

Street Spin Foam: A Preliminary Formulation

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Abstract

Movement through a city is usually modelled by fixing a geometry, placing agents inside it, and treating the worth of a location as a number attached to that location. This paper develops a representation in which relations carry the state instead. A street district is written as a compact lattice gauge theory of valuation on a two-complex, in the spin-foam representation of covariant loop quantum gravity. Persons, places and stores become one-cells, that is, worldlines, and the two-cells become the histories of the relations among them, so that being somewhere is a relation with a history rather than a coordinate a person carries. An entity is then a constraint on how the relations meeting at it may combine, rather than a bearer of state.

Two consequences follow. Supplying a group and an action makes the spins, the intertwiners and the vertex amplitude follow from a character expansion rather than from stipulation, so that a large collection of local weights reduces to a small number of global choices; the expansion is verified against an independent numerical evaluation. Pedestrians then enter as charged matter, minimally coupled, and Kolmogorov's criterion yields a prediction sharper than the statement that valuation affects movement: circulation is sourced by the topological part of the valuation field alone, so that a region of large local disagreement and zero winding carries zero net circulation. A model whose forces are gradients of potentials leaves that pattern inaccessible, which makes the difference testable rather than interpretive.

A case study on the street network of Ginza, Tokyo, with constructed commercial content, illustrates what the formalism does. The gauge structure changes simulated behaviour against a flat-connection control; circulation tracks the winding number while leaving a matched non-topological predictor inert; an effective distance read off correlation decay departs from street distance; encounter order is inert under an abelian connection and becomes material as soon as the valuation axes separate; and a walk on the complex sees an effective two-dimensional geometry over about a decade and a half of scale. One prediction fails, and its diagnosis is the most transferable result: it was imported from a confining gauge theory that the model leaves uninstantiated.

The cell assignment admits two orientations, with entities on edges or objects on faces, and a counting argument bears on which of them leaves a configuration space to sample. Under the second, persons enter the spin foam directly, on condition that the one-cells are occasions gathering several parties rather than pairwise relations.

The contribution is a representation in which relations rather than sites carry the state, a route by which local weights follow from a group and an action, a falsifiable point of difference from gradient-based models, and a criterion for choosing the cell assignment. Estimating the couplings from observation is identified as the next stage.

Keywords: lattice gauge theory; spin foam; two-complex; urban morphology; pedestrian dynamics; valuation; relational ontology.

Discussion Paper Note

This paper is a preliminary discussion paper intended to share an evolving idea and invite further dialogue, criticism, revision, and independent development. Its definitions, distinctions, and formal constructions remain provisional. Circulation across scholarly and practical communities is part of the purpose of releasing the manuscript at this stage.

The author treats the viewpoints, concepts, and lines of reasoning presented here as contributions to a shared field of inquiry. Similar or related ideas may have appeared in other intellectual, cultural, and disciplinary traditions. The borrowing of spin-foam and lattice gauge vocabulary from mathematical physics is stated as such throughout, and the manuscript separates what the formalism supplies from what the author stipulates. The manuscript therefore states its known antecedents, separates the researcher-origin proposal from later formal reconstruction, and leaves historical priority open pending a systematic originality review.

The arguments should be understood as provisional and historically situated. Readers are encouraged to question, test, revise, extend, reinterpret, or independently develop the ideas presented here. Where appropriate, acknowledgment of this paper as one point of encounter in the development of a related idea is appreciated. Such acknowledgment records an intellectual route; the ideas themselves remain available for criticism, revision, and independent development.

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Status. This working draft records an evolving stage of the author’s position and is circulated for discussion. Definitions, section structure, formal statements, and numbering remain subject to revision. The case study of Section 8 is an illustration rather than a measurement: the street geometry is taken from OpenStreetMap, while the commercial content, the agents and the coupling constants are constructed to exhibit the formalism. Estimation of the couplings from observation remains a future research stage, alongside the questions left open in Section 10.

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Statement on the Use of Language Models. The exploratory discussion, formal criticism, implementation and drafting of this paper were conducted in dialogue with Anthropic’s Claude. The model contributed criticism of an earlier design, the accompanying code, the execution of the reported experiments, and drafting in L^AT_EX. The author selected the research question, directed and approved the theoretical commitments and epistemic status of the claims, and bears sole responsibility for the manuscript, including its definitions, formal constructions, calculations, arguments, conclusions, and errors. Authorship credit remains with the human author. References have been checked against publisher records through their digital object identifiers, which are printed with each entry that carries one. Two entries carry none: the Kelly volume, which predates the identifier system and whose edition was confirmed from catalogue records, and the street data, which is cited by retrieval route.

Reproducibility. The reported figures and numbers regenerate from a recorded seed. Code, data, the step-by-step build and the full implementation record, including the errors corrected during it, are at `street-spinfoam/` in the accompanying repository. Thirty-five invariants are enforced as tests, six of which check this manuscript rather than the code.

Related Research Programme. This paper accompanies the `street-spinfoam` module of the Social Spin Foam repository and stands to one side of that programme’s main line. The parent work develops a two-complex representation of social facts for power analysis and governance decision, and records as one of its own limits that the labels carry no group, so that each local weight is estimated or stipulated. The present paper takes a narrower case, a street district, and supplies a group and an action, so that the spins, intertwiners and vertex amplitude follow from a character expansion. The $SU(N)$ extension, the coarse-graining analysis, and empirical estimation remain later stages.

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1 Introduction

Movement through a city is usually modelled by fixing a geometry and placing agents within it. A street network supplies the space, a potential supplies the forces, and the numbers that weight one location against another are supplied by the analyst. Representations of this kind are mature and productive: social force models reproduce crowd behaviour from few parameters [9], space syntax relates spatial configuration to observed movement [10], and the physics of spatial networks characterises street systems at scale [3, 5]. They share two commitments. The space exists prior to what happens in it, and the worth of a location is a scalar attached to that location.

A different commitment makes relations primary. What a shopfront is worth is constituted by what stands beside it, what can be seen from it, who passes, and what else carries its name elsewhere in the district. On that reading the quantity belongs to the relation rather than to the site, and an entity becomes a constraint on the relations meeting at it rather than a bearer of state. Formalisms built to treat relations as primary objects are therefore of interest, and among them the spin foams of covariant loop quantum gravity carry three things at once: heterogeneous local states, constraints on how those states may be combined, and a dynamics that changes the combinatorial structure itself rather than only the labels on a fixed one [1, 16, 18].

That availability carries a corresponding risk. A spin foam is a labelled two-complex, and a labelled two-complex can be given the name on no further grounds. The label earns the name *spin*, and the complex the name *foam*, when the labels are irreducible representations of a group, the intertwiners are invariant tensors, and the total weight is therefore unchanged by a local change of frame. Without the group the object is a Markov random field on a decorated complex: serviceable, with a weaker claim on the vocabulary. Borrowing the words is inexpensive; earning them requires supplying the structure that gives them content.

This paper supplies that structure and reports its consequences. Section 4 fixes a cell assignment in which persons, places and stores are one-cells and the relations among them are two-cells. Section 5 identifies the gauge freedom with the absence of an absolute valuation frame, and curvature with the winding of that frame around a closed circuit, an integer attached to each such circuit. Section 6 obtains the spins, the intertwiners and the vertex amplitude by character expansion from a single action, and verifies the expansion against an independent numerical evaluation. Section 7 couples pedestrians as charged matter and derives a prediction about circulation from Kolmogorov's criterion. Section 8 runs the resulting model on a case study, the street network of Ginza with constructed commercial content, against matched controls. Readers who do not work with gauge theory will find the machinery stated in Section 3.

The paper contributes (1) **a relational cell assignment**, in which relations carry the state and entities carry constraints on how the relations meeting at them may combine, so that persons and places become the same kind of cell and the split between background and matter that a fixed geometry imposes falls away; (2) **local weights obtained by derivation**, since given a group and an action the character expansion produces the spins, the intertwiners and the vertex amplitude, reducing a large collection of local stipulations to a small number of global ones; (3) **a falsifiable point of difference from gradient models**, because the connection enters antisymmetrically and circulation is therefore sourced by the topological part of the valuation field alone; (4) **a counting criterion for the cell assignment**, since the same three cells can be read in either order and §4.4 shows which reading leaves a configuration space to sample; and (5) **a record of what the formalism refused**, since running the model corrected several statements in the design that generated it, and §8.7 reports the corrections rather than absorbing them quietly.

Section 2 places the construction against the literatures it draws on, Section 9 states what the present work leaves undone, and Section 10 states the questions it leaves unresolved.

2 Related Work

The construction draws on four bodies of work. Its position is stated against each in turn, since each borrowing is partial.

