

Transformations between Structural and Spectral-Temporal Representations in Generative-Relational Governance

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Abstract

This paper develops a transformation framework connecting structural and spectral-temporal representations of governance within a generative-relational approach. Building on two previously distinguished representational coordinates, Type-I governance describes intervention through structural objects such as rules, dynamical processes, relational structures, and generative backgrounds, while Type-II governance describes intervention through temporal objects such as timescales, spectral components, phase relations, synchronization, resonance, polyfrequency organization, cross-frequency coupling, and spectral regimes. The present paper studies the relations between these representations rather than introducing an additional taxonomy. Structural-to-temporal transformation is modeled as a generative chain in which structural configurations produce trajectories that become observable through measurement and are subsequently represented in spectral-temporal form. Because distinct structural systems can generate equivalent temporal representations, the inverse relation is generally set-valued and gives rise to representation-relative equivalence classes and problems of structural identifiability. The framework further develops induced temporal operators, structural lifting of temporal targets, interventional identifiability, local tangent-space transformations, structural-to-temporal sensitivity maps, approximate reconstruction, and information preservation across representations. Particular attention is given to conditions under which a structural intervention induces a well-defined temporal transformation and to the multiplicity of structural realizations capable of achieving the same temporal objective. The resulting framework establishes a representational theory of generative-relational governance in which structural and spectral-temporal descriptions are complementary, partially transformable, and generally non-invertible. It provides a foundation for analyzing how governance knowledge, intervention design, and empirical inference change across representational domains.

Keywords: generative-relational governance; governance representation; structural representation; spectral-temporal representation; representation transformation; identifiability; structural lifting; interventional equivalence; tangent-space transformation; inverse problem

Conceptual and Formal Note

This note specifies the representational commitments, transformation relations, and formal boundaries used throughout the paper. Its purpose is to clarify the relation between the structural Type-I representation and the spectral-temporal Type-II representation before the detailed transformation framework is developed. The note treats transformation as a generative and observation-dependent process, distinguishes forward representation from inverse inference, and introduces the equivalence, intervention, lifting, and local-transformation concepts used in later sections.

Structural and Spectral-Temporal Representational Domains

The paper begins from two previously distinguished representations of generative-relational governance. The structural Type-I representation describes the organization through which a governed system generates its evolution. A governed structural system is represented by Equation 1.

$$\mathfrak{S}_t = (X_t, x_t, R_t, F_t, C_t, \mathcal{B}_t). \quad (1)$$

Here X_t denotes the relevant state space, x_t the current state, R_t the rule structure, F_t the dynamical process, C_t the relational structure, and $\mathcal{B}_t$ the generative background.

The spectral-temporal Type-II representation describes temporally organized features generated through the evolution and observation of such a system. A general Type-II representation is denoted by Equation 2.

$$\Theta_t = (\tau_t, \Omega_t, \phi_t, \mathcal{L}_t, \mathcal{R}_t, \mathcal{I}_t, \mathcal{H}_t, \mathcal{C}_t, \Sigma_t), \quad (2)$$

where the components can represent characteristic timescales, spectral support, phase relations, locking relations, resonant-response structures, interference relations, harmonic or polyfrequency organization, cross-frequency relations, and spectral-regime structure.

Equation 2 defines a representational domain rather than a requirement that every empirical system possess every component. The applicable Type-II coordinates depend on the temporal structure supported by the model and observations.

Generative Transformation Chain

The relation between the two representations is mediated by generated trajectories and observation. The structural representation therefore does not transform directly into a spectrum or time-frequency representation.

The complete forward chain is represented by Equation 3.

$$\mathfrak{S} \xrightarrow{\mathcal{D}} x(\cdot) \xrightarrow{\mathcal{O}} y(\cdot) \xrightarrow{\mathcal{Q}} \Theta. \quad (3)$$

In Equation 3, $\mathcal{D}$ denotes dynamical realization, $\mathcal{O}$ observation or measurement, and $\mathcal{Q}$ temporal representation.

The composite structural-to-temporal transformation is therefore represented by Equation 4.

$$\mathcal{T}_{\mathcal{O}, \mathcal{Q}, W} = \mathcal{Q}_W \circ \mathcal{O} \circ \mathcal{D}. \quad (4)$$

The corresponding Type-II representation generated from a structural system is given by Equation 5.

$$\Theta = \mathcal{T}_{\mathcal{O}, \mathcal{Q}, W}(\mathfrak{S}), \quad (5)$$

where W denotes the relevant temporal window or localization structure.

The transformation is therefore conditional on the system realization, observable, measurement architecture, representation method, and temporal resolution. It should not be interpreted as a universal transform acting directly on the structural coordinates $(R, F, C, \mathcal{B})$.

Representation-Relative Equivalence

Distinct structural systems can generate the same Type-II representation under a specified transformation. The paper therefore introduces a representation-relative equivalence relation.

For a fixed transformation $\mathcal{T}$, structural equivalence is defined by Equation 6.

$$\mathfrak{S}_1 \sim_{\mathcal{T}} \mathfrak{S}_2 \iff \mathcal{T}(\mathfrak{S}_1) = \mathcal{T}(\mathfrak{S}_2). \quad (6)$$

The corresponding equivalence class is represented by Equation 7.

$$[\mathfrak{S}]_{\mathcal{T}} = \{ \mathfrak{S}' \mid \mathcal{T}(\mathfrak{S}') = \mathcal{T}(\mathfrak{S}) \}. \quad (7)$$

Equation 7 identifies the structural systems that remain indistinguishable under the selected spectral-temporal representation.

Equivalence is therefore relative to the observation and representation architecture. Increasing temporal resolution, adding observables, or adding phase, coupling, or regime information can refine the equivalence partition and reduce structural ambiguity.

Inverse Relation and Structural Compatibility

The inverse problem begins from an observed spectral-temporal representation and asks which structural systems remain compatible with it.

For an observed representation Θ^{obs} , the structural compatibility set is represented by Equation 8.

$$\mathfrak{I}_{\mathcal{T}}(\Theta^{\text{obs}}) = \{ \mathfrak{S} \mid \mathcal{T}(\mathfrak{S}) = \Theta^{\text{obs}} \}. \quad (8)$$

The inverse relation in Equation 8 is generally set-valued. Structural reconstruction becomes unique only under additional identifiability conditions.

This asymmetry is fundamental to the paper. The forward problem concerns the generation of temporal representation from structural organization. The inverse problem concerns the restriction of a structural compatibility set using temporal evidence.

Forward Temporal Multiplicity

A structural system can also generate more than one Type-II representation when initial conditions, disturbances, contexts, observation windows, or parameter configurations vary.

The forward temporal set associated with a structural system is represented by Equation 9.

$$\mathfrak{T}(\mathfrak{S}) = \{ \Theta \mid \Theta = \mathcal{T}_{\eta}(\mathfrak{S}), \eta \in \mathcal{E} \}, \quad (9)$$

where η collects the relevant initial, contextual, observational, or representational conditions.

Equation 9 prevents the transformation framework from assuming that one structural configuration has one invariant spectral-temporal image.

Structural Intervention and Induced Temporal Transformation

The relation between Type-I and Type-II governance can also be studied at the level of intervention.

Let a structural intervention be represented by Equation 10.

$$\mathcal{U}_I : \mathfrak{S}^- \longrightarrow \mathfrak{S}^+. \quad (10)$$

Applying the representation transformation before and after the intervention produces the diagrammatic relation summarized by Equation 11.

$$\mathcal{T} \circ \mathcal{U}_I : \mathfrak{S}^- \longrightarrow \Theta^+. \quad (11)$$

A structural intervention induces a well-defined operator on the Type-II representation only when its action is compatible with the equivalence classes generated by $\mathcal{T}$.

The required consistency condition is represented by Equation 12.

$$\mathfrak{S}_1 \sim_{\mathcal{T}} \mathfrak{S}_2 \implies \mathcal{U}_I(\mathfrak{S}_1) \sim_{\mathcal{T}} \mathcal{U}_I(\mathfrak{S}_2). \quad (12)$$

When Equation 12 holds, an induced Type-II operator $\bar{\mathcal{U}}_{II}$ can be defined through Equation 13.

$$\bar{\mathcal{U}}_{II}(\mathcal{T}(\mathfrak{S})) = \mathcal{T}(\mathcal{U}_I(\mathfrak{S})). \quad (13)$$

The resulting transformation relation satisfies Equation 14.

$$\mathcal{T} \circ \mathcal{U}_I = \bar{\mathcal{U}}_{II} \circ \mathcal{T}. \quad (14)$$

Failure of the condition in Equation 12 constitutes an interventional form of representational non-identifiability. Two structurally distinct systems can be temporally indistinguishable before intervention and temporally distinguishable after the same structural intervention.

Temporal Targets and Structural Lifting

The reverse intervention problem begins with a desired transformation in the Type-II domain and asks which structural interventions can realize it.

Let a desired temporal transformation be represented by Equation 15.

$$\mathcal{V}_{II} : \Theta^- \longrightarrow \Theta^+. \quad (15)$$

The structural lifting set associated with this temporal target is represented by Equation 16.

$$\text{Lift}_{\mathcal{T}}(\mathcal{V}_{II}) = \{\mathcal{U}_I \mid \mathcal{T} \circ \mathcal{U}_I = \mathcal{V}_{II} \circ \mathcal{T}\}. \quad (16)$$

Equation 16 is generally set-valued. A single Type-II objective can admit several Type-I realizations through different rules, dynamical processes, relational structures, or generative backgrounds.

The multiplicity of lifts makes the two taxonomies jointly necessary. Temporal equivalence of interventions does not imply structural equivalence, and structurally different realizations can carry different resource, institutional, distributive, and generative consequences.

Local and Tangent-Space Transformation

Global transformation can remain nonlinear and non-invertible while local relations between structural and temporal variation remain informative.

Let $\mathcal{M}_I$ denote a local manifold of structural representations and $\mathcal{M}_{II}$ a local manifold of spectral-temporal representations. The differential of the transformation is represented by Equation 17.

$$D\mathcal{T}_{\mathfrak{S}} : T_{\mathfrak{S}}\mathcal{M}_I \longrightarrow T_{\Theta}\mathcal{M}_{II}. \quad (17)$$

For sufficiently small structural variation, the corresponding temporal variation is approximated by Equation 18.

$$\delta\Theta \approx D\mathcal{T}_{\mathfrak{S}} \delta\mathfrak{S}. \quad (18)$$

Under finite-dimensional local coordinates $z \in \mathbb{R}^p$ and $\theta \in \mathbb{R}^q$, the local sensitivity map is represented by Equation 19.

$$J_{I \rightarrow II} = \frac{\partial \theta}{\partial z}. \quad (19)$$

Equation 19 provides a local description of which structural perturbations affect which temporal coordinates.

The null space of the local map is represented by Equation 20.

$$\ker J_{I \rightarrow \Pi} = \{\delta z \mid J_{I \rightarrow \Pi} \delta z = 0\}. \quad (20)$$

Directions in Equation 20 represent locally distinguishable structural changes that remain invisible in the selected Type-II coordinates.

Approximate Inversion

When a desired local Type-II change is specified, structural intervention can be formulated as an approximate inverse problem.

Let $\delta\theta^*$ denote a desired local temporal change. The admissible structural adjustment can be selected through Equation 21.

$$\delta z^* \in \arg \min_{\delta z \in \mathcal{U}_{\text{adm}}} \|J_{I \rightarrow \Pi} \delta z - \delta\theta^*\|. \quad (21)$$

Here $\mathcal{U}_{\text{adm}}$ denotes the set of structurally admissible interventions.

Equation 21 permits local governance even when a global inverse transformation is unavailable. The admissible set can encode resource, institutional, legal, viability, reversibility, and generative constraints.

Information Preservation and Loss

Transformation between representational domains generally changes the information available for governance analysis. A Type-II representation can preserve temporal regularities while discarding structural distinctions that do not affect the selected observable within the selected resolution.

The information preserved by a representation is therefore task-relative. Structural distinctions that are irrelevant for one temporal objective can be essential for another intervention or normative evaluation.

Representational refinement can be expressed through partitions of the structural domain. Let $\mathcal{P}_{\mathcal{T}}$ denote the partition generated by the equivalence relation $\sim_{\mathcal{T}}$. A representation $\mathcal{T}_2$ is structurally finer than $\mathcal{T}_1$ when the condition in Equation 22 holds.

$$\mathcal{P}_{\mathcal{T}_2} \preceq \mathcal{P}_{\mathcal{T}_1}, \quad (22)$$

where every equivalence class under $\mathcal{T}_2$ is contained within an equivalence class under $\mathcal{T}_1$.

Equation 22 formalizes the idea that additional observables or temporal coordinates can increase structural distinguishability.

Time-Varying Transformation Structure

Structural systems, observation architectures, and spectral-temporal representations can all evolve. The transformation operator should therefore be allowed to vary through time.

A time-dependent transformation is represented by Equation 23.

$$\Theta_t = \mathcal{T}_t(\mathfrak{S}_t). \quad (23)$$

Consequently, representational equivalence can also evolve. Two systems that are indistinguishable under one historical observation regime can become distinguishable after changes in measurement, temporal resolution, structural organization, or modal accessibility.

The paper therefore treats equivalence classes, inverse sets, sensitivity maps, and structural lifts as potentially time-dependent objects.

Formal Status of the Transformation Framework

The transformation framework does not claim a universal Fourier-like duality between Type-I and Type-II governance. Fourier, wavelet, time-frequency, modal, phase, and related analytical methods can participate in the representation operator $\mathcal{Q}$ when their assumptions are supported.

The relation developed here is broader:

structural organization $\longrightarrow$ generated trajectory $\longrightarrow$ observation $\longrightarrow$ spectral-temporal representation.

The reverse direction is correspondingly an inference, reconstruction, or lifting problem whose uniqueness depends on additional structural, observational, and model conditions.

The terms *transformation*, *equivalence*, *identifiability*, *lifting*, and *local inversion* are used in this representational sense throughout the paper.

Scope of the Present Contribution

The present paper studies relations between two previously developed representational coordinates of generative-relational governance. It does not introduce a third governance taxonomy.

Its principal objects are the forward transformation $\mathcal{T}$, representation-relative equivalence classes, structural compatibility sets, intervention-induced temporal operators, structural lifts of temporal targets, local tangent maps, approximate inverse problems, and information preservation across representations.

The framework also distinguishes four analytical tasks that recur throughout the paper:

1. structural-to-temporal representation;
2. temporal-to-structural inference;
3. structural intervention with induced temporal transformation;
4. temporal governance objectives with structural realization.

These tasks share a common representational architecture while imposing different conditions on observability, identifiability, intervention, and reconstruction. The following sections develop each of these relations in greater detail.

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1 Introduction

Governance can be represented through the structures that generate system evolution and through the temporal organization exhibited by that evolution. These representational perspectives answer different analytical questions. A structural representation describes rules, dynamical processes, relational structures, and generative backgrounds through which governance acts. A spectral-temporal representation describes characteristic timescales, frequencies, phases, synchronization relations, resonance, interference, polyfrequency organization, cross-frequency coupling, and spectral regimes. The present paper studies the transformation relations between these two representational domains.

Temporality is already deeply embedded in political and administrative analysis. Historical institutional research emphasizes sequence, duration, path dependence, and processes whose consequences unfold over extended periods [10]. Public-administration and policy research likewise recognizes timing, speed, duration, time horizons, and differentiated timescapes as consequential dimensions of governance [6]. These traditions establish the importance of time for governance analysis. The spectral-temporal perspective extends this concern by distinguishing several temporal objects that can remain analytically different even when they occur within the same institutional process.

Formal analysis introduces an additional representational issue. Dynamical systems can exhibit oscillatory, multiscale, synchronized, and nonstationary behavior, while time-frequency analysis provides several methods for representing temporal variation whose frequency content changes through time [2, 11]. Such representations can reveal regularities that are difficult to express through structural description alone. Their interpretation nevertheless depends on the processes that generated the observations. Similar temporal patterns can arise from different underlying mechanisms, and different temporal patterns can emerge from structurally similar systems under different initial conditions, contexts, disturbances, or observation procedures.

The preceding generative-relational governance papers distinguished these perspectives as Type-I and Type-II governance representations. Type-I classifies governance according to the structural objects directly transformed by intervention. Its domain includes state-and-rule structure, dynamical process, relational structure, and generative background. Type-II classifies governance according to the spectral-temporal objects or relations directly transformed by intervention. Its domain includes timescale, spectral-selective, phase, synchronization and entrainment, resonance, superposition and interference, harmonic and polyfrequency, cross-frequency, and spectral-regime governance.

The distinction creates a new theoretical problem. Once two representations of governance have been specified, their relation cannot be assumed to be self-evident. A structural configuration does not transform directly into a frequency spectrum in the same sense that a time-domain signal can be mapped into a Fourier representation. Rules, relations, dynamical laws, and generative backgrounds first generate trajectories. Those trajectories are then observed through a particular measurement architecture, after which a spectral-temporal representation can be constructed.

The resulting forward relation is represented by Equation 24.

$$\mathfrak{S} \xrightarrow{\mathcal{D}} x(\cdot) \xrightarrow{\mathcal{O}} y(\cdot) \xrightarrow{\mathcal{Q}} \Theta. \quad (24)$$

In Equation 24, $\mathfrak{S}$ denotes the structural representation, $\mathcal{D}$ the dynamical realization through which structural conditions generate a trajectory, $\mathcal{O}$ the observation process, $\mathcal{Q}$ the selected temporal representation, and Θ the resulting spectral-temporal description.

This chain makes the transformation generative and observation-dependent. The relevant transformation is therefore the composition of dynamical realization, measurement, and representation. For a selected observation architecture and temporal window W , the composite operator is represented by Equation 25.

$$\mathcal{T}_{\mathcal{O}, \mathcal{Q}, W} = \mathcal{Q}_W \circ \mathcal{O} \circ \mathcal{D}. \quad (25)$$

Equation 25 provides the central forward operator of the paper. Its form immediately implies that the relation between Type-I and Type-II representations depends on more than structural organization.

Initial conditions, contextual parameters, observation selection, temporal resolution, windowing, and representation method can all affect the resulting Type-II description.

This observation also changes the inverse problem. Recovering a structural system from a spectral-temporal representation is generally closer to system identification and inverse inference than to an ordinary inverse coordinate transform. System-identification theory emphasizes that inference from observed input-output behavior depends on model classes, available data, experimental conditions, and identifiability [8]. The present framework extends this general concern to the relation between structural and spectral-temporal governance representations.

For an observed temporal representation Θ^{obs} , the corresponding structural compatibility set is represented by Equation 26.

$$\mathfrak{I}_{\mathcal{T}}(\Theta^{\text{obs}}) = \left\{ \mathfrak{S} \mid \mathcal{T}(\mathfrak{S}) = \Theta^{\text{obs}} \right\}. \quad (26)$$

Equation 26 is generally set-valued. Distinct structural systems can therefore remain indistinguishable within a selected spectral-temporal representation.

This structural multiplicity motivates a representation-relative notion of equivalence. For a fixed transformation $\mathcal{T}$, two structural systems belong to the same temporal equivalence class when they generate the same represented temporal organization. This relation is introduced by Equation 27.

$$\mathfrak{S}_1 \sim_{\mathcal{T}} \mathfrak{S}_2 \iff \mathcal{T}(\mathfrak{S}_1) = \mathcal{T}(\mathfrak{S}_2). \quad (27)$$

Equation 27 shifts the inverse problem from the immediate search for one hidden structural truth toward the characterization of structural systems that remain observationally compatible under the selected representation.

Representation-relative equivalence is important for governance because observational equivalence does not guarantee interventional equivalence. Two systems can generate the same temporal pattern and respond differently to the same structural intervention. Temporal observation can therefore be sufficient for some governance decisions while remaining insufficient for others.

This problem becomes explicit when interventions are introduced. Let $\mathcal{U}_I$ denote an intervention in the structural domain. A corresponding Type-II operator can be defined independently of the hidden structural realization only when structurally equivalent systems remain equivalent after the intervention. The required condition is represented by Equation 28.

$$\mathfrak{S}_1 \sim_{\mathcal{T}} \mathfrak{S}_2 \implies \mathcal{U}_I(\mathfrak{S}_1) \sim_{\mathcal{T}} \mathcal{U}_I(\mathfrak{S}_2). \quad (28)$$

When Equation 28 holds, the structural intervention descends to a well-defined transformation in the spectral-temporal representation. When it fails, systems that appear temporally identical before intervention can diverge temporally after the same structural intervention. The paper refers to this problem as *interventional identifiability*.

The reverse governance problem is equally important. A policymaker, institution, or other governance actor can begin from a desired temporal objective, such as modifying cadence, reducing synchronization, detuning a resonant relation, preserving temporal niches, weakening cross-frequency coupling, or changing a spectral regime. Such a Type-II objective does not specify which structural intervention should implement it.

The set of structural interventions capable of realizing a temporal transformation $\mathcal{V}_{II}$ is represented by Equation 29.

$$\text{Lift}_{\mathcal{T}}(\mathcal{V}_{II}) = \{ \mathcal{U}_I \mid \mathcal{T} \circ \mathcal{U}_I = \mathcal{V}_{II} \circ \mathcal{T} \}. \quad (29)$$

Equation 29 is also generally set-valued. Similar temporal objectives can potentially be realized through changes in rules, dynamical processes, relational couplings, infrastructural arrangements, or other structural supports. These realizations can differ substantially in cost, reversibility, institutional distribution, power, and generative consequences even when their represented temporal effects are similar.

The multiplicity in both directions provides one reason to preserve Type-I and Type-II as distinct analytical coordinates. Type-II representation cannot generally replace structural analysis, and Type-I representation cannot generally replace temporal analysis. Their relation is complementary, context-dependent, and only partially invertible.

The paper further develops a local form of this relation. Global structural-to-temporal transformations can be nonlinear, history-dependent, and difficult to invert. Within a sufficiently restricted neighborhood, however, small structural perturbations can sometimes be related to small temporal changes through a local differential map. This approach permits analysis of structural sensitivity even when global reconstruction remains unavailable.

The local transformation is represented by Equation 30.

$$\delta\Theta \approx D\mathcal{T}_{\Theta} \delta\mathfrak{S}. \quad (30)$$

Equation 30 provides the formal basis for the paper’s treatment of tangent-space transformation, local identifiability, null directions, and approximate structural inversion.

The local perspective has an important governance interpretation. A governance actor may lack sufficient information to reconstruct the complete structural system and can still possess useful knowledge about how selected structural changes are likely to alter selected temporal properties. Local governance can therefore remain possible under conditions where global inverse reconstruction is unavailable.

The transformation framework developed here consequently addresses several related problems. It studies how structural systems generate spectral-temporal representations, how temporal observations constrain possible structural explanations, how representation-relative equivalence classes arise, how structural interventions induce temporal transformations, how temporal objectives can be lifted into structural intervention sets, how local sensitivity supports partial inversion, and how information is preserved or lost across representational domains.

These problems also establish a distinction between transformation and translation. Transformation concerns a mathematically or operationally specified relation between representations. Translation concerns the interpretive use of one representation to inform another. A Type-II diagnosis can therefore guide Type-I investigation even when no exact inverse operator exists. Similarly, Type-I structural knowledge can constrain expectations about temporal behavior without uniquely determining one spectral-temporal trajectory.

The framework does not introduce a Type-III governance taxonomy. Its object is the relation between two already distinguished representations. It also does not posit a universal structural-spectral duality. Fourier transforms, time-frequency methods, phase reconstruction, modal decomposition, and related techniques can enter the representation operator $\mathcal{Q}$ when appropriate, while the full structural-to-temporal relation remains broader than any individual signal transformation.

The paper adopts three methodological commitments throughout. First, representation is treated as model-relative and observation-relative. Second, inverse inference is distinguished from forward generation. Third, formal transformation claims are limited to the structural, observational, and temporal conditions under which the required mathematical objects are defined.

The remainder of the paper develops this programme in stages. Section 2 specifies the structural and spectral-temporal representational domains and the generated trajectories connecting them. Section 3 develops the forward transformation from structural organization to temporal representation. Subsequent sections examine representational equivalence, structural identifiability, forward temporal multiplicity, intervention-induced operators, structural lifting, local and tangent-space transformations, approximate reconstruction, information preservation, and transformation under evolving structural conditions. The later sections connect these formal results with governance design, epistemic and operational constraints, empirical research, and the broader implications of maintaining structural and spectral-temporal representations as complementary coordinates of generative-relational governance.

2 Representational Foundations of Generative-Relational Governance

This section establishes the representational domains used throughout the paper. Its objective is to specify the structural Type-I representation, the spectral-temporal Type-II representation, the trajectories connecting them, the role of observation and measurement, the transformation operators acting between representational stages, and the conditions under which particular representations are applicable. The discussion proceeds from generative structure to realized trajectories, observed trajectories, and constructed temporal representations. This ordering is important because subsequent claims concerning equivalence, identifiability, induced operators, and structural lifting depend on distinctions established at each stage.

2.1 Structural Representation

This subsection specifies the structural domain from which the transformation problem begins. Its objective is to represent the objects through which a governed system generates evolution while retaining the distinctions required by the Type-I taxonomy.

A structural system at time t is represented by Equation 31.

$$\mathfrak{S}_t = (X_t, x_t, R_t, F_t, C_t, \mathcal{B}_t). \quad (31)$$

In Equation 31, X_t denotes the relevant state space, $x_t \in X_t$ the realized state, R_t the rule structure, F_t the dynamical process, C_t the relational structure, and $\mathcal{B}_t$ the generative background.

The components correspond to analytically different structural supports. Rules can determine admissible transitions, permissions, obligations, or decision procedures. Dynamical processes determine how states evolve under specified conditions. Relational structures determine which components can interact and through what couplings. Generative backgrounds condition the possibility, accessibility, cost, persistence, or stability of the processes represented at the other structural levels.

For transformation analysis, the structural system can be embedded in a broader structural domain. This domain is represented by Equation 32.

$$\mathcal{M}_I = \{\mathfrak{S} : \mathfrak{S} \text{ satisfies the structural conditions of the selected model class}\}. \quad (32)$$

Equation 32 defines the admissible structural configurations considered by a particular analysis.

The domain $\mathcal{M}_I$ can be finite-dimensional, infinite-dimensional, discrete, continuous, hybrid, or partially specified. The transformation framework does not require one universal structural state space. What matters is that the structural objects relevant to the selected governance problem are represented with enough precision to define the dynamical realization that follows.

The structural representation should also be distinguished from one realized history. A single $\mathfrak{S}$ can support several trajectories under different initial states, disturbances, parameter realizations, or exogenous conditions. This multiplicity becomes central to the forward transformation problem developed in Section 3.

2.2 Spectral-Temporal Representation

This subsection specifies the target representational domain used by Type-II governance. Its objective is to organize temporal information without requiring every system to support every possible spectral-temporal object.

A general Type-II representation at time t is represented by Equation 33.

$$\Theta_t = (\tau_t, \Omega_t, \phi_t, \mathcal{L}_t, \mathcal{R}_t, \mathcal{I}_t, \mathcal{H}_t, \mathcal{C}_t, \Sigma_t). \quad (33)$$

In Equation 33, τ_t collects characteristic timescales, Ω_t the relevant spectral or modal support, ϕ_t phase coordinates or phase relations, $\mathcal{L}_t$ locking and entrainment relations, $\mathcal{R}_t$ resonant-response structures, $\mathcal{I}_t$ interference and superpositional relations, $\mathcal{H}_t$ harmonic and polyfrequency organization, $\mathcal{C}_t$ cross-frequency or modulation relations, and Σ_t higher-order spectral-regime organization.

The corresponding representational domain is introduced in Equation 34.

$$\mathcal{M}_{\text{II}} = \{\Theta : \Theta \text{ is supported by the selected temporal model and observation system}\}. \quad (34)$$

Equation 34 makes the Type-II domain conditional on the representation that can actually be constructed.

The domain can therefore vary across studies. A governance process observed through sparse administrative records may support characteristic durations and recurrence rates while providing insufficient structure for phase reconstruction. Dense temporal data can support local spectra, phase relations, coherence, or cross-frequency analysis. Nonstationary processes can require local or time-varying representations [12, 2].

A useful distinction is therefore made between the full conceptual domain of Type-II objects and the subset supported by one empirical representation. For model M , the applicable Type-II coordinates are represented by Equation 35.

$$\mathcal{A}_{\text{II}}(M) \subseteq \{\text{T, S, P, L, R, I, H, C, } \}. \quad (35)$$

Equation 35 prevents stronger spectral-temporal language from being applied where its required formal objects are absent.

2.3 Generated Trajectories

This subsection introduces generated trajectories as the intermediate object between structural organization and temporal representation. Its objective is to separate structural specification from the histories that the structure can generate.

Let η collect initial conditions, exogenous inputs, disturbances, contextual parameters, and other realization conditions. The dynamical realization operator is represented by Equation 36.

$$\mathcal{D} : (\mathfrak{S}, \eta) \mapsto x_{\mathfrak{S}, \eta}(\cdot). \quad (36)$$

Equation 36 makes explicit that a trajectory is generated jointly by structural organization and realization conditions.

The set of trajectories compatible with one structural system is represented by Equation 37.

$$\mathfrak{X}(\mathfrak{S}) = \{x_{\mathfrak{S}, \eta}(\cdot) \mid \eta \in \mathcal{E}_{\mathfrak{S}}\}, \quad (37)$$

where $\mathcal{E}_{\mathfrak{S}}$ denotes the admissible realization conditions for $\mathfrak{S}$.

Equation 37 captures structural multiplicity at the level of system evolution.

This distinction is especially important in nonlinear and multiscale dynamical systems. Different initial conditions can lead toward different attractors, transient structures, or regime transitions, while small parameter changes can alter qualitative behavior [5, 7]. A structural representation therefore defines conditions for possible evolution rather than one invariant temporal history.

For governance analysis, the relevant trajectory can contain more than the nominal state variable. A trajectory descriptor can combine state, interaction, resource, decision, and event processes. Such an enriched trajectory is represented by Equation 38.

$$\xi(t) = (x(t), e(t), r(t), a(t), q(t)), \quad (38)$$

where the additional components can denote events, relational activity, resource flows, actions, or other temporally evolving observables required by the study.

Equation 38 allows the transformation framework to operate on governance processes whose relevant temporal organization is distributed across several state and event variables.

Generated trajectories also preserve an important causal ordering. Structural objects constrain or generate possible evolution, while temporal features are subsequently extracted from realized or observed

evolution. The spectral-temporal representation therefore remains downstream from the generative realization even when it becomes the primary object of governance analysis.

2.4 Observation and Measurement

This subsection specifies the observation layer separating generated trajectories from empirical temporal representations. Its objective is to make explicit the information selection, aggregation, noise, and temporal resolution introduced by measurement.

Let $x(\cdot)$ denote a generated trajectory and $y(\cdot)$ its observed representation. The observation relation is represented by Equation 39.

$$y(\cdot) = \mathcal{O}[x(\cdot); \psi], \quad (39)$$

where ψ denotes observation parameters such as selected variables, sampling resolution, aggregation, delay, noise structure, and measurement window.

Equation 39 permits the observation operator to differ substantially from direct state measurement.

A conventional state-space observation model provides one special case. It is represented by Equation 40.

$$y_k = h(x(t_k)) + \nu_k, \quad (40)$$

where h is an observation function and ν_k represents measurement error or unresolved variation.

Equation 40 illustrates how distinct underlying states can become observationally similar when the observation map compresses structural information.

Observation can also be event-based. If only the times $\{t_1, t_2, \dots\}$ at which specified events occur are recorded, the observable can be represented by Equation 41.

$$y_E(t) = \sum_k \delta(t - t_k). \quad (41)$$

Equation 41 represents an idealized event train whose temporal organization can support cadence, inter-event interval, or spectral analysis.

Observation is consequential for inverse inference because information absent from $y(\cdot)$ cannot generally be recovered through temporal representation alone. System identification therefore depends on what is observed, how it is excited, and which model class is entertained [8]. State reconstruction from temporal observations likewise requires specific dynamical and observational conditions [16].

The observation layer can therefore enlarge structural equivalence classes. Two structural systems distinguishable at the full-state level can become indistinguishable after aggregation, coarse temporal sampling, or observation of only a common output variable.

The dependence of observation on scale is represented by Equation 42.

$$y_{\Delta, W} = \mathcal{O}_{\Delta, W}[x(\cdot)], \quad (42)$$

where Δ denotes temporal resolution and W the observation window.

Equation 42 will later support resolution-relative equivalence and information-loss analysis.

2.5 Representation Operators

This subsection specifies the operators that transform observed trajectories into Type-II representations. Its objective is to distinguish the generic representation map from the particular analytical method used to construct temporal coordinates.

Let $\mathcal{Q}$ denote a temporal representation operator acting on the observed process. The representation stage is expressed by Equation 43.

$$\Theta = \mathcal{Q}[y(\cdot)]. \quad (43)$$

Equation 43 includes a broad class of possible temporal descriptions.

For a stationary or approximately stationary process, one component of $\mathcal{Q}$ can be a frequency-domain representation. For nonstationary processes, a localized time-frequency representation can be introduced. A generic local complex representation is expressed by Equation 44.

$$Z(t, \omega) = A(t, \omega)e^{i\phi(t, \omega)}. \quad (44)$$

Equation 44 separates local amplitude and phase where such a decomposition is defined. Time-frequency analysis supplies several constructions of this general kind, with different localization and resolution properties [2, 3].

The complete transformation from structural representation to Type-II representation is represented by Equation 45.

$$\mathcal{T}_{\eta, \psi, \mathcal{Q}} = \mathcal{Q} \circ \mathcal{O}_{\psi} \circ \mathcal{D}_{\eta}. \quad (45)$$

The resulting temporal representation is given by Equation 46.

$$\Theta = \mathcal{T}_{\eta, \psi, \mathcal{Q}}(\mathfrak{S}). \quad (46)$$

Equations 45 and 46 make explicit that temporal representation depends jointly on structural configuration, realization conditions, observation architecture, and representation choice.

Several different operators can therefore be constructed from the same observed process. Their outputs can be represented collectively by Equation 47.

$$\Theta = (\mathcal{Q}_1[y], \mathcal{Q}_2[y], \dots, \mathcal{Q}_m[y]). \quad (47)$$

Equation 47 provides the basis for later analysis of multi-representation fusion and representational refinement.

Representation operators are analytical constructions. A Fourier transform, wavelet transform, phase estimator, coherence measure, state-space decomposition, or modal estimator can help construct Θ , while these methods remain distinct from the governance mechanisms classified within Type-II.

2.6 Representational Scope and Applicability

This subsection consolidates the conditions under which structural and spectral-temporal transformations are meaningful. Its objective is to define the scope of later claims concerning equivalence, inversion, lifting, and local transformation.

A transformation statement requires specification of at least four domains: a structural model class, admissible realization conditions, an observation architecture, and a temporal representation family. These conditions are collected in Equation 48.

$$\Xi = (\mathcal{M}_I, \mathcal{E}, \mathcal{O}, \mathcal{Q}). \quad (48)$$

Equation 48 defines the representational context within which a transformation $\mathcal{T}_{\Xi}$ is interpreted.

The corresponding transformation is represented by Equation 49.

$$\mathcal{T}_{\Xi} : \mathcal{M}_I \longrightarrow \mathcal{M}_{II}^{\Xi}. \quad (49)$$

Equation 49 emphasizes that the Type-II image depends on the representational context Ξ .

A change in Ξ can therefore change the equivalence relation among structural systems even when the structural systems themselves remain unchanged. Higher sampling resolution can expose a previously invisible fast mode. A longer observation horizon can reveal a slow cycle. Addition of phase

information can separate systems with similar frequency content. Observation of another state variable can distinguish systems that were previously output-equivalent.

For later comparison, the image of the structural domain under one representation is represented by Equation 50.

$$\text{Im}(\mathcal{T}_{\Xi}) = \{\Theta \in \mathcal{M}_{\Pi}^{\Xi} \mid \exists \mathfrak{S} \in \mathcal{M}_I : \Theta = \mathcal{T}_{\Xi}(\mathfrak{S})\}. \quad (50)$$

Equation 50 defines the set of Type-II representations actually reachable from the selected structural domain under the selected representational context.

This image can occupy only a restricted part of the conceptual Type-II domain. A particular class of structural systems may generate no meaningful oscillatory modes, no stable phase description, or no identifiable cross-frequency relations. Such absence defines a legitimate boundary of application.

Table 1 summarizes the stages and objects introduced in this section.

Table 1: Representational Foundations of Structural and Spectral-Temporal Governance

Representational Stage	Principal Object	Analytical Function	Dependence
Structural Domain	$\mathfrak{S} \in \mathcal{M}_I$	Represents rules, dynamics, relations, states, and generative backgrounds	Model class and structural resolution
Dynamical Realization	$x_{\mathfrak{S}, \eta}(\cdot)$	Generates system trajectories from structural conditions	Initial conditions, context, disturbances, and parameters
Observation	$y = \mathcal{O}_{\psi}[x]$	Selects and records temporally accessible information	Variables, sampling, aggregation, delay, noise, and window
Temporal Representation	$\Theta = \mathcal{Q}[y]$	Constructs timescale, spectral, phase, coupling, and regime descriptions	Representation method and applicability conditions
Composite Transformation	$\mathcal{T} = \mathcal{Q} \circ \mathcal{O} \circ \mathcal{D}$	Connects Type-I structure with Type-II representation	All preceding structural, realization, observation, and representation conditions
Representational Context	$\Xi = (\mathcal{M}_I, \mathcal{E}, \mathcal{O}, \mathcal{Q})$	Defines the domain within which transformation claims are interpreted	Research design and model specification

The architecture summarized in Table 1 establishes the main methodological boundary of the paper. A Type-II representation is produced through a chain of generation, observation, and representation. Each stage can preserve some distinctions while compressing others. Consequently, questions of structural equivalence and identifiability are meaningful only relative to a specified transformation context.

The next section develops the forward transformation in greater detail. It examines dynamical realization, observation mapping, temporal representation mapping, composition of the transformation operator, dependence on context and parameters, temporal-window effects, and families of structural-to-temporal transformations.

3 Structural-to-Temporal Transformation

This section develops the forward transformation from a Type-I structural representation to a Type-II spectral-temporal representation. Its objective is to specify how structural organization generates trajectories, how those trajectories become observable, how temporal representations are constructed, and how these stages combine into a context-dependent transformation operator. The section also distinguishes several sources of transformation dependence and introduces a family-level representation of

structural-to-temporal maps. The analysis proceeds from generative realization to observation, representation, composition, contextual dependence, temporal resolution, and transformation families.

3.1 Generative Transformation Chain

This subsection formalizes the complete forward path between the two representational domains. Its objective is to preserve the intermediate stages through which structural information becomes temporal information.

Let $\mathfrak{S} \in \mathcal{M}_I$ denote a structural system, let $\eta \in \mathcal{E}_{\mathfrak{S}}$ collect realization conditions, let ψ denote observation conditions, and let $\mathcal{Q}$ denote the selected temporal representation operator. The forward generative chain is represented by Equation 51.

$$\mathfrak{S} \xrightarrow{\mathcal{D}_{\eta}} x_{\mathfrak{S},\eta}(\cdot) \xrightarrow{\mathcal{O}_{\psi}} y_{\mathfrak{S},\eta,\psi}(\cdot) \xrightarrow{\mathcal{Q}} \Theta_{\mathfrak{S},\eta,\psi,\mathcal{Q}}. \quad (51)$$

Equation 51 separates four analytically different objects: structural organization, realized dynamics, observed dynamics, and temporal representation.

The distinction is consequential because representational equivalence can emerge at several stages. Two structural systems can generate the same trajectory under specified conditions. Distinct trajectories can become indistinguishable after observation. Distinct observations can also produce the same selected spectral-temporal representation.

For later use, the four-stage domains are represented by Equation 52.

$$\mathcal{M}_I \xrightarrow{\mathcal{D}} \mathfrak{X} \xrightarrow{\mathcal{O}} \mathcal{Y} \xrightarrow{\mathcal{Q}} \mathcal{M}_{II}, \quad (52)$$

where $\mathfrak{X}$ denotes a trajectory domain and $\mathcal{Y}$ an observation domain.

Equation 52 provides the basic architecture for the equivalence and identifiability analysis developed in subsequent sections.

The forward transformation is therefore generative in a specific sense. Structural organization constrains or produces possible trajectories, and Type-II representations are constructed from realized and observed temporal behavior. This ordering allows temporal descriptions to inform structural analysis while preserving the distinction between a generative mechanism and its represented temporal consequences.

3.2 Dynamical Realization

This subsection develops the first transformation stage. Its objective is to represent how a structural configuration generates one or more possible trajectories under specified realization conditions.

For a deterministic dynamical model, one possible realization takes the form represented by Equation 53.

$$\dot{x}(t) = F(x(t), R, C, \mathcal{B}, u(t); \vartheta), \quad x(t_0) = x_0, \quad (53)$$

where $u(t)$ denotes exogenous input and ϑ collects model parameters.

Equation 53 illustrates one way in which rules, relational couplings, background conditions, and dynamical processes jointly determine system evolution.

A discrete-time realization can instead be represented by Equation 54.

$$x_{k+1} = F_k(x_k, R_k, C_k, \mathcal{B}_k, u_k; \vartheta_k). \quad (54)$$

Equation 54 accommodates governance systems whose relevant temporal structure is organized around decisions, events, reviews, or institutional transitions.

Hybrid and switching systems can require both continuous and discrete components. A generic hybrid realization is represented by Equation 55.

$$\begin{aligned}\dot{x}(t) &= F_{\sigma(t)}(x(t), u(t)), \\ \sigma(t^+) &= \mathcal{G}(\sigma(t^-), x(t), e(t)),\end{aligned}\tag{55}$$

where $\sigma(t)$ denotes the active dynamical regime and $e(t)$ an event process.

Equation 55 is useful when institutional rules activate different dynamical processes across emergencies, review stages, policy phases, or threshold events.

The transformation framework also accommodates stochastic realization. One general representation is given by Equation 56.

$$dx_t = F(x_t; \mathfrak{S}) dt + G(x_t; \mathfrak{S}) dW_t,\tag{56}$$

where W_t denotes a stochastic driving process.

Equation 56 makes the generated trajectory a realization from a structural probability law.

These alternatives show that the transformation framework does not require a single dynamical ontology. Continuous, discrete, hybrid, multiscale, and stochastic descriptions can all serve as the realization stage when appropriate to the governance problem. Nonlinear systems can further exhibit multiple attractors, bifurcations, transient behavior, and sensitivity to initial conditions, making one structural configuration compatible with several qualitatively different temporal realizations [5].

For multiscale systems, the realization can contain explicitly separated fast and slow variables. One canonical representation is given by Equation 57.

$$\begin{aligned}\varepsilon \dot{x} &= f(x, z; \mathfrak{S}), \\ \dot{z} &= g(x, z; \mathfrak{S}), \quad 0 < \varepsilon \ll 1.\end{aligned}\tag{57}$$

Equation 57 illustrates how structural organization can generate distinct characteristic timescales before any spectral-temporal representation is constructed [7].

The output of dynamical realization is consequently better understood as a set of possible histories. The realization map is represented generally by Equation 58.

$$\mathcal{D} : (\mathfrak{S}, \eta) \mapsto x_{\mathfrak{S}, \eta}(\cdot).\tag{58}$$

Equation 58 will later allow structural multiplicity and temporal multiplicity to be studied separately.

