

Trajectory and Attribution in Generative Relational Systems

A Preliminary Position Paper on Datability, Criticality, and the Endogenous Predisposition

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Abstract

In one class of injury a set of factors, each individually incapable of destabilising the injured system, acts over a long period, reduces its capacity to recover from disturbance, and permits a later disturbance of ordinary magnitude to produce a severe outcome. Doctrine processes multi-causal injury with tolerable success and processes this class badly, and the reason is one of representation rather than of proof. The paper isolates one requirement, named datability: a change of state carrying legal significance must be assignable to a point in time. A dynamical presentation of a bistable system under slow forcing shows that the onset of criticality withholds what the requirement demands, for two reasons that hold of the system rather than of the observer, and six doctrinal operations fail together with it. Four consequences follow. Each factor is necessary and the conjunction of all of them remains insufficient, so the but-for test and the balance of probabilities come apart and the refinement requiring a sufficient set survives only by admitting a term attributable to nobody. The antecedent conduct and the terminal disturbance are excluded by different doctrines, and the threshold that decides which acts leave a record is the threshold that decides which acts carry significance, so the two exclusions are one filter counted twice. The predisposition on which reduction doctrines operate is a product of the conduct at issue, so those doctrines deduct the quantity that constitutes the wrong. Near a boundary of stability the outcome is completed by the amplification of the injured system, which doctrine reads through two settled figures that both discharge the defendants. The paper then separates two limitations of the classical apparatus. One concerns the resources available for recording and interpreting a history and would be relieved by their extension. The other concerns what a determinate contribution would require, namely closure of the set of variables bearing on the case, a declared class of admissible trajectories, and identifiability of the couplings, and it is unaffected by any capacity of the tribunal. Against the second the paper proposes findings of position, whose value is fixed by the record where the quantities are not, and observes that such findings are binary as a verdict is binary. It closes with the migration of the case to supervisory obligation, the price of that migration, and one route that dispenses with the representation of trajectories. Conditions of refutation are given throughout.

Keywords: legal causation; attribution; critical transition; cumulative harm; predisposition; limitation; supervisory obligation; generative relational systems.

A note on the standing of this paper. This paper states one position and defends it in a form intended to permit its own refutation. It neither asserts that the doctrines discussed in §2 were designed for the class of case stated in §1 nor that they fail at the tasks for which they were designed. Its contribution is narrow by intention: one representational requirement, the doctrinal operations that depend upon it, and three consequences that follow where the requirement goes unsatisfied. The framework of representational requirements on legal causation, and the thesis that the structure of legal causal representation is a projection of the structure of the available judgment, are taken from a companion position paper and are assumed here rather than re-derived. The epistemic conditions on which the supervisory route of §11 depends are the subject of a sister paper. The dynamical material in §3 is elementary, and it is meant to be. Objection, correction, and counter-evidence are welcome at huangwanhong@serendip.ngo.

Draft. This is a preprint draft. It is circulated for comment, and its arguments, formulations, and open questions are subject to revision. Correction and criticism are welcome at huangwanhong@serendip.ngo.

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Use of generative artificial intelligence. An AI assistant (Anthropic’s Claude) was used extensively in preparing this manuscript: for literature survey, for drafting prose from the author’s specifications, for adversarial review of the argument, for the numerical simulations underlying the figures, for consistency auditing, and for typesetting. Formulations and objections arose in the course of that exchange as well as from the author. Every claim, argument, and citation was reviewed and decided by the author, who is responsible for the content and for any errors. Bibliographic details, and in particular the law report and journal citations, are to be verified against published sources before reliance.

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1 INTRODUCTION

This section states the class of case with which the paper is concerned, isolates the feature that distinguishes it from ordinary multi-causal injury, and sets out the order of the argument. The method throughout is doctrinal, with the dynamical material of §3 supplying the properties against which doctrine is tested; each claim is stated against an identified legal operation and is accompanied by the observation that would defeat it.

The facts of the class are familiar and its structure is not. Within a workplace or a school, several factors act upon one person over a long period. Occasional slighting remarks come from one quarter, sustained inattention from those charged with supervision, withdrawal of ordinary interaction from peers, and pressure arriving from outside the organisation. Each factor, taken singly and observed over any short interval, leaves the person functioning, and recovery from each occurrence is complete or nearly so. After a year, a remark of ordinary weight is followed by severe harm.

Definition 1.1. (The criticality case). *A criticality case is one in which factors acting over an extended period leave the injured system on a stable attractor throughout, reduce that system's capacity to recover from disturbance, bring the system near a boundary of stability, and permit a disturbance of ordinary magnitude to carry the system across that boundary into a qualitatively distinct state in which the harm is realised.*

Ordinary multi-causal injury is handled with tolerable success. Where several factors combine to produce an outcome, the law possesses joint and several liability, contribution between tortfeasors, apportionment, and in the criminal setting a developed apparatus for the realisation of a created danger through intervening events. The criticality case differs in one respect, and the whole of the argument depends on stating that respect exactly.

Claim 1.2. (The endogenous receiving state). *In the criticality case the state of the system that receives the disturbance is a product of the factors at issue. Legal causal doctrine holds the condition of the injured party constant as a background against which conduct varies. Here the conduct and the susceptibility are one history observed twice.*

The claim is easily mistaken for a claim about complexity or about difficulty of proof. It is a claim about representation. A doctrine that varies conduct against a fixed background has a place for the acts and a place for the condition, and it possesses no operation that converts the first into the second over time. Sections 3 and 4 establish the property of the system and the requirement of doctrine that meet at this point.

The order of what follows is: what existing doctrine and scholarship settle (§2); the dynamical presentation of the class and the three properties it yields (§3); the requirement of datability and the operations that depend on it (§4); the location of trajectories within existing doctrine (§5); the structure of necessity and sufficiency among the factors (§6); the failure of

both ends of the process (§7); the pathology of the endogenous predisposition (§8); the completion of the outcome by the injured system (§9); the migration to supervisory obligation and its price (§11); and the conditions under which each claim fails (§12).

2 RELATION TO PRIOR WORK ON CUMULATIVE INJURY

This section identifies the bodies of doctrine and scholarship that already address cumulative injury, states what each achieves, and narrows the claim of the paper to the residue. The method is to concede first and to claim afterwards, since the value of the position depends entirely on the accuracy of the concession.

Cumulative exposure litigation has solved the problem of dating an injury produced by a long process. In the occupational disease cases the loss is commonly treated as arising when the condition manifests, which relocates the commencement of the limitation period away from the exposure and towards the appearance of the disease. Continuing tort doctrine narrates accumulation directly, and the law of progressive disease has long separated the moment of injury from the moment of exposure (Hart & Honoré, 1985). Devices for accumulation therefore exist, and any claim that they are absent is false.

Doctrine has also relaxed the causal requirement where the dynamics of an injury defeat proof of individual contribution. The mesothelioma line substituted a material contribution to risk for causation of the harm, charging a defendant with contribution to a hazard (Fairchild, 2002; Barker, 2006; Compensation Act, 2006). Legislative correction followed, and the episode establishes that courts revise the causal rule where the underlying process resists it.

Rate is represented as well. Administrative criteria for the recognition of work-related disease commonly operate on a quantity of exposure per unit time supported by a cumulative window, and a general claim that legal instruments are blind to rate is defeated by them. Such criteria are administrative and supervisory in character, and §11 returns to that feature.

Structural accounts of responsibility are developed at length. The social connection model locates responsibility in participation in structural processes that produce unjust outcomes (Young, 2011); complicity accounts address collective harms composed of individually insignificant contributions (Kutz, 2000); distributed accounts extend to systems in which no participant satisfies the standard conditions of moral agency (Floridi, 2016); shared responsibility is a settled subject in international law (Nollkaemper & Jacobs, 2013). A relational critique of legal responsibility exists already (Norrie, 2000), and the imposition of risk has been analysed as a wrong in its own right (Oberdiek, 2017). The correlativity of a private law claim, under which the plaintiff's right and the defendant's duty are two aspects of one relation, marks the boundary that these accounts press against (Weinrib, 1995).

The dynamical and organisational literatures supply the vocabulary of the phenomenon. Critical transitions in ecological, financial, and social systems have been characterised, together

with the statistical signatures that precede them (Scheffer, 2009a; Scheffer et al., 2009b; Kuehn, 2011). The organisational literature on the incubation of disaster describes a period during which discrepant events accumulate unnoticed against prevailing beliefs about hazard, which is a close description of the class treated here (Turner, 1978).

Claim 2.1. (The residue after concession). *Legal doctrine possesses devices for accumulation, for contribution to a hazard, for rate under administrative criteria, and for structural participation. The operation it lacks is one that converts a history of conduct into a change in the stability properties of the injured system, since a change of that kind is assignable to no point in time.*

3 THE DYNAMICAL PRESENTATION OF THE CRITICALITY CASE

This section gives an elementary dynamical presentation of the class defined in §1, states the three properties on which the doctrinal argument depends, and fixes the limits of what the presentation delivers. The method is analytic for the properties that admit of derivation and numerical for the illustrations; the figures report simulations of the system stated below and carry no empirical claim.

Two disclaimers are entered before anything else. The formalism carries no claim that a person is a particle, and the variables below stand for the condition of a relational system and not for a psychological quantity. The presentation is used to establish that certain properties hold of any system in the class, so that the doctrinal argument may proceed against properties rather than against metaphors. Where the doctrinal argument requires only the qualitative property, the quantitative form is given for definiteness and may be discarded.

