circuit through the Other, signification, identifications, demand, desire, fantasy, drive, jouissance, and castration. Fink devotes a sustained close reading to “Subversion of the Subject” and the graph; Van Haute reconstructs the essay line by line; Eidelsztein follows the graph from simpler to more complex forms and explicitly discusses graph theory and topology (Fink, 2004; Van Haute, 2002; Eidelsztein, 2009). These works support interpretive depth while also demonstrating that a single paraphrase cannot exhaust the graph.

Table 1: Literature layers, argumentative functions, and authority limits. The table organizes the review protocol and supplies no independent evidence.

Layer	Principal materials	Function in the manuscript	Authority limit
Primary psycho-analytic	Lacan’s <i>Écrits</i> and Seminars V–VI	Establish vocabulary, graph construction, and textual relations among demand, desire, fantasy, drive, and the Other	Textual authority leaves interpretive disputes open
Specialist interpretation	Van Haute, Fink, Eidelsztein, Chiesa, Johnston	Reconstruct difficult passages, track conceptual development, and expose alternative readings	Commentary remains answerable to primary texts
Philosophical comparison	Hegel, Kojève, Deleuze and Guattari, Merleau-Ponty	Clarify recognition, negativity, production, embodiment, and possibility	Comparison creates contrast and carries no genealogical identity by itself
Psychological and ecological comparison	Lewin, Gibson, contemporary work on <i>après-coup</i>	Supply neighboring concepts of life-space, affordance, and retrospective meaning formation	Psychological concepts require separate operationalization
Physical and mathematical source	Spin foams, GFT, coarse-graining, Kron reduction, resistance geometry	Supply exact structures, derivations, and examples of emergence	Physical success remains within the physical source domain
Model methodology	Hesse, Frigg and Hartmann, Suárez	Regulate analogy, representation, and surrogate inference	Methodological warrant cannot validate target semantics

2.3 Interpretive plurality and declared commitments

This subsection records disagreements that matter for the formal proposal. First, the word *graph* may designate a diagram, a pedagogical machine, a matheme-like arrangement, or a mathematical graph. Eidelsztein distinguishes Lacan’s graph from the topological surfaces employed elsewhere in Lacan’s teaching and questions a simple identification of the graph with topology (Eidelsztein, 2009). The present manuscript consequently begins with a diagrammatic reading. It introduces graph theory only through an explicit authorial reconstruction in Section 9.

Second, the relation between Lacan’s 1950s emphasis on the Symbolic and the later prominence of the Real remains historically and philosophically complex. Johnston describes a shift in emphasis around 1959–1960 while preserving continuities across Lacan’s itinerary (Johnston, 2026). Chiesa likewise reads the Lacanian subject as systematically revised across registers and periods (Chiesa, 2007). The present paper treats the graph as historically situated and resists projecting every later Lacanian formulation back into its 1960 configuration.

Third, *desire of the Other* supports several grammatical relations: desire addressed to the Other, desire for the Other, desire shaped through the Other’s field, and the enigmatic desire attributed to the Other. The graph holds these relations in tension. The proposed model adopts a relational thesis: a subject’s orientation is constituted through dependencies whose terms exceed private preference. This thesis remains weaker than any assertion that the Other fully determines desire.

Fourth, retroaction carries multiple meanings. It can designate the later punctuation of an utterance, the retrospective constitution of an event’s psychic meaning, the revision of a narrative, or a causal feedback process implemented in ordinary temporal order. A systematic review of *Nachträglichkeit/après-coup* identifies metapsychological, traumatic, clinical, and meaning-making uses, while also reporting an absence of direct empirical studies of the construct in the reviewed

literature (Mezzalana et al., 2023). The formal treatment in Section 9.4 models a restricted semantic dependence and preserves the differences among these senses.

2.4 Comparative literature and conceptual distance

This subsection explains the inclusion of neighboring traditions. Saussure supplies the signifier–signified distinction and a structural account of value; Jakobson develops selection and combination through the poles of metaphor and metonymy (de Saussure, 2011; Jakobson, 1971). Lacan transforms these resources within psychoanalysis. Their presence in the genealogy therefore supports a claim of transformation, together with a warning against treating Lacanian signification as standard linguistic semantics.

Lewin’s topological psychology represents possible movement, regions, barriers, and valences in a person’s life-space (Lewin, 1936). Gibsonian affordance theory relates possibilities for action to an organism–environment system (Gibson, 1979). Merleau-Ponty’s phenomenology gives the lived body a constitutive role in spatiality, motility, and situation (Merleau-Ponty, 2012). Each tradition offers a serious predecessor to any language of subjective possibility geometry. The present model differs by making symbolic and relational history central and by asking how the effective geometry itself may emerge across scale.

Deleuze and Guattari provide an essential philosophical objection. Their concept of desiring-production contests a picture of desire organized primarily by lack and places desire within productive natural, social, and psychic processes (Deleuze and Guattari, 1983). The spacetime proposal can accommodate this objection only if geometry records both generative opening and structured obstruction. A pure deficit metric would prejudge the dispute. Section 4 therefore adopts a plural target semantics.

2.5 Review limits and revision procedure

This subsection fixes the review’s current boundaries and revision path. The verified corpus is weighted toward English-language editions and philosophical interpretations. French- and Spanish-language scholarship on the graph merits a dedicated later pass. Clinical case literature, feminist criticism, queer theory, and postcolonial criticism require deeper treatment before the framework can claim broad psychoanalytic adequacy. The current paper uses these omissions as explicit research obligations.

The review will be revised by claim rather than by citation count. A new source earns admission when it changes an interpretation, supplies evidence, identifies a predecessor, or exposes a failure condition. Bibliographic accumulation alone carries little argumentative value. This procedure accords with a model-based paper whose main risk lies in illicit transfer across domains.

3 Philosophical Ground of Historically Constituted Possibility

This section develops the philosophical problem that motivates the formalism. Its objective is to distinguish movement among available options from the historical constitution of optionhood itself. The discussion proceeds through carrier spaces, embodied and ecological possibility, social-symbolic mediation, temporal depth, and the status of geometrical language.

3.1 Carrier spaces and constituted alternatives

This subsection isolates the principal ontological distinction. A conventional state-space model specifies a set X , a state $x_t \in X$, and a transition rule $T : X \rightarrow X$. The rule selects or generates a trajectory within an already specified carrier. Such a model can represent learning by changing a policy, a vector field, or transition probabilities while leaving the identity and topology of X fixed.

Subject formation raises a further question. An encounter may produce a distinction that the earlier subject could not articulate. A practice may render a project thinkable; an injury may close an avenue that remains physically available; a later phrase may reorganize the significance of a long sequence of earlier events. These transformations concern the partition, adjacency, salience, and identity of possibilities. Formally, the relevant process has the schematic form

$$(X_t, \mathcal{T}_t, d_t, \mathcal{A}_t) \xrightarrow{h_{t:t'}} (X_{t'}, \mathcal{T}_{t'}, d_{t'}, \mathcal{A}_{t'}), \quad (3.1)$$

where $h_{t:t'}$ denotes a relational history, $\mathcal{T}$ a topology or weaker adjacency structure, d a declared distance or cost relation, and $\mathcal{A}$ a family of admissible actions or articulations.

Equation (3.1) is a problem statement. It leaves open whether subjective possibilities form sets with stable identity, whether a metric is justified, and whether transformations admit a common comparison map. The equation also marks an important asymmetry: a newly constituted alternative can change the description of the earlier carrier. Retrospective redescriptions therefore belong to the object of inquiry.

Definition 3.1 (Constitutive history). *A relational history h is constitutive for a possibility structure $\mathcal{P}$ when variation in h , under stated background conditions, changes the individuation, adjacency, accessibility, valuation, or interpretability of possibilities represented in $\mathcal{P}$. The definition is model-relative and becomes empirical only after a measurement procedure is supplied.*

The definition distinguishes constitutive history from a history that merely selects one route through a fixed map. Both relations can coexist. A person can choose among familiar alternatives while also participating in practices that alter what later counts as an alternative.

3.2 Embodied situation and practical reach

This subsection situates the proposed geometry relative to embodied and ecological accounts. Merleau-Ponty’s analysis of bodily spatiality treats the lived body as the organizing perspective of practical engagement; spatiality is experienced through motility and situation as well as represented through objective coordinates (Merleau-Ponty, 2012). Gibson’s ecological approach places affordances in a reciprocal organism–environment relation and analyzes perception through opportunities for action (Gibson, 1979). These traditions correct any purely propositional picture of possibility.

A spacetime of desire must therefore represent at least three forms of reach. *Motor-practical reach* concerns bodily capacities and environmental support. *Symbolic reach* concerns actions and positions that can be articulated, recognized, promised, prohibited, or institutionally sustained. *Libidinal reach* concerns trajectories that attract, repel, compel, or organize repetition even when explicit endorsement points elsewhere. A single scalar distance can summarize one projection of these relations. It cannot preserve all three by default.

The embodied correction also prevents a simplistic internalism. The proposed geometry is neither a map stored inside an isolated mind nor a direct copy of an external environment. It is an effective relation reconstructed from embodied, material, social, and symbolic interactions. A staircase can be physically present, motorically inaccessible, institutionally restricted, symbolically charged, and libidinally avoided. These modes share an object while generating different possibility relations.

3.3 Life-space, valence, and historical reconstruction

This subsection connects the proposal to Lewin’s field and topological psychology. Lewin’s life-space brings person and psychological environment into one field and uses regions, boundaries, locomotion, and valence to analyze possible action (Lewin, 1936). This is a major ancestor for any geometry of subjective possibilities. It also supplies a warning: pictorial regions and arrows can remain suggestive unless their empirical and mathematical roles are controlled.

The present proposal retains four Lewinian intuitions. Possibility is organized relationally; barriers and attractive regions can structure conduct; relevant geometry belongs to the person’s effective situation; and field organization may change. It adds three emphases. First, symbolic histories participate in producing the field. Second, the subject can be divided across incompatible circuits, so a single present-centered field may conceal divergent orientations. Third, effective structure is reconstructed at a scale, which makes coarse-graining an explicit operation.

The temporal difference matters. Lewin’s principle of contemporaneity directs explanation toward the present field. A historical reconstruction can respect that insight by treating history as sedimented in present relations, while also testing whether two systems with the same present summary respond differently because their hidden organizations differ. The finite result in Subsection 11.6 supplies exactly such a test. Equal observed endpoint geometry can coexist with divergent response to a common perturbation.

3.4 Symbolic mediation and the field of the Other

This subsection explains why ecological possibility alone is insufficient for the target problem. Speaking subjects inherit languages, kinship terms, institutions, classifications, prohibitions, promises, and roles that precede any particular act of choice. Lacan’s Symbolic and the big Other name dimensions of this transindividual order, while his account of the subject stresses dependence on signifiers and a split between conscious self-presentation and unconscious formations (Lacan, 2006; Johnston, 2026).

Symbolic mediation changes possibility in several ways. It supplies distinctions through which alternatives become identifiable. It assigns positions from which speech receives different authority. It establishes consequences through law and institution. It also transmits unresolved demands and fantasies. A socially permitted act can remain subjectively unavailable; a prohibited act can organize desire with exceptional intensity. Accessibility and endorsement consequently require separate variables.

The big Other is also internally incomplete within Lacanian theory. The point $S(\bar{A})$ on the complete graph marks a signifier of lack in the Other, which blocks any interpretation of symbolic order as a complete code capable of settling desire. The model must preserve this incompleteness. Treating the Other as an omniscient central node would violate the very source used to motivate relational mediation.

3.5 Temporality, sedimentation, and retrospective constitution

This subsection separates temporal registers that the word *history* can blur. Event order records when encounters occur. Storage order records when traces are encoded and modified. narrative order records how a subject recounts events. Logical order records dependencies among positions in a formal construction. Analytic order records how later interventions punctuate earlier speech. Coarse-graining order records the sequence of averaging and elimination operations.

Table 2 provides the minimum vocabulary for a rigorous account of retroaction. The construction excludes physical information transmission from an event at T to an earlier time $t < T$. It permits

Table 2: Temporal registers in the spacetime-of-desire programme. Each register supports a distinct ordering relation and a distinct evidential task.

Register	Index or order	Modeled dependence	Principal caution
Event chronology	$t_1 < t_2 < \dots$	Material and communicative succession	Chronology alone supplies little semantic structure
Trace update	$z_{t+1} = U(z_t, e_t)$	Storage, consolidation, and revision	Stored traces differ from complete historical events
Utterance completion	$0 \leq t \leq T$	Later words punctuate earlier elements of one sequence	Retrospective meaning is compatible with forward physical causation
Narrative reconstruction	$N_k \mapsto N_{k+1}$	Re-description of earlier events under later distinctions	Narrative coherence supplies neither truth nor therapeutic value automatically
Logical articulation	$p \prec q$	Structural dependence among graph positions	Logical order need not be developmental chronology
Scale reconstruction	$\ell_1 \preceq \ell_2$	Elimination and aggregation of degrees of freedom	Order of operations can change the effective result

a representation evaluated at T to assign a new semantic value to a trace originating at t . It can also permit a later update to alter current parameters that govern future action. The distinction is philosophically substantial: retrospective constitution concerns what an event becomes within a structured history, while causal order concerns the production of later physical states.

3.6 Geometrical vocabulary and ontological restraint

This subsection determines what the word *geometry* commits the paper to. The minimal commitment is relational: there are positions and structured comparisons of adjacency, separation, path, obstruction, or neighborhood. Stronger commitments require additional properties. A metric requires nonnegativity, identity of indiscernibles, symmetry, and the triangle inequality. A topology requires a declared carrier and family of open sets satisfying the topology axioms. A manifold requires local Euclidean structure and compatibility conditions. Lorentzian spacetime requires a differentiable manifold with an appropriate metric signature and causal structure.

The proposed model earns a genuine metric in one finite positive graph sector through effective resistance. Other sectors may yield directed costs, quasi-metrics, multiple incompatible metrics, or typed reachability relations. The global phrase *spacetime of desire* therefore names a research programme whose local mathematical realizations must declare their warrant. It carries no automatic assertion of manifoldness, Lorentzian signature, physical time, or quantum ontology.

Claim 3.2 (Layered possibility thesis). *Historically constituted subjective possibility is adequately represented only when practical, symbolic, and libidinal relations remain distinguishable and their interactions are explicitly modeled. The claim is philosophical and revisable. Its current support consists in conceptual analysis and the limits of one-coordinate alternatives.*

The layered thesis provides a criterion for the later mathematics. A model that assigns one distance to every relevant relation may offer a useful projection, yet it cannot claim comprehensive representation of desire. A mature framework should permit several coupled geometries and record the maps that connect them.

4 Genealogies and Rival Conceptions of Desire

This section positions the paper’s use of *desire* within several philosophical lineages. Its role is critical: it prevents the formalism from treating desire as a self-evident synonym for preference, utility, lack, or drive. The discussion follows recognition, linguistic mediation, productive desire, and a plural semantics for modeling.

4.1 Recognition and mediated self-relation

This subsection presents the Hegelian and Kojévian background relevant to Lacan. Hegel’s account of self-consciousness develops recognition through a relation in which each self’s certainty encounters another self-consciousness. Kojève’s influential lectures emphasize human desire as desire directed through another desire and connect recognition to negating action, labor, and historical transformation (Hegel, 1977; Kojève, 1969). Lacan attended Kojève’s seminars, and contemporary philosophical scholarship treats the Hegelian–Kojévian encounter as a formative component of his intellectual trajectory (Johnston, 2026).

This genealogy matters because desire then exceeds a private vector toward an object. The object can function through the place it occupies in another’s desire, through the recognition it promises, or through the position it allows the subject to take. The relation is triangular and can become recursive: I orient toward what I take the Other to want, while my interpretation of that wanting is already shaped by the symbolic positions available to me.

Recognition also introduces conflict between visibility and transformation. A subject can seek confirmation of an existing identity, yet recognition can require inhabiting a position defined by another. Successful recognition in one register may consolidate alienation in another. The model therefore distinguishes stabilization of an ego position from expansion of a subject’s generative possibilities.

4.2 Need, demand, and the remainder of desire

This subsection states the Lacanian triad that constrains the manuscript. Need concerns bodily requirements. Demand places need into an address to another and carries a demand for love or recognition. Desire emerges in the difference opened when need passes through demand and language (Lacan, 2006; Johnston, 2026). The often-cited subtraction formula can be written schematically as

$$\text{desire} \sim \text{demand} - \text{need}, \tag{4.1}$$

provided the symbol “minus” is read as structural remainder rather than numerical subtraction.

Equation (4.1) prohibits a direct identification of desire with biological appetite. It also makes desire dependent on an address. The caregiver’s interpretation, the available signifiers, and the symbolic rules of requesting enter the formation of what can subsequently be wanted. Satisfaction of the requested object can leave the demand for love unresolved, sustaining a further movement of desire.

Desire also moves metonymically along signifying chains. A succession of objects can occupy the position of a promise whose complete fulfillment recedes. Metonymy here names more than serial consumer choice. It concerns the displacement through which one signifier or object inherits an unresolved relational function from another. A geometry of desire must therefore represent substitution, detour, and recurrent passage, together with local attractions.

4.3 Fantasy, object cause, and structured orientation

This subsection introduces fantasy as a condition of orientation. Lacan’s matheme $\$ \diamond a$ relates the barred subject to object a , the object-cause of desire (Lacan, 2006; Johnston, 2026). Fantasy supplies a scene or frame in which the subject takes a position relative to what is imagined to answer the Other’s desire. The lozenge condenses several possible logical relations and leaves the relation underdetermined.

Object a should therefore remain distinct from an ordinary goal node. An empirical object can serve as a stand-in or support for the object-cause, while the causal function exceeds the object’s positive properties. If the model places a at one vertex, the vertex must be typed as a functional placeholder and accompanied by a rule that allows different empirical objects to instantiate or support it. A fixed object label would collapse cause into target.

Fantasy also filters the possibility field. It can render one route compelling, another invisible, and a third threatening. This filtering resembles a deformation of weights or accessibility, although the resemblance alone supplies no measurement rule. The formal proposal treats fantasy as a possible operator on relational weights and leaves its empirical identification open.

4.4 Desiring-production and generative opening

This subsection presents the strongest philosophical counterposition in the corpus. Deleuze and Guattari’s *Anti-Oedipus* develops desiring-production across natural, social, and psychic processes and challenges an account centered on desire as lack (Deleuze and Guattari, 1983). On this view, desire produces connections, flows, arrangements, and subjects; a subject can appear as a residue of production rather than its sovereign origin.

The challenge exposes a danger in spatial metaphors. If desire becomes a force moving toward an absent object, the model can reproduce a restricted intentional schema. If distance measures only deprivation, the metric encodes lack as an unquestioned ontology. A generative approach must also represent the production of vertices, relations, distinctions, and carriers. Group-field-like generation becomes relevant precisely at this point: the formalism can include changing combinatorics rather than motion across one fixed graph.