3.3 Observation Mapping

This subsection develops the second transformation stage. Its objective is to specify how generated trajectories become observable and how measurement can preserve, aggregate, distort, delay, or remove temporal information.

A general observation map is represented by Equation 59.

$$y(\cdot) = \mathcal{O}_{\psi}[x(\cdot)],\tag{59}$$

where ψ collects the parameters and design choices defining the observation architecture.

Equation 59 can include variable selection, sampling, spatial aggregation, institutional reporting procedures, temporal aggregation, measurement error, missingness, delay, and censoring.

A sampled state-space observation provides a useful special case. It is represented by Equation 60.

$$y_k = h(x(t_k)) + \nu_k.\tag{60}$$

Equation 60 makes the observed temporal record dependent on both the state-observation function h and the sampling sequence $\{t_k\}$.

Aggregation can itself change represented temporal structure. Let $\Delta > 0$ denote an aggregation interval. A window-aggregated observation is represented by Equation 61.

$$\bar{y}_\Delta(t) = \frac{1}{\Delta} \int_{t-\Delta}^t y(s) ds. \quad (61)$$

Equation 61 can attenuate fast variation and alter apparent temporal dependence.

Observation delay can be represented separately. For delay $\ell \geq 0$, the delayed observable is represented by Equation 62.

$$y^{(\ell)}(t) = h(x(t - \ell)). \quad (62)$$

Equation 62 is relevant when governance records become available after reporting, validation, institutional review, or transmission delay.

Observation can also be distributed across several channels. A multi-observable architecture is represented by Equation 63.

$$\mathbf{y}(t) = (h_1(x(t)), h_2(x(t)), \dots, h_m(x(t)))^\top. \quad (63)$$

Equation 63 can reduce structural ambiguity when different components expose different aspects of the underlying dynamics.

System identification and dynamical reconstruction both emphasize that observability and identifiability depend on the relation between system dynamics and the available measurements [8, 16]. The transformation framework therefore treats observation as an active representational stage.

This distinction allows later non-identifiability to be localized. Structural information can disappear because different systems generate the same trajectory, because different trajectories generate the same observation, or because the subsequent temporal representation compresses the remaining differences.

3.4 Temporal Representation Mapping

This subsection develops the third transformation stage. Its objective is to represent how observed temporal records are mapped into the Type-II objects used for governance analysis.

The generic temporal representation map is represented by Equation 64.

$$\mathcal{Q} : \mathcal{Y} \longrightarrow \mathcal{M}_{\text{II}}, \quad y(\cdot) \longmapsto \Theta. \quad (64)$$

Equation 64 can contain several analytical components whose applicability depends on the observation.

For a frequency-domain representation, one possible component is the Fourier transform represented by Equation 65.

$$Y(\omega) = \int_{-\infty}^{\infty} y(t) e^{-i\omega t} dt. \quad (65)$$

Equation 65 can support frequency and spectral analysis under suitable assumptions.

For temporally localized structure, a windowed transform can be represented by Equation 66.

$$Z_y(t, \omega) = \int_{-\infty}^{\infty} y(s) w^*(s - t) e^{-i\omega s} ds, \quad (66)$$

where w denotes a localization window.

Equation 66 provides one generic form of local time-frequency representation [2].

Wavelet representations provide another family of localized mappings. A continuous wavelet coefficient is represented by Equation 67.

$$W_y(a, b) = \frac{1}{\sqrt{|a|}} \int_{-\infty}^{\infty} y(t) \psi^*\left(\frac{t - b}{a}\right) dt. \quad (67)$$

Equation 67 represents temporal information across scale a and location b [3].

A local complex representation can further support amplitude and phase coordinates. Such a representation is expressed by Equation 68.

$$Z(t, \omega) = A(t, \omega)e^{i\phi(t, \omega)}. \quad (68)$$

Equation 68 supplies temporal coordinates from which phase, locking, interference, modulation, or other Type-II relations can be constructed when their applicability conditions are satisfied.

The representation map can therefore be composite. A Type-II descriptor is represented by Equation 69.

$$\Theta = (Q_\tau[y], Q_\Omega[y], Q_\phi[y], Q_L[y], Q_R[y], Q_I[y], Q_H[y], Q_C[y], Q_\Sigma[y]). \quad (69)$$

Equation 69 makes explicit that different Type-II coordinates can be produced by different analytical suboperators.

The temporal representation stage therefore involves model choice. Spectral peaks, instantaneous phase, modal decomposition, coherence, and regime descriptors are constructed representations whose interpretive value depends on the observed system and analytical assumptions. Nonstationary processes make this dependence especially important [12].

3.5 Composite Transformation Operator

This subsection consolidates the three forward stages into one transformation operator. Its objective is to provide a compact mathematical object for later equivalence, identifiability, intervention, and lifting analysis.

For fixed realization condition η , observation architecture ψ , and temporal representation $\mathcal{Q}$, the composite transformation is represented by Equation 70.

$$\mathcal{T}_{\eta, \psi, \mathcal{Q}} = \mathcal{Q} \circ \mathcal{O}_\psi \circ \mathcal{D}_\eta. \quad (70)$$

Equation 70 maps a structural system into a Type-II representation according to Equation 71.

$$\mathcal{T}_{\eta, \psi, \mathcal{Q}} : \mathcal{M}_I \longrightarrow \mathcal{M}_{II}, \quad \mathfrak{S} \longmapsto \Theta_{\mathfrak{S}, \eta, \psi, \mathcal{Q}}. \quad (71)$$

Equation 71 defines the principal forward map used in the remainder of the paper.

The operator composition also allows information loss to be attributed to a specific stage. Let $\mathcal{D}$, $\mathcal{O}$, and $\mathcal{Q}$ each induce an equivalence relation over their respective input domains. The progressive loss of distinguishability can be represented by Equation 72.

$$\mathfrak{S}_1 \sim_{\mathcal{D}} \mathfrak{S}_2 \implies \mathfrak{S}_1 \sim_{\mathcal{O} \circ \mathcal{D}} \mathfrak{S}_2 \implies \mathfrak{S}_1 \sim_{\mathcal{T}} \mathfrak{S}_2. \quad (72)$$

Equation 72 expresses the persistence of indistinguishability once an earlier transformation stage has collapsed a difference.

The converse implication does not generally follow. Two systems that remain distinguishable at the trajectory stage can become equivalent after measurement or temporal representation. The resulting distinction will allow later sections to separate generative equivalence, observational equivalence, and representational equivalence.

The composite transformation also provides the proper location for Fourier-like methods. They can form part of $\mathcal{Q}$, while the complete Type-I-to-Type-II transformation remains the broader composition expressed in Equation 70.

3.6 Context and Parameter Dependence

This subsection develops the contextual dependence of the forward transformation. Its objective is to show how the same structural representation can generate different temporal descriptions under changing realization, environmental, observational, or representational conditions.

Let χ denote contextual variables that influence realization while remaining external to the structural coordinates emphasized by the selected Type-I model. The context-dependent transformation is represented by Equation 73.

$$\Theta = \mathcal{T}(\mathfrak{S}; \eta, \chi, \psi, \mathcal{Q}). \quad (73)$$

Equation 73 makes contextual dependence explicit.

A parametric family of structural systems can be represented by Equation 74.

$$\mathfrak{S}(\vartheta) \in \mathcal{M}_I, \quad \vartheta \in \mathcal{P}. \quad (74)$$

The corresponding family of temporal representations is given by Equation 75.

$$\Theta(\vartheta) = \mathcal{T}(\mathfrak{S}(\vartheta)). \quad (75)$$

Equation 75 provides the basis for later sensitivity and tangent-space analysis.

Parameter changes can produce smooth temporal changes within one dynamical regime and qualitative reorganization near bifurcation or critical boundaries. The local relation can therefore coexist with globally discontinuous transformation structure [5].

Context also affects structural comparison. Two systems can be temporally similar under one contextual condition and diverge under another. A context-indexed equivalence relation is represented by Equation 76.

$$\mathfrak{S}_1 \sim_{\mathcal{T}, \chi} \mathfrak{S}_2 \iff \mathcal{T}_\chi(\mathfrak{S}_1) = \mathcal{T}_\chi(\mathfrak{S}_2). \quad (76)$$

Equation 76 makes representational equivalence conditional on the context in which systems are realized and observed.

This context dependence has an important governance consequence. Temporal similarity observed under ordinary operation can provide limited evidence about how systems will behave under crisis, resource scarcity, institutional transition, or other conditions that alter the relevant realization regime.

3.7 Window and Resolution Dependence

This subsection develops the dependence of Type-II representation on temporal window and resolution. Its objective is to clarify how distinguishability can change even when the structural system and generated trajectory remain unchanged.

Let $W = [t_0, t_1]$ denote an observation window and Δ a temporal resolution. The windowed transformation is represented by Equation 77.

$$\Theta_{W, \Delta} = \mathcal{T}_{W, \Delta}(\mathfrak{S}). \quad (77)$$

Equation 77 makes temporal scale part of the representation operator.

A mode with characteristic period τ_i can remain weakly represented when the observation horizon $H_W = t_1 - t_0$ is short relative to the mode. The corresponding horizon ratio is represented by Equation 78.

$$\rho_i^W = \frac{H_W}{\tau_i}. \quad (78)$$

Equation 78 provides a simple descriptor of whether the observation window contains sufficient temporal extent to expose repeated expression of the mode.

Sampling resolution imposes a complementary condition. For approximately regular sampling interval Δ , the nominal sampling frequency is represented by Equation 79.

$$f_s = \frac{1}{\Delta}. \quad (79)$$

Equation 79 determines which faster temporal features can be resolved without aliasing under conventional sampling assumptions.

Resolution can also affect phase and synchronization inference. Coarse sampling can preserve a low-frequency recurrence pattern while obscuring within-cycle phase relations. A longer window can expose stable locking while a short local window can reveal transient phase drift.

For two representation contexts (W_1, Δ_1) and (W_2, Δ_2) , their induced equivalence relations can therefore differ. This dependence is represented by Equation 80.

$$\sim_{\mathcal{T}_{W_1, \Delta_1}} \neq \sim_{\mathcal{T}_{W_2, \Delta_2}}. \quad (80)$$

Equation 80 expresses the possibility that structural distinguishability changes with observational resolution.

Time-frequency and wavelet methods can reduce some limitations of globally stationary representation by localizing temporal information, while their resolution properties remain governed by the selected representation [2, 3]. Window and resolution dependence therefore remains part of the transformation itself.

3.8 Transformation Families

This subsection consolidates the forward transformation into a family of context-indexed maps. Its objective is to replace the assumption of one universal Type-I-to-Type-II transformation with a structured family of representations.

Let Ξ denote a complete transformation context containing realization conditions, observation architecture, temporal window, resolution, and representation operator. The transformation family is represented by Equation 81.

$$\mathcal{T} = \{\mathcal{T}_\Xi : \Xi \in \mathcal{C}\}, \quad (81)$$

where $\mathcal{C}$ denotes the admissible set of representational contexts.

Equation 81 treats structural-to-temporal representation as a family rather than a single invariant map.

For one structural system, the corresponding family of temporal images is represented by Equation 82.

$$\text{Rep}(\mathcal{S}) = \{\mathcal{T}_\Xi(\mathcal{S}) \mid \Xi \in \mathcal{C}\}. \quad (82)$$

Equation 82 collects the temporal representations accessible under different admissible contexts.

Transformation families can be organized according to the stage that varies. A realization family changes initial conditions, disturbances, or contextual parameters. An observation family changes variables, aggregation, sampling, or delay. A temporal-representation family changes the analytical coordinates extracted from the same observed process. A compound family changes several stages simultaneously.

The family decomposition is represented by Equation 83.

$$\mathcal{T} = \mathcal{T}_\mathcal{D} \cup \mathcal{T}_\mathcal{O} \cup \mathcal{T}_\mathcal{Q} \cup \mathcal{T}_{\text{comp}}, \quad (83)$$

where the four subsets denote realization-varying, observation-varying, representation-varying, and compound transformation families.

Equation 83 is a bookkeeping decomposition. The subsets can overlap when more than one transformation component changes.

Table 2 summarizes the forward transformation architecture developed in this section.

Table 2: Structural-to-Temporal Transformation Architecture

Transformation Component	Formal Object	Representational Role	Principal Dependence
Dynamical Realization	$\mathcal{D}_\eta$	Generates trajectories from structural organization	Initial conditions, parameters, inputs, disturbances, and dynamical regime
Observation Mapping	$\mathcal{O}_\psi$	Converts generated trajectories into observable records	Variable selection, sampling, aggregation, delay, and measurement process
Temporal Representation	$\mathcal{Q}$	Constructs Type-II temporal coordinates	Spectral, phase, modal, coupling, and regime representation choices
Composite Transformation	$\mathcal{T} = \mathcal{Q} \circ \mathcal{O} \circ \mathcal{D}$	Maps Type-I structural systems into Type-II representations	All realization, observation, and representation conditions
Context Dependence	$\mathcal{T}_\chi$	Conditions temporal images on environmental and institutional context	External state, parameter regime, and realization conditions
Window and Resolution	$\mathcal{T}_{W,\Delta}$	Determines accessible temporal scales and distinguishability	Observation horizon, sampling interval, and localization
Transformation Family	$\{\mathcal{T}_\Xi\}_{\Xi \in \mathfrak{C}}$	Represents multiple admissible structural-to-temporal maps	Selection of transformation context

The architecture summarized in Table 2 establishes the forward side of the representation problem. Structural organization generates possible trajectories, trajectories become available through observation, and temporal representation maps those observations into Type-II coordinates. Each stage can preserve, amplify, suppress, or collapse distinctions relevant to subsequent governance analysis.

A central consequence follows from this architecture. Structural-to-temporal transformation is generally a family of context-dependent generative maps. Consequently, the inverse problem should be formulated relative to a specified transformation rather than to Type-II representation in the abstract.

The next section develops this consequence through *Representational Equivalence*. It defines structural equivalence under a selected temporal representation, constructs the resulting equivalence classes, distinguishes observation-, resolution-, and model-relative equivalence, and examines how additional representations can refine the partition of structurally compatible systems.

4 Representational Equivalence

This section develops equivalence relations induced by the transformation architecture established in Sections 2 and 3. Its objective is to formalize when structurally distinct systems become indistinguishable under a selected spectral-temporal representation, identify the stage at which such indistinguishability arises, and describe how observation, resolution, model choice, and additional representations alter the resulting equivalence classes. The section proceeds from structural equivalence under a fixed temporal transformation to equivalence classes, observation-relative and resolution-relative equivalence, model-relative equivalence, refinement of representational partitions, and equivalence across multiple representations.

4.1 Structural Equivalence under Temporal Representation

This subsection defines the principal equivalence relation used throughout the paper. Its objective is to distinguish equality of structural systems from indistinguishability under a selected structural-to-temporal transformation.

Let $\mathcal{T}_\Xi$ denote a structural-to-temporal transformation under representational context Ξ . Two structural systems are temporally equivalent under this transformation when they generate the same Type-II representation. This relation is defined by Equation 84.

$$\mathfrak{S}_1 \sim_{\mathcal{T}_\Xi} \mathfrak{S}_2 \iff \mathcal{T}_\Xi(\mathfrak{S}_1) = \mathcal{T}_\Xi(\mathfrak{S}_2). \quad (84)$$

Equation 84 defines equivalence relative to a transformation rather than asserting structural identity.

Provided that equality in the Type-II codomain is itself well defined, $\sim_{\mathcal{T}_\Xi}$ is reflexive, symmetric, and transitive. The relation therefore partitions the structural domain $\mathcal{M}_I$ into equivalence classes.

Structural distinction and temporal distinction consequently occupy different levels. It is possible that

$$\mathfrak{S}_1 \neq \mathfrak{S}_2$$

while

$$\mathfrak{S}_1 \sim_{\mathcal{T}_\Xi} \mathfrak{S}_2.$$

The distinction can arise because the systems generate the same realized trajectory, because observation removes differences between their trajectories, or because the temporal representation compresses differences that remain observable.

The transformation chain established in Equation 52 allows these sources to be separated formally.

Two structural systems are dynamically equivalent under fixed realization conditions when their generated trajectories coincide. This relation is defined by Equation 85.

$$\mathfrak{S}_1 \sim_{\mathcal{D}_\eta} \mathfrak{S}_2 \iff \mathcal{D}_\eta(\mathfrak{S}_1) = \mathcal{D}_\eta(\mathfrak{S}_2). \quad (85)$$

Two generated trajectories are observationally equivalent when the selected observation operator produces identical observations. This relation is defined by Equation 86.

$$x_1(\cdot) \sim_{\mathcal{O}_\psi} x_2(\cdot) \iff \mathcal{O}_\psi[x_1(\cdot)] = \mathcal{O}_\psi[x_2(\cdot)]. \quad (86)$$

Two observed processes are temporally representationally equivalent when the selected temporal representation operator maps them to the same Type-II object. This relation is defined by Equation 87.

$$y_1(\cdot) \sim_{\mathcal{Q}} y_2(\cdot) \iff \mathcal{Q}[y_1(\cdot)] = \mathcal{Q}[y_2(\cdot)]. \quad (87)$$

Equations 85–87 locate indistinguishability at three successive stages of the transformation chain.

An equivalence created at an earlier stage persists through subsequent deterministic mappings. This implication is represented by Equation 88.

$$\mathfrak{S}_1 \sim_{\mathcal{D}_\eta} \mathfrak{S}_2 \implies \mathfrak{S}_1 \sim_{\mathcal{O}_\psi \circ \mathcal{D}_\eta} \mathfrak{S}_2 \implies \mathfrak{S}_1 \sim_{\mathcal{T}_\Xi} \mathfrak{S}_2. \quad (88)$$

Equation 88 gives a stage-sensitive account of information loss. Later equivalence can arise without earlier equivalence, because observation or temporal representation can collapse distinctions that remain present upstream.

4.2 Temporal Equivalence Classes

This subsection develops the equivalence classes induced by a fixed structural-to-temporal transformation. Its objective is to replace a presumption of unique structural reconstruction with a set-valued description of structurally compatible systems.

For a structural system $\mathfrak{S}$, its temporal equivalence class under $\mathcal{T}_{\Xi}$ is represented by Equation 89.

$$[\mathfrak{S}]_{\mathcal{T}_{\Xi}} = \{\mathfrak{S}' \in \mathcal{M}_I \mid \mathcal{T}_{\Xi}(\mathfrak{S}') = \mathcal{T}_{\Xi}(\mathfrak{S})\}. \quad (89)$$

Equation 89 collects all structural systems that remain indistinguishable under the selected temporal representation.

The structural domain can consequently be represented through the quotient induced by this equivalence relation. The quotient domain is defined by Equation 90.

$$\mathcal{M}_I / \sim_{\mathcal{T}_{\Xi}} = \{[\mathfrak{S}]_{\mathcal{T}_{\Xi}} : \mathfrak{S} \in \mathcal{M}_I\}. \quad (90)$$

Equation 90 represents the structural domain at the resolution supplied by the selected Type-II transformation.

The transformation can then be viewed as acting on equivalence classes rather than on individually distinguishable structural systems. The induced class-level map is represented by Equation 91.

$$\tilde{\mathcal{T}}_{\Xi} : \mathcal{M}_I / \sim_{\mathcal{T}_{\Xi}} \longrightarrow \text{Im}(\mathcal{T}_{\Xi}), \quad [\mathfrak{S}]_{\mathcal{T}_{\Xi}} \longmapsto \mathcal{T}_{\Xi}(\mathfrak{S}). \quad (91)$$

Equation 91 is well defined because all members of an equivalence class share the same image under $\mathcal{T}_{\Xi}$.

This quotient perspective is important for inverse inference. An observed Type-II representation identifies at most an equivalence class unless further information subdivides that class. The inverse problem therefore begins with a structural compatibility set rather than with one uniquely determined structural configuration.

Equivalence classes can vary substantially in size and internal heterogeneity. One class can contain structurally similar parameter variants, while another can contain systems that differ across rules, relational structures, dynamical mechanisms, or generative backgrounds. The size of a class alone therefore provides limited information about the substantive distance among its members.

For this reason, later identifiability analysis will distinguish uniqueness, local indistinguishability, partial structural recovery, and recovery of particular mechanisms or parameters.

4.3 Observation-Relative Equivalence

This subsection develops equivalence induced specifically by the observation architecture. Its objective is to show how variable selection, aggregation, delay, and measurement structure can change which structural systems remain distinguishable before temporal representation is applied.

Let $\mathcal{O}_{\psi_1}$ and $\mathcal{O}_{\psi_2}$ denote two observation architectures applied to the same generated trajectory domain. The corresponding composite transformations are represented by Equation 92.

$$\mathcal{T}_{\psi_1} = \mathcal{Q} \circ \mathcal{O}_{\psi_1} \circ \mathcal{D}, \quad \mathcal{T}_{\psi_2} = \mathcal{Q} \circ \mathcal{O}_{\psi_2} \circ \mathcal{D}. \quad (92)$$

Equation 92 generates two potentially different structural equivalence relations.

Observation-relative equivalence can therefore be expressed by Equation 93.

$$\mathfrak{S}_1 \sim_{\psi} \mathfrak{S}_2 \iff \mathcal{Q}[\mathcal{O}_{\psi}[\mathcal{D}(\mathfrak{S}_1)]] = \mathcal{Q}[\mathcal{O}_{\psi}[\mathcal{D}(\mathfrak{S}_2)]] . \quad (93)$$

Equation 93 makes the observation architecture part of the equivalence criterion.

Additional observables can refine structural distinguishability. Suppose $\mathbf{y}^{(1)}$ contains one observed process and $\mathbf{y}^{(2)}$ contains that process together with additional channels. Their observation structures can be represented by Equation 94.

$$\mathbf{y}^{(2)}(t) = \left(\mathbf{y}^{(1)}(t), \mathbf{y}^{\text{add}}(t) \right). \quad (94)$$

When the additional channels preserve all information contained in the first observation and add further discriminatory information, the resulting equivalence classes can become smaller.

Observation can also enlarge equivalence classes through aggregation. If $\mathcal{A}$ is an aggregation operator, the composed observation $\mathcal{A} \circ \mathcal{O}$ can identify trajectories that remain distinguishable under $\mathcal{O}$.

This possibility is represented by Equation 95.

$$\mathcal{O}[x_1] \neq \mathcal{O}[x_2], \quad \mathcal{A} \circ \mathcal{O}[x_1] = \mathcal{A} \circ \mathcal{O}[x_2]. \quad (95)$$

Equation 95 identifies observation-induced loss of distinguishability independently of the later spectral-temporal representation.

Observation-relative equivalence is especially consequential for governance because institutional data are rarely neutral samples of a complete system. Reporting procedures, administrative categories, legal documentation, monitoring infrastructure, and resource constraints determine which trajectories become visible. Structural inference from Type-II representation therefore inherits the boundaries of the observation architecture.

4.4 Resolution-Relative Equivalence

This subsection develops equivalence as a function of temporal resolution and observation horizon. Its objective is to formalize how structural distinguishability changes when faster or slower temporal organization enters or leaves the observable representation.

Let $\mathcal{T}_{W,\Delta}$ denote a transformation constructed with observation window W and temporal resolution Δ . Resolution-relative equivalence is defined by Equation 96.

$$\mathfrak{S}_1 \sim_{W,\Delta} \mathfrak{S}_2 \iff \mathcal{T}_{W,\Delta}(\mathfrak{S}_1) = \mathcal{T}_{W,\Delta}(\mathfrak{S}_2). \quad (96)$$

Equation 96 makes temporal resolution part of the equivalence relation.

Two systems can therefore be equivalent at one scale and distinguishable at another. A coarse representation can preserve a shared slow cycle while suppressing differences in fast fluctuations. A short observation horizon can preserve fast dynamics while providing insufficient evidence for different slow modes.

For two resolutions Δ_1 and Δ_2 , a refinement relation can arise when the finer representation preserves the information available at the coarser resolution and adds additional temporal detail. This relation is expressed by Equation 97.

$$\Delta_2 < \Delta_1 \text{ and sufficient information preservation} \implies \mathcal{P}_{W,\Delta_2} \preceq \mathcal{P}_{W,\Delta_1}, \quad (97)$$

where $\mathcal{P}_{W,\Delta}$ denotes the structural partition induced by $\mathcal{T}_{W,\Delta}$.

Equation 97 is conditional because higher nominal sampling resolution does not automatically produce a strictly more informative representation. Noise, changed measurement procedures, finite windows, and representation choices can prevent monotonic refinement.

Observation horizon introduces a complementary effect. Let $W_1 \subset W_2$ denote nested windows. A longer window can expose slow recurrence, quasiperiodicity, regime switching, or persistent synchronization that remains undetermined within the shorter interval.

When the longer observation preserves the shorter record and adds usable information, the induced partitions can satisfy the relation in Equation 98.

$$W_1 \subset W_2 \implies \mathcal{P}_{W_2} \preceq \mathcal{P}_{W_1}, \quad (98)$$

subject to the measurement and stationarity conditions required by the selected representation.

Resolution-relative equivalence therefore gives a precise meaning to the claim that structural distinguishability is scale dependent. A structural difference can be present while remaining outside the temporal scale resolved by a given governance observation system.

4.5 Model-Relative Equivalence

This subsection develops equivalence relative to the structural and temporal model classes used by the analyst. Its objective is to distinguish indistinguishability in the world from indistinguishability within a selected modeling vocabulary.

Let M denote a representational model specifying the admissible structural domain, realization mechanism, observation architecture, and temporal representation. The associated transformation is represented by Equation 99.

$$\mathcal{T}^{(M)} : \mathcal{M}_I^{(M)} \longrightarrow \mathcal{M}_{II}^{(M)}. \quad (99)$$

Model-relative equivalence is then defined by Equation 100.

$$\mathfrak{S}_1 \sim_M \mathfrak{S}_2 \iff \mathcal{T}^{(M)}(\mathfrak{S}_1) = \mathcal{T}^{(M)}(\mathfrak{S}_2). \quad (100)$$

Equation 100 emphasizes that equivalence can depend on which structural and temporal variables the model permits.

A model that records only characteristic timescales can identify systems that a richer phase-sensitive model separates. A model that represents one aggregate network can collapse distinctions visible in a multilayer relational representation. A stationary spectral model can identify processes that a time-localized representation distinguishes through changing modal structure.

Model-relative equivalence is therefore epistemically modest. It states that two systems cannot be distinguished within a specified representational framework. It does not establish that they are identical under every possible observation or model.

This distinction becomes important when equivalence classes are used for governance decisions. An intervention can be safely selected from an equivalence class only when the distinctions omitted by the model are irrelevant to the intervention consequences under consideration. The interventional conditions required for this claim are developed in Section 7.

4.6 Refinement of Representational Partitions

This subsection develops an ordering among representations according to the structural distinctions they preserve. Its objective is to formalize how additional observables, temporal coordinates, or modeling detail can refine the partition of the structural domain.

Every transformation $\mathcal{T}$ induces a partition $\mathcal{P}_{\mathcal{T}}$ of the structural domain. This partition is represented by Equation 101.

$$\mathcal{P}_{\mathcal{T}} = \{[\mathfrak{S}]_{\mathcal{T}} : \mathfrak{S} \in \mathcal{M}_I\}. \quad (101)$$

Equation 101 records structural distinguishability at the resolution of the selected transformation.

A representation $\mathcal{T}_2$ is finer than $\mathcal{T}_1$ when every equivalence class generated by $\mathcal{T}_2$ is contained within an equivalence class generated by $\mathcal{T}_1$. This relation is defined by Equation 102.

$$\mathcal{P}_{\mathcal{T}_2} \preceq \mathcal{P}_{\mathcal{T}_1} \iff \forall C_2 \in \mathcal{P}_{\mathcal{T}_2}, \exists C_1 \in \mathcal{P}_{\mathcal{T}_1} : C_2 \subseteq C_1. \quad (102)$$

Equation 102 provides a partial ordering of representations by structural distinguishability.

Strict refinement occurs when at least one equivalence class is subdivided. This condition is represented by Equation 103.

$$\mathcal{P}_{\mathcal{T}_2} \prec \mathcal{P}_{\mathcal{T}_1} \iff \mathcal{P}_{\mathcal{T}_2} \preceq \mathcal{P}_{\mathcal{T}_1} \wedge \mathcal{P}_{\mathcal{T}_2} \neq \mathcal{P}_{\mathcal{T}_1}. \quad (103)$$

A finer representation need not be universally preferable. Additional temporal detail can increase measurement cost, computational burden, sensitivity to noise, or model dependence. The useful degree of refinement therefore depends on the governance task.

This task dependence can be expressed by introducing a governance-relevant structural distinction set $\mathcal{G}$. A representation can be sufficient for a task even when it does not uniquely identify every structural system, provided that it distinguishes all structural differences that alter the relevant decision.

The paper returns to this relation in Section 11, where representational information is evaluated relative to governance tasks rather than through maximum structural resolution alone.

4.7 Equivalence across Multiple Representations

This subsection develops equivalence when several temporal representations are used jointly. Its objective is to formalize how complementary representations can reduce structural ambiguity.

Let $\mathcal{T}_1, \dots, \mathcal{T}_m$ denote several transformations applied to the same structural domain. Their joint representation is defined by Equation 104.

$$\mathcal{T}_{\text{joint}}(\mathfrak{S}) = (\mathcal{T}_1(\mathfrak{S}), \mathcal{T}_2(\mathfrak{S}), \dots, \mathcal{T}_m(\mathfrak{S})). \quad (104)$$

Equation 104 combines information supplied by several representational views.

Two structural systems are equivalent under the joint representation when they are equivalent under every component representation. This relation is defined by Equation 105.

$$\mathfrak{S}_1 \sim_{\text{joint}} \mathfrak{S}_2 \iff \bigwedge_{k=1}^m [\mathfrak{S}_1 \sim_{\mathcal{T}_k} \mathfrak{S}_2]. \quad (105)$$

The corresponding joint equivalence class is represented by Equation 106.

$$[\mathfrak{S}]_{\text{joint}} = \bigcap_{k=1}^m [\mathfrak{S}]_{\mathcal{T}_k}. \quad (106)$$

Equation 106 shows that combining representations can only preserve or reduce the set of structurally compatible systems when each component is retained without information loss.

A useful special case combines temporal coordinates that discriminate different structural properties. One representation can emphasize slow timescales, another phase relations, another cross-frequency coupling, and another regime transitions. Their intersection can distinguish systems that remain equivalent under each representation separately.

The refinement generated by adding one representation is expressed by Equation 107.

$$[\mathfrak{S}]_{\mathcal{T}_1, \dots, \mathcal{T}_m} \subseteq [\mathfrak{S}]_{\mathcal{T}_1, \dots, \mathcal{T}_{m-1}}. \quad (107)$$

Equation 107 establishes monotonic refinement at the level of exact joint information.

Representations can nevertheless be redundant. If $\mathcal{T}_m$ does not subdivide any equivalence class already generated by the preceding representations, its addition does not improve structural distinguishability. This condition is represented by Equation 108.

$$\mathcal{P}_{\mathcal{T}_1, \dots, \mathcal{T}_m} = \mathcal{P}_{\mathcal{T}_1, \dots, \mathcal{T}_{m-1}}. \quad (108)$$

Equation 108 defines representational redundancy relative to the current structural domain.

The complementary case occurs when the added representation strictly refines the joint partition. This condition is represented by Equation 109.

$$\mathcal{P}_{\mathcal{T}_1, \dots, \mathcal{T}_m} \prec \mathcal{P}_{\mathcal{T}_1, \dots, \mathcal{T}_{m-1}}. \quad (109)$$

Equation 109 gives a structural meaning to representational complementarity: the additional representation makes previously indistinguishable systems distinguishable.

Table 3: Equivalence Relations across the Structural-to-Temporal Transformation

Equivalence Domain	Defining Relation	Source of Indistinguishability	Analytical Consequence
Dynamical Equivalence	$\mathcal{D}(\mathfrak{S}_1) = \mathcal{D}(\mathfrak{S}_2)$	Different structures generate the same realized trajectory	Structural differences are absent from the realized history
Observational Equivalence	$\mathcal{O}[x_1] = \mathcal{O}[x_2]$	Measurement, aggregation, or variable selection collapses trajectory differences	Differences remain structurally present and empirically inaccessible through the selected observation
Temporal Representational Equivalence	$\mathcal{Q}[y_1] = \mathcal{Q}[y_2]$	Temporal representation compresses differences among observed processes	Different observations share the same Type-II description
Composite Structural Equivalence	$\mathcal{T}(\mathfrak{S}_1) = \mathcal{T}(\mathfrak{S}_2)$	One or several transformation stages collapse structural differences	Inverse reconstruction yields an equivalence class
Observation-Relative Equivalence	$\sim_\psi$	Observation architecture determines visible distinctions	Changing observables can alter structural compatibility
Resolution-Relative Equivalence	$\sim_{W,\Delta}$	Temporal window and sampling scale determine accessible modes	Structural distinguishability changes across temporal scales
Model-Relative Equivalence	$\sim_M$	Model vocabulary and assumptions determine represented distinctions	Equivalence remains conditional on the selected model
Joint Equivalence	$\bigcap_k [\mathfrak{S}]_{\mathcal{T}_k}$	Several representations constrain the same structural domain	Complementary representations can refine structural compatibility

Table 3 summarizes the principal equivalence relations developed in this section.

The relations summarized in Table 3 establish equivalence as a layered and representation-relative property. Structural systems can become indistinguishable through dynamical realization, observation, temporal representation, or combinations of these stages. Consequently, a Type-II description identifies a structural equivalence class whose boundaries depend on the transformation context.

This result provides the foundation for the inverse problem developed in the next section. Section 5 examines when these equivalence classes can be reduced to unique structural systems, when only local or partial identification is possible, which parameters or mechanisms remain recoverable, and how additional temporal coordinates can increase structural identifiability.

5 Structural Identifiability from Spectral-Temporal Representation

This section develops the inverse side of the transformation framework. Its objective is to determine which structural properties can be recovered from a spectral-temporal representation, under which representational conditions such recovery is unique, and how structural ambiguity can persist even when the Type-II description is highly informative. The section proceeds from structural compatibility sets to global and local identifiability, partial structural recovery, parameter and mechanism identifiability, structural underdetermination, and refinement through additional temporal coordinates. The analysis treats identifiability as relative to a specified transformation context and distinguishes exact structural recovery from practically useful partial inference.

5.1 Structural Compatibility Sets

This subsection defines the inverse object associated with a structural-to-temporal transformation. Its objective is to represent all structural systems that remain compatible with an observed Type-II representation before stronger identifiability conditions are imposed.

Let $\mathcal{T}_\Xi$ denote a fixed structural-to-temporal transformation under context Ξ , and let Θ^{obs} denote an observed spectral-temporal representation. The exact structural compatibility set is represented by Equation 110.

$$\mathcal{I}_\Xi(\Theta^{\text{obs}}) = \left\{ \mathfrak{S} \in \mathcal{M}_I \mid \mathcal{T}_\Xi(\mathfrak{S}) = \Theta^{\text{obs}} \right\}. \quad (110)$$

Equation 110 is the principal inverse object used throughout this section.

For an exactly represented temporal observation, the compatibility set is identical to the equivalence class associated with any structural system that generates Θ^{obs} . This relation is represented by Equation 111.

$$\Theta^{\text{obs}} = \mathcal{T}_\Xi(\mathfrak{S}) \implies \mathcal{I}_\Xi(\Theta^{\text{obs}}) = [\mathfrak{S}]_{\mathcal{T}_\Xi}. \quad (111)$$

Equation 111 connects inverse inference directly with the representational equivalence classes developed in Section 4.

Empirical observations generally include estimation error, measurement noise, finite-window effects, or model mismatch. An approximate compatibility set can therefore be defined using a distance d_{II} on the relevant Type-II representation. This set is represented by Equation 112.

$$\mathcal{I}_\Xi^\varepsilon(\Theta^{\text{obs}}) = \left\{ \mathfrak{S} \in \mathcal{M}_I \mid d_{II}(\mathcal{T}_\Xi(\mathfrak{S}), \Theta^{\text{obs}}) \leq \varepsilon \right\}. \quad (112)$$

Equation 112 introduces a tolerance ε whose empirical meaning depends on the uncertainty and resolution of the study.

The compatibility-set formulation avoids requiring premature selection of one structural explanation. It also separates model admissibility from identifiability. A structural system can be compatible with the observed Type-II representation while remaining indistinguishable from several other systems within the selected transformation context.

This inverse formulation is consistent with the broader logic of system identification, where inferential uniqueness depends on model class, observation, excitation, and available data [8].

5.2 Global Identifiability

This subsection defines global structural identifiability. Its objective is to identify the strongest case in which one Type-II representation determines a unique structural system over the complete admissible structural domain.

A structural system $\mathfrak{S}^*$ is globally identifiable under $\mathcal{T}_\Xi$ when its temporal representation has a singleton compatibility set. This condition is represented by Equation 113.

$$\mathcal{I}_\Xi(\mathcal{T}_\Xi(\mathfrak{S}^*)) = \{\mathfrak{S}^*\}. \quad (113)$$

Equation 113 states that no other admissible structural system produces the same Type-II representation under the selected transformation.

The transformation itself is globally structurally identifiable on $\mathcal{M}_I$ when it is injective. This condition is represented by Equation 114.

$$\mathcal{T}_\Xi(\mathfrak{S}_1) = \mathcal{T}_\Xi(\mathfrak{S}_2) \implies \mathfrak{S}_1 = \mathfrak{S}_2 \quad \forall \mathfrak{S}_1, \mathfrak{S}_2 \in \mathcal{M}_I. \quad (114)$$

Equation 114 represents the strongest form of recoverability considered in this paper.

Global identifiability is demanding because the composite transformation contains several opportunities for information compression. Distinct structures can generate identical trajectories, observation can aggregate distinct trajectories, and temporal representation can retain only selected properties of the observed process.

A full spectral-temporal description can therefore remain structurally non-injective. Knowledge of frequency content, phase, coupling, and regime structure can substantially reduce structural ambiguity while still leaving several structurally distinct generators compatible with the same temporal description.

Global identifiability should consequently be established rather than presumed. In many governance applications, a weaker form of identification is sufficient for the decision problem.

5.3 Local Identifiability

This subsection develops local structural identifiability. Its objective is to capture cases in which a structural system can be uniquely distinguished from nearby alternatives even though globally different structural systems remain compatible with the same Type-II representation.

Let $\mathcal{N}_\delta(\mathfrak{S}^*)$ denote a neighborhood of $\mathfrak{S}^*$ within the structural domain. Local identifiability is represented by Equation 115.

$$\mathcal{I}_\Xi(\mathcal{T}_\Xi(\mathfrak{S}^*)) \cap \mathcal{N}_\delta(\mathfrak{S}^*) = \{\mathfrak{S}^*\}. \quad (115)$$

Equation 115 permits structurally distant systems to share the same temporal representation while excluding nearby alternatives.

Under finite-dimensional local coordinates $z \in \mathbb{R}^p$ for the structural domain and $\theta \in \mathbb{R}^q$ for the Type-II representation, the local transformation is represented by a Jacobian. This Jacobian is introduced by Equation 116.

$$J_{I \rightarrow \Pi}(z^*) = \left. \frac{\partial \theta}{\partial z} \right|_{z=z^*}. \quad (116)$$

Equation 116 describes first-order sensitivity of temporal coordinates to structural variation near the selected system.

When the structural coordinates have dimension p , full column rank of the local Jacobian provides a useful regularity condition for first-order local distinguishability. This condition is represented by Equation 117.

$$\text{rank}(J_{I \rightarrow \Pi}(z^*)) = p. \quad (117)$$

Equation 117 should be interpreted within the differentiable local model in which the Jacobian is defined. It is a local sensitivity criterion and does not establish global uniqueness.

Failure of full column rank identifies structural directions that remain invisible to first order in the selected temporal representation. The corresponding local null space is represented by Equation 118.

$$\mathcal{N}_{\text{null}}(z^*) = \ker J_{I \rightarrow \Pi}(z^*). \quad (118)$$

Equation 118 provides a formal bridge to the tangent-space transformation developed later in Section 9.

Local identifiability is especially useful for governance because intervention often concerns perturbations around the current system rather than complete reconstruction of every globally possible structural alternative.

5.4 Partial Identifiability

This subsection develops partial structural identifiability. Its objective is to represent situations in which the complete structural system remains ambiguous while a selected structural component, projection, or property is uniquely determined by the Type-II representation.

Let Π_A denote a projection or structural feature map extracting the component A of interest from $\mathfrak{S}$. The identifiable image of the compatibility set is represented by Equation 119.

$$\Pi_A \left[\mathcal{I}_{\Xi}(\Theta^{\text{obs}}) \right] = \left\{ \Pi_A(\mathfrak{S}) \mid \mathfrak{S} \in \mathcal{I}_{\Xi}(\Theta^{\text{obs}}) \right\}. \quad (119)$$

Equation 119 records all values of the selected structural property compatible with the observed temporal representation.

The structural property A is identifiable when this projected set is a singleton. This condition is represented by Equation 120.

$$\left| \Pi_A \left[\mathcal{I}_{\Xi}(\Theta^{\text{obs}}) \right] \right| = 1. \quad (120)$$

Equation 120 permits useful structural knowledge even when $|\mathcal{I}_{\Xi}(\Theta^{\text{obs}})| > 1$.

For the Type-I representation, partial identification can concern a rule, dynamical parameter, relational edge, coupling pattern, background resource, boundary condition, or other structural property. Several complete systems can remain compatible with one Type-II representation while sharing the same value of the property relevant to a governance decision.

This distinction is important because governance rarely requires recovery of the complete ontological specification of a system. A temporal diagnosis can be sufficient when every compatible structural system implies the same decision-relevant structural feature.

A task-relative identifiable projection is therefore more practically useful than global injectivity in many applications.

5.5 Parameter Identifiability

This subsection develops parameter identifiability within a specified structural model class. Its objective is to distinguish uncertainty about continuous or discrete structural parameters from uncertainty about the model's broader mechanism class.

Let a structural family be parameterized by $\vartheta \in \mathcal{P}$ according to $\mathfrak{S}(\vartheta)$. The parameter-to-temporal map is represented by Equation 121.

$$\mathcal{G}_{\Xi} : \vartheta \mapsto \mathcal{T}_{\Xi}(\mathfrak{S}(\vartheta)). \quad (121)$$

Equation 121 restricts the inverse problem to variation within one structural family.

For an observed temporal representation, the compatible parameter set is represented by Equation 122.

$$\mathcal{P}_{\Xi}(\Theta^{\text{obs}}) = \left\{ \vartheta \in \mathcal{P} \mid \mathcal{G}_{\Xi}(\vartheta) = \Theta^{\text{obs}} \right\}. \quad (122)$$

Equation 122 contains all parameter values consistent with the selected Type-II representation.

A parameter vector ϑ^* is globally identifiable within the model family when the compatible parameter set is a singleton. This condition is represented by Equation 123.

$$\mathcal{P}_{\Xi}(\mathcal{G}_{\Xi}(\vartheta^*)) = \{\vartheta^*\}. \quad (123)$$

Equation 123 concerns uniqueness within the specified structural model class.

Parameter identifiability can be stronger than full structural identifiability because the model class has already restricted the structural possibilities. This strength also introduces model dependence. A parameter can be identifiable conditional on one structural family while an entirely different family produces the same temporal representation.