3.1 The bistable presentation and its parameters

Let the condition of the injured system be summarised by a variable x whose fast dynamics are governed by a potential V , and let the cumulative effect of the factors be summarised by a control parameter μ that varies slowly by comparison with the relaxation of x . Taking the elementary bistable form, with additive noise of intensity σ ,

$$dx = (-x^3 + x + \mu(t)) dt + \sigma dW, \quad \dot{\mu} > 0, \quad V(x) = \frac{1}{4}x^4 - \frac{1}{2}x^2 - \mu x. \quad (1)$$

For $|\mu| < \mu_c = 2/(3\sqrt{3})$ the deterministic system has two stable fixed points $x_l < x_u$ separated by a saddle x_s , these being the three roots of $x^3 - x = \mu$. Write

$$\Delta(\mu) = x_s - x_l, \quad \tau(\mu) = \frac{1}{|\lambda(x_l)|} = \frac{1}{3x_l^2 - 1}, \quad (2)$$

for the margin separating the attractor from the boundary of its basin and for the characteristic time of recovery from a small disturbance. At $\mu = \mu_c$ the lower attractor and the saddle merge

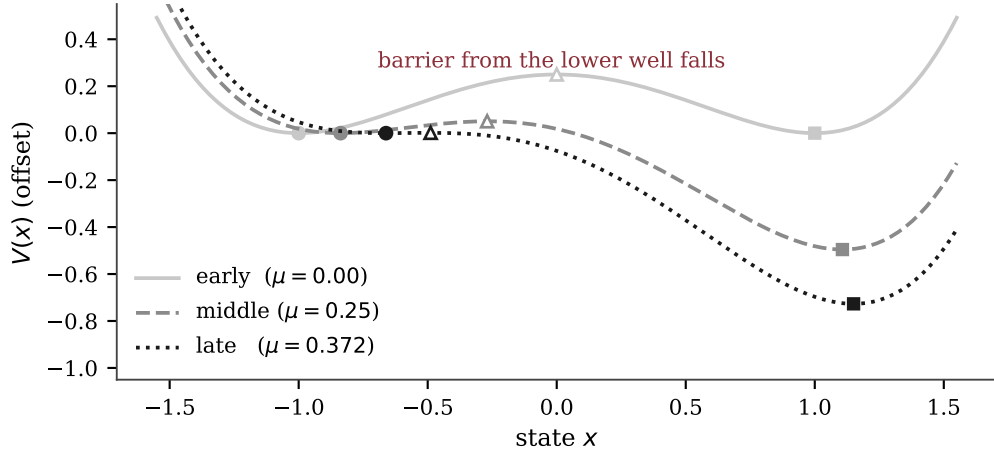


Figure 1: Deformation of the potential under slow forcing. The lower attractor (circle) persists at every stage, the saddle (triangle) approaches it, and the upper attractor (square) remains available throughout. The system is stable at every stage, and the quantity that changes is the margin Δ , which is a property of the system and is no event in its history.

at $x = -1/\sqrt{3}$ and the system moves to x_u , which is finite. Near that merger the system reduces to the normal form of a saddle-node bifurcation, so that $\tau \sim (\mu_c - \mu)^{-1/2}$ and $\Delta \sim (\mu_c - \mu)^{1/2}$, and the lengthening of τ on approach to a boundary of stability is a general property of that class (Wissel, 1984; Scheffer et al., 2009b; Kuehn, 2011).

The choice of the bistable form over the unfolded saddle-node normal form $dx = (r + x^2)dt$ is required by Definition 1.1 and is worth stating, since the two agree on every local scaling. The normal form possesses one attractor and one saddle, and trajectories carried beyond the saddle leave every bounded set, so it supplies no second state for the system to arrive at. Definition 1.1 requires a qualitatively distinct state that persists, and (1) supplies one. Figure 1 shows the deformation of V as μ rises, with the lower attractor persisting throughout and the barrier separating it from the upper attractor falling.

Three properties follow, and each is used in a later section. A fourth is added below, and a fifth in the phase plane.

Property 3.1. (Constancy of the attractor). *Throughout the interval in which the factors act, the system occupies a stable attractor and returns to it after each disturbance. Observation of the system at any time during the interval, conducted by asking whether the system is functioning, returns an affirmative answer. The change accumulated over the interval is a change in Δ , which is a property of the dynamics and is not an event in the history.*

Property 3.2. (Sufficiency of an ordinary disturbance). *A disturbance of fixed magnitude a carries the system across the boundary when $a > \Delta(r)$. Since Δ falls to zero at the merger, the magnitude sufficient to produce the transition becomes arbitrarily small, and the same disturbance*

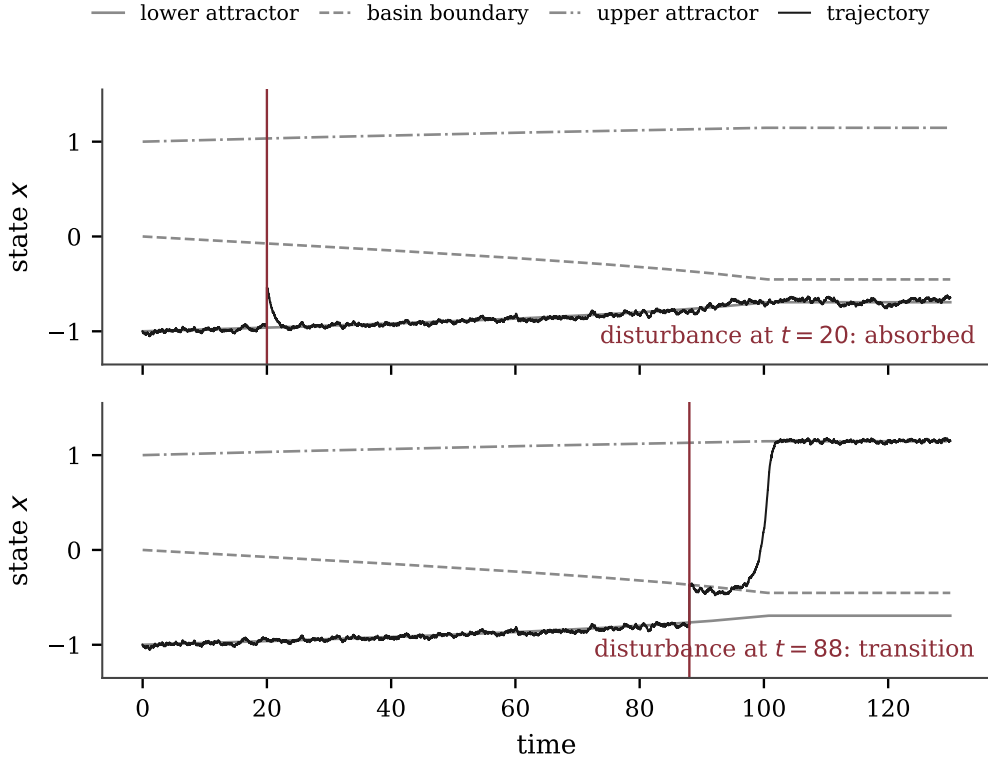


Figure 2: One disturbance of fixed magnitude applied at two times. The trajectory, the noise realisation, and the amplitude of the disturbance are identical in the two panels. In the upper panel the system absorbs the disturbance and returns to the attractor; in the lower panel the same disturbance completes the transition. The magnitude of the disturbance carries no information about the magnitude of its consequence.

that is absorbed at an early stage is sufficient at a late one.

Figure 2 shows the two applications of one disturbance. The upper panel and the lower panel differ in the time of application and in nothing else.

Property 3.3. (Absence of a threshold under noise). *With $\sigma > 0$ the transition may occur at any $\mu < \mu_c$, and the rate at which it occurs rises continuously as Δ decreases, in the elementary case in proportion to $\exp(-2\Delta V/\sigma^2)$ with barrier height $\Delta V = V(x_s) - V(x_l)$, which falls to zero at the merger. There exists no value of μ whose crossing constitutes the onset of criticality. What exists is a hazard, and a hazard has no date.*

Figure 3 shows the two consequences of Property 3.3 that matter for §4. The left panel shows the continuity of the hazard. The right panel shows the distribution of transition times across realisations of the noise that differ in nothing else, from which it follows that the time of transition is a property of the realisation and the condition of the system is not recoverable from it.

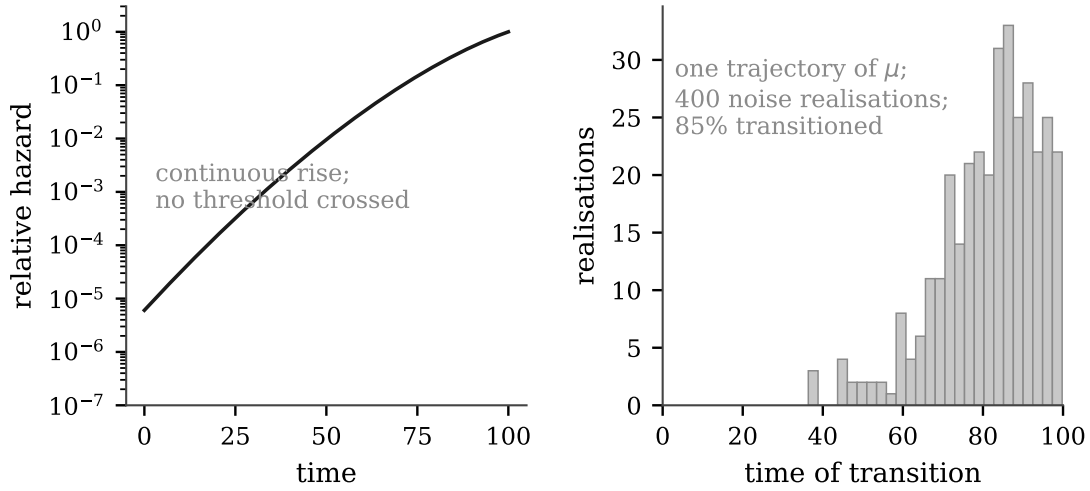


Figure 3: The hazard and its realisations. Left: relative hazard under the model of (1), rising continuously with no crossing of any threshold. Right: the times at which the transition occurred across realisations of the noise, with one and the same trajectory of the slow variable. The spread is a property of the noise, so the same condition of the system yields transition at widely separated times and yields no transition at all in a portion of the realisations.