Models of Movement on a Fixed Geometry. Spatial interaction models derive flows between zones from a maximum-entropy argument with the geometry given [21]. Social force models place agents in a continuous space under potentials [9], and cellular automaton models discretise the same picture onto a lattice with a floor field standing for accumulated attraction [6]. Space syntax relates the configuration of a street system to observed movement through measures computed on the network [10], and the statistical physics of spatial networks characterises street systems by degree, centrality and block structure [3, 5]. These share the two commitments named in Section 1: the geometry precedes what happens in it, and the desirability of a location is a scalar attached to that location. The present model keeps the street network as a substrate but moves the state onto the relations, which is what allows a person and a shopfront to be cells of the same kind.

Statistical Physics Applied to Social Systems. Opinion, voter and Ising-like models on social networks form a developed literature [7], and the alignment action of Section 5.3 is recognisably of that family. The difference lies in the frame freedom. A spin system on a fixed lattice measures its variables against a common reference, whereas the model here admits an independent change of reference at each entity and admits as results only the quantities that survive it. The requirement is a restriction rather than a generalisation, and Section 5.1 gives the reading of it: valuation vocabulary is comparative, so a quantity depending on one party's zero point is not a statement about the district.

Gauge and Higher-Order Structure on Graphs Outside Physics. Combinatorial Hodge theory decomposes pairwise comparison data on a graph into gradient, curl and harmonic components, and interprets the curl as the inconsistency of the comparisons around a triangle [11]. The winding of Section 5.2 is the same failure-to-close, with two differences: the group there is the additive reals, so the curl is a real number rather than an integer topological charge, and the decomposition is applied to fixed data rather than coupled to a dynamics. Separately, the study of higher-order networks argues that relations among more than two parties are not reducible to the pairwise relations they contain [4]. Section 4.4 reaches a version of that conclusion from the Gauss law rather than from representational adequacy: a two-party occasion forces its members' spins equal, so the configuration space it generates is trivial.

Spin Foams. The formalism is taken from covariant loop quantum gravity [1, 16, 18], and the borrowing is deliberately partial. Absent here are the Lorentzian signature, the simplicity constraints that relate the representations to a discrete geometry, the continuum limit, and the complex amplitudes. The weights of Section 5.3 are real and positive, so the object constructed is a statistical model in the shape of a spin foam rather than a quantum one, and the sum carries no interference. The retained part is the one that does the work in this setting: representations on faces, invariant tensors on edges, and a total weight invariant under a local change of frame.

3 Background

This section states the machinery the paper uses, for a reader comfortable with linear algebra and elementary probability who has not worked with lattice gauge theory. Readers who have may proceed to §4. Standard treatments are Kogut [13] for the lattice theory and Baez [1] or Perez [16] for spin foams.

3.1 Two-Complexes and Boundary Maps

A *two-complex* consists of cells of dimension zero, one and two, called vertices, edges and faces, together with maps recording which lower cells bound each higher one. Write ∂_1 for the map sending an edge to its endpoint minus its start, and ∂_2 for the map sending a face to the signed sum of the edges around its boundary, the sign recording the direction in which each edge is traversed. The defining condition is

$$\partial_1 \partial_2 = 0, \quad (1)$$

read as *the boundary of a boundary vanishes*: walking once around a closed face enters and leaves each corner in balance, so the endpoints cancel in pairs. A collection of faces whose boundary vanishes, $\partial_2 j = 0$, is called a *two-cycle*; the set of these is written $\ker \partial_2$, and it is where this model's configurations live.

Ordering matters. A face records its boundary as an oriented, signed sequence rather than an unordered set, since the products defined below are taken in the order the boundary is traversed.

3.2 Connections, Holonomy and Gauge Invariance

Fix a group G . A *connection* attaches an element $g_e \in G$ to each edge, read as the translation between conventions in force at the edge's two ends. Multiplying these in order around a closed circuit gives the *holonomy* of that circuit, a single group element recording the net translation accumulated by going around.

A *gauge transformation* chooses an arbitrary $\phi(v) \in G$ at each vertex independently, and replaces g_e by $\phi(v_1) g_e \phi(v_2)^{-1}$. This is the formal counterpart of each party changing its own convention with no coordination. Holonomies around closed circuits are conjugated by such a change, so their *class functions*, meaning functions constant on conjugacy classes, are unaffected. Quantities of that kind are called *gauge invariant*, and a gauge theory treats them as the only physically meaningful ones.

3.3 Actions and the Gibbs Measure

An *action* is a real-valued function S that scores a configuration: lower S carries more weight. It occupies the position a negative log-likelihood occupies in statistics. Given an action, configurations are weighted by the *Gibbs measure*

$$P[g] = \frac{e^{-S[g]}}{Z}, \quad Z = \int \prod_e dg_e e^{-S[g]}, \quad (2)$$

where Z , the *partition function*, normalises and the integral runs over the group with its invariant (Haar) measure. Wilson's construction [22] takes S to be a sum over faces of a class function of that face's holonomy, which by §3.2 makes S gauge invariant by construction. A theory built this way is a *lattice gauge theory*.

Supplying an action is what distinguishes a model from a description. Without one, each local weight is a separate stipulation; with one, the weights follow from a single choice.

3.4 From a Lattice Gauge Theory to a Spin Foam

For a compact group, the Peter-Weyl theorem expands a class function in the *characters* χ_j of the irreducible representations, indexed by a label j . For the circle group $U(1)$ these labels are the integers; for $SU(2)$ they are the spins $0, \frac{1}{2}, 1, \dots$, which is where the name comes from.

Expanding face by face and then integrating out the edge variables converts an integral over group elements into a sum over labels. Each edge integral is an average over the group,

which projects onto the subspace of tensors left fixed by the group action; a basis vector of that subspace is called an *intertwiner*. There remains a sum over labelled two-complexes carrying

- a *spin* j_f on each face, the representation label surviving the expansion;
- an *intertwiner* i_e on each edge, recording how the representations meeting there may be combined;
- a *vertex amplitude* A_v , the number obtained by contracting the intertwiners that meet at a vertex.

That sum is a *spin foam*. In quantum gravity the same object arises as the history swept out by a spin network, and the vocabulary is inherited from that setting; here it is used for the structure alone, with no claim that a street is a quantum system.

3.5 Detailed Balance and Circulation

A stochastic dynamics satisfies *detailed balance* when, in its stationary state, transitions between each pair of configurations balance in both directions. Kolmogorov's criterion [12] states the condition combinatorially: detailed balance holds exactly when, around each closed cycle of configurations, the product of forward transition rates equals the product of backward rates. A dynamics satisfying it carries no net probability current, hence no persistent circulation, whatever the shape of its weights. Producing circulation therefore requires breaking that equality, and §7 identifies which term in this model breaks it.

4 The Model: Streets as Two-Complexes

4.1 Cell Assignment

The complex $\mathcal{F} = (V, E, F, \partial_2, \partial_1)$ assigns kinds of history to dimensions rather than kinds of thing, as set out in Table 1. A person, a place, a store and a brand are edges. They differ in their labels and in the statistics of their worldlines, and share an ontological category; an implementation accordingly needs one cell type and dispatches on kind nowhere. Attention becomes a *face*, the history of a relation between a person and something else, rather than a property the person holds.

A relation's history is a **bigon** (Fig. 1). If entities a and b stand in a relation over an interval that opens at event v_1 and closes at v_2 , the face bounded by their two worldline segments satisfies

$$\partial_2 f = e_a|_{[v_1, v_2]} - e_b|_{[v_1, v_2]}, \quad (3)$$

and (1) holds because both segments run between the same two events, so the endpoints cancel.

4.2 Cycles and the Configuration Space

Closed circuits of relations are the objects the spin sector is built from. Let $x_1, \dots, x_n, x_1$ be a cycle of entities joined by relation faces f_i between x_i and x_{i+1} . By (3) the boundaries telescope,

$$\partial_2 \left(\sum_i f_i \right) = \sum_i (e_{x_i} - e_{x_{i+1}}) = 0, \quad (4)$$

so a circuit of relations is an element of $\ker \partial_2$. Section 6 shows that this kernel is the configuration space of the spins, which gives such circuits an operational role beyond bookkeeping: in a district the natural circuits are the city blocks, and §8.1 reports how far they span the kernel in one case.

Cell	History Recorded	Label Carried
vertex v (0-cell)	an <i>event</i>	amplitude A_v
edge e (1-cell)	an <i>entity's worldline</i>	intertwiner $i_e \in \text{Inv}(\bigotimes_{f \supset e} V_{j_f})$
face f (2-cell)	a <i>relation's worldsheet</i>	spin $j_f \in \text{Irr}(G)$

Table 1: The cell assignment. The dimension of a cell records what kind of history it is. The objects in the third column are those of §3.4.