System-identification theory provides a broader methodological context for this distinction between model structure, parameter inference, and observed behavior [8].

5.6 Mechanism Identifiability

This subsection develops identifiability at the level of structural mechanisms. Its objective is to determine whether a Type-II representation can identify the generative mechanism relevant to governance even when detailed parameters or complete structural configuration remain uncertain.

Let $\mu(\mathfrak{S})$ denote a mechanism label or mechanism-level structural descriptor. Examples can include centralized forcing, mutual coupling, threshold switching, feedback regulation, shared-resource dependence, or another mechanism defined by the structural model.

The mechanism-compatible set is represented by Equation 124.

$$\mathfrak{M}_{\Xi}(\Theta^{\text{obs}}) = \left\{ \mu(\mathfrak{S}) \mid \mathfrak{S} \in \mathfrak{I}_{\Xi}(\Theta^{\text{obs}}) \right\}. \quad (124)$$

Equation 124 collects all structural mechanisms compatible with the observed temporal representation.

The mechanism is identifiable when the mechanism-compatible set is a singleton. This condition is represented by Equation 125.

$$\left| \mathfrak{M}_{\Xi}(\Theta^{\text{obs}}) \right| = 1. \quad (125)$$

Equation 125 allows parameter uncertainty within a common mechanism while identifying the mechanism itself.

This distinction is particularly important for spectral-temporal governance. Observed synchronization can arise from mutual coupling, asymmetric entrainment, common forcing, or shared scheduling. Similar spectral concentration can arise from suppression of competing modes, stronger excitation of a dominant mode, measurement aggregation, or structural collapse.

A Type-II pattern therefore acquires stronger causal significance when the compatibility set is restricted to one mechanism class through additional structural, observational, or interventional evidence.

Mechanism identifiability also provides a natural boundary between temporal diagnosis and structural causal explanation. A temporal representation can identify a governance-relevant pattern while leaving the generative mechanism open.

5.7 Structural Underdetermination

This subsection develops structural underdetermination as the persistence of multiple substantively different Type-I systems within the compatibility set. Its objective is to characterize ambiguity more carefully than through a binary identifiable or unidentifiable distinction.

A basic underdetermination condition is represented by Equation 126.

$$|\mathcal{I}_{\Xi}(\Theta^{\text{obs}})| > 1. \quad (126)$$

Equation 126 establishes non-uniqueness but provides limited information about the structural difference among compatible systems.

When the structural domain supports a meaningful distance d_1 , the diameter of the compatibility set can be used as one descriptor of structural ambiguity. This diameter is represented by Equation 127.

$$D_1(\Theta^{\text{obs}}) = \sup_{\mathfrak{S}_1, \mathfrak{S}_2 \in \mathcal{I}_{\Xi}(\Theta^{\text{obs}})} d_1(\mathfrak{S}_1, \mathfrak{S}_2). \quad (127)$$

Equation 127 is meaningful only when the structural metric has a justified interpretation.

A compatibility set can be large in cardinality and structurally narrow, as with a continuum of nearby parameter values. Another can contain relatively few systems that differ across major structural mechanisms. The geometry and composition of the compatibility set can therefore matter more than its raw size.

Structural underdetermination can also be decomposed according to Type-I components. Let $\Pi_R, \Pi_F, \Pi_C, \Pi_B$ denote projections onto rule, dynamical, relational, and background structures. The component-wise ambiguity profile is represented by Equation 128.

$$\mathbf{A}_I = (\Pi_R[\mathcal{I}_{\Xi}], \Pi_F[\mathcal{I}_{\Xi}], \Pi_C[\mathcal{I}_{\Xi}], \Pi_B[\mathcal{I}_{\Xi}]). \quad (128)$$

Equation 128 records where structural ambiguity remains concentrated.

A Type-II representation can therefore identify relational structure strongly while leaving rule structure ambiguous, or identify a dynamical mechanism while leaving its generative background weakly constrained.

This decomposition supports more precise governance judgment than a single global identifiability label.

5.8 Information Gain through Additional Temporal Coordinates

This subsection develops the refinement of structural inference through additional Type-II information. Its objective is to formalize how adding timescale, phase, coupling, regime, or other temporal coordinates can reduce the structural compatibility set.

Let $\Theta^{(1)}$ denote an initial Type-II representation and let q_{add} denote an additional temporal coordinate or representation. The augmented representation is represented by Equation 129.

$$\Theta^{(2)} = (\Theta^{(1)}, q_{\text{add}}). \quad (129)$$

Equation 129 retains all previous temporal information while adding a further discriminating coordinate.

The corresponding compatibility sets satisfy the inclusion represented by Equation 130.

$$\mathcal{I}(\Theta^{(2)}) \subseteq \mathcal{I}(\Theta^{(1)}). \quad (130)$$

Equation 130 follows when the augmented representation preserves the original information exactly.

The additional coordinate produces strict structural information gain when the inclusion is strict. This condition is represented by Equation 131.

$$\mathcal{I}(\Theta^{(2)}) \subsetneq \mathcal{I}(\Theta^{(1)}). \quad (131)$$

Equation 131 provides a set-theoretic definition of additional structural distinguishability.

For example, spectral amplitude alone can leave two systems compatible, while phase information separates them. Frequency and phase can remain insufficient until coupling direction is observed. A stationary spectrum can identify several systems that become distinguishable after time-localized regime information is included.

The marginal structural refinement produced by temporal coordinate q can be represented abstractly by Equation 132.

$$\Delta\mathcal{I}_q = \mathcal{I}(\Theta) \setminus \mathcal{I}(\Theta, q). \quad (132)$$

Equation 132 identifies the structural alternatives excluded by the additional temporal coordinate.

An additional coordinate can also be structurally redundant. Redundancy is represented by Equation 133.

$$\mathcal{I}(\Theta, q) = \mathcal{I}(\Theta). \quad (133)$$

Equation 133 indicates that the new temporal information does not further distinguish the admissible structural systems.

The value of additional temporal information is therefore task dependent. A coordinate can provide no global refinement while identifying a particular structural mechanism or decision-relevant parameter. Conversely, substantial global refinement can be operationally unnecessary when all remaining structural alternatives imply the same admissible intervention.

Table 4 summarizes the principal identifiability concepts developed in this section.

The distinctions summarized in Table 4 show that structural inference from Type-II representation is intrinsically graded. Complete structural recovery is one possible endpoint. Local recovery, parameter recovery, mechanism identification, and identification of decision-relevant structural projections can remain possible under broader conditions.

The resulting inverse problem is therefore better represented through the geometry and projections of a compatibility set than through a binary distinction between knowledge and ignorance. A Type-II representation can constrain the Type-I domain substantially while leaving several structural realizations compatible with the available evidence.

The next section develops the complementary forward multiplicity problem. Section 6 examines the set of Type-II representations accessible from one structural system, including dependence on initial conditions, context, parameters, interventions, and dynamical regime.

Table 4: Structural Identifiability from Spectral-Temporal Representation

Identifiability Domain	Formal Object	Recoverable Content	Residual Ambiguity
Structural Compatibility	$\mathcal{I}_{\Xi}(\Theta^{\text{obs}})$	All Type-I systems consistent with the selected Type-II representation	Complete compatibility set remains available
Global Identifiability	$\mathcal{I}_{\Xi}(\Theta) = \{\mathfrak{S}\}$	Complete structural configuration	None within the specified model domain
Local Identifiability	Singleton compatibility within a neighborhood	Structural configuration near the reference system	Structurally distant alternatives can remain
Partial Identifiability	$\Pi_A[\mathcal{I}_{\Xi}]$	Selected structural component or property	Other structural coordinates remain unresolved
Parameter Identifiability	$\mathcal{P}_{\Xi}(\Theta)$	Parameters within a specified structural family	Alternative model families can remain compatible
Mechanism Identifiability	$\mathfrak{M}_{\Xi}(\Theta)$	Generative mechanism class	Parameter and implementation detail can remain uncertain
Structural Underdetermination	$ \mathcal{I}_{\Xi} > 1$	Range of compatible structural explanations	Ambiguity can span several Type-I layers
Temporal Information Gain	$\mathcal{I}(\Theta, q) \subseteq \mathcal{I}(\Theta)$	Additional structural distinctions exposed by new Type-II coordinates	Redundant coordinates can provide no further refinement

6 Temporal Reachability from Structural Representation

This section develops the forward multiplicity of the transformation framework. Its objective is to characterize the set of spectral-temporal representations that can be generated from one structural system when initial conditions, contextual conditions, parameters, interventions, and dynamical regimes vary. The section complements the inverse identifiability analysis of Section 5: structural identifiability asks which Type-I systems are compatible with a Type-II representation, whereas temporal reachability asks which Type-II representations remain accessible from a specified Type-I system or structural family. The discussion proceeds through forward temporal sets, initial-state dependence, contextual dependence, parameter variation, intervention-conditioned reachability, regime-dependent reachability, and the temporal multiplicity of structural systems.

6.1 Forward Temporal Sets

This subsection defines the principal forward set associated with a structural representation. Its objective is to replace the assumption of a unique temporal image with a set of Type-II representations generated under admissible realization and representational conditions.

Let $\mathfrak{S} \in \mathcal{M}_1$ denote a fixed structural system, and let $\eta \in \mathcal{E}_{\mathfrak{S}}$ collect admissible realization conditions. For a fixed observation and temporal representation context, the forward temporal set is represented by Equation 134.

$$\mathfrak{T}_{\Xi}(\mathfrak{S}) = \{\mathcal{T}_{\Xi, \eta}(\mathfrak{S}) \mid \eta \in \mathcal{E}_{\mathfrak{S}}\}. \quad (134)$$

Equation 134 contains all Type-II representations reachable from the selected structure under the admissible realization conditions included in the model.

When the representational context is also allowed to vary, the broader representation set is given by Equation 135.

$$\mathfrak{T}(\mathfrak{S}) = \{\mathcal{T}_{\Xi, \eta}(\mathfrak{S}) \mid \Xi \in \mathfrak{C}, \eta \in \mathcal{E}_{\mathfrak{S}}\}. \quad (135)$$

Equation 135 includes variation arising from both system realization and representation context.

The distinction between these two sets is important. Variation within $\mathfrak{T}_{\Xi}(\mathfrak{S})$ reflects different temporal realizations under a fixed analytical representation. Variation introduced only after changing Ξ can arise from altered observation, temporal resolution, or representation method.

A particular temporal representation Θ^* is forward reachable from $\mathfrak{S}$ under context Ξ when the condition in Equation 136 holds.

$$\Theta^* \in \mathfrak{T}_{\Xi}(\mathfrak{S}). \quad (136)$$

Equation 136 is an existential statement: at least one admissible realization condition generates the target Type-II representation.

The stronger condition of robust temporal reachability requires the target representation to remain accessible across a specified set of realization conditions. Let $\mathcal{E}^* \subseteq \mathcal{E}_{\mathfrak{S}}$ denote such a set. Robust reachability is represented by Equation 137.

$$\forall \eta \in \mathcal{E}^*, \quad \mathcal{T}_{\Xi, \eta}(\mathfrak{S}) \in \mathcal{N}_{\varepsilon}(\Theta^*), \quad (137)$$

where $\mathcal{N}_{\varepsilon}(\Theta^*)$ denotes an admissible neighborhood of the target representation.

Equation 137 distinguishes the existence of one favorable realization from persistence of a temporal objective across a range of admissible conditions.

The geometry of the forward temporal set can therefore matter for governance. A structurally fixed system can have a narrow temporal image, a broad continuous image, several disconnected temporal regions, or several qualitatively different spectral regimes.

6.2 Initial-State Dependence

This subsection develops temporal multiplicity generated through variation in initial state. Its objective is to show how one structural system can produce different Type-II representations even when its rules, dynamics, relations, and generative background remain fixed.

Let $x_0 \in X_0^{\text{adm}}$ denote an admissible initial state. The initial-state-conditioned temporal representation is represented by Equation 138.

$$\Theta(x_0) = \mathcal{T}_{\Xi}(\mathfrak{S}; x_0). \quad (138)$$

Equation 138 defines a map from admissible initial conditions to Type-II representations. The corresponding initial-state temporal set is represented by Equation 139.

$$\mathfrak{T}_{x_0}(\mathfrak{S}) = \left\{ \Theta(x_0) \mid x_0 \in X_0^{\text{adm}} \right\}. \quad (139)$$

Equation 139 isolates temporal multiplicity produced through initial-state variation.

This dependence is especially consequential in nonlinear systems possessing multiple attractors, long transients, or basin-sensitive dynamics [5]. Two initial states can evolve toward different long-run modes even though the governing structural system remains unchanged.

Let $\mathcal{A}_1, \dots, \mathcal{A}_m$ denote attractors or recurrent dynamical regions accessible within the same structural system. Their basins are represented by Equation 140.

$$\mathcal{B}_k = \left\{ x_0 \in X_0^{\text{adm}} \mid x(t; x_0) \longrightarrow \mathcal{A}_k \right\}. \quad (140)$$

Equation 140 partitions part of the initial-state domain according to asymptotic dynamical outcome when such attractor structure is applicable.

Each basin can correspond to a different family of Type-II representations. This relation is represented by Equation 141.

$$\mathfrak{T}_k = \{ \mathcal{T}_{\Xi}(\mathfrak{S}; x_0) \mid x_0 \in \mathcal{B}_k \}. \quad (141)$$

Equation 141 associates dynamical basins with temporal images.

The temporal multiplicity generated by initial-state variation has an important inferential implication. Observation of one Type-II realization does not establish that the structural system can generate only that temporal organization. A structurally unchanged system can support other rhythms, locking relations, or spectral regimes that remain unrealized under the observed initial condition.

Initial-state dependence also matters for intervention. A structural intervention can have different temporal consequences depending on the basin, transient region, or local phase-space neighborhood from which it is applied.

6.3 Contextual Dependence

This subsection develops temporal reachability under changing external and institutional context. Its objective is to represent how structurally similar or structurally fixed systems can generate different temporal organizations when their realization environment changes.

Let $\chi \in \mathcal{C}_{\text{adm}}$ denote an admissible contextual condition. The context-conditioned temporal representation is represented by Equation 142.

$$\Theta_{\chi} = \mathcal{T}_{\Xi}(\mathfrak{S}; \chi). \quad (142)$$

Equation 142 makes the temporal image dependent on the context in which the structural system is realized.

The context-conditioned forward set is represented by Equation 143.

$$\mathfrak{T}_{\chi}(\mathfrak{S}) = \{ \mathcal{T}_{\Xi}(\mathfrak{S}; \chi) \mid \chi \in \mathcal{C}_{\text{adm}} \}. \quad (143)$$

Equation 143 collects the Type-II representations accessible under admissible contextual variation.

Context can include environmental forcing, demand intensity, resource availability, external shocks, institutional environment, or other conditions treated as exogenous to the structural coordinates of the selected model.

A context can also reveal temporal modes that remain weakly expressed under ordinary operation. Let $A_k(\chi)$ denote the represented amplitude or prominence of temporal mode k under context χ . Context-dependent modal visibility is represented by Equation 144.

$$A_k = A_k(\chi). \quad (144)$$

Equation 144 allows an underlying structural mode to become more or less visible as contextual conditions change.

This possibility complicates structural inference from ordinary-operation data. Two systems can appear temporally equivalent in one context while diverging sharply under another. The context-indexed relation is represented by Equation 145.

$$\mathfrak{S}_1 \sim_{\chi_1} \mathfrak{S}_2, \quad \mathfrak{S}_1 \not\sim_{\chi_2} \mathfrak{S}_2. \quad (145)$$

Equation 145 identifies context-dependent loss and recovery of structural distinguishability.

For governance, this means that temporal equivalence established during routine conditions can be insufficient for reasoning about crisis, transition, scarcity, or unusually high load. Contextual variation can act as a natural probe of hidden structural differences.

6.4 Parameter Variation

This subsection develops temporal reachability within a parameterized structural family. Its objective is to characterize how continuous or discrete structural variation maps into temporal variation before explicit governance intervention is introduced.

Let $\mathfrak{S}(\vartheta)$ denote a structural family parameterized by $\vartheta \in \mathcal{P}$. The temporal image of the parameter family is represented by Equation 146.

$$\mathfrak{T}_{\mathcal{P}} = \{\mathcal{T}_{\Xi}(\mathfrak{S}(\vartheta)) \mid \vartheta \in \mathcal{P}\}. \quad (146)$$

Equation 146 defines the Type-II region reachable through variation of the selected structural parameters.

For differentiable parameter-to-temporal maps, local variation is represented by Equation 147.

$$\delta\Theta \approx J_{\vartheta \rightarrow \Theta} \delta\vartheta, \quad J_{\vartheta \rightarrow \Theta} = \frac{\partial\Theta}{\partial\vartheta}. \quad (147)$$

Equation 147 describes first-order temporal sensitivity to parameter variation.

The image can change smoothly across some parameter regions and discontinuously across others. A qualitative temporal transition at parameter value ϑ_c can be represented by Equation 148.

$$\lim_{\vartheta \rightarrow \vartheta_c^-} \Sigma(\vartheta) \neq \lim_{\vartheta \rightarrow \vartheta_c^+} \Sigma(\vartheta), \quad (148)$$

where $\Sigma(\vartheta)$ denotes the relevant Type-II regime descriptor.

Equation 148 captures a qualitative change in temporal regime across a structural parameter boundary.

Such behavior is familiar in nonlinear dynamical systems, where changes in control parameters can reorganize equilibria, oscillations, attractors, or stability structure [5]. The governance interpretation requires additional care because a model parameter can represent a rule, coupling, capacity, threshold, or another structural quantity.

Parameter variation therefore provides a bridge between descriptive forward reachability and intervention. An intervention becomes a governance action when the parameter change is produced through an identified admissible structural operator.

6.5 Intervention-Conditioned Temporal Sets

This subsection introduces governance intervention into the forward reachability problem. Its objective is to identify the Type-II representations that become accessible after admissible structural interventions are applied to a reference system.

Let $\mathcal{U}_I^{\text{adm}}$ denote the admissible set of Type-I structural interventions. The intervention-conditioned structural set is represented by Equation 149.

$$\mathfrak{R}_I(\mathfrak{S}) = \left\{ \mathcal{U}_I(\mathfrak{S}) \mid \mathcal{U}_I \in \mathcal{U}_I^{\text{adm}} \right\}. \quad (149)$$

Equation 149 contains the structural configurations reachable through admissible governance action. The corresponding intervention-conditioned temporal set is represented by Equation 150.

$$\mathfrak{R}_{II}(\mathfrak{S}) = \left\{ \mathcal{T}_{\Xi}[\mathcal{U}_I(\mathfrak{S})] \mid \mathcal{U}_I \in \mathcal{U}_I^{\text{adm}} \right\}. \quad (150)$$

Equation 150 defines the spectral-temporal region reachable through the admissible Type-I governance interventions.

A desired temporal target Θ^* is structurally reachable when the condition in Equation 151 holds.

$$\Theta^* \in \mathfrak{R}_{II}(\mathfrak{S}). \quad (151)$$

Equation 151 states that at least one admissible structural intervention can realize the target under the selected transformation context.

For approximate targets, reachability can be defined relative to a tolerance. The approximate condition is represented by Equation 152.

$$\exists \mathcal{U}_I \in \mathcal{U}_I^{\text{adm}} : d_{II}(\mathcal{T}_{\Xi}[\mathcal{U}_I(\mathfrak{S})], \Theta^*) \leq \varepsilon. \quad (152)$$

Equation 152 is useful when temporal objectives specify ranges, tolerances, or acceptable spectral regimes.

The reachability set can also be indexed by intervention budget. Let $c(\mathcal{U}_I)$ denote a context-specific intervention cost and B an admissible budget. Budget-constrained temporal reachability is represented by Equation 153.

$$\mathfrak{R}_{II}^B(\mathfrak{S}) = \left\{ \mathcal{T}_{\Xi}[\mathcal{U}_I(\mathfrak{S})] \mid \mathcal{U}_I \in \mathcal{U}_I^{\text{adm}}, c(\mathcal{U}_I) \leq B \right\}. \quad (153)$$

Equation 153 makes practical reachability dependent on available intervention resources.

This distinction becomes central to structural lifting. A Type-II target can be conceptually desirable and dynamically feasible while remaining outside the set reachable through admissible Type-I interventions.

6.6 Regime-Dependent Reachability

This subsection develops temporal reachability conditional on the system's current dynamical or spectral regime. Its objective is to show how the same structural intervention can have different reachable temporal consequences across different regions of the system's evolution.

Let $\rho \in \mathcal{R}$ denote the current regime, where a regime can be defined through a structural, dynamical, or Type-II criterion appropriate to the model. The regime-conditioned temporal set is represented by Equation 154.

$$\mathfrak{R}_{\text{II}}(\mathfrak{S} \mid \rho) = \left\{ \mathcal{T}_{\Xi}[\mathcal{U}_{\text{I}}(\mathfrak{S})] \mid \mathcal{U}_{\text{I}} \in \mathfrak{U}_{\text{I}}^{\text{adm}}(\rho) \right\}. \quad (154)$$

Equation 154 allows the admissible intervention set and temporal image to depend on the current regime.

Two regimes can therefore possess different temporal reachable sets even within the same broader structural system. This relation is represented by Equation 155.

$$\mathfrak{R}_{\text{II}}(\mathfrak{S} \mid \rho_1) \neq \mathfrak{R}_{\text{II}}(\mathfrak{S} \mid \rho_2). \quad (155)$$

Equation 155 captures regime-dependent governance opportunity.

Near a critical transition, the local sensitivity of Type-II coordinates to structural perturbation can increase, decrease, or become strongly anisotropic. Let J_{ρ} denote the local structural-to-temporal sensitivity matrix within regime ρ . Regime dependence of local sensitivity is represented by Equation 156.

$$J_{\rho} = \left. \frac{\partial \Theta}{\partial z} \right|_{\rho}. \quad (156)$$

Equation 156 provides a local representation of how intervention effectiveness can change across regimes.

The reachability geometry can also become path dependent. After a transition into another attractor or institutional regime, returning structural parameters to earlier values need not immediately restore the previous temporal organization. Hysteresis and basin dependence provide formal examples of such behavior in nonlinear systems [5].

Regime-dependent reachability therefore introduces a temporal dimension to governance opportunity itself. A target that is easily reachable before a transition can become costly, slow, or inaccessible after the system enters a different regime.

6.7 Temporal Multiplicity of Structural Systems

This subsection consolidates the forward multiplicity results of the section. Its objective is to distinguish structural equivalence from the range of temporal organizations that one structural system can generate.

For each structural system $\mathfrak{S}$, the forward temporal set $\mathfrak{T}(\mathfrak{S})$ can contain one or several Type-II representations. Temporal multiplicity is represented by Equation 157.

$$|\mathfrak{T}(\mathfrak{S})| > 1. \quad (157)$$

Equation 157 identifies a structural system capable of generating more than one temporal representation under the admissible realization conditions.

For continuous Type-II domains, cardinality alone is rarely informative. A diameter can be introduced when the temporal domain possesses an appropriate distance. The temporal reachability diameter is represented by Equation 158.

$$D_{\text{II}}(\mathfrak{S}) = \sup_{\Theta_1, \Theta_2 \in \mathfrak{T}(\mathfrak{S})} d_{\text{II}}(\Theta_1, \Theta_2). \quad (158)$$

Equation 158 provides one measure of the extent of temporal variation generated by one structural system when the distance d_{II} has substantive meaning.

A further distinction concerns overlap between the forward temporal sets of different structures. For two systems $\mathfrak{S}_1$ and $\mathfrak{S}_2$, their temporal overlap is represented by Equation 159.

$$\mathfrak{O}_{12} = \mathfrak{T}(\mathfrak{S}_1) \cap \mathfrak{T}(\mathfrak{S}_2). \quad (159)$$

Equation 159 identifies Type-II representations that can be generated by both structural systems.

The overlap can be nonempty even when the systems also generate distinct temporal organizations. This produces a relation richer than ordinary pointwise equivalence. Two structural systems can coincide temporally under some realization conditions and diverge under others.

A complete forward-set equivalence can therefore be defined by Equation 160.

$$\mathfrak{S}_1 \sim_{\mathfrak{T}} \mathfrak{S}_2 \iff \mathfrak{T}(\mathfrak{S}_1) = \mathfrak{T}(\mathfrak{S}_2). \quad (160)$$

Equation 160 is stronger than equality of one realized Type-II observation. It requires the two structures to possess the same admissible temporal image under the selected forward-set construction.

This distinction is consequential for governance. Two systems that appear equivalent under one observed trajectory can have very different temporal possibility sets. One can possess latent alternative regimes, stronger response to contextual change, or greater structural capacity for temporal reconfiguration.

The forward-set perspective therefore shifts part of governance analysis from the current temporal state toward the temporal possibilities generated by the underlying structure.

Table 5 summarizes the principal forms of forward temporal multiplicity developed in this section.

Table 5: Temporal Reachability from Structural Representation

Reachability Domain	Formal Object	Source of Temporal Variation	Governance Significance
Forward Temporal Set	$\mathfrak{T}_{\Xi}(\mathfrak{S})$	Admissible realization conditions	Defines the temporal possibilities generated by one structure
Initial-State Reachability	$\mathfrak{T}_{x_0}(\mathfrak{S})$	Different initial states and dynamical basins	Observed temporal behavior can represent one among several latent realizations
Contextual Reachability	$\mathfrak{T}_{\chi}(\mathfrak{S})$	External and institutional context	Temporal equivalence can change across operating conditions
Parameter Reachability	$\mathfrak{T}_{\mathcal{P}}$	Variation within a structural parameter family	Links structural sensitivity with temporal change
Intervention-Conditioned Reachability	$\mathfrak{R}_{\Pi}(\mathfrak{S})$	Admissible Type-I interventions	Defines temporal targets achievable through structural governance
Regime-Dependent Reachability	$\mathfrak{R}_{\Pi}(\mathfrak{S} \mid \rho)$	Current dynamical or spectral regime	Governance opportunity varies across system regimes
Forward-Set Overlap	$\mathfrak{T}(\mathfrak{S}_1) \cap \mathfrak{T}(\mathfrak{S}_2)$	Shared temporal realizations across different structures	Pointwise temporal similarity can conceal different latent possibility sets
Forward-Set Equivalence	$\mathfrak{T}(\mathfrak{S}_1) = \mathfrak{T}(\mathfrak{S}_2)$	Equality of complete temporal possibility sets	Provides a stronger equivalence criterion than one observed trajectory

The relations summarized in Table 5 establish the forward counterpart to structural identifiability. The inverse problem associates one temporal representation with a set of structurally compatible systems. The forward problem associates one structural system with a set of temporally compatible realizations. These two set-valued relations form the basic many-to-many architecture connecting Type-I and Type-II representations.

This architecture also prepares the intervention problem developed in the next section. Section 7 examines when a Type-I intervention induces a well-defined operator on Type-II equivalence classes, when temporally indistinguishable structures remain indistinguishable after intervention, and when intervention reveals structural distinctions hidden by the pre-intervention representation.

7 Structural Interventions and Induced Temporal Operators

This section develops the transformation of governance interventions across the Type-I and Type-II representational domains. Its objective is to determine when a structural intervention admits a well-defined spectral-temporal representation, how structural interventions can be compared through their temporal effects, and how intervention can expose structural differences that remain hidden under ordinary observation. The section proceeds from structural intervention operators to representation-preserving conditions, induced spectral-temporal operators, commutative transformation structure, intervention equivalence, interventional identifiability, divergence within pre-intervention equivalence classes, and composition of induced operators.

The analysis distinguishes two questions. The first concerns forward intervention representation: given a Type-I intervention, which temporal transformation does it generate? The second concerns interventional identifiability: can responses to one or more interventions distinguish structural systems that share the same pre-intervention Type-II representation? These questions require stronger conditions than ordinary structural-to-temporal transformation because the transformation must remain consistent across all structural realizations represented by the same Type-II state.

7.1 Structural Intervention Operators

This subsection specifies intervention in the Type-I structural domain. Its objective is to represent governance action as a transformation of structural systems before examining whether that action admits a representation in the Type-II domain.

Let $\mathcal{M}_I$ denote the admissible structural domain. A structural intervention is represented by Equation 161.

$$\mathcal{U}_I : \mathcal{M}_I \longrightarrow \mathcal{M}_I, \quad \mathfrak{S}^- \longmapsto \mathfrak{S}^+. \quad (161)$$

Equation 161 represents a governance action that transforms one or more structural coordinates of the governed system.

Using the Type-I structural representation, the intervention can be written component-wise as Equation 162.

$$\mathcal{U}_I : (X, x, R, F, C, \mathcal{B}) \longmapsto (X^+, x^+, R^+, F^+, C^+, \mathcal{B}^+). \quad (162)$$

Equation 162 permits interventions with single-layer or multilayer direct structural support.

The direct Type-I support of an intervention can be retained through the mapping introduced in the preceding structural taxonomy. For present purposes, the support profile is represented by Equation 163.

$$\Lambda_I(\mathcal{U}_I) \subseteq \{\text{SR}, \text{D}, \text{REL}, \text{B}\}. \quad (163)$$

Equation 163 records whether the intervention directly transforms state-and-rule, dynamical-process, relational-structural, or generative-background components.

An admissible intervention family is represented by Equation 164.

$$\mathcal{U}_1^{\text{adm}} = \{\mathcal{U}_1^{(1)}, \mathcal{U}_1^{(2)}, \dots\}. \quad (164)$$

Equation 164 can be restricted by legal, institutional, resource, viability, technical, or generative conditions.

The structural intervention and the representation operator remain distinct objects. A structural intervention changes the system from which future trajectories are generated. The representation operator subsequently maps that changed system through dynamical realization, observation, and temporal representation.

For a fixed transformation context Ξ , the post-intervention Type-II representation is given by Equation 165.

$$\Theta^+ = \mathcal{T}_\Xi [\mathcal{U}_1(\mathfrak{S}^-)]. \quad (165)$$

Equation 165 defines the forward temporal consequence of a structural intervention for a specified structural realization.

This pointwise relation always provides a temporal outcome when the composite transformation is defined. A stronger condition is required before one Type-II operator can be assigned independently of which structurally compatible realization underlies the observed temporal state.

7.2 Representation-Preserving Intervention Conditions

This subsection establishes the condition under which a Type-I intervention can act consistently on Type-II equivalence classes. Its objective is to identify when the post-intervention temporal representation depends only on the pre-intervention Type-II state rather than on unresolved structural differences within its equivalence class.

Let $\sim_{\mathcal{T}}$ denote the structural equivalence relation induced by a fixed transformation $\mathcal{T}$. The representation-preserving intervention condition is represented by Equation 166.

$$\mathfrak{S}_1 \sim_{\mathcal{T}} \mathfrak{S}_2 \implies \mathcal{U}_1(\mathfrak{S}_1) \sim_{\mathcal{T}} \mathcal{U}_1(\mathfrak{S}_2). \quad (166)$$

Equation 166 requires the intervention to preserve every equivalence class of the pre-intervention representation at the level relevant to the induced temporal map.

Using the defining transformation relation, the same condition can be written directly in Type-II coordinates. This equivalent form is represented by Equation 167.

$$\mathcal{T}(\mathfrak{S}_1) = \mathcal{T}(\mathfrak{S}_2) \implies \mathcal{T}[\mathcal{U}_1(\mathfrak{S}_1)] = \mathcal{T}[\mathcal{U}_1(\mathfrak{S}_2)]. \quad (167)$$

Equation 167 is the central well-definedness condition of this section.

The condition has a direct governance interpretation. Suppose two structural systems cannot be distinguished using the currently available Type-II representation. If the same structural intervention produces the same post-intervention temporal representation in both systems, the unresolved structural difference is irrelevant to the temporal consequence represented by the selected Type-II coordinates.

If the condition fails, the current Type-II state does not contain sufficient information to predict the represented temporal consequence of the intervention uniquely.

The failure condition is represented by Equation 168.

$$\exists \mathfrak{S}_1, \mathfrak{S}_2 : \mathcal{T}(\mathfrak{S}_1) = \mathcal{T}(\mathfrak{S}_2) \quad \wedge \quad \mathcal{T}[\mathcal{U}_1(\mathfrak{S}_1)] \neq \mathcal{T}[\mathcal{U}_1(\mathfrak{S}_2)]. \quad (168)$$

Equation 168 describes an intervention that splits at least one pre-intervention equivalence class.

This splitting relation is theoretically important because ordinary observational equivalence and interventionally relevant equivalence become distinct. Two systems can support the same observed temporal organization while containing different structural response mechanisms.

The representation-preserving condition can also be restricted to a subset of the structural domain. Let $\mathcal{D}_{\mathcal{U}} \subseteq \mathcal{M}_I$ denote a domain of interest. Local or domain-specific preservation is represented by Equation 169.

$$\forall \mathfrak{S}_1, \mathfrak{S}_2 \in \mathcal{D}_{\mathcal{U}}, \quad \mathfrak{S}_1 \sim_{\mathcal{T}} \mathfrak{S}_2 \implies \mathcal{U}_I(\mathfrak{S}_1) \sim_{\mathcal{T}} \mathcal{U}_I(\mathfrak{S}_2). \quad (169)$$

Equation 169 permits an induced Type-II operator to be valid within a restricted governance regime even when global preservation across the complete structural domain is unavailable.

7.3 Induced Spectral-Temporal Operators

This subsection constructs the Type-II operator generated by a representation-preserving Type-I intervention. Its objective is to formalize the temporal action of structural governance independently of the particular structural representative chosen from an equivalence class.

Assume that Equation 166 holds. An induced Type-II operator can then be defined by Equation 170.

$$\bar{\mathcal{U}}_{II}(\mathcal{T}(\mathfrak{S})) = \mathcal{T}[\mathcal{U}_I(\mathfrak{S})]. \quad (170)$$

Equation 170 is independent of the choice of $\mathfrak{S}$ within its equivalence class precisely because the preservation condition holds.

The induced operator therefore acts on the image of the structural representation according to Equation 171.

$$\bar{\mathcal{U}}_{II} : \text{Im}(\mathcal{T}) \longrightarrow \text{Im}(\mathcal{T}). \quad (171)$$

Equation 171 describes the exact Type-II operator associated with the structural intervention under the selected representation context.

The same construction can be expressed through the quotient structural domain developed in Section 4. Let $\pi_{\mathcal{T}}$ denote the quotient projection. This projection is represented by Equation 172.

$$\pi_{\mathcal{T}} : \mathcal{M}_I \longrightarrow \mathcal{M}_I / \sim_{\mathcal{T}}, \quad \mathfrak{S} \longmapsto [\mathfrak{S}]_{\mathcal{T}}. \quad (172)$$

Equation 172 maps structural systems to their temporal equivalence classes.

The structural intervention induces a class-level operator represented by Equation 173.

$$\tilde{\mathcal{U}}_I([\mathfrak{S}]_{\mathcal{T}}) = [\mathcal{U}_I(\mathfrak{S})]_{\mathcal{T}}. \quad (173)$$

Equation 173 is well defined under the same representation-preserving condition.

This quotient formulation clarifies the meaning of a Type-II intervention. The induced temporal operator acts on information preserved by the representation. Structural distinctions within one equivalence class remain suppressed whenever they do not alter the represented temporal consequence.

A structural intervention can consequently admit a useful induced temporal operator even when the complete structural system remains unidentifiable.

7.4 Commutative Transformation Structure

This subsection develops the commutative structure connecting Type-I and Type-II interventions. Its objective is to express formally when structural intervention followed by representation yields the same Type-II result as representation followed by the induced temporal operator.

The commutative relation is represented by Equation 174.

$$\mathcal{T} \circ \mathcal{U}_I = \bar{\mathcal{U}}_{II} \circ \mathcal{T}. \quad (174)$$

Equation 174 is the central transformation relation between intervention domains.

The corresponding diagram can be expressed by Equation 175.

$$\begin{array}{ccc}
\mathfrak{S} & \xrightarrow{\mathcal{U}_I} & \mathcal{U}_I(\mathfrak{S}) \\
\mathcal{T} \downarrow & & \downarrow \mathcal{T} \\
\Theta & \xrightarrow{\bar{\mathcal{U}}_{II}} & \Theta^+.
\end{array} \tag{175}$$

Equation 175 summarizes the two equivalent paths when the induced operator exists.

The relation should be interpreted as representation consistency. It does not imply that Type-I and Type-II interventions are ontologically identical. The structural intervention changes the generative organization of the system. The induced Type-II operator records the represented temporal consequence of that structural change.

Approximate commutativity can be useful when empirical representations contain noise, estimation error, or model approximation. Let d_{II} denote a justified distance on the temporal representation. Approximate commutativity is represented by Equation 176.

$$d_{II}(\mathcal{T}[\mathcal{U}_I(\mathfrak{S})], \bar{\mathcal{U}}_{II}[\mathcal{T}(\mathfrak{S})]) \leq \varepsilon. \tag{176}$$

Equation 176 permits an induced temporal model to approximate structural intervention within a specified tolerance.

The tolerance must remain tied to the governance task and measurement model. A deviation negligible for broad timescale classification can remain consequential for phase-sensitive or critical-regime intervention.

7.5 Intervention Equivalence

This subsection develops equivalence among structurally different interventions. Its objective is to identify when different Type-I governance actions produce the same represented Type-II transformation.

Let $\mathcal{U}_I^{(a)}$ and $\mathcal{U}_I^{(b)}$ denote two structural interventions. They are pointwise temporally equivalent at structural system $\mathfrak{S}$ when the condition in Equation 177 holds.

$$\mathcal{T}[\mathcal{U}_I^{(a)}(\mathfrak{S})] = \mathcal{T}[\mathcal{U}_I^{(b)}(\mathfrak{S})]. \tag{177}$$

Equation 177 identifies equivalent represented temporal consequences at one structural system.

A stronger intervention equivalence over structural domain $\mathcal{D}$ is represented by Equation 178.

$$\mathcal{U}_I^{(a)} \sim_{\mathcal{T}, \mathcal{D}} \mathcal{U}_I^{(b)} \iff \forall \mathfrak{S} \in \mathcal{D}, \mathcal{T}[\mathcal{U}_I^{(a)}(\mathfrak{S})] = \mathcal{T}[\mathcal{U}_I^{(b)}(\mathfrak{S})]. \tag{178}$$

Equation 178 defines temporal equivalence of structural interventions relative to the selected representation and domain.

When both interventions induce Type-II operators, their Type-II equivalence can be represented by Equation 179.

$$\bar{\mathcal{U}}_{II}^{(a)} = \bar{\mathcal{U}}_{II}^{(b)} \quad \text{on } \mathcal{T}(\mathcal{D}). \tag{179}$$

Equation 179 means that the selected Type-II representation cannot distinguish the temporal actions of the two structural interventions within the specified domain.

The interventions can nevertheless differ substantially in Type-I support. This structural difference is represented by Equation 180.

$$\Lambda_I(\mathcal{U}_I^{(a)}) \neq \Lambda_I(\mathcal{U}_I^{(b)}). \tag{180}$$

Equation 180 can coexist with Equation 179.

This coexistence provides a central reason to retain both taxonomies. A temporal objective such as reducing synchronization can potentially be realized through modification of rules, couplings, feedback dynamics, or generative backgrounds. The Type-II outcome can be similar while structural cost, institutional distribution, reversibility, and generative consequences differ.

Temporal equivalence of interventions therefore defines an equivalence class of structural realizations rather than one uniquely preferred governance action.

7.6 Interventional Identifiability

This subsection develops structural identifiability obtained through active intervention. Its objective is to formalize how responses to selected structural perturbations can refine equivalence classes that remain broad under passive temporal observation.

Let $\mathcal{U}^P = \{\mathcal{U}_1, \dots, \mathcal{U}_m\}$ denote a family of diagnostic or governance-relevant structural interventions. The interventional response signature of structural system $\mathfrak{S}$ is represented by Equation 181.

$$\mathcal{R}_{\mathcal{U}^P}(\mathfrak{S}) = (\mathcal{T}[\mathcal{U}_1(\mathfrak{S})], \mathcal{T}[\mathcal{U}_2(\mathfrak{S})], \dots, \mathcal{T}[\mathcal{U}_m(\mathfrak{S})]). \quad (181)$$

Equation 181 records the represented temporal responses generated by a family of interventions.

Two structural systems are interventionally equivalent under the probe family when their response signatures coincide. This relation is defined by Equation 182.

$$\mathfrak{S}_1 \sim_{\mathcal{U}^P} \mathfrak{S}_2 \iff \mathcal{R}_{\mathcal{U}^P}(\mathfrak{S}_1) = \mathcal{R}_{\mathcal{U}^P}(\mathfrak{S}_2). \quad (182)$$

Equation 182 induces an interventional partition of the structural domain.

Passive temporal observation and interventional response can be combined. The joint observation-intervention signature is represented by Equation 183.

$$\mathcal{J}_{\mathcal{U}^P}(\mathfrak{S}) = (\mathcal{T}(\mathfrak{S}), \mathcal{R}_{\mathcal{U}^P}(\mathfrak{S})). \quad (183)$$

Equation 183 retains both the pre-intervention Type-II representation and the responses to the selected interventions.

A structural system is globally interventionally identifiable relative to the probe family when the joint signature is unique over the admissible structural domain. This condition is represented by Equation 184.

$$\mathcal{J}_{\mathcal{U}^P}(\mathfrak{S}_1) = \mathcal{J}_{\mathcal{U}^P}(\mathfrak{S}_2) \implies \mathfrak{S}_1 = \mathfrak{S}_2. \quad (184)$$

Equation 184 defines interventional identifiability relative to the selected structural domain, representation, and probe family.

Intervention can therefore refine structural compatibility beyond passive observation. If $\mathcal{J}(\Theta)$ denotes the passive compatibility set and $\mathcal{J}_{\mathcal{U}^P}(\Theta, \mathcal{R})$ the compatibility set after response information is included, their relation is represented by Equation 185.

$$\mathcal{J}_{\mathcal{U}^P}(\Theta, \mathcal{R}) \subseteq \mathcal{J}(\Theta). \quad (185)$$

Equation 185 expresses the potential information gain from intervention.

The inclusion becomes strict when at least one pair of passively equivalent structural systems responds differently to the probe family. This condition is represented by Equation 186.

$$\exists \mathfrak{S}_1, \mathfrak{S}_2 \in [\mathfrak{S}]_{\mathcal{T}} : \mathcal{R}_{\mathcal{U}^P}(\mathfrak{S}_1) \neq \mathcal{R}_{\mathcal{U}^P}(\mathfrak{S}_2). \quad (186)$$

Equation 186 identifies a probe family that reveals hidden structural differences.

The governance use of diagnostic intervention requires additional normative and operational conditions. Some interventions can be too costly, irreversible, unsafe, or institutionally unavailable for identification purposes. Interventional identifiability is therefore constrained by the set of admissible probes rather than by every mathematically conceivable perturbation.

7.7 Interventional Divergence within Representation Classes

This subsection develops the divergence of post-intervention temporal behavior among systems that share the same pre-intervention representation. Its objective is to characterize the practical significance of hidden structural heterogeneity within a Type-II equivalence class.