Property 3.4. (Dependence on the trajectory). *Whether a disturbance of given magnitude produces the transition is settled by the value of Δ at the moment of application, and Δ is fixed by the whole prior trajectory of μ . Systems receiving identical disturbances at identical times therefore reach different terminal states where their trajectories differ, and a system receiving no disturbance whatever reaches the same terminal state as one that received a disturbance, where its trajectory carries it to the merger.*

Figure 4 exhibits Property 3.4 in the two registers in which the case is ordinarily described. The left panel shows the branches of stable and unstable states against the control parameter, with three histories traced upon them. The right panel shows the same three histories in time.

3.2 The account in the phase plane

The presentation of (1) carries one state variable, and the geometry it describes is easier to read in the plane. Two portraits are given. The first places the fast variable against the slow one and shows the mechanism of the transition. The second gives a two-dimensional state space in which the boundary of the basin is a curve, which is the form the account takes for a system described by more than one relational quantity.

Figure 5 shows the extended phase plane of the fast-slow system, in which μ is treated as a further coordinate advancing slowly. The set of points at which the fast dynamics vanish is the S-shaped critical manifold, whose two outer arms attract and whose middle arm repels. The system rides the lower arm as μ advances, so that the state changes continuously and remains

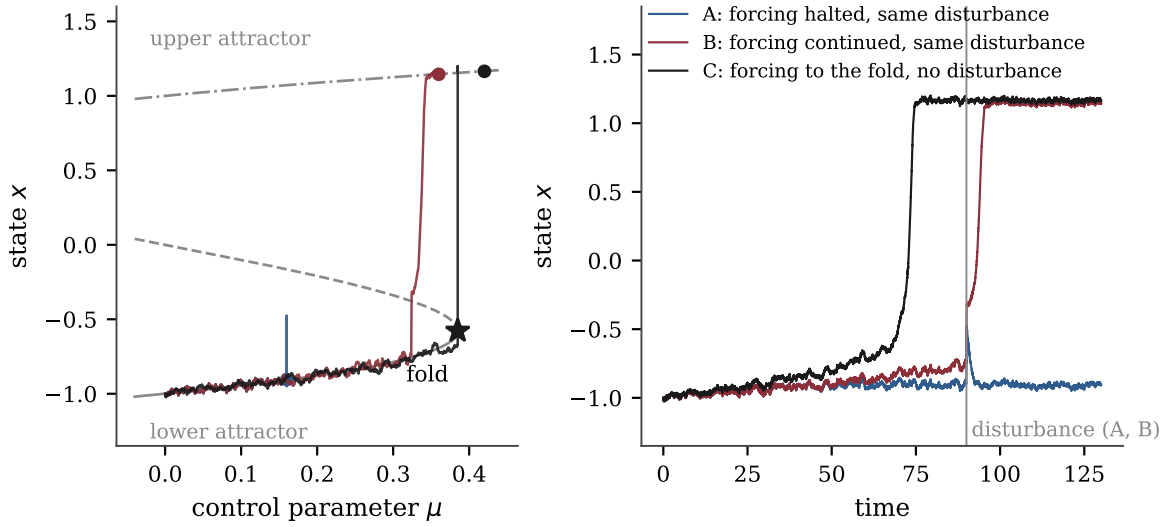


Figure 4: Three histories under one disturbance. Left: the stable branches (solid and dash-dotted), the saddle (dashed), and the merger, with the three histories traced against the control parameter. Right: the same three in time. History A halts the forcing and absorbs the disturbance. History B receives an identical disturbance at an identical time and arrives at the upper attractor. History C receives no disturbance and arrives at the same place. The terminal state is settled by the trajectory and is settled by the disturbance in no case.

at a fixed point of the fast dynamics throughout. At the fold the lower arm terminates, and the fast dynamics carry the state to the upper arm. The whole of the accumulated history is the traverse of the lower arm, and the whole of the harm is the jump, and the two are governed by different dynamics on different time scales.

A system described by one quantity has a basin whose boundary is a point, and the margin is a distance along a line. Where the injured system is described by two quantities, the basin has a boundary that is a curve, and a disturbance is a vector. Figure 6 gives the portrait for the planar system obtained by coupling the state x to a second relational quantity y , with dynamics derived from a potential and with parameters chosen so that two attractors persist.

Property 3.5. (Direction of the disturbance). *Where the injured system is described by more than one quantity, the boundary of the basin is a curve whose distance from the attractor varies with direction. Whether a disturbance carries the state across the boundary is therefore settled by its direction as well as by its magnitude, and disturbances of equal magnitude delivered at one moment to one system may differ in outcome.*

The two portraits agree on Properties 3.1 to 3.4 and add Property 3.5, which the one-dimensional presentation is unable to express. The doctrinal consequence is recorded in §6 and in §7: a measure of the gravity of an act that runs on magnitude alone predicts the outcome in neither case.

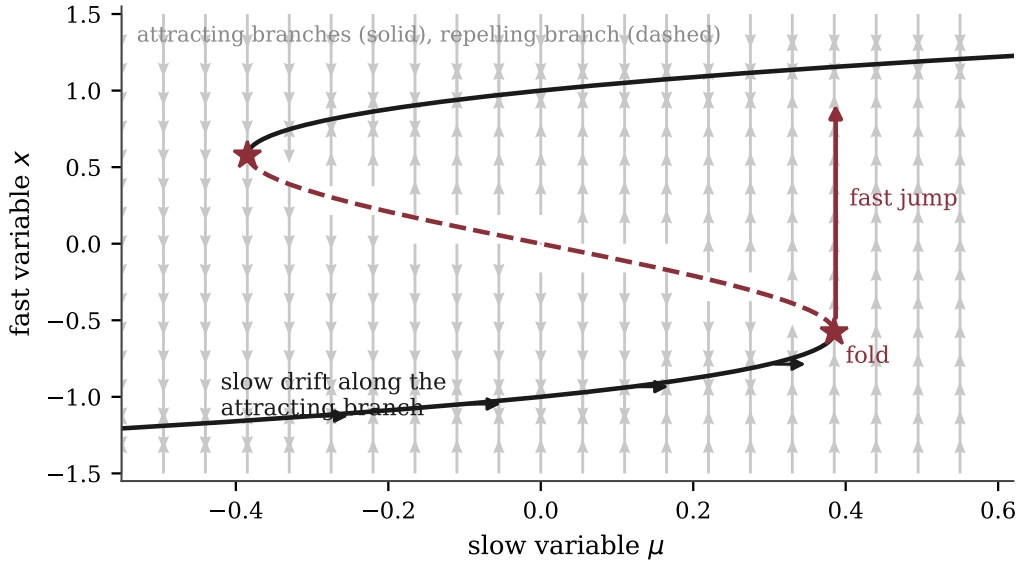


Figure 5: The fast–slow phase plane. Solid arms of the critical manifold attract and the dashed arm repels; short arrows give the direction of the fast dynamics at each point. The system advances along the lower arm under the slow drift, which is the interval in which the factors act, and departs at the fold, which is the interval in which the harm is realised. The state occupies a fixed point of the fast dynamics at every moment before the fold.

3.3 Limits of the presentation

The presentation supplies properties and supplies no measurements. It does not identify the variable x in any social system, it does not estimate r , and it does not license the inference from an observed harm to a critical transition, which requires evidence of the kind discussed in the sister paper. The form of (1) is chosen because the loss of one of two stable states under slow forcing occurs generically through the merger of that state with a saddle, and the scalings of (2) together with Property 3.3 hold for that class; systems in other classes may deviate, and the doctrinal argument requires only Properties 3.1 to 3.5, each of which may be asserted of a candidate case and denied of it on evidence.

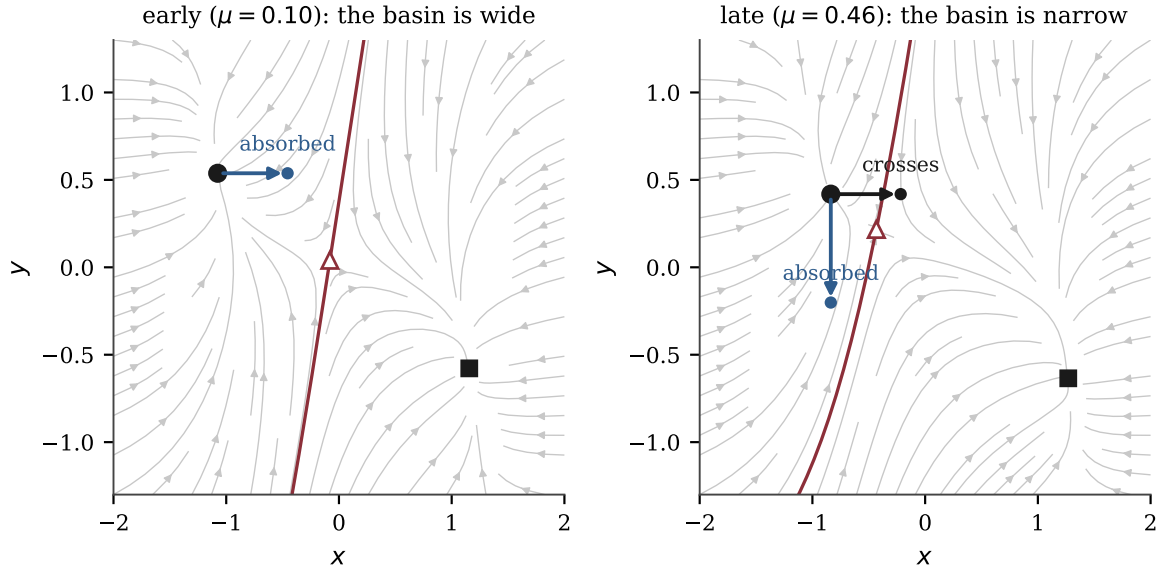


Figure 6: Basins in a two-dimensional state space. Circles mark the lower attractor, squares the upper, triangles the saddle, and the heavy curve is the boundary between the basins, which is the stable manifold of the saddle. Light curves give the flow. Left: at an early value of the slow parameter the basin is wide and a disturbance is absorbed. Right: at a late value the boundary has moved close to the attractor, the same disturbance carries the state into the upper basin, and a disturbance of the same magnitude in a different direction is absorbed.