Lacanian and schizoanalytic perspectives can be held in productive tension without premature synthesis. The Lacanian graph emphasizes signifying mediation, division, lack in the Other, fantasy, and drive. Desiring-production emphasizes immanent connection, social investment, machinic assemblage, and productive multiplicity. The spacetime programme can test model variants associated with both perspectives. Agreement on a relational and generative carrier does not erase disagreement about negativity, the signifier, subjectivity, and the organization of production.

4.5 Preference, intention, drive, and desire

This subsection differentiates concepts commonly merged in computational models. A preference ranks outcomes under a declared comparison procedure. An intention organizes action around a practical commitment. A need names a requirement whose frustration carries bodily or functional consequences. A demand is an addressed articulation. Desire, in the Lacanian register, is mediated by the Other and cannot be exhausted by an explicit ranking. Drive organizes repetitive circuits and obtains satisfaction in their movement. Jouissance names forms of enjoyment whose relation to pleasure and regulation is complex (Johnston, 2026; Evans, 1996).

The distinctions imply several formal constraints. Self-report cannot serve as a complete observation of desire. Stable preference does not entail stable fantasy. Repeated behavior can arise from drive, institutional constraint, habit, or material necessity. A scalar reward function may

reproduce observed choices while misidentifying their organization. The model therefore treats “desire” as an interpretation of a structured pattern across heterogeneous observations.

Definition 4.1 (Plural desire semantics). *A plural desire semantics is a family*

$$\Sigma_D = (\Sigma_{\text{need}}, \Sigma_{\text{demand}}, \Sigma_{\text{desire}}, \Sigma_{\text{fantasy}}, \Sigma_{\text{drive}}, \Sigma_{\text{production}}) \quad (4.2)$$

together with declared relations among its components. Each component specifies target predicates, admissible observations, and interpretive limits. The family permits overlap while withholding identity among components.

Plural semantics makes the formal programme modular. The finite resistance metric can operationalize one aspect of obstruction or access. A directed cycle can represent one aspect of repetition. A generative rule can represent production of new relational structures. A typed diagram can preserve distinctions among statement, enunciation, demand, and fantasy. Their integration remains a substantive theoretical achievement rather than a consequence of shared notation.

Open Problem 4.2 (Compatibility of lack and production). *Construct and compare model classes in which lack functions as a structural incompleteness, desiring-production functions as relational generation, and neither term is reduced to a scalar deficit. Identify observations that could favor, connect, or separate these classes.*

5 Conceptual Scope and Interpretive Commitments

This section defines the target of the model and separates four levels of commitment. The structure proceeds from a scale-indexed possibility geometry to a working definition of the spacetime of desire, followed by a classification of mathematical, interpretive, and empirical claims.

Definition 5.1 (Scale-indexed possibility geometry). *Let B_ℓ be a set of retained subjective positions, articulations, projects, or action-relevant states at resolution ℓ . A scale-indexed possibility geometry is a tuple*

$$\mathcal{G}^{(\ell)} = (B_\ell, d_\ell, \mathcal{A}_\ell, \mathcal{F}_\ell), \quad (5.1)$$

where d_ℓ measures an explicitly defined form of separation or obstruction, $\mathcal{A}_\ell$ records admissible transitions, and $\mathcal{F}_\ell$ records retained fields or contextual variables.

The definition permits finite metric spaces, graphs, manifolds, stratified spaces, and set-valued reconstructions. The current paper establishes only a finite resistance metric and a threshold transition relation. Continuum and manifold interpretations require further results.

Definition 5.2 (Spacetime of desire). *A spacetime of desire at scale ℓ is*

$$\mathcal{S}_D^{(\ell)} = (\mathcal{G}^{(\ell)}, \mathcal{D}_\ell, \mathcal{O}_\ell, \mathcal{U}_\ell), \quad (5.2)$$

where $\mathcal{D}_\ell$ governs trajectories, $\mathcal{O}_\ell$ maps modeled histories to observable articulations or actions, and $\mathcal{U}_\ell$ updates the later relational substrate. The word desire applies when an independently argued psychoanalytic bridge relates these structures to historically mediated subjective orientation.

Definition 5.2 assigns a structural role to time. Time appears as ordered relational updating and trajectory formation. A physical clock, developmental index, conversational turn, institutional stage, or event order can instantiate that role after its invariance and measurement conditions are stated.

Claim 5.3 (Historically constituted possibility). *Relational history may affect later subjective trajectories through two distinct channels:*

$$h_\tau \mapsto \mathcal{G}_\tau^{(\ell)}, \quad (h_\tau, \gamma_{\tau:\tau+1}) \mapsto h_{\tau+1}. \quad (5.3)$$

The first channel reconstructs a current geometry from history. The second incorporates a realized trajectory into later history. The claim is a model hypothesis until target variables and intervention tests are supplied.

The model has four distinct statuses. Its graph-theoretic results are mathematical consequences under explicit assumptions. Its interpretation of conductance as transition support is a formal target hypothesis. Its Lacanian relation is a philosophical interpretation. Its claims about people or clinical processes are empirical questions. Support and priority must be established separately at every level.

The distinction follows a general discipline in philosophy of modeling. Analogical, idealized, and toy models select limited structural similarities and can distort many features of their targets (Frigg and Hartmann, 2025). P036 therefore treats transfer as a partial map with declared disanalogies and failure conditions.

6 Lacanian Architecture of Subject and Desire

This section reconstructs the conceptual architecture required to read the graph of desire. Its role is preparatory: the diagram becomes intelligible only after subject, ego, Other, signifier, statement, enunciation, fantasy, drive, and jouissance have been differentiated. The discussion follows these dependencies and ends with a temporal account of punctuation and retroaction.

6.1 Ego, subject, and division

This subsection distinguishes the agencies condensed by everyday first-person language. Lacan’s mirror-stage theory treats the ego as an identification with an image of unity. That unity offers coordination and recognizable form, while its source in an external image and the ratification of others gives it an alienating structure. The ego is therefore an object of identification and a position in an imaginary relation. It cannot serve automatically as the source of every utterance, symptom, or desire (Johnston, 2026; Chiesa, 2007).

The subject of the unconscious appears through gaps, substitutions, slips, symptoms, and effects of the signifier. Lacan’s barred-subject notation, \$, marks a division that prevents transparent self-presence. The bar records a division within signifying representation, distinct from damaged substance or a second hidden person. It marks the incompatibility between a subject represented within the signifying order and any complete capture of subjectivity by that representation.

This distinction changes the modeling target. A computational state vector attributed to an individual can summarize observations associated with the ego, the subject, the body, or the social position. The vector itself supplies no adjudication among them. A Lacanian interpretation requires a pattern of discordance: for example, a repeated formation whose direction diverges from the speaker’s endorsed account, or a change in signification produced by a later element of speech. The paper therefore refuses a one-to-one identification between a modeled agent and Lacan’s subject.

Definition 6.1 (Subject-indexed record). *A subject-indexed record is a temporally ordered family*

$$\mathcal{R}_u = \{(e_k, a_k, r_k, c_k)\}_{k=1}^n, \quad (6.1)$$

where e_k is an encounter or utterance event, a_k an action, r_k an articulation or report, and c_k a declared context. The index u links observations for analysis and carries no assumption of a unitary underlying agent.

Definition 6.1 creates a data container without resolving the psychoanalytic subject. It permits conflict among reports, actions, and recurrent formations to remain visible.

6.2 Otherness and symbolic precedence

This subsection identifies the forms of otherness that organize the graph. Lacan distinguishes the little other associated with imaginary counterparts from the big Other associated with language, law, symbolic positions, and the locus to which speech is addressed. The big Other can be instantiated through people and institutions, yet it exceeds any single empirical person. It is a structural locus through which an utterance becomes intelligible and consequential (Lacan, 2006; Evans, 1996).

Symbolic precedence means that a newborn enters an order already populated by names, expectations, kinship relations, narratives, and prohibitions. This precedence enables subject formation and imposes heteronomous coordinates. The relation has no simple external-to-internal direction. Caregivers speak from symbolic positions, the child responds through available distinctions, and these responses can alter the local field. The Other is transmitted through relations and practices.

The graph also marks incompleteness in the Other. $S(\bar{A})$, frequently printed as $S(A)$ in varying typographic conventions, designates a signifier of lack in the Other. The paper preserves the notation in the supplied figure and treats the bar as a mark of incompleteness. A complete dictionary translating every desire into a settled signification would erase this position and dissolve a central Lacanian constraint.

6.3 Signifying chains and punctuation

This subsection develops the mechanism represented by crossings in the graph. Saussure's structural linguistics makes signs depend on differential relations and distinguishes signifier from signified (de Saussure, 2011). Lacan reverses and transforms the familiar diagram, emphasizes the signifier's priority, and uses the chain to describe the production and displacement of signification. Jakobson's poles of metaphor and metonymy provide an important linguistic resource for Lacan's account of condensation-like substitution and displacement-like combination (Jakobson, 1971).

An utterance unfolds diachronically. An early word often receives a determinate function only after later material arrives. Syntax offers an ordinary illustration: a sentence beginning with an ambiguous phrase can be resegmented or reinterpreted at completion. Psychoanalytic punctuation extends this temporal structure to equivocation, symptom, and intervention. A later signifier can alter the effective value of an earlier element without changing the earlier acoustic event.

The graph's crossing points represent functions rather than instants alone. One chain traverses another and receives punctuation through the traversal. A point de capiton, commonly rendered as quilting point or anchoring point, locally arrests the sliding of signification. The arrest is effective and revisable. It supplies a momentary organization rather than a final metalanguage.

Definition 6.2 (Retrospective punctuation). *Given a signifier trace $\sigma_{0:T}$, a retrospective punctuation at completion T is an operator*

$$\Pi_T : \sigma_{0:T} \mapsto m_T, \tag{6.2}$$

where $m_T(t)$ assigns an interpreted role at time T to trace elements originating at $t \leq T$. The operator changes the current interpretation of a stored trace and preserves physical event order.

This definition is an authorial formalization. It captures one temporal feature of the graph and leaves clinical interpretation, unconscious determination, and truth outside its mathematical content.

6.4 Statement and enunciation

This subsection articulates the two levels represented in the complete graph. The statement concerns what is said as an articulated sequence. Enunciation concerns the position and act from which saying occurs, including the address, implication, and desire that the statement can partly conceal. The distinction prevents semantic content from exhausting the event of speech.

An utterance such as “I am entirely indifferent” may function at the level of statement as a report. Repetition, timing, addressee, hesitation, and surrounding associations may position the speaker differently at the level of enunciation. The example supports an analytic distinction and supplies no diagnosis. A formal model should represent the possibility that identical propositional contents occur within different enunciative structures.

The two levels also complicate observation. Transcripts preserve words and timing with varying accuracy. Enunciation is inferred through contextual and relational patterns; it is never a directly measured hidden variable merely because the model names one. The paper therefore treats upper-layer variables as interpretive hypotheses constrained by observations, comparison, and contestable theory.

6.5 Demand, desire, and address

This subsection connects the need–demand–desire triad to the Other. Demand translates a need into signifiers addressed to another. This translation introduces conventions, expectations, recognition, and a request for love. The fulfilled object can answer one component while leaving the relational address open. Desire names a remainder and movement sustained through this mismatch (Lacan, 2006; Van Haute, 2002).

The address has two directions. The subject asks something of the Other, and the Other’s demand is experienced as directed toward the subject. The Italian phrase *Che vuoi?*, which Lacan draws from Cazotte and uses in the exposition of the graph, condenses the enigma of what the Other wants. The enigma carries personal, social, and structural dimensions. A parent, institution, language, or intimate partner can occupy the Other’s place in a local relation, while the structural function remains irreducible to that occupant’s conscious intention.

Desire consequently involves interpretation of an interpretation. The subject infers the Other’s desire, organizes a response through fantasy, and encounters the effects of that response in later relations. Misrecognition can become generative: an attributed demand can produce a durable trajectory even when the empirical other never formulated it. The model must therefore distinguish transmitted signal, attributed demand, inferred position, and resulting update.

6.6 Fantasy and the object-cause

This subsection specifies the role of $\$ \diamond a$ in the graph. Fantasy stages a relation between the divided subject and object a . It provides a frame through which the enigma of the Other’s desire receives a workable, if unstable, response. Object a functions as cause of desire rather than as a final object whose possession terminates desire (Johnston, 2026; Fink, 2004).

The formal significance is architectural. A fantasy operator can select, amplify, suppress, or recode possible relations. It can stabilize an orientation across changing empirical objects. This

suggests a family of transformations

$$\mathcal{F}_{u,\tau} : W(h) \longmapsto W_{u,\tau}^{\mathcal{F}}(h), \quad (6.3)$$

where $W(h)$ is a history weight and $\mathcal{F}_{u,\tau}$ is a subject- and history-indexed filter. Equation (6.3) is a modeling proposal. It presents a limited numerical surrogate for one filtering function of fantasy.

Fantasy also supplies relative stability. Because it organizes the subject’s position, the same relational form can recur across different partners or situations. Such recurrence can generate an effective geometry in which diverse surface events occupy nearby positions. The empirical task would be to demonstrate invariant relational structure without presuming it.

6.7 Drive, circuit, and jouissance

This subsection differentiates drive from goal pursuit. Lacan’s account emphasizes the drive’s circuit around an object and the satisfaction obtained through the circuit itself. The matheme $\$ \diamond D$ on the graph relates the barred subject to demand at the level of drive. Jouissance designates forms of enjoyment whose excess, repetition, and relation to suffering exceed a simple pleasure-maximizing account (Lacan, 2006; Johnston, 2026).

A trajectory model centered on shortest paths would therefore miss a central possibility. Repeated detours may sustain a circuit whose function lies in recurrence. A high-cost route can remain compelling. Local improvement of a reward proxy can strengthen the circuit rather than release it. Directed cycles, return probabilities, and recurrence classes provide mathematical resources, while their psychoanalytic interpretation requires an independent bridge.

The distinction between desire and drive also affects the word *spacetime*. Desire can be represented through shifting orientation and unreachable completion. Drive emphasizes circulation around a structural gap. A mature geometry requires path-sensitive quantities in addition to endpoint distance.

6.8 Logical time and retrospective determination

This subsection places the graph’s returns within Lacan’s broader concern with logical temporality. The stages of the graph are pedagogical and logical articulations. Their order supports understanding of dependencies and does not prescribe a universal biological sequence. Within speech, a later signifier can determine how an earlier element functions in the completed statement. Within a life history, later symbolic resources can transform the meaning and consequences of earlier traces.

Après-coup offers a related, broader psychoanalytic vocabulary. Contemporary review literature treats it as a nonlinear process implicated in retrospective constitution of experience, trauma, clinical work, and meaning-making (Mezzalana et al., 2023). The concepts should remain distinct: grammatical or semantic retroaction within an utterance supplies a tractable formal case, while traumatic aftermath and analytic reconstruction add affective, bodily, interpersonal, and clinical dimensions.

The paper adopts a two-time notation. Event time t indexes occurrence and trace formation. Evaluation time $T \geq t$ indexes the current interpretive organization. A value $m_T(t)$ can vary with T even though the event at t remains fixed. This notation will support the formal reconstruction in Section 9.4.

7 The Complete Graph of Desire

This section gives the graph of desire sustained attention as a conceptual object. Its objective is to reconstruct the diagram’s pedagogical build-up, identify its positions and vectors, and establish constraints on any mathematical reinterpretation. The discussion moves from the elementary cell to the complete graph, then follows the lower chain, upper chain, vertical trajectories, and return movements.

7.1 Pedagogical construction sequence

This subsection explains why the complete graph should be read through its construction. Lacan presents a succession of figures in “Subversion of the Subject,” while Seminars V and VI supply longer developments (Lacan, 2006, 2017, 2019). The elementary cell begins from the crossing of a signifying chain and an intentional or subjective vector. Two intersections permit a later point to determine the function of an earlier point. This is the diagrammatic kernel of retroactive signification.

The next stages differentiate the Other as a locus of the signifier, the message or signification produced through the chain, the ego and ideal identifications, and the question opened by the Other’s desire. The upper storey then introduces the chain of enunciation, demand at the level of drive, fantasy, desire, jouissance, castration, and lack in the Other. Complexity arises by articulation and doubling, rather than by the accumulation of independent modules.

The construction sequence has three methodological implications. First, crossings perform conceptual work; they cannot be replaced by a list of nodes. Second, arrow direction matters because the graph encodes anticipatory and retroactive relations. Third, the two storeys distinguish modes of articulation while remaining connected through the subject’s path. Any reconstruction that flattens the graph into an undirected proximity network loses these properties.

7.2 Diagram and notation

This subsection presents the complete graph used throughout the paper. Figure 1 reproduces the project-supplied TikZ reconstruction, whose labels follow a common English/French presentation of Lacan’s complete graph. Variants in typography occur across editions and commentaries. The interpretation below follows the structural relations and records the main notational alternatives.

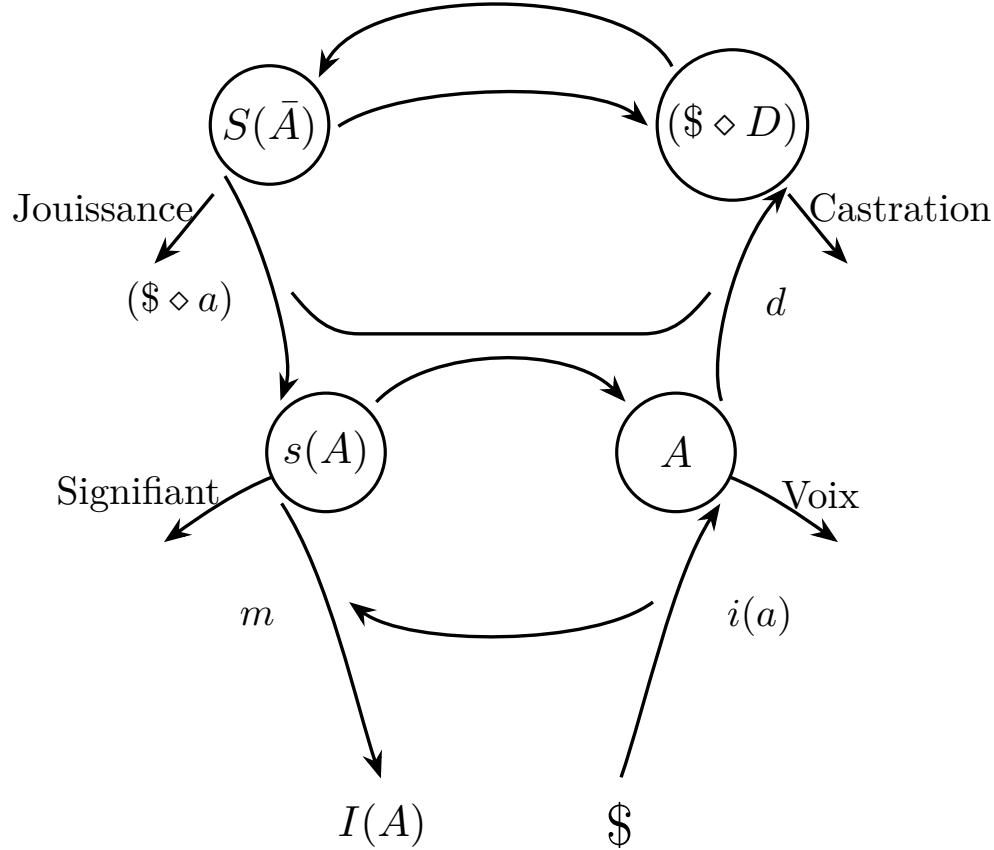
Table 3 gives only the first pass. Each position receives its sense through paths that enter and leave it. The symbol a also changes contextual role across Lacan’s teaching, and the relation between specular other $i(a)$ and object a requires care. The table therefore serves as a navigation aid.