Let $C = [\mathfrak{S}]_{\mathcal{T}}$ denote a pre-intervention structural equivalence class. Under intervention $\mathcal{U}_I$, the set of possible post-intervention temporal representations is represented by Equation 187.

$$\mathfrak{R}_{\mathcal{U}}(C) = \{\mathcal{T}[\mathcal{U}_I(\mathfrak{S}')] \mid \mathfrak{S}' \in C\}. \quad (187)$$

Equation 187 collects every represented temporal consequence compatible with the unresolved pre-intervention structure.

The intervention admits a unique Type-II prediction on class C when the response set is a singleton. This condition is represented by Equation 188.

$$|\mathfrak{R}_{\mathcal{U}}(C)| = 1. \quad (188)$$

Equation 188 is equivalent to representation preservation on the selected equivalence class.

Interventional divergence occurs when the response set contains several elements. This condition is represented by Equation 189.

$$|\mathfrak{R}_{\mathcal{U}}(C)| > 1. \quad (189)$$

Equation 189 expresses temporal uncertainty generated by unresolved structural heterogeneity.

When the Type-II domain possesses a justified distance d_{II} , the extent of divergence can be described by the diameter of the response set. This quantity is represented by Equation 190.

$$D_{\mathcal{U}}^{\text{resp}}(C) = \sup_{\Theta_1, \Theta_2 \in \mathfrak{R}_{\mathcal{U}}(C)} d_{\text{II}}(\Theta_1, \Theta_2). \quad (190)$$

Equation 190 measures the range of possible represented temporal outcomes generated by the structural ambiguity within the class.

This quantity has a direct decision interpretation. A broad response set indicates that current Type-II knowledge can support weak prediction of the intervention's temporal consequence even when the pre-intervention temporal state itself is known precisely.

Interventional divergence also reveals a distinction between observational sufficiency and decision sufficiency. An equivalence class can be adequate for describing the current temporal state while remaining too coarse for selecting a structural intervention.

A decision-relevant equivalence relation can therefore be defined relative to a contemplated intervention. Two systems are interventionally equivalent for $\mathcal{U}$ when their post-intervention temporal consequences coincide. This relation is represented by Equation 191.

$$\mathfrak{S}_1 \sim_{\mathcal{U}, \mathcal{T}} \mathfrak{S}_2 \iff \mathcal{T}[\mathcal{U}(\mathfrak{S}_1)] = \mathcal{T}[\mathcal{U}(\mathfrak{S}_2)]. \quad (191)$$

Equation 191 provides a task-dependent notion of structural sufficiency.

This relation can be considerably coarser than complete structural identifiability. Governance can therefore proceed without full reconstruction when all structurally compatible systems produce an acceptable temporal response to the contemplated intervention.

7.8 Composition of Induced Operators

This subsection develops sequential composition of structural interventions and their induced Type-II operators. Its objective is to identify when composition in the structural domain is preserved by temporal representation.

Let $\mathcal{U}_I$ and $\mathcal{V}_I$ be two structural interventions that each preserve the relevant representation equivalence classes. Their sequential structural composition is represented by Equation 192.

$$\mathcal{W}_I = \mathcal{V}_I \circ \mathcal{U}_I. \quad (192)$$

Equation 192 applies $\mathcal{U}_I$ first and $\mathcal{V}_I$ second.

When the required preservation conditions hold throughout the relevant domain, the induced temporal operators compose according to Equation 193.

$$\overline{\mathcal{W}}_{II} = \overline{\mathcal{V}}_{II} \circ \overline{\mathcal{U}}_{II}. \quad (193)$$

Equation 193 shows that sequential structural intervention can be represented through sequential Type-II operators under suitable consistency conditions.

The complete commutative relation is represented by Equation 194.

$$\mathcal{T} \circ \mathcal{V}_I \circ \mathcal{U}_I = \overline{\mathcal{V}}_{II} \circ \overline{\mathcal{U}}_{II} \circ \mathcal{T}. \quad (194)$$

Equation 194 permits a structural intervention sequence to be analyzed through the Type-II representation when each induced operator remains well defined over the states reached by the preceding interventions.

The order of structural interventions can matter. Structural noncommutativity is represented by Equation 195.

$$\mathcal{V}_I \circ \mathcal{U}_I \neq \mathcal{U}_I \circ \mathcal{V}_I. \quad (195)$$

Equation 195 can generate corresponding temporal order dependence.

When the Type-II representation preserves that distinction, the induced operators satisfy the relation in Equation 196.

$$\overline{\mathcal{V}}_{II} \circ \overline{\mathcal{U}}_{II} \neq \overline{\mathcal{U}}_{II} \circ \overline{\mathcal{V}}_{II}. \quad (196)$$

Equation 196 records temporal path dependence visible in the selected representation.

A further possibility arises when structural order dependence is compressed by the Type-II representation. This condition is represented by Equation 197.

$$\mathcal{V}_I \circ \mathcal{U}_I \neq \mathcal{U}_I \circ \mathcal{V}_I, \quad \mathcal{T} \circ \mathcal{V}_I \circ \mathcal{U}_I = \mathcal{T} \circ \mathcal{U}_I \circ \mathcal{V}_I. \quad (197)$$

Equation 197 identifies structural sequence differences that remain invisible in the selected Type-II representation.

This distinction will become important for structural lifting. Two intervention sequences can appear temporally equivalent while producing different structural conditions for subsequent governance, different reversibility, or different generative possibilities.

Table 6 summarizes the principal relations developed in this section.

The relations summarized in Table 6 establish a stronger connection between structural and spectral-temporal governance than passive representation alone. A structural intervention admits a unique Type-II operator when unresolved structural differences do not alter its represented temporal consequence. When this condition fails, intervention exposes information absent from the pre-intervention Type-II state.

The resulting distinction separates three cases. In the first, structurally different systems remain temporally equivalent before and after intervention. In the second, structural systems are temporally equivalent before intervention and diverge afterward. In the third, different structural interventions generate equivalent Type-II transformations despite acting on different structural supports. These cases respectively correspond to representation-preserving intervention, interventional identification, and multiple structural realization of a temporal governance effect.

The next section develops the reverse intervention problem. Section 8 begins with a desired Type-II transformation and studies the set of Type-I interventions capable of realizing it, including structural multiplicity, admissibility, temporal reachability, minimal realization, and composite lifting.

Table 6: Structural Interventions and Induced Spectral-Temporal Operators

Intervention Relation	Formal Object	Representational Function	Governance Significance
Structural Intervention	$\mathcal{U}_I : \mathfrak{S}^- \mapsto \mathfrak{S}^+$	Transforms Type-I structural organization	Defines the generative change preceding temporal response
Representation Preservation	$\mathfrak{S}_1 \sim_{\mathcal{T}} \mathfrak{S}_2 \Rightarrow \mathcal{U}(\mathfrak{S}_1) \sim_{\mathcal{T}} \mathcal{U}(\mathfrak{S}_2)$	Preserves temporal equivalence under intervention	Permits a unique Type-II intervention representation
Induced Temporal Operator	$\bar{\mathcal{U}}_{II}$	Represents the Type-II action generated by structural intervention	Allows temporal prediction without selecting a structural representative
Commutative Structure	$\mathcal{T} \circ \mathcal{U}_I = \bar{\mathcal{U}}_{II} \circ \mathcal{T}$	Connects intervention across representational domains	Formalizes consistency of structural and temporal descriptions
Intervention Equivalence	$\mathcal{T} \circ \mathcal{U}_a = \mathcal{T} \circ \mathcal{U}_b$	Groups structurally different interventions by temporal consequence	One Type-II objective can admit several Type-I realizations
Interventional Identifiability	$\mathcal{J}_{\mathcal{U}^P}(\mathfrak{S})$	Uses intervention responses to refine structural compatibility	Active perturbation can reveal structure hidden by passive observation
Interventional Divergence	$\mathfrak{R}_{\mathcal{U}}([\mathfrak{S}]_{\mathcal{T}})$	Represents possible post-intervention outcomes within one equivalence class	Current temporal knowledge can be insufficient for intervention prediction
Operator Composition	$\bar{\mathcal{V}}_{II} \circ \bar{\mathcal{U}}_{II}$	Represents sequential structural interventions in Type-II space	Preserves intervention order when the representation remains sufficiently informative

8 Temporal Targets and Structural Lifting

This section develops the reverse intervention problem of the transformation framework. Its objective is to begin from a desired Type-II spectral-temporal transformation and determine which Type-I structural interventions can realize it. The section distinguishes temporal targets from their structural implementations, defines lifting relations and realization sets, examines multiplicity among structurally different realizations, introduces structural and admissibility constraints, characterizes reachable temporal transformations, and develops minimal and composite lifting. The analysis preserves the distinction between equivalence in represented temporal effect and equivalence in structural governance.

8.1 Spectral-Temporal Target Operators

This subsection specifies governance objectives directly in the Type-II representational domain. Its objective is to represent desired changes in timescale, phase, locking, resonance, polyfrequency organization, cross-frequency relations, or spectral regime independently of the structural intervention eventually selected to realize them.

Let $\mathcal{M}_{\text{II}}$ denote the selected Type-II representation domain. A temporal target operator is represented by Equation 198.

$$\mathcal{V}_{\text{II}} : \mathcal{D}_{\text{II}} \longrightarrow \mathcal{M}_{\text{II}}, \quad (198)$$

where $\mathcal{D}_{\text{II}} \subseteq \mathcal{M}_{\text{II}}$ denotes the temporal domain over which the objective is defined.

For a current temporal representation Θ^- , the target transformation is represented by Equation 199.

$$\Theta^* = \mathcal{V}_{\text{II}}(\Theta^-). \quad (199)$$

Equation 199 specifies the desired Type-II outcome without yet specifying how the underlying structure should be changed.

A target can also be set-valued. Let $\mathcal{G}_{\text{II}}(\Theta^-)$ denote an admissible or desirable temporal target region. This target region is represented by Equation 200.

$$\mathcal{G}_{\text{II}}(\Theta^-) \subseteq \mathcal{M}_{\text{II}}. \quad (200)$$

Equation 200 allows governance objectives to specify ranges rather than one exact temporal state.

Examples include a bounded synchronization level, a phase window, a minimum degree of temporal diversity, a resonance-avoidance region, a target band of cross-frequency coupling, or a collection of viable spectral regimes.

The distinction between point targets and target regions is important because many governance objectives are tolerant rather than exact. A system can remain satisfactory across a family of temporal organizations, and forcing a single temporal configuration can impose unnecessary structural cost.

A temporal target can also concern change rather than final state. Let Δ_{II} denote a desired change operator. The corresponding target condition is represented by Equation 201.

$$\Theta^+ - \Theta^- \in \Delta_{\text{II}}, \quad (201)$$

when subtraction is meaningful in the selected local coordinates.

Equation 201 is useful for local objectives such as reducing phase dispersion, increasing a timescale, or weakening a particular coupling relation without prescribing the complete final temporal state.

8.2 Structural Realization Sets

This subsection defines the set of structural interventions capable of realizing a selected temporal objective. Its objective is to represent structural multiplicity explicitly rather than select one implementation prematurely.

Let $\mathfrak{S}$ denote the current structural system and $\mathcal{U}_I^{\text{adm}}$ the admissible family of structural interventions. For a point target Θ^* , the structural realization set is represented by Equation 202.

$$\mathfrak{L}(\Theta^* \mid \mathfrak{S}) = \left\{ \mathcal{U}_I \in \mathcal{U}_I^{\text{adm}} \mid \mathcal{T}[\mathcal{U}_I(\mathfrak{S})] = \Theta^* \right\}. \quad (202)$$

Equation 202 collects all admissible Type-I interventions that realize the selected Type-II target from the current structural system.

For a target region $\mathcal{G}_{II}$, the realization set is represented by Equation 203.

$$\mathfrak{L}(\mathcal{G}_{II} \mid \mathfrak{S}) = \left\{ \mathcal{U}_I \in \mathcal{U}_I^{\text{adm}} \mid \mathcal{T}[\mathcal{U}_I(\mathfrak{S})] \in \mathcal{G}_{II} \right\}. \quad (203)$$

Equation 203 is often the more practical governance object.

The realization set can be empty, singleton, or contain several structural interventions. These cases are represented by Equation 204.

$$|\mathfrak{L}| = 0, \quad |\mathfrak{L}| = 1, \quad |\mathfrak{L}| > 1. \quad (204)$$

Equation 204 corresponds respectively to structural non-realizability, unique realization, and multiple structural realization within the specified admissible intervention domain.

A nonempty realization set therefore establishes more than temporal desirability. It shows that the target is structurally implementable through at least one admissible intervention under the selected transformation model.

8.3 Lifting Relations

This subsection formalizes the relation between Type-II target operators and Type-I structural interventions. Its objective is to define when a structural operator realizes a temporal operator across a domain rather than at one individual system.

Let $\mathcal{V}_{II}$ denote a temporal target operator and $\mathcal{U}_I$ a structural intervention. A structural lift of $\mathcal{V}_{II}$ through transformation $\mathcal{T}$ satisfies the relation represented by Equation 205.

$$\mathcal{T} \circ \mathcal{U}_I = \mathcal{V}_{II} \circ \mathcal{T} \quad (205)$$

on the structural domain over which the lift is claimed.

Equation 205 is the structural lifting condition.

The complete lifting set of a temporal operator is represented by Equation 206.

$$\text{Lift}_{\mathcal{T}}(\mathcal{V}_{II}) = \{ \mathcal{U}_I \mid \mathcal{T} \circ \mathcal{U}_I = \mathcal{V}_{II} \circ \mathcal{T} \}. \quad (206)$$

Equation 206 defines lifting at the operator level.

This condition is stronger than pointwise target realization. A structural intervention can realize a desired temporal target at one system while failing to induce the same temporal operator across other systems in the domain.

Pointwise lifting at $\mathfrak{S}$ is represented by Equation 207.

$$\mathcal{T}[\mathcal{U}_I(\mathfrak{S})] = \mathcal{V}_{II}[\mathcal{T}(\mathfrak{S})]. \quad (207)$$

Equation 207 is sufficient when the governance problem concerns one identified structural system.

A domain-level lift requires the relation to hold for every system in a specified set $\mathcal{D} \subseteq \mathcal{M}_I$. This condition is represented by Equation 208.

$$\forall \mathfrak{S} \in \mathcal{D}, \quad \mathcal{T}[\mathcal{U}_I(\mathfrak{S})] = \mathcal{V}_{II}[\mathcal{T}(\mathfrak{S})]. \quad (208)$$

Equation 208 permits lifts whose validity is restricted to a particular regime, institution class, or local structural region.

Approximate lifting can be introduced when exact commutation is unnecessary or empirically unavailable. Let d_{II} denote a Type-II discrepancy measure. Approximate lifting is represented by Equation 209.

$$d_{II}(\mathcal{T}[\mathcal{U}_I(\mathfrak{S})], \mathcal{V}_{II}[\mathcal{T}(\mathfrak{S})]) \leq \varepsilon \quad \forall \mathfrak{S} \in \mathcal{D}. \quad (209)$$

Equation 209 defines a tolerance-dependent lifting relation.

8.4 Multiple Structural Realizations

This subsection develops the multiplicity of Type-I interventions capable of realizing the same Type-II objective. Its objective is to show why temporal equivalence does not determine a unique governance architecture.

Suppose $\mathcal{U}_I^{(1)}$ and $\mathcal{U}_I^{(2)}$ both lift the same temporal operator. Their common relation is represented by Equation 210.

$$\mathcal{T} \circ \mathcal{U}_I^{(1)} = \mathcal{V}_{II} \circ \mathcal{T} = \mathcal{T} \circ \mathcal{U}_I^{(2)}. \quad (210)$$

Equation 210 establishes temporal equivalence of the two structural interventions under the selected representation.

Their structural supports can nevertheless differ. This possibility is represented by Equation 211.

$$\Lambda_I(\mathcal{U}_I^{(1)}) \neq \Lambda_I(\mathcal{U}_I^{(2)}). \quad (211)$$

Equation 211 means that the same temporal objective can be realized through structurally different governance mechanisms.

For example, a synchronization objective can potentially be realized through rule changes, coupling changes, feedback changes, or background infrastructure. A reduction in temporal concentration can similarly arise through changes in allocation rules, relational topology, process dynamics, or access conditions.

The set of structural supports represented among all lifts of a temporal target is defined by Equation 212.

$$\mathfrak{S}_\Lambda(\mathcal{V}_{II}) = \{\Lambda_I(\mathcal{U}_I) \mid \mathcal{U}_I \in \text{Lift}_{\mathcal{T}}(\mathcal{V}_{II})\}. \quad (212)$$

Equation 212 records the structural diversity of possible realizations.

The multiplicity of lifts is normatively and operationally important because temporally equivalent interventions can distribute costs, authority, risk, reversibility, and generative consequences differently.

The transformation framework therefore separates temporal target selection from structural realization selection.

8.5 Structural Constraints on Temporal Targets

This subsection develops the structural constraints that limit which Type-II objectives can be realized. Its objective is to distinguish a mathematically describable temporal target from a structurally feasible governance target.

Let $\mathfrak{U}_I^{\text{adm}}(\mathfrak{S})$ denote the structural interventions admissible from system $\mathfrak{S}$. The structurally feasible temporal image is represented by Equation 213.

$$\mathfrak{F}_{II}(\mathfrak{S}) = \left\{ \mathcal{T}[\mathcal{U}_I(\mathfrak{S})] \mid \mathcal{U}_I \in \mathfrak{U}_I^{\text{adm}}(\mathfrak{S}) \right\}. \quad (213)$$

Equation 213 is the set of temporal states that can be reached under the currently admissible structural intervention set.

A target Θ^* is structurally feasible when the condition in Equation 214 holds.

$$\Theta^* \in \mathfrak{F}_{\text{II}}(\mathfrak{S}). \quad (214)$$

Equation 214 is equivalent to nonemptiness of the point-target realization set.

Admissibility can be decomposed into several structural constraints. Let $\mathfrak{U}^L$, $\mathfrak{U}^R$, $\mathfrak{U}^V$, and $\mathfrak{U}^G$ denote intervention sets satisfying legal-institutional, resource, viability, and generative constraints. Their intersection is represented by Equation 215.

$$\mathfrak{U}_I^{\text{adm}} = \mathfrak{U}^L \cap \mathfrak{U}^R \cap \mathfrak{U}^V \cap \mathfrak{U}^G. \quad (215)$$

Equation 215 illustrates how a structurally possible intervention can remain inadmissible within the broader governance system.

Additional constraints can be introduced according to the application. The framework does not require these four sets to exhaust admissibility.

Structural constraints can also make a temporal target feasible only from some regions of the structural domain. This dependence is represented by Equation 216.

$$\mathfrak{F}_{\text{II}}(\mathfrak{S}_1) \neq \mathfrak{F}_{\text{II}}(\mathfrak{S}_2). \quad (216)$$

Equation 216 means that structurally different systems can possess different temporal opportunity sets even when their current Type-II representations are similar.

8.6 Reachable Temporal Transformations

This subsection develops reachability at the level of temporal operators. Its objective is to identify which Type-II transformations can be induced by the available family of Type-I governance interventions.

Let $\mathfrak{U}_I^{\text{adm}}$ denote the admissible structural intervention family. The corresponding set of inducible temporal operators is represented by Equation 217.

$$\mathfrak{V}_{\text{II}}^{\text{reach}} = \left\{ \overline{\mathcal{U}}_{\text{II}} \mid \mathcal{U}_I \in \mathfrak{U}_I^{\text{adm}} \text{ and } \overline{\mathcal{U}}_{\text{II}} \text{ is well defined} \right\}. \quad (217)$$

Equation 217 defines the operator space available to governance under the current structural intervention capacity.

A temporal target operator is reachable when the condition in Equation 218 holds.

$$\mathcal{V}_{\text{II}} \in \mathfrak{V}_{\text{II}}^{\text{reach}}. \quad (218)$$

Equation 218 requires at least one admissible Type-I lift.

Reachability can also be approximate. The distance from a desired temporal operator to the reachable operator set is represented abstractly by Equation 219.

$$d_{\text{reach}}(\mathcal{V}_{\text{II}}) = \inf_{\mathcal{W} \in \mathfrak{V}_{\text{II}}^{\text{reach}}} d_{\text{op}}(\mathcal{V}_{\text{II}}, \mathcal{W}), \quad (219)$$

where d_{op} is an application-specific discrepancy between temporal operators.

Equation 219 represents how closely available structural governance can approximate a desired Type-II transformation.

This distinction is useful when exact control of the temporal system is unavailable. Governance can select a structurally realizable operator whose temporal effect remains sufficiently close to the desired target.

The reachable operator set can change when structural capacity changes. Institutional reform, new infrastructure, additional resources, or altered legal authority can enlarge or restrict $\mathfrak{V}_{\text{II}}^{\text{reach}}$.

Temporal governance capacity is therefore itself partly generated by the Type-I structural system.

8.7 Minimal Structural Realizations

This subsection develops selection among multiple lifts. Its objective is to identify structurally economical realizations of a temporal target while preserving explicit dependence on the criterion used to define minimality.

Let $c_I(\mathcal{U})$ denote a context-specific structural intervention cost. A minimum-cost realization of temporal target $\mathcal{V}_{II}$ is represented by Equation 220.

$$\mathcal{U}_I^* \in \arg \min_{\mathcal{U}_I \in \text{Lift}_{\mathcal{T}}(\mathcal{V}_{II})} c_I(\mathcal{U}_I). \quad (220)$$

Equation 220 defines minimality only with respect to the specified cost function.

Structural magnitude can supply another criterion. Let d_I denote a justified structural distance. A minimum-displacement lift is represented by Equation 221.

$$\mathcal{U}_I^* \in \arg \min_{\mathcal{U}_I \in \text{Lift}_{\mathcal{T}}(\mathcal{V}_{II})} d_I(\mathcal{S}, \mathcal{U}_I(\mathcal{S})). \quad (221)$$

Equation 221 identifies a lift that changes the current structural configuration as little as possible according to the selected metric.

These criteria need not select the same intervention. A low-cost intervention can produce larger structural displacement, while a locally minimal change can be costly or difficult to reverse.

A multidimensional intervention profile can therefore be represented by Equation 222.

$$\mathbf{C}_I(\mathcal{U}) = (c_{\text{resource}}, c_{\text{structural}}, c_{\text{latency}}, c_{\text{reversal}}, c_{\text{risk}}, c_{\text{generative}}). \quad (222)$$

Equation 222 retains several selection dimensions without requiring a universal scalar aggregation.

Minimal structural realization should therefore be interpreted as criterion-relative. The transformation framework identifies the lift set, while governance theory must supply the principles through which candidate lifts are evaluated.

8.8 Composite Lifting

This subsection develops structural lifting for temporal objectives composed of several Type-II transformations. Its objective is to represent governance architectures in which one structural intervention is insufficient or in which temporal objectives must be realized sequentially, jointly, or conditionally.

Let $\mathcal{V}_{II}^{(1)}$ and $\mathcal{V}_{II}^{(2)}$ denote two temporal target operators. Their sequential composition is represented by Equation 223.

$$\mathcal{V}_{II}^{(2:1)} = \mathcal{V}_{II}^{(2)} \circ \mathcal{V}_{II}^{(1)}. \quad (223)$$

Equation 223 applies the first temporal transformation before the second.

Suppose structural interventions $\mathcal{U}_I^{(1)}$ and $\mathcal{U}_I^{(2)}$ lift the corresponding Type-II operators. Their structural composition is represented by Equation 224.

$$\mathcal{U}_I^{(2:1)} = \mathcal{U}_I^{(2)} \circ \mathcal{U}_I^{(1)}. \quad (224)$$

Under the required commutative conditions, the composite lift satisfies Equation 225.

$$\mathcal{T} \circ \mathcal{U}_I^{(2:1)} = \mathcal{V}_{II}^{(2:1)} \circ \mathcal{T}. \quad (225)$$

Equation 225 establishes lifting of the composite temporal objective.

Composite lifting can fail even when individual targets are separately liftable. The first structural intervention can alter the structural domain in ways that make the second lift inadmissible or change the transformation context.

The sequential admissibility condition is represented by Equation 226.

$$\mathcal{U}_I^{(2)} \in \mathcal{U}_I^{\text{adm}} \left(\mathcal{U}_I^{(1)}(\mathfrak{S}) \right). \quad (226)$$

Equation 226 requires the second intervention to remain admissible after the first intervention has changed the system.

Parallel temporal objectives can also be represented. Let $\mathcal{V}_{II}^A$ and $\mathcal{V}_{II}^B$ denote two objectives that must be satisfied jointly. Their structural realization set is represented by Equation 227.

$$\mathfrak{L}(\mathcal{V}_{II}^A, \mathcal{V}_{II}^B) = \text{Lift}_{\mathcal{T}}(\mathcal{V}_{II}^A) \cap \text{Lift}_{\mathcal{T}}(\mathcal{V}_{II}^B). \quad (227)$$

Equation 227 contains structural interventions that realize both temporal objectives under the selected formulation.

The intersection can be empty even when both individual lift sets are nonempty. Such incompatibility indicates a structural conflict among temporal objectives.

Conditional lifting can be represented by allowing the selected structural intervention to depend on the current Type-II or structural state. A conditional lift architecture is represented by Equation 228.

$$\mathcal{U}_I = \mathcal{U}_I(\mathfrak{S}, \Theta, \chi), \quad (228)$$

where χ collects relevant contextual conditions.

Equation 228 permits different structural realizations of the same temporal objective across different system conditions.

Table 7 summarizes the principal lifting relations developed in this section.

The relations summarized in Table 7 establish structural lifting as the reverse governance problem associated with the Type-I–Type-II transformation. A temporal objective defines an image or operator in the Type-II domain, while its structural realization is generally a set of admissible Type-I interventions.

This multiplicity has two consequences. First, temporal feasibility does not identify one structural governance architecture. Second, selecting among temporally equivalent lifts requires criteria that extend beyond the temporal target itself. Structural cost, legal admissibility, reversibility, institutional distribution, power, and generative effects can all distinguish candidate realizations that remain equivalent within the selected Type-II representation.

The next section develops a local version of the same problem. Section 9 replaces global transformation and lifting with tangent-space relations, structural-to-temporal Jacobians, local sensitivity, null directions, and locally realizable temporal changes.

Table 7: Temporal Targets and Structural Lifting

Lifting Domain	Formal Object	Analytical Function	Governance Significance
Temporal Target	$\mathcal{V}_{\text{II}}$	Specifies a desired transformation in Type-II space	Separates temporal objective from structural implementation
Structural Realization Set	$\mathcal{L}(\Theta^* \mid \mathfrak{S})$	Collects Type-I interventions that realize a selected target	Makes structural multiplicity explicit
Operator Lift	$\text{Lift}_{\mathcal{T}}(\mathcal{V}_{\text{II}})$	Identifies structural operators inducing the temporal target operator	Connects Type-II objective with Type-I governance
Multiple Structural Realization	$ \text{Lift}_{\mathcal{T}} > 1$	Represents several structural implementations of one temporal effect	Temporally equivalent interventions can differ institutionally and normatively
Structural Feasibility	$\mathfrak{F}_{\text{II}}(\mathfrak{S})$	Defines temporal states reachable through admissible structural interventions	Separates desirable targets from implementable targets
Reachable Temporal Operators	$\mathfrak{T}_{\text{II}}^{\text{reach}}$	Defines the temporal transformations induced by current structural capacity	Represents temporal governance capability
Minimal Realization	$\arg \min_{\mathcal{U} \in \text{Lift}} c(\mathcal{U})$	Selects among multiple lifts according to a stated criterion	Supports resource-, risk-, or reversibility-sensitive intervention choice
Composite Lifting	$\mathcal{U}^{(2)} \circ \mathcal{U}^{(1)}$	Realizes compound temporal objectives through structural sequences	Captures order dependence, compatibility, and conditional implementation

9 Local and Tangent-Space Transformations

This section develops a local transformation framework connecting Type-I structural variation with Type-II spectral-temporal variation. Its objective is to identify conditions under which small structural changes can be mapped into small temporal changes even when the global structural-to-temporal transformation remains nonlinear, many-to-one, or globally non-invertible. The section introduces local representation maps, tangent spaces, structural- to-temporal Jacobians, sensitivity structure, rank conditions, null directions, local structural equivalence, and local temporal control. The resulting framework provides a formal basis for governance under incomplete global knowledge by allowing locally informative transformation relations to remain available.

9.1 Local Representation Maps

This subsection restricts the global transformation $\mathcal{T}$ to a neighborhood of a reference structural system. Its objective is to separate local transformation behavior from global representational multiplicity.

Let $\mathfrak{S}^* \in \mathcal{M}_I$ denote a reference structural system and let $\Theta^* = \mathcal{T}(\mathfrak{S}^*)$ denote its Type-II representation. A structural neighborhood of radius δ is represented by Equation 229.

$$\mathcal{N}_\delta^I(\mathfrak{S}^*) = \{\mathfrak{S} \in \mathcal{M}_I \mid d_I(\mathfrak{S}, \mathfrak{S}^*) < \delta\}, \quad (229)$$

where d_I denotes a structural distance when such a distance is available and meaningful.

The local transformation is the restriction represented by Equation 230.

$$\mathcal{T}_{\text{loc}} = \mathcal{T}|_{\mathcal{N}_\delta^I(\mathfrak{S}^*)}. \quad (230)$$

Equation 230 permits the transformation to be studied in a neighborhood where the relevant structural and temporal coordinates are sufficiently regular.

A corresponding temporal neighborhood can be defined around Θ^* . This neighborhood is represented by Equation 231.

$$\mathcal{N}_\varepsilon^{\text{II}}(\Theta^*) = \{\Theta \in \mathcal{M}_{\text{II}} \mid d_{\text{II}}(\Theta, \Theta^*) < \varepsilon\}. \quad (231)$$

Equation 231 defines the region of temporal variation treated as locally comparable with the reference system.

The local transformation can therefore be written as Equation 232.

$$\mathcal{T}_{\text{loc}} : \mathcal{N}_\delta^I(\mathfrak{S}^*) \longrightarrow \mathcal{N}_\varepsilon^{\text{II}}(\Theta^*). \quad (232)$$

Equation 232 does not imply that every point in the temporal neighborhood is reachable. It defines the local codomain within which the transformation is analyzed.

Local analysis is especially useful when global transformation contains multiple regimes, disconnected images, or structural equivalence classes that are large at the system-wide level. A local region can remain regular enough for differential analysis even when the complete transformation is strongly nonlinear.

9.2 Tangent Spaces of Structural and Temporal Representations

This subsection introduces tangent representations of small structural and temporal variations. Its objective is to construct linear spaces in which local transformation sensitivity can be analyzed.

Let $T_{\mathfrak{S}^*}\mathcal{M}_I$ denote the tangent space of admissible structural perturbations at $\mathfrak{S}^*$. A structural perturbation is represented by Equation 233.

$$\delta\mathfrak{S} \in T_{\mathfrak{S}^*}\mathcal{M}_I. \quad (233)$$

Equation 233 denotes an infinitesimal or sufficiently small structural direction within the selected local model.

Similarly, let $T_{\Theta^*}\mathcal{M}_{\text{II}}$ denote the tangent space of temporal variations around Θ^* . A temporal perturbation is represented by Equation 234.

$$\delta\Theta \in T_{\Theta^*}\mathcal{M}_{\text{II}}. \quad (234)$$

Equation 234 can contain local changes in timescales, frequencies, phases, coupling coefficients, locking measures, spectral weights, or other Type-II coordinates supported by the local representation.

The differential of the transformation at Θ^* is represented by Equation 235.

$$DT_{\Theta^*} : T_{\Theta^*}\mathcal{M}_{\text{I}} \longrightarrow T_{\Theta^*}\mathcal{M}_{\text{II}}. \quad (235)$$

Equation 235 is the tangent-space transformation between structural and temporal representations.

For sufficiently small structural perturbations, the corresponding temporal change is approximated by Equation 236.

$$\delta\Theta = DT_{\Theta^*}[\delta\Theta] + o(\|\delta\Theta\|). \quad (236)$$

Equation 236 separates the first-order temporal effect from higher-order terms.

The local framework therefore does not require a globally linear governance system. It requires only a region in which the selected transformation admits a meaningful local differential approximation.

9.3 Structural-to-Temporal Jacobians

This subsection introduces finite-dimensional coordinate representations of the tangent transformation. Its objective is to express local structural sensitivity through a Jacobian matrix whose entries connect particular Type-I and Type-II coordinates.

Let $z = (z_1, \dots, z_p)$ denote local structural coordinates and $\theta = (\theta_1, \dots, \theta_q)$ local Type-II coordinates. The local representation map is expressed by Equation 237.

$$\theta = \mathcal{T}_{\text{loc}}(z). \quad (237)$$

The structural-to-temporal Jacobian at reference point z^* is represented by Equation 238.

$$J_{\text{I} \rightarrow \text{II}}(z^*) = \left[\frac{\partial \theta_i}{\partial z_j} \right]_{\substack{i=1, \dots, q \\ j=1, \dots, p}}. \quad (238)$$

Equation 238 records the first-order effect of each selected structural coordinate on each selected temporal coordinate.

The local transformation then takes the matrix form represented by Equation 239.

$$\delta\theta \approx J_{\text{I} \rightarrow \text{II}}(z^*) \delta z. \quad (239)$$

Equation 239 is the principal local structural-to-temporal relation used in this section.

The Jacobian can be partitioned according to Type-I structural supports. Let

$$z = (z_{\text{SR}}, z_{\text{D}}, z_{\text{REL}}, z_{\text{B}}).$$

The corresponding block Jacobian is represented by Equation 240.

$$J_{\text{I} \rightarrow \text{II}} = [J_{\text{SR}} \mid J_{\text{D}} \mid J_{\text{REL}} \mid J_{\text{B}}]. \quad (240)$$

Equation 240 makes the structural location of temporal sensitivity explicit.

A complementary partition can be formed by Type-II family. Let the temporal coordinates be grouped as

$$\theta = (\theta_T, \theta_S, \theta_P, \theta_L, \theta_R, \theta_I, \theta_H, \theta_C, \theta).$$

The resulting Jacobian block $J_{\alpha\beta}$ describes the local effect of Type-I structural block β on Type-II family α .

This block structure provides a local quantitative counterpart to the joint Type-I–Type-II classification developed in the preceding papers.

9.4 Local Sensitivity Structure

This subsection develops sensitivity descriptors derived from the local Jacobian. Its objective is to distinguish structurally influential, temporally sensitive, weakly coupled, and direction-dependent local relations.

For temporal coordinate θ_i and structural coordinate z_j , the elementary local sensitivity is represented by Equation 241.

$$S_{ij} = \left. \frac{\partial \theta_i}{\partial z_j} \right|_{z=z^*}. \quad (241)$$

Equation 241 records the signed first-order effect of structural variation z_j on temporal coordinate θ_i .

When scale comparability is required, a normalized sensitivity can be introduced. One possible local elasticity is represented by Equation 242.

$$E_{ij} = \left. \frac{z_j}{\theta_i} \frac{\partial \theta_i}{\partial z_j} \right|_{z=z^*}, \quad (242)$$

provided $z_j \neq 0$ and $\theta_i \neq 0$.

Equation 242 is optional and should be used only where its scaling has substantive meaning.

The sensitivity of temporal coordinate θ_i to a structural direction v is represented by Equation 243.

$$D_v \theta_i = \nabla_z \theta_i \cdot v. \quad (243)$$

Equation 243 allows governance analysis to consider coordinated structural perturbations rather than one-coordinate changes.

A structural direction can influence several temporal coordinates at once. The corresponding temporal response vector is represented by Equation 244.

$$r(v) = J_{I \rightarrow II} v. \quad (244)$$

Equation 244 describes the local Type-II signature of the structural direction v .

The local sensitivity matrix can therefore reveal structural interventions that appear similar in Type-I magnitude while producing very different temporal consequences, and temporal objectives that are especially sensitive to a small subset of structural coordinates.

9.5 Rank and Local Identifiability

This subsection develops the relation between Jacobian rank and local structural identifiability. Its objective is to identify which structural directions can be distinguished through first-order Type-II variation.

Let the structural coordinate dimension be p and temporal coordinate dimension be q . The local transformation rank is represented by Equation 245.

$$r^* = \text{rank}(J_{I \rightarrow II}(z^*)). \quad (245)$$

Equation 245 gives the number of locally independent structural directions whose effects are represented within the selected Type-II coordinates.

When $r^* = p$ and $q \geq p$, the local Jacobian has full column rank. This condition is represented by Equation 246.

$$\text{rank}(J_{I \rightarrow \Pi}) = p. \quad (246)$$

Equation 246 is a first-order condition for local structural distinguishability within the selected coordinate model.

When

$$r^* < p,$$

at least one nonzero structural direction is invisible to first order in the selected Type-II representation. The dimension of the local invisible structural subspace follows from Equation 247.

$$\dim \ker J_{I \rightarrow \Pi} = p - r^*. \quad (247)$$

Equation 247 quantifies the number of locally unresolved structural directions in the finite-dimensional model.

The rank can also vary across structural space. This dependence is represented by Equation 248.

$$r(z) = \text{rank}(J_{I \rightarrow \Pi}(z)). \quad (248)$$

Equation 248 allows local identifiability to change as the system evolves or moves across structural regimes.

A governance system can therefore be well identified in one region and weakly identified in another even under the same observation architecture.

9.6 Null Directions

This subsection develops structural perturbations that remain invisible in the selected Type-II coordinates. Its objective is to formalize local structural change without first-order temporal change.

The local null space is represented by Equation 249.

$$\mathcal{K}_{\mathfrak{S}^*} = \ker J_{I \rightarrow \Pi}(z^*). \quad (249)$$

Equation 249 contains structural perturbations whose first-order temporal effect vanishes.

For $\delta z \in \mathcal{K}_{\mathfrak{S}^*}$, the first-order relation is represented by Equation 250.

$$J_{I \rightarrow \Pi} \delta z = 0. \quad (250)$$

Equation 250 indicates local invisibility rather than global temporal equivalence.

Higher-order effects can remain present. For a smooth transformation, the second-order expansion along a null direction is represented by Equation 251.

$$\delta \theta \approx \frac{1}{2} \delta z^\top H_{\mathcal{T}} \delta z, \quad (251)$$

where $H_{\mathcal{T}}$ collects the relevant second-order derivatives of the temporal coordinates.

Equation 251 shows that a structurally invisible first-order perturbation can become temporally visible at higher order.

Null directions can arise for several reasons. The selected temporal representation can omit the affected mode, the structural perturbation can be compensated by another structural change, the observation process can remove the resulting difference, or the current operating point can lie at a local insensitivity condition.

A null direction can therefore be specific to the current representation and operating point. A structural perturbation invisible under one Type-II coordinate set can become visible when additional temporal coordinates are added.

9.7 Locally Equivalent Structural Perturbations

This subsection develops equivalence among distinct structural perturbations that generate the same local temporal response. Its objective is to provide a tangent-space analogue of the global intervention-equivalence relation.

Let δz_1 and δz_2 denote two structural perturbations. They are locally Type-II equivalent when the condition in Equation 252 holds.

$$\delta z_1 \sim_J \delta z_2 \iff J_{I \rightarrow II} \delta z_1 = J_{I \rightarrow II} \delta z_2. \quad (252)$$

Equation 252 groups structurally distinct local interventions by their first-order Type-II consequence.

The difference between two locally equivalent perturbations lies in the null space. This relation is represented by Equation 253.

$$\delta z_1 \sim_J \delta z_2 \iff \delta z_1 - \delta z_2 \in \ker J_{I \rightarrow II}. \quad (253)$$

Equation 253 identifies the null space as the local source of structural multiplicity for a given temporal effect.

For desired local temporal change $\delta \theta^*$, the set of structural perturbations that realize the target is represented by Equation 254.

$$\mathcal{L}_{\text{loc}}(\delta \theta^*) = \{\delta z \mid J_{I \rightarrow II} \delta z = \delta \theta^*\}. \quad (254)$$

Equation 254 is the local analogue of the global structural lifting set.

If δz_0 is one solution, the full affine solution set is represented by Equation 255.

$$\mathcal{L}_{\text{loc}}(\delta \theta^*) = \delta z_0 + \ker J_{I \rightarrow II}. \quad (255)$$

Equation 255 makes local structural multiplicity explicit.

This result provides a particularly clear mathematical expression of the complementarity between Type-I and Type-II governance. A desired local temporal change generally identifies an equivalence class of structural perturbations rather than one unique structural intervention.

9.8 Local Temporal Control

This subsection develops the local reachability of desired Type-II changes. Its objective is to determine which temporal directions can be generated by small admissible Type-I interventions around the current structural system.

The image of the local Jacobian is represented by Equation 256.

$$\mathcal{R}_{\text{loc}}^{\text{II}} = \text{Im}(J_{I \rightarrow II}). \quad (256)$$

Equation 256 contains the Type-II directions reachable to first order through local structural perturbations.

A desired temporal perturbation $\delta \theta^*$ is locally reachable when the condition in Equation 257 holds.

$$\delta \theta^* \in \text{Im}(J_{I \rightarrow II}). \quad (257)$$

Equation 257 is the local analogue of global temporal reachability.

When structural interventions are restricted to an admissible tangent set $\mathcal{U}_{\text{loc}}^{\text{adm}}$, the locally admissible temporal image is represented by Equation 258.

$$\mathcal{R}_{\text{loc}}^{\text{adm}} = \left\{ J_{I \rightarrow II} \delta z \mid \delta z \in \mathcal{U}_{\text{loc}}^{\text{adm}} \right\}. \quad (258)$$

Equation 258 distinguishes mathematically reachable directions from governance-admissible directions.

A desired temporal change can therefore be locally reachable in the unconstrained tangent space and remain inaccessible under legal, resource, institutional, or generative constraints.

For an approximate target $\delta\theta^*$, a local structural intervention can be selected by solving the constrained problem represented by Equation 259.

$$\delta z^* \in \arg \min_{\delta z \in \mathcal{U}_{\text{loc}}^{\text{adm}}} \|J_{\text{I} \rightarrow \text{II}} \delta z - \delta\theta^*\|. \quad (259)$$

Equation 259 provides a local governance design problem when exact temporal realization is unavailable.

A weighted version can prioritize selected Type-II coordinates. Let W_θ denote a positive semidefinite weighting operator. The weighted local control problem is represented by Equation 260.

$$\delta z^* \in \arg \min_{\delta z \in \mathcal{U}_{\text{loc}}^{\text{adm}}} \left[(J_{\text{I} \rightarrow \text{II}} \delta z - \delta\theta^*)^\top W_\theta (J_{\text{I} \rightarrow \text{II}} \delta z - \delta\theta^*) \right]. \quad (260)$$

Equation 260 permits governance priorities to enter the local temporal design problem explicitly.

The local controller can also regularize structural magnitude. One regularized objective is represented by Equation 261.

$$\delta z^* \in \arg \min_{\delta z \in \mathcal{U}_{\text{loc}}^{\text{adm}}} \left\{ \|J_{\text{I} \rightarrow \text{II}} \delta z - \delta\theta^*\|^2 + \lambda \|\delta z\|^2 \right\}, \quad \lambda \geq 0. \quad (261)$$

Equation 261 balances temporal approximation with structural intervention magnitude.

The local formulation is especially valuable under severe information constraints. Complete structural inversion can remain unavailable while locally estimated sensitivities still support finite structural adjustments whose temporal effects can be monitored and revised.

Table 8 summarizes the principal local objects developed in this section.