4 THE REQUIREMENT OF DATABILITY AND THE OPERATIONS DEPENDENT ON IT

This section states the representational requirement on which the paper turns, derives it from an interest that legal systems protect independently, exhibits the doctrinal operations that depend on it, and applies Properties 3.1 to 3.3 to it. The method is the derivation of a representational requirement from the structure of the available judgment, following the projection thesis of the companion paper.

Proposition 4.1. (Datability). *Legal doctrine requires that a change of state carrying legal significance be assignable to a point or to a bounded interval in time.*

Proposition 4.2. (Datability from repose). *The requirement follows from the interest in repose that limitation protects. A limitation period requires a clock, a clock requires a moment at which it starts, and the security that limitation confers is available only where that moment is determinate. Datability is the price of repose, in the manner in which the acyclicity of legal causal findings is the price of finality.*

The requirement is invisible in ordinary cases because ordinary injuries arrive with their dates attached. Its reach becomes visible when the operations that depend on it are collected. Table 1 sets these out.

Table 1: Doctrinal operations dependent on the datability of the change of state

Operation	Content of the dependence
Commencement of limitation	The period runs from the occurrence of the loss or from knowledge of it, and each requires a determinate moment.
Time of the foreseeability assessment	Foreseeability is assessed as at the time of the conduct, which requires that the state of affairs then obtaining be specifiable.
Time of the negligence assessment	The standard of care is applied to circumstances as they stood at an identified time.
Occurrence of the loss	Liability attaches upon occurrence of damage, which the pleading must locate.
Reference date for quantification	Damages are assessed by reference to a stated date, from which interest and adjustment run.
Individuation under continuing tort	Continuing tort doctrine narrates a process by decomposing it into acts, each carrying its own date.

The criticality case withholds the required moment, and the reason must be stated precisely, since the natural formulation defeats itself. To say that the moment of entry into a critical state is unknown invites the reply that difficulty of proof is the ordinary condition of litigation and is met by presumptions, burdens, and expert evidence. Property 3.3 supports a stronger statement.

Claim 4.3. (Undefinedness of the onset). *In a system subject to noise there exists no threshold whose crossing constitutes entry into a critical state, and what exists is a probability of transition rising continuously with the condition of the system. Doctrine requires a boundary and the system supplies a hazard. The moment is undefined, and its undefinedness is a property of the system rather than a limit upon the observer.*

Claim 4.4. (Retrospective ascription of criticality). *Whether a given state was critical depends in part on the disturbances that subsequently arrived, since one realisation of the noise leaves the system intact where another carries it across. Criticality is a dispositional property, and dispositional properties are ascribed in retrospect. The right panel of Figure 3 exhibits the ascription problem directly, since the realisations shown differ in the date of transition and agree in every property of the system.*

Two consequences follow, and the second is the more serious. Each of the operations in Table 1 fails at once, so the difficulty is not localised in limitation doctrine where it is most visible. And any line drawn by a court through the trajectory is constituted after the conduct that it governs.

Corollary 4.5. (A boundary constituted after the conduct). *Attribution in the criticality case would require the actors to conform to a boundary constituted after their conduct. The objection rests on legal certainty and on the possibility of guidance, which are first-order legal values, and it stands independently of any claim concerning the complexity of the system.*

5 THE LOCATION OF TRAJECTORIES WITHIN EXISTING DOCTRINE

This section distinguishes two processes commonly described by one word, locates the trajectory within the doctrines where it presently appears, and states the resulting position. The method is comparative across doctrinal sites, since the position turns on where trajectories appear and where they are absent.

The first distinction concerns what accumulates. In cumulative exposure the accumulating quantity is a dose. A dose is a scalar in a single unit, it is additive across sources, and its additivity is what permits apportionment between defendants. In the criticality case the quantity of interest is Δ , the margin separating the system from the boundary of its basin. That quantity has no natural unit and it is a property of the dynamics. Its resistance to apportionment holds even where the factors are additive in the control parameter, which is the strongest case for apportionment that the class permits.

Proposition 5.1. (Concavity of the margin). *Let the factors act additively on the control parameter, $\mu = \sum_i \mu_i$. Then Δ is a strictly decreasing and strictly concave function of μ , with $d\Delta/d\mu \rightarrow -\infty$ as $\mu \rightarrow \mu_c$. The reduction of margin produced by a given increment of conduct is therefore settled by the quantity already accumulated and grows without bound on approach to the merger.*

Corollary 5.2. (Order dependence of apportionment). *Apportionment of a concave quantity by marginal contribution assigns to each factor a share settled by the order in which the factors are counted, so apportionment by marginal contribution is order dependent. Averaging over orders restores order independence and requires the tribunal to fix a distribution over counterfactual orderings, which is a further decision the doctrine leaves unmade. Under any rule taking the final contribution at the margin, the last factor absorbs the greater part of the reduction, and that factor is the one excluded by the proportionality requirement of Proposition 7.2.*

Figure 7 exhibits Proposition 5.1 and Corollary 5.2 on three factors acting additively on the control parameter. The left panel traces the reduction of margin produced by each factor in one order, and the right panel gives the reduction attributed to each factor under each of the six orders. The share attributed to a factor varies by a factor of about two according to the order alone, with the sizes of the factors held fixed.

Proposition 5.3. (Dose and margin). *Legal doctrine processes accumulation, because a dose is a scalar in one unit and is additive, so that apportionment is well defined and order independent. It halts at the loss of resilience, because the margin is concave in the accumulated forcing by Proposition 5.1, so that apportionment among the contributions is order dependent by Corollary 5.2.*

The second distinction concerns where trajectories appear in doctrine. Any suggestion that

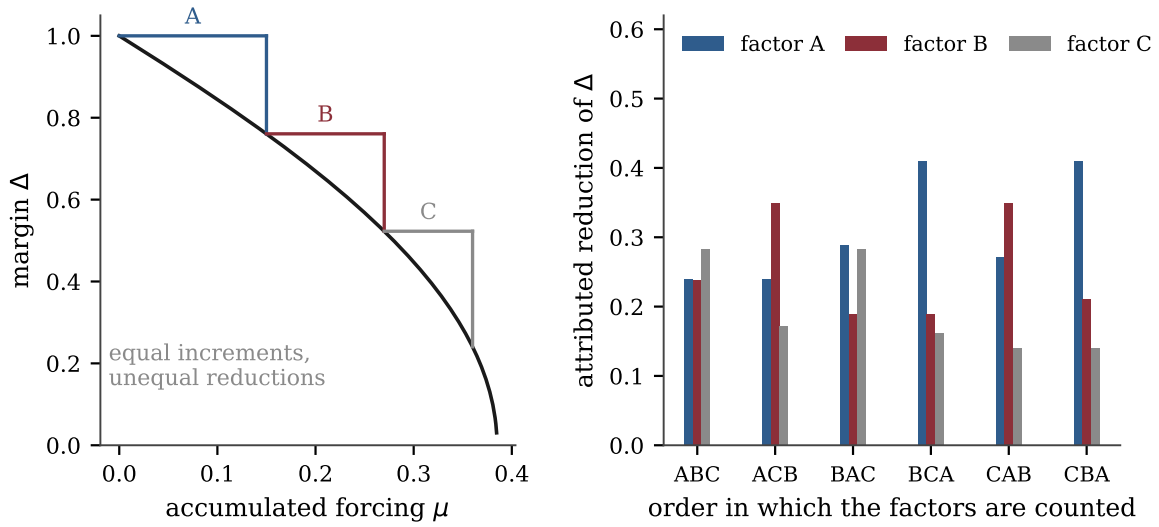


Figure 7: Order dependence of apportionment by marginal contribution. Left: three factors of fixed size applied in one order, with the reduction of margin produced by each. Since the margin is concave, equal increments of forcing produce unequal reductions, and later increments produce larger ones. Right: the reduction attributed to each factor under each of the six orders in which the three may be counted. The sizes of the factors are identical in all six, and the attributed shares are not.

legal reasoning is blind to the history of a factor is refuted immediately by sentencing, where the circumstances leading to the offence, the history of the offender, and the development of a habit are examined in detail; by the doctrine of predisposition, which considers the constitution of the injured party; and by the assessment of contributory conduct. Trajectories are represented, and the position must be stated with regard to the side of the judgment on which they appear.

Claim 5.4. (Trajectory on the mitigating side). *Legal doctrine represents trajectories on the side of mitigation and reduction, where a history enters as circumstance, predisposition, or extenuation. It represents a trajectory as a ground of attribution only under a further condition.*

Claim 5.5. (The condition of single authorship). *A trajectory grounds attribution where it has a single author, as in liability for a condition brought about by the defendant's own antecedent conduct and in the assumption of risk through a prior undertaking. Where the trajectory is distributed across many actors, it appears on the mitigating side alone.*

The explanation of Claim 5.4 lies in §4. The mitigating side is the region of legal judgment that dispenses with a determinate quantity and a determinate date. Sentencing, extenuation, and reduction for predisposition are exercises of overall evaluative discretion in which the tribunal is under no obligation to state by how much a given element moved the outcome, nor when it did so. The attributive side requires both. A trajectory therefore finds accommodation where the mapping from history to condition may remain unstated, and finds none where the

mapping must be produced.

6 NECESSITY AND INSUFFICIENCY AMONG THE FACTORS

This section states the structure of necessity and sufficiency that holds among the factors in a criticality case, exhibits it numerically, and draws the consequences for the tests by which legal doctrine establishes causal connection. The method is to compute the probability of the outcome under each subset of the factors in the system of (1), and then to apply to the resulting structure the but-for test, the requirement of proof on the balance of probabilities, and the necessary-element-of-a-sufficient-set refinement in turn.

Definition 6.1. (Factor decomposition). *Let factors A , B , and C act additively on the control parameter, each raising μ by a fixed amount over the interval, and let the harm consist in the departure of the state from the lower basin within a stated horizon. Since (1) carries noise, the outcome under any given subset of the factors is a random variable, and the quantity settled by the subset is the probability of the harm and not the harm.*

Figure 8 reports that probability for each of the seven non-empty subsets, under one intensity of noise and one horizon, together with the margins that each subset leaves.