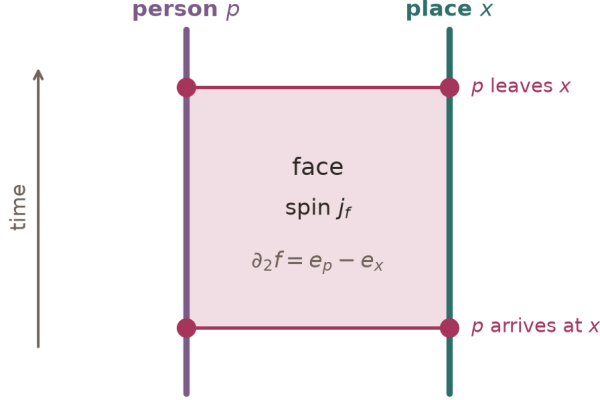


Figure 1: A relation's history is a bigon: two worldline segments and two events, with $\partial_2 f = e_p - e_x$. A person is an edge, exactly as a place is. This figure is a schematic and is purely mathematical.

4.3 The Face-Degree Condition

In a lattice gauge theory a link lies in several faces. Here a worldline segment lies in as many faces as the entity has *concurrent* relations, and where that number is one the Gauss law of §6 forces $j_f = 0$, emptying the gauge sector while a simulation continues to run.

Definition 4.1 (Face-Degree Condition). *Each one-cell must lie in at least three two-cells.*

The condition constrains the relation repertoire rather than the geometry. A place with two adjacencies and no other relation fails it, so a model of this kind requires relations beyond adjacency: occupancy, sightlines, affiliation. Section 8.2 reports what this costs in practice.

4.4 Two Orientations, and Which One Leaves Room to Sample

The three cells can be read in either order. The assignment of Table 1 puts entities on one-cells and relations on two-cells. The reverse puts *objects* on two-cells, so that a person carries a spin directly, and *occasions* on one-cells, so that what constrains the objects is a situation several parties are jointly in. The second reading has an attraction the first lacks: the holonomy around the occasions bounding one person measures the incoherence among that person's own engagements, which is a property of the person rather than of the street.

A counting argument separates them. Since $\dim \ker \partial_2 = n_2 - \text{rank } \partial_2$ and the rank of ∂_2 is bounded by the number of one-cells,

$$\dim \ker \partial_2 \geq n_2 - n_1, \quad (5)$$

so an excess of two-cells over one-cells guarantees a configuration space at least that large. The implication runs one way only. Where one-cells are the more numerous the bound is vacuous,

and whether room to sample survives then depends on the rank rather than on the counts, which is a matter for measurement rather than for arithmetic. Section 8.6 reports both cases.

A second constraint acts in the same direction: a one-cell lying in only two two-cells has its neighbours' spins forced equal by the Gauss law of §6, so the face-degree condition of §4.3 applies to whichever cell carries the intertwiner.

Under the first orientation the excess called for in (5) is of relations over entities, which a relational description supplies without effort. Under the second it is of objects over occasions, and a pairwise relation promoted to an occasion supplies neither the excess nor the arity, since it gathers two parties while pairs outnumber parties. The second orientation therefore calls for a small number of occasions, each gathering many parties. Section 8.6 reports the measurement, and the reading it suggests is that the pairwise relation is the artefact: the object is the occasion, meaning this person, that storefront, this stretch of pavement, at this hour.

5 The Gauge Sector

5.1 Valuation Frames and Gauge Freedom

“Expensive”, “prestigious” and “fashionable” are defined relative to a local comparison class, and different parties at the same place carry different classes. A statement about the district is meaningful when it survives an independent change of frame at each entity, which is the gauge transformation of §3.2. This formalises the position that valuation admits no view from nowhere, while leaving room for statements that hold regardless of viewpoint: the quantities surviving such a change are the ones a claim about the district can rest on.

The connection g_e lives on an entity's own worldline and transports its frame along its history. Around the bigon (3),

$$h_f = g_a g_b^{-1}, \quad (6)$$

whose class functions are invariant. The reading is direct: the curvature of a relation is the mismatch between how the two related parties' frames drifted while the relation held. An identity holonomy means the encounter left their relative valuation where it began.

5.2 Choice of Gauge Group and the Static Limit

We take $G = U(1)$, the circle group: compact, so the partition function exists without gauge fixing, and with the integers as representation labels, so the spin foam is an integer flux model with a Kronecker-delta constraint and no group integration at runtime.

A limitation appears here that an earlier design of this model overlooked. The gauge transformation acts at zero-cells, that is, at events. In a static slab there are only two events, so the freedom degenerates to a single global rotation and $h_f = \theta_a - \theta_b$ telescopes to zero around a circuit, if θ is treated as a real number. Read naively, the static gauge sector is flat and empty.

Compactness recovers it, which is the reason $U(1)$ rather than $\mathbb{R}$ was the default. Since θ is defined modulo 2π , the winding of a circuit c ,

$$n_c = \frac{1}{2\pi} \sum_{f \in c} \text{wrap}(\theta_a - \theta_b) \in \mathbb{Z}, \quad \text{wrap} : \text{into } (-\pi, \pi], \quad (7)$$

is an integer that survives relabelling, and it carries the curvature. In social terms it counts the whole turns of valuation frame accumulated along a closed circuit of the district, independently of where the circuit began and of each entity's arbitrary zero.

The static model is therefore an XY, or Villain, model on the relational graph [19, 14], whose topological defects carry the curvature and whose dual is the integer flux model of §6. Local-in-time gauge invariance returns once events subdivide the worldlines.

5.3 The Alignment Action

Following §3.3,

$$S[g] = \sum_{f \in F} S_f(h_f), \quad S_f(h) = -\beta_f \operatorname{Re} \chi_1(h), \quad (8)$$

requiring only that S_f be a class function of h_f , hence gauge invariant, and bounded below. The coupling β_f is the alignment stiffness of a relation: how strongly it holds the two parties to a common frame. So (8) is a social conformity model derived from the gauge action rather than posited, with β_f saying how hard the relation pulls.

The weights are real and positive and the measure is that of (2). Amplitudes and interference stay out: with real positive factors an exponent κ on $|\mathcal{A}|^\kappa$ is absorbed by reparametrisation, making it a flat direction in the likelihood rather than a parameter.

6 The Spin-Foam Representation

6.1 Character Expansion

The step that supplies the group structure is the expansion of §3.4, applied to the lattice gauge theory of (8) [22, 13]:

$$Z = \int \prod_e dg_e \prod_f e^{-S_f(h_f)}, \quad e^{-S_f(h)} = \sum_{j \in \operatorname{Irr}(G)} d_j c_j^{(f)} \chi_j(h), \quad (9)$$

with d_j the dimension of the representation j . Substituting, each g_e appears once for each face containing e , and the edge integral is the group average that projects onto the invariant subspace, $\int dg \otimes_{f \supset e} \rho_{j_f}(g) = \sum_{i_e} |i_e\rangle \langle i_e|$. Contracting the remaining index lines at the vertices leaves

$$Z = \sum_{\{j_f\}} \sum_{\{i_e\}} \prod_f d_{j_f} c_{j_f}^{(f)} \prod_v A_v(\{j_f\}, \{i_e\}), \quad (10)$$

with A_v the invariant contraction of the intertwiners meeting at v . The spins are derived, the intertwiners are derived, and the vertex amplitude is derived: it is a group-theoretic contraction rather than $\exp[-\beta \mathcal{E}_v]$ with $\mathcal{E}_v$ chosen by an analyst.

For $U(1)$ with the Wilson action, $c_j = I_j(\beta)$ is a modified Bessel function and the edge integral collapses to a constraint:

$$\int_0^{2\pi} \frac{d\theta_e}{2\pi} \exp \left[i\theta_e \sum_{f \supset e} \epsilon_{f_e} j_f \right] = \delta((\partial_2 j)_e), \quad Z = \sum_{\substack{j \in C_2(\mathcal{F}; \mathbb{Z}) \\ \partial_2 j = 0}} \prod_f I_{j_f}(\beta_f). \quad (11)$$

The delta function is the Gauss law referred to above, and the configuration space is the integer two-cycles of §4.2: integers, local constraints, positive weights.