The relations summarized in Table 8 show that local transformation can remain informative under conditions in which global inversion is unavailable. The Jacobian identifies which structural perturbations become visible through selected Type-II coordinates, the null space identifies locally hidden structural directions, and the image identifies locally reachable temporal directions.

The local framework also reveals an important dual multiplicity. A desired temporal perturbation can have several structurally distinct local realizations, while a structural perturbation can affect several temporal coordinates simultaneously. Local governance is therefore naturally many-to-many even before global nonlinearities are considered.

The next section develops *Approximate Inversion and Structural Reconstruction*. It extends the local transformation framework toward generalized inverse relations, regularized reconstruction, set-valued reconstruction, partial observation, uncertainty, structural constraints, and stability of the reconstructed structural representation.

Table 8: Local and Tangent-Space Structural-to-Temporal Transformations

Local Domain	Formal Object	Analytical Function	Governance Significance
Local Representation	$\mathcal{T}_{\text{loc}}$	Restricts the transformation to a structural neighborhood	Permits regular local analysis under globally complex transformation
Tangent Transformation	$D\mathcal{T}_{\mathfrak{S}^*}$	Maps structural tangent directions into temporal tangent directions	Connects small Type-I changes with small Type-II changes
Structural-to-Temporal Jacobian	$J_{\text{I} \rightarrow \text{II}}$	Represents first-order coordinate sensitivities	Identifies which structural supports influence which temporal coordinates
Sensitivity Structure	S_{ij}	Measures local effect of structural coordinate j on temporal coordinate i	Supports intervention prioritization and diagnostic analysis
Local Rank	$\text{rank}(J)$	Measures the number of structurally distinguishable local directions	Links temporal information with local structural identifiability
Null Directions	$\ker J$	Identifies structural changes invisible to first order in Type-II space	Exposes hidden structural variation and local intervention multiplicity
Local Perturbation Equivalence	$\delta z_1 \sim_J \delta z_2$	Groups structural perturbations with identical first-order temporal effects	Defines local structural realization classes
Local Temporal Reachability	$\text{Im}(J)$	Defines Type-II directions reachable through small structural change	Identifies locally feasible temporal objectives
Constrained Local Control	$\arg \min_{\delta z \in \mathcal{U}_{\text{adm}}} \ J\delta z - \delta\theta^*\ $	Selects admissible structural perturbations for desired temporal changes	Supports governance under incomplete global inversion

10 Approximate Inversion and Structural Reconstruction

This section develops reconstruction of Type-I structural information from Type-II spectral-temporal representations when an exact global inverse is unavailable, nonunique, unstable, or only partially defined. Its objective is to replace the idea of a universal inverse transform with a hierarchy of inverse relations involving approximation, generalized inverses, regularization, compatibility sets, partial observation, uncertainty, structural constraints, and stability. The section proceeds from approximate inverse relations to local generalized inverses, regularized reconstruction, set-valued reconstruction, reconstruction under partial observation, uncertainty representation, structural priors and constraint sets, and stability of reconstructed structural descriptions.

10.1 Approximate Inverse Relations

This subsection introduces approximate structural reconstruction from spectral-temporal representation. Its objective is to formalize inverse inference when exact equality between observed and reconstructed Type-II representations is stronger than the observation and model support.

Let Θ^{obs} denote an observed Type-II representation and let $\hat{\mathfrak{S}}$ denote a reconstructed structural system. An exact reconstruction would satisfy the relation represented by Equation 262.

$$\mathcal{T}(\hat{\mathfrak{S}}) = \Theta^{\text{obs}}. \quad (262)$$

Equation 262 defines consistency with the observed representation under the selected transformation.

When the observation contains estimation error or the model provides only an approximate representation, reconstruction can instead minimize discrepancy in the Type-II domain. The basic inverse problem is represented by Equation 263.

$$\hat{\mathfrak{S}} \in \arg \min_{\mathfrak{S} \in \mathcal{M}_I} d_{II}(\mathcal{T}(\mathfrak{S}), \Theta^{\text{obs}}), \quad (263)$$

where d_{II} denotes an application-specific discrepancy on the selected Type-II representation.

Equation 263 defines reconstruction as optimization over structurally admissible explanations.

The minimum achievable representation discrepancy is represented by Equation 264.

$$\varepsilon^* = \inf_{\mathfrak{S} \in \mathcal{M}_I} d_{II}(\mathcal{T}(\mathfrak{S}), \Theta^{\text{obs}}). \quad (264)$$

Equation 264 provides a model-relative measure of how closely the structural domain can reproduce the observed Type-II representation.

A positive ε^* can indicate measurement uncertainty, representational approximation, structural model misspecification, or omission of contextual variables.

For tolerance ε , the approximate inverse set is represented by Equation 265.

$$\mathcal{I}^\varepsilon(\Theta^{\text{obs}}) = \left\{ \mathfrak{S} \in \mathcal{M}_I \mid d_{II}(\mathcal{T}(\mathfrak{S}), \Theta^{\text{obs}}) \leq \varepsilon \right\}. \quad (265)$$

Equation 265 generalizes the exact compatibility set introduced in Section 5.

The size and geometry of $\mathcal{I}^\varepsilon$ depend jointly on temporal observation, transformation model, structural domain, and tolerance. Reconstruction is therefore a relation among these objects rather than an intrinsic property of the observed Type-II state alone.

10.2 Generalized Inverse Operators

This subsection develops generalized inverse relations for local structural-to-temporal maps. Its objective is to reconstruct structural perturbations from desired or observed temporal perturbations when the local Jacobian is rectangular or rank deficient.

Recall the local transformation represented by Equation 239:

$$\delta\theta \approx J_{I \rightarrow II} \delta z.$$

For a locally observed temporal change $\delta\theta^{\text{obs}}$, a generalized inverse operator J^+ can provide one structural estimate. This relation is represented by Equation 266.

$$\hat{\delta z} = J_{I \rightarrow II}^+ \delta\theta^{\text{obs}}. \quad (266)$$

Equation 266 provides one local representative of the structural changes compatible with the observed Type-II change.

When the temporal change lies in the image of the Jacobian, the reconstructed response satisfies the projection relation represented by Equation 267.

$$JJ^+ \delta\theta^{\text{obs}} = \delta\theta^{\text{obs}} \quad \text{for } \delta\theta^{\text{obs}} \in \text{Im}(J). \quad (267)$$

Equation 267 expresses exact local temporal reconstruction within the reachable temporal subspace.

When $\delta\theta^{\text{obs}}$ contains components outside $\text{Im}(J)$, the generalized inverse yields a projected approximation. This relation is represented by Equation 268.

$$\hat{\delta\theta} = JJ^+ \delta\theta^{\text{obs}}. \quad (268)$$

Equation 268 identifies the component of the observed Type-II change explainable through the selected local structural model.

Structural nonuniqueness remains whenever the Jacobian possesses a nontrivial null space. The complete local solution family is represented by Equation 269.

$$\delta z = J^+ \delta\theta + (I - J^+ J) w, \quad (269)$$

where w is an arbitrary structural vector of compatible dimension.

Equation 269 decomposes the reconstruction into one particular solution and an unresolved structural component lying in the null space.

This expression provides a local formalization of structural underdetermination. Type-II information determines the component visible through the Jacobian while leaving null-space structural variation unresolved.

A generalized inverse should therefore be interpreted as a reconstruction rule that selects one representative from a local structural compatibility class. Additional criteria are required when the selected representative carries substantive governance meaning.

10.3 Regularized Reconstruction

This subsection develops regularization for structurally ambiguous or unstable inverse problems. Its objective is to combine Type-II consistency with additional criteria that favor structurally plausible, stable, or parsimonious reconstructions.

Let $\mathcal{R}_I(\mathfrak{S})$ denote a structural regularization functional and $\lambda \geq 0$ a regularization parameter. A regularized global reconstruction is represented by Equation 270.

$$\hat{\mathfrak{S}}_\lambda \in \arg \min_{\mathfrak{S} \in \mathcal{M}_I} \left\{ d_{II}^2 \left(\mathcal{T}(\mathfrak{S}), \Theta^{\text{obs}} \right) + \lambda \mathcal{R}_I(\mathfrak{S}) \right\}. \quad (270)$$

Equation 270 balances temporal fit with a structural criterion.

Within the local linear approximation, quadratic regularization yields the problem represented by Equation 271.

$$\hat{\delta z}_\lambda = \arg \min_{\delta z} \left\{ \left\| J \delta z - \delta\theta^{\text{obs}} \right\|^2 + \lambda \left\| L \delta z \right\|^2 \right\}, \quad (271)$$

where L specifies the structural directions or combinations subject to regularization.

Equation 271 allows regularization to penalize overall structural magnitude or selected structural variations.

When the required matrices are well defined, the corresponding regularized solution is represented by Equation 272.

$$\hat{\delta z}_\lambda = \left(J^\top J + \lambda L^\top L \right)^{-1} J^\top \delta \theta^{\text{obs}}. \quad (272)$$

Equation 272 provides one explicit local reconstruction under quadratic regularization.

Regularization can encode several kinds of structural preference. A norm penalty can favor smaller structural displacement. A graph penalty can favor smooth relational change. A temporal continuity penalty can favor structural evolution close to the preceding estimate. A sparsity-oriented penalty can favor explanations involving fewer structural coordinates.

The choice of regularization is therefore substantive. It introduces information beyond the Type-II observation itself.

For this reason, the reconstructed structural system should be interpreted as conditional on both the representation evidence and the selected regularization principle.

10.4 Set-Valued Reconstruction

This subsection develops reconstruction that retains structural multiplicity. Its objective is to preserve the compatibility set when available information does not justify selecting one structural representative.

The exact reconstruction relation can be written as a set-valued inverse operator. This operator is represented by Equation 273.

$$\mathcal{T}_{\text{set}}^{-1}(\Theta) = \{ \mathfrak{S} \in \mathcal{M}_I \mid \mathcal{T}(\mathfrak{S}) = \Theta \}. \quad (273)$$

Equation 273 coincides with the exact structural compatibility set.

The approximate set-valued inverse is represented by Equation 274.

$$\mathcal{T}_\varepsilon^{-1}(\Theta) = \{ \mathfrak{S} \in \mathcal{M}_I \mid d_{\text{II}}(\mathcal{T}(\mathfrak{S}), \Theta) \leq \varepsilon \}. \quad (274)$$

Equation 274 retains all structural explanations satisfying the selected temporal tolerance.

A reconstruction can then be reported through structural projections rather than one complete system. For structural feature map Π_A , the reconstructed feature set is represented by Equation 275.

$$\hat{\mathcal{A}} = \Pi_A \left[\mathcal{T}_\varepsilon^{-1} \left(\Theta^{\text{obs}} \right) \right]. \quad (275)$$

Equation 275 expresses uncertainty only in the structural dimensions relevant to the selected feature.

This set-valued approach is particularly appropriate when several structurally different systems remain equally compatible with the Type-II representation. A single point estimate can conceal this ambiguity and create the appearance of stronger structural knowledge than the observation supports.

Set-valued reconstruction also connects directly with governance design. When every structural system in the reconstructed set admits the same decision-relevant intervention, further reconstruction can be unnecessary for that task.

10.5 Reconstruction under Partial Observation

This subsection develops structural reconstruction when the Type-II representation is constructed from only part of the system's observable trajectory. Its objective is to distinguish uncertainty created by structural multiplicity from uncertainty created by restricted observation.

Let $\mathcal{O}_P$ denote a partial observation operator. The resulting temporal representation is represented by Equation 276.

$$\Theta_P = \mathcal{Q} \circ \mathcal{O}_P \circ \mathcal{D}(\mathfrak{S}). \quad (276)$$

Equation 276 defines a partial structural-to-temporal transformation.

The corresponding structural compatibility set is represented by Equation 277.

$$\mathcal{I}_P(\Theta_P^{\text{obs}}) = \left\{ \mathfrak{S} \in \mathcal{M}_I \mid \mathcal{T}_P(\mathfrak{S}) = \Theta_P^{\text{obs}} \right\}. \quad (277)$$

Equation 277 contains structural systems compatible with the restricted temporal observation.

When a richer observation operator $\mathcal{O}_R$ contains all information available under $\mathcal{O}_P$ and adds additional channels, the compatibility sets can satisfy Equation 278.

$$\mathcal{I}_R(\Theta_R^{\text{obs}}) \subseteq \mathcal{I}_P(\Theta_P^{\text{obs}}). \quad (278)$$

Equation 278 represents structural refinement through additional observation.

Partial observation can affect different Type-I components unevenly. For example, aggregate system output can strongly constrain dynamical timescales while providing weak information about relational topology. Event timing can identify institutional cadence while leaving generative background structure largely unresolved.

The component-wise reconstruction profile can be represented by Equation 279.

$$\mathbf{I}_P = (\Pi_R[\mathcal{I}_P], \Pi_F[\mathcal{I}_P], \Pi_C[\mathcal{I}_P], \Pi_B[\mathcal{I}_P]). \quad (279)$$

Equation 279 records the remaining structural compatibility by Type-I component.

Reconstruction under partial observation therefore benefits from explicit reporting of which structural dimensions are constrained by the available Type-II evidence and which remain weakly identified.

10.6 Reconstruction Uncertainty

This subsection develops uncertainty representation for structural reconstruction. Its objective is to separate uncertainty arising from measurement, temporal representation, model specification, and structural non-identifiability.

Let Θ^{obs} be an estimated temporal representation with uncertainty descriptor $\mathcal{U}_\Theta$. The induced structural uncertainty set is represented by Equation 280.

$$\mathcal{U}_\mathfrak{S} = \{ \mathfrak{S} \in \mathcal{M}_I \mid \mathcal{T}(\mathfrak{S}) \in \mathcal{U}_\Theta \}. \quad (280)$$

Equation 280 propagates temporal uncertainty into the structural domain.

A broader reconstruction uncertainty decomposition can be represented by Equation 281.

$$\mathcal{U}_{\text{rec}} = (\mathcal{U}_{\text{meas}}, \mathcal{U}_{\text{rep}}, \mathcal{U}_{\text{model}}, \mathcal{U}_{\text{struct}}, \mathcal{U}_{\text{context}}), \quad (281)$$

where the components denote uncertainty arising from measurement, representation choice, model specification, structural nonuniqueness, and contextual conditions.

Equation 281 retains these sources as distinct analytical objects.

For a probabilistic reconstruction model, uncertainty can be represented by a conditional structural distribution. This relation is represented by Equation 282.

$$p(\mathfrak{S} \mid \Theta^{\text{obs}}, M), \quad (282)$$

where M denotes the selected structural and representational model.

Equation 282 is a generic probabilistic representation and does not prescribe a particular inferential procedure.

A point estimate can then be accompanied by uncertainty over structural features. For feature $A = \Pi_A(\mathfrak{S})$, the induced distribution is represented by Equation 283.

$$p\left(A \mid \Theta^{\text{obs}}, M\right). \quad (283)$$

Equation 283 permits decision-relevant structural inference even when full-system uncertainty remains substantial.

Uncertainty should therefore accompany reconstruction throughout the transformation framework. A precise Type-II estimate can coexist with broad structural uncertainty when the inverse relation remains many-to-one.

10.7 Structural Priors and Constraint Sets

This subsection develops the role of structural knowledge introduced before or alongside temporal reconstruction. Its objective is to formalize how legal, institutional, physical, relational, historical, and generative knowledge can reduce the inverse domain.

Let $\mathcal{C}_I$ denote the structural constraint set supported by independent knowledge. The constrained inverse set is represented by Equation 284.

$$\mathcal{I}_{\text{con}}\left(\Theta^{\text{obs}}\right) = \mathcal{I}\left(\Theta^{\text{obs}}\right) \cap \mathcal{C}_I. \quad (284)$$

Equation 284 removes temporal explanations incompatible with independently established structural conditions.

The constraint set can itself be decomposed. One representation is given by Equation 285.

$$\mathcal{C}_I = \mathcal{C}_R \cap \mathcal{C}_F \cap \mathcal{C}_C \cap \mathcal{C}_B \cap \mathcal{C}_H, \quad (285)$$

where the components constrain rule, dynamical, relational, generative-background, and historical structure.

Equation 285 illustrates how knowledge from Type-I analysis can improve inversion of Type-II observations.

In a probabilistic formulation, structural prior information can be represented by Equation 286.

$$p\left(\mathfrak{S} \mid M\right). \quad (286)$$

Equation 286 encodes structural information available before the current Type-II observation is incorporated.

The resulting structural inference combines temporal evidence with structural knowledge. This relation is represented abstractly by Equation 287.

$$p\left(\mathfrak{S} \mid \Theta^{\text{obs}}, M\right) \propto p\left(\Theta^{\text{obs}} \mid \mathfrak{S}, M\right) p\left(\mathfrak{S} \mid M\right). \quad (287)$$

Equation 287 provides a generic probabilistic expression for combining Type-II evidence with Type-I structural information.

Structural constraints also prevent the inverse problem from being treated as purely temporal. A reconstruction that reproduces the observed spectrum while violating known institutional rules, relational dependencies, or resource constraints remains structurally inadmissible.

This is another form of complementarity between the two representations. Type-II evidence constrains temporal compatibility, while Type-I knowledge constrains structural plausibility.

10.8 Reconstruction Stability

This subsection develops stability of structural reconstruction under small changes in Type-II observation. Its objective is to distinguish identifiability from practical reconstructability.

Let $\mathcal{R}$ denote a reconstruction rule mapping Type-II representations to structural estimates. This rule is represented by Equation 288.

$$\mathcal{R} : \Theta \longmapsto \widehat{\mathfrak{S}}. \quad (288)$$

Equation 288 can represent generalized inversion, regularized estimation, constrained optimization, or another selected reconstruction procedure.

A local reconstruction is stable when nearby Type-II observations produce nearby structural estimates. One local stability condition is represented by Equation 289.

$$d_I(\mathcal{R}(\Theta_1), \mathcal{R}(\Theta_2)) \leq K d_{II}(\Theta_1, \Theta_2), \quad (289)$$

for observations within a specified neighborhood and finite constant K .

Equation 289 expresses Lipschitz-type stability of the reconstruction rule.

Large values of K indicate that small temporal changes can produce large changes in the reconstructed structure. Such sensitivity can make practical reconstruction unreliable even when the inverse is formally unique.

For local linear reconstruction, singular values of the Jacobian provide a useful sensitivity description. Let the singular values be represented by Equation 290.

$$\sigma_1 \geq \sigma_2 \geq \dots \geq \sigma_r > 0. \quad (290)$$

Equation 290 describes the nonzero local sensitivity directions of the structural-to-temporal map.

A small minimum nonzero singular value implies strong amplification of observation error along the corresponding inverse direction. A local conditioning descriptor is represented by Equation 291.

$$\kappa_J = \frac{\sigma_{\max}}{\sigma_{\min}}, \quad (291)$$

when the relevant nonzero singular values are defined.

Equation 291 provides one local measure of inverse sensitivity.

Structural reconstruction can therefore exhibit three different limitations: nonuniqueness, instability, and model mismatch. These limitations should be reported separately.

A useful reconstruction profile is represented by Equation 292.

$$\mathbf{Q}_{\text{rec}} = (Q_{\text{fit}}, Q_{\text{id}}, Q_{\text{stab}}, Q_{\text{unc}}, Q_{\text{adm}}), \quad (292)$$

where the components denote temporal fit, identifiability, stability, uncertainty, and structural admissibility.

Equation 292 retains several dimensions of reconstruction quality without collapsing them into a universal scalar score.

Table 9 summarizes the principal inverse relations developed in this section.

The relations summarized in Table 9 show that inversion between Type-II and Type-I representations is generally a reconstruction problem rather than an ordinary reverse transform. Exact structural recovery requires restrictive conditions. Approximate inversion, set-valued reconstruction, regularization, and structural constraints provide more general interfaces between the two representational domains.

A further distinction follows from this analysis. Identifiability concerns whether the available representation distinguishes structural alternatives. Stability concerns how strongly reconstruction changes when the observed representation changes. A structural system can therefore be uniquely identifiable within a model and remain difficult to reconstruct reliably when the inverse relation is poorly conditioned.

The next section develops *Information Preservation across Representations*. It examines which structural distinctions survive the transformation into Type-II space, which temporal distinctions are lost through coarse-graining, how multiple representations can provide complementary information, and when a representation preserves sufficient information for a specified governance task.

Table 9: Approximate Inversion and Structural Reconstruction

Reconstruction Domain	Formal Object	Analytical Function	Principal Limitation
Approximate Inverse	$\arg \min_{\mathfrak{S}} d_{\Pi}(\mathcal{T}(\mathfrak{S}), \Theta^{\text{obs}})$	Finds structural explanations approximating the observed Type-II representation	Several structures can attain similar temporal fit
Generalized Inverse	$J^+ \delta \theta$	Provides a local structural representative from Type-II variation	Null-space structural components remain unresolved
Regularized Reconstruction	Temporal fit plus structural penalty	Stabilizes inversion and introduces structural preference	Result depends on the selected regularization principle
Set-Valued Reconstruction	$\mathcal{T}_{\varepsilon}^{-1}(\Theta)$	Retains all structurally compatible explanations	Can remain too broad for some governance decisions
Partial-Observation Reconstruction	$\mathfrak{I}_P(\Theta_P)$	Reconstructs structure from restricted observation channels	Unobserved structural dimensions can remain weakly constrained
Reconstruction Uncertainty	$\mathcal{U}_{\mathfrak{S}}$ or $p(\mathfrak{S} \mid \Theta, M)$	Represents structural uncertainty induced by temporal evidence and model choice	Requires explicit uncertainty and model assumptions
Structural Constraints	$\mathfrak{I}(\Theta) \cap \mathcal{C}_I$	Combines Type-II evidence with independent Type-I knowledge	Inference remains conditional on the validity of the constraint set
Reconstruction Stability	$d_I(\mathcal{R}(\Theta_1), \mathcal{R}(\Theta_2)) \leq K d_{\Pi}(\Theta_1, \Theta_2)$	Evaluates sensitivity of structural estimates to temporal perturbation	A unique inverse can remain operationally unstable

11 Information Preservation across Representations

This section develops the information-theoretic structure of transformation between Type-I structural and Type-II spectral-temporal representations without assuming a universal scalar measure of information. Its objective is to identify which distinctions are preserved, which are collapsed, how coarse-graining and temporal resolution alter structural distinguishability, how complementary representations can recover distinctions lost by individual representations, and when a representation preserves sufficient information for a specified governance task. The section uses equivalence partitions, compatibility sets, and task-relative distinctions as its principal formal objects. This approach keeps information preservation tied to the representational and governance problems developed in the preceding sections.

11.1 Representational Information Content

This subsection develops a structural notion of information content induced by a representation. Its objective is to compare representations according to the structural distinctions they preserve without requiring a universal entropy measure or probability distribution over the structural domain.

Let $\mathcal{T}$ be a structural-to-temporal transformation defined over $\mathcal{M}_I$. As established in Section 4, the transformation induces the partition represented by Equation 293.

$$\mathcal{P}_{\mathcal{T}} = \{[\mathfrak{S}]_{\mathcal{T}} : \mathfrak{S} \in \mathcal{M}_I\}. \quad (293)$$

Equation 293 represents the structural distinctions preserved by the selected representation.

Two systems lying in different cells of $\mathcal{P}_{\mathcal{T}}$ remain distinguishable through the representation. Two systems lying in the same cell are structurally distinct possibilities that the representation does not separate.

The information content of a representation can therefore be compared through partition refinement. Let $\mathcal{T}_1$ and $\mathcal{T}_2$ be two representations over the same structural domain. The representational-information preorder is defined by Equation 294.

$$\mathcal{T}_2 \succeq_{\text{info}} \mathcal{T}_1 \iff \mathcal{P}_{\mathcal{T}_2} \preceq \mathcal{P}_{\mathcal{T}_1}. \quad (294)$$

Equation 294 means that $\mathcal{T}_2$ preserves every structural distinction preserved by $\mathcal{T}_1$ and possibly additional distinctions.

Strictly greater structural distinguishability is represented by Equation 295.

$$\mathcal{T}_2 \succ_{\text{info}} \mathcal{T}_1 \iff \mathcal{P}_{\mathcal{T}_2} \prec \mathcal{P}_{\mathcal{T}_1}. \quad (295)$$

Equation 295 identifies a representation that subdivides at least one structural equivalence class preserved by the coarser representation.

Two representations can also be incomparable. This relation occurs when each preserves some distinctions that the other collapses. Representational incomparability is expressed by Equation 296.

$$\mathcal{P}_{\mathcal{T}_1} \not\preceq \mathcal{P}_{\mathcal{T}_2} \quad \text{and} \quad \mathcal{P}_{\mathcal{T}_2} \not\preceq \mathcal{P}_{\mathcal{T}_1}. \quad (296)$$

Equation 296 is important because richer representation cannot always be described by one scalar ordering. A phase-sensitive representation can distinguish systems that remain equivalent under spectral amplitude, while another representation can preserve slow-timescale information that the phase-sensitive representation omits.

Representational information content is therefore naturally partially ordered. The relevant question concerns which structural distinctions a representation preserves for the governance task under consideration.

11.2 Structural Information Loss

This subsection develops structural information loss across the forward transformation. Its objective is to identify how several Type-I systems can collapse into the same Type-II representation and to localize the stage at which the loss occurs.

The complete transformation chain is recalled by Equation 297.

$$\mathcal{M}_I \xrightarrow{\mathcal{D}} \mathfrak{X} \xrightarrow{\mathcal{O}} \mathcal{Y} \xrightarrow{\mathcal{Q}} \mathcal{M}_{II}. \quad (297)$$

Equation 297 contains three transformations at which distinguishability can be reduced.

For a structural system $\mathfrak{S}$, the terminal structural ambiguity produced by the complete transformation is represented by its equivalence class in Equation 298.

$$\mathcal{L}_I(\mathfrak{S}; \mathcal{T}) = [\mathfrak{S}]_{\mathcal{T}}. \quad (298)$$

Equation 298 treats structural information loss as the set of structural alternatives that become indistinguishable from the reference system.

The source of that ambiguity can be decomposed through stage-specific relations. Let $\mathcal{P}_{\mathcal{D}}$, $\mathcal{P}_{\mathcal{O} \circ \mathcal{D}}$, and $\mathcal{P}_{\mathcal{T}}$ denote the partitions induced after dynamical realization, observation, and temporal representation. Their successive relation is represented by Equation 299.

$$\mathcal{P}_{\mathcal{D}} \preceq \mathcal{P}_{\mathcal{O} \circ \mathcal{D}} \preceq \mathcal{P}_{\mathcal{T}}. \quad (299)$$

Equation 299 expresses progressive coarsening when each subsequent deterministic transformation acts only on the information supplied by the preceding stage.

The first transition can lose structural information because different structural systems generate the same realized trajectory. The second can lose information because observation aggregates or omits distinctions present in the trajectories. The third can lose information because the selected Type-II representation retains only particular temporal properties of the observed process.

When the structural domain admits a meaningful metric d_I , one possible descriptor of residual structural ambiguity is the equivalence-class diameter. This quantity is represented by Equation 300.

$$D_{\mathcal{T}}(\mathfrak{S}) = \sup_{\mathfrak{S}_1, \mathfrak{S}_2 \in [\mathfrak{S}]_{\mathcal{T}}} d_I(\mathfrak{S}_1, \mathfrak{S}_2). \quad (300)$$

Equation 300 provides a metric-dependent measure of the structural range collapsed by the representation.

The substantive composition of the class remains important even when such a metric is available. A class containing nearby parameter variants differs from a class containing systems with distinct rules, relational structures, or generative backgrounds.

Structural information loss should therefore be reported through the equivalence structure and relevant Type-I projections rather than through one universal scalar quantity.

11.3 Temporal Information Loss

This subsection develops information loss within the temporal representation stage itself. Its objective is to distinguish compression of observed temporal behavior from structural ambiguity generated earlier in the transformation chain.

Let $y(\cdot) \in \mathcal{Y}$ denote an observed process and $\mathcal{Q}$ a Type-II representation operator. Two observed processes are equivalent under $\mathcal{Q}$ when the condition in Equation 301 holds.

$$y_1 \sim_{\mathcal{Q}} y_2 \iff \mathcal{Q}[y_1] = \mathcal{Q}[y_2]. \quad (301)$$

Equation 301 defines the temporal distinctions removed by the representation operator.

The temporal equivalence class associated with observed process y is represented by Equation 302.

$$[y]_{\mathcal{Q}} = \{y' \in \mathcal{Y} \mid \mathcal{Q}[y'] = \mathcal{Q}[y]\}. \quad (302)$$

Equation 302 contains observed temporal histories that share the same selected Type-II description.

Examples of temporal information loss can occur when a representation retains power spectrum while discarding phase, retains mean recurrence interval while discarding inter-event variability, or retains one dominant mode while suppressing weaker modes.

A partial Type-II representation can be described through projection from a richer temporal representation. Let Θ^{full} denote the richer object and $\Pi_{\mathcal{Q}}$ a temporal projection. The reduced representation is represented by Equation 303.

$$\Theta^{\text{red}} = \Pi_{\mathcal{Q}} \left(\Theta^{\text{full}} \right). \quad (303)$$

Equation 303 makes explicit which temporal coordinates are discarded.

The information lost under the projection can be represented abstractly by the unresolved fiber defined in Equation 304.

$$\mathcal{F}_{\Pi_{\mathcal{Q}}} \left(\Theta^{\text{red}} \right) = \left\{ \Theta^{\text{full}} \mid \Pi_{\mathcal{Q}} \left(\Theta^{\text{full}} \right) = \Theta^{\text{red}} \right\}. \quad (304)$$

Equation 304 contains the richer temporal descriptions that remain compatible with the reduced representation.

The same Type-II family can therefore be represented at several informational resolutions. Frequency content alone, frequency plus amplitude, frequency plus phase, and frequency plus cross-modal relations preserve progressively different temporal distinctions.

The choice of representation should follow the governance task rather than a presumption that maximal temporal detail is always desirable.

11.4 Coarse-Graining and Aggregation

This subsection develops coarse-graining as an explicit transformation of structural, observational, or temporal information. Its objective is to show how aggregation can simplify governance representation while altering distinguishability and inverse reconstruction.

Let $\mathcal{G}$ denote a coarse-graining operator acting on an observed or represented temporal object. The coarse representation is represented by Equation 305.

$$\Theta^{\text{coarse}} = \mathcal{G} \left(\Theta^{\text{fine}} \right). \quad (305)$$

Equation 305 maps several fine-scale states into a lower-resolution representation.

The structural transformation induced by coarse-graining is represented by Equation 306.

$$\mathcal{T}_{\text{coarse}} = \mathcal{G} \circ \mathcal{T}_{\text{fine}}. \quad (306)$$

Equation 306 makes the relation between fine and coarse representations explicit.

The corresponding structural partitions satisfy the relation represented by Equation 307.

$$\mathcal{P}_{\mathcal{T}_{\text{fine}}} \preceq \mathcal{P}_{\mathcal{T}_{\text{coarse}}}. \quad (307)$$

Equation 307 means that coarse-graining can preserve existing equivalences or merge previously distinguishable structural classes.

Temporal aggregation provides a common example. For aggregation window Δ , the averaged process is represented by Equation 308.

$$\bar{y}_{\Delta}(t) = \frac{1}{\Delta} \int_{t-\Delta}^t y(s) ds. \quad (308)$$

Equation 308 suppresses variation whose timescale is short relative to the aggregation window.

Aggregation can also be structural. A relational network can be represented through communities, an institutional hierarchy through organizational levels, or several decision processes through one aggregate policy cycle. Such coarse structural models can generate useful temporal representations while removing distinctions relevant to local intervention.

Coarse-graining therefore involves a tradeoff between tractability and distinguishability. A representation can become easier to observe, compute, and communicate while increasing the size of the structural compatibility set.

The appropriate coarse-graining level depends on the governance action to be supported.

11.5 Resolution and Distinguishability

This subsection develops the relation between temporal resolution and structural distinguishability. Its objective is to connect sampling, observation horizon, and time-frequency localization with the refinement of structural equivalence classes.

Let ρ denote a generic resolution parameter that can include sampling interval, temporal window, spectral bandwidth, or localization scale. The resolution-dependent transformation is represented by Equation 309.

$$\mathcal{T}_\rho : \mathcal{M}_I \longrightarrow \mathcal{M}_{II}^{(\rho)}. \quad (309)$$

Equation 309 makes structural distinguishability dependent on the resolution at which temporal information is represented.

For two resolutions ρ_1 and ρ_2 , a genuine refinement relation requires the higher-resolution representation to preserve the lower-resolution information while adding new distinctions. This condition is represented by Equation 310.

$$\mathcal{P}_{\mathcal{T}_{\rho_2}} \preceq \mathcal{P}_{\mathcal{T}_{\rho_1}}. \quad (310)$$

Equation 310 defines resolution refinement in terms of structural distinguishability rather than nominal sampling density alone.

A nominally finer measurement does not guarantee Equation 310. Increased noise, changed pre-processing, shorter observation windows, or different representation methods can remove distinctions preserved by the coarser representation.

Time-frequency analysis makes this issue especially visible because temporal and spectral localization depend on the representation and selected window [2, 3].

A structural pair can therefore possess a distinguishability scale. Let $\rho^*(\mathfrak{S}_1, \mathfrak{S}_2)$ denote a threshold representation scale at which the pair first becomes distinguishable within a specified ordered family of representations. This threshold is represented abstractly by Equation 311.

$$\rho^*(\mathfrak{S}_1, \mathfrak{S}_2) = \inf \{ \rho : \mathcal{T}_\rho(\mathfrak{S}_1) \neq \mathcal{T}_\rho(\mathfrak{S}_2) \}. \quad (311)$$

Equation 311 is meaningful only when the resolution family possesses an appropriate ordering.

The concept nevertheless captures an important governance point. Some structural distinctions become visible only after observations reach the temporal scale at which their effects are expressed.

Increasing resolution therefore has value when the newly exposed distinction can alter inference or intervention. Resolution beyond the task-relevant threshold can add cost without changing the governance decision.

11.6 Complementary Representations

This subsection develops complementarity among Type-II representations. Its objective is to distinguish representations that duplicate the same structural distinctions from representations that expose different dimensions of the structural system.

Let $\mathcal{T}_1$ and $\mathcal{T}_2$ be two transformations over the same structural domain. Their joint equivalence class is represented by Equation 312.

$$[\mathfrak{S}]_{1,2} = [\mathfrak{S}]_{\mathcal{T}_1} \cap [\mathfrak{S}]_{\mathcal{T}_2}. \quad (312)$$

Equation 312 contains structural systems compatible with both representations simultaneously.

The second representation contributes additional structural information at $\mathfrak{S}$ when the relation in Equation 313 holds.

$$[\mathfrak{S}]_{1,2} \subsetneq [\mathfrak{S}]_{\mathcal{T}_1}. \quad (313)$$

Equation 313 defines complementarity relative to the first representation and the reference structural system.

Mutual complementarity occurs when each representation distinguishes structural alternatives that the other collapses. This condition is represented by Equation 314.

$$\mathcal{P}_{\mathcal{T}_1} \not\subseteq \mathcal{P}_{\mathcal{T}_2} \quad \text{and} \quad \mathcal{P}_{\mathcal{T}_2} \not\subseteq \mathcal{P}_{\mathcal{T}_1}. \quad (314)$$

Equation 314 describes representations that preserve different structural distinctions.

For example, a frequency representation can distinguish systems by modal content while a phase-sensitive representation distinguishes systems by temporal alignment. A local time-frequency representation can expose regime change that a global spectrum compresses. A cross-frequency representation can distinguish systems that share marginal spectra while differing in intermodal dependence.

Complementarity therefore differs from simple richness. A representation can contain many temporal coordinates while adding little structural distinguishability if those coordinates are redundant with information already available.

The value of a complementary representation lies in the structural alternatives it removes from the compatibility set.

11.7 Multi-Representation Fusion

This subsection develops the combination of several representations into a joint structural inference. Its objective is to formalize how multiple spectral-temporal views can be integrated while preserving their distinct observation and uncertainty structures.

Let $\mathcal{T}_1, \dots, \mathcal{T}_m$ denote several structural-to-temporal transformations. The fused representation is represented by Equation 315.

$$\Theta_{\text{fuse}}(\mathfrak{S}) = (\mathcal{T}_1(\mathfrak{S}), \dots, \mathcal{T}_m(\mathfrak{S})). \quad (315)$$

Equation 315 preserves all component representations as separate coordinates.

The exact fused structural compatibility set is represented by Equation 316.

$$\mathcal{I}_{\text{fuse}} = \bigcap_{k=1}^m \mathcal{I}_k \left(\Theta_k^{\text{obs}} \right). \quad (316)$$

Equation 316 contains structural systems consistent with every included representation.

When each representation has its own tolerance ε_k , an approximate fused set is represented by Equation 317.

$$\mathcal{I}_{\text{fuse}}^\varepsilon = \bigcap_{k=1}^m \left\{ \mathfrak{S} : d_k \left(\mathcal{T}_k(\mathfrak{S}), \Theta_k^{\text{obs}} \right) \leq \varepsilon_k \right\}. \quad (317)$$

Equation 317 allows each representation to retain its own uncertainty model.

A weighted reconstruction can instead combine representation discrepancies. One such objective is represented by Equation 318.

$$\hat{\mathfrak{S}} \in \arg \min_{\mathfrak{S} \in \mathcal{M}_I} \sum_{k=1}^m w_k d_k^2 \left(\mathcal{T}_k(\mathfrak{S}), \Theta_k^{\text{obs}} \right), \quad w_k \geq 0. \quad (318)$$

Equation 318 permits differences in reliability, relevance, or measurement precision among representations to be expressed explicitly.

Fusion can also expose inconsistency. If the exact compatibility sets have an empty intersection, the condition in Equation 319 holds.

$$\bigcap_{k=1}^m \mathcal{J}_k \left(\Theta_k^{\text{obs}} \right) = \emptyset. \quad (319)$$

Equation 319 indicates that the representations, transformation assumptions, structural model, or observations cannot all be simultaneously satisfied.

Such inconsistency should be diagnostically useful. It can reveal nonstationarity, changing structural conditions, observation error, or model misspecification.

Multi-representation fusion therefore supports both refinement and model criticism.

11.8 Sufficient Representations for Governance Tasks

This subsection develops task-relative sufficiency as the principal governance criterion for representational adequacy. Its objective is to determine when a representation preserves all structural distinctions that can alter a specified governance decision, even when complete structural reconstruction remains unavailable.

Let $\mathcal{A}$ denote a governance task and let $\mathcal{D}_{\mathcal{A}}$ be a decision rule mapping structural systems to the governance action appropriate under the selected normative and operational framework. This decision rule is represented by Equation 320.

$$\mathcal{D}_{\mathcal{A}} : \mathcal{M}_I \longrightarrow \mathfrak{A}_{\mathcal{A}}, \quad (320)$$

where $\mathfrak{A}_{\mathcal{A}}$ denotes the admissible decision or intervention set for task $\mathcal{A}$.

A Type-II representation $\mathcal{T}$ is sufficient for governance task $\mathcal{A}$ when all structurally equivalent systems under the representation imply the same task-relevant decision. This condition is represented by Equation 321.

$$\mathcal{T}(\mathfrak{S}_1) = \mathcal{T}(\mathfrak{S}_2) \implies \mathcal{D}_{\mathcal{A}}(\mathfrak{S}_1) = \mathcal{D}_{\mathcal{A}}(\mathfrak{S}_2). \quad (321)$$

Equation 321 defines task-relative representational sufficiency.

The same condition can be expressed over structural compatibility sets. For every observed temporal representation Θ , sufficiency requires the decision image of the compatibility set to be a singleton. This condition is represented by Equation 322.

$$|\{\mathcal{D}_{\mathcal{A}}(\mathfrak{S}) \mid \mathfrak{S} \in \mathcal{J}(\Theta)\}| = 1. \quad (322)$$

Equation 322 permits governance to proceed without complete structural identification.

This definition can also be intervention specific. For contemplated structural intervention $\mathcal{U}$, a representation is sufficient for predicting the selected Type-II consequence when the condition in Equation 323 holds.

$$\mathfrak{S}_1 \sim_{\mathcal{T}} \mathfrak{S}_2 \implies \mathcal{T}[\mathcal{U}(\mathfrak{S}_1)] = \mathcal{T}[\mathcal{U}(\mathfrak{S}_2)]. \quad (323)$$

Equation 323 reproduces the representation-preserving condition of Section 7 as a task-specific information criterion.

A representation can therefore be insufficient for complete reconstruction and sufficient for one governance decision. Another decision can require a finer representation because previously irrelevant structural distinctions become consequential.

Let $\mathcal{P}_{\mathcal{A}}$ denote the partition of the structural domain induced by the task decision rule. This task partition is represented by Equation 324.

$$\mathcal{P}_{\mathcal{A}} = \{\mathcal{D}_{\mathcal{A}}^{-1}(a) : a \in \mathfrak{A}_{\mathcal{A}}\}. \quad (324)$$

Equation 324 groups structural systems that lead to the same governance decision.

Representational sufficiency can then be stated through partition refinement. The required relation is represented by Equation 325.

$$\mathcal{P}_{\mathcal{T}} \preceq \mathcal{P}_{\mathcal{A}}. \quad (325)$$

Equation 325 means that the representation distinguishes systems at least as finely as required by the governance task.

This condition provides a practical stopping rule for representation refinement. Additional observation, resolution, or temporal coordinates are unnecessary for the specified decision once every remaining structural ambiguity lies within a single task-equivalent class.

The result also avoids equating maximal information with optimal governance. Higher-resolution representation can consume observation, computation, time, and institutional resources without changing the decision. Under limited governance capacity, the appropriate representation can therefore be the least demanding representation that remains sufficient for the task.

Table 10 summarizes the principal information relations developed in this section.

The relations summarized in Table 10 show that representational information is best understood relative to the distinctions required by inference and governance. A representation can preserve substantial temporal detail while collapsing structural distinctions that matter for intervention. A comparatively coarse representation can also be sufficient when all unresolved structural alternatives imply the same governance action.

The resulting framework therefore separates three objectives: maximizing structural distinguishability, reconstructing a selected structural feature, and preserving enough information for a specified governance task. These objectives can require different observation architectures and different Type-II representations.

The next section develops *Transformation under Nonstationarity and Structural Evolution*. It examines how the transformation operator, equivalence classes, structural compatibility sets, and local sensitivity relations change when the structural system and its representational environment evolve through time.