Proposition 6.2. (Joint necessity). *Under the parameters of Figure 8 no single factor and no pair of factors raises the probability of the harm above zero, while the conjunction of the three raises it to a positive value. Removal of any one factor returns the probability to zero. Each factor is therefore a necessary condition of the harm, and each is individually incapable of producing it.*

Proposition 6.3. (Insufficiency of the conjunction). *The conjunction of all three factors leaves the probability of the harm strictly between zero and one. What the conjunction accomplishes is the reduction of the margin, and the transition is completed by a realisation of the noise. The conjunction is therefore necessary for the harm and insufficient for it, and the insufficiency holds however the factors are enlarged short of carrying the system past the merger.*

The two propositions are ordinarily satisfied together in litigation and come apart here, and the coming apart is what the doctrinal tests register.

Corollary 6.4. (But-for satisfied, the balance of probabilities failed). *Each factor satisfies the but-for test, since the harm would have been of vanishing probability in its absence. The conjunction of every factor nonetheless leaves the harm more likely to be absent than present, so a requirement that the conduct rendered the harm more likely than not fails while the but-for test succeeds. Doctrine treats these as one enquiry, and in a criticality case they diverge.*

Corollary 6.5. (A sufficient set containing no one's conduct). *The refinement under which a factor causes an outcome where it is a necessary element of a set sufficient for that outcome requires that a sufficient set exist. No set of the factors is sufficient, by Proposition 6.3. The smallest sufficient*

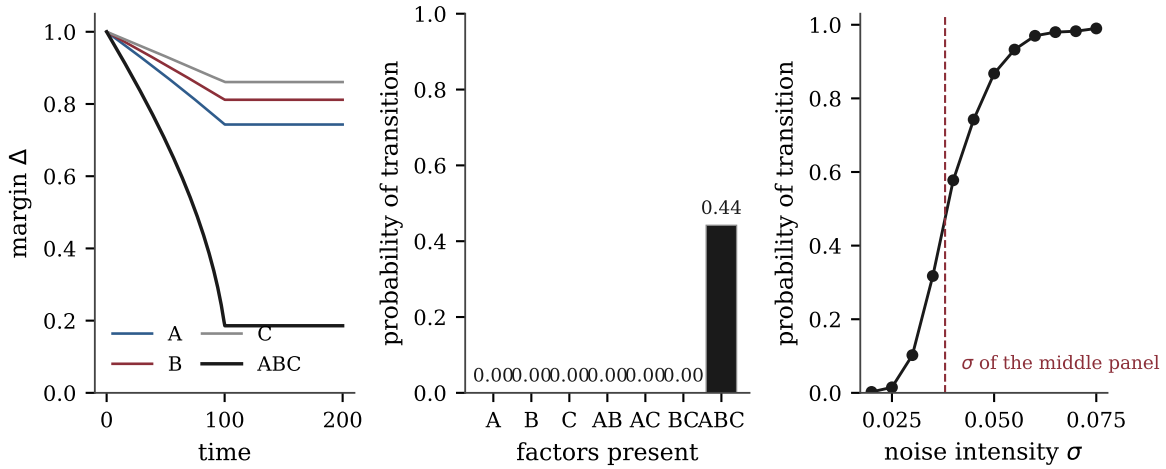


Figure 8: Necessity and insufficiency among the factors. Left: the margin over time under each single factor and under the conjunction. Middle: the probability of transition under each of the seven subsets, over five hundred realisations of the noise. Each single factor and each pair leaves the probability at zero; the conjunction raises it to a value strictly below one half. Right: the probability under the conjunction against the intensity of the noise, which is a property of the setting and of no party's conduct.

set therefore contains a realisation of the noise, which is the conduct of no person and the constitution of no party. The refinement survives in this class by including a term that is attributable to nobody.

Claim 6.6. (The variance of the setting). *The right panel of Figure 8 shows the probability of the harm rising with the intensity of the noise, with the factors and the injured system held fixed. The same conduct directed at the same person in a quieter setting produces no harm. Doctrine possesses a category for the constitution of the injured party and possesses none for the variance of the setting in which the conduct occurs, so a determinant of the outcome falls outside every established head of allocation.*

The route through contribution to risk is the doctrinal response already available for a structure of this kind, since it charges a defendant with the increase of a hazard where proof of causation of the harm is unavailable (Fairchild, 2002; Barker, 2006; Compensation Act, 2006). Its present limits are the obstacle: it is confined to a narrow class of injury, and it requires that the several exposures be of a single agent, whereas the factors here are heterogeneous and act upon one quantity only through their aggregate effect on the margin.

7 THE EXCLUSION OF BOTH ENDS OF THE PROCESS

This section shows that the antecedent conduct and the terminal disturbance fail under different doctrines, states the resulting condition of the case, and explains why the failure is invisible from within any single doctrine. The method is to take each end of the process in turn and apply the doctrine governing it.

At the antecedent end each factor falls below the threshold of legal significance. A remark of ordinary weight, an omission by a person under no duty to act, and the withdrawal of voluntary interaction are lawful when taken singly. Where the conduct consists in the withholding of what was never owed, the difficulty is compounded. To these substantive obstacles the passage of time adds a procedural one, since by the date of the harm the earlier conduct will often fall outside the limitation period, whose commencement §4 has shown to be indeterminate in this class.

Proposition 7.1. (Failure at the antecedent end). *The antecedent conduct fails on two independent grounds. Each act falls below the threshold of significance that attribution requires, and the period within which the aggregate would be actionable commences at a moment that the process withholds.*

At the terminal end the obstacle has the opposite form. By Property 3.2 the magnitude sufficient to complete the transition is arbitrarily small. Doctrines governing the relation between a created danger and its realisation, and those requiring a proportion between conduct and outcome, assess whether the danger created by the defendant was realised in the harm. Where the disturbance is of ordinary magnitude and the outcome is catastrophic, the assessment returns a negative answer.

Proposition 7.2. (Failure at the terminal end). *At criticality the sufficient disturbance is arbitrarily small by construction. Any doctrine requiring proportion between the created danger and the realised harm therefore excludes the terminal actor as a matter of the structure of the case, and Figure 2 exhibits the structural character of the exclusion, since the two panels differ in nothing that doctrine measures.*

Figure 9 places one history in the two registers in which it may be described. The upper two panels give the register of the system: the trajectory, and beneath it the margin and the hazard, both continuous and neither crossing anything. The lower panel gives the register in which doctrine works: a series of dated acts of stated magnitude, to which a threshold of significance and a limitation period are applied. The two registers are computed from the same run. Ten acts of identical magnitude are delivered; the first nine are absorbed and the tenth produces the transition, since the margin has fallen below the magnitude in the interval. Every quantity that settles the outcome sits in the upper register, and every quantity that doctrine measures sits in the lower one.

7.1 Evidentiary sampling under a threshold of record

The two failures just stated are ordinarily read as independent, since one concerns the significance of the acts and the other the proportion between the final act and the outcome. A third consideration bears on the first of them and shows that the appearance of independent

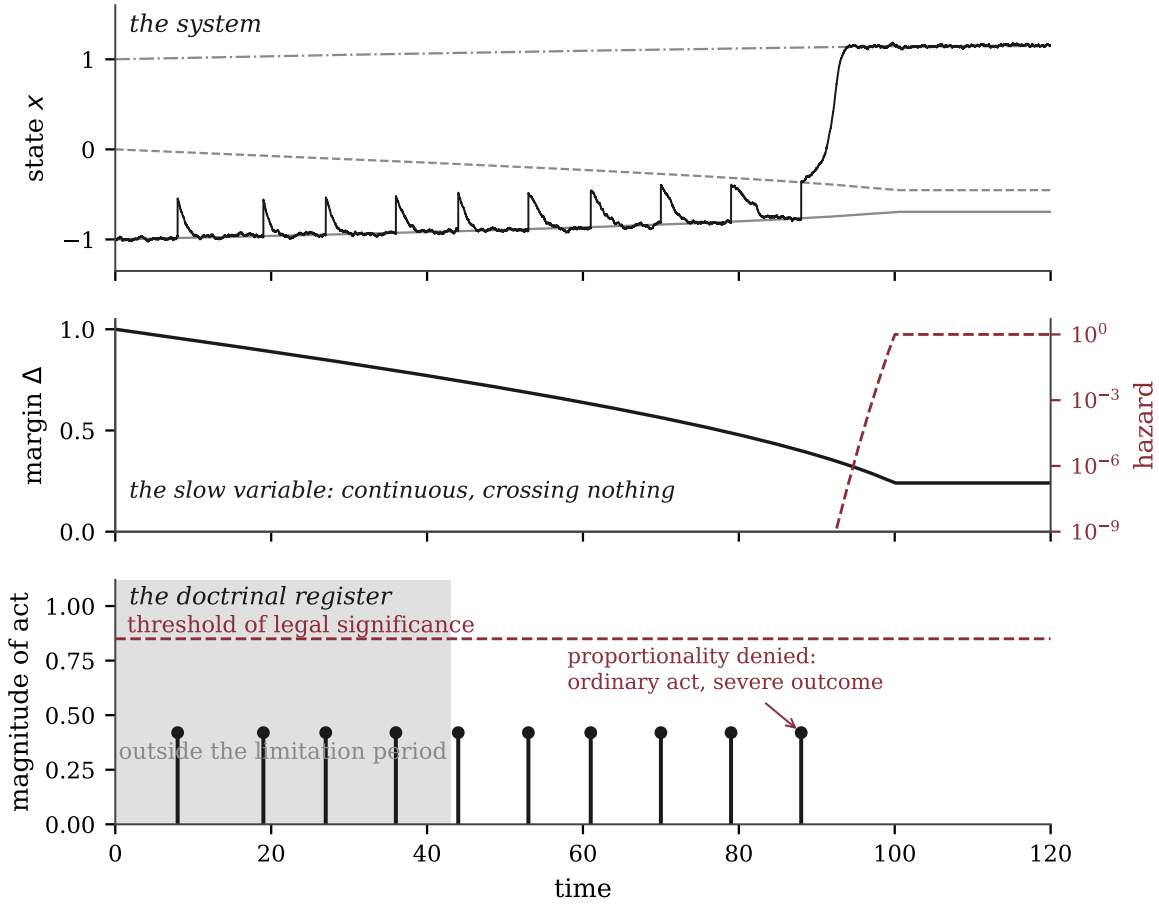


Figure 9: One history in two registers. Above: the trajectory against the attractors and the saddle. Middle: the margin and the hazard, both continuous, with no value whose crossing marks an onset. Below: the same history as doctrine receives it, as ten dated acts of identical magnitude, each below the threshold of significance, with the earlier acts falling outside the limitation period and the last excluded by the requirement of proportion between the act and the outcome. The acts in the lower panel carry the dates that the operations of Table 1 require, and the quantity that changed over the interval appears in the middle panel alone.

support is spurious. It concerns the manner in which a history reaches a tribunal.