7.3 Lower signifying chain

This subsection follows the lower horizontal trajectory from signifier to voice through the crossings A and $s(A)$. The chain represents articulated discourse at the level of statement. A functions as the treasury or locus of signifiers; $s(A)$ marks the signification produced through punctuation. The path’s return direction represents the dependence of earlier elements on later completion.

The endpoint labeled *Voix* indicates voice as the remainder or object associated with the chain, rather than the acoustic carrier alone. The endpoint labeled *Signifiant* indicates entry of signifying material. Between them, the chain passes through the Other and the production of signification. A linear message-passing model captures succession but misses the retroactive closure through which the sequence becomes a statement.

Figure 1: Complete graph of desire, redrawn in TikZ from the standard diagram in Lacan’s “Subversion of the Subject.” The figure is used as a conceptual source. Section 9 introduces a separate mathematical reconstruction.



The lower chain also intersects the imaginary route $i(a) \rightarrow m$. The ego receives stabilization through an image, while the signifying chain crosses and reorganizes this imaginary relation. A self-description can therefore be both an ego construction and an effect of the Other’s signifiers. The graph represents their articulation without identifying them.

7.4 Imaginary identification and symbolic ideal

This subsection reads the left lower descent $s(A) \rightarrow I(A)$ together with the route $i(a) \rightarrow m$. The ideal ego belongs to the image of desirable wholeness; the ego ideal belongs to the symbolic point from which the subject is evaluated. The distinction supports different modes of normativity. One concerns resemblance to an image; the other concerns a position within the Other from which recognition and judgment appear to issue.

In developmental and social contexts, these modes interact. A child, student, worker, lover, or citizen can form an image of who one should be while also orienting toward a symbolic authority imagined to confer legitimacy. The relation can generate stable paths of effort and avoidance. It can also generate conflict when the image of completion and the symbolic demand diverge.

For modeling, $i(a)$, m , and $I(A)$ should remain distinct nodes or types. Merging them into a single self-representation erases the difference between image, ego organization, and symbolic ideal. This distinction will motivate the typed node set in Equation (9.1).

Table 3: Principal positions in Figure 1. Interpretations are concise orientation statements and remain answerable to Lacan’s text and specialist commentary.

Symbol	Working designation	Structural role in the present reading
$\$$	Barred subject	Subject divided through signifying representation and positioned at the start of the upward circuit
A	Big Other	Locus of the signifier, symbolic code, and address
$s(A)$	Signification of the Other	Message or signification produced through punctuation of the lower chain
$I(A)$	Ego ideal	Symbolic identification and point from which the subject experiences itself as seen or evaluated
$i(a)$	Specular image	Imaginary form supporting identification
m	Ego	Imaginary organization formed through identification
d	Desire	Position of desire in relation to fantasy and the Other’s demand
$\$ \diamond a$	Fantasy	Structured relation between divided subject and object-cause
$\$ \diamond D$	Drive	Subject in relation to demand at the upper level
$S(\bar{A})$	Signifier of lack in the Other	Mark of symbolic incompleteness and the absence of a final guaranteeing signifier

7.5 Upper chain of enunciation

This subsection follows the upper horizontal trajectory from jouissance to castration through $S(\bar{A})$ and $\$ \diamond D$. The upper chain concerns enunciation and the drive. It represents an articulation whose terms include the bodily and libidinal consequences of signification, the circuit of demand, and the limit marked by castration.

The left position $S(\bar{A})$ marks that the Other lacks a final signifier capable of guaranteeing the whole symbolic order. The right position $\$ \diamond D$ places the barred subject in relation to demand at the level of drive. Their relation prevents the upper circuit from becoming a second complete code above the lower code. Incompletion and repetition are internal to the structure.

Jouissance and castration mark boundaries of the upper chain. Jouissance indicates an enjoyment that exceeds regulated pleasure and can appear through repetition, suffering, or transgression. Castration indicates the symbolic limit through which unrestricted completion is renounced or shown to be unavailable. The graph thereby connects desire to limit without reducing desire to a simple response to prohibition.

7.6 Desire, fantasy, and the enigma of the Other

This subsection examines the pair d and $\$ \diamond a$ situated beside the upper paths. Desire appears where the subject encounters the question of the Other’s desire. Fantasy supplies a structured response by positioning the subject relative to an object-cause. The path through fantasy can stabilize a relation while preserving its constitutive incompleteness.

The phrase *Che vuoi?* can be read along several directions: the subject’s demand to know the Other’s desire, the Other’s demand as experienced by the subject, and the subject’s encounter with its own desire through the Other. The graph’s strength lies in holding these directions together. A survey question asking “what do you want?” captures only one explicit response and may itself occupy the position of an Other’s demand.

The geometry of this relation is therefore relational and reflexive. The subject’s path depends on an attributed position of the Other; that attributed position is inferred through a history already

filtered by fantasy; and the resulting action changes the later relational history. This reflexive loop motivates the update architecture introduced below.

7.7 Vertical trajectory and doubled traversal

This subsection traces the right ascent from $\$$ through A to $\$ \diamond D$ and the left descent from $S(\bar{A})$ through $s(A)$ to $I(A)$. These vertical trajectories connect the two storeys. The upward movement begins from the barred subject, traverses the locus of the Other, and reaches the drive relation. The downward movement passes from lack in the Other through produced signification toward symbolic identification.

The doubled traversal rejects a picture in which speech merely expresses a completed inner intention. The subject enters the signifying network and receives effects from it. Meaning, identification, and demand emerge at crossings. The graph thus diagrams a circulation in which the subject is both represented and altered.

The arrows should be read as oriented dependencies within a conceptual construction. They are neither anatomical pathways nor direct causal arrows. A later formal model can assign update rules to selected edges, provided each assignment is defended and the source-target difference remains visible.

7.8 Returns, short circuits, and repeated formations

This subsection interprets the curved returns and internal cross-connections. Return paths make retroaction visible: completion feeds back into signification, and the upper circuit returns across the relation of enunciation. Internal arcs allow shorter circuits through identification or fantasy. Such a circuit can bypass a fuller traversal and produce a recurrent settlement.

The idea of a short circuit should remain structurally precise. It describes a path that closes a relation through a restricted subset of positions, often producing stability at the cost of leaving other articulations untraversed. The concept becomes formally meaningful when a path algebra, transition system, or dynamical rule is supplied. Until then, it serves as a diagrammatic interpretation.

Repeated formations can be represented through path families. Let Γ_{uv} denote admissible paths from graph position u to v . A recurrence statistic may take the form

$$\rho(C) = \limsup_{T \rightarrow \infty} \frac{1}{T} \sum_{t=1}^T \mathbf{1}\{\gamma_t \text{ traverses cycle } C\}. \quad (7.1)$$

The statistic records modeled repetition. Psychoanalytic drive enters only when a separate interpretation connects the recurring circuit to satisfaction, demand, bodily organization, and clinical material.

7.9 Diagrammatic achievements and formal absences

This subsection states what the complete graph contributes and what it leaves mathematically open. The graph achieves a dense articulation of distinctions that ordinary preference models flatten. It separates statement and enunciation, ego and subject, image and symbolic ideal, desire and drive, empirical object and object-cause, completeness and lack in the Other. It also represents crossings, directed paths, returns, and layered structure.

The diagram supplies no numerical edge weights, probability law, metric, state-transition semantics, dimensional analysis, observation model, or parameter-estimation procedure. Its arrows are conceptually typed rather than mathematically uniform. Its positions need not be mutually

Table 4: Source structures, target roles, and transfer burdens

Source work	frame-	Imported structure	Proposed target role	Principal transfer burden
Lacanian theory		divided subject, signifier, Other, demand, desire, fantasy	interpretive account of mediated orientation and partial observability	specialist interpretation and construct-level bridge
Spin foam		labeled two-complex, boundary data, local amplitudes, history sum	compositional representation of relational histories	explicit target semantics for cells, labels, and weights
Group field theory		field on group data, interactions, generated complexes, collective regimes	generator of variable relational structures and effective sectors	transformation meaning, measure, convergence, and empirical role
Coarse-graining		retained boundary data and scale-dependent effective description	passage from hidden mediation to observable geometry	preservation criteria, stability, and scale selection
Network reduction		Laplacian, Schur complement, resistance metric	exact finite model of effective adjacency and obstruction	linearity, positivity, symmetry, and measurement validity

exclusive states. Consequently, an adjacency matrix copied directly from the drawing would create a new model while concealing the act of construction.

Claim 7.1 (Graphical constraint). *Any formal descendant of Lacan’s graph that is used in this research programme should preserve, represent by an explicit alternative, or openly omit the distinctions between statement and enunciation, ego and subject, Other and empirical other, desire and drive, fantasy and goal, and symbolic completion and lack in the Other.*

The claim is methodological. It establishes a fidelity ledger rather than a theorem about Lacan. Section 9 will implement a partial descendant and record its omissions.

Open Problem 7.2 (Complete-graph comparison). *Construct competing formalizations of the complete graph as a typed directed multigraph, a diagram in a category of relations, a process network, and a sheaf-like system of local interpretations. Compare which Lacanian distinctions each formalization preserves and which inferences each licenses.*

8 Antecedent Frameworks and Transfer Boundaries

This section identifies the source traditions used by the proposal and records the limits of each transfer. The discussion follows Table 4, moving from psychoanalytic concepts to discrete histories, field-generated complexes, coarse-graining, and finite network geometry.

8.1 Psychoanalytic conceptual source

This subsection locates the word *desire* without turning Lacanian diagrams into mathematical graphs. Lacan’s account places desire within signifying and intersubjective structures, including the distinction among need, demand, and desire, the role of the Other, the divided subject, and fantasy (Lacan, 2006). Johnston’s philosophical overview emphasizes the mediation of desire through the Other and the historical shifts among the Imaginary, Symbolic, and Real in Lacan’s work (Johnston, 2026).

The transfer retains two limited propositions. Subjective articulation can disclose a structured projection of a subject’s history. Relational and symbolic mediation can shape orientations that later appear as the subject’s own. The mathematical graph introduced below is a new target model. Lacan’s graph of desire functions as conceptual ancestry and interpretive constraint.

8.2 Discrete-history architecture

This subsection identifies the source of the complex and history language. Spin-foam research formulates quantum-gravitational dynamics through labeled discrete structures, amplitudes, boundary states, and sums over histories (Perez, 2013). The source literature also preserves strong cautions about observables, dynamics, discretization, and semiclassical recovery. P036 imports the compositional pattern. It assigns target meanings independently.

8.3 Generated-complex architecture

This subsection identifies the stronger role of group field theory. GFT is a quantum field theory on group data whose perturbative structures can reproduce spin-foam complexes and amplitudes (Freidel, 2005; Oriti, 2016). GFT cosmology supplies physical examples in which collective states support effective macroscopic dynamics under defined approximation schemes (Gielen et al., 2013, 2014). P036 uses this as methodological precedent for a micro-to-effective question. The target field, labels, and effective observables require their own definitions.

8.4 Scale reconstruction

This subsection states the role of coarse-graining. Spin-foam coarse-graining seeks effective coarse amplitudes from finer structures and emphasizes boundary data, regulator dependence, and open continuum questions (Steinhaus, 2020). The present finite model adopts the boundary-retention logic. It uses a graph Schur complement as its exact reduction operation. This finite operation supplies an effective geometry while leaving continuum emergence open.

Assumption 8.1 (Transfer certificate). *Every transferred structure is accompanied by a record*

$$\mathcal{T} = (S, T, \mu, \mathcal{I}_+, \mathcal{I}_-, \mathcal{V}, \mathcal{F}), \quad (8.1)$$

where S and T are source and target structures, μ is a partial mapping, $\mathcal{I}_+$ lists preserved inferences, $\mathcal{I}_-$ lists disanalogies, $\mathcal{V}$ lists validation tasks, and $\mathcal{F}$ lists failure conditions.

Table 4 is the compact certificate for the present draft. Later work should expand each row before adding richer group, representation, quantum, or clinical claims.

9 Formal Reconstruction of the Lacanian Diagram

This section constructs a mathematical descendant of the graph of desire. Its objective is limited and explicit: it represents typed positions, directed dependencies, layered observation, retrospective punctuation, and latent mediation. The construction proceeds from a typed directed multigraph to path semantics, semantic retroaction, a positive surrogate sector, and a fidelity ledger. Every object in this section belongs to the present proposal unless a source is cited for a standard mathematical operation.

9.1 Reconstruction status and type discipline

This subsection declares the relation between Figure 1 and the new formal objects. The visual graph contains heterogeneous arrows and mathemes. A standard graph $G = (V, E)$ records incidence and possibly direction; it gives all edges a common mathematical type unless additional structure is supplied. The present reconstruction therefore uses a typed directed multigraph and keeps a separate interpretation map.

Definition 9.1 (Typed Lacanian diagram). *Let*

$$\mathbb{G}_L = (V_L, E_L, s, t, \tau_V, \tau_E, \iota_L) \quad (9.1)$$

be a typed directed multigraph, where $s, t : E_L \rightarrow V_L$ give edge source and target, $\tau_V : V_L \rightarrow \mathsf{T}_V$ and $\tau_E : E_L \rightarrow \mathsf{T}_E$ assign node and edge types, and ι_L maps selected formal elements to positions and paths in Figure 1. The principal node set is

$$V_L = \{\$, A, s(A), I(A), i(a), m, d, \$ \diamond a, \$ \diamond D, S(\bar{A})\}. \quad (9.2)$$

The mapping ι_L is partial because the external labels, crossings, arrows, and curved trajectories can require entities beyond the ten principal positions. Multiple edges are permitted because two positions may participate in more than one conceptual relation. Type sets can include

$$\begin{aligned} \mathsf{T}_V = \{ & \text{subject-position, Other-locus, signification, identification,} \\ & \text{desire, fantasy, drive, limit} \}, \end{aligned} \quad (9.3)$$

$$\begin{aligned} \mathsf{T}_E = \{ & \text{signifying-chain, subject-vector, punctuation-return, identification,} \\ & \text{fantasy-support, drive-circuit, interpretive-link} \}. \end{aligned} \quad (9.4)$$

These types organize the author’s reading. They are authored interpretive categories and leave alternative formal semantics open. Their value lies in making interpretive decisions inspectable.

Table 5 turns fidelity into a set of local obligations. A future model can reject one of these devices, yet it should state how the associated distinction is preserved, revised, or abandoned.

9.2 Layers, paths, and typed composition

This subsection defines the graph’s two storeys and the composition of paths. Let V_L^{lo} contain positions primarily associated with statement and identification, and let V_L^{up} contain positions associated with enunciation, desire, fantasy, and drive. Some positions, especially $\$$ and A , mediate the layers. A layer map

$$\lambda : V_L \longrightarrow \mathcal{P}(\{\text{lo, up, bridge}\}) \quad (9.5)$$

allows overlapping assignments.

A typed path $p = e_n \circ \cdots \circ e_1$ is admissible when $t(e_k) = s(e_{k+1})$ and a compatibility predicate $C_\tau(\tau_E(e_k), \tau_E(e_{k+1})) = 1$ holds for each composition. This predicate matters because a signifying return, imaginary identification, and drive circuit should not compose indiscriminately. Let

$$\mathsf{P}_{uv}^\alpha = \{p : u \rightsquigarrow v \mid p \text{ admissible and of composite type } \alpha\} \quad (9.6)$$

denote a family of paths between positions.

The path construction supports three kinds of comparison. Incidence asks whether positions are connected. Reachability asks whether at least one compatible directed path exists. Path-sensitive

Table 5: Fidelity ledger for the typed reconstruction in Equation (9.1). “Represented” records a mathematical feature; “deferred” records a formal obligation.

Lacanian distinction	Reconstruction status	Formal device	Residual burden
Statement/enunciation	represented	layer map $\lambda : V_L \rightarrow \{L, U\}$	justify observables assigned to each layer
Ego/subject	represented	separate vertex types and projections	prevent vertex ontology from reifying agencies
Other/empirical other	represented partially	Other-locus type and contextual occupant map	model institutional and interpersonal mediation
Desire/drive	represented	distinct nodes, paths, and recurrence statistics	connect path properties to clinical interpretation
Fantasy/ordinary goal	represented partially	path-weight transformation $\mathcal{F}$	identify stable relational form across objects
Lack in the Other	represented symbolically	distinguished vertex $S(\bar{A})$ and missing closure condition	formulate incompleteness without creating a privileged central node
Retroactive punctuation	represented	evaluation-time kernel and punctuation operator	estimate kernels from sequential evidence
Jouissance/castration	deferred	boundary labels only	construct a formal role that preserves bodily and limit dimensions

comparison asks whether different routes induce different effects. The third is essential for drive and retroaction. Endpoint identity alone can conceal the circuit through which an endpoint was reached.

Assign each edge an operator T_e between typed state spaces. For a path $p = e_n \circ \dots \circ e_1$, define

$$T_p = T_{e_n} \dots T_{e_1}. \quad (9.7)$$

If two paths $p, q : u \rightsquigarrow v$ yield $T_p \neq T_q$, the reconstruction is path-dependent. For a loop $c : u \rightsquigarrow u$, T_c is a holonomy-like return operator. The term *holonomy-like* records a structural analogy. A genuine connection on a bundle and its curvature require further construction.

Definition 9.2 (Relational return defect). *For two admissible paths $p, q : u \rightsquigarrow v$ in a normed representation space, define*

$$\Delta_{p,q}(x) = \|T_p x - T_q x\|. \quad (9.8)$$

The quantity is a relational return defect relative to the chosen representation, norm, and input x .

The defect measures history-sensitive discrepancy inside a model. Its interpretation as conflict, symptom, or subjective division requires evidence beyond the norm.