6.2 Triviality of the Vertex Amplitude under $U(1)$

The intertwiner space at a $U(1)$ edge is one-dimensional when the charges balance and empty otherwise, so $A_v \equiv 1$: with a single intertwiner per edge the vertex has one option available to it. The whole content of the $U(1)$ vertex is the Gauss law. This is a simplification and equally a limitation: the vertex amplitude remains untested in this work, because $U(1)$ supplies only a trivial one. Under $SU(2)$ the space $\operatorname{Inv}(\otimes_f V_{j_f})$ can be larger, by an amount set by which representations meet at the edge rather than by how many. Four faces carrying spin one-half give a two-dimensional space and six give a five-dimensional one, while an odd number of spin-one-half faces gives an empty space, since half-integer spins couple to the trivial

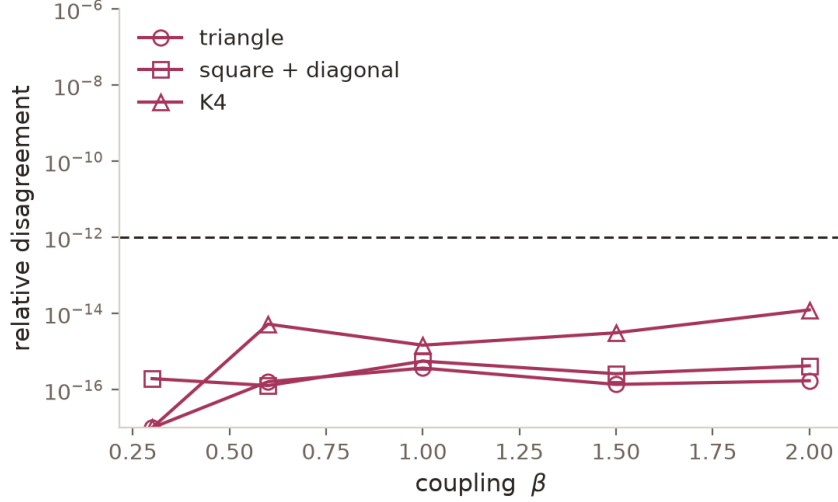


Figure 2: Two independent evaluations of Z agree to machine precision on each complex tested, across couplings β ; the dashed line marks 10^{-12} . The character expansion is thereby a derivation rather than a claim. This figure is purely mathematical.

representation only in even numbers. Mixed assignments interpolate: two spin one-half faces and one spin-one face give a one-dimensional space. The dimensions are obtained by Haar integration of the characters and checked against fourteen known couplings. Where the space is more than one-dimensional the edge carries a choice, and the vertex contraction becomes a $\{3nj\}$ -symbol.

6.3 Numerical Verification of the Expansion

Whether (10) is a derivation or a claim is decidable. The frame-angle integral and the spin-foam sum are evaluated independently: quadrature over angles on one side, a sum over integer two-cycles with modified Bessel weights on the other, with I_n itself computed by quadrature from its integral representation, so that no library stands between the claim and the check. Over three complexes and five couplings the worst relative disagreement is 1.3×10^{-14} (Fig. 2). This verification is independent of the district studied below.

7 Pedestrian Dynamics and Minimal Coupling

Events fire one at a time by the next-reaction method [8], so the clock is derived from the rates rather than imposed, and the maximum-weight-independent-set problem that a synchronous update would require is avoided.

A person at place x moves to a neighbour y with weight

$$w_y = \underbrace{\exp[\beta \cos(\theta_x - \theta_y)]}_{\text{scalar potential}} \cdot \underbrace{\exp[-q \text{wrap}(\theta_x - \theta_y)]}_{\text{minimal coupling}}. \quad (12)$$

The two factors do different work, and the difference carries the model's prediction. The first is symmetric under exchanging x and y , so by Kolmogorov's criterion (§3.5) it leaves circulation untouched regardless of strength. The second is the minimal coupling $D_\mu = \partial_\mu - qA_\mu$, the standard way a connection enters the motion of a charged particle.

Around a closed circuit the two factors behave differently. A circuit visits the same places whichever way it is traversed, so the scalar potential contributes the same product in both directions and cancels from their ratio. The minimal coupling does not cancel: the step from x

to y contributes $\exp[-q \text{wrap}(\theta_x - \theta_y)]$ and the reverse step its reciprocal, so each step enters the ratio squared, and

$$\prod_c \frac{w_{\text{fwd}}}{w_{\text{bwd}}} = \exp \left[-2q \sum_{f \in c} \text{wrap}(\theta_a - \theta_b) \right] = e^{-4\pi q n_c}, \quad (13)$$

with n_c the winding (7) of that circuit. For non-zero charge the equality of forward and backward products therefore fails when the winding is non-zero, and by an amount the winding fixes.

The model therefore predicts more than the statement that valuation affects movement: circulation is sourced by the topological part of the frame field alone, so that a region of large local frame disagreement and zero winding carries zero net circulation. This follows from which factor is antisymmetric rather than being built in, and it is falsifiable.

Remark 7.1 (Circulation Requires a Driven System). By §3.5 a closed equilibrated simulation shows zero circulation by construction, whatever the curvature, since circulation is a non-equilibrium quantity. A test of the prediction therefore requires a driven system, with agents entering and leaving. Reporting circulation from an undriven run would report sampling noise.

8 Case Study: Ginza

The remaining sections illustrate the formalism on one district. The street geometry is taken from OpenStreetMap; the commercial content, the agents and the coupling constants are constructed to exhibit the model rather than to describe the place. The case study establishes the reach of the formalism and its limits on a substrate with realistic connectivity.

8.1 Street Geometry from OpenStreetMap

Places and their adjacencies are taken from a bounded study area of Ginza, Chūō-ku, Tokyo [15]: the extract is reduced to its largest component, degree-one spurs are pruned, degree-two chains are contracted, and links on commercially classified streets are subdivided so that a place is a junction or a shopfront position. Bounded faces of the planar embedding are extracted by the standard rotation-system walk and give the city blocks.

Two properties of the substrate matter. First, by (4) each block is an element of $\ker \partial_2$, so the blocks supply short, local generators of the configuration space. Second, the walkable network departs from planarity: over this study area the embedding has genus 24, so the blocks span 634 of the 682 independent cycles and the remaining 48 run through crossings that a planar embedding conceals.

8.2 Constructed Commercial Content

Onto the substrate a constructed layer is placed: stores on commercial frontage, sightlines within a capped range, and brands. A brand is admitted as an entity only if it has at least three locations, which is forced rather than chosen. A one-location brand entity would hold a single relation and violate the face-degree condition of §4.3, and the single-location case is better represented by an unbranded store, which is the control the experiment of §8.5 requires.

The face-degree condition binds. On the bare street skeleton 403 of 1362 places fail it, holding only two adjacencies; adding occupancy and sightlines and then reducing to the 3-core of the relational graph restores it, at a cost of 68 places. The final complex has 1397 entities and 3374 relations, with $\dim \ker \partial_2 = 1978$.

8.3 Sampled Spin Configurations

Figure 3 shows a patch of the foam reconstructed from a recorded event history, and Fig. 4 a spin configuration sampled from (11) on the full complex: 392 of 3374 faces carry $j_f \neq 0$,

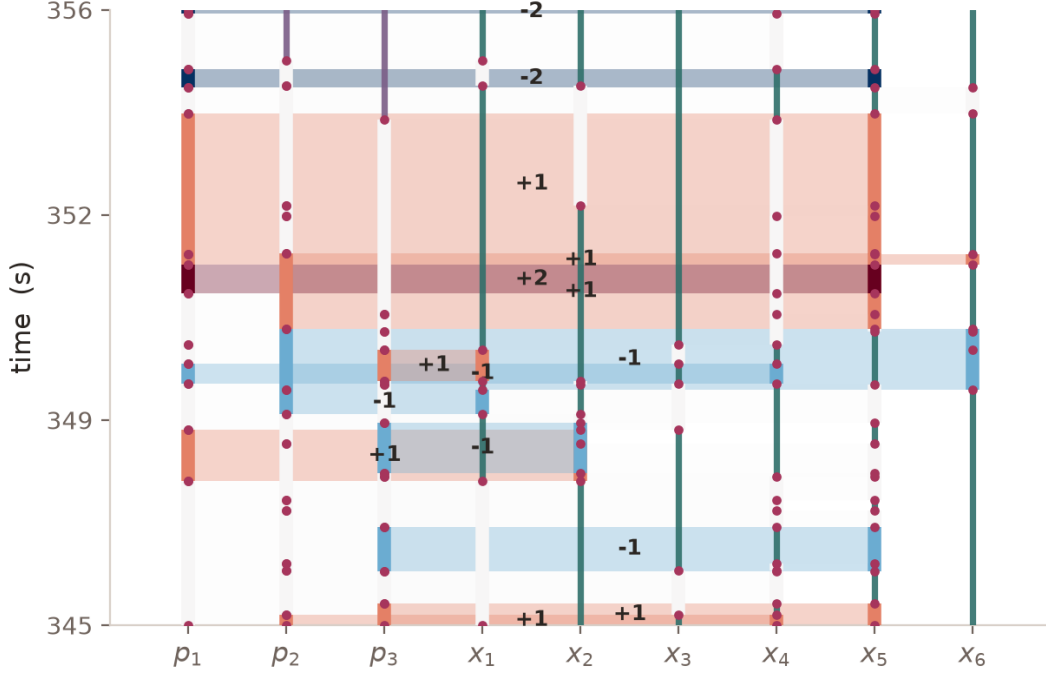


Figure 3: A patch of the foam reconstructed from a recorded event history, with time on the vertical axis. Vertical strands are worldlines, persons p_i and places x_i , drawn identically because they are the same kind of cell; shaded bands are the bigons of (3), labelled by their integer spins; dots are events. The net flux at each worldline vanishes. *Synthetic content on real geometry.*