Proposition 7.3. (Evidentiary sampling). *A tribunal recovers a finite set of points from a continuous history, and the points recovered are those that produced a record. A record arises where the effect of an act was salient enough and lasted long enough to be noticed, complained of, or written down. The material before the tribunal is therefore a sample of the history taken under a threshold of salience, and the reconstruction is an interpolation between the sampled points.*

Two consequences follow, and the second concerns the appearance of independent confirmation.

Corollary 7.4. (One threshold applied twice). *The threshold that governs which acts generate*

a record and the threshold that governs which acts carry legal significance are the same threshold. In a criticality case the accumulated change occurs below it. The absence of evidence and the insignificance of the acts are therefore one filter counted twice, and the finding that the record discloses nothing before the terminal event is guaranteed by the same criterion that denies the acts their significance.

Corollary 7.5. (A manufactured date). *Since the earliest recorded point is settled by the level at which a record arises, the date that the tribunal assigns to the beginning of the process is a property of the recording practice. Claim 4.3 states that the process supplies no date; the present corollary states what fills the gap, namely a date produced by the evidentiary filter and read as a fact about the injured system.*

Figure 10 exhibits the three points on one run. The acts are identical in magnitude throughout. The duration for which each leaves a visible effect grows across the interval, since the recovery time lengthens as the margin narrows, so the later acts generate records where the earlier ones of the same magnitude did not. The record consequently thickens towards the end of the interval and invites the reading that the conduct escalated.

Claim 7.6. (An artefact of escalation). *Where the conduct is constant and the recovery time lengthens, the density of the record rises across the interval. A tribunal reading the record as a description of the conduct finds an escalation that did not occur, and the quantity that in fact changed is the one the record fails to represent.*

7.2 Topological divergence of the reconstruction

Proposition 7.3 states that the material before a tribunal is a finite sample. Two further operations act upon that sample before a finding emerges, and their joint effect is a difference of kind between the reconstructed history and the history. The method of this subsection is to treat the two operations separately and then to state what is preserved and what is lost.

Proposition 7.7. (The sampled point enters in an available register). *A point enters the record in whatever register was available to whoever recorded it. Registers for events are available, since an event has a date, an author, and a description. A register for a reduction in the capacity of a relational system to recover from disturbance is unavailable, so a report in those terms is either re-described as a report of events or discounted for want of a form. The unavailability is a property of the shared interpretive resources rather than of the reporter, and the discounting of the reporter's credibility and the absence of a concept in which the experience could be rendered are the two forms this takes (Fricker, 2007).*

Proposition 7.8. (The connection is supplied by the repertoire). *The links between sampled points are supplied by the tribunal from its own repertoire, since no link is given in the evidence.*

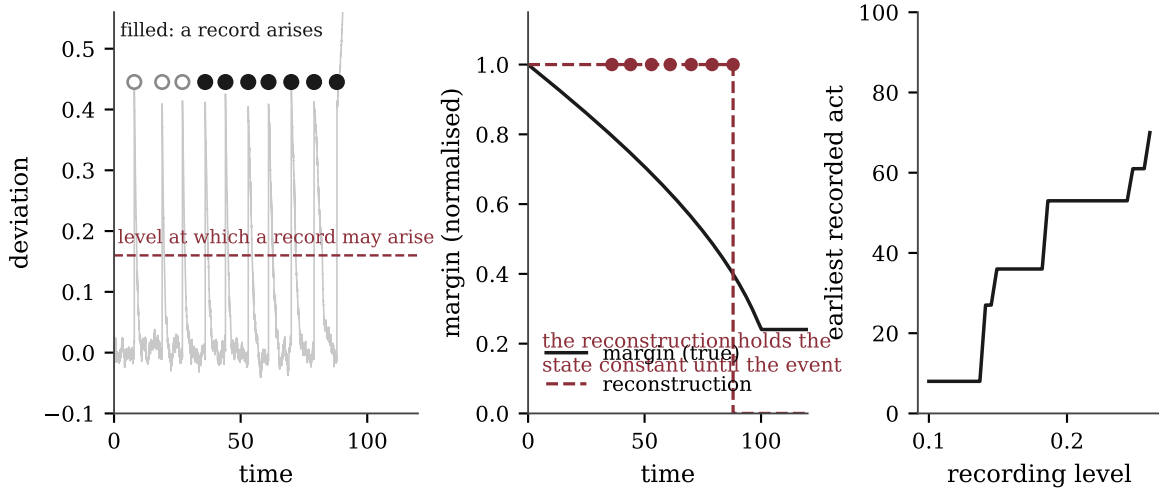


Figure 10: What threshold-triggered sampling recovers. Left: the deviation of the injured system under ten acts of identical magnitude, with the level at which a record may arise; filled markers indicate the acts that left a record and open markers those that did not. Right of it: the true margin against the reconstruction available from the recorded points, which holds the state constant until the terminal event. Right: the earliest recorded act as the level at which a record arises is varied, which places the beginning of the process anywhere in the first two thirds of the interval according to the recording practice alone.

That repertoire contains chains running from conduct to result, and it contains no term for a change in the stability of the system through which the chain runs. The connections drawn are therefore of the only kind available, and their availability settles the shape of the reconstruction.

Figures 11 and 12 display the result of the two operations. The consequence is stated in terms of what is preserved, since a difference in quantity would be repaired by more evidence and a difference of the present kind would not.

Proposition 7.9. (Invariants that differ). *Three quantities preserved under the equivalences appropriate to each object differ between the history and its reconstruction. In the state space, and under topological equivalence of flows, the number of attractors falls from two to one and the codimension-one manifold separating the basins is absent, so the final displacement stands as a movement within one basin where in the history it was a crossing between two. In the causal structure, and under isomorphism of graphs, the cycle rank falls from two to zero, since the edges running from the state to the effect of a later act are exactly those for which no sampled point can be produced.*

Claim 7.10. (Refinement does not converge). *A difference in the quantity of evidence is repaired by more evidence. A difference in the quantities of Proposition 7.9 is not, since they are unchanged by any refinement of the sample conducted within the repertoire of Proposition 7.8. The limitation of the classical apparatus in this class of case is therefore qualitative, and the direction of*

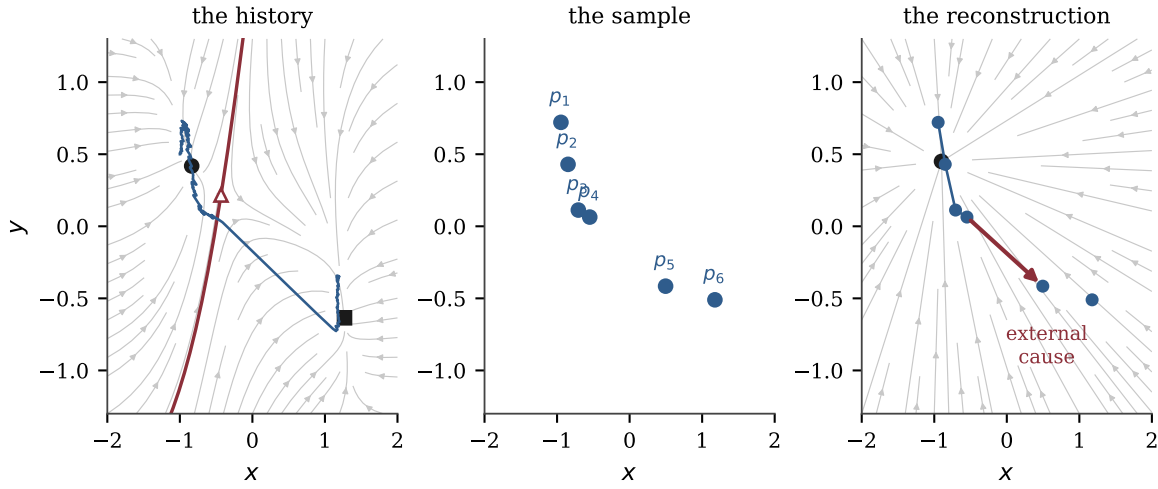


Figure 11: History, sample, and reconstruction in the state space. Left: the flow, the two attractors, the saddle, the boundary between the basins, and the path taken. Middle: the points that entered the record. Right: the reconstruction, in which the repertoire supplies one attractor and the departure from it is rendered as an external cause. The number of attractors, the presence of a boundary between basins, and the standing of the final displacement as a crossing all differ between the first panel and the third.