Table 10: Information Preservation across Structural and Spectral-Temporal Representations

Information Domain	Formal Object	Representational Function	Governance Significance
Representational Content	$\mathcal{P}_{\mathcal{T}}$	Records structural distinctions preserved by a representation	Provides a non-scalar comparison of representational informativeness
Structural Information Loss	$[\mathfrak{S}]_{\mathcal{T}}$	Collects structural alternatives collapsed by the transformation	Defines residual Type-I ambiguity after Type-II representation
Temporal Information Loss	$[y]_{\mathcal{Q}}$	Collects observed temporal processes sharing one Type-II description	Identifies distinctions removed by temporal representation
Coarse-Graining	$\mathcal{T}_{\text{coarse}} = \mathcal{G} \circ \mathcal{T}_{\text{fine}}$	Reduces representational detail	Trades structural distinguishability for tractability
Resolution	$\mathcal{T}_{\rho}$	Controls temporal distinctions accessible to representation	Determines the scale at which structural differences become visible
Complementary Representation	$[\mathfrak{S}]_1 \cap [\mathfrak{S}]_2$	Combines distinct structural distinctions preserved by different views	Reduces ambiguity that remains under either representation separately
Multi-Representation Fusion	$\bigcap_k \mathcal{J}_k$	Integrates several temporal representations into one structural inference	Supports refinement and detection of representational inconsistency
Task Sufficiency	$\mathcal{P}_{\mathcal{T}} \preceq \mathcal{P}_{\mathcal{A}}$	Preserves all structural distinctions relevant to a selected decision	Allows governance without complete structural reconstruction

12 Transformation under Nonstationarity and Structural Evolution

This section extends the transformation framework to systems whose structural organization, generated dynamics, observation architecture, or spectral-temporal representation changes through time. Its objective is to replace a fixed transformation $\mathcal{T} : \mathcal{M}_I \rightarrow \mathcal{M}_{II}$ with an evolving family of transformations whose domains, images, equivalence classes, sensitivities, and reconstruction properties can change along the history of the governed system. The section develops time-varying structural systems, time-varying representation operators, moving equivalence classes, regime-dependent transformations, structural change accompanied by temporal persistence, temporal change accompanied by structural persistence, transformation across critical transitions, and historical dependence. Nonstationary spectral analysis and nonlinear dynamical systems provide formal resources for representing several of these relations [12, 2, 5].

12.1 Time-Varying Structural Systems

This subsection develops the structural side of an evolving transformation. Its objective is to represent governance systems whose state spaces, rules, dynamics, relations, or generative backgrounds change during the observation and intervention horizon.

A time-varying Type-I structural system is represented by Equation 326.

$$\mathfrak{S}_t = (X_t, x_t, R_t, F_t, C_t, \mathcal{B}_t). \quad (326)$$

Equation 326 allows every structural component to evolve through time.

Structural evolution can itself be generated through a higher-order process. A generic structural evolution law is represented by Equation 327.

$$\dot{\mathfrak{S}}_t = \mathcal{G}(\mathfrak{S}_t, x_t, u_t, \chi_t), \quad (327)$$

where u_t denotes intervention or external input and χ_t collects relevant contextual conditions.

Equation 327 treats structural change as part of the system history rather than as a sequence of unrelated static models.

For discrete institutional change, the evolution can instead be represented by Equation 328.

$$\mathfrak{S}_{k+1} = \mathcal{G}_k(\mathfrak{S}_k, e_k, u_k), \quad (328)$$

where e_k denotes an event capable of triggering structural revision.

Equation 328 accommodates legal reform, organizational restructuring, resource reallocation, network rewiring, institutional learning, and other event-centered structural changes.

The structural domain can also evolve. The admissible Type-I domain at time t is represented by Equation 329.

$$\mathcal{M}_I(t) = \{\mathfrak{S} : \mathfrak{S} \text{ is admissible under the structural conditions at time } t\}. \quad (329)$$

Equation 329 permits available institutional forms, resources, legal capacities, or relational structures to change through history.

This evolving-domain formulation matters for lifting and reachability. An intervention unavailable at time t_1 can become admissible at t_2 , while an earlier structural realization can become inaccessible after institutional or infrastructural change.

Structural evolution therefore changes both the system generating temporal behavior and the space of governance actions available to modify that behavior.

12.2 Time-Varying Representation Operators

This subsection develops transformations whose observation and spectral-temporal representation components evolve through time. Its objective is to distinguish change in the governed system from change in the means through which that system becomes temporally represented.

The time-dependent structural-to-temporal relation is represented by Equation 330.

$$\Theta_t = \mathcal{T}_t(\mathfrak{S}_t). \quad (330)$$

Equation 330 permits both the structural input and the transformation itself to change.

The composite transformation can be decomposed according to Equation 331.

$$\mathcal{T}_t = \mathcal{Q}_t \circ \mathcal{O}_t \circ \mathcal{D}_t. \quad (331)$$

Equation 331 identifies three potentially evolving stages: dynamical realization, observation, and temporal representation.

A change in $\mathcal{O}_t$ can result from new sensors, altered reporting procedures, revised administrative categories, changed sampling cadence, or expansion of observable variables. A change in $\mathcal{Q}_t$ can result from altered temporal resolution, movement from global to local spectral representation, addition of phase or coupling coordinates, or revision of the model used to define spectral regimes.

For nonstationary processes, a local representation can be indexed explicitly by time. A generic local temporal representation is given by Equation 332.

$$Z(t, \omega) = A(t, \omega) e^{i\phi(t, \omega)}. \quad (332)$$

Equation 332 permits local spectral amplitude and phase to evolve rather than requiring one invariant global spectrum. Nonstationary spectral analysis and time-frequency methods provide established foundations for such localized representations [12, 2].

Changes in representation must be separated from changes in the system. Suppose the structural system remains fixed while the representation changes. This case is represented by Equation 333.

$$\mathfrak{S}_{t_1} = \mathfrak{S}_{t_2}, \quad \mathcal{T}_{t_1} \neq \mathcal{T}_{t_2}. \quad (333)$$

Equation 333 allows two different Type-II descriptions to arise from an unchanged structural system because the observation or representation architecture has changed.

This distinction is essential for historical comparison. Apparent temporal change across archival periods can partly reflect changes in measurement, recording, aggregation, or analytic representation.

12.3 Moving Equivalence Classes

This subsection develops representation equivalence when the transformation changes through time. Its objective is to represent structural indistinguishability as a moving relation whose classes can merge, split, or change composition.

At time t , structural equivalence is represented by Equation 334.

$$\mathfrak{S}_1 \sim_{\mathcal{T}_t} \mathfrak{S}_2 \iff \mathcal{T}_t(\mathfrak{S}_1) = \mathcal{T}_t(\mathfrak{S}_2). \quad (334)$$

Equation 334 defines a time-indexed partition of the structural domain.

The corresponding equivalence class is represented by Equation 335.

$$[\mathfrak{S}]_{\mathcal{T}_t} = \{\mathfrak{S}' \in \mathcal{M}_1(t) \mid \mathcal{T}_t(\mathfrak{S}') = \mathcal{T}_t(\mathfrak{S})\}. \quad (335)$$

Equation 335 allows structural compatibility to evolve even for a fixed reference system.

An equivalence class splits between t_1 and t_2 when systems previously indistinguishable become distinguishable. This condition is represented by Equation 336.

$$\mathfrak{S}_1 \sim_{\mathcal{T}_{t_1}} \mathfrak{S}_2, \quad \mathfrak{S}_1 \not\sim_{\mathcal{T}_{t_2}} \mathfrak{S}_2. \quad (336)$$

Equation 336 can occur because the systems themselves evolve differently or because the later representation reveals distinctions previously hidden.

The complementary merging relation is represented by Equation 337.

$$\mathfrak{S}_1 \not\sim_{\mathcal{T}_{t_1}} \mathfrak{S}_2, \quad \mathfrak{S}_1 \sim_{\mathcal{T}_{t_2}} \mathfrak{S}_2. \quad (337)$$

Equation 337 identifies loss of distinguishability through system convergence, aggregation, changed observation, or representational compression.

The partition itself can therefore be represented as a time-dependent object. This relation is given by Equation 338.

$$\mathcal{P}_t = \mathcal{P}_{\mathcal{T}_t}. \quad (338)$$

Equation 338 provides a compact representation of changing structural distinguishability.

Moving equivalence classes have an epistemic consequence. Structural inference valid at one historical time can become obsolete after the transformation or structural domain changes. Identifiability should therefore be indexed by the period and representation conditions under which it was established.

12.4 Regime-Dependent Transformations

This subsection develops transformations whose local behavior depends on the system regime. Its objective is to represent changes in structural-to-temporal mapping across different dynamical or spectral regions.

Let $\rho \in \mathcal{R}$ denote a regime label. A regime-conditioned transformation is represented by Equation 339.

$$\mathcal{T}^{(\rho)} : \mathcal{M}_I^{(\rho)} \longrightarrow \mathcal{M}_{II}^{(\rho)}. \quad (339)$$

Equation 339 permits the transformation law itself to differ across regimes.

For a structural system located in regime $\rho(t)$, the Type-II representation is given by Equation 340.

$$\Theta_t = \mathcal{T}^{(\rho(t))}(\mathfrak{S}_t). \quad (340)$$

Equation 340 makes transformation dependence on the active regime explicit.

The local structural-to-temporal Jacobian can also vary by regime. This dependence is represented by Equation 341.

$$J^{(\rho)} = \left. \frac{\partial \Theta}{\partial z} \right|_{\rho}. \quad (341)$$

Equation 341 allows identical structural perturbations to generate different temporal responses in different regimes.

For regimes ρ_1 and ρ_2 , the sensitivity difference is represented by Equation 342.

$$J^{(\rho_1)} \neq J^{(\rho_2)}. \quad (342)$$

Equation 342 implies that a local intervention rule calibrated in one regime can become inaccurate after a regime transition.

The rank of the transformation can change as well. This relation is represented by Equation 343.

$$\text{rank} \left(J^{(\rho_1)} \right) \neq \text{rank} \left(J^{(\rho_2)} \right). \quad (343)$$

Equation 343 means that structural directions identifiable or controllable in one regime can become hidden or inaccessible in another.

Regime-dependent transformation therefore links spectral-regime governance with representation theory. A regime transition can alter both the temporal state of the system and the map through which structural change becomes temporally expressed.

12.5 Structural Change with Temporal Persistence

This subsection develops trajectories through which Type-I structure changes while the selected Type-II representation remains approximately persistent. Its objective is to distinguish structural evolution hidden within a stable temporal image.

Let $\mathfrak{S}_{t_1}$ and $\mathfrak{S}_{t_2}$ denote two structural configurations separated in time. Structural change is represented by Equation 344.

$$\mathfrak{S}_{t_1} \neq \mathfrak{S}_{t_2}. \quad (344)$$

Temporal persistence under the selected representation is represented by Equation 345.

$$d_{\text{II}}(\Theta_{t_1}, \Theta_{t_2}) \leq \varepsilon. \quad (345)$$

Equation 345 allows structural evolution to remain weakly visible in the selected Type-II coordinates. The combined relation is represented by Equation 346.

$$\mathfrak{S}_{t_1} \neq \mathfrak{S}_{t_2}, \quad \mathcal{T}_{t_1}(\mathfrak{S}_{t_1}) \approx \mathcal{T}_{t_2}(\mathfrak{S}_{t_2}). \quad (346)$$

Equation 346 identifies structural change concealed by temporal persistence.

Several mechanisms can generate this relation. Structural changes can lie within local null directions, different changes can compensate one another, the representation can be too coarse to expose them, or the system can actively reorganize to preserve a temporal regime despite changing internal structure.

Compensatory structural change can be represented locally by Equation 347.

$$J_{\text{I} \rightarrow \text{II}}(\delta z_1 + \delta z_2) \approx 0. \quad (347)$$

Equation 347 represents structural changes whose first-order temporal effects approximately cancel.

Temporal persistence therefore provides limited evidence for structural persistence. A stable cadence, spectrum, or synchronization regime can coexist with substantial changes in rules, relations, dynamical mechanisms, or generative backgrounds.

This distinction is important for governance monitoring because temporal stability can conceal changing future reachability or changing responses to intervention.

12.6 Temporal Change with Structural Persistence

This subsection develops the complementary case in which the selected Type-I structure remains approximately persistent while Type-II temporal organization changes. Its objective is to identify temporal change generated through realization conditions, context, state evolution, or observation rather than through a structural transformation.

Structural persistence over the selected interval is represented by Equation 348.

$$d_{\text{I}}(\mathfrak{S}_{t_1}, \mathfrak{S}_{t_2}) \leq \delta. \quad (348)$$

Equation 348 treats the structural system as unchanged within the tolerance relevant to the study.

Temporal change is represented by Equation 349.

$$d_{\text{II}}(\Theta_{t_1}, \Theta_{t_2}) > \varepsilon. \quad (349)$$

Equation 349 identifies a consequential change in the selected Type-II representation. The combined relation is represented by Equation 350.

$$\mathfrak{S}_{t_1} \approx \mathfrak{S}_{t_2}, \quad \Theta_{t_1} \neq \Theta_{t_2}. \quad (350)$$

Equation 350 can arise through changes in initial state, external forcing, transient dynamics, contextual parameters, stochastic realization, or observation architecture.

This relation was already implicit in the forward temporal set developed in Section 6. The present section places it within an evolving history.

Temporal change under structural persistence has a governance implication. A change in spectral regime does not automatically establish that rules, relations, or generative backgrounds have changed. Structural intervention should therefore be preceded by analysis of whether the observed Type-II transition arose from structural change, contextual change, state evolution, or representational change.

The distinction also prevents Type-II change from being treated as a direct proxy for Type-I reform.

12.7 Transformation across Critical Transitions

This subsection develops structural-to-temporal transformation near critical transitions. Its objective is to characterize conditions in which small structural or contextual changes can produce large temporal reorganization and in which local transformation models become rapidly obsolete.

Let λ denote a control or contextual parameter and λ_c a transition value associated with a qualitative change in system dynamics. The temporal regime on each side of the transition is represented by Equation 351.

$$\Sigma^- = \lim_{\lambda \rightarrow \lambda_c^-} \Sigma(\lambda), \quad \Sigma^+ = \lim_{\lambda \rightarrow \lambda_c^+} \Sigma(\lambda). \quad (351)$$

Equation 351 permits Σ^- and Σ^+ to occupy qualitatively different Type-II regimes.

The corresponding transformation sensitivity can change sharply near the transition. This change is represented by Equation 352.

$$J(\lambda) = \frac{\partial \Theta}{\partial z}(\lambda). \quad (352)$$

Equation 352 allows the local Jacobian to vary as the system approaches the transition.

A finite local approximation is reliable only while the higher-order remainder remains sufficiently small. The local transformation error is represented by Equation 353.

$$r(\delta z) = \mathcal{T}(z + \delta z) - \mathcal{T}(z) - J(z)\delta z. \quad (353)$$

Equation 353 measures the deviation from the first-order tangent approximation.

Near a transition, the admissible neighborhood within which $\|r(\delta z)\|$ remains small can contract substantially. This behavior limits the horizon over which tangent-space governance can be extrapolated.

Critical-transition research has developed indicators that can sometimes provide advance evidence of declining resilience or approaching qualitative change, while their interpretation remains model- and context-dependent [14]. The transformation framework treats such indicators as evidence about evolving system conditions rather than as universal identifiers of one structural mechanism.

A transition can also reorganize identifiability. Let $\mathfrak{I}^-$ and $\mathfrak{I}^+$ denote compatibility sets before and after a transition. Their relation is represented by Equation 354.

$$\mathfrak{I}^-(\Theta^-) \neq \mathfrak{I}^+(\Theta^+). \quad (354)$$

Equation 354 means that a transition can expose previously hidden structural differences or compress previously observable ones.

Critical transitions therefore affect the governed system, its temporal regime, the local transformation geometry, and the epistemic relation between Type-I and Type-II representations simultaneously.

12.8 Historical Dependence and Path Structure

This subsection develops historical dependence in the transformation between structural and temporal representations. Its objective is to represent cases in which the current structural and temporal state does not contain enough information to determine future transformation because the path through which the system arrived there remains consequential.

Institutional and political analysis has long emphasized sequencing, duration, and path-dependent development [10]. Within the present framework, historical dependence enters when transformation at time t depends on a segment of the preceding structural or temporal history.

A history-dependent transformation is represented by Equation 355.

$$\Theta_t = \mathcal{T}_t [\mathfrak{S}_{[0,t]}, \Theta_{[0,t]}, \chi_{[0,t]}], \quad (355)$$

where the interval-indexed terms denote the relevant structural, temporal, and contextual histories.

Equation 355 permits current temporal organization to depend on more than the instantaneous structural state.

Two systems can therefore occupy similar current states while possessing different future transformation relations because their histories differ. This condition is represented by Equation 356.

$$\mathfrak{S}_t^{(1)} \approx \mathfrak{S}_t^{(2)}, \quad \Theta_t^{(1)} \approx \Theta_t^{(2)}, \quad \mathcal{T}_{t:t+\Delta}^{(1)} \neq \mathcal{T}_{t:t+\Delta}^{(2)}. \quad (356)$$

Equation 356 represents hidden historical dependence in future structural-to-temporal response.

Hysteresis provides one formal example. Let λ be a varying control parameter. Different transition values under increasing and decreasing λ can be represented by Equation 357.

$$\lambda_{\uparrow} \neq \lambda_{\downarrow}. \quad (357)$$

Equation 357 indicates a path-dependent transition structure.

Historical dependence can also enter through structural accumulation. Repeated interventions can alter rules, relations, resources, expectations, or institutional capacities in ways that change later transformation operators.

The structural history generated by a sequence of interventions is represented by Equation 358.

$$\mathfrak{S}_n = \mathcal{U}_n \circ \mathcal{U}_{n-1} \circ \cdots \circ \mathcal{U}_1 (\mathfrak{S}_0). \quad (358)$$

Equation 358 makes intervention order part of the resulting structural state.

When intervention operators do not commute, alternative histories can produce different structures and temporal possibility sets even when some intermediate Type-II observations appear similar.

The temporal reachable set should consequently be allowed to depend on history. This dependence is represented by Equation 359.

$$\mathfrak{R}_{\text{II}} = \mathfrak{R}_{\text{II}} (\mathfrak{S}_t, \mathcal{H}_t), \quad (359)$$

where $\mathcal{H}_t$ denotes the relevant historical path.

Equation 359 means that the same current structural description can support different future temporal options when relevant historical variables have been omitted from the instantaneous state representation.

This possibility also identifies a modeling issue. If historical dependence is persistent and consequential, the structural state can be augmented to include sufficient memory variables. The enriched structural state is represented by Equation 360.

$$\tilde{\mathfrak{S}}_t = (\mathfrak{S}_t, m_t), \quad (360)$$

where m_t collects historical information required for the selected transformation problem.

Equation 360 converts part of the apparent path dependence into an expanded state representation when such augmentation is feasible.

Historical dependence therefore has both substantive and representational forms. Some path effects arise because the system genuinely retains historically generated structure. Others appear because the selected Type-I representation omits state variables carrying that history.

Table 11 summarizes the principal relations developed in this section.

Table 11: Transformation under Nonstationarity and Structural Evolution

Evolution Domain	Formal Object	Transformation Function	Governance Significance
Time-Varying Structure	$\mathfrak{S}_t$	Represents evolving rules, dynamics, relations, and generative backgrounds	Changes both temporal generation and available structural interventions
Time-Varying Transformation	$\mathcal{T}_t = \mathcal{Q}_t \circ \mathcal{O}_t \circ \mathcal{D}_t$	Allows realization, observation, and representation to evolve	Separates system change from representational change
Moving Equivalence Classes	$[\mathfrak{S}]_{\mathcal{T}_t}$	Tracks changing structural indistinguishability	Earlier structural inference can lose validity as representation evolves
Regime-Dependent Transformation	$\mathcal{T}^{(\rho)}$	Conditions structural-to-temporal mapping on the active regime	Intervention sensitivity and identifiability can change across regimes
Structural Change with Temporal Persistence	$\mathfrak{S}_{t_1} \neq \mathfrak{S}_{t_2}, \Theta_{t_1} \approx \Theta_{t_2}$	Represents hidden structural evolution under stable temporal appearance	Temporal stability can conceal changing future response
Temporal Change with Structural Persistence	$\mathfrak{S}_{t_1} \approx \mathfrak{S}_{t_2}, \Theta_{t_1} \neq \Theta_{t_2}$	Represents temporal reorganization under stable selected structure	Type-II change does not by itself establish Type-I reform
Critical Transition	$\Sigma^- \rightarrow \Sigma^+$	Reorganizes temporal regime and local transformation geometry	Local models, reachability, and identifiability can change rapidly
Historical Dependence	$\mathcal{T}_t[\mathfrak{S}_{[0,t]}, \mathcal{H}_t]$	Makes current transformation conditional on system history	Intervention order and accumulated structure shape future temporal possibility

The relations summarized in Table 11 show that the Type-I–Type-II transformation is itself a dynamic object. Structural systems evolve, observation architectures change, temporal representations move across scales, equivalence classes split and merge, and local sensitivities can change across dynamical regimes.

This extension also qualifies several earlier results. A reconstruction can be accurate at one time and obsolete later. A structural lift can cease to realize the intended Type-II operator after the system enters another regime. A null direction can become visible as the local Jacobian changes. A task-sufficient representation can become insufficient after the relevant decision partition changes.

The transformation framework should therefore be interpreted historically. Representational suffi-

ciency, identifiability, reachability, and intervention equivalence are properties of a system under specified structural, observational, and temporal conditions during a specified interval.

The next section develops *Governance Design across Representational Domains*. It uses the preceding results to examine structural design for temporal objectives, temporal diagnosis for structural intervention, selection among multiple structural realizations, admissible intervention sets, viability, reversibility, generative constraints, and multi-objective governance design.

13 Governance Design across Representational Domains

This section develops governance design using the transformation relations established between Type-I structural and Type-II spectral-temporal representations. Its objective is to connect temporal objectives and diagnoses with structurally admissible interventions while preserving the distinction between representation, diagnosis, intervention, and normative selection. The section proceeds through structural design for temporal objectives, temporal diagnosis for structural intervention, intervention selection under multiple realizations, admissible intervention sets, viability constraints, reversibility and revisability, generative constraints, and multi-objective governance design.

The resulting design problem is inherently cross-representational. Type-II analysis can identify a temporal condition to sustain, alter, avoid, or investigate. Type-I analysis identifies the structural locations and mechanisms through which intervention can occur. Structural lifting then generates one or several candidate interventions, while governance design selects among these candidates using operational, institutional, viability, reversibility, generative, and normative criteria.

13.1 Structural Design for Temporal Objectives

This subsection develops structural intervention design beginning from a desired Type-II objective. Its objective is to combine temporal target specification, structural lifting, admissibility, and intervention selection within one formal design relation.

Let Θ^- denote the current Type-II representation and let $\mathcal{G}_{II}(\Theta^-)$ denote an acceptable temporal target region. A structural intervention $\mathcal{U}_I$ satisfies the temporal objective when the post-intervention representation lies within this region. This condition is represented by Equation 361.

$$\mathcal{T}[\mathcal{U}_I(\mathcal{G})] \in \mathcal{G}_{II}(\Theta^-). \quad (361)$$

Equation 361 defines temporal success without yet selecting a unique Type-I realization. The corresponding structural candidate set is represented by Equation 362.

$$\mathcal{U}_I^\Theta(\mathcal{G}) = \left\{ \mathcal{U}_I \in \mathcal{U}_I^{\text{adm}} \mid \mathcal{T}[\mathcal{U}_I(\mathcal{G})] \in \mathcal{G}_{II}(\Theta^-) \right\}. \quad (362)$$

Equation 362 is the governance design set generated by the temporal objective.

The structural supports represented in this set can span several Type-I layers. The support diversity of the candidate set is represented by Equation 363.

$$\mathcal{L}_\Theta = \{ \Lambda_I(\mathcal{U}_I) \mid \mathcal{U}_I \in \mathcal{U}_I^\Theta \}. \quad (363)$$

Equation 363 records which Type-I structural layers can realize the selected temporal objective.

A temporal target such as reducing excessive synchronization can therefore admit interventions through rules, dynamical feedback, relational coupling, or generative-background conditions. The temporal objective identifies a desired Type-II transformation. It does not determine which structural route should carry the intervention.

Governance design consequently involves two distinct operations:

$$\Theta^- \longrightarrow \mathcal{G}_{II}$$

for temporal objective specification, and

$$\mathcal{G}_{\text{II}} \longrightarrow \mathcal{U}_I^\Theta$$

for structural realization.

The second relation is generally set-valued, as established in Section 8. Governance design begins once this structural multiplicity has been made explicit.

13.2 Temporal Diagnosis for Structural Intervention

This subsection develops the diagnostic use of Type-II representation before structural intervention. Its objective is to distinguish temporal evidence about system behavior from the structural governance mechanisms selected in response to that evidence.

Let $\mathcal{D}_{\text{II}}$ denote a diagnostic operator applied to the observed Type-II representation. The temporal diagnostic state is represented by Equation 364.

$$\Delta_{\text{diag}} = \mathcal{D}_{\text{II}} \left(\Theta^{\text{obs}} \right). \quad (364)$$

Equation 364 can encode evidence such as timescale mismatch, phase drift, increasing synchronization, resonance, interference, cross-frequency coupling, or approaching spectral-regime change.

The diagnostic operator is analytically distinct from the governance intervention. This separation is represented schematically by Equation 365.

$$\Theta^{\text{obs}} \xrightarrow{\mathcal{D}_{\text{II}}} \Delta_{\text{diag}} \xrightarrow{\mathcal{I}} \mathcal{U}_I^{\text{cand}} \xrightarrow{\mathcal{S}} \mathcal{U}_I^*, \quad (365)$$

where $\mathcal{I}$ maps diagnostic information into candidate structural interventions and $\mathcal{S}$ denotes the subsequent selection procedure.

Equation 365 separates three operations: temporal diagnosis, generation of structural candidates, and governance selection.

This separation prevents analytical tools from being classified as governance mechanisms merely because they inform governance. A time-frequency transform, phase estimate, synchronization index, or critical-transition indicator can support diagnosis while leaving the actual intervention at the level of rules, dynamics, relations, or generative backgrounds.

Temporal diagnosis also remains structurally underdetermined when several Type-I explanations are compatible with the same observed pattern. Let $\mathcal{J}(\Delta_{\text{diag}})$ denote the structural compatibility set associated with the diagnosis. This set is represented by Equation 366.

$$\mathcal{J}(\Delta_{\text{diag}}) = \{ \mathfrak{S} \in \mathcal{M}_I \mid \mathfrak{S} \text{ is compatible with the diagnostic evidence} \}. \quad (366)$$

Equation 366 makes explicit that a temporal pattern can support several structural interpretations.

Structural intervention can nevertheless proceed when the diagnostic representation is task sufficient in the sense developed in Section 11. For contemplated decision rule $\mathcal{D}_{\mathcal{A}}$, the condition is represented by Equation 367.

$$|\{ \mathcal{D}_{\mathcal{A}}(\mathfrak{S}) \mid \mathfrak{S} \in \mathcal{J}(\Delta_{\text{diag}}) \}| = 1. \quad (367)$$

Equation 367 permits intervention without complete structural reconstruction when all structurally compatible explanations support the same decision.

13.3 Intervention Selection under Multiple Realizations

This subsection develops intervention selection when several Type-I realizations satisfy the same Type-II objective. Its objective is to preserve structural multiplicity through the selection stage and evaluate candidate interventions according to dimensions that the temporal target itself does not determine.

Let

$$\mathcal{U}_1^\Theta = \{\mathcal{U}_1, \dots, \mathcal{U}_m\}$$

denote candidate structural interventions realizing the desired temporal objective.

Each candidate can be associated with an evaluation vector. This vector is represented by Equation 368.

$$\mathbf{E}(\mathcal{U}) = (E_{\text{temp}}, E_{\text{resource}}, E_{\text{legal}}, E_{\text{viability}}, E_{\text{reversal}}, E_{\text{risk}}, E_{\text{generative}}, E_{\text{distribution}}). \quad (368)$$

Equation 368 retains temporal, operational, institutional, and normative dimensions as separate coordinates.

Temporal equivalence among candidates is represented by Equation 369.

$$\mathcal{T}[\mathcal{U}_a(\mathcal{G})] = \mathcal{T}[\mathcal{U}_b(\mathcal{G})]. \quad (369)$$

Equation 369 does not imply equality of the remaining entries of $\mathbf{E}(\mathcal{U})$.

Selection can therefore be formulated through a partial order rather than an immediate scalar utility function. Let $\preceq_{\mathcal{C}}$ denote an ordering defined by the governance criteria $\mathcal{C}$. A candidate is dominated when another candidate is at least as acceptable on every relevant criterion and preferable on at least one.

The set of nondominated interventions is represented by Equation 370.

$$\mathcal{U}_1^{\text{ND}} = \{\mathcal{U} \in \mathcal{U}_1^\Theta \mid \nexists \mathcal{V} \in \mathcal{U}_1^\Theta : \mathcal{V} \prec_{\mathcal{C}} \mathcal{U}\}. \quad (370)$$

Equation 370 preserves candidate interventions when the evaluation dimensions cannot be justified through one common scalar metric.

A scalar objective can still be used when the weighting rule is substantively defensible. One generic form is represented by Equation 371.

$$\mathcal{U}^* \in \arg \min_{\mathcal{U} \in \mathcal{U}_1^\Theta} \sum_{k=1}^m w_k c_k(\mathcal{U}), \quad w_k \geq 0. \quad (371)$$

Equation 371 makes the normative and operational weights explicit rather than embedding them invisibly within the temporal objective.

Selection among structural lifts is therefore a governance problem in its own right. The transformation framework supplies the candidate relation, while institutional and normative reasoning determines which realization is acceptable.

13.4 Admissible Structural Intervention Sets

This subsection develops admissibility as a constraint on structural governance design. Its objective is to distinguish dynamically conceivable interventions from actions available within the actual legal, institutional, resource, informational, and technical environment.

Let $\mathcal{U}_1^{\text{all}}$ denote the broad space of structurally definable interventions. The admissible intervention set is represented by Equation 372.

$$\mathcal{U}_1^{\text{adm}} \subseteq \mathcal{U}_1^{\text{all}}. \quad (372)$$

Equation 372 makes practical governance capacity a restricted subset of theoretical structural possibility.

A decomposed admissibility condition is represented by Equation 373.

$$\mathcal{U}_1^{\text{adm}} = \mathcal{U}^L \cap \mathcal{U}^R \cap \mathcal{U}^K \cap \mathcal{U}^I \cap \mathcal{U}^V \cap \mathcal{U}^G, \quad (373)$$

where the sets correspond respectively to legal-institutional authority, resource feasibility, technical or organizational capacity, informational feasibility, viability, and generative constraints.

Equation 373 is an extensible decomposition. Applications can add or remove constraint domains according to the governance system.

Admissibility can vary through time. The time-dependent intervention set is represented by Equation 374.

$$\mathfrak{U}_1^{\text{adm}} = \mathfrak{U}_1^{\text{adm}}(t, \mathfrak{S}_t, \chi_t). \quad (374)$$

Equation 374 allows institutional authority, resources, information, and structural capacity to evolve.

Admissibility is therefore itself a Type-I property. A generative-background change can enlarge the future intervention set even if it produces only a modest immediate Type-II change. Infrastructure, institutional capacity, and information architecture can be valuable partly because they expand future governance possibility.

This observation connects structural governance with adaptive and experimentalist approaches in which institutional arrangements can preserve capacity for learning and revision under changing conditions [4, 13].

13.5 Viability Constraints

This subsection develops viability as a constraint on the trajectory generated during and after structural intervention. Its objective is to prevent evaluation of an intervention solely through achievement of a terminal Type-II target.

Let $\mathcal{K}_t$ denote the set of structurally and operationally viable states at time t . A controlled trajectory $\mathfrak{S}_t^{\mathcal{U}}$ is viable over interval $[t_0, t_f]$ when the condition in Equation 375 holds.

$$\mathfrak{S}_t^{\mathcal{U}} \in \mathcal{K}_t \quad \forall t \in [t_0, t_f]. \quad (375)$$

Equation 375 requires the intervention path to remain within the selected viability region.

The set of interventions satisfying the temporal target and viability constraints is represented by Equation 376.

$$\mathfrak{U}_1^{\Theta, V} = \{\mathcal{U} \in \mathfrak{U}_1^{\Theta} \mid \mathfrak{S}_t^{\mathcal{U}} \in \mathcal{K}_t \forall t \in [t_0, t_f]\}. \quad (376)$$

Equation 376 excludes temporal realizations whose transition path violates the governance system's viability requirements.

Viability can concern continuity of essential services, preservation of institutional capacity, resource bounds, legal commitments, safety margins, or maintenance of conditions necessary for continued system operation.

A Type-II target can therefore be reachable as a final state and remain unacceptable because the structural path required to reach it passes through an inadmissible region.

Temporal representation can assist viability monitoring. Let Θ_t contain temporal indicators related to the approach toward a boundary of the viable region. A temporal warning set is represented by Equation 377.

$$\mathcal{W}_{\text{II}} = \{\Theta : d_{\text{II}}(\Theta, \partial\mathcal{K}_{\text{II}}) \leq \varepsilon\}, \quad (377)$$

where $\partial\mathcal{K}_{\text{II}}$ denotes a representation of the relevant viability boundary when such a mapping is justified.

Equation 377 gives temporal diagnosis a role in maintaining viable structural trajectories while keeping the diagnostic representation distinct from the intervention itself.

13.6 Reversibility and Revisability

This subsection develops reversibility and revisability as distinct properties of governance interventions. Its objective is to evaluate whether structural changes can be physically or institutionally reversed and whether the governance process preserves the capacity to revise decisions as knowledge and conditions evolve.

A structural intervention $\mathcal{U}$ is exactly reversible on domain $\mathcal{D}$ when an admissible inverse intervention exists. This condition is represented by Equation 378.

$$\exists \mathcal{U}^{-1} \in \mathcal{U}_I^{\text{adm}} : \mathcal{U}^{-1} \circ \mathcal{U}(\mathfrak{S}) = \mathfrak{S} \quad \forall \mathfrak{S} \in \mathcal{D}. \quad (378)$$

Equation 378 represents a strong form of structural reversal.

Many governance interventions support only approximate reversal. For structural distance d_I , approximate reversibility is represented by Equation 379.

$$d_I(\mathcal{U}^{-1}[\mathcal{U}(\mathfrak{S})], \mathfrak{S}) \leq \delta. \quad (379)$$

Equation 379 permits residual structural change after attempted reversal.

Revisability is broader than reversibility. A governance process is revisable when it preserves an admissible family of future interventions capable of altering the current trajectory in response to new information.

Let $\mathcal{U}_{\text{future}}^{\text{adm}}(\mathfrak{S}^+)$ denote the intervention set available after current intervention $\mathcal{U}$. A simple revisability descriptor is represented by Equation 380.

$$\mathfrak{R}_{\text{rev}}(\mathcal{U}; \mathfrak{S}) = \mathcal{U}_{\text{future}}^{\text{adm}}(\mathcal{U}(\mathfrak{S})). \quad (380)$$

Equation 380 represents the governance options remaining after the intervention.

Two temporally equivalent interventions can therefore differ substantially in revisability. This possibility is represented by Equation 381.

$$\mathcal{T}[\mathcal{U}_a(\mathfrak{S})] = \mathcal{T}[\mathcal{U}_b(\mathfrak{S})], \quad \mathfrak{R}_{\text{rev}}(\mathcal{U}_a; \mathfrak{S}) \neq \mathfrak{R}_{\text{rev}}(\mathcal{U}_b; \mathfrak{S}). \quad (381)$$

Equation 381 shows that current Type-II equivalence can conceal different future governance possibility.

Revisability is particularly important under model uncertainty and nonstationarity. Experimentalist and adaptive governance approaches similarly emphasize iterative learning, revision, and institutional adjustment [4, 13].

Within the present framework, revisability is treated structurally: it concerns preservation of future generative and intervention possibilities rather than a general preference for frequent policy change.

13.7 Generative Constraints

This subsection develops generative constraints on structural intervention. Its objective is to preserve conditions under which relevant actors, relations, institutions, and system components can continue generating, learning, adapting, participating, and revising future configurations.

Let $\mathcal{G}(\mathfrak{S})$ denote a vector of context-specific generative conditions. This vector is represented by Equation 382.

$$\mathcal{G}(\mathfrak{S}) = (g_1(\mathfrak{S}), g_2(\mathfrak{S}), \dots, g_n(\mathfrak{S})). \quad (382)$$

Equation 382 can include access to resources, capacity to act, availability of information, relational connectivity, institutional participation, option diversity, or other generative conditions defined for the application.

A basic generative admissibility condition can be represented through lower bounds. This condition is given by Equation 383.

$$g_i(\mathcal{U}(\mathfrak{S})) \geq \underline{g}_i \quad \forall i \in \mathcal{I}_G. \quad (383)$$

Equation 383 prevents selected generative conditions from falling below application-specific admissible thresholds.

A path-sensitive generative constraint is represented by Equation 384.

$$g_i(\mathfrak{S}_t^{\mathcal{U}}) \geq \underline{g}_i(t) \quad \forall t \in [t_0, t_f]. \quad (384)$$

Equation 384 evaluates generative conditions throughout the intervention process.

Generative constraints are distinct from maximizing generativity. The framework does not require every generative quantity to be increased indefinitely. Competing generative conditions can exist across actors, timescales, and system levels, and some generative processes can undermine others.

For this reason, a candidate intervention can be evaluated through a multi-actor generative profile. This profile is represented by Equation 385.

$$\mathbf{G}_{\mathcal{U}} = (\mathcal{G}_1[\mathcal{U}(\mathfrak{S})], \dots, \mathcal{G}_m[\mathcal{U}(\mathfrak{S})]). \quad (385)$$

Equation 385 preserves heterogeneity among affected actors or subsystems.

Two interventions with the same Type-II target can therefore differ sharply in whose generative conditions they preserve, weaken, or expand. This is a principal reason why temporal optimization cannot substitute for normative and relational governance judgment.

13.8 Multi-Objective Governance Design

This subsection consolidates governance design as a constrained multi-objective problem across Type-I and Type-II domains. Its objective is to combine temporal objectives with structural admissibility, viability, reversibility, revisability, generative conditions, and distributional considerations without reducing these domains automatically to one scalar objective.

Let the governance objective vector associated with structural intervention $\mathcal{U}$ be represented by Equation 386.

$$\mathbf{J}(\mathcal{U}) = (J_{\text{II}}, J_{\text{cost}}, J_{\text{viability}}, J_{\text{reversal}}, J_{\text{revisability}}, J_{\text{generative}}, J_{\text{distribution}}). \quad (386)$$

Equation 386 retains Type-II performance as one component of a broader governance design problem.

The feasible design set is represented by Equation 387.

$$\mathfrak{F}_{\text{gov}} = \mathfrak{U}_1^{\Theta} \cap \mathfrak{U}_1^{\text{adm}} \cap \mathfrak{U}_1^V \cap \mathfrak{U}_1^G, \quad (387)$$

where the sets denote temporal-target realization, structural admissibility, viability, and generative acceptability.

Equation 387 can be extended with additional application-specific constraints.

A multi-objective design can then be represented by Equation 388.

$$\underset{\mathcal{U} \in \mathfrak{F}_{\text{gov}}}{\text{minimize}} \mathbf{J}(\mathcal{U}), \quad (388)$$

where the vector optimization notation denotes comparison under an explicitly defined decision relation rather than automatic scalarization.

Equation 388 separates feasibility from selection. Feasibility identifies interventions satisfying hard constraints. Selection addresses tradeoffs among interventions that remain admissible.

Governance under uncertainty can further evaluate candidate interventions across the structural compatibility set. Let $\mathfrak{I}(\Theta^{\text{obs}})$ denote the currently plausible structural systems. A robust candidate set is represented by Equation 389.

$$\mathcal{U}_I^{\text{rob}} = \left\{ \mathcal{U} \in \mathfrak{F}_{\text{gov}} \mid \mathcal{U} \text{ satisfies the required constraints for every } \mathfrak{S} \in \mathfrak{I}(\Theta^{\text{obs}}) \right\}. \quad (389)$$

Equation 389 provides one conservative response to unresolved structural ambiguity.

A less conservative formulation can preserve outcome sets explicitly. For candidate intervention $\mathcal{U}$, the uncertainty-conditioned outcome set is represented by Equation 390.

$$\mathfrak{O}(\mathcal{U} \mid \Theta^{\text{obs}}) = \left\{ \mathbf{J}(\mathcal{U}; \mathfrak{S}) \mid \mathfrak{S} \in \mathfrak{I}(\Theta^{\text{obs}}) \right\}. \quad (390)$$

Equation 390 retains the range of governance consequences compatible with current structural knowledge.

Multi-objective governance design can therefore remain set-valued at several levels: several structural systems can explain the observed Type-II state, several interventions can realize the same temporal target, and each intervention can generate a range of consequences under unresolved structural uncertainty.

The role of governance design is consequently to manage these multiplicities through explicit constraints, evidence, revisability, and selection criteria. It does not require collapsing them prematurely into one inferred structure, one temporal objective, or one scalar measure of value.

Table 12 summarizes the principal design relations developed in this section.

The relations summarized in Table 12 complete the transition from representation theory to governance design. Type-II analysis can identify temporal patterns, constraints, and objectives. Type-I analysis identifies the structural mechanisms, supports, and intervention capacities through which those temporal objectives can be realized. Structural lifting connects these domains while preserving the multiplicity of possible realizations.

The resulting governance architecture can be summarized by the transformation chain represented in Equation 391.

$$\Theta^{\text{obs}} \longrightarrow \Delta_{\text{diag}} \longrightarrow \mathcal{G}_{\text{II}} \longrightarrow \mathcal{U}_I^{\Theta} \longrightarrow \mathfrak{F}_{\text{gov}} \longrightarrow \mathcal{U}_I^* \longrightarrow \mathfrak{S}^+ \longrightarrow \Theta^+. \quad (391)$$

Equation 391 separates diagnosis, temporal objective formation, structural lifting, feasibility filtering, intervention selection, structural transformation, and subsequent temporal response.

The chain is iterative under nonstationarity and incomplete knowledge. Post-intervention observation generates a new Type-II representation, which can alter structural compatibility, intervention admissibility, and the appropriate governance objective. Governance across the two representational domains is therefore naturally recursive and revisable.

The next section develops the *Epistemic and Operational Conditions* governing this process. Section 14 examines observation architecture, sampling and temporal resolution, model and transformation uncertainty, finite-data identifiability, computational feasibility, decision horizon, and representation selection under limited information.

Table 12: Governance Design across Structural and Spectral-Temporal Representational Domains

Design Domain	Formal Object	Design Function	Governance Significance
Structural Design for Temporal Objectives	$\mathcal{U}_I^\Theta$	Collects structural interventions realizing a Type-II target	Separates temporal objective from structural implementation
Temporal Diagnosis	$\mathcal{D}_\Pi(\Theta^{\text{obs}})$	Extracts temporally represented evidence relevant to intervention	Keeps diagnostic methods distinct from governance mechanisms
Multiple-Realization Selection	$\mathbf{E}(\mathcal{U})$	Compares temporally equivalent structural interventions	Introduces structural, operational, and normative selection criteria
Structural Admissibility	$\mathcal{U}_I^{\text{adm}}$	Restricts theoretical interventions to available governance capacity	Connects intervention possibility with legal, resource, informational, and institutional conditions
Viability	$\mathcal{S}_t^{\mathcal{U}} \in \mathcal{K}_t$	Constrains the complete intervention trajectory	Prevents terminal temporal objectives from concealing inadmissible transition paths
Reversibility and Revisability	$\mathfrak{R}_{\text{rev}}(\mathcal{U}; \mathcal{S})$	Represents reversal and future intervention possibilities	Preserves capacity for correction under uncertainty and structural evolution
Generative Constraints	$\mathcal{G}[\mathcal{U}(\mathcal{S})]$	Evaluates preservation of relevant generative conditions	Distinguishes temporal performance from effects on future agency and system possibility
Multi-Objective Design	$\mathbf{J}(\mathcal{U})$	Combines temporal, structural, operational, and normative design dimensions	Supports governance under multiple admissible objectives and unresolved structural uncertainty

14 Epistemic and Operational Conditions

This section develops the epistemic and operational conditions under which the structural-to-temporal transformation framework can support empirical inference and governance action. Its objective is to specify how observation architecture, temporal resolution, model uncertainty, transformation uncertainty, finite data, computational limits, decision horizon, and representation choice constrain what can be identified, reconstructed, and acted upon. The discussion treats these conditions as part of the governance problem itself because the availability of information, analytical capacity, and decision time determines which representational distinctions can be made operationally relevant. The section proceeds from observation architecture and sampling to model and transformation uncertainty, finite-data identifiability, computational feasibility, decision horizon, and representation selection under limited information.