Label	Test	Control	Outcome
D0	round-trip of the character expansion	not applicable	pass , 1.3×10^{-14}
D1	effect of the gauge structure	flat connection	pass , $2.95 \times$
D2	circulation against winding number	permutation null	pass , $r = -0.740$, $p < 0.0005$
D2b	circulation against local disagreement	matched predictor	pass , $r = +0.005$, $p = 0.91$
D3	flux inside a three-location brand	scattered, unaffiliated	fail (§8.5)
D4	order of encounters	abelian endpoint	pass , 0 to 1.59 rad
D5	effective geometry	square lattice	pass , $d_s \approx 1.8$
D6	effective distance from correlation decay	street distance	pass , $\xi = 1.97$ hops

Table 2: Case-study results. D0, verified in §6.3, is listed for completeness and is independent of the district. Thirty-five invariants are additionally enforced in tests, including (1) together with its ability to fail and the face-degree floor of §4.3.

forming closed loops around city blocks, with $\max |(\partial_2 j)_e| = 0$ over the 1397 entities. The Gauss law holds by construction, since each Monte Carlo move adds ± 1 times a basis element of $\ker \partial_2$ and a basis element is already a two-cycle, so the constraint sits in the move set rather than being imposed by rejection. That is the operational payoff of (4), and the reason the blocks matter for sampling as well as for bookkeeping.

8.4 Experiments and Controls

Each test is run against a matched control. Table 2 summarises; the stipulated couplings and the seed are fixed throughout. Agents are injected at the study area’s subway entrances and removed after a finite dwell, which supplies the drive that the remark in §7 shows to be necessary.

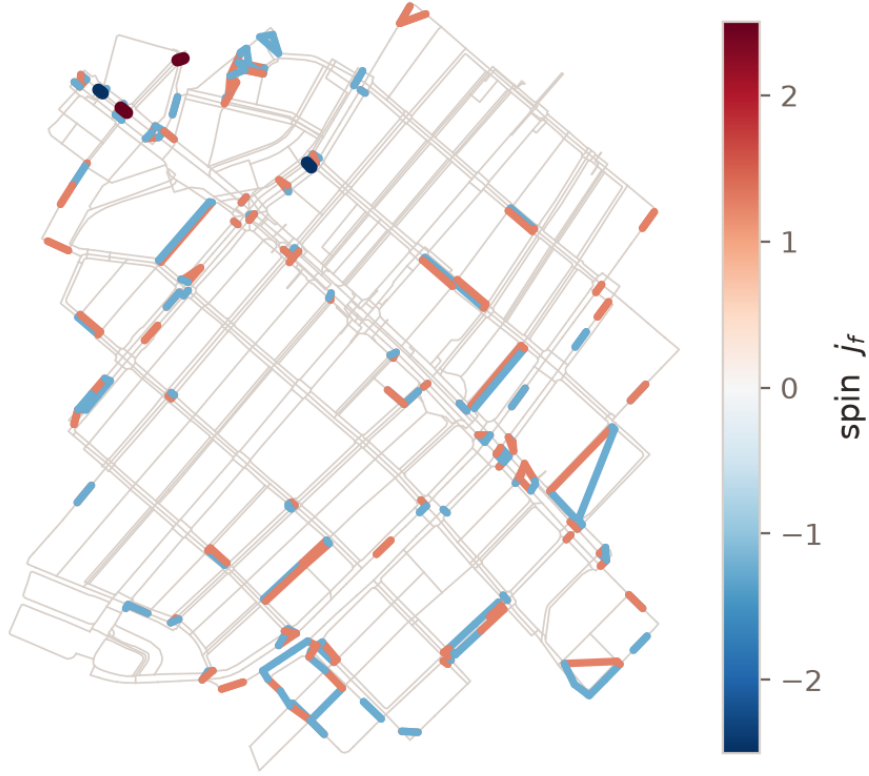


Figure 4: A spin configuration sampled from (11) on the full complex. Each coloured line is one relation-face, drawn between the two entities it relates; the flux forms closed loops around city blocks, which is the appearance of an integer two-cycle. *Synthetic content on real geometry.*

Effect of the Gauge Structure. Identical complex, drive, event count and seed; the control has each valuation frame equal, so each holonomy is the identity. Mean $|C|$ is 0.108 against 0.037, a factor of 2.95, and the concentration of where agents go more than doubles (Gini 0.674 against 0.249). A null result here would have indicated that the gauge apparatus was decoration, and reporting that and stopping would then have been the appropriate response.

Circulation Tracks the Winding Number. Circulation against winding number gives $r = -0.740$ ($p < 0.0005$, 2000 permutations); blocks at $n_c = -1, 0, +1$ have mean circulation $+0.401, -0.006, -0.419$. Against the matched non-topological predictor, mean absolute local frame mismatch with the winding discarded, $r = +0.005$ ($p = 0.91$). The second number carries the result, because it is the one the symmetric factor of (12) was predicted to leave inert (Fig. 5). A model whose forces are gradients of potentials leaves this pattern out of reach, as does a non-topological reading of the frame field.

Effective Distance from Correlation Decay. The *connected correlator* of two quantities is their covariance, the mean of their product with the product of their means subtracted, and in a system with a characteristic scale it decays exponentially with separation. Over a 50-sample equilibrium ensemble the connected frame correlations decay as $e^{-d/\xi}$ with correlation length $\xi = 1.97$ hops, over the range where the correlator is resolved above the ensemble noise floor. Setting $d_{\text{eff}} = -\xi \log C_c$ gives a distance departing from street distance with a spread of 2.02 hops: pairs at the same street distance sit at very different effective distances. The

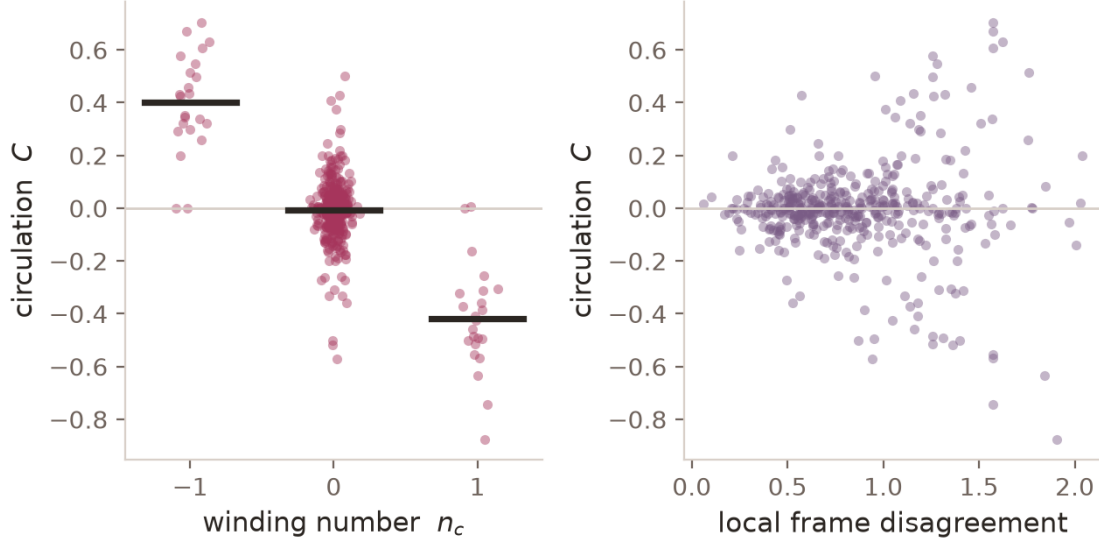


Figure 5: Circulation is sourced by the topological part of the frame field alone. *Left:* circulation against the winding number of each block, with horizontal bars giving group means. *Right:* the same circulation against mean local frame disagreement, with the winding discarded. *Synthetic content on real geometry.*

provenance matters more than the number, since it is read off the model, where a construction from analyst-supplied cost coefficients would return whatever was put in.