9.3 Observation projections and inferential restraint

This subsection separates accessible records from latent structure. Let X_L be the full state assigned to the typed diagram, Y_{st} an observation space for statements, Y_{act} an action record, and Y_{ctx} contextual data. Define

$$\pi_{\text{st}} : X_L \rightarrow Y_{\text{st}}, \quad \pi_{\text{act}} : X_L \rightarrow Y_{\text{act}}, \quad \pi_{\text{ctx}} : X_L \rightarrow Y_{\text{ctx}}. \quad (9.9)$$

No direct projection to “the unconscious” is postulated. Upper-layer variables are inferential constructs used to explain patterns across observations. In the language of scientific representation,

the formal model supports surrogate reasoning only when an interpretation rule connects its demonstrations to the target (Suarez, 2004). The ability to calculate a hidden state does not establish that the state corresponds to a psychoanalytic structure.

Two latent configurations $x, x' \in X_L$ are observationally equivalent for a probe family Π when

$$\pi(x) = \pi(x') \quad \text{for every } \pi \in \Pi. \quad (9.10)$$

The equivalence is probe-relative. New contexts, interventions, or longitudinal observations can separate previously equivalent configurations.

9.4 Semantic retroaction kernel

This subsection gives a restricted formal realization of retrospective punctuation. Let $\xi(s) \in \mathbb{R}^d$ encode features of a signifier trace at event time $s \in [0, T]$. Let $m_0(t)$ be a provisional semantic representation formed at t . A completion-sensitive interpretation is

$$m_T(t) = m_0(t) + \int_t^T K_{\text{ret}}(t, s) \xi(s) ds, \quad 0 \leq t \leq T, \quad (9.11)$$

where $K_{\text{ret}}(t, s)$ is a matrix-valued retrospective kernel.

The upper integration limit expresses an evaluation performed at T . The value assigned to an earlier trace can depend on later elements of the completed sequence. Equation (9.11) is implementable through ordinary forward-time storage. Define a trace store $z_T = \{\xi(s) : 0 \leq s \leq T\}$; at completion, an interpreter computes $m_T = R_T(z_T)$. Every physical update occurs at or before T .

Proposition 9.3 (Forward implementation of retrospective interpretation). *For any finite sequence $\xi_1, \dots, \xi_T$ and kernels $K_{t,s}$ defined for $t \leq s \leq T$, the retrospective assignment*

$$m_T(t) = m_0(t) + \sum_{s=t}^T K_{t,s} \xi_s \quad (9.12)$$

admits a forward-causal implementation with finite storage of the sequence and an evaluation at time T .

Proof. At event time s , append ξ_s to a store $z_s = (z_{s-1}, \xi_s)$. This operation depends only on the previous store and the current event. At T , compute every sum in Equation (9.12) from z_T . The assignment changes the representation available at T for elements indexed by earlier times. It creates each physical state from data available at its own time. Hence retrospective semantic dependence and forward causal implementation coexist. $\square$

The proposition supplies a conceptual hygiene result. It blocks a false inference from retroactive meaning to backward physical causation. It also leaves stronger psychoanalytic claims open. Stored sequence features, affective traces, repression, and clinical reconstruction require richer models.

Definition 9.4 (Punctuation sensitivity). *For two completions $\xi_{1:T}$ and $\xi'_{1:T'}$ sharing a prefix through t , define*

$$\mathcal{P}_t(\xi, \xi') = \|m_T(t) - m'_{T'}(t)\|. \quad (9.13)$$

The quantity measures how strongly later completion changes the modeled interpretation of the shared earlier trace.

Punctuation sensitivity can be estimated in linguistic experiments that manipulate sentence completion or discourse context. Such studies would validate a semantic mechanism and still leave the Lacanian bridge as a separate interpretive task.

9.5 Latent mediation and effective lower-layer geometry

This subsection develops a positive weighted surrogate that connects the graph’s two-storey architecture to exact elimination. Direction and edge type are temporarily suppressed after this loss is declared. Partition variables into a lower observable sector L and an upper latent sector U . Let a symmetric positive semidefinite Laplacian take block form

$$\mathcal{L} = \begin{pmatrix} L_{LL} & L_{LU} \\ L_{UL} & L_{UU} \end{pmatrix}, \quad L_{UL} = L_{LU}^\top, \quad (9.14)$$

with L_{UU} invertible on the internal subspace. Eliminating U produces

$$L_{\text{eff}} = L_{LL} - L_{LU}L_{UU}^{-1}L_{UL}. \quad (9.15)$$

Proposition 9.5 (Latent-layer contribution). *If $L_{LU} \neq 0$, the effective lower-layer operator in Equation (9.15) generally differs from L_{LL} . The correction*

$$\Delta_L = L_{LU}L_{UU}^{-1}L_{UL} \quad (9.16)$$

is positive semidefinite when L_{UU} is positive definite.

Proof. For any lower-layer vector x ,

$$x^\top \Delta_L x = (L_{UL}x)^\top L_{UU}^{-1}(L_{UL}x) \geq 0, \quad (9.17)$$

because L_{UU}^{-1} is positive definite. When $L_{UL}x \neq 0$ for some x , the correction acts nontrivially. Therefore the effective lower operator can encode mediation through eliminated upper variables. $\square$

This proposition is mathematical. Its interpretive use is modest: relations visible at the level of statement can be effectively altered by pathways through a latent layer. Calling that layer enunciation, fantasy, or desire demands independent evidence. The symmetric surrogate also loses arrow direction, heterogeneous edge types, and symbolic incompleteness.

9.6 Fantasy-conditioned reweighting

This subsection gives one formal role to fantasy while avoiding its identification with a numerical parameter. Let $w_e(h) > 0$ be a baseline support on edge e under history h . Let a fantasy-conditioned field $f_{u,h}(e) \in \mathbb{R}$ produce

$$w_e^{\mathcal{F}}(h) = w_e(h) \exp\{f_{u,h}(e)\}. \quad (9.18)$$

The exponential preserves positivity in the finite resistance sector. Positive f amplifies modeled support and negative f attenuates it.

The field can be stable across changing empirical objects if edges are typed by relational role rather than object identity. For example, multiple interactions can instantiate a common relation such as “seeking recognition from an unavailable authority.” This label remains a hypothesis produced through interpretation. It must not be assigned from surface behavior alone.

Fantasy-conditioned reweighting can interact with punctuation. Let P_T update the semantic typing of edges after an utterance and let F_h reweight edges under a current fantasy hypothesis. Their order can differ:

$$P_T \circ F_h \neq F_{P_T(h)} \circ P_T. \quad (9.19)$$

Equation (9.19) is a conjectured noncommutativity. It expresses a substantive possibility: a later interpretation can change which relational pattern counts as recurrent, thereby changing the inferred fantasy operator itself.

9.7 Drive-sensitive path functionals

This subsection adds path functionals that distinguish circulation from destination. Let a realized trajectory be $\gamma = (e_1, \dots, e_n)$. Define endpoint cost, repetition load, and circuit return by

$$C_{\text{end}}(\gamma) = d(s(e_1), t(e_n)), \quad (9.20)$$

$$C_{\text{rep}}(\gamma) = \sum_{c \in \mathcal{C}_*} \omega_c N_c(\gamma), \quad (9.21)$$

$$R_{\text{circ}}(\gamma) = \sum_{k < j} \mathbf{1}\{t(e_k) = t(e_j)\} r_{kj}, \quad (9.22)$$

where $\mathcal{C}_*$ is a declared family of cycles, N_c counts traversals, and r_{kj} weights returns. Two paths with the same endpoints can differ in repetition load and circuit return.

Proposition 9.6 (Endpoint insufficiency for circuit-sensitive models). *If two admissible paths $p, q : u \rightsquigarrow v$ satisfy $N_c(p) \neq N_c(q)$ for at least one cycle c with $\omega_c \neq 0$, then any representation retaining only endpoints identifies paths that the repetition functional in Equation (9.21) separates.*

Proof. The endpoint representation assigns both paths the pair (u, v) . Their repetition-load difference contains the nonzero term $\omega_c[N_c(p) - N_c(q)]$, together with any other cycle differences. A choice of weights avoiding exact cancellation yields unequal values. Thus endpoint data is insufficient for that circuit-sensitive representation. $\square$

The proposition justifies retaining path history when drive-like repetition is under investigation. It supplies no criterion for deciding which cycles are clinically significant.

9.8 Incompletion and closure constraints

This subsection translates $S(\bar{A})$ into a formal obligation. A model with a global semantic decoder

$$D_{\text{tot}} : Y_{\text{st}} \longrightarrow X_L \quad \text{such that} \quad D_{\text{tot}} \circ \pi_{\text{st}} = \text{id}_{X_L} \quad (9.23)$$

would make statement observations sufficient to recover the full latent state. Such a decoder conflicts with the intended incompletion and division. The programme therefore imposes a nonclosure condition: for the chosen statement probe, there exist $x \neq x'$ with

$$\pi_{\text{st}}(x) = \pi_{\text{st}}(x'). \quad (9.24)$$

The condition is weak and common in latent-variable models. Its Lacanian relevance depends on how the equivalence is generated. Measurement noise alone would satisfy Equation (9.24) and carry little psychoanalytic meaning. A stronger version should derive nonidentifiability from the model's representational structure, such as the impossibility of one signifier representing the subject without relation to another signifier.

Open Problem 9.7 (Structural nonclosure). *Define a categorical, logical, or information-theoretic condition in which the representation of a subject within one signifying relation necessarily produces residual underdetermination across another. Compare this condition with ordinary statistical latent-variable nonidentifiability.*

Table 6: Relational objects and proposed target semantics

Object	Mathematical role	Primary target reading	Alternative reading
V_R	vertices	relational events or distinguishable positions	articulated states only
E_R	oriented or typed edges	admissible continuation or mediation	observed association
F_R	faces or higher cells	composed relational history and consistency	higher-order interaction
∂	boundary operator	incidence and composition	empirical interface
Λ_R	label assignment	relational, symbolic, or contextual degrees of freedom	measurement annotations
W_R	history weight	compatibility, propensity, or formal amplitude	unnormalized score
B	retained boundary	observed, intervention, or comparison interface	selected coarse scale

9.9 Scope of the reconstruction

This subsection consolidates the results and losses. The typed multigraph preserves heterogeneity and direction; path operators preserve route dependence; the retrospective kernel represents completion-sensitive meaning; Schur reduction demonstrates latent mediation in a positive surrogate; fantasy-conditioned fields represent stable relational filtering; recurrence functionals distinguish circuits from endpoints; and nonclosure blocks a total decoder.

The reconstruction omits clinical technique, transference, bodily zones of drive, sexuation, the later topology of knots, and the full status of jouissance. It also leaves open whether numerical parameters can be measured with adequate reliability. These absences define the boundary of the current formal claim. The construction should be treated as a family of disciplined questions made calculable, rather than as a mathematical capture of Lacanian psychoanalysis.

10 Relational-Field Translation and Complex Generation

This section constructs the bridge from the typed reconstruction of Lacan’s diagram to a generative relational-complex model. Its role is to separate semantic constraint, history generation, weighting, and observation before these operations enter the finite effective-geometry calculation. The construction proceeds through a typed relational substrate, a partial realization relation, fixed- and varying-complex ensembles, and a group-field-like generator. Each stage records the information that it preserves and the information that remains outside its scope.

10.1 Typed relational substrate

This subsection defines the microscopic target-domain objects. It first introduces a typed relational complex, then adds transformations and labels, and finally states two semantic alternatives whose empirical consequences require comparison. Table 6 keeps the mathematical role of each object separate from its proposed target reading.

Definition 10.1 (Typed relational complex). *Let*

$$\mathcal{C}_R = (V_R, E_R, F_R, \partial) \tag{10.1}$$

be a finite two-dimensional cell complex or another explicitly declared incidence structure. A semantic map

$$\Sigma_R : V_R \sqcup E_R \sqcup F_R \longrightarrow S_V \sqcup S_E \sqcup S_F \quad (10.2)$$

assigns target types compatible with incidence and boundary relations.

Incidence supplies adjacency and composition. A metric, topology beyond the declared complex, psychological mechanism, or psychoanalytic meaning enters through additional structure. This restriction prevents the word *relational* from serving as a complete semantics.

Definition 10.2 (Transformation-compatible labeling). *Let G_R be a declared group, groupoid, or category of admissible transformations acting on a label space $\mathbb{L}_R$. A label assignment belongs to*

$$\Lambda_R \in \text{Adm}_{G_R}(\mathcal{C}_R, \mathbb{L}_R; B). \quad (10.3)$$

One useful decomposition is

$$\Lambda_R = \{\lambda_f, \kappa_e, \xi_v\}_{f \in F_R, e \in E_R, v \in V_R}, \quad (10.4)$$

where each label family has its own codomain and transformation law.

The transformation structure is substantive. It specifies which changes preserve the modeled object, which changes move it to another state, and which label differences can appear in observation. A future empirical version could use groupoids when admissible transformations vary across local contexts.

Assumption 10.3 (Minimal semantic discipline). *Every primitive has a declared codomain, observation relation, intervention relation, and failure test. Two semantic assignments that produce the same observable and interventional consequences remain underidentified by those probes.*

Two semantic branches deserve comparison. An *event-history branch* lets vertices denote events, edges denote admissible continuations, and faces denote composed histories. An *articulation branch* lets vertices denote observed statements or acts and treats hidden mediation through latent labels and cells. The first offers a richer generative picture. The second is closer to available data. Their adequacy depends on different evidence.

10.2 Translation architecture and preservation profile

This subsection defines the formal relation between the Lacanian source diagram and relational histories. Its objective is to prevent a direct identification of mathemes with vertices, psychological concepts with field variables, or directed signifying paths with symmetric conductances. The construction uses a set-valued realization relation because one structural role can admit several relational realizations and one relational history can participate in several interpretations.

Let Tok_L denote the collection of selected typed vertices, directed edges, paths, layer crossings, return relations, circuits, and punctuation states constructed from $\mathcal{G}_L$ in Section 9. For a declared boundary B , let

$$\text{Hist}_R(B) = \{(\mathcal{C}_R, \Lambda_R; B) : \Lambda_R \in \text{Adm}_{G_R}(\mathcal{C}_R, \mathbb{L}_R; B)\} \quad (10.5)$$

be the class of admissibly labeled relational complexes prior to weighting.

Definition 10.4 (Partial relational realization). *A partial relational realization of the typed Lacanian reconstruction is a relation*

$$\mathfrak{T}_R \subseteq D_L \times \text{Hist}_R(B), \quad D_L \subseteq \text{Tok}_L, \quad (10.6)$$

Table 7: Translation ledger from the typed desire graph to relational histories

Source structure	Preservation obligation	Candidate relational realization	Principal loss risk
Directed signifying path	Retain source, target, order, and edge type	Oriented edge sequence with typed composition	Symmetric projection erases address and order
Statement–enunciation layering	Retain distinct observation maps and cross-layer mediation	Typed strata with latent variables and declared projections	A single state vector collapses heterogeneous registers
Retroactive punctuation	Separate event time from evaluation time	Completion-sensitive kernel or posterior retyping of an earlier trace	Ordinary recurrence can be mistaken for semantic retroaction
Fantasy relation	Condition trajectories through a stable relational frame	History- and subject-indexed edge or path reweighting	A scalar parameter reifies an interpretive construct
Drive circuit	Preserve route, return, and repetition beyond endpoints	Path functional on typed cycles	Endpoint geometry deletes circuit structure
Symbolic nonclosure	Preserve residual underdetermination under observation	Noninjective probe grounded in representational structure	Measurement noise supplies only trivial nonidentifiability

whose fibre

$$\mathfrak{T}_R(q) = \{h \in \text{Hist}_R(B) : (q, h) \in \mathfrak{T}_R\} \quad (10.7)$$

contains the relational histories admitted as realizations of the selected source structure q . Every admitted pair carries an explicit preservation profile and loss record.

The domain D_L is declared because the paper formalizes selected dependencies within the desire graph. The relation is set-valued because a formal role such as return, mediation, or circuit persistence underdetermines its microscopic realization. It also permits overlap between fibres: one history can exhibit several source structures without reducing them to one symbol.

For each admitted pair (q, h) , define the preservation profile

$$\Pi(q, h) = (P_{\text{dir}}, \Pi_{\text{lay}}, \Pi_{\text{ret}}, \Pi_{\text{circ}}, \Pi_{\text{nc}})(q, h), \quad (10.8)$$

where the components record the status of direction, layer distinction, retrospective dependence, circuit sensitivity, and representational nonclosure. Each component takes one of four methodological values: preserved, approximated, lost, or open. These values serve as audit labels, with a role distinct from numerical observables and truth values for psychoanalytic propositions.

Table 7 functions as a transfer certificate. A later calculation may use a lossy projection, provided that the lost component is stated and the conclusion is restricted accordingly. The finite resistance sector, for example, preserves mediated connectivity and boundary response while losing direction, phase, cell incidence, and most semantic typing.

The architecture has three distinct channels:

$$\mathcal{G}_L \xrightarrow{\mathfrak{T}_R} \text{Adm}_L(\text{Hist}_R(B)), \quad \text{semantic-constraint channel}, \quad (10.9)$$

$$S_R[\varphi_R] \xrightarrow{\text{Exp}} \{W_R(h) : h \in \text{Hist}_R(B)\}, \quad \text{generative-weight channel}, \quad (10.10)$$

$$h \xrightarrow{P_G} G_h \xrightarrow{K_B} \mathcal{S}_{D,h}^{(B)}, \quad \text{effective-reconstruction channel}. \quad (10.11)$$

Here Adm_L denotes histories satisfying the declared subset of Lacanian-inspired structural constraints. Compatibility among interpretation, generation, and reconstruction remains a separate model obligation for Equations (10.9)–(10.11).

Assumption 10.5 (Separated semantic and generative roles). *The realization relation $\mathfrak{T}_R$ constrains which target histories can express a selected source structure. The action S_R and its kernels determine*

how histories receive formal weight. Semantic interpretation belongs to the constraint channel; dynamical weighting belongs to the generative channel. Their coordination requires an explicit realization argument.

This separation gives the proposed transfer a testable failure mode. If the dynamics systematically generates histories that violate the declared preservation profile, the chosen action fails to realize the intended model even when its mathematics is consistent. If many incompatible semantic assignments fit the same generated and observed histories, the interpretation remains underidentified.

10.3 History weights and fixed-complex ensembles

This subsection introduces weights on a fixed relational complex. Its objective is to separate an unnormalized compatibility, a physical-style amplitude, and a probability distribution, since these codomains support different operations and interpretations. The varying-complex and field-generated constructions follow only after this fixed-complex level is explicit.