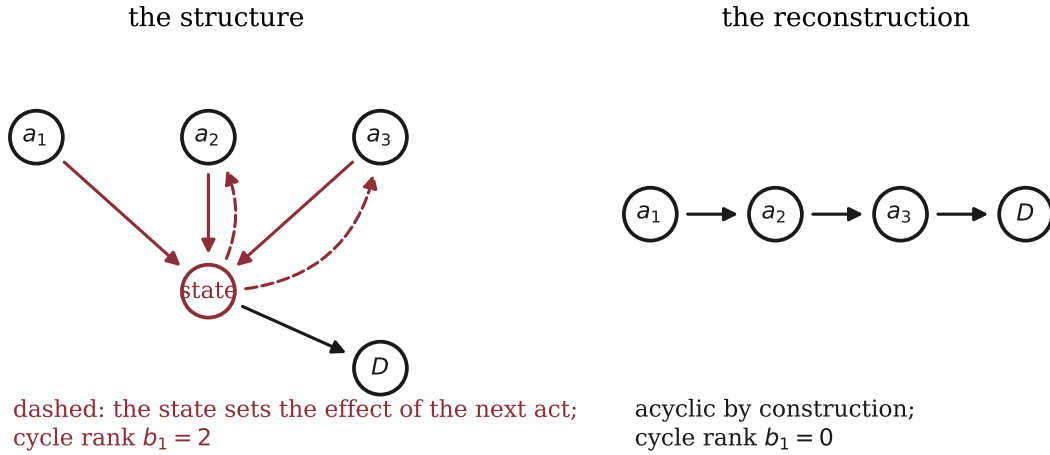


Figure 12: The causal structure and its reconstruction. Left: the acts bear on the state, and the state settles the effect of each subsequent act, so two of the edges run backwards and the cycle rank is two. Right: the reconstruction, which is acyclic because a finding must terminate, and whose cycle rank is zero. The edges that carry the cycles are those for which no sampled point can be produced, since a change in the state generates no event.

improvement lies in the repertoire and in the registers of Proposition 7.7 rather than in the density of the record.

The difficulty stated in this subsection concerns the resources presently available and

would be relieved by their extension. Section 10 states a difficulty that the extension would leave standing, and proposes what may be sought in its place.

Corollary 7.11. (Causal saturation with attributive vacancy). *The criticality case is causally saturated, since every element of the process contributed, and attributively vacant, since each end is excluded by a different doctrine. Neither doctrine appears defective from within, so the vacancy is invisible at the level of any single legal operation, and by Corollary 7.4 the support the two limbs appear to give one another is spurious.*

8 THE ENDOGENEITY OF THE PREDISPOSITION

This section examines the doctrines that operate upon the constitution of the injured party, shows that they share a presupposition that the criticality case defeats, and states the consequence for the assessment of damages. The method is to treat two doctrines conventionally opposed as one structure carrying two signs.

Two rules govern the case in which the condition of the injured party contributed to the extent of the harm. Under the first, a defendant takes the injured party as found, and an unusual susceptibility that magnifies the harm is charged to the defendant. Under the second, an unusual susceptibility reduces the award, whether by apportionment, by analogy with the provisions on contributory fault, or by treating part of the loss as attributable to the condition (Smith, 1962; Hart & Honoré, 1985).

The rules carry qualifications already, and these must be stated before any further one is proposed. Systems that permit reduction commonly distinguish a condition amounting to a disease or disorder from ordinary variation within the population, and they decline reduction for the latter. Every qualification of this kind turns on the character of the feature and leaves untouched the question of how the feature came about, so the existing limits constrain the argument of this section without answering it.

Claim 8.1. (One structure with two signs). *The rule that a defendant takes the injured party as found and the rule reducing damages for predisposition are one doctrine with opposite signs. Each asks whether an amplification present in the injured party is to be charged to the defendant or discounted from the award.*

Claim 8.2. (The presupposition of exogeneity). *Both rules presuppose that the amplification was present independently of the conduct at issue. Each is an allocation rule for a susceptibility taken as given.*

Corollary 8.3. (Deduction of the wrong). *In the criticality case the susceptibility is a product of the conduct at issue, by Claim 1.2 and Property 3.1. Application of the reduction rule therefore removes from the award the quantity that constitutes the wrong, and a doctrine designed to prevent*

a defendant from bearing another's misfortune operates to relieve the defendant of the consequences of the defendant's own contribution.

The case law contains an authority reaching the correct result on a narrower ground. Where an employee was injured through the employer's breach, developed a depressive illness in consequence, and later took his own life, the House of Lords declined to treat the death as falling outside the employer's liability, and the majority declined any reduction for the injured party's own conduct (Corr, 2008). The rationales offered concerned foreseeability and the impairment of the injured party's capacity. The rationale available on the present analysis is one of endogeneity, and it is the more general of the two, since it reaches susceptibilities of any kind where those susceptibilities were themselves produced by the conduct complained of, without turning on the degree of impairment or on what was foreseeable at the outset.

One further support is available, and it lies outside the law of damages. Administrative criteria for the recognition of work-related disease proceed from the premise that an underlying condition forms and worsens along a natural course, and they treat the disease as work related where the burden of the work aggravated that condition markedly beyond that course. The distinction between the progression a predisposition would have undergone of itself and the aggravation produced by the conduct is therefore already drawn, and it is operated as a matter of routine. What this section asks of the law of damages is the transposition of a distinction that administrative practice already applies.

Proposition 8.4. (Argument by extension). *The rationale of the authority just discussed generalises from an impairment of capacity produced by the breach to susceptibilities of any kind produced by the conduct at issue. The extension operates within existing doctrine and requires no legislative amendment.*

9 AMPLIFICATION BY THE INJURED SYSTEM

This section addresses the feature of the criticality case that is most consequential for the defence, states the two settled figures through which doctrine reads that feature, and shows that the authority discussed in §8 has already crossed the relevant ground. The method is again to identify the governing doctrine and to derive the extension from its stated rationale.

By (2) a disturbance of amplitude a delivered at the attractor produces an excursion whose integral over time is $a\tau$ in the linear regime, and τ rises without bound as the margin narrows. The amplification is therefore a property of the condition that the antecedent conduct produced, and the amplitude of the disturbance settles it in no part. The disturbance supplies an occasion and the magnitude of the outcome is furnished within the system. In the human case this appears as a response disproportionate to the occasion, and in severe cases as an act

of the injured party.

Claim 9.1. (Self-amplification). *In a criticality case the harm is produced through amplification performed by the injured system. The integrated excursion following a disturbance of amplitude a is $a\tau$, and τ is a property of the state that the antecedent conduct produced, so the contribution of the injured system to the magnitude of the outcome rises without bound as the margin narrows.*

Claim 9.2. (Two discharging readings). *Doctrine possesses two settled readings of an outcome completed by the injured party. Under the first, a free and voluntary act of the injured party terminates the chain of attribution. Under the second, the contribution of the injured party's constitution reduces the award. Both readings relieve the defendants, so the structure of the criticality case supplies the defence with an affirmative argument.*

This is the point at which the class is most exposed, and the exposure increases as a case approaches the structure of Definition 1.1. The closer the terminal disturbance comes to insignificance, the stronger the argument that the outcome belongs to the injured party.

Proposition 9.3. (Installed gain). *Where the amplification that completed the transition was produced by the conduct at issue, the completion of the outcome by the injured system leaves the attribution intact. The authority discussed in §8 applies this reasoning, since the final act was that of the injured party and the attribution was maintained on the ground that the condition from which the act proceeded had been produced by the breach (Corr, 2008).*

The proposition generalises an existing holding and carries the same limits. It requires that the production of the amplifying condition by the conduct be established, and it leaves untouched cases in which the condition arose independently. Section 12 states what would defeat it.

10 ATTRIBUTION ON INVARIANTS OF POSITION

This section states the limitation that survives every improvement in the resources of §7, and proposes what a tribunal may seek once the demand for a determinate quantity of contribution is abandoned. The method is to state three conditions on that demand, to observe that the case supplies none of them, and then to identify the findings whose value is fixed by the record notwithstanding.

10.1 Conditions on a determinate contribution

This subsection states what a finding of determinate contribution would require of the case and of the tribunal, and separates the requirements that a better-resourced tribunal could meet from those it could not. The method is to enumerate the conditions and to ask of each what supplies it.

Proposition 10.1. (Three conditions on a determinate contribution). *A finding that a factor contributed a determinate quantity requires three things. It requires CLOSURE of the set of variables bearing on the margin, since a contribution is defined only against a fixed remainder. It requires a declared CLASS of admissible trajectories, since finitely many sampled points are consistent with uncountably many continuous histories and are made to determine one only by a restriction stated in advance. And it requires the IDENTIFIABILITY of the couplings among the variables from the evidence available, since the effect of a factor runs through them.*

Corollary 10.2. (The conditions are prior to the tribunal). *The three conditions concern what the case supplies rather than what a tribunal can do, so a tribunal of unlimited capacity, whose reconstruction preserved every structure of §7, would face them unchanged. The first is supplied by nothing available, since the variables bearing on the margin of a relational system extend to whatever couples with it, and no principle internal to the case delimits that set. The difficulties of §7 are remediable and this one is not, so the limitation of the classical apparatus in this class is of two kinds and only the lesser is a matter of resources.*

10.2 Findings fixed by the record

This subsection identifies the findings whose value is settled by the record notwithstanding the conditions of the preceding subsection, and reports a computation separating them from the findings the record leaves open. The method is to enumerate models consistent with one record and to compare the findings across them.