Order of Encounters. Section 5.2 takes the connection abelian, which should make the sequence of a walk immaterial while leaving its content intact. The claim is tested on the walks the simulation produces, by permuting the steps of each walk so that the multiset of encounters is held fixed and only their order varies. Each step is given a rotation whose angle is the frame mismatch the $U(1)$ model already carries and whose axis lies in a cone of half-angle α about a common direction, so that α interpolates between an abelian and an isotropic connection.

Under $U(1)$ as built, reordering moves the endpoint by at most 1.8×10^{-15} radians over 300 walks, which is the arithmetic saying that phases commute. The $SU(2)$ sweep begins at exactly zero and rises to 1.589 radians when the axes are isotropic (Fig. 6). The result beyond the confirmation is how quickly order begins to matter: an axis spread of 0.05 radians, valuation registers almost aligned, already separates a twelve-step walk from its permutations by 0.10 radians. Order-sensitivity does not wait for a strongly non-abelian group.

An Effective Geometry, and the Scale It Holds At. A complex behaves like a manifold of dimension d where a walk on it returns to its origin as $P(t) \sim t^{-d/2}$, so the spectral dimension $d_s(t) = -2 \frac{d \log P}{d \log t}$ is the quantity to measure, and the test is whether it settles rather than drifting. The return probability is obtained from the spectrum of the walk operator, so no sampling noise enters. A square lattice of comparable size, whose spectral dimension is known to be two, serves as the control against which settling is judged.

Over walk lengths from 8 to 200 the street graph gives $d_s = 1.80$ and the full relational complex $d_s = 1.89$, against 1.77 for the lattice, each drifting by about 0.3 across the band (Fig. 7). An effective two-dimensional geometry therefore exists, over roughly a decade and a half of scale. The fall beyond $t \approx 300$ appears in the lattice as well, which identifies it as the walk having explored a finite complex rather than as a property of the district.

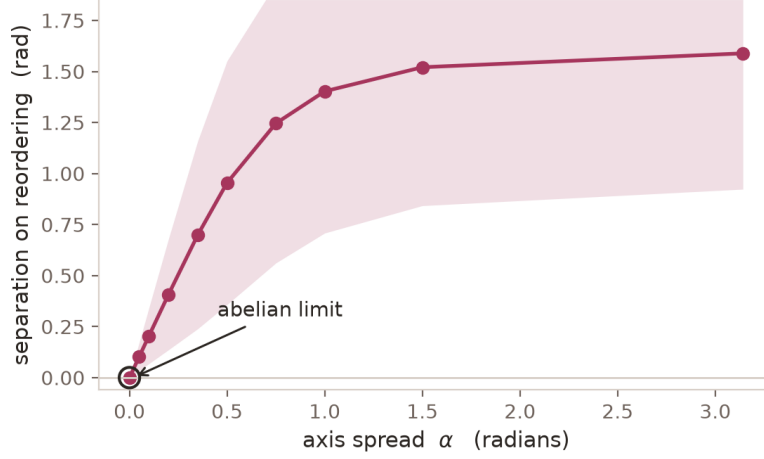


Figure 6: Separation between a walk and its reorderings, against the spread of the rotation axes. The multiset of encounters is held fixed, so the quantity plotted is the effect of order alone. The abelian endpoint at $\alpha = 0$ is exactly zero. *Synthetic content on real geometry.*

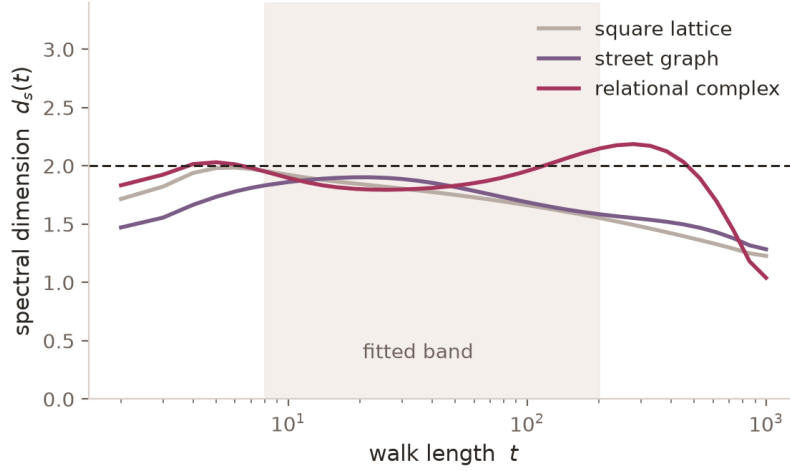


Figure 7: Spectral dimension against walk length. The shaded band is the range over which the dimension is read. The lattice, whose spectral dimension is two, drifts and falls in the same way, which is what separates a finite-size artefact from a property of the complex. *Synthetic content on real geometry.*

8.5 Flux Inside a Three-Location Brand

A brand with three nearby locations is the case that motivated the exercise: the intuition that such a configuration is one distributed structure rather than three stores, and that the region it encloses is somehow under its influence. Formally the expectation was a Steiner point. In a *confining* gauge theory, separated charges are joined by tubes of flux whose energy grows with their length, so a three-charge configuration minimises total tube length over an interior junction and the flux fills the enclosed region rather than sitting at the storefronts [22, 17].

The test compares three configurations differing only in the affiliation layer, on the identical complex, geometry, store set and seed sequence: a compact affiliated triangle (254/254/251 m, enclosing 59 places and 26 block circuits), three affiliated stores far apart, and the same three triangle stores with affiliation cut. The observable is the density of windings inside the region relative to the district rate, and also within a corridor band along the three connections,

Configuration	Interior	Corridor
triangle (affiliated, compact)	0.93 ± 0.10	0.90 ± 0.07
scattered (affiliated, far apart)	1.14 ± 0.09	0.99 ± 0.06
unaffiliated (same three, affiliation cut)	1.18 ± 0.11	1.00 ± 0.07

Table 3: Flux density inside the enclosed region and along the corridor band, each as a ratio to the district rate, over forty seeds per configuration, given as mean $\pm$ standard error. The compact affiliated triangle carries less flux than either control, reversing the predicted direction.

which separates the Steiner reading from the flux-tube reading. Forty independent seeds per configuration give Table 3.

The measured direction is reversed. The affiliated compact triangle carries less interior flux than either control ($p = 0.12$ and $p = 0.10$), and a coherent mechanism exists for that which is the opposite of the one assumed: pinning three distant stores to a shared brand frame imposes coherence across the region between them, which expels windings rather than concentrating them.

The diagnosis carries further than the failed test. The model actually built is an XY, or Villain, model: it has vortices, and it lacks flux tubes between charges, so the mechanism by which an enclosed region would fill is absent. The prediction is unentailed by the model rather than tested and refuted. Testing it properly requires the dynamic foam with the connection on relations, which this work leaves for later.

The power of the test is stated with the result, because a null is worth as much as the effect it could have seen and no more: seed-to-seed standard deviations of 0.61 and 0.41 give minimum detectable differences of 0.38 and 0.26 at 80% power with forty seeds. The null therefore rules out an effect above about $0.26\times$, and leaves smaller ones open.

Remark 8.1 (Reversal of Sign Under a Larger Sample). At ten seeds the differences were $+0.11$ and $+0.20$, the direction predicted. At forty they are -0.21 and -0.25 . Only the sample size changed. A trend that reverses when the seed count is quadrupled was noise throughout, which indicates why the single-run version of this comparison was uninformative.

8.6 Persons as Faces

Under the orientation of Table 1 a person walks on the valuation field without appearing in the complex that carries it. The reverse orientation of §4.4 puts persons in directly, and the counting law (5) says what it costs.

Table 4 reports the measurement. Promoting each pairwise relation to an occasion leaves dimker $\partial_2 = 1$, a single global mode that leaves the sampler no room, and the arity floor of §4.3 fails on each occasion at once. Grouping the relations into place-situations clears both constraints. Giving each recorded stay its own occasion then adds persons as faces but reverses the counting law, with sixty-five thousand one-cells against seventeen hundred two-cells, and the configuration space is squeezed back to 47. An occasion per city block per band of time satisfies both constraints, gathering the block’s places, the stores standing on them, and whoever passed through: median arity 23, and a configuration space of 786 dimensions carrying 300 persons alongside the places, stores and brands (Fig. 8).

The construction turns on two points. (1) **Closure fixes the signs.** The signs around each object-face sum to zero, in direct analogue with the bigon’s $e_a - e_b$, so an object of odd degree has one membership dropped. (2) **The reduction iterates.** Enforcing the arity floor proceeds like the 3-core reduction: removing an occasion lowers the degree of the objects in it, which changes which membership is dropped to close each boundary, which can push a further occasion below the floor.