Definition 10.6 (Relational-history model). *A relational-history model is*

$$\mathfrak{R} = (\mathcal{C}_R, \Lambda_R, W_R; B), \quad (10.12)$$

where

$$W_R : (\mathcal{C}_R, \Lambda_R; B) \longrightarrow \mathbb{K} \quad (10.13)$$

has a declared codomain $\mathbb{K}$. The fixed-complex generating quantity is

$$Z_R(\mathcal{C}_R \mid B) = \sum_{\Lambda_R \in \text{Adm}_{G_R}(\mathcal{C}_R; B)} W_R(\mathcal{C}_R, \Lambda_R; B). \quad (10.14)$$

If $\mathbb{K} = \mathbb{R}_{\geq 0}$, the sum exists, and $0 < Z_R < \infty$, normalization gives

$$p_R(\Lambda_R \mid \mathcal{C}_R, B) = \frac{W_R(\mathcal{C}_R, \Lambda_R; B)}{Z_R(\mathcal{C}_R \mid B)}. \quad (10.15)$$

Complex or signed weights require an observation rule that handles phase or cancellation. Division by Z_R alone supplies a normalized complex weight; probabilistic interpretation requires further structure.

10.4 Varying-complex ensembles and compositional locality

This subsection allows relational organization itself to vary. Its role is to distinguish uncertainty over labels on one carrier from an ensemble whose incidence structure, cell composition, and boundary realization can change. The construction first states the varying-complex sum and then records a conditional local factorization that can support complex generation.

Definition 10.7 (Varying-complex ensemble). *For a boundary condition B , let $\mathfrak{C}_B$ be a family of compatible complexes and μ_C a declared measure or counting weight. Define*

$$\mathfrak{Z}_R(B) = \sum_{\mathcal{C}_R \in \mathfrak{C}_B} \mu_C(\mathcal{C}_R) \sum_{\Lambda_R \in \text{Adm}_{G_R}(\mathcal{C}_R; B)} W_R(\mathcal{C}_R, \Lambda_R; B). \quad (10.16)$$

In the finite nonnegative branch, this expression defines a joint distribution after normalization.

The varying-complex ensemble expresses a central claim of the research programme: relational organization itself can vary. Symmetry factors, equivalence under relabeling, boundary composition, convergence, and the empirical interpretation of complex variation remain explicit obligations.

Assumption 10.8 (Compositional locality ansatz). *A spin-foam-like fixed-complex factorization has the form*

$$W_R(\mathcal{C}_R, \Lambda_R; B) = \prod_{f \in F_R} A_f(\lambda_f) \prod_{e \in E_R} A_e(\kappa_e, \lambda_{f \supset e}) \prod_{v \in V_R} A_v(\xi_v, \kappa_{e \supset v}, \lambda_{f \supset v}; B). \quad (10.17)$$

Each factor has a declared target meaning and compatibility domain.

Equation (10.17) states a conditional locality structure. It is useful when local composition supports global histories. Long-range dependence, global constraints, and context-dependent factorization can require nonlocal factors or additional variables.

10.5 Group-field generator and interaction structure

This subsection specifies the field-theoretic level in enough detail to expose its formal obligations. Its objective is to identify which parts of a GFT architecture perform generative work: the field domain supplies boundary data for elementary building blocks, the kinetic kernel relates or propagates those data, interaction kernels prescribe gluing patterns, and the perturbative expansion generates weighted complexes. These roles derive from the GFT source architecture and receive new target semantics only through the realization relation in Equation (10.6) (Freidel, 2005; Oriti, 2016).

Definition 10.9 (Group-field-like generator). *Let*

$$\varphi_R : G_R^d \times \Xi_R \longrightarrow \mathbb{C} \quad (10.18)$$

be a target-domain relational field with the optional diagonal invariance

$$\varphi_R(g_1 h, \dots, g_d h; \xi) = \varphi_R(g_1, \dots, g_d; \xi), \quad h \in G_R. \quad (10.19)$$

Write $1 = (\mathbf{g}_1, \xi_1)$, with $\mathbf{g}_1 \in G_R^d$ and $\xi_1 \in \Xi_R$, and let $d\mu(1)$ be a declared product measure. For a family $\mathfrak{B}$ of interaction types, define the schematic action

$$\begin{aligned} S_R[\varphi_R, \overline{\varphi_R}] &= \int d\mu(1) d\mu(2) \overline{\varphi_R(1)} \mathcal{K}_R(1, 2) \varphi_R(2) \\ &+ \sum_{\beta \in \mathfrak{B}} \frac{\lambda_\beta}{\text{sym}(\beta)} \int \prod_{a=1}^{n_\beta} d\mu(a) \mathcal{V}_\beta(1, \dots, n_\beta) \prod_{a \in \beta_+} \varphi_R(a) \prod_{b \in \beta_-} \overline{\varphi_R(b)}. \end{aligned} \quad (10.20)$$

The partition functional is

$$\mathcal{Z}_R = \int \mathcal{D}\varphi_R \mathcal{D}\overline{\varphi_R} e^{-S_R[\varphi_R, \overline{\varphi_R}]}. \quad (10.21)$$

Here $\mathcal{K}_R$ is the kinetic kernel, $\mathcal{V}_\beta$ is the interaction kernel for gluing type β , n_β is its valence, and β_+ and β_- specify the occurrences of the field and its conjugate.

The desired perturbative organization is

$$\mathcal{Z}_R \sim \sum_{\Gamma \in \mathfrak{G}_R} \frac{\prod_{\beta \in \mathfrak{B}} \lambda_\beta^{N_\beta(\Gamma)}}{\text{sym}(\Gamma)} \mathcal{A}_R(\Gamma), \quad (10.22)$$

where $\mathfrak{G}_R$ is the generated family of Feynman-like complexes, $N_\beta(\Gamma)$ counts vertices of interaction type β , and $\mathcal{A}_R(\Gamma)$ is assembled from propagators and vertex kernels. This expression remains formal until the measure, regularization, kernels, interaction combinatorics, convergence regime, and target semantics have been fixed. Its distinctive role is generative: interactions build complexes, while Equation (10.14) evaluates labels on one complex.

The coupling constants λ_β regulate the relative contribution of interaction types within the formal expansion. A target interpretation might associate one interaction type with a class of address, response, identification, prohibition, or institutional mediation. Such an association enters through $\mathfrak{T}_R$ and requires independent evidence. Combinatorics determines generated structure; the declared realization relation and supporting evidence determine target meaning.

The kinetic and interaction terms also bear different conceptual burdens. The inverse of a suitable kinetic operator supplies a propagator and thereby constrains continuation between elementary relational data. An interaction kernel specifies how several elementary data sets compose at a generated event. Their combination can produce recurrent, branching, or layered complexes. A model of desire would need to show how its selected kernels implement the preservation obligations in Table 7, especially directed address, cross-layer mediation, circuit persistence, and nonclosure.

Assumption 10.10 (Perturbative realization discipline). *For every generated complex Γ admitted as a target history, there is a declared extraction*

$$E_R : \Gamma \longmapsto (\mathcal{C}_R, \Lambda_R; B) \in \text{Hist}_R(B) \quad (10.23)$$

and a recorded value of $\Pi(q, E_R\Gamma)$ for each source structure q claimed to be realized. Generated complexes outside the admitted semantic domain remain formal histories without the target interpretation.

Equation (10.23) makes the transfer auditable. It also separates two forms of multiplicity. The perturbative expansion can generate many complexes with different combinatorics and weights. The realization relation can assign several of those complexes to the same source structure. Their conjunction yields a model of distributed realization with a many-to-many relation between source structures and target histories.

10.6 Positive-sector interface and information loss

This subsection defines the interface between the complex-generating architecture and the exact finite calculations. Its role is to state the additional restrictions required before a relational history can induce a positive weighted graph. The interface is a projection with a nontrivial fibre, so the mathematical results at the next level apply to equivalence classes of histories under that projection.

Let $\text{Hist}_R^+(B) \subseteq \text{Hist}_R(B)$ contain histories for which a declared projection produces finite, connected, undirected, positive weighted graphs. Define

$$P_G : \text{Hist}_R^+(B) \longrightarrow \text{Graph}_+, \quad h \longmapsto G_h = (V_h, E_h, c_h). \quad (10.24)$$

For each candidate edge $\{i, j\}$, its conductance is determined by a declared extraction rule

$$c_{ij}(h) = \psi_{ij}(\mathcal{C}_R, \Lambda_R, W_R; B) > 0. \quad (10.25)$$

The function ψ_{ij} can summarize frequency, transition support, compatibility, or another target quantity. The chosen interpretation fixes the construct being measured. Positivity, symmetry, and connectedness enter as additional sector assumptions at this projection stage.

The projection induces the equivalence relation

$$h \sim_{P_G} \tilde{h} \iff P_G(h) = P_G(\tilde{h}). \quad (10.26)$$

Distinct orientations, phases, higher-cell structures, label assignments, or semantic histories can therefore share one positive graph. Any conclusion derived solely from G_h is invariant across the fibre $P_G^{-1}(G_h)$. A claim that distinguishes members of this fibre requires a richer observable or a return to the relational-complex level.

The finite derivations below use the nonnegative graph branch. They therefore establish graph-level results without depending on the functional integral in Equation (10.21). A future paper can connect the two levels by deriving a positive graph ensemble or an effective action from a specified relational field model.

11 Effective Geometry, Scale, and Historical Differentiation

This section develops the exact finite sector as one continuous derivation. Its role is to show how a positive relational graph induces a boundary geometry, how the result depends on reduction and aggregation order, and how currently equivalent histories can support different future trajectories after a shared update. The discussion proceeds from the graph projection through variational elimination, mediated adjacency, nested scales, ensemble order, observational equivalence, and historical differentiation.

11.1 Positive graph sector and boundary data

This subsection specifies the finite domain on which the exact results hold. It restates the output of Equation (10.24) as a weighted Laplacian and declares the semantic interpretation of conductance and resistance. The resulting sector is narrower than the relational-field model because it retains positive symmetric transmission structure.

Definition 11.1 (Positive graph sector). *For an admitted history h , define*

$$P_G(h) = G_h = (V_h, E_h, c_h), \quad (11.1)$$

where G_h is finite, connected, and undirected, and $c_h : E_h \rightarrow \mathbb{R}_{>0}$. Its weighted Laplacian is

$$(L_h)_{ij} = \begin{cases} \sum_{k: \{i,k\} \in E_h} c_{ik}, & i = j, \\ -c_{ij}, & \{i, j\} \in E_h, \\ 0, & \text{otherwise.} \end{cases} \quad (11.2)$$

Assumption 11.2 (Linearized target semantics). *The conductance c_{ij} represents a local linearized capacity for relational transmission, compatibility, or transition support. Effective resistance represents aggregate obstruction across all available paths. This semantic assignment defines the toy target model and remains subject to construct validation.*

Let $B \subset V_h$ be a nonempty retained set and $I = V_h \setminus B$ the internal set. Order the vertices as (B, I) :

$$L_h = \begin{pmatrix} L_{BB} & L_{BI} \\ L_{IB} & L_{II} \end{pmatrix}. \quad (11.3)$$

Connectedness and nonempty B make the grounded block L_{II} positive definite. The inverse in the next definition therefore exists.

11.2 Boundary reduction and variational characterization

This subsection derives the effective boundary operator from an energy-minimization problem. Its objective is to show that the Schur complement is the unique quadratic response obtained when internal variables equilibrate relative to fixed boundary values. The derivation also states precisely which internal information the reduced operator removes.

Definition 11.3 (Kron effective geometry). *The reduced Laplacian on B is*

$$L_h^K = L_{BB} - L_{BI}L_{II}^{-1}L_{IB}. \quad (11.4)$$

For $i, j \in B$, define effective-resistance distance

$$R_{ij}^h = (e_i - e_j)^\top L_h^+(e_i - e_j), \quad (11.5)$$

where L_h^+ is the Moore–Penrose pseudoinverse. The finite effective geometry is

$$\mathcal{S}_{D,h}^{(B)} = (B, R^h|_{B \times B}, L_h^K). \quad (11.6)$$

Proposition 11.4 (Boundary-energy equivalence). *For every fixed boundary vector x_B , define the graph energy*

$$\mathcal{E}_h(x_B, x_I) = \frac{1}{2} \begin{pmatrix} x_B \\ x_I \end{pmatrix}^\top L_h \begin{pmatrix} x_B \\ x_I \end{pmatrix}. \quad (11.7)$$

Under the assumptions of the positive graph sector,

$$\min_{x_I} \mathcal{E}_h(x_B, x_I) = \frac{1}{2} x_B^\top L_h^K x_B, \quad x_I^* = -L_{II}^{-1} L_{IB} x_B. \quad (11.8)$$

Proof. Expanding Equation (11.7) gives

$$\mathcal{E}_h = \frac{1}{2} x_B^\top L_{BB} x_B + x_B^\top L_{BI} x_I + \frac{1}{2} x_I^\top L_{II} x_I. \quad (11.9)$$

The positive definiteness of L_{II} makes the expression strictly convex in x_I . Its stationarity condition is

$$\nabla_{x_I} \mathcal{E}_h = L_{IB} x_B + L_{II} x_I = 0, \quad (11.10)$$

which has the unique solution $x_I^* = -L_{II}^{-1} L_{IB} x_B$. Substitution into Equation (11.9) yields

$$\mathcal{E}_h(x_B, x_I^*) = \frac{1}{2} x_B^\top (L_{BB} - L_{BI} L_{II}^{-1} L_{IB}) x_B. \quad (11.11)$$

The operator in parentheses is L_h^K by Equation (11.4). $\square$

Proposition 11.4 provides an operational interpretation. The reduced graph reproduces the minimum energy or boundary response after internal equilibration. It preserves the declared boundary quadratic form and effective resistances. Internal values, path direction, phase, and semantic labels remain in the eliminated fibre. The proposition therefore establishes effective equivalence relative to a specified probe and leaves microscopic identity unresolved.

11.3 Mediated adjacency calculation

This subsection evaluates the general reduction on the smallest path that can generate a new boundary adjacency. Its role is to display every elimination step and to distinguish an emergent effective edge from a microscopic edge. The calculation also supplies the common finite example used later for ensemble order and historical differentiation.

Effective resistance is a metric on a connected positive weighted graph (Klein and Randić, 1993; Devriendt, 2022). Kron reduction yields another Laplacian and preserves effective resistances among retained nodes under the stated assumptions (Dörfler and Bullo, 2013). The retained set B plays the role of scale: different retained sets can yield different effective adjacencies while preserving the boundary responses declared for each reduction.

Proposition 11.5 (Mediated effective adjacency). *Let $G(a, b)$ be the path $A - X - C$, with conductance $a > 0$ on $A - X$, conductance $b > 0$ on $X - C$, retained set $B = \{A, C\}$, and internal set $I = \{X\}$. Eliminating X produces an effective $A - C$ edge with conductance*

$$c_{AC}^K = \frac{ab}{a+b} \quad (11.12)$$

and resistance

$$R_{AC} = \frac{1}{a} + \frac{1}{b}. \quad (11.13)$$

Proof. In the order $(A, C; X)$, the blocks are

$$L_{BB} = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}, \quad L_{BI} = \begin{pmatrix} -a \\ -b \end{pmatrix}, \quad L_{II} = a + b. \quad (11.14)$$

Internal flow balance gives

$$L_{IB}x_B + L_{II}x_I = 0, \quad x_I = -L_{II}^{-1}L_{IB}x_B. \quad (11.15)$$

Substitution into the boundary flow equation produces Equation (11.4). Direct multiplication yields

$$L^K(a, b) = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} - \frac{1}{a+b} \begin{pmatrix} a^2 & ab \\ ab & b^2 \end{pmatrix} = \frac{ab}{a+b} \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}. \quad (11.16)$$

A two-node Laplacian with edge conductance c has the final matrix form in Equation (11.16). Its resistance is $1/c$, which gives Equation (11.13). $\square$

The endpoint edge is absent from the microscopic graph and present in the reduced graph. This is a precise finite meaning of emergent adjacency. The edge summarizes mediation through X ; the reduction preserves relevant endpoint response while discarding the internal state. The result establishes a finite metric space. It leaves dimension, differentiability, causal signature, and continuum approximation open.

Remark 11.6 (Directionality boundary). Effective resistance is symmetric. Desire, address, recognition, and influence can be directional. A directed Laplacian, asymmetric cost, controlled reachability relation, or nonlinear response model would therefore be required when directionality carries explanatory content.

Table 8: Dimensionless normalized witness for reduction order and shared-update separation

History	first edge	second edge	c_{AC}^K	R_{AC}	$c'_{AC}(\delta = 1)$	$c' \geq 1$
h_1	1	3	3/4	4/3	6/5	yes
h_2	3	1	3/4	4/3	4/5	below threshold
Microscopic mean	2	2	1	1	6/5	yes

11.4 Aggregation and reduction order

This subsection examines the relation between an ensemble of histories and the coarse-graining map. Its objective is to distinguish the expected geometry of individually reduced histories from the geometry obtained after averaging microscopic parameters. An exact parameter-family result establishes order sensitivity and supplies a methodological constraint on any reported average geometry.

Let K denote the Kron map from the three-node Laplacian to its retained two-node Laplacian. For $a, b > 0$ with $a \neq b$, consider the equal-probability mirrored histories

$$h_1 = G(a, b), \quad h_2 = G(b, a). \quad (11.17)$$

Proposition 11.7 (Aggregation–reduction order sensitivity). *For the equal-probability ensemble in Equation (11.17),*

$$\mathbb{E}[K(L_h)] \neq K(\mathbb{E}[L_h]). \quad (11.18)$$

More precisely, the difference between the conductance obtained by microscopic averaging before reduction and the expected history-wise conductance is

$$\frac{a+b}{4} - \frac{ab}{a+b} = \frac{(a-b)^2}{4(a+b)} > 0. \quad (11.19)$$

Proof. Proposition 11.5 gives

$$c^K(a, b) = c^K(b, a) = \frac{ab}{a+b}. \quad (11.20)$$

The history-wise reduction therefore has expected conductance $ab/(a+b)$. The mean microscopic conductances are

$$(\bar{a}, \bar{b}) = \frac{1}{2}(a, b) + \frac{1}{2}(b, a) = \left(\frac{a+b}{2}, \frac{a+b}{2} \right). \quad (11.21)$$

Writing $m = (a+b)/2$, Proposition 11.5 gives $c^K(m, m) = m/2 = (a+b)/4$. Subtraction yields Equation (11.19). $\square$

The difference arises from the nonlinearity of $ab/(a+b)$. It vanishes exactly when $a = b$, which is also the case in which the mirrored histories coincide at the conductance level. An empirical model that reports an average relational geometry must therefore specify its order of aggregation and reduction.

Table 8 gives one dimensionless arithmetic witness by selecting $a = 1$, $b = 3$, and later $\delta = 1$, $\theta = 1$. These values are normalized model inputs chosen for transparent calculation. Their status is empirically uncalibrated and independent of a psychoanalytic scale.

Remark 11.8 (Status of the numerical witness). The rational values in Table 8 verify the symbolic derivations at one convenient parameter choice. Their algebraic exactness remains confined to the toy model. Empirical precision and a defensible unit for relation or subjective possibility require a validated conductance extraction ψ_{ij} , measurement units, an estimation procedure, and uncertainty analysis.