Definition 10.3. (Invariant finding). *Consider the set of models of the injured system that are consistent with the record. A finding is invariant where its value is the same across that set, and it is underdetermined where it varies within it.*

Proposition 10.4. (Separation of quantities from invariants). *The record leaves the quantitative contribution of a factor underdetermined and fixes several findings of position. Figure 13 reports both over three hundred models drawn from unknown couplings, unknown noise intensity and an unmodelled exogenous term, and retained only where the resulting history agrees with the record. The share attributed to one factor varies across those models by a factor of about five, while the findings that two states were available, that a boundary between them was crossed, and that no path of return remained hold in every one of them. One further finding, whether the prior state was effectively eliminated, holds in fewer than half, which marks the boundary of what the record settles.*

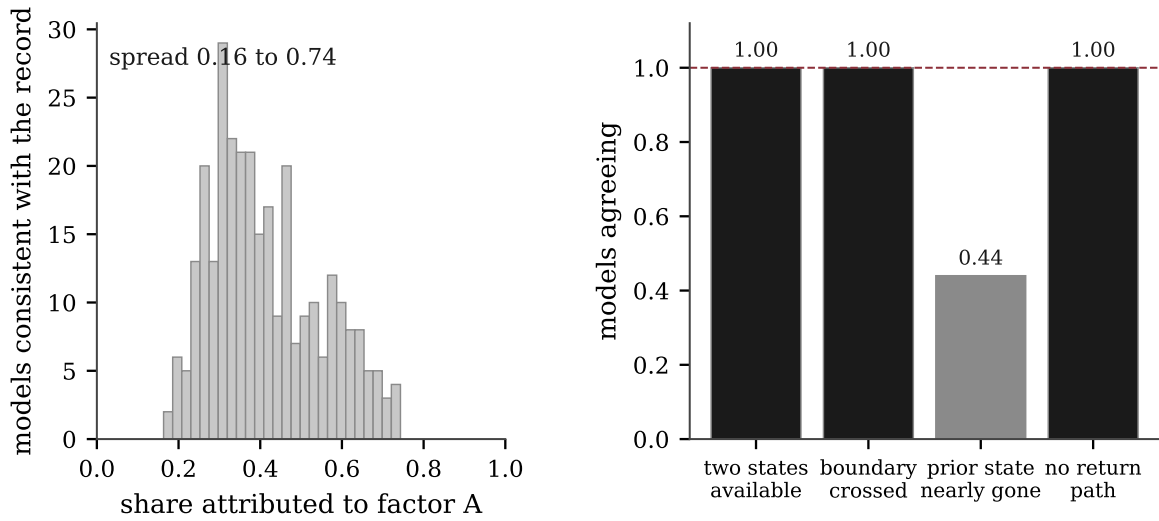


Figure 13: Quantities and invariants across the models consistent with one record. Left: the share of the reduction of margin attributed to one factor, over three hundred models agreeing with the record, spread across most of the available range. Right: the proportion of the same models agreeing on each finding of position. Three of the four are settled by the record and the fourth is not, so the separation proposed here is a partition of the findings and no general licence.

10.3 Findings of position and their doctrinal counterparts

This subsection sets out the findings of position available in a criticality case, states the counterpart of each in existing doctrine, and gives the reason for preferring findings of this form. The method is to pair each finding with the doctrinal question it answers, and then to argue from the form of the available judgment.

Proposition 10.5. (Four candidate findings). *Four findings of position are available in a criticality case, and each has a counterpart in existing doctrine. Whether a state available to the injured party was eliminated corresponds to the closing of a possibility, for which the law of competition already has an apparatus. Whether a boundary between states was crossed corresponds to the occurrence of damage. Whether the path of return was destroyed corresponds to the impossibility of restitution, and it is the question the remedial repertoire is least equipped to ask. Whether the structure ceased to admit revision corresponds to the loss of the capacity of the injured party to have an account of himself entered and corrected.*

Claim 10.6. (The form of an invariant matches the form of the judgment). *Each of the findings of Proposition 10.5 is binary and a verdict is binary. On the thesis that legal causal representation is a projection of the structure of the available judgment, the quantities are the material that the remedial form is unable to carry and the findings of position are the material it is able to carry without modification. An invariant standard is therefore no second best, and it is the standard whose form the existing judgment already has.*

Claim 10.7. (What an invariant standard gives up). *A finding of position attributes and does not apportion, since a binary finding carries no share. Liability established on such findings is therefore joint, or the obligation is a threshold obligation, and any division among those responsible must be drawn from a source other than the causal enquiry. The proposal of this section trades apportionment for attributability, and the trade is to be defended and not concealed.*

11 THE INHERITANCE OF THE PROBLEM BY SUPERVISORY OBLIGATION

This section examines the proposal that the criticality case be handled through supervisory obligations imposed on organisations, states the price of that migration, identifies the difficulty that supervision inherits, and sets out the one route that dispenses with the representation of trajectories. The method is to state the advantages first and the costs afterwards, since the costs determine whether the migration constitutes a solution.

The migration is attractive for reasons internal to the projection thesis. Criticality is representable where the available judgment is continuing, forward-looking, and non-terminal, and supervisory instruments have that form. Obligations of the kind exist across systems. The employer's obligation to attend to the safety of those working under its direction, and the obligations imposed on schools to maintain standing arrangements for the detection and handling of bullying, address the maintenance of conditions and require the prediction of no particular outcome.

Proposition 11.1. (Inheritance by supervision). *Supervisory instruments carry the problem of trajectory representation in modified form. They require an estimate of the condition of the supervised system, which is the subject of the sister paper. They operate over a boundary fixed by institutional and territorial criteria while the generative process crosses such boundaries. They are attainable only through structural proxies and are therefore exposed to the displacement of the objective by its indicator. And they are themselves generative, since a supervised organisation reorganises its recording practices under supervision and thereby alters the system that is measured.*

To these is added a cost that concerns the injured party directly. Supervisory instruments are forward-looking and impersonal. They regulate a capacity and they attribute no wrong.

Claim 11.2. (The price of migration). *Migration of the criticality case to supervisory obligation relieves the individual participants. The organisation bears a capacity obligation, the participants whose conduct constituted the trajectory bear nothing, and the outcome is least acceptable in the class of case that motivated the inquiry.*

One route avoids both the vacancy of §7 and the price of Claim 11.2, and it does so by declining to represent the trajectory at all.

Claim 11.3. (Relocation of the injury). *The injury may be located in the degradation of the relation itself. On this construction each act of exclusion constitutes an independent interference with*

the personal interest of the injured party, actionable when it occurs and derived from no terminal outcome.

Corollary 11.4. (Restoration of datability). *Under Claim 11.3 each act carries its own date, so the operations of Table 1 are supplied with what they require. The threshold difficulty of Proposition 7.1 is dissolved by declining the threshold, since the wrong is complete in the act.*

The resulting division of labour assigns to adjudication the individual acts, reconceived as interferences complete in themselves, and assigns to supervision the trajectory and the maintenance of the conditions under which trajectories of this kind form. Each carries its own burden. The residual difficulty is that the severity of the terminal harm remains unattributed under Claim 11.3, and §12 records this as open.

12 CONDITIONS OF REFUTATION AND OPEN QUESTIONS

This section states, for each principal claim, the observation that would defeat it, and records the questions the paper leaves open. The method follows the requirement that a position be stated in a form permitting its own refutation.

Proposition 4.1 and Claim 5.4 fail together upon the exhibition of a doctrine in which a distributed trajectory operates as a ground of attribution, with the change of condition neither assigned to a date nor decomposed into dated acts. Claim 5.5 predicts that any candidate will be found to have a single author, and a counterexample with many authors defeats it.

Proposition 5.1 fails upon the exhibition of a system in the class whose margin is affine in the accumulated forcing, and Corollary 5.2 fails upon the exhibition of an order-independent apportionment rule for a concave quantity that requires no distribution over orderings. Proposition 5.3 fails upon the exhibition of an apportionment rule operating on a quantity lacking a common unit across contributions; the mesothelioma line is the nearest candidate and does not presently satisfy the condition, since it apportions exposure and leaves stability untouched.

The reading of Figure 9 fails if the acts composing a candidate case are shown to be separable into a class above the threshold of significance, in which case ordinary doctrine reaches them and the analysis of this paper is unnecessary for that case. Properties 3.1 to 3.5 fail of a candidate case upon evidence that the injured system was already outside its basin during the interval, that the terminal disturbance was of a magnitude sufficient in itself, that the transition admits of a threshold description robust to the noise present in the case, or, for Property 3.5, that the injured system is adequately described by a single quantity, in which case direction has no content. Each is an empirical question about the case and is to be contested on evidence.

Propositions 6.2 and 6.3 are assertions about a parameterised system and are established for the parameters reported. They fail of a candidate case upon evidence that one factor alone

would have carried the injured system across the boundary, in which case the case is one of ordinary single causation, or upon evidence that the conjunction was sufficient with certainty, in which case the noise term is idle and Corollaries 6.4 and 6.5 lapse with it. Claim 6.6 fails upon the exhibition of a head of allocation addressed to the variance of the setting.

Proposition 7.3 and Corollary 7.4 fail upon the exhibition of a recording practice whose threshold is set independently of the threshold of legal significance, in which case the two failures are indeed independent and the argument of §7 is weakened to that extent. Corollary 7.5 fails where an independent and continuous record of the injured system exists, and §11 and the sister paper concern the conditions under which such a record is held. Claim 7.6 is testable directly against records in which the magnitude of the acts is separately established.

Propositions 7.8 and 7.9 fail upon the exhibition of an evidentiary form in which a reduction of recoverable capacity may be reported directly and received, and of a repertoire containing a term for a change in the stability of a system. Proposition 7.9 fails if the reconstruction in a given case is shown to preserve the quantities named, and Claim 7.10 falls with it. Each is a claim about the resources presently available and would be defeated by an instance of their use.

Corollary 8.3 fails if the reduction doctrines are shown to contain a qualification excluding susceptibilities produced by the conduct at issue. The qualifications presently available turn on the character of the feature and are silent on how the feature arose, so the corollary stands against the law as it is and would fall to an authority addressed to origin. Proposition 8.4 fails if the authority relied upon is shown to rest on a rationale confined to impairment of capacity, in which case the extension requires legislation.

Proposition 9.3 fails where the amplifying condition is shown to have arisen independently of the conduct, and it is silent on the allocation of the outcome where the condition has mixed origins.

Proposition 10.1 fails upon the exhibition of a principle internal to a case that closes the set of variables bearing on the margin, and Corollary 10.2 falls with it. Proposition 10.4 is a claim about one parameterised family and fails where the invariants named are shown to vary across the models consistent with a given record, which is assessable case by case and is the assessment the proposal requires in each instance. Claim 10.6 fails where a remedial form is exhibited that carries shares of a non-additive quantity, and Claim 10.7 states the cost that any defence of the proposal must meet.

Three questions are left open. The first concerns the estimation of the condition of a system under supervision, which Proposition 11.1 identifies as inherited and which the sister paper addresses. The second concerns the severity of the terminal harm under Claim 11.3, which locates the wrong in the individual acts and leaves the aggregate outcome without an attributive home. The third concerns the point at which the regress of generation terminates, since the factors composing the trajectory are themselves products of prior generative processes, and

this paper offers no normative ground for arresting the regress at any particular remove.

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