14.1 Observation Architecture

This subsection develops observation architecture as the empirical interface between generated system trajectories and Type-II representation. Its objective is to characterize which system variables become visible, at which locations and times they are recorded, and how the observation design affects subsequent structural inference.

Let $\mathfrak{O}$ denote the complete observation architecture. One abstract representation is given by Equation 392.

$$\mathfrak{O} = (\mathcal{V}_{\text{obs}}, \mathcal{S}_{\text{obs}}, \mathcal{H}_{\text{obs}}, \mathcal{A}_{\text{obs}}, \mathcal{N}_{\text{obs}}), \quad (392)$$

where $\mathcal{V}_{\text{obs}}$ denotes observed variables, $\mathcal{S}_{\text{obs}}$ sampling structure, $\mathcal{H}_{\text{obs}}$ observation horizon, $\mathcal{A}_{\text{obs}}$ aggregation and preprocessing, and $\mathcal{N}_{\text{obs}}$ measurement uncertainty.

Equation 392 makes observation a designed system rather than a transparent window onto the governed process.

For a latent trajectory $x(t)$, the observed process can be written as Equation 393.

$$y_k = h_k(x(t_k)) + \nu_k, \quad (393)$$

where h_k can vary across observation channels and ν_k represents measurement error.

Equation 393 allows observation itself to change across time, institutions, sensors, reporting procedures, or data sources.

The structural information available through the observation architecture can be represented by the induced compatibility set. For observed record Y^{obs} , this set is given by Equation 394.

$$\mathfrak{I}_{\mathfrak{O}}(Y^{\text{obs}}) = \left\{ \mathfrak{S} \in \mathcal{M}_1 \mid \mathcal{O}_{\mathfrak{O}} \circ \mathcal{D}(\mathfrak{S}) \text{ is compatible with } Y^{\text{obs}} \right\}. \quad (394)$$

Equation 394 shows that structural identifiability can be constrained before a spectral-temporal representation is constructed.

Observation architecture can therefore be evaluated through the distinctions it preserves. Let $\mathcal{P}_{\mathfrak{O}}$ denote the structural partition induced by the observation architecture. Richer observation architecture provides epistemic gain when its partition strictly refines that of an existing architecture. This relation is represented by Equation 395.

$$\mathcal{P}_{\mathfrak{O}_2} \prec \mathcal{P}_{\mathfrak{O}_1}. \quad (395)$$

Equation 395 provides a task-relevant criterion for adding observation channels.

A new variable, sensor, report, or temporal record is therefore useful when it removes structural alternatives relevant to the inference or governance task. Increasing data volume alone does not guarantee such refinement.

Observation architecture is also institutionally distributed. Different actors can observe different parts of the governed system, at different resolutions and under different reporting rules. A distributed observation architecture is represented by Equation 396.

$$\mathfrak{O}_{\text{dist}} = \left\{ \mathfrak{O}^{(1)}, \dots, \mathfrak{O}^{(m)} \right\}. \quad (396)$$

Equation 396 permits governance knowledge to emerge through the fusion of heterogeneous observation systems.

Such heterogeneity can improve structural distinguishability, while it can also introduce incompatible temporal scales, definitions, missingness patterns, or measurement conventions. Observation design therefore precedes many of the representational questions that later appear as problems of spectral interpretation or structural reconstruction.

14.2 Sampling and Temporal Resolution

This subsection develops the operational relation between sampling, observation horizon, and Type-II temporal resolution. Its objective is to identify which temporal modes can be observed reliably and how sampling choices constrain subsequent structural inference.

For approximately regular sampling interval Δt , the sampling frequency is represented by Equation 397.

$$f_s = \frac{1}{\Delta t}. \quad (397)$$

Equation 397 determines the nominal rate at which the observed process is discretized.

Under conventional band-limited sampling assumptions, temporal components above the corresponding resolvable range can be aliased into lower frequencies [9]. Consequently, an observed low-frequency pattern can sometimes reflect unresolved higher-frequency structure.

The observation horizon also constrains accessible slow dynamics. Let

$$H = t_{\text{end}} - t_{\text{start}}.$$

For a temporal mode with characteristic period τ_i , the number of represented cycles is given by Equation 398.

$$N_i = \frac{H}{\tau_i}. \quad (398)$$

Equation 398 provides a simple descriptor of the temporal evidence available for recurring behavior at scale τ_i .

A long-horizon process can therefore remain weakly identifiable within a short record even when fast variation is densely sampled. Conversely, a long observation horizon with coarse sampling can preserve slow cycles while removing fast phase or modulation structure.

For time-localized representations, temporal and frequency resolution also depend on the selected window or basis. A windowed representation can be written as Equation 399.

$$Z_W(t, \omega) = \mathcal{Q}_W[y](t, \omega), \quad (399)$$

where W controls the local temporal support.

Equation 399 emphasizes that local spectral detail depends on the representation's temporal localization properties [2, 3].

Sampling design can therefore be expressed as a multiscale problem. Let $\mathcal{R}_{\text{temp}}$ denote the set of temporal scales that the observation architecture can meaningfully resolve. This set is represented by Equation 400.

$$\mathcal{R}_{\text{temp}} = \left\{ \tau : \tau \text{ is distinguishable under } \mathfrak{O} \text{ and } \mathcal{Q} \right\}. \quad (400)$$

Equation 400 is preferable to treating temporal resolution as a single scalar when the observation system contains several sampling rates or representation windows.

The operational requirement is therefore alignment between the temporal scales relevant to governance and the temporal scales accessible to observation. When a governance decision concerns a mode outside $\mathcal{R}_{\text{temp}}$, Type-II inference can become structurally misleading regardless of the sophistication of the subsequent analysis.

14.3 Model Uncertainty

This subsection develops uncertainty concerning the structural model used to generate and interpret the transformation. Its objective is to distinguish uncertainty within one model from uncertainty over which structural model should be used.

Let $\mathfrak{M}$ denote an admissible family of structural models. The model family is represented by Equation 401.

$$\mathfrak{M} = \{M_1, M_2, \dots, M_K\}. \quad (401)$$

Equation 401 permits different models to specify different structural variables, dynamical mechanisms, observation relations, or contextual dependencies.

Each model induces its own transformation. This family is represented by Equation 402.

$$\mathfrak{T}_{\mathfrak{M}} = \{\mathcal{T}^{(M)} : M \in \mathfrak{M}\}. \quad (402)$$

Equation 402 makes structural reconstruction conditional on model specification.

For observed Type-II representation Θ^{obs} , each model generates its own compatibility set. This collection is represented by Equation 403.

$$\mathfrak{J}_{\mathfrak{M}}(\Theta^{\text{obs}}) = \{\mathcal{J}^{(M)}(\Theta^{\text{obs}}) : M \in \mathfrak{M}\}. \quad (403)$$

Equation 403 preserves variation in structural inference across plausible models.

Model agreement can be represented by the intersection of compatible structural conclusions. For structural feature map Π_A , the model-robust feature set is represented by Equation 404.

$$\mathcal{A}_{\text{rob}} = \bigcap_{M \in \mathfrak{M}} \Pi_A \left[\mathcal{J}^{(M)}(\Theta^{\text{obs}}) \right]. \quad (404)$$

Equation 404 identifies feature values supported across the selected model family when the intersection is nonempty.

Model disagreement is itself informative. If competing structural models reproduce the same Type-II observation while implying different intervention responses, the representation can be sufficient for description and insufficient for governance design.

Model uncertainty should therefore remain separate from parameter uncertainty. System identification similarly distinguishes assumptions about model structure from estimation of parameters within a selected model [8].

14.4 Transformation Uncertainty

This subsection develops uncertainty in the relation linking Type-I and Type-II representations. Its objective is to account for incomplete knowledge of dynamical realization, observation, temporal representation, and their composition even when a structural model has been selected.

Let $\hat{\mathcal{T}}$ denote an estimated transformation and $\mathcal{T}^*$ the transformation that would describe the system under the chosen representational assumptions. Their discrepancy is represented abstractly by Equation 405.

$$\mathcal{E}_{\mathcal{T}} = \mathcal{T}^* - \hat{\mathcal{T}}, \quad (405)$$

when the operator difference is meaningful in the selected representation.

Equation 405 can contain error from dynamical misspecification, observation error, finite-window estimation, parameter uncertainty, representation approximation, or contextual dependence.

A more general formulation represents the transformation as a set. The admissible transformation family is given by Equation 406.

$$\mathfrak{T}_{\text{adm}} = \{\mathcal{T} : \mathcal{T} \text{ is compatible with current structural and empirical evidence}\}. \quad (406)$$

Equation 406 allows inference to preserve uncertainty over the transformation itself.

The structural compatibility set then expands across possible transformations. This robust inverse set is represented by Equation 407.

$$\mathfrak{I}_{\text{rob}}(\Theta^{\text{obs}}) = \bigcup_{\mathcal{T} \in \mathfrak{T}_{\text{adm}}} \mathcal{T}^{-1}(\Theta^{\text{obs}}). \quad (407)$$

Equation 407 contains all structural systems compatible with the observation under at least one currently admissible transformation.

For intervention prediction, transformation uncertainty generates an outcome set. Given structural intervention $\mathcal{U}$, the set of predicted temporal outcomes is represented by Equation 408.

$$\mathfrak{O}_{\mathcal{T}}(\mathcal{U}, \mathfrak{S}) = \{\mathcal{T}[\mathcal{U}(\mathfrak{S})] \mid \mathcal{T} \in \mathfrak{T}_{\text{adm}}\}. \quad (408)$$

Equation 408 expresses uncertainty in Type-II intervention consequence generated by incomplete knowledge of the transformation relation.

This uncertainty is particularly consequential near regime changes, where the local Jacobian, applicable model, or observation relation can change rapidly. Transformation uncertainty therefore links the epistemic problem directly with the nonstationary framework developed in Section 12.

14.5 Identifiability under Finite Data

This subsection develops practical identifiability when only a finite temporal record is available. Its objective is to distinguish theoretical identifiability of a transformation from the ability to discriminate structural alternatives using the available observations.

Let Y_N denote a temporal record containing N observations. The finite-data structural compatibility set is represented by Equation 409.

$$\mathfrak{I}_N(Y_N) = \{\mathfrak{S} \in \mathcal{M}_I \mid \mathfrak{S} \text{ remains compatible with } Y_N \text{ under the selected model}\}. \quad (409)$$

Equation 409 defines practical structural ambiguity at sample size N .

With additional informative data, the compatibility set can contract. Under a nested evidence sequence, the refinement relation is represented by Equation 410.

$$\mathfrak{I}_{N+M} \subseteq \mathfrak{I}_N, \quad (410)$$

when the additional observations preserve the earlier evidence and impose further valid constraints.

Equation 410 is conditional because more data can reveal model misspecification, nonstationarity, or previously omitted regime change rather than simply refine one fixed compatibility set.

Finite-data identifiability can be represented through distinguishability at a selected tolerance. Two structural systems are empirically indistinguishable under record Y_N when the discrepancy between their predicted temporal representations remains below the effective resolution. This relation is represented by Equation 411.

$$\mathfrak{S}_1 \sim_{N,\varepsilon} \mathfrak{S}_2 \iff d_{\text{II}} \left(\hat{\Theta}_N(\mathfrak{S}_1), \hat{\Theta}_N(\mathfrak{S}_2) \right) \leq \varepsilon. \quad (411)$$

Equation 411 introduces empirical resolution directly into the equivalence relation.

This distinction is essential because a structurally injective theoretical map can be practically uninformative when finite observations cannot resolve the difference among nearby temporal images.

A practical identifiability profile can therefore be represented by Equation 412.

$$\mathbf{I}_{\text{prac}} = (I_{\text{model}}, I_{\text{obs}}, I_{\text{sample}}, I_{\text{resolution}}, I_{\text{stability}}). \quad (412)$$

Equation 412 separates model, observation, sample, resolution, and reconstruction-stability contributions to practical structural knowledge.

The relevant governance question is therefore often whether the available data distinguish the structural alternatives that would change the contemplated decision, rather than whether asymptotic identification would be possible under unlimited observation.

14.6 Computational Feasibility

This subsection develops computational feasibility as an operational constraint on transformation, reconstruction, and intervention design. Its objective is to distinguish mathematically defined operations from analyses that can be completed with available computational resources within the relevant decision interval.

Let $\mathcal{A}$ denote an analytical procedure for representation, reconstruction, simulation, or optimization. Its computational resource profile is represented by Equation 413.

$$\mathbf{C}_{\mathcal{A}} = (C_{\text{time}}, C_{\text{memory}}, C_{\text{data}}, C_{\text{communication}}). \quad (413)$$

Equation 413 records several operational resources required by the procedure.

Let $\mathbf{B}_{\text{comp}}$ denote the corresponding available computational budget. Operational feasibility requires the resource profile to remain within this budget. This condition is represented abstractly by Equation 414.

$$\mathbf{C}_{\mathcal{A}} \preceq \mathbf{B}_{\text{comp}}. \quad (414)$$

Equation 414 treats computational feasibility as multidimensional rather than reducing it automatically to runtime alone.

The feasible analytical set is represented by Equation 415.

$$\mathfrak{A}_{\text{feas}} = \{ \mathcal{A} \mid \mathbf{C}_{\mathcal{A}} \preceq \mathbf{B}_{\text{comp}} \}. \quad (415)$$

Equation 415 defines the analytical procedures available under current operational constraints.

Computational feasibility can change the appropriate representation. A high-dimensional structural reconstruction can be theoretically informative and operationally unavailable, while a lower-dimensional Type-II diagnostic can be calculated within the decision horizon.

Approximation then becomes a governance resource rather than merely a mathematical concession. Let $\mathcal{A}_{\text{approx}}$ denote an approximate procedure and $\epsilon_{\mathcal{A}}$ its representation or prediction error. A feasible approximation condition is represented by Equation 416.

$$\mathcal{A}_{\text{approx}} \in \mathfrak{A}_{\text{feas}}, \quad \epsilon_{\mathcal{A}} \leq \epsilon_{\text{task}}. \quad (416)$$

Equation 416 requires computational feasibility together with accuracy sufficient for the governance task.

This relation provides an operational counterpart to task-sufficient representation. The analytically richest model is unnecessary when a simpler procedure preserves all distinctions required for the decision.

14.7 Decision Horizon

This subsection develops the relation between available decision time and the depth of analysis that can be completed before action becomes necessary. Its objective is to represent governance under finite epistemic and computational time.

Let H_D denote the remaining decision horizon. The decision deadline is represented by Equation 417.

$$H_D = t_{\text{deadline}} - t_{\text{now}}. \quad (417)$$

Equation 417 defines the time available for observation, inference, computation, deliberation, and implementation.

Let τ_{obs} , τ_{inf} , τ_{comp} , τ_{delib} , and τ_{impl} denote the corresponding operational durations. A basic temporal feasibility condition is represented by Equation 418.

$$\tau_{\text{obs}} + \tau_{\text{inf}} + \tau_{\text{comp}} + \tau_{\text{delib}} + \tau_{\text{impl}} \leq H_D. \quad (418)$$

Equation 418 constrains which epistemic procedures can contribute before action is required.

A long horizon can permit additional observation, model comparison, interventional probing, structural reconstruction, and evaluation of multiple lifts. A short horizon can require local inference, reduced-order representation, finite-step simulation, or robust action across the current compatibility set.

The appropriate epistemic depth can therefore be represented as a horizon-dependent set. This set is given by Equation 419.

$$\mathfrak{A}(H_D) = \{\mathcal{A} : \tau_{\mathcal{A}} \leq H_D\}. \quad (419)$$

Equation 419 contains analytical procedures capable of returning decision-relevant information before the deadline.

The decision horizon itself can shrink endogenously when the system approaches a critical transition or irreversible boundary. Let $H_C(t)$ denote an estimated horizon to a consequential system transition. The effective governance horizon is represented by Equation 420.

$$H_{\text{eff}} = \min(H_D, H_C). \quad (420)$$

Equation 420 reflects situations in which the system's dynamics impose a tighter deadline than the formal decision process.

Near critical transitions, indicators can sometimes provide evidence of changing system stability, while their reliability remains dependent on the underlying process and available data [14]. Under such conditions, waiting for globally complete structural reconstruction can carry its own governance cost.

The decision problem therefore includes the value of additional information relative to the time consumed in obtaining it.

14.8 Representation Selection under Limited Information

This subsection consolidates the epistemic and operational conditions into a representation-selection problem. Its objective is to identify a Type-I, Type-II, or joint representation that preserves sufficient information for the governance task while respecting observation, computational, and decision constraints.

Let $\mathfrak{R}$ denote the admissible family of representations. A representation $R \in \mathfrak{R}$ is associated with an epistemic-operational profile represented by Equation 421.

$$\mathbf{E}(R) = (E_{\text{dist}}, E_{\text{unc}}, E_{\text{obs}}, E_{\text{comp}}, E_{\text{time}}, E_{\text{rev}}), \quad (421)$$

where the components denote structural distinguishability, uncertainty, observation burden, computational burden, time requirement, and support for revisable decision making.

Equation 421 preserves several dimensions of representational adequacy.

For governance task $\mathcal{A}$, define $\mathfrak{R}_{\mathcal{A}}^{\text{su}}f$ as the representations satisfying the task-sufficiency criterion introduced in Section 11. This set is represented by Equation 422.

$$\mathfrak{R}_{\mathcal{A}}^{\text{su}}f = \{R \in \mathfrak{R} \mid \mathcal{P}_R \preceq \mathcal{P}_{\mathcal{A}}\}. \quad (422)$$

Equation 422 contains representations that preserve every structural distinction required by the selected governance decision.

Operational feasibility further restricts this set. Let $\mathfrak{R}^{\text{obs}}$, $\mathfrak{R}^{\text{comp}}$, and $\mathfrak{R}^{H_D}$ denote representations compatible with available observation, computational resources, and decision horizon. The feasible task-sufficient set is represented by Equation 423.

$$\mathfrak{R}_{\mathcal{A}}^{\text{feas}} = \mathfrak{R}_{\mathcal{A}}^{\text{su}}f \cap \mathfrak{R}^{\text{obs}} \cap \mathfrak{R}^{\text{comp}} \cap \mathfrak{R}^{H_D}. \quad (423)$$

Equation 423 defines the representations both sufficient and usable under current conditions.

A parsimonious representation can then be selected from this set according to a stated burden criterion. Let $c_R(R)$ denote representational cost. A minimum-burden sufficient representation is represented by Equation 424.

$$R^* \in \arg \min_{R \in \mathfrak{R}_{\mathcal{A}}^{\text{feas}}} c_R(R). \quad (424)$$

Equation 424 provides an operational stopping principle for representational refinement.

When $\mathfrak{R}_{\mathcal{A}}^{\text{feas}}$ is empty, the current information architecture cannot support the desired decision at the required level of discrimination. This condition is represented by Equation 425.

$$\mathfrak{R}_{\mathcal{A}}^{\text{feas}} = \emptyset. \quad (425)$$

Equation 425 indicates a need to change at least one element of the governance problem: acquire additional information, extend the decision horizon, increase computational capacity, reduce the decision's required precision, restrict the intervention set, or select an action robust across the unresolved structural alternatives.

The representation-selection problem can therefore be summarized by Equation 426.

$$\text{governance task} \longrightarrow \text{required distinctions} \longrightarrow \text{task-sufficient representations} \longrightarrow \text{operationally feasible representations} \quad (426)$$

Equation 426 reverses a common analytical order. Representation is selected according to the distinctions needed for the governance task and the conditions under which those distinctions can actually be observed and computed.

Table 13 summarizes the principal conditions developed in this section.

The relations summarized in Table 13 show that representational adequacy is jointly epistemic and operational. A representation can be mathematically rich and empirically unavailable, structurally informative and computationally infeasible, or theoretically identifying and too slow for the decision horizon.

The resulting governance principle is therefore task-relative and resource-relative. The appropriate representation preserves the distinctions required for the current decision while remaining observable, computable, and available within the effective governance horizon.

This conclusion also clarifies the role of uncertainty. Model uncertainty, transformation uncertainty, finite-data ambiguity, and computational approximation need not prevent governance when the remaining uncertainty lies within a task-equivalent structural class or when an intervention remains acceptable across the complete compatibility set. Additional knowledge becomes necessary when unresolved structural alternatives imply materially different governance consequences.

The next section develops an *Empirical and Computational Research Programme*. It translates the formal framework into paired structural-temporal datasets, intervention-centered designs, longitudinal

Table 13: Epistemic and Operational Conditions of Structural–Temporal Governance

Condition Domain	Formal Object	Epistemic or Operational Function	Governance Significance
Observation Architecture	$\mathfrak{O}$	Determines which generated dynamics become empirically visible	Constrains structural inference before Type-II analysis begins
Sampling and Resolution	$\mathcal{R}_{\text{temp}}$	Defines temporal scales accessible to the observation and representation system	Prevents unresolved modes from being treated as absent system structure
Model Uncertainty	$\mathfrak{M}$	Represents alternative structural explanations and model families	Separates within-model identification from uncertainty over structural mechanism
Transformation Uncertainty	$\mathfrak{T}_{\text{adm}}$	Represents uncertainty in structural-to-temporal mappings	Produces sets of compatible structural explanations and intervention outcomes
Finite-Data Identifiability	$\mathfrak{I}_N$	Represents structural ambiguity under finite observations	Distinguishes theoretical injectivity from practical structural discrimination
Computational Feasibility	$\mathfrak{A}_{\text{feas}}$	Restricts analytical procedures to available computational capacity	Makes approximate and reduced representations legitimate governance resources
Decision Horizon	H_{eff}	Limits the time available for observation, inference, deliberation, and implementation	Determines feasible epistemic depth before action
Representation Selection	$\mathfrak{R}_{\mathcal{A}}^{\text{feas}}$	Identifies task-sufficient representations available under operational constraints	Provides a stopping rule for information acquisition and model refinement

analysis, comparative structural realizations, forward simulation, reconstruction experiments, local sensitivity estimation, equivalence validation, transformation benchmarking, and iterative revision of the representational framework.

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15 Empirical and Computational Research Programme

This section develops an empirical and computational research programme for evaluating the transformation framework. Its objective is to translate the formal relations established in the preceding sections into observable data structures, intervention designs, longitudinal analyses, simulation studies, reconstruction experiments, sensitivity estimates, equivalence tests, and benchmarking procedures. The programme treats Type-I structural information and Type-II spectral-temporal information as paired but distinct empirical objects. It therefore evaluates both the forward transformation $\mathfrak{S} \rightarrow \Theta$ and the inverse relations used for structural inference, while preserving uncertainty about observation, model structure, and transformation context.

The programme is methodological rather than tied to one empirical domain. Applications can concern administrative systems, organizations, public services, infrastructure, socio-ecological governance, conflict processes, or other systems for which both structural and temporal observations can be defined responsibly. The principal empirical requirement is explicit documentation of how structural variables, temporal observations, and interventions are operationalized.

15.1 Structural-Temporal Paired Data

This subsection develops the basic empirical data structure required to study relations between Type-I and Type-II representations. Its objective is to construct observations in which structural configurations can be associated with generated or observed temporal representations under documented transformation conditions.

Let the i -th empirical unit at observation period t be represented by a paired record. This record is defined by Equation 427.

$$\mathcal{Z}_{i,t} = (\mathfrak{S}_{i,t}, Y_{i,t}, \Theta_{i,t}, \Xi_{i,t}), \quad (427)$$

where $\mathfrak{S}_{i,t}$ is the Type-I structural representation, $Y_{i,t}$ the observed temporal record, $\Theta_{i,t}$ the derived Type-II representation, and $\Xi_{i,t}$ the documented transformation context.

Equation 427 preserves the distinction between raw or processed temporal observation and its Type-II representation.

The empirical dataset is then represented by Equation 428.

$$\mathcal{D}_{\text{pair}} = \{\mathcal{Z}_{i,t}\}_{i=1,\dots,N; t \in \mathcal{T}_i}. \quad (428)$$

Equation 428 can contain cross-sectional, longitudinal, experimental, observational, or mixed empirical records.

Structural observations should retain the Type-I decomposition where empirically meaningful. One possible operational record is represented by Equation 429.

$$\widehat{\mathfrak{S}}_{i,t} = (\widehat{x}_{i,t}, \widehat{R}_{i,t}, \widehat{F}_{i,t}, \widehat{C}_{i,t}, \widehat{B}_{i,t}). \quad (429)$$

Equation 429 makes explicit that empirical structural information can itself be estimated rather than directly observed.

The Type-II record can similarly contain only the applicable temporal coordinates. This record is represented by Equation 430.

$$\widehat{\Theta}_{i,t} = (\widehat{\theta}_T, \widehat{\theta}_S, \widehat{\theta}_P, \widehat{\theta}_L, \widehat{\theta}_R, \widehat{\theta}_I, \widehat{\theta}_H, \widehat{\theta}_C, \widehat{\theta})_{i,t}. \quad (430)$$

Equation 430 does not require every coordinate to be present in every study. Applicability should follow the temporal structure and measurement architecture of the empirical system.

Paired data should also document transformation conditions. A minimum transformation metadata vector is represented by Equation 431.

$$\Xi_{i,t} = (\eta_{i,t}, \psi_{i,t}, W_{i,t}, \Delta_{i,t}, Q_{i,t}, M_{i,t}), \quad (431)$$

where the entries record realization conditions, observation architecture, window, temporal resolution, representation procedure, and model specification.

Equation 431 is essential for distinguishing structural change from changes created by measurement or representation.

The empirical programme therefore requires more than a dataset of spectra and institutional covariates. It requires an auditable relation among structural description, temporal observation, representation procedure, and relevant context.

15.2 Intervention-Centered Designs

This subsection develops empirical designs centered on structural interventions and their temporal responses. Its objective is to evaluate induced Type-II operators and to determine whether intervention can refine structural distinctions hidden under passive observation.

Let $\mathcal{U}_k$ denote intervention k . A minimal pre-post intervention record is represented by Equation 432.

$$\mathcal{Z}_{i,k}^{\text{int}} = (\Theta_i^-, \Theta_i^+, \mathcal{U}_k, \Theta_i^-, \Theta_i^+). \quad (432)$$

Equation 432 records both the structural change and its represented temporal consequence.

The empirical temporal response to the intervention can be represented by Equation 433.

$$\Delta\Theta_{i,k} = \Theta_{i,k}^+ - \Theta_i^-, \quad (433)$$

when subtraction is meaningful in the chosen Type-II coordinates.

For nonlinear or regime-valued representations, the response can instead be recorded through a discrepancy or transition operator. A general response descriptor is represented by Equation 434.

$$R_{i,k} = \mathcal{R}_{\text{II}}(\Theta_i^-, \Theta_{i,k}^+). \quad (434)$$

Equation 434 permits phase changes, regime transitions, synchronization changes, or other non-vector temporal responses to be studied.

Intervention-centered designs are particularly useful for testing the representation-preserving condition developed in Section 7. Suppose two systems are temporally equivalent before intervention. The empirical test concerns whether the condition in Equation 435 is approximately satisfied.

$$d_{\text{II}}(\Theta_{1,k}^+, \Theta_{2,k}^+) \leq \varepsilon \quad \text{given} \quad d_{\text{II}}(\Theta_1^-, \Theta_2^-) \leq \varepsilon_0. \quad (435)$$

Equation 435 evaluates whether pre-intervention temporal equivalence survives the structural intervention.

Repeated interventions can generate an empirical response signature. This signature is represented by Equation 436.

$$\hat{\mathcal{R}}_i = (R_{i,1}, R_{i,2}, \dots, R_{i,m}). \quad (436)$$

Equation 436 can be compared across structurally uncertain systems to evaluate interventional identifiability.

Experimental manipulation will often be unavailable or inappropriate in governance settings. Natural experiments, phased implementation, institutional reforms, policy discontinuities, shocks, and historical interventions can provide alternative empirical variation when their causal interpretation is carefully justified. The framework therefore distinguishes an intervention-centered research design from an assumption of randomized intervention.

15.3 Longitudinal Transformation Analysis

This subsection develops longitudinal analysis of evolving structural-to-temporal relations. Its objective is to estimate how structural systems, temporal representations, equivalence relations, and transformation parameters change across time.

A longitudinal structural-temporal sequence is represented by Equation 437.

$$\mathcal{H}_i = \{(\mathfrak{S}_{i,t}, \Theta_{i,t}, \Xi_{i,t})\}_{t=1}^{T_i}. \quad (437)$$

Equation 437 retains both system change and transformation-context change over the observation history.

A time-local transformation estimate can be represented by Equation 438.

$$\hat{\Theta}_{i,t} = \hat{\mathcal{T}}_{i,t}(\hat{\mathfrak{S}}_{i,t}). \quad (438)$$

Equation 438 permits the structural-to-temporal mapping itself to vary through time.

Transformation drift can then be represented abstractly by Equation 439.

$$\Delta\mathcal{T}_t = d_{\mathcal{T}}(\hat{\mathcal{T}}_{t+1}, \hat{\mathcal{T}}_t), \quad (439)$$

where $d_{\mathcal{T}}$ is a study-specific comparison between estimated transformations.

Equation 439 can be used to detect periods in which a previously calibrated representation becomes unreliable.

Longitudinal analysis should also test the two persistence relations developed in Section 12. Structural change with temporal persistence can be operationalized through Equation 440.

$$d_I(\mathfrak{S}_t, \mathfrak{S}_{t+\Delta}) > \delta_I, \quad d_{II}(\Theta_t, \Theta_{t+\Delta}) \leq \delta_{II}. \quad (440)$$

Temporal change with structural persistence can be operationalized through Equation 441.

$$d_I(\mathfrak{S}_t, \mathfrak{S}_{t+\Delta}) \leq \delta_I, \quad d_{II}(\Theta_t, \Theta_{t+\Delta}) > \delta_{II}. \quad (441)$$

Equations 440 and 441 provide empirical diagnostics for separating structural and temporal change.

Localized spectral methods can be useful when the temporal process itself is nonstationary [12, 2]. Longitudinal transformation analysis should nevertheless preserve the distinction between a changing observed spectrum and a changing structural generator.

15.4 Comparative Structural Realizations

This subsection develops comparative research across structurally different systems that generate similar Type-II representations. Its objective is to study multiple structural realization empirically rather than treating it solely as a formal possibility.

Let $\mathcal{C}_{\Theta}$ denote a sample of systems whose Type-II representations lie within an empirical equivalence tolerance around Θ^* . This sample is represented by Equation 442.

$$\mathcal{C}_{\Theta^*}^{\varepsilon} = \{i \mid d_{II}(\Theta_i, \Theta^*) \leq \varepsilon\}. \quad (442)$$

Equation 442 defines a temporally matched comparative set.

Structural heterogeneity within this set can then be studied through Type-I projections. For structural component A , the observed realization set is represented by Equation 443.

$$\mathcal{S}_A(\Theta^*) = \{\Pi_A(\mathfrak{S}_i) \mid i \in \mathcal{C}_{\Theta^*}^{\varepsilon}\}. \quad (443)$$

Equation 443 identifies structural variation compatible with approximately similar temporal organization.

Comparative research can then ask whether the matched systems respond similarly to perturbation. For intervention $\mathcal{U}_k$, response dispersion is represented by Equation 444.

$$D_k^{\text{resp}} = \sup_{i,j \in \mathcal{C}_{\Theta^*}^{\varepsilon}} d_{\text{II}} \left(\Theta_{i,k}^+, \Theta_{j,k}^+ \right). \quad (444)$$

Equation 444 provides an empirical measure of interventional divergence within an approximately equivalent pre-intervention temporal class.

A small response dispersion supports the use of a common induced Type-II operator within that empirical class. A large response dispersion indicates that temporally similar systems contain structural differences consequential for intervention.

Comparative structural realizations can therefore connect comparative institutional research with the representation-equivalence framework.

15.5 Simulation of Forward Transformations

This subsection develops computational simulation of the forward mapping from Type-I structure to Type-II temporal representation. Its objective is to evaluate how structural variables and realization conditions generate temporal patterns under controlled computational variation.

Let $\widehat{\mathcal{D}}$ denote a computational dynamical realization model. Simulated trajectory r is represented by Equation 445.

$$x^{(r)}(\cdot) = \widehat{\mathcal{D}} \left(\mathfrak{S}; \eta^{(r)} \right). \quad (445)$$

Equation 445 allows initial conditions, stochastic inputs, parameters, or contextual variables to vary across simulations.

The simulated Type-II representation is represented by Equation 446.

$$\Theta^{(r)} = \mathcal{Q} \circ \mathcal{O} \left[x^{(r)}(\cdot) \right]. \quad (446)$$

Equation 446 reproduces the same realization-observation-representation chain used for empirical data.

The simulated forward temporal set is represented by Equation 447.

$$\widehat{\mathfrak{Z}}_R(\mathfrak{S}) = \left\{ \Theta^{(r)} \right\}_{r=1}^R. \quad (447)$$

Equation 447 approximates the temporal possibility set generated by the structural model.

Simulation can also vary structural configurations systematically. Let $\{\mathfrak{S}^{(1)}, \dots, \mathfrak{S}^{(K)}\}$ denote a structural design set. The computational transformation sample is represented by Equation 448.

$$\widehat{\mathcal{D}}_{\text{sim}} = \left\{ \left(\mathfrak{S}^{(k)}, \Theta^{(k,r)} \right) : k = 1, \dots, K, r = 1, \dots, R \right\}. \quad (448)$$

Equation 448 supports sensitivity, equivalence, reachability, and reconstruction experiments.

For nonlinear systems, simulation is particularly useful for exploring multiple attractors, transient dynamics, bifurcations, and regime changes [5]. Simulation results remain conditional on the specified structural and dynamical model and therefore provide model-based evidence rather than direct empirical identification.

15.6 Reconstruction Experiments

This subsection develops experiments for evaluating the inverse reconstruction methods introduced in Section 10. Its objective is to measure how accurately and stably Type-I structural information can be recovered from Type-II representations under controlled levels of ambiguity, noise, and incomplete observation.

Let $\mathfrak{S}^{(k)}$ denote a known structural configuration used to generate synthetic or benchmark data, and let $\Theta^{(k)}$ denote the resulting temporal representation. A reconstruction algorithm $\mathcal{R}$ produces the estimate represented by Equation 449.

$$\widehat{\mathfrak{S}}^{(k)} = \mathcal{R} \left(\Theta^{(k)} \right). \quad (449)$$

Equation 449 allows the reconstructed structure to be compared with the known generator.

When a meaningful structural distance is available, reconstruction error can be represented by Equation 450.

$$E_I^{(k)} = d_1 \left(\widehat{\mathfrak{S}}^{(k)}, \mathfrak{S}^{(k)} \right). \quad (450)$$

Equation 450 evaluates point reconstruction.

For set-valued reconstruction, success should instead be evaluated through coverage. Let $\widehat{\mathfrak{J}}^{(k)}$ denote the reconstructed compatibility set. Coverage is represented by Equation 451.

$$C^{(k)} = \mathbf{1} \left\{ \mathfrak{S}^{(k)} \in \widehat{\mathfrak{J}}^{(k)} \right\}. \quad (451)$$

Equation 451 tests whether the reconstruction retains the true structural generator.

Reconstruction experiments should vary observation quality and temporal representation. A general experimental design is represented by Equation 452.

$$E_{\text{rec}} = E_{\text{rec}} (N, \sigma_\nu, W, \Delta, Q, M, \lambda), \quad (452)$$

where the factors denote sample size, observation noise, temporal window, resolution, representation operator, model family, and regularization parameter.

Equation 452 allows practical identifiability and stability to be studied systematically.

Reconstruction experiments should distinguish at least three failure modes: the true structure is outside the selected model family, several structures are intrinsically equivalent under the representation, or the available data are insufficient to resolve structures that are theoretically distinguishable.

This distinction prevents algorithmic reconstruction error from being confused with representational non-identifiability.

15.7 Local Sensitivity Estimation

This subsection develops empirical and computational estimation of local structural-to-temporal sensitivity. Its objective is to estimate the Jacobian and related local geometry without assuming that the complete global transformation is known.

For structural coordinate z_j , a finite perturbation h_j can be used to estimate the effect on temporal coordinate θ_i . A central finite-difference estimate is represented by Equation 453.

$$\widehat{J}_{ij} = \frac{\theta_i(z + h_j e_j) - \theta_i(z - h_j e_j)}{2h_j}, \quad (453)$$

where e_j denotes the unit direction associated with structural coordinate z_j .

Equation 453 can be implemented in simulation or in empirical settings where controlled small perturbations are available.

When direct perturbation is unavailable, local sensitivity can be estimated from naturally occurring structural variation. A local regression representation is given by Equation 454.

$$\delta\theta_t = J_{\text{loc}} \delta z_t + \varepsilon_t. \quad (454)$$

Equation 454 treats the Jacobian as an estimated local relation rather than a known derivative.

The estimated singular-value decomposition is represented by Equation 455.

$$\hat{J} = U \Sigma V^\top. \quad (455)$$

Equation 455 can identify strongly visible structural directions, weakly visible directions, and approximate local null spaces.

An empirical null direction v can be defined relative to tolerance ε_J . This condition is represented by Equation 456.

$$\|\hat{J}v\| \leq \varepsilon_J \|v\|. \quad (456)$$

Equation 456 avoids treating small estimated singular values as exact mathematical zeros.

Local sensitivity estimation should be repeated across structural and temporal regimes because J can vary substantially with the operating point. The empirical Jacobian can therefore be represented as a field,

$$\hat{J} = \hat{J}(z, t, \rho), \quad (457)$$

where ρ denotes the active regime.

Equation 457 provides an empirical route toward the regime-dependent tangent governance framework developed earlier.

15.8 Validation of Representation Equivalence

This subsection develops empirical validation of structural equivalence under Type-II representation. Its objective is to determine whether systems grouped as equivalent by a representation remain sufficiently similar under additional observations, alternative representations, contexts, and interventions.

Let $\hat{\mathcal{C}}_\Theta$ denote an empirically estimated equivalence class. This class is represented by Equation 458.

$$\hat{\mathcal{C}}_\Theta = \{\Theta_i : d_{\text{II}}(\Theta_i, \Theta) \leq \varepsilon\}. \quad (458)$$

Equation 458 uses an empirical tolerance because exact equality is rarely meaningful in observed data.

Equivalence should then be challenged through a richer representation $\mathcal{T}'$. The refinement rate is represented by Equation 459.

$$R_{\text{split}} = \frac{\text{number of classes in } \hat{\mathcal{C}}_\Theta \text{ distinguished by } \mathcal{T}'}{|\hat{\mathcal{C}}_\Theta|}. \quad (459)$$

Equation 459 is one descriptive measure of how strongly the additional representation subdivides the original empirical class.

Interventional validation provides a stronger test. Let $\mathcal{U}$ be a common intervention applied across members of the estimated equivalence class. The post-intervention dispersion is represented by Equation 460.

$$D_{\mathcal{U}} = \sup_{i,j \in \hat{\mathcal{C}}_\Theta} d_{\text{II}}(\Theta_i^+, \Theta_j^+). \quad (460)$$

Equation 460 evaluates whether observational equivalence is also adequate for intervention prediction.

Contextual validation can be performed similarly. For context family $\mathcal{C}$, the maximum contextual divergence is represented by Equation 461.

$$D_{\mathcal{C}} = \sup_{\chi \in \mathcal{C}} \sup_{i,j \in \hat{\mathcal{C}}_\Theta} d_{\text{II}}(\Theta_i(\chi), \Theta_j(\chi)). \quad (461)$$

Equation 461 tests whether equivalence observed under one condition persists across relevant contextual variation.

Representation equivalence should therefore be validated according to its intended use. A class adequate for description can be too coarse for intervention prediction, crisis analysis, or another governance task.

15.9 Transformation Benchmarking

This subsection develops benchmark procedures for comparing alternative structural-to-temporal transformations. Its objective is to evaluate representations according to structural distinguishability, predictive performance, reconstruction quality, robustness, computational burden, and task sufficiency.

Let $\mathfrak{T}_{\text{bench}}$ denote a set of candidate transformations. This set is represented by Equation 462.

$$\mathfrak{T}_{\text{bench}} = \{\mathcal{T}_1, \dots, \mathcal{T}_K\}. \quad (462)$$

Equation 462 can contain different observation architectures, resolutions, temporal representations, model classes, or combinations thereof.

Each transformation can be assigned a multidimensional benchmark profile. This profile is represented by Equation 463.

$$\mathbf{B}(\mathcal{T}_k) = (B_{\text{dist}}, B_{\text{pred}}, B_{\text{rec}}, B_{\text{stab}}, B_{\text{rob}}, B_{\text{comp}}, B_{\text{task}})_k. \quad (463)$$

Equation 463 records structural distinguishability, temporal prediction, reconstruction, stability, robustness, computational burden, and task-sufficiency performance.

One benchmark can evaluate forward prediction. Given held-out structural system $\mathfrak{S}_i$, the prediction discrepancy is represented by Equation 464.

$$E_{\text{forward}}^{(k)} = \frac{1}{N_{\text{test}}} \sum_{i \in \mathcal{I}_{\text{test}}} d_{\text{II}}(\mathcal{T}_k(\mathfrak{S}_i), \Theta_i^{\text{obs}}). \quad (464)$$

Equation 464 evaluates predictive accuracy in Type-II space.

Another benchmark can evaluate task sufficiency. Let $\mathcal{A}$ be the governance task and $\hat{\mathcal{P}}_{\mathcal{T}_k}$ the empirically estimated partition. A task-sufficiency indicator is represented by Equation 465.

$$B_{\text{task}}^{(k)} = \mathbf{1} \left\{ \hat{\mathcal{P}}_{\mathcal{T}_k} \preceq \mathcal{P}_{\mathcal{A}} \right\}. \quad (465)$$

Equation 465 evaluates whether the representation preserves the distinctions required by the selected decision.

Benchmarking should preserve multiple criteria because a transformation with the best reconstruction accuracy can require data or computation unavailable within the decision horizon. A somewhat coarser representation can be preferable when it remains task sufficient and substantially more feasible.

Transformation benchmarking therefore connects formal representation theory with operational representation selection.

15.10 Taxonomic and Representational Revision

This subsection develops revision of the Type-I and Type-II frameworks in response to empirical evidence. Its objective is to preserve the taxonomies as revisable analytical systems while avoiding proliferation of categories in response to every empirical anomaly.

Let $\mathfrak{F}^{(n)}$ denote the complete representation framework at revision stage n . The revision process is represented by Equation 466.

$$\mathfrak{F}^{(n+1)} = \mathcal{R}_{\text{framework}}(\mathfrak{F}^{(n)}, \mathcal{E}^{(n)}), \quad (466)$$

where $\mathcal{E}^{(n)}$ denotes empirical and computational evidence accumulated at the current stage.

Equation 466 treats taxonomy development as an iterative research process.