11.5 Nested scale structure and consistency

This subsection separates two multiscale properties that the preceding counterexample can otherwise obscure. Sequential elimination across nested boundary sets is consistent in the positive finite sector, while ensemble expectation can remain order-sensitive because the Schur complement is nonlinear. The distinction provides an exact baseline for later approximate coarse-graining.

Let $C \subseteq B \subseteq V_h$ be nonempty retained sets. Write $K_B(L_h)$ for elimination of $V_h \setminus B$, and write $K_C(K_B(L_h))$ for subsequent elimination of $B \setminus C$.

Proposition 11.9 (Nested boundary consistency). *For a connected positive weighted graph and nonempty nested retained sets $C \subseteq B \subseteq V_h$,*

$$K_C(K_B(L_h)) = K_C(L_h). \quad (11.22)$$

The equality concerns the reduced boundary operator on C .

Proof. Let $\mathcal{E}_h(x_C, x_{B \setminus C}, x_{V_h \setminus B})$ be the Dirichlet energy from Equation (11.7). Proposition 11.4 identifies Kron reduction with minimization over eliminated variables. Sequential reduction therefore gives

$$\begin{aligned} \frac{1}{2} x_C^\top K_C(K_B(L_h)) x_C &= \min_{x_{B \setminus C}} \min_{x_{V_h \setminus B}} \mathcal{E}_h(x_C, x_{B \setminus C}, x_{V_h \setminus B}) \\ &= \min_{x_{V_h \setminus C}} \mathcal{E}_h(x_C, x_{V_h \setminus C}) \\ &= \frac{1}{2} x_C^\top K_C(L_h) x_C. \end{aligned} \quad (11.23)$$

The equality holds for every x_C , so the two symmetric boundary operators coincide. $\square$

Proposition 11.9 shows that compatible finite elimination has a coherent scale order. The result relies on the same positive connected graph and equilibrium-response assumptions as Proposition 11.4. Changing the projection P_G , updating weights between reductions, or comparing different semantic boundary choices defines another operation and requires a separate consistency analysis.

A retained-node geometry represents responses after a declared elimination. Altering B , averaging microscopic parameters, or averaging already reconstructed distances therefore defines different procedures. Agreement among them requires a theorem or approximation bound.

Open Problem 11.10 (Controlled multiscale consistency). *Find conditions on a nested retained family $B_0 \supset B_1 \supset \dots$, an ensemble $p(h)$, and reductions K_{B_r} such that sequential and direct reductions agree, while ensemble aggregation has a quantified commutation error:*

$$\|\mathbb{E}[K_{B_r}(L_h)] - K_{B_r}(\mathbb{E}[L_h])\| \leq \varepsilon_r. \quad (11.24)$$

Nested reduction is exact under Proposition 11.9. The ensemble-order problem remains because expectation interacts with a nonlinear Schur complement. Larger models should report both issues separately.

11.6 Observation-equivalent histories

This subsection turns the static effective geometry into a minimal identification problem. Its objective is to formalize equality under a current probe while preserving possible differences inside the projection fibre. It first defines general observational equivalence and then specializes the definition to the two path histories used above.

Definition 11.11 (Probe-relative history equivalence). *Let $\mathcal{O} : \text{Hist}_R^+(B) \rightarrow Y$ be a declared observation map. Define*

$$h \sim_{\mathcal{O}} \tilde{h} \iff \mathcal{O}(h) = \mathcal{O}(\tilde{h}). \quad (11.25)$$

The equivalence class $[h]_{\mathcal{O}}$ contains histories unresolved by the current probe.

The relation in Equation (11.25) records observational resolution and limits the asserted equality to the observed output. Histories within one class may retain different labels, paths, cell structures, and responses to later events. A richer probe can refine the partition, while a coarser probe can merge previously distinct classes.

Definition 11.12 (Endpoint observation). *For the history family $G(a, b)$, let the current probe observe only*

$$\mathcal{O}(h) = R_{AC}(h). \quad (11.26)$$

Histories h and $\tilde{h}$ are equivalent for this probe when $\mathcal{O}(h) = \mathcal{O}(\tilde{h})$.

The two histories in Equation (11.17) satisfy

$$\mathcal{O}(h_1) = \mathcal{O}(h_2) = \frac{1}{a} + \frac{1}{b}. \quad (11.27)$$

Their endpoint observation and current reduced Laplacian agree. Their internal allocations of conductance differ.

11.7 Shared updates and future reachability

This subsection introduces one controlled perturbation and follows its consequences through the reduced geometry. Its role is to establish an intervention-relative distinction between histories that the current endpoint probe identifies. The same named local update is applied to each history, after which threshold reachability supplies a minimal prospective observable.

For an update U and a metric d_Y on the observation space, define the response separation

$$\Delta_{\mathcal{O}, U}(h, \tilde{h}) = d_Y(\mathcal{O}(Uh), \mathcal{O}(U\tilde{h})). \quad (11.28)$$

Two currently equivalent histories are interventionally separated by U when $h \sim_{\mathcal{O}} \tilde{h}$ and $\Delta_{\mathcal{O}, U}(h, \tilde{h}) > 0$. This definition converts hidden-history difference into a comparison of responses under a declared operation.

Definition 11.13 (Shared local update). *For $\delta > 0$, define*

$$U_{\delta}(a, b) = (a + \delta, b). \quad (11.29)$$

The update strengthens the same named relation $A - X$ in every admitted history.

The update is a deliberately minimal model of a new relational event. Its substantive meaning could be repeated address, recognition, learning, trust, practice, or another process that increases one local transition capacity. An application must fix this meaning before estimation.

Definition 11.14 (Threshold reachability). *For $\theta > 0$, define the one-step endpoint reachable set*

$$\mathcal{K}_{\theta}(h) = \begin{cases} \{A, C\}, & c_{AC}^K(h) \geq \theta, \\ \{A\}, & c_{AC}^K(h) < \theta. \end{cases} \quad (11.30)$$

The threshold represents a declared minimum effective support for the prospective transition from A to C .

Proposition 11.15 (Hidden-history feedback separation). *Let $h_1 = G(a, b)$, $h_2 = G(b, a)$, $0 < a < b$, and $\delta > 0$. The histories are equivalent under the current endpoint observation in Equation (11.26). After the common update U_δ , their effective conductances satisfy*

$$c'_1(\delta) - c'_2(\delta) = \frac{\delta(b-a)}{a+b+\delta} > 0. \quad (11.31)$$

For every threshold satisfying

$$\frac{a(b+\delta)}{a+b+\delta} < \theta \leq \frac{b(a+\delta)}{a+b+\delta}, \quad (11.32)$$

their future reachable sets differ:

$$\mathcal{K}_\theta(U_\delta h_1) = \{A, C\}, \quad \mathcal{K}_\theta(U_\delta h_2) = \{A\}. \quad (11.33)$$

Proof. Current equivalence follows from Equation (11.27). After the update,

$$c'_1(\delta) = \frac{b(a+\delta)}{a+b+\delta}, \quad c'_2(\delta) = \frac{a(b+\delta)}{a+b+\delta}. \quad (11.34)$$

Subtracting gives Equation (11.31). The positivity of the gap makes the interval in Equation (11.32) nonempty. Definition 11.14 then gives Equation (11.33). $\square$

Proposition 11.15 gives a finite formal content to the phrase *history shapes possibility*. Equal current endpoint geometry leaves hidden organization unresolved. A shared event couples to that organization and generates different later accessibility. The result depends on the update location and threshold. Those dependencies define a candidate test once measurement semantics, intervention design, and calibration have been supplied.

The values $a = 1$, $b = 3$, $\delta = 1$, and $\theta = 1$ in Table 8 instantiate the symbolic result: the updated conductances are $6/5$ and $4/5$. Their role is verification by example. The proof and the separation claim depend on the inequalities in Equations (11.31) and (11.32).

The proposition also separates current-state sufficiency from current-state description. The endpoint resistance is a sufficient description for the present boundary-response probe in this toy family. It is insufficient for predicting the response to an update that acts on a hidden component. A predictive state representation must therefore include either the relevant internal allocation or an uncertainty distribution over members of $[h]_\mathcal{O}$.

For a probability distribution $p(h \mid y)$ over histories compatible with current observation y , the prospective reachability probability is

$$\Pr(C \in \mathcal{K}_\theta(Uh) \mid y) = \sum_{h \in [h_y]_\mathcal{O}} \mathbf{1}\{C \in \mathcal{K}_\theta(Uh)\} p(h \mid y). \quad (11.35)$$

Equation (11.35) extends the two-history proof to uncertainty over hidden organization. Its empirical use requires a justified history model, observation likelihood, update semantics, and threshold. The equation provides an estimand only after those components have been specified.

11.8 Integrated feedback architecture and interpretive scope

This subsection consolidates the finite derivation and reconnects it to the larger relational-field programme. Its objective is to state what the calculation establishes, where information enters and leaves, and which psychoanalytic interpretations remain conditional. The resulting feedback cycle distinguishes reconstruction, transition, realized event, and historical update.

The full feedback architecture is

$$\mathfrak{R}_\tau \xrightarrow{\mathfrak{E}_\ell} \mathcal{S}_{D,\tau}^{(\ell)} \xrightarrow{\mathcal{D}_\ell} \gamma_{\tau:\tau+1} \xrightarrow{\mathfrak{U}} \mathfrak{R}_{\tau+1}. \quad (11.36)$$

Here $\mathfrak{E}_\ell$ reconstructs geometry, $\mathcal{D}_\ell$ specifies motion or transition, and $\mathfrak{U}$ incorporates realized events into later relational structure. In the toy model, $\mathfrak{E}$ is Kron reduction, $\mathcal{D}$ is threshold reachability, and $\mathfrak{U} = U_\delta$.

The continuous argument of this section can now be stated in five steps. First, a declared projection extracts a positive graph from a richer relational history. Second, internal equilibration induces a boundary operator and effective-resistance geometry. Third, nested reductions preserve the boundary energy under compatible elimination. Fourth, ensemble averaging and reduction can produce different geometries. Fifth, an update acting inside a current observation class can separate later reachable sets. Each step has its own assumptions, and only the final conjunction supports the restricted claim that history-dependent organization can shape prospective accessibility.

The Lacanian interpretation enters through the realization and preservation profile of Section 10.2. A model may read hidden mediation as a structural analogue of the distinction between statement and enunciation, path-sensitive recurrence as a limited representation of drive-like circuits, or history-conditioned weights as one aspect of fantasy. These readings remain conditional because the finite Laplacian sector loses direction and much of the graph's semantic heterogeneity. The exact graph results demonstrate a mechanism of mediated geometry and historical differentiation. They leave the psychoanalytic identity of that mechanism open to textual, clinical, and empirical assessment.

Remark 11.16 (Retroactive reconstruction). Later evidence may change the effective reconstruction assigned to an earlier history. This is a change in modeled interpretation, retained variables, or posterior description. A claim about alteration of physical past events would require a separate physical theory and evidence.

12 Psychoanalytic Interpretation of the Formal Architecture

This section returns from the mathematical constructions to Lacan's graph of desire. Its role is to evaluate the interpretive bridge at the level of each major concept, including signifier, divided subject, Other, demand, desire, fantasy, object a , drive, and symbolic incompleteness. The discussion proceeds through local correspondences, disanalogies, and formal obligations. The resulting interpretation remains a revisable philosophical proposal whose clinical use requires a further evidential programme.

12.1 Interpretive bridge and status sequence

This subsection establishes the sequence by which a mathematical result can acquire psychoanalytic relevance. The finite graph theorem concerns conductances, Schur complements, and effective resistance. The target model interprets these quantities as one surrogate for relational support and obstruction. A Lacanian interpretation then asks whether the target structure illuminates signifying mediation and historically organized possibility. Empirical or clinical claims form a later stage with their own observations, rival explanations, and ethical controls.

Lacan's graph of desire belongs to a theory of the divided speaking subject, signifying chains, the Other, demand, desire, and fantasy (Lacan, 2006). It is already a conceptual diagram with its own internal logic. The weighted networks developed in Section 11 are newly proposed mathematical objects. Any Lacanian meaning assigned to their vertices and edges requires the realization relation and preservation audit developed in Section 10.

Three correspondences motivate further work. First, the observation map $\mathcal{O}$ formalizes a distinction between observable articulation and the relational history that supports it. This is compatible with treating an enunciated statement as an incomplete disclosure of a divided subject. Second, the history-to-geometry map expresses the possibility that the Other and signifying mediation shape which subjective transitions become available. Third, the feedback map expresses how new articulations and encounters can reorganize later possibilities.

The term *desire* consequently enters through an interpretive projection

$$\Pi_L : (\mathcal{S}_{D,h}^{(\ell)}, \gamma, \mathcal{O}(h)) \dashrightarrow A_L, \quad (12.1)$$

where A_L is a space of carefully defined Lacanian descriptions. The dashed arrow records partiality, possible failure, and interpretive plurality.

Claim 12.1 (Interpretive compatibility). *The finite model is compatible with a minimal Lacanian proposition when relational mediation affects effective accessibility and observed articulation leaves that mediation partly unresolved. Compatibility supports a research question. Textual equivalence and clinical validation require separate evidence.*

Several source concepts remain outside the toy model. Effective resistance has symmetry and additivity properties generated by linear network flow. Lacanian desire involves lack, metonymy, fantasy, object a , demand, and relations among registers whose structure exceeds a scalar obstruction metric (Lacan, 2006; Johnston, 2026). The Real requires a treatment beyond noise, missing data, or an unobserved node. The Symbolic requires structure beyond arbitrary labels. An interpretation of the Imaginary through geometric embedding requires a separate argument.

The paper therefore uses the following interpretive hierarchy:

$$\begin{aligned} \text{graph theorem} &\implies \text{target-model mechanism} \\ &\implies \text{psychoanalytic interpretation} \implies \text{empirical or clinical claim.} \end{aligned} \quad (12.2)$$

Every arrow requires additional premises. Later stages can fail while earlier mathematical results remain valid.

12.2 Signifying chains and semantic time

This subsection relates the graph’s elementary crossing to the two-time construction in Section 9.4. Its objective is to identify a precise temporal structure that survives formal transfer while preserving the difference between physical causation and retrospective signification. The discussion moves from linguistic sequence to punctuation, historical rearticulation, and the limits of the kernel model.

Saussurean linguistics supplies an important background to Lacan’s emphasis on differential signifying relations, while Jakobson’s analysis of metaphor and metonymy supplies a more specific linguistic resource for substitution and displacement (de Saussure, 2011; Jakobson, 1971). Lacan reorganizes these resources within a psychoanalytic theory of the subject. In the elementary cell of the graph, a signifying sequence crosses a subjective vector twice. The second crossing gives the first crossing its articulated value. The graph thereby represents a sequence whose completed sense can reconfigure the function of an earlier element (Lacan, 2006, 2017; Fink, 2004).

The kernel in Equation (9.11) isolates this minimal structure. Event time orders the arrival of trace elements. Evaluation time indexes the current interpretation of those elements. The construction permits $m_{T_1}(t) \neq m_{T_2}(t)$ for $t < T_1 < T_2$, while the record of occurrence remains ordered. This distinction gives formal content to the proposition that a later signifier can alter what an earlier signifier has come to mean within a completed chain.

Punctuation carries an additional relational dimension. A sequence is spoken to an addressee, interpreted within conventions, and heard from a position in the symbolic order. A period, reply, silence, institutional ruling, or analytic intervention can function as a punctuation event because it changes the relevant unit and its attribution. The kernel therefore requires contextual indices in any developed version:

$$m_T^{u,o,c}(t) = m_0^{u,o,c}(t) + \int_t^T K_{\text{ret}}^{u,o,c}(t, s) \xi(s) ds, \quad (12.3)$$

where u identifies a subject-indexed record, o identifies the position of address, and c records the relevant linguistic and institutional context. These indices represent analytic distinctions. They carry no presumption that an empirical person, a symbolic position, and an institutional function coincide.

The contextual kernel also shows why a purely chronological archive is insufficient. Two utterance histories can contain identical word tokens and differ in speaker position, addressee, genre, timing, or consequences. Their semantic organization can therefore diverge. Conversely, different surface tokens can instantiate one recurring relational form. A mature observation model must represent token sequence, timing, embodied expression, addressee, and contextual uptake while permitting contestable interpretations of their relation.

The broader psychoanalytic notion of *après-coup* extends beyond sentence completion. The systematic review by Mezzalana et al. (2023) organizes the literature around retrospective constitution, trauma, clinical use, and meaning-making, and reports an absence of direct empirical tests within the reviewed body. That result recommends a staged programme. Linguistic punctuation can provide an experimentally tractable mechanism. Autobiographical reconstruction adds memory, affect, embodiment, and social narration. Clinical retroaction adds transference, intervention, and therapeutic context. A result at the first stage supplies a component for the later stages and leaves their distinctive burdens intact.

The kernel model also reaches a clear boundary. Linear superposition treats each later element as contributing through an additive operator. Signifying chains can exhibit threshold effects, reinterpretation of units, emergence of new categories, and changes in the kernel itself. Equation (9.19) represents one such interaction. A later punctuation can change which edges and events belong to the relevant pattern, so the operation transforms the state space used by the earlier interpretation. This stronger case requires variable carriers or typed rewriting systems in addition to linear kernels.

Claim 12.2 (Two-time semantic discipline). *A formal account of retroactive signification should distinguish event time, evaluation time, and contextual position. The distinction permits retrospective semantic constitution within a forward-causal implementation and exposes the further assumptions required for psychological or clinical interpretation.*

12.3 Subject division and observation structure

This subsection connects the barred subject to the manuscript’s observation maps. Its objective is to preserve a disciplined gap between subject-indexed data and the subject of psychoanalysis. The discussion separates ego report, action, enunciation, recurrent formation, and latent formal state.

Lacan’s account places the subject within signifying representation and treats this representation as divisive. The ego offers an image of coherence; the subject appears through effects that can interrupt this coherence. Specialist expositions differ in emphasis, yet they converge on the importance of distinguishing the ego from the subject and conscious self-knowledge from the effects of signification (Evans, 1996; Chiesa, 2007; Johnston, 2026). The model therefore treats a first-person statement as one observation channel among several.

Let the observation family for a subject-indexed record be

$$\Pi_u = \{\pi_{\text{report}}, \pi_{\text{action}}, \pi_{\text{timing}}, \pi_{\text{address}}, \pi_{\text{return}}, \pi_{\text{context}}\}. \quad (12.4)$$

Each projection retains a declared feature of the record. A report projection can encode propositional content and lexical choice. An action projection records conduct under specified conditions. A timing projection records pauses, order, and recurrence. An address projection records the position toward which speech is directed. A return projection records the recurrence of typed paths. A context projection records material and institutional conditions. None of these channels receives privileged access to desire solely through its name.