Assignment	Two-Cells	One-Cells	Mean Arity	$\dim \ker \partial_2$
entities on edges, relations on faces	3374	1397	n/a	1978
objects on faces, pairwise occasions	1397	3374	1.8	1
objects on faces, place-situations	1397	1022	5.1	376
objects on faces, one occasion per stay	1697	65430	4.5	47
objects on faces, block situations	1697	1219	24.8	786

Table 4: The two orientations measured on the same world. The last row carries 1041 places, 337 stores, 19 brands and 300 persons as two-cells, with $\partial_1 \partial_2 = 0$ and the arity floor both satisfied.

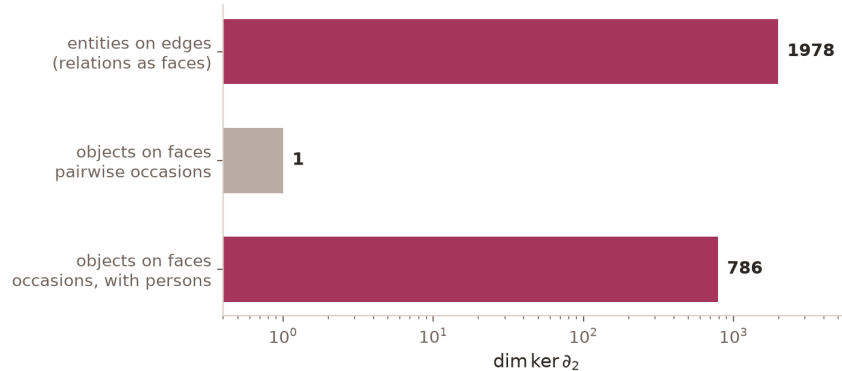


Figure 8: Size of the configuration space under each reading. A pairwise relation fails as an occasion: with two parties the Gauss law forces their spins equal, and the space collapses to the global mode. *Synthetic content on real geometry.*

8.7 Corrections Arising from the Case Study

Running the case study corrected four statements in the design that generated it.

1. **The configuration space is on the foam, and not on the planar map.** Built with blocks as two-cells, $\ker \partial_2 = 0$ identically, because the bounded faces of a planar embedding are linearly independent. The telescoping argument (4) is correct and is a statement about the foam, where entities are one-cells and relations are two-cells; there the dimension is 1978. The dimension shift separates a rich configuration space from an empty one.
2. **The street network departs from planarity**, at genus 24 over the study area (§8.1).
3. **In the static slab the local gauge freedom degenerates**, and compactness rather than the gauge group is what keeps the curvature non-trivial (§5.2).
4. **The Steiner-point prediction is unentailed** by the model built (§8.5).

Six implementation errors are also on record, and they share a property: each ran without error and produced a plausible figure. A complex of the wrong dimension; a symmetric move rule that leaves circulation untouched regardless of strength; a cache keyed on everything except the input that mattered; a figure whose passing state was a blank rectangle, indistinguishable from a plot that had silently failed; the raw correlator standing in for the connected one, so that a fitted correlation length reported the magnetisation instead; and a fit run through the ensemble noise floor. Each was caught only by an invariant that could fail or a control that could disagree.

9 Limitations

- **Lack of Empirical Investigation.** The case study is an illustration, so the reported numbers are properties of the model. Confronting the model with observation, through footfall counts, tenancy records and frontage surveys, is the next step, and this paper leaves it undone.
- **Derivation and Finding Are Distinct.** That (10) is correct speaks to the formalism rather than to the subject matter.
- **Arbitrariness Is Relocated Rather Than Eliminated.** Supplying G and S_f replaces hundreds of stipulated local weights with two global stipulations: a real gain, and a bounded one.
- **The Vertex Amplitude Remains Untested.** The group $U(1)$ supplies only a trivial vertex amplitude, so this work exercises none (§6.2). The $SU(2)$ transport used for the order test carries holonomies rather than intertwiners, so it leaves the vertex amplitude equally untouched.
- **The Effective Geometry Holds Over a Band.** The spectral dimension settles between walk lengths of 8 and 200 and falls away beyond, so the geometry is a statement about a range of scales rather than about the complex as a whole.
- **The Second Orientation Is Built but Not Driven.** Section 8.6 constructs the assignment in which persons are faces and samples its configuration space, and stops there. Re-running the dynamics under it, where curvature would be a property of a person rather than of the street, would put the two orientations to a comparison the present work sets up without performing.
- **Dual Use of the Counterfactual Machinery.** The machinery that predicts a crowding hazard is the machinery that optimises the capture of attention. A deployment should state which of the two purposes it serves, and on whose behalf. The demonstrator runs on constructed agents, which argues for keeping the demonstrator constructed rather than licensing a relaxation.

10 Open Questions

The limitations of Section 9 concern work the present paper leaves undone. The questions collected here are of a different kind: the construction does not settle them, and each admits an answer that would count against it.

10.1 Questions About the Formalism

Which Group the Subject Matter Selects. The choice of $U(1)$ in §5.2 was made for tractability, and it carries the substantive claim that valuation is one-dimensional and cyclic. Whether a district requires a non-abelian group is decidable in principle, because the two differ observably: under an abelian connection the order of a person's encounters leaves the outcome unchanged, and the measurement of §8.4 shows that a modest departure from commuting axes already separates a walk from its permutations. Ordered encounter data would settle the choice, and the result would be informative in either direction, since a finding that order does not matter would support the simpler group.

Whether the Complex Should Itself Be Summed Over. In the gravitational setting the partition function is not evaluated on a single two-complex; a sum over complexes, or

a refinement limit, carries part of the dynamics. In the present model the complex is built from the substrate and the constructed layer and then held fixed, so which relations exist is an input rather than a variable. Allowing it to vary would make the relational structure of a district dynamical, with shops opening and closing and sightlines appearing as a frontage clears. Whether such a sum can be defined here with a controlled weight on each complex, and whether it converges to a stable description, is unresolved. A finding that results move sharply with the constructed relation repertoire would indicate that the fixed complex does more of the work than the action does.

What the Boundary States Represent. A spin foam is bounded by spin networks, and its amplitude is read as a transition between them. The slab used here carries an initial and a final time boundary whose labels have been treated as bookkeeping. Whether a boundary spin network admits a reading as the relational state of a district at an instant, and whether a weight between two such states is interpretable as anything about the interval between them, remains open.

Whether the Vertex Amplitude Can Be Given Content. Under $U(1)$ the vertex carries the Gauss law and little else (§6.2). A non-abelian group enlarges the intertwiner space and turns the vertex contraction into a $\{3nj\}$ -symbol. Whether that weight corresponds to a property of a social event, rather than arriving as a formal consequence carried along by the choice of group, is untested, and until it is tested the vertex has not earned its place in the construction.

10.2 Questions About Encoding Cultural Content

Which Map Takes Cultural Attributes to Representations. Each entity here carries one angle, so cultural position is represented as a single cyclic coordinate, and each relation carries one spin standing for the stiffness with which the relation holds two parties to a common frame. Cultural position plausibly has more than one dimension, since price tier, formality, novelty and subcultural affiliation need not vary together, and the paper selects among the available encodings on no evidence. A product group $U(1)^k$ supplies independent coordinates. A non-abelian group is called for where two attributes fail to commute, meaning that acquiring one and then the other leaves a position different from the reverse order. Higher representations could carry the salience of a relation, or could index a discrete ladder of taste levels, and those are different models with different consequences. Estimating such a map from evidence, whether from tenancy records, signage, price data or imagery, is the harder half of the question. A negative answer is available: should an unconstrained real vector attached to each entity fit observations as well as a group-valued encoding does, the group structure would be decoration, and the invariance requirement it imposes would be a cost without a return.

How Persistent Identity Is Assigned. An entity is a worldline, which presumes that it remains the same entity along its length. A store under new ownership, a brand after repositioning, and a person whose taste has shifted are cases where the presumption is doubtful, and where the worldline is cut changes the complex and the quantities computed on it. The formalism supplies no criterion for the cut, and the choice is currently made by the modeller without argument.

Where the Boundaries of an Entity Lie. A place is a junction or a frontage position, fixed by the subdivision of §8.1. Interiors, floors and the arrangement of units within a building lie outside the representation, and a finer decomposition would change the face-degree structure and the effective dimension. Whether a scale exists at which the description is stable, or whether the reported quantities move with an arbitrary choice of granularity, is open.