Division appears formally as structured discordance among projections across time. A report can endorse one project while action repeatedly approaches another position. A recurrent phrase can appear in different contexts and attract divergent later punctuation. An intervention can change the relation between report and action without changing either marginal distribution immediately. Such patterns motivate latent relational hypotheses. They provide neither a direct measurement of the unconscious nor a diagnostic conclusion.

This distinction guards against a familiar computational substitution. When a model estimates a latent vector z_u from an individual's data, the vector becomes a statistical construct defined by the model, loss function, and training distribution. Calling z_u a subject, an unconscious, or a desire would add an ontological interpretation unsupported by the estimation procedure. A valid bridge requires invariance tests, intervention-sensitive consequences, and comparison with rival constructs such as habit, strategic concealment, institutional constraint, memory error, and changing material opportunity.

The nonclosure condition in Equation (9.24) formalizes one aspect of this restraint. Several latent configurations can agree under a chosen observation family. Additional data can refine the equivalence classes, yet the Lacanian position $S(\bar{A})$ demands a stronger thought: completeness may fail through the structure of signification itself. Any statement represents the subject through available signifiers and within an address. Further representation changes the chain rather than merely revealing a fixed object in full. The formal open problem is therefore to model reflexive observation whose act participates in the represented relation.

An appropriate next step is a typed observation update

$$(x_t, \Pi_t, o_t) \xrightarrow{\text{utterance}} (x_{t+1}, \Pi_{t+1}, o_{t+1}), \quad (12.5)$$

in which an utterance changes the latent organization, the available probes, and the position of address. Equation (12.5) remains a model obligation. It states why passive state estimation captures only one limited case of speaking subjectivity.

12.4 The Other and distributed mediation

This subsection analyzes the relation between Lacan's big Other and the manuscript's relational substrate. Its objective is to retain symbolic precedence, interpersonal occupancy, institutional force, and incompleteness without converting the Other into a central database or environmental node. The discussion distinguishes structural locus, local occupant, network mediation, and reflexive change.

The big Other names the locus of signifiers, law, convention, and address. An empirical other can occupy this place in a particular relation, while the structural function exceeds that person's intention. A teacher can speak with institutional authority; a parent can convey kinship positions and inherited expectations; a lover can become the addressee through whom the subject encounters the question of desirability. The same person can occupy several positions, and several people or artifacts can jointly sustain one position (Lacan, 2006; Evans, 1996).

This structure supports a distributed formal representation. Let P denote empirical participants and artifacts, Q symbolic positions, and $J_t : P \rightrightarrows Q$ a time-indexed multivalued occupancy relation. A symbolic edge between positions can be enacted through many participant histories. Conversely, one participant event can activate several positions. The effective support of a symbolic relation may be written schematically as

$$w_{qq'}(t) = \mathcal{A}_{qq'}(\{h_p(t) : p \in J_t^{-1}(q) \cup J_t^{-1}(q')\}, \mathcal{I}_t), \quad (12.6)$$

where $\mathcal{I}_t$ records institutional and material arrangements and $\mathcal{A}_{qq'}$ is a declared aggregation rule. This construction treats symbolic efficacy as enacted and distributed. It avoids locating the Other inside one mind or one machine.

The construction also clarifies symbolic precedence. A newborn, newcomer, or novice enters a network with existing names, roles, prohibitions, and expectations. The initial possibility geometry is consequently inherited through relations. Participation can later transform that network. Precedence and co-constitution coexist: an order precedes a particular subject, and subjects reproduce or alter its local enactment through their trajectories.

Incompletion constrains the aggregation rule. A centralized decoder that assigns final meaning to every position would reproduce an omniscient Other. The distinguished node $S(\bar{A})$ marks the absence of such guarantee. Formally, multiple compatible extensions should remain for some finite histories, and adding an interpretation can alter the field of admissible interpretations. This condition differs from ordinary missing data because increased observation may generate new signifying distinctions together with new uncertainty.

The network vocabulary also carries a political risk. Measures of centrality, influence, or transmission can make institutional authority appear natural and quantitatively settled. Yet the meaning of an edge depends on recognition, sanction, material resources, and contested interpretation. A network map can therefore assist inquiry only when its construction remains open to those whose relations it represents. The model should record who defines the positions, who supplies observations, who receives authority from the result, and how alternative mappings can be compared.

12.5 Demand, desire, and fantasy-conditioned possibility

This subsection develops the connection among address, desire, fantasy, and effective possibility. Its objective is to distinguish an empirical goal from an object-cause and to show how fantasy can organize a stable relational form across changing objects. The analysis then states which part of this organization can enter the weighted formal sector.

Demand is articulated in signifiers and directed to the Other. The object explicitly requested can answer a bodily or practical requirement, while the address also carries relations of recognition and love. Desire persists through the remainder introduced by this passage. Lacan's formula of fantasy, $\$ \diamond a$, positions the divided subject relative to object a , whose function concerns the cause of desire (Lacan, 2006, 2019; Fink, 2004). The formula therefore resists a direct translation into an agent maximizing reward over object states.

Consider a sequence of empirical objects $o_1, o_2, \dots$ that support one relational role r . The objects can include achievements, partners, institutions, or signs of approval. A goal model assigns utilities $U(o_k)$. A fantasy-conditioned model also represents the subject position, the attributed demand of the Other, and the recurring scene through which the object becomes causally effective. One schematic factorization is

$$w_e(h, u) = w_e^{\text{mat}}(h), w_e^{\text{sym}}(h, u), \exp\{f_{u,h}(r_e)\}, \quad (12.7)$$

where material support, symbolic availability, and fantasy-conditioned relational role remain separate factors. The factorization is a modeling hypothesis and allows interactions or alternative links in later versions.

The relation to geometry follows from effective accessibility. An empirically nearby option can acquire high modeled obstruction because its relational role threatens the fantasy frame. A materially difficult route can acquire strong support because it repeats a scene of recognition, rescue, refusal, or unattainability. Distance in this context means the result of a declared reconstruction across several relations. It has no automatic relation to conscious effort, physical separation, moral worth, or therapeutic desirability.

Object a imposes an additional constraint. The model may type a vertex or role as a placeholder for an object-causal function, yet the function cannot be exhausted by the positive features of one empirical object. A candidate operationalization should therefore test substitutions. If several objects with divergent attributes organize the same relational trajectory, a role-level model may gain support. If the pattern follows stable object features, ordinary preference or reinforcement models may provide the better account.

Fantasy also affects observation. The fantasy frame can shape which events count as evidence, how the Other's response is interpreted, and which self-description becomes coherent. The interaction between punctuation and fantasy in Equation (9.19) expresses this reciprocity. New signification changes the inferred relational pattern, while the current pattern changes the uptake of new signifiers. A fixed filter captures only a local approximation to this coupled process.

Claim 12.3 (Role persistence hypothesis). *A fantasy-conditioned account earns comparative value when a stable relational role predicts substitutions, repetitions, and intervention responses across changing empirical objects more accurately than object-feature, preference, habit, and institutional-constraint rivals.*

The claim is empirical in its developed form. Its current contribution is methodological: it translates a psychoanalytic distinction into a comparative burden rather than treating the vocabulary as self-validating.

12.6 Drive circuits and path-sensitive satisfaction

This subsection interprets the recurrence functionals in relation to drive. Its objective is to distinguish destination value from satisfaction attached to a circuit. The discussion moves from endpoint equivalence to repetition, interruption, and rival mechanisms.

A shortest-path model assumes that detours are costs relative to a destination. A drive-sensitive account allows a trajectory to derive its organization from circling, delay, return, and repeated encounter with a limit. In Lacanian terms, the drive's satisfaction concerns the circuit around its object, while the graph places $\$ \diamond D$ on the upper chain of enunciation and demand (Lacan, 2006; Johnston, 2026). The mathematical consequence is modest but decisive: endpoint data lacks the information required to represent route-dependent recurrence.

The functionals in Equations (9.21) and (9.22) separate paths with common endpoints. A more developed model can distinguish at least four quantities:

$$\begin{aligned} G(\gamma) &= \text{declared endpoint gain}, & J(\gamma) &= \text{circuit-attached modeled satisfaction}, \\ H(\gamma) &= \text{repetition burden}, & Q(\gamma) &= \text{material and interpersonal consequence}. \end{aligned} \quad (12.8)$$

These quantities can covary or diverge. A path can yield little endpoint gain and high recurrence. It can carry modeled satisfaction together with severe consequence. It can also reflect coercion or

scarcity with little evidence for a drive interpretation. Keeping the registers separate prevents the vocabulary of *jouissance* from absorbing every costly repetition.

Intervention supplies a possible discriminator. A goal-directed mechanism should respond predictably when the endpoint becomes easier to obtain. A habit mechanism should respond to cue restructuring and repeated alternative practice. An institutional-constraint mechanism should respond to changes in sanction and resource. A drive-sensitive mechanism predicts that direct delivery of the apparent object can preserve or relocate the circuit because the circuit carries the relevant satisfaction. These patterns remain broad and require domain-specific specification.

The graph's curved returns also complicate path counting. Two cycles can share edges and acquire different meaning from their order, address, or punctuation. A free monoid of edge strings records order but ignores equivalence under interpretation. A category of typed paths can encode admissible composition, while a higher structure may be required when a passage changes the types of later passages. The mathematical programme should therefore compare Markov chains, semi-Markov models, automata with memory, path categories, and rewriting systems before invoking a more elaborate field theory.

Drive also limits normative inference. A recurrent circuit can sustain a life, organize a creative practice, or produce harm. Generativity and repetition remain distinct dimensions. The manuscript therefore evaluates formal patterns through declared consequences and participant-sensitive inquiry, while psychoanalytic interpretation concerns the organization of satisfaction and demand.

12.7 Symbolic incompleteness and model closure

This subsection examines the role of $S(\bar{A})$ in limiting formal closure. Its objective is to distinguish statistical uncertainty, practical ignorance, logical incompleteness, and Lacanian lack in the Other. The distinctions prevent a familiar error in which any missing value becomes a formalization of the Real or symbolic lack.

Statistical uncertainty concerns distributions over parameters or states. Practical ignorance concerns information unavailable to an investigator. Logical incompleteness concerns limits internal to a formal system under specified conditions. Lacanian lack in the Other concerns the absence of a final signifier or guarantee capable of completing the symbolic order. These forms can interact, yet their meanings and warrants remain different.

Equation (9.24) supplies only observation-level underdetermination. Several latent states produce the same statement projection. This property occurs in many ordinary models and therefore offers weak evidence for a Lacanian connection. A stronger descendant would make representation reflexive: the operation that identifies the subject changes the signifying relation through which identification occurs. Let R_s be representation by signifier s . Structural nonclosure would require a residual transformation $\delta_s(x)$ such that

$$R_s(x) = y, \text{quad} x \xrightarrow{R_s} x' = x + \delta_s(x), \text{quad} R_s(x') \neq y \quad \text{for some admissible } x, s. \quad (12.9)$$

The expression is schematic because addition may lack meaning on the target carrier. Its conceptual role is clear: representation participates in subject formation, so exhaustive recovery through a passive decoder becomes structurally unstable.

This constraint also protects the project from predictive totalization. A system that claims to infer an individual's true desire from comprehensive data would treat resistance to the model as further evidence for its hidden-state interpretation. Such a procedure becomes self-sealing. A responsible framework preserves contestability, admits observational equivalence, registers rival semantics, and allows a represented person to challenge the categories and consequences of representation.

The Real requires similar restraint. Noise, outliers, adversarial inputs, and unobserved nodes are ordinary formal phenomena. Calling them the Real adds little unless the construction explains a limit generated by the symbolic organization at issue. The present model leaves the Real outside its demonstrated sector. This omission narrows the claim and identifies a serious future obligation.

12.8 Emergent geometry and subject formation

This subsection connects the psychoanalytic reading to the paper’s central claim about generated possibility spaces. Its objective is to show why the emergence of geometry concerns more than parameter adaptation and why the generated carrier remains an effective model rather than a literal psychic spacetime. The discussion integrates symbolic inheritance, fantasy, recurrence, and historical update.

Subject formation occurs within relations that precede and address the subject. Names, kinship positions, languages, narratives, institutions, and material arrangements contribute distinctions through which possibilities become articulable. Repeated response and identification sediment some paths. Fantasy organizes recurring roles. Encounters and later punctuation can open or close distinctions and reinterpret earlier histories. The resulting possibility structure is simultaneously inherited, enacted, and revisable.

The group-field-like architecture enters at this level. Its relevant feature is generated combinatorics: the formal carrier can vary across histories rather than remaining a fixed graph with changing numbers. In the target domain, a new node can represent the constitution of a previously unavailable articulation or project; a new edge can represent a newly enactable transition; a higher cell can represent a compatibility relation among several passages. Each interpretation requires an explicit semantic rule. The success of variable-complex methods in source physics establishes mathematical precedents for generated structure and continuum reconstruction, while philosophy of emergent spacetime emphasizes the conceptual labor required to recover empirical significance from nonspatiotemporal bases (Oriti, 2016; Gielen et al., 2014; Huggett and Wuthrich, 2025).

The psychoanalytic contribution is a set of constraints on what gets generated. The carrier should permit divided observation, address through the Other, signifying substitution, fantasy-conditioned paths, drive-like recurrence, and incomplete closure. These constraints distinguish the programme from a generic adaptive network. They also show why a direct copy of a quantum-gravity action would have little target meaning. The relational cells, labels, weights, and observables must be redefined for histories of speaking, embodiment, and institution.

The exact finite example offers one local mechanism. Eliminating an internal mediator changes effective relations among retained positions. Equal present distances can conceal histories that respond differently to a later encounter. This result supports the possibility of historically sedimented effective geometry. It leaves subject genesis, semantic innovation, and psychoanalytic structure as additional layers. The typed Lacanian diagram and variable-complex ensemble propose how these layers might be connected.

The term *spacetime* therefore operates at three strengths. At minimal strength, it names ordered relational change. At intermediate strength, it names a reconstructed geometry with defined adjacency, path, and scale. At maximal strength, it would name a continuum with derived dimensional, metric, and causal structure. The present results reach the intermediate strength in a finite positive sector. The wider programme remains conditional upon new derivations and evidence.

Claim 12.4 (Historically generated possibility). *The central philosophical hypothesis states that relational history can constitute the individuation and effective adjacency of subjective possibilities, in addition to influencing movement among already individuated alternatives. The finite graph*

demonstrates one mechanism of mediated adjacency; the Lacanian graph supplies constraints for a richer theory of signifying and libidinal constitution.

12.9 Field-theoretic analogy and transfer discipline

This subsection evaluates the use of spin-foam and group-field-theoretic formalisms within the psychoanalytic target. Its objective is to preserve the productive analogy while preventing physical vocabulary from carrying unearned authority. The discussion applies the transfer certificate of Equation (8.1) to generation, amplitude, boundary, scale, and emergence.

Models and analogies support inference through selected similarities and controlled idealization. Their evidential value depends on the source-target mapping, the inferences preserved under that mapping, and the disanalogies that constrain transfer (Hesse, 1966; Suarez, 2004; Frigg and Hartmann, 2025). P036 uses this discipline to separate mathematical borrowing from ontological identification.

The preserved structural roles are limited. A labeled complex can represent a composed relational history. A boundary can represent the variables retained for a question or observational interface. A local weight can represent compatibility or propensity. A sum over complexes can represent uncertainty or variation in relational organization. A field-theoretic expansion can generate combinatorial structures. Coarse-graining can reconstruct an effective relation among retained variables.

The principal disanalogies are equally important. Quantum amplitudes, gauge symmetries, group representations, locality, dimensionality, and continuum limits possess source-domain meanings derived through physical theory. Subjective histories contain signification, embodiment, institutional power, interpretation, and first-person contestability. A reused symbol acquires target meaning only through an independent definition and validation rule. Mathematical elegance in the source theory supplies motivation and techniques; it carries no direct evidence for the target ontology.

The term *field* also requires care. Lewin’s psychological field, a classical field over a carrier, a quantum field, and the symbolic field of the Other have different formal and conceptual structures. Their common vocabulary points toward relational organization and distributed dependence. It leaves their equations, observables, and ontological commitments distinct. The manuscript therefore uses *group-field-like* for the target generator until a group action, measure, interaction kernel, and controlled expansion are fully defined.

Transfer becomes productive when it creates testable obligations. A generated-complex model should outperform a fixed graph under interventions that create or remove distinctions. Group or groupoid labels should encode invariances with empirical significance. Boundary composition should support stable decomposition across contexts. Coarse-grained observables should remain robust under declared changes of resolution. Collective regimes should yield consequences unavailable to unconstrained latent-state models. Failure across these tests favors simpler network, state-space, linguistic, or narrative models.

The transfer discipline thereby protects both domains. Quantum-gravity formalisms retain their physical specificity. Psychoanalytic concepts retain their textual, clinical, and philosophical burdens. The new model stands between them as an authored construction whose value depends on derivation, interpretation, comparison, and evidence.

Open Problem 12.5 (Non-scalar desire geometry). *Construct a directed, typed, and possibly higher-order effective geometry that can represent address, signifying substitution, fantasy-mediated trajectory constraints, and incompatible registers while preserving identifiable observables and explicit failure conditions.*

Table 9: Objection ledger for the spacetime-of-desire programme. Each response is provisional and each revision condition constrains future model development.

Objection domain	Provisional response	Revision condition
Physicalization	Source and target semantics are separated by transfer certificates	remove physical terminology whose formal role remains ornamental
Diagrammatic flattening	Typed edges, layers, paths, and omissions preserve selected distinctions	abandon adjacency reconstruction when crossings and returns lose their conceptual function
Reification	Formal states are model constructs linked to observations through partial maps	revise any variable treated as a discovered psychic entity solely through model fit
Hermeneutic circularity	Rival semantics and intervention-sensitive predictions constrain interpretation	withdraw constructs that absorb every counterinstance through redescription
Clinical scope	The present warrant is textual, philosophical, and mathematical	require independent clinical expertise, governance, and evidence before clinical use
Normative authority	Model outputs require participant contestability and consequence-sensitive review	prohibit individual classification when challenge and appeal are unavailable
Restricted Lacanian coverage	The paper treats the graph of desire during a defined period of Lacan’s teaching	revise claims that depend on later topology, sexuation, discourse theory, or sinthome
Theory pluralism	Lacanian, ecological, field-theoretic, and productive-desire accounts remain distinguishable	prefer a simpler rival when it explains the target consequences with fewer transfers

13 Philosophical Objections and Revision Conditions

This section subjects the proposal to adversarial philosophical review. Its objective is to identify errors that could survive correct mathematics, including category mistakes, diagrammatic flattening, reification, interpretive circularity, empirical underdetermination, normative misuse, and restricted coverage of Lacan. Table 9 states the objection ledger, after which the subsections develop the strongest cases and their consequences for revision.

Table 9 treats objections as design constraints rather than ceremonial caveats. A successful response specifies an operation, comparison, or withdrawal condition. General assurances add little protection against conceptual overreach.

13.1 Physicalization and category transfer

This subsection addresses the concern that quantum-gravity language physicalizes desire. The objection has substantial force because the title joins a psychoanalytic concept to spin-foam and group-field-theoretic formalisms, whose scientific authority can migrate rhetorically even when their evidence remains source-specific.

The proposal responds through level separation. Spin foams and GFT contribute architectures for labeled complexes, weighted histories, generated combinatorics, boundaries, and effective reconstruction. The target model independently defines relational events, labels, weights, observations, and updates. The finite theorems apply to a positive graph sector whose assumptions are explicit.

A psychoanalytic interpretation enters through a partial map. This sequence prevents a theorem about a Laplacian from becoming evidence that desire is physically quantum.

The objection nevertheless survives whenever terminology outruns construction. Terms such as *amplitude*, *gauge*, *phase*, *condensate*, and *spacetime* can suggest properties that the target model has yet to derive. The appropriate response is local discipline. Each term needs a symbol, domain, operation, interpretation, and validation burden. Terms lacking this ledger should be replaced by plainer target-domain vocabulary.

The title itself remains intentionally provocative and therefore increases the burden of clarification. Its defensible reading is methodological: emergent-spacetime research offers a formal precedent for treating geometry as reconstructed from relational structures. Its strongest reading, a common ontology of physical spacetime and desire, lies beyond the present argument. Later work should revise the title if readers repeatedly infer the stronger position despite explicit distinctions.

13.2 Diagrammatic form and graph reduction

This subsection addresses the transformation of Lacan's diagram into a mathematical graph. The objection states that visual adjacency in the graph of desire lacks a uniform mathematical meaning. Crossings, arrows, labels, curves, and spatial placement participate in an exposition whose logic exceeds graph incidence. A direct adjacency matrix would therefore flatten the source object.

The typed reconstruction answers part of this objection. Vertex and edge types prevent universal composition. The layer map preserves the two storeys. Directed paths preserve orientation. The semantic kernel represents retroactive punctuation. Return operators preserve route dependence. The fidelity table records omitted concepts. These devices turn a copied image into an explicit interpretive construction.

Several difficulties remain. The graph's four stages form a pedagogical sequence, while the complete diagram condenses their result. A static multigraph omits the act of construction. The diagram also uses crossing as a relation between chains, whereas a conventional graph crossing has no mathematical effect unless declared as a vertex or higher cell. Curved returns can indicate retroaction, logical dependence, or circuit, and these meanings may require different edge types. Spatial proximity in the printed figure may be typographic rather than conceptual.

A stronger formal descendant should therefore model the construction sequence as a diagram transformation

$$\mathbb{G}_L^{(1)} \xrightarrow{\Phi_1} \mathbb{G}_L^{(2)} \xrightarrow{\Phi_2} \mathbb{G}_L^{(3)} \xrightarrow{\Phi_3} \mathbb{G}_L^{(4)}, \quad (13.1)$$

where each Φ_k adds a distinction and records its interpretive justification. The sequence can then be compared with a single complete-graph representation. Failure to recover the conceptual function of a crossing or return would count against the reconstruction.

13.3 Geometry, topology, and metaphorical reach

This subsection evaluates the strength of the manuscript's spatial vocabulary. Its objective is to keep adjacency, metric, topology, manifold, and spacetime distinct. The objection concerns both mathematics and rhetoric: a relational diagram can appear geometric while satisfying few geometrical axioms.

The finite resistance sector earns a metric on retained vertices under the stated positivity and connectedness assumptions. The typed directed graph earns reachability relations and path families. The varying-complex ensemble earns combinatorial variation. These are genuine formal structures. They jointly support an intermediate use of geometry as scale-indexed effective relational organization.

Topology enters only after a carrier and collection of open sets are defined, or after an equivalent structure such as a neighborhood or closure system is supplied. The present global model has several carriers and typed relations whose integration remains open. Manifold and spacetime language would require further local, dimensional, and causal properties. The research programme should therefore report its mathematical level in each result and reserve stronger terms for derived structures.

The philosophical value of the spatial vocabulary survives this restraint. It directs attention to neighborhood, boundary, detour, accessibility, and the historical constitution of a field. Lewin's topological psychology, Merleau-Ponty's bodily spatiality, and Gibson's affordances offer important precedents for relationally organized possibility (Lewin, 1936; Merleau-Ponty, 2012; Gibson, 1979). Their differences also demonstrate that shared spatial language supports several ontologies and methods.

13.4 Subject individuation and formal reification

This subsection addresses the risk that nodes, vectors, and latent states turn theoretical distinctions into inner entities. A graph vertex can invite the picture of a stored mental module labeled "desire" or "Other." The source diagram instead articulates positions and relations within a theory of signification and subjectivity.

The formal response is type discipline. A vertex represents a position in the model, and its interpretation can be functional, relational, observational, or heuristic. A subject-indexed record groups observations for analysis. It carries no theorem about personal unity. The big Other is represented through symbolic positions and occupancy relations. Object a is represented as a role whose empirical supports can vary.

The practical response requires model auditing. Every fitted latent variable should be reported with its generating assumptions, observation channels, identifiability class, and rival interpretations. Stability across datasets can support a construct while leaving its ontology open. Intervention response can strengthen a mechanism claim while leaving psychoanalytic terminology contestable. Participant testimony can inform interpretation while remaining one situated form of evidence.

Reification becomes especially dangerous when an institution uses the model to classify people. A predicted trajectory can feed back into opportunity, surveillance, treatment, or exclusion. Such feedback changes the relational field and can make the prediction self-confirming. The model must therefore represent institutional deployment as part of the history update rather than as an external application step.

13.5 Interpretive circularity and comparative constraint

This subsection addresses the possibility that every outcome can be redescribed as evidence for desire, fantasy, or resistance. Such flexibility can preserve a vocabulary while dissolving empirical constraint. The response requires prospective rival models and local consequences.

For each application, the project should specify at least one ordinary goal model, one habit or learning model, one material or institutional constraint model, and one history-sensitive relational model. The Lacanian branch earns support only through consequences that differentiate it. Candidate consequences include role persistence across object substitution, path-sensitive response to direct goal delivery, context-dependent divergence between statement and enunciation probes, and retroactive restructuring under controlled completion.

Interpretation also requires a stopping rule. When a candidate fantasy changes after every counterinstance, the model risks absorbing noise through unconstrained redescription. A constrained analysis should define the permissible transformations, penalize complexity, evaluate held-out

histories, and record interpretations supplied before outcome inspection. Qualitative inquiry can use analogous safeguards through case comparison, deviant cases, independent interpretation, and transparent revision histories.

Psychoanalytic inquiry may seek singular meaning whose value cannot be reduced to population prediction. The demand for comparative constraint still applies in a suitable form. An interpretation should illuminate connections, permit consequences, survive confrontation with material, and remain revisable by later speech. Formalization can assist by exposing assumptions and sequence. It cannot replace analytic judgment or the subject's participation.

13.6 Clinical scope and evidential transitions

This subsection specifies the distance between philosophical reconstruction and clinical use. The current literature corpus establishes textual concepts, specialist interpretations, formal precedents, and one review of retroactivity. It supplies limited direct evidence for measurement of desire-graph variables or therapeutic consequences.

Clinical development would require collaboration with qualified practitioners, transparent governance, and a research design suited to sensitive longitudinal material. Construct validity would need evidence that operational variables track the intended phenomena. Reliability would need evidence that coding and inference remain stable enough for the proposed use. Comparative validity would need evidence against simpler explanations. Consequence assessment would need evidence concerning benefits, harms, and distributional effects.

The graph can initially function as a structured case-research aid. It can prompt distinctions among statement, enunciation, demand, fantasy, and recurrence. Such use should preserve uncertainty and alternatives. Automated diagnosis, treatment selection, or risk scoring would require much stronger evidence and safeguards. The presence of precise equations raises the communication burden because formal precision can be mistaken for clinical validation.

13.7 Restricted corpus and Lacanian development

This subsection states the historical scope of the Lacanian material. The paper centers the graph of desire as developed around Seminar V, Seminar VI, and "Subversion of the Subject" (Lacan, 2017, 2019, 2006). Specialist works by Fink, Van Haute, Eidelsztein, Chiesa, Evans, and Johnston support reconstruction and expose interpretive choices (Fink, 2004; Van Haute, 2002; Eidelsztein, 2009; Chiesa, 2007; Evans, 1996; Johnston, 2026).

Lacan's teaching develops beyond this corpus through discourse theory, sexuation, topology, knots, the sinthome, and changing accounts of the Real and jouissance. These developments can alter the interpretation of structure, subject, and formalization. The present paper treats them as future research domains. It therefore offers a graph-centered reconstruction rather than a comprehensive mathematical Lacanianism.

This restriction matters for the word *topology*. Specialist discussion of the graph can use topological language, while the graph itself remains a diagram rather than one of Lacan's later topological surfaces or knots (Eidelsztein, 2009). A later paper should compare graph transformations with surfaces and knotting only after establishing the relevant mathematical and textual transitions.

The current model also gives limited treatment to sexual difference and castration. Their appearance as boundary labels in Figure 1 preserves their place while leaving their formal role open. A model that removes these concepts could become a theory of signifying mediation inspired by Lacan. A model that retains a strong Lacanian claim must engage their structural roles and the extensive debates surrounding them.

Table 10: Current claims, statuses, and revision triggers

Statement	Current status	Revision trigger
Kron reduction creates a mediated boundary edge	exact finite result	violation of positivity, connectedness, or linear elimination assumptions
Resistance defines retained-node distance	established graph result	target application requires a different geometry
Aggregation and reduction can diverge	exact finite counterexample	restricted ensemble class yields a commutation theorem
Equal current geometry can yield different future reachability	exact toy-model result	update and threshold lack target validity or broader robustness
Relational history shapes subjective possibility	conditional target conjecture	interventional data favor fixed-geometry or alternative mechanisms
Lacanian theory supports the interpretation	bounded conceptual compatibility	primary or specialist analysis exposes a structural conflict
GFT architecture adds explanatory value	research-program hypothesis	simpler fixed graph models match every relevant consequence

13.8 Normative consequences and participatory constraint

This subsection develops the ethical implications of modeling desire. The central risk arises when a model claims authority over a person’s self-interpretation, future possibilities, or treatment. The model could become an occupant of the Other’s place and reshape the field it purports to observe.

Four safeguards follow. First, construct labels should remain contestable and carry uncertainty. Second, consequential use should disclose the observation and interpretation maps. Third, affected persons should have meaningful opportunities to challenge records, categories, and consequences. Fourth, deployment effects should enter the update model because classification can alter later opportunities and self-relations.

The safeguards also constrain research communication. Visualizations should distinguish observed edges, inferred edges, and interpretive edges. Model confidence should remain separate from clinical importance. Group summaries should not be converted automatically into individual judgments. Explanations should state rival accounts and the conditions under which the preferred interpretation changes.

The ethical point connects directly to symbolic theory. An authoritative prediction can function as a signifier addressed to a subject within an institution. It can produce identification, resistance, compliance, or exclusion. Evaluating the model therefore requires analysis of its performative position in addition to statistical accuracy.

14 Identification and Comparative Models

This section states what the present construction identifies and compares its contribution with simpler rivals. The analysis distinguishes mathematical recovery, observation-level identification, explanatory priority, and empirical support. Table 10 records the status and revision trigger of every principal statement.

For a probe family $\mathcal{P}$, define observation equivalence by

$$h_1 \sim_{\mathcal{O}, \mathcal{P}} h_2 \iff \mathcal{O}^p(h_1) = \mathcal{O}^p(h_2) \text{ for every } p \in \mathcal{P}. \quad (14.1)$$

The relation depends on the probe family. It equals gauge equivalence only when a separate theorem connects the two.

Proposition 14.1 (Probe-relative nonidentification). *Suppose $h_1 \sim_{\mathcal{O}, \mathcal{P}} h_2$ and a common admissible update U gives*

$$\mathcal{K}_\theta(Uh_1) \neq \mathcal{K}_\theta(Uh_2). \quad (14.2)$$

Then the current probes fail to identify the latent history sufficiently for prediction under U .

Proof. The observed current data assign both histories to the same equivalence class. Equation (14.2) assigns different interventional predictions to members of that class. A point prediction therefore requires additional observation, a distribution over class members, or a selection assumption. $\square$

Proposition 14.1 is elementary, yet it has a substantive consequence. Predictive adequacy should be evaluated under interventions that couple differently to hidden relational organization. A cross-sectional match to current articulation supplies limited evidence for a history-sensitive geometry.

14.1 Simpler formal rivals

This subsection evaluates explanatory economy. A weighted graph, a retained set, a Schur complement, and an update fully support the exact theorems. The weighted-graph model is therefore the principal baseline for the finite results. Spin-foam amplitudes and the GFT functional integral enter the wider complex-generation programme.

A fixed effective graph rival places L^K directly at the observational level and updates that graph without hidden nodes. It can match current endpoint behavior. The histories in Proposition 11.15 distinguish the models only when the intervention is defined at the hidden relational level. A standard state-space rival can also reproduce any finite observed trace by expanding its latent state. Model comparison must consequently examine intervention structure, generalization across scales, and constraints on latent complexity.

Spin-foam and GFT architecture gains comparative relevance under stronger conditions. Variable complex structure must carry target meaning. Boundary composition must support empirically useful decompositions. Group labels or symmetry constraints must restrict admissible models. Collective or continuum regimes must yield novel, stable consequences. Until such conditions are met, the architecture organizes the research programme and the graph sector carries the demonstrated mathematics.

14.2 Empirical identification

This subsection identifies a minimal longitudinal design. An application would require repeated observations of candidate positions, local relational interventions, and later transitions. Conductances could be treated as latent parameters estimated from response patterns. Competing observation maps should be fixed before outcome comparison. The primary estimands would include

$$\Delta R_{ij}(U) = R_{ij}(Uh) - R_{ij}(h), \quad (14.3)$$

$$\Delta P_{ij}(U, T) = P(j \text{ reached by } T \mid U, h) - P(j \text{ reached by } T \mid h), \quad (14.4)$$

with uncertainty propagated across latent histories, retained sets, and semantic assignments.

Ethical constraints enter at the design stage. Interventions on relationships, speech, attachment, or vulnerability can create harm. Observational and naturally occurring designs, simulation,

synthetic data, and clinically governed retrospective studies may provide safer initial tests. Ethical authorization requires independent review, participant protection, and domain-appropriate governance.

15 Research Programme and Failure Conditions

This section converts the proposal into a staged research programme. It separates mathematical development, interpretive development, empirical evaluation, and criteria for revision or abandonment.

15.1 Mathematical development

This subsection extends the exact finite sector while preserving tractability. The next graph-level tasks are loops, parallel mediators, heterogeneous edge types, signed or directed influence, time-varying conductance, and uncertainty over retained sets. Each extension should retain a complete derivation and identify which resistance and Kron properties survive.

The complex-level tasks are stronger. A model should specify how two-cells constrain compositional history, how boundaries compose, and how local weights generate global consistency. The group-field-like branch requires a defined group or groupoid, field domain, kinetic kernel, interaction combinatorics, measure, and controlled expansion. An effective regime should be derived from those structures through a specified approximation. The existence of successful GFT condensate constructions in quantum cosmology demonstrates a source-domain strategy (Gielen et al., 2013); it supplies a demanding precedent for derivational clarity.

Open Problem 15.1 (Complex-to-metric reconstruction). *Give sufficient conditions on a relational model, retained boundary B , and coarse-graining map K_ℓ such that the reconstructed metric or pseudometric exists, is stable under declared perturbations, and supports trajectory predictions that differ from a fixed-geometry baseline.*

Open Problem 15.2 (Field-to-ensemble derivation). *Construct $S_R[\varphi_R]$ so that a controlled expansion produces a declared family of relational complexes and weights, then derive a positive or observation-compatible effective graph sector with quantified approximation error.*

15.2 Interpretive development

This subsection specifies the psychoanalytic tasks. Primary-text study should determine which relations among signifier, demand, desire, fantasy, object a , and the Other can be represented without scalar reduction. Specialist review should examine whether the model’s historical feedback clarifies retroaction or merely redescribes it. The relation among effective geometry and the Real, Symbolic, and Imaginary requires separate treatment. The current draft reserves these as interpretive obligations.

15.3 Empirical development

This subsection specifies a conservative progression from simulation to observation. A first simulation can generate latent networks, apply predefined interventions, perform Kron reduction, and compare history-wise with pre-aggregated predictions. A second stage can test recovery under noisy and partial observations. A third stage can analyze ethically obtained longitudinal data with preregistered rival models. Clinical interpretation requires additional governance, domain expertise, and evidence.

Useful outcomes include both support and failure. Support would require stable reconstruction, interventional discrimination, and explanatory gain across held-out settings. Failure can reveal that the retained variables, linear semantics, hidden-history update, or geometric language is poorly chosen.

15.4 Revision and abandonment conditions

This subsection states the conditions under which the framework should change substantially. Revision is required when directed or nonlinear effects dominate the symmetric linear network, when retained-set choices produce unstable conclusions, when semantic assignments remain empirically indistinguishable, or when psychoanalytic interpretation conflicts with primary and specialist scholarship.

The GFT branch should be abandoned for this target when simpler graph or dynamical models reproduce every consequence of variable complexes, group constraints, and collective regimes. The geometric framing should be abandoned when accessibility and obstruction resist measurement with acceptable construct validity or when nongeometric models consistently provide clearer predictions. The term *desire* should be narrowed or replaced when its Lacanian bridge remains ornamental.

16 Conclusion

This section consolidates the paper’s achieved result and preserves the boundary between demonstration and programme. The contribution is a typed route from Lacanian source constraints through generated relational histories to an effective possibility geometry, together with exact finite results concerning boundary response, mediated emergence, scale consistency, order dependence, and hidden-history feedback.

The spacetime of desire is defined as a scale-indexed effective geometry, trajectory law, observation map, and historical update. Spin-foam and group-field-theoretic work supplies a source architecture for labeled complexes, weighted histories, generated combinatorics, boundary data, and micro-to-effective reasoning. Explicit transfer certificates preserve the source-target distinction.

The finite graph sector supplies the current mathematical result. Dirichlet-energy minimization yields the Schur boundary operator. Internal-node elimination creates an effective boundary edge and preserves effective-resistance geometry. Compatible nested reductions produce the same final boundary operator as direct elimination. A two-history ensemble shows that microscopic averaging and coarse-graining can produce different geometries. The same histories show that equal current endpoint geometry can yield different future reachability under one common update. A conditional distribution over the current observation class extends that comparison to uncertain hidden histories. These results establish precise possibilities within the model.

The philosophical proposal is that subjective possibility may be historically constituted at the level of effective geometry as well as trajectory. The psychoanalytic proposal is that this architecture may illuminate mediated desire, divided articulation, and historical reorganization. Empirical and clinical warrant remain future achievements. The framework is designed to become more specific through successful derivation and evidence, and to contract when simpler or better models prevail.

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