10.3 Questions About Computation and Scale

Whether Quantum Hardware Would Contribute Anything. The model is a compact lattice gauge theory, and lattice gauge theories of this kind are among the established targets for quantum simulation, in both digital and analogue forms [20, 2]. That prospect does not transfer to the model as built. Section 5.3 keeps the weights real and positive, so the equilibrium problem carries no sign problem and classical Monte Carlo samples it directly, which would leave quantum hardware addressing a difficulty the model does not have. Two circumstances would change the assessment. Real-time evolution of the driven system, as distinct from its stationary distribution, is where classical sampling of a lattice gauge theory becomes hard, and the dynamics of §7 is stated in real time. Giving the amplitudes a complex phase, which would make the object a quantum spin foam rather than a statistical model in the shape of one, would reintroduce the sign problem as well. Whether a complex phase admits a defensible reading in this setting is doubtful, since interference between social histories is not an evident phenomenon and §5.3 notes that the exponent on a real positive amplitude is absorbed by reparametrisation. The question that comes first is therefore about motivation rather than about hardware: whether a reason exists to make the model quantum, rather than whether a machine could run such a model once it were made.

How the Description Behaves Under Coarse-Graining. The spectral dimension settles near 1.8 over roughly a decade and a half of walk length (§8.4). Whether a renormalisation-group treatment yields a description stable under coarse-graining, or whether a commercial district is intrinsically multi-scale, with the effective dimension drifting as blocks are merged, is unresolved. The second outcome would confine the model to a band of scales that would then have to be stated with it. A related practical question is whether the construction survives enlargement from a district to a city, where the relation repertoire grows faster than the entity count and the cost of sampling grows with it.

10.4 Questions About Validation

Whether the Winding Has an Observable Counterpart. The model makes circulation depend on an integer rather than on the magnitude of local disagreement, which is its sharpest point of difference from gradient models. Whether anything in a district corresponds to that integer is untested. A negative result is available: persistent directional asymmetry in footfall around blocks, uncorrelated with the topological quantities recoverable from valuation data, would indicate that the winding is an artefact of the representation rather than a feature of the place.

The Standing of the Two Cell Assignments. Both orientations of §4.4 are constructible, and the counting argument selects one on the ground that it leaves a configuration space to sample. That is a statement about tractability rather than about which assignment better describes a district. Whether a person is better represented as a face carrying a spin, or as an edge constraining the relations that meet at them, remains open, and comparing the two on behaviour rather than on dimension would be the way to address it.

Whether Derivation from an Action Is the Favourable Trade. Section 9 records that supplying a group and an action relocates arbitrariness rather than removing it. Whether the relocation is a gain depends on a question no argument internal to the formalism can answer: whether observed valuation data can be fitted by weights of the derived form. Should the derived family prove too rigid to fit, the freedom that a stipulated model retains would be the more honest position, and the trade would have gone the wrong way.

11 Conclusion

A street district can be written as a compact lattice gauge theory of valuation on a two-complex whose one-cells are the worldlines of persons, places and stores alike and whose two-cells are the histories of the relations among them. Its spin-foam representation is the form in which the measure is manifestly positive, the constraints are local, and the sampling is possible, rather than an ornament on that statement. Curvature acquires a direct reading: the failure of a valuation frame to return to itself around a closed circuit, an integer attached to that circuit.

On the case examined the formalism has measurable effect. The gauge structure changes behaviour against a matched flat control by a factor of 2.95; circulation tracks a topological invariant and leaves its non-topological shadow inert; effective distance is read off the model rather than supplied to it. It also fails to produce an effect it was expected to produce, and the diagnosis of that failure, a prediction imported from a theory the model leaves uninstantiated, is the result most worth carrying forward.

Section 8.7 records the corrections that running the model imposed on the design, and they suggest the general point. A physics-borrowing model of a city is disciplined by whether its formal machinery can be made to contradict something, rather than by the plausibility of its analogy, since analogies are plausible by construction. The least costly place to find out is an implementation required to run controls alongside each claim.

The next stages follow from the limitations. Estimating the couplings from observation would turn the case study into a measurement. Driving the second orientation would let the two readings of the cell assignment be compared on behaviour rather than on the size of their configuration spaces. A genuine $SU(N)$ spin foam, with intertwiners and a vertex amplitude rather than transport alone, would give the vertex something to weigh. The present contribution is narrower and prior to those: the apparatus can be made to answer, and to return a negative answer where one is due.

Acknowledgements

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References

- [1] Baez, John C., “Spin foam models,” *Classical and Quantum Gravity* 15(7), 1998, 1827–1858. doi:10.1088/0264-9381/15/7/004.
- [2] Bañuls, Mari Carmen, Rainer Blatt, Jacopo Catani, Alessio Celi, Juan Ignacio Cirac, Marcello Dalmonte, Leonardo Fallani, Karl Jansen, et al., “Simulating lattice gauge theories within quantum technologies,” *The European Physical Journal D* 74(8), 2020. doi:10.1140/epjd/e2020-100571-8.
- [3] Barthélemy, Marc, “Spatial networks,” *Physics Reports* 499(1–3), 2011, 1–101. doi:10.1016/j.physrep.2010.11.002.
- [4] Battiston, Federico, Giulia Cencetti, Iacopo Iacopini, and Vito Latora, “Networks beyond pairwise interactions: structure and dynamics,” *Physics Reports* 874, 2020, 1–92. doi:10.1016/j.physrep.2020.05.004.
- [5] Batty, Michael, *The New Science of Cities*, MIT Press, Cambridge MA, 2013. doi:10.7551/mitpress/9399.001.0001.
- [6] Burstedde, Carsten, Kai Klauck, Andreas Schadschneider, and Johannes Zittartz, “Simulation of pedestrian dynamics using a two-dimensional cellular automaton,” *Physica A* 295(3–4), 2001, 507–525. doi:10.1016/S0378-4371(01)00141-8.

- [7] Castellano, Claudio, Santo Fortunato, and Vittorio Loreto, “Statistical physics of social dynamics,” *Reviews of Modern Physics* 81(2), 2009, 591–646. doi:10.1103/RevModPhys.81.591.
- [8] Gillespie, Daniel T., “Exact stochastic simulation of coupled chemical reactions,” *Journal of Physical Chemistry* 81(25), 1977, 2340–2361. doi:10.1021/j100540a008.
- [9] Helbing, Dirk, and Péter Molnár, “Social force model for pedestrian dynamics,” *Physical Review E* 51(5), 1995, 4282–4286. doi:10.1103/PhysRevE.51.4282.
- [10] Hillier, Bill, and Julianne Hanson, *The Social Logic of Space*, Cambridge University Press, Cambridge, 1984. doi:10.1017/CB09780511597237. A paperback edition appeared in 1989.
- [11] Jiang, Xiaoye, Lek-Heng Lim, Yuan Yao, and Yinyu Ye, “Statistical ranking and combinatorial Hodge theory,” *Mathematical Programming* 127(1), 2011, 203–244. doi:10.1007/s10107-010-0419-x.
- [12] Kelly, Frank P., *Reversibility and Stochastic Networks*, Wiley, Chichester, 1979. (Source of the reversibility criterion attributed to Kolmogorov used at (13).)
- [13] Kogut, John B., “An introduction to lattice gauge theory and spin systems,” *Reviews of Modern Physics* 51(4), 1979, 659–713. doi:10.1103/RevModPhys.51.659.
- [14] Kosterlitz, J. M., and D. J. Thouless, “Ordering, metastability and phase transitions in two-dimensional systems,” *Journal of Physics C: Solid State Physics* 6(7), 1973, 1181–1203. doi:10.1088/0022-3719/6/7/010.
- [15] OpenStreetMap contributors, *Planet dump*, retrieved via the Overpass API, 2026. <https://www.openstreetmap.org>
- [16] Perez, Alejandro, “The spin-foam approach to quantum gravity,” *Living Reviews in Relativity* 16, 2013, article 3. doi:10.12942/lrr-2013-3.
- [17] Polyakov, Alexander M., “Quark confinement and topology of gauge theories,” *Nuclear Physics B* 120(3), 1977, 429–458. doi:10.1016/0550-3213(77)90086-4.
- [18] Rovelli, Carlo, and Francesca Vidotto, *Covariant Loop Quantum Gravity*, Cambridge University Press, Cambridge, 2014. doi:10.1017/CB09781107706910.
- [19] Villain, Jacques, “Theory of one- and two-dimensional magnets with an easy magnetization plane. II. The planar, classical, two-dimensional magnet,” *Journal de Physique* 36(6), 1975, 581–590. doi:10.1051/jphys:01975003606058100.
- [20] Wiese, Uwe-Jens, “Ultracold quantum gases and lattice systems: quantum simulation of lattice gauge theories,” *Annalen der Physik* 525(10–11), 2013, 777–796. doi:10.1002/andp.201300104.
- [21] Wilson, Alan G., “A statistical theory of spatial distribution models,” *Transportation Research* 1(3), 1967, 253–269. doi:10.1016/0041-1647(67)90035-4.
- [22] Wilson, Kenneth G., “Confinement of quarks,” *Physical Review D* 10(8), 1974, 2445–2459. doi:10.1103/PhysRevD.10.2445.