Revision can occur at several levels. The structural domain can require revised operationalization. A Type-II family can require a narrower applicability condition. The transformation chain can require an additional context variable. A previously assumed equivalence can fail under intervention. A local transformation can require nonlinear terms. None of these cases automatically requires a new first-level governance category.

A candidate taxonomic revision should therefore satisfy a structural necessity condition. Let $\mathcal{C}_{\text{existing}}$ denote the compositional capacity of the existing taxonomy. A new category is motivated only when the observed governance object cannot be represented adequately through existing categories and their combinations. This requirement is expressed by Equation 467.

$$\mathcal{O}_{\text{new}} \notin \mathcal{C}_{\text{existing}} \quad \text{under the stated representational scope.} \quad (467)$$

Equation 467 places a high threshold on taxonomic expansion.

Representational revision has a lower threshold. Observation procedures, temporal coordinates, transformation models, and empirical thresholds can be revised without changing the first-level taxonomy.

The distinction is represented by Equation 468.

$$\mathcal{R}_{\text{framework}} = (\mathcal{R}_{\text{tax}}, \mathcal{R}_{\text{rep}}, \mathcal{R}_{\text{op}}, \mathcal{R}_{\text{emp}}), \quad (468)$$

where the components denote taxonomic, representational, operationalization, and empirical-estimation revision.

Equation 468 allows evidence to improve the framework while preserving category stability where possible.

Revision should also maintain provenance. Let v_n denote framework version n , and let Δ_n record the changes and their evidential justification. The revision ledger is represented by Equation 469.

$$\mathcal{L}_{\text{rev}} = \{(v_n, \Delta_n, \mathcal{E}_n)\}_{n=1}^N. \quad (469)$$

Equation 469 makes representational evolution itself inspectable and revisable.

This procedure is consistent with the broader generative-relational commitment to provisional formalization. The taxonomies provide stable analytical coordinates, while their operationalizations and transformation relations remain open to empirical correction.

Table 14 summarizes the research programme developed in this section.

The research domains summarized in Table 14 provide an empirical route through the principal claims of the transformation framework. Forward transformation can be studied through paired observations and simulation. Structural multiplicity can be examined through comparative realizations. Inverse relations can be tested through reconstruction experiments. Local governance relations can be estimated through sensitivity analysis. Equivalence classes can be challenged through new representations, contexts, and interventions.

The programme also establishes a distinction between empirical failure and taxonomic failure. A particular transformation model can fail while the Type-I and Type-II representational coordinates remain useful. A temporal estimator can fail while the underlying Type-II category remains meaningful. A proposed equivalence class can split under additional evidence without requiring an additional first-level taxonomy. Taxonomic revision is warranted only when the representational object itself cannot be accommodated through the existing structural or spectral-temporal architecture.

The next section develops the *Implications for Generative-Relational Governance*. It examines the complementarity of the two representational coordinates, structural multiplicity under temporal similarity, temporal multiplicity under structural similarity, intervention under representational ambiguity, temporal power across structural realizations, generativity across representational domains, and the revisability of governance representations.

Table 14: Empirical and Computational Research Programme for Structural–Temporal Transformation

Research Domain	Principal Object	Empirical or Computational Function	Framework Contribution
Structural-Temporal Paired Data	$\mathcal{D}_{\text{pair}}$	Links Type-I structural observations with Type-II temporal representations	Provides the basic empirical substrate for transformation analysis
Intervention-Centered Designs	$\mathcal{Z}^{\text{int}}$	Observes structural intervention and subsequent temporal response	Tests induced operators and interventional identifiability
Longitudinal Analysis	$\mathcal{H}_i$	Tracks structural, temporal, and transformation change through time	Evaluates nonstationarity and moving equivalence classes
Comparative Structural Realizations	$\mathcal{C}_{\Theta}^{\varepsilon}$	Compares structurally different systems with similar temporal representations	Tests multiple realization and interventional divergence
Forward Simulation	$\hat{\mathfrak{T}}_R(\mathfrak{S})$	Generates temporal possibility sets from specified structural models	Evaluates reachability, regimes, and forward multiplicity
Reconstruction Experiments	$\hat{\mathfrak{S}} = \mathcal{R}(\Theta)$	Tests structural recovery under controlled ambiguity and noise	Evaluates identifiability, regularization, uncertainty, and stability
Local Sensitivity Estimation	$\hat{J}_{\text{I} \rightarrow \text{II}}$	Estimates structural-to-temporal sensitivity around observed operating points	Operationalizes tangent-space transformation and local control
Equivalence Validation	$\hat{\mathcal{C}}_{\Theta}$	Challenges temporal equivalence through richer observation, context, and intervention	Tests the validity and scope of representation classes
Transformation Benchmarking	$\mathbf{B}(\mathcal{T})$	Compares transformations across predictive, reconstructive, computational, and task criteria	Supports empirically grounded representation selection
Framework Revision	$\mathfrak{F}^{(n+1)} = \mathcal{R}_{\text{framework}}(\mathfrak{F}^{(n)}, \mathcal{E}^{(n)})$	Updates operationalizations and formal relations in response to evidence	Preserves taxonomic stability while permitting representational revision

16 Implications for Generative-Relational Governance

This section develops the conceptual implications of the transformation framework for generative-relational governance. Its objective is to clarify what follows once Type-I structural and Type-II spectral-temporal taxonomies are treated as complementary representational domains connected through context-dependent, partially invertible, and frequently set-valued transformations. The section examines complementarity between the two representations, structural multiplicity under temporal similarity, temporal multiplicity under structural similarity, intervention under representational ambiguity, temporal power across alternative structural realizations, generativity across representational domains, and the revisability of governance representations.

The principal implication is that governance knowledge cannot generally be located within one representational domain alone. Structural representation describes the objects and mechanisms through which governance is organized, while spectral-temporal representation describes temporal modes and relations through which those structures become dynamically expressed. The transformation framework provides an interface between these descriptions while preserving the information, ambiguities, and normative questions specific to each.

16.1 Complementarity of Type-I and Type-II Representations

This subsection develops the complementarity of the two governance representations. Its objective is to clarify why structural and spectral-temporal descriptions supply different forms of information and why neither representation generally subsumes the other.

Type-I governance represents the direct structural support of intervention. For structural intervention $\mathcal{U}$, its Type-I coordinate is represented by Equation 470.

$$\Lambda_I(\mathcal{U}) \subseteq \{\text{SR}, \text{D}, \text{REL}, \text{B}\}. \quad (470)$$

Equation 470 identifies whether governance directly transforms state-and-rule, dynamical-process, relational-structural, or generative-background components.

Type-II governance represents the spectral-temporal objects or relations directly transformed. Its coordinate is represented by Equation 471.

$$\Lambda_{II}(\mathcal{U}) \subseteq \{\text{T}, \text{S}, \text{P}, \text{L}, \text{R}, \text{I}, \text{H}, \text{C}, \}. \quad (471)$$

Equation 471 identifies temporal support through timescale, spectral selection, phase, synchronization and entrainment, resonance, interference, harmonic and polyfrequency structure, cross-frequency relations, or spectral regime.

The joint representation of an intervention is therefore given by Equation 472.

$$\Gamma(\mathcal{U}) = (\Lambda_I(\mathcal{U}), \Lambda_{II}(\mathcal{U})). \quad (472)$$

Equation 472 treats the two taxonomies as orthogonal analytical coordinates.

The transformation framework adds a third object to this joint description: the relation through which structural intervention generates represented temporal consequence. This relation is represented by Equation 473.

$$\mathfrak{G} = (\mathfrak{S}, \Theta, \mathcal{T}). \quad (473)$$

Equation 473 records structural representation, temporal representation, and the transformation relation connecting them.

This triple provides a more complete description of governance than either taxonomy considered separately. Type-I information can identify the institutional or relational location of intervention while leaving its temporal consequences uncertain. Type-II information can identify temporal organization

while leaving its structural realization underdetermined. Knowledge of $\mathcal{T}$ provides the conditional relation through which one representation can inform the other.

Complementarity therefore has an epistemic structure. The structural representation contributes information about generative organization. The spectral-temporal representation contributes information about temporal expression. The transformation contributes information about their relation.

The three objects can also fail independently. A structural model can be mispecified while temporal measurement remains accurate. A temporal representation can be poorly selected while the structural description remains useful. The transformation relation can be uncertain even when both representations are individually well documented.

This separation supports representational humility. Disagreement about one coordinate does not require abandonment of the entire governance framework.

16.2 Structural Multiplicity and Temporal Similarity

This subsection develops the implications of multiple Type-I structures sharing similar Type-II representations. Its objective is to clarify how temporal similarity can coexist with institutional, relational, dynamical, and generative-background heterogeneity.

For temporal representation Θ , the structural compatibility set is represented by Equation 474.

$$\mathcal{J}(\Theta) = \{\mathfrak{S} \in \mathcal{M}_I \mid \mathcal{T}(\mathfrak{S}) = \Theta\}. \quad (474)$$

Equation 474 can contain structurally heterogeneous systems even when their represented temporal organization is identical.

The internal Type-I diversity of the set can be represented through component projections. This profile is given by Equation 475.

$$\mathbf{H}_I(\Theta) = (\Pi_R[\mathcal{J}(\Theta)], \Pi_F[\mathcal{J}(\Theta)], \Pi_C[\mathcal{J}(\Theta)], \Pi_B[\mathcal{J}(\Theta)]). \quad (475)$$

Equation 475 identifies which structural dimensions remain heterogeneous within one temporal class.

This multiplicity has an interpretive implication. Similar rhythms, synchronization patterns, waiting distributions, spectral concentrations, or regime transitions can emerge through different structural pathways. A common Type-II pattern therefore identifies a temporal phenomenon before it identifies one structural cause.

Structural multiplicity also produces a comparative research opportunity. Systems displaying similar temporal organization can be compared through their Type-I realizations to identify which structural differences remain compatible with the same temporal effect.

The stronger comparison concerns intervention. Let $\mathcal{U}$ be a common structural intervention. The post-intervention response set is represented by Equation 476.

$$\mathfrak{R}_{\mathcal{U}}(\Theta) = \{\mathcal{T}[\mathcal{U}(\mathfrak{S})] \mid \mathfrak{S} \in \mathcal{J}(\Theta)\}. \quad (476)$$

Equation 476 reveals whether temporal similarity remains sufficient for predicting intervention consequence.

A narrow response set indicates that the unresolved structural multiplicity has limited consequence for the contemplated action. A broad response set indicates that the apparent temporal similarity conceals governance-relevant structural differences.

Temporal similarity therefore has several possible levels of practical strength: descriptive similarity, predictive similarity, intervention similarity, and similarity of future temporal possibility sets.

These levels should be distinguished empirically.

16.3 Temporal Multiplicity and Structural Similarity

This subsection develops the complementary implication that one Type-I structure can generate several Type-II temporal organizations. Its objective is to show how structural similarity can coexist with temporal heterogeneity through state, context, regime, history, or stochastic realization.

For structural system $\mathfrak{S}$, the forward temporal set is represented by Equation 477.

$$\mathfrak{T}(\mathfrak{S}) = \{\mathcal{T}_{\eta, \chi}(\mathfrak{S}) \mid (\eta, \chi) \in \mathcal{E}_{\text{adm}}\}. \quad (477)$$

Equation 477 contains the temporal representations generated from one structural system under admissible realization and contextual conditions.

Temporal multiplicity implies that one observed trajectory occupies only part of the structural system's temporal possibility space. The observed representation is represented by Equation 478.

$$\Theta^{\text{obs}} \in \mathfrak{T}(\mathfrak{S}). \quad (478)$$

Equation 478 does not determine the remaining members of $\mathfrak{T}(\mathfrak{S})$.

This distinction changes the interpretation of governance stability. Stable current temporal behavior can coexist with latent alternative regimes. Structural systems can contain multiple attractors, transient modes, or context-dependent temporal responses [5].

A useful structural property is therefore the geometry of the forward temporal set rather than only its current member. Let $\mathcal{G}_{\text{safe}}$ denote a temporally admissible region. A structural system's temporal robustness can be represented by Equation 479.

$$\mathfrak{T}(\mathfrak{S}) \subseteq \mathcal{G}_{\text{safe}}. \quad (479)$$

Equation 479 is a stronger condition than observing the present temporal state inside the admissible region.

The implication for governance is significant. Structural evaluation should consider which temporal organizations the system can generate under plausible future conditions, including stress, scarcity, transition, or changing participation.

A structurally unchanged institution can therefore become temporally problematic without requiring prior institutional reform. Conversely, temporal instability can sometimes be addressed by changing state or context within the existing structure.

The distinction prevents structural reform from becoming an automatic response to every Type-II change.

16.4 Intervention Choice under Representational Ambiguity

This subsection develops governance choice when structural reconstruction remains incomplete. Its objective is to identify conditions under which action can proceed safely across a structural compatibility set and conditions under which additional information becomes decision relevant.

Let $\mathfrak{J}(\Theta^{\text{obs}})$ denote the current structural compatibility set and let $\mathfrak{U}^{\text{adm}}$ denote available structural interventions.

For intervention $\mathcal{U}$, the uncertainty-conditioned temporal outcome set is represented by Equation 480.

$$\mathfrak{D}_{\text{II}}(\mathcal{U} \mid \Theta^{\text{obs}}) = \left\{ \mathcal{T}[\mathcal{U}(\mathfrak{S})] \mid \mathfrak{S} \in \mathfrak{J}(\Theta^{\text{obs}}) \right\}. \quad (480)$$

Equation 480 retains prediction uncertainty generated by unresolved structure.

Let $\mathcal{G}_{\text{II}}$ denote the acceptable temporal outcome region. A robustly temporally admissible intervention satisfies the condition represented by Equation 481.

$$\mathfrak{D}_{\text{II}}(\mathcal{U} \mid \Theta^{\text{obs}}) \subseteq \mathcal{G}_{\text{II}}. \quad (481)$$

Equation 481 permits structural ambiguity to remain when every compatible structure generates an acceptable temporal consequence.

A broader governance condition can include structural and generative constraints as well. Let $\mathcal{A}_{\text{gov}}(\mathfrak{S}, \mathcal{U})$ be an indicator of intervention admissibility under the relevant governance criteria. Robust admissibility is represented by Equation 482.

$$\mathcal{A}_{\text{gov}}(\mathfrak{S}, \mathcal{U}) = 1 \quad \forall \mathfrak{S} \in \mathfrak{I}(\Theta^{\text{obs}}). \quad (482)$$

Equation 482 establishes a task-specific reason why full structural identification can be unnecessary.

Additional information becomes operationally valuable when it changes the admissible intervention set. Let $\mathfrak{I}_1$ and $\mathfrak{I}_2 \subseteq \mathfrak{I}_1$ denote compatibility sets before and after information acquisition. The decision-relevant information condition is represented by Equation 483.

$$\mathcal{U}_{\text{gov}}(\mathfrak{I}_2) \neq \mathcal{U}_{\text{gov}}(\mathfrak{I}_1). \quad (483)$$

Equation 483 identifies information that alters governance possibility or selection.

This provides a practical epistemic criterion. Information has governance value when it changes which interventions remain admissible, how their consequences are ranked, or whether further action should be deferred.

The framework therefore supports selective information acquisition. It directs attention toward distinctions capable of changing governance action.

16.5 Temporal Power across Structural Realizations

This subsection develops the implications of structural multiplicity for temporal power. Its objective is to show how capacities to shape time can be realized through different Type-I mechanisms and why similar Type-II outcomes can carry different distributions of power.

Temporal power is understood here as relational capacity to shape temporal conditions affecting participation, coordination, access, waiting, pace, sequence, synchronization, interruption, or temporal opportunity. Social and political research has long shown that schedules, waiting, acceleration, and temporal discipline can organize asymmetrical relations [17, 15, 1, 6].

Let actors or relational positions be indexed by $a \in \mathcal{A}$. A temporal opportunity vector for actor a is represented by Equation 484.

$$\mathbf{O}_a = (o_a^{\text{access}}, o_a^{\text{waiting}}, o_a^{\text{pace}}, o_a^{\text{phase}}, o_a^{\text{coord}}, o_a^{\text{revision}}). \quad (484)$$

Equation 484 is an application-dependent representation of temporal conditions affecting an actor's capacity to participate and generate future action.

A structural intervention changes these temporal conditions through the structural-to-temporal transformation. The actor-specific change is represented by Equation 485.

$$\Delta \mathbf{O}_a = \mathbf{O}_a[\mathcal{T}(\mathcal{U}(\mathfrak{S}))] - \mathbf{O}_a[\mathcal{T}(\mathfrak{S})]. \quad (485)$$

Equation 485 permits temporal effects to be distributed unevenly across relational positions.

Two structural interventions can generate a similar aggregate Type-II outcome while distributing temporal opportunities differently. This possibility is represented by Equation 486.

$$\Theta_a^+ \approx \Theta_b^+, \quad \left\{ \Delta \mathbf{O}_i^{(a)} \right\}_{i \in \mathcal{A}} \neq \left\{ \Delta \mathbf{O}_i^{(b)} \right\}_{i \in \mathcal{A}}. \quad (486)$$

Equation 486 identifies distributional difference hidden by aggregate temporal similarity.

This relation is particularly important for synchronization. Increased coordination can reduce waiting for some participants while requiring others to reorganize their schedules around a dominant temporal center. Reduced system-level variance can therefore coexist with asymmetric temporal burdens.

Temporal power also has a structural realization. Let $\mathcal{P}^T$ denote a temporal power relation and $\text{Lift}_{\mathcal{T}}(\mathcal{P}^T)$ its compatible structural realizations. This relation is represented by Equation 487.

$$\text{Lift}_{\mathcal{T}}(\mathcal{P}^T) = \{\mathcal{P}_1 : \mathcal{P}_1 \text{ generates the selected temporal power relation}\}. \quad (487)$$

Equation 487 emphasizes that one temporal asymmetry can be generated through several institutional, relational, or infrastructural arrangements.

Temporal power should therefore be analyzed jointly through temporal effects and structural realization.

16.6 Generativity across Representation Domains

This subsection develops the relation between generativity and the two governance representations. Its objective is to clarify how temporal organization can condition generative possibility while preserving the distinction between generativity, justice, and structural governance.

Within the generative-relational framework, generativity concerns the conditions through which actors and relational systems can continue producing, learning, adapting, relating, revising, and generating future possibilities.

Let $\mathbf{G}(\mathfrak{S}, \Theta)$ denote a context-dependent generative condition vector. This vector is represented by Equation 488.

$$\mathbf{G} = \mathbf{G}(\mathfrak{S}, \Theta). \quad (488)$$

Equation 488 allows generativity to depend jointly on structural organization and temporal conditions.

The Type-I domain can influence generativity through access to resources, rules of participation, relational topology, information architecture, institutional capacities, and generative backgrounds. The Type-II domain can influence generativity through available time, coordination opportunities, phase compatibility, waiting, recurrence, interruption, temporal diversity, and regime persistence.

The change in generative conditions associated with intervention $\mathcal{U}$ is represented by Equation 489.

$$\Delta \mathbf{G}_{\mathcal{U}} = \mathbf{G}(\mathcal{U}(\mathfrak{S}), \mathcal{T}[\mathcal{U}(\mathfrak{S})]) - \mathbf{G}(\mathfrak{S}, \mathcal{T}(\mathfrak{S})). \quad (489)$$

Equation 489 combines structural and temporal changes within one generative evaluation.

This formulation permits temporally similar interventions to generate different effects on generativity. Let $\mathcal{U}_a$ and $\mathcal{U}_b$ be Type-II-equivalent interventions. Their generative effects can satisfy Equation 490.

$$\mathcal{T}[\mathcal{U}_a(\mathfrak{S})] = \mathcal{T}[\mathcal{U}_b(\mathfrak{S})], \quad \Delta \mathbf{G}_{\mathcal{U}_a} \neq \Delta \mathbf{G}_{\mathcal{U}_b}. \quad (490)$$

Equation 490 shows that temporal equivalence does not establish generative equivalence.

The converse can also occur. Different temporal configurations can sustain similar generative conditions when actors possess different modes of temporal organization compatible with continued participation and adaptation.

Generativity therefore supplies neither a universal Type-II target nor a scalar temporal optimum.

A generative viability region can instead be represented by Equation 491.

$$\mathcal{K}_G = \{(\mathfrak{S}, \Theta) : \mathbf{G}(\mathfrak{S}, \Theta) \in \mathcal{G}_{\text{adm}}\}, \quad (491)$$

where $\mathcal{G}_{\text{adm}}$ denotes the contextually specified domain of acceptable generative conditions.

Equation 491 supports governance that maintains conditions for continued generation and revision without requiring unbounded increase of every generative variable.

Generativity also remains distinct from justice. A system can sustain high productive capacity while distributing temporal opportunities, recognition, resources, or burdens unjustly. Generative evaluation

should therefore be combined with procedural, relational, and distributive judgment where those dimensions are relevant.

The transformation framework consequently treats generativity as one governance criterion among several, albeit one particularly important for preserving future relational possibility.

16.7 Revisability of Governance Representations

This subsection develops revisability at the level of the governance representations themselves. Its objective is to ensure that Type-I descriptions, Type-II descriptions, transformation models, equivalence classes, and intervention relations remain open to revision as the governed system and available evidence evolve.

Let the representation framework at time t be represented by Equation 492.

$$\mathfrak{F}_t = \left(\mathcal{M}_I^{(t)}, \mathcal{M}_{II}^{(t)}, \mathcal{T}_t, \mathcal{P}_t, \mathcal{U}_t^{\text{adm}} \right). \quad (492)$$

Equation 492 contains the structural domain, temporal domain, transformation relation, equivalence partition, and admissible intervention set currently used for governance analysis.

New evidence $\mathcal{E}_{t+1}$ can produce a revised framework according to Equation 493.

$$\mathfrak{F}_{t+1} = \mathcal{R}(\mathfrak{F}_t, \mathcal{E}_{t+1}). \quad (493)$$

Equation 493 treats representational revision as part of governance learning.

Revision can affect different components independently. This decomposition is represented by Equation 494.

$$\Delta \mathfrak{F} = \left(\Delta \mathcal{M}_I, \Delta \mathcal{M}_{II}, \Delta \mathcal{T}, \Delta \mathcal{P}, \Delta \mathcal{U}^{\text{adm}} \right). \quad (494)$$

Equation 494 prevents changes in one part of the framework from being interpreted automatically as failure of the other parts.

A temporal representation can be revised while the first-level Type-II taxonomy remains stable. An observation architecture can change while the structural taxonomy remains stable. An equivalence class can split after new evidence while both representational domains remain useful. A transformation model can be replaced without creating an additional taxonomy.

Revisability also concerns retained provenance. Let $\mathcal{H}_{\mathfrak{F}}$ denote the history of representation states. This history is represented by Equation 495.

$$\mathcal{H}_{\mathfrak{F}} = \{\mathfrak{F}_0, \mathfrak{F}_1, \dots, \mathfrak{F}_t\}. \quad (495)$$

Equation 495 permits earlier classifications, assumptions, and transformation relations to remain inspectable after revision.

The revision process can itself be subject to a preservation condition. Let $\mathcal{K}_{\text{rev}}$ denote the set of representation states that preserve traceability, comparability, and recoverability of previous analytical decisions. A revision is representationally admissible when the condition in Equation 496 holds.

$$\mathfrak{F}_{t+1} \in \mathcal{K}_{\text{rev}}. \quad (496)$$

Equation 496 gives provenance and revisability a formal place within governance representation.

This approach also changes the status of taxonomy. Type-I and Type-II are stable analytical coordinates whose empirical application remains revisable. Their stability supports cumulative inquiry, while representational revision allows the framework to respond to new system conditions and evidence.

Table 15 summarizes the principal implications developed in this section.

The implications summarized in Table 15 establish the broader significance of the transformation framework. Structural and spectral-temporal representations provide different views of the same governance process. Their relation is frequently many-to-many, context dependent, historically variable,

Table 15: Implications of Structural–Temporal Transformation for Generative-Relational Governance

Implication Domain	Formal Object	Representational Meaning	Governance Consequence
Type-I–Type-II Complementarity	$(\mathfrak{S}, \Theta, \mathcal{T})$	Combines structural organization, temporal expression, and their transformation	Supports joint analysis without collapsing one taxonomy into the other
Structural Multiplicity	$\mathfrak{I}(\Theta)$	Several Type-I structures can share one Type-II representation	Temporal similarity alone can leave governance-relevant structural ambiguity
Temporal Multiplicity	$\mathfrak{I}(\mathfrak{S})$	One Type-I structure can generate several Type-II realizations	Current temporal state represents only part of the structural possibility space
Representational Ambiguity	$\mathfrak{D}_{\Pi}(\mathcal{U} \mid \Theta)$	Retains intervention outcomes across compatible structural explanations	Supports task-relative action without complete structural reconstruction
Temporal Power	$\{\Delta \mathbf{O}_a\}_{a \in \mathcal{A}}$	Represents distribution of temporal opportunities and burdens	Aggregate temporal similarity can conceal relational asymmetry
Generativity	$\mathbf{G}(\mathfrak{S}, \Theta)$	Evaluates generative conditions jointly across structural and temporal domains	Temporal objectives remain subject to relational, generative, and justice considerations
Representational Revisability	$\mathfrak{F}_{t+1} = \mathcal{R}(\mathfrak{F}_t, \mathcal{E}_{t+1})$	Allows models, transformations, and equivalence classes to evolve with evidence	Preserves cumulative analysis while supporting correction and institutional learning

and partially observable. Governance knowledge therefore emerges through the coordinated use of both representations and through explicit analysis of the transformation relation connecting them.

Several consequences follow. Temporal similarity does not guarantee structural similarity. Structural similarity does not guarantee one temporal future. Temporal objectives can have several structural realizations. Representational ambiguity can remain compatible with action when all plausible structures support the same decision. Similar aggregate temporal outcomes can distribute temporal power and generative conditions differently. The representations themselves require revision as systems, observations, and governance tasks evolve.

The transformation framework therefore extends generative-relational governance beyond classification. It provides a formal language for moving among structural explanation, temporal diagnosis, intervention design, uncertainty, and revision while preserving the distinctions required by each domain.

The next section develops the *Discussion*. Section 17 examines the scope of the transformation framework, its relation to causal explanation, the epistemic meaning of identifiability, the relation between local and global transformation, structural realization of temporal governance, the status of the mathematical analogies employed throughout the paper, principal limitations, and the resulting research trajectory.

17 Discussion

This section interprets the transformation framework developed throughout the paper and clarifies its conceptual scope, epistemic commitments, and formal limits. Its objective is to consolidate the relation between Type-I structural and Type-II spectral-temporal representations without converting that relation into an additional governance taxonomy or a universal mathematical duality. The discussion proceeds through the scope of the framework, the relation between representation and causal explanation, identifiability and governance knowledge, local and global transformation, structural realization of temporal governance, the status of the formal analogies employed in the paper, principal limitations, and the resulting research trajectory.

The central argument can be summarized through the structural-to-temporal chain represented by Equation 497.

$$\mathfrak{S} \xrightarrow{\mathcal{D}} x(\cdot) \xrightarrow{\mathcal{O}} y(\cdot) \xrightarrow{\mathcal{Q}} \Theta. \quad (497)$$

Equation 497 establishes the Type-I–Type-II relation as a generative and representational sequence. Structural organization produces possible dynamics, observation makes selected aspects of those dynamics empirically available, and temporal representation constructs the Type-II objects used for analysis and governance.

The composite transformation is consequently represented by Equation 498.

$$\mathcal{T} = \mathcal{Q} \circ \mathcal{O} \circ \mathcal{D}. \quad (498)$$

Equation 498 provides the formal center of the present paper. The discussion below examines what can and cannot be inferred from this representation.

17.1 Scope of the Transformation Framework

This subsection clarifies the analytical scope of the transformation framework. Its objective is to distinguish the present contribution from a universal theory of governance dynamics, a direct structural–spectral isomorphism, or an additional first-level taxonomy.

The paper begins from two previously distinguished governance representations. Type-I identifies structural objects directly transformed through governance, while Type-II identifies temporal modes and temporal relations directly transformed. The present framework studies the relation between these representations.

Its principal object is therefore the triple represented by Equation 499.

$$\mathfrak{G} = (\mathfrak{S}, \Theta, \mathcal{T}), \quad (499)$$

where $\mathfrak{S}$ is a Type-I structural representation, Θ a Type-II spectral-temporal representation, and $\mathcal{T}$ the context-dependent transformation connecting them.

Equation 499 should be understood as an analytical architecture. It does not assert that every governance system possesses one uniquely correct structural model, one uniquely correct spectral-temporal representation, or one invariant transformation between the two.

The transformation is indexed by realization, observation, representation, and contextual conditions. A more explicit form is represented by Equation 500.

$$\Theta = \mathcal{T}_{\Xi}(\mathfrak{S}), \quad (500)$$

where Ξ collects the conditions under which the structural system becomes dynamically realized, observed, and temporally represented.

Equation 500 places model and measurement dependence inside the formalism rather than treating them as secondary complications.

The framework is therefore most appropriate when at least three conditions hold. First, a meaningful structural representation of the governed system can be constructed. Second, temporally ordered observations permit one or more Type-II properties to be represented. Third, the research or governance problem requires analysis of how structural organization and temporal organization are related.

Some governance questions will remain adequately addressed within Type-I or Type-II alone. A legal classification problem can require detailed structural analysis with little benefit from spectral representation. A descriptive temporal study can remain useful without structural reconstruction. The transformation framework becomes especially relevant when inference or intervention must move between these domains.

The paper also avoids treating every temporal pattern as oscillatory in a strict physical sense. Type-II includes timescale, recurrence, sequence, phase, coupling, regime, and other temporal structures where their operationalization is justified. Spectral and dynamical language functions as a formal vocabulary whose applicability must be demonstrated for the empirical system.

The resulting scope is therefore intentionally bounded. The paper provides a representation theory for relations between two governance taxonomies rather than a complete ontology of social, institutional, political, or ecological systems.

17.2 Representation and Causal Explanation

This subsection examines the relation between structural-to-temporal representation and causal explanation. Its objective is to distinguish a successful transformation model from a demonstrated causal account of the mechanisms generating the observed Type-II pattern.

A transformation of the form

$$\mathcal{T}(\mathfrak{S}) = \Theta$$

establishes that the selected structural representation maps to the selected temporal representation under the specified realization, observation, and representation model. This relation alone does not establish that every structural coordinate appearing in $\mathfrak{S}$ causally contributes to Θ .

The transformation chain helps locate this distinction. A structural variable can enter the model, affect a latent trajectory, disappear under observation, or remain statistically associated with a represented temporal coordinate through another mechanism.

A causal structural claim therefore requires additional assumptions or evidence beyond representational compatibility.

The distinction can be formalized by separating predictive transformation from intervention response. A predictive relation is represented by Equation 501.

$$\Theta = \mathcal{T}(\mathfrak{S}). \quad (501)$$

A governance-relevant intervention relation is represented by Equation 502.

$$\Theta^+ = \mathcal{T}[\mathcal{U}_1(\mathfrak{S})]. \quad (502)$$

Equation 502 asks how the temporal representation changes after a specified structural intervention.

Intervention-centered evidence can therefore strengthen claims about structural mechanisms by testing whether temporal responses vary as predicted. This logic is compatible with the broader system-identification distinction between reproducing observed behavior and determining the structure and parameters of the underlying model [8].

The framework also distinguishes mechanism identification from full structural identification. Suppose several structural systems generate the same Type-II representation but all instantiate the same governance-relevant mechanism. The mechanism can remain identifiable even when detailed structural reconstruction is unavailable.

Conversely, accurate prediction of Θ can coexist with mechanism ambiguity.

The causal interpretation of a Type-II pattern should therefore remain proportional to the structural evidence supporting it. Synchronization does not by itself determine whether the relevant mechanism is mutual coupling, common forcing, asymmetric entrainment, or an externally imposed schedule. Spectral concentration does not by itself establish which structural process produced the concentration.

The transformation framework contributes by making these ambiguities explicit. It separates temporal pattern recognition, structural compatibility, mechanism identification, and intervention response into distinct analytical stages.

17.3 Identifiability and Governance Knowledge

This subsection interprets structural identifiability as a graded form of governance knowledge. Its objective is to replace a binary distinction between complete knowledge and ignorance with several levels of inference relevant to different governance tasks.

The inverse structural relation is represented by Equation 503.

$$\mathcal{I}(\Theta) = \{\mathfrak{S} \in \mathcal{M}_1 \mid \mathcal{T}(\mathfrak{S}) = \Theta\}. \quad (503)$$

Equation 503 is generally set-valued.

The strongest epistemic condition is global structural identification, represented by Equation 504.

$$|\mathcal{I}(\Theta)| = 1. \quad (504)$$

Equation 504 is useful when it can be established, while the framework does not require it as a universal precondition for governance.

A weaker condition concerns identification of a structural feature $\Pi_A(\mathfrak{S})$. Feature-level identifiability is represented by Equation 505.

$$|\Pi_A[\mathcal{I}(\Theta)]| = 1. \quad (505)$$

Equation 505 permits a decision-relevant structural property to be known while the remaining system remains underdetermined.

A still more directly operational condition is task sufficiency. For decision rule $\mathcal{D}_A$, the condition is represented by Equation 506.

$$|\{\mathcal{D}_A(\mathfrak{S}) \mid \mathfrak{S} \in \mathcal{I}(\Theta)\}| = 1. \quad (506)$$

Equation 506 means that unresolved structural ambiguity does not alter the decision supported by the representation.

These relations suggest a hierarchy of epistemic demands,

$$\text{full structural identification} \implies \text{relevant-feature identification} \implies \text{task-sufficient discrimination}, \quad (507)$$

for tasks whose decision structure permits the corresponding implications.

Equation 507 should not be interpreted as a universal logical ordering across every possible feature and task. Its purpose is to distinguish progressively weaker forms of structural knowledge that can remain operationally adequate.

This graded account is especially important under finite data, changing systems, and limited decision horizons. Seeking complete structural reconstruction can consume time and observational resources without changing the intervention choice.

Additional information is governance relevant when it changes the set of admissible or preferred actions. This criterion connects epistemology with the decision problem directly.

The framework therefore supports an epistemic stopping rule: representation can stop being refined when every structural distinction that remains hidden is irrelevant to the decision currently being made.

This principle remains provisional because the relevant decision can change. A representation sufficient for routine allocation can become insufficient for crisis intervention, structural reform, or long-horizon institutional design.

17.4 Local and Global Transformation

This subsection interprets the relation between global transformation and local tangent-space approximation. Its objective is to clarify how local governance knowledge can remain useful even when the global map is nonlinear, non-injective, discontinuous across regimes, or computationally intractable.

The global transformation is represented by Equation 508.

$$\mathcal{T} : \mathcal{M}_I \longrightarrow \mathcal{M}_{II}. \quad (508)$$

Equation 508 can contain several equivalence classes, regime boundaries, disconnected temporal images, and nontrivial dependence on initial state and context.

Near a reference structural configuration Θ^* , a local differential can nevertheless exist. This local map is represented by Equation 509.

$$D\mathcal{T}_{\Theta^*} : T_{\Theta^*}\mathcal{M}_I \longrightarrow T_{\Theta^*}\mathcal{M}_{II}. \quad (509)$$

Equation 509 provides the tangent approximation developed in Section 9.

The corresponding first-order governance relation is represented by Equation 510.

$$\delta\Theta \approx J_{I \rightarrow II} \delta z. \quad (510)$$

Equation 510 can support local sensitivity analysis, short-step intervention, and approximate lifting.

The coexistence of global ambiguity and local informativeness is a central feature of the framework. A Type-II representation can fail to determine the complete structural system while changes near the current operating point remain predictably related to structural perturbations.

This relation can be expressed by Equation 511.

$$|\mathcal{J}(\Theta^*)| > 1, \quad \text{rank}(J_{I \rightarrow II}(z^*)) > 0. \quad (511)$$

Equation 511 represents a system that is globally underdetermined while retaining locally informative structural-to-temporal directions.

This provides a formal basis for finite-step governance under severe epistemic limits. When the system is sufficiently far from major regime boundaries, local intervention can be followed by observation and re-estimation rather than relying on one long-horizon global prediction.

The usefulness of the local model remains regime dependent. Near bifurcations, critical transitions, or discontinuous institutional changes, the neighborhood over which a tangent approximation remains accurate can contract rapidly [5, 14].

Local governance therefore requires repeated verification of the operating region and transformation sensitivity.

The distinction between local and global transformation also cautions against extrapolation. A locally effective structural direction need not remain effective across distant structural states. A local null direction can become temporally visible after the Jacobian changes. A locally reachable temporal target can remain globally inaccessible because of structural constraints along the path.

Local analysis should consequently support iterative governance rather than serve as evidence of a globally linear system.

17.5 Structural Realization of Temporal Governance

This subsection interprets the structural lifting results as a central governance consequence of the Type-I–Type-II distinction. Its objective is to clarify why temporal objectives and structural interventions should remain separate analytical objects.

For temporal target operator $\mathcal{V}_{II}$, the set of structural lifts is represented by Equation 512.

$$\text{Lift}_{\mathcal{T}}(\mathcal{V}_{II}) = \{\mathcal{U}_I : \mathcal{T} \circ \mathcal{U}_I = \mathcal{V}_{II} \circ \mathcal{T}\}. \quad (512)$$

Equation 512 is generally set-valued.

The implication is stronger than a technical statement about inverse maps. Temporal governance objectives can be structurally plural. The same desired change in synchronization, phase organization, timescale, spectral concentration, or regime can potentially be generated through different rules, dynamical mechanisms, relational structures, or generative backgrounds.

The structural differences among lifts can be represented through Equation 513.

$$\Lambda_I(\mathcal{U}_a) \neq \Lambda_I(\mathcal{U}_b), \quad \mathcal{U}_a, \mathcal{U}_b \in \text{Lift}_{\mathcal{T}}(\mathcal{V}_{II}). \quad (513)$$

Equation 513 identifies structurally different realizations of the same represented temporal objective.

Selection among these lifts requires information that the Type-II objective does not contain. Candidate interventions can differ in legal authority, resource demand, distribution of power, institutional burden, reversibility, future option preservation, and effects on generative conditions.

Consequently,

$$\text{temporal equivalence} \not\Rightarrow \text{governance equivalence}. \quad (514)$$

Equation 514 expresses one of the paper's principal conclusions.

The inverse relation is also important. A single structural intervention can produce several Type-II consequences simultaneously. Changing one relational coupling can alter synchronization, phase, resonance, and spectral-regime structure together.

The relation between structural support and temporal support is therefore many-to-many. This relation is represented schematically by Equation 515.

$$\Lambda_I(\mathcal{U}) \Longleftrightarrow_{\mathcal{T}} \Lambda_{II}(\mathcal{U}). \quad (515)$$

Equation 515 denotes a transformation-mediated relation and should not be interpreted as a direct bijection.

This many-to-many structure explains why Type-I and Type-II remain useful as separate taxonomies. Collapsing them would conceal both multiple structural realization and multiple temporal consequence.

17.6 Formal Analogies and Mathematical Restraint

This subsection clarifies the formal status of the mathematical language used throughout the paper. Its objective is to preserve the analytical usefulness of concepts drawn from dynamical systems, signal processing, geometry, inverse problems, and control while limiting claims to the structures actually specified by the model.

The framework uses terms such as state space, trajectory, attractor, bifurcation, frequency, phase, synchronization, resonance, tangent space, Jacobian, kernel, image, inverse, lifting, and equivalence class. These terms have established mathematical meanings in their originating disciplines [5, 2, 11, 8].

Their use in governance requires one of two statuses.

The first status is formal application. A concept is formally applied when the required mathematical object has been defined for the governance system. For example, a Jacobian is meaningful when structural and temporal coordinates are differentiable locally. A spectrum is meaningful when an appropriate time-indexed observable and representation operator have been specified. Synchronization is meaningful when phase or another justified locking relation can be operationalized.

The second status is conceptual analogy. A concept functions analogically when it organizes reasoning without establishing that the complete mathematical structure of the originating theory is present.

The distinction can be represented by the classification in Equation 516.

$$\text{Status}(C) \in \{\text{Formal}, \text{Operational}, \text{Analogical}\}. \quad (516)$$

Equation 516 distinguishes formally defined, empirically operationalized, and conceptually analogical uses of a borrowed concept C .

A formal term should be promoted from analogy to application only when its required objects and relations are specified.

This restraint is particularly important for Fourier language. The paper does not propose a universal direct Fourier transform from

$$(R, F, C, \mathcal{B})$$

to

$$\Theta.$$

Fourier and related time-frequency operations can enter the final representation stage $\mathcal{Q}$ after a structural system has generated a trajectory and the trajectory has become observable.

The relevant relation remains

$$\mathfrak{S} \rightarrow x(\cdot) \rightarrow y(\cdot) \rightarrow \Theta,$$

with spectral analysis operating principally on $y(\cdot)$ or a related temporal object.

The same restraint applies to tangent spaces and differential geometry. Writing

$$D\mathcal{T}_{\mathfrak{S}} : T_{\mathfrak{S}}\mathcal{M}_I \rightarrow T_{\Theta}\mathcal{M}_{II}$$

requires local representation spaces for which tangent constructions are meaningful. In finite-dimensional empirical applications, this can reduce to ordinary local coordinates and Jacobian sensitivity.

Likewise, terms such as field, manifold, gauge, or criticality should retain their disciplinary meaning where used formally and remain explicitly analogical where only conceptual structure is borrowed.

Mathematical restraint strengthens the framework because it preserves falsifiability and operational clarity. The purpose of formalization is to make assumptions and relations inspectable, rather than to lend physical authority to a social description.

17.7 Limitations

This subsection identifies the principal limitations of the present framework. Its objective is to delimit what the current formalism establishes and identify where empirical, mathematical, and normative development remains necessary.

A first limitation concerns structural specification. The Type-I system

$$\mathfrak{S} = (X, x, R, F, C, \mathcal{B})$$

is itself a representation. Empirical systems can contain omitted relations, latent background conditions, changing boundaries, and contested categories. Structural reconstruction cannot recover distinctions absent from the structural model class.

A second limitation concerns observation. The transformation depends on $\mathcal{O}$, and governance systems frequently provide incomplete, aggregated, strategically produced, or institutionally heterogeneous records. Observation-induced equivalence can therefore dominate later Type-II analysis.

A third limitation concerns temporal representation. Different spectral, phase, timescale, and regime methods preserve different aspects of the same observed trajectory. Time-frequency methods additionally involve localization and resolution choices [12, 2, 3]. Type-II representation is consequently method dependent.

A fourth limitation concerns nonstationarity. Structural systems and transformation operators can evolve simultaneously. Equivalence classes, Jacobian rank, reachable sets, and structural lifts can therefore change during the governance process.

A fifth limitation concerns identifiability. A rich Type-II representation does not guarantee unique structural reconstruction. Multiple realization is expected in many systems, and inverse relations can remain set-valued even with accurate observations.

A sixth limitation concerns causal inference. Representation compatibility, predictive fit, and temporal similarity do not by themselves establish causal mechanisms. Intervention-centered or otherwise causally informative evidence is required for stronger structural claims.

A seventh limitation concerns mathematical regularity. Differentiability, metric structure, compactness, linear approximation, or the existence of well-conditioned generalized inverses cannot be presumed across every governance application.

An eighth limitation concerns empirical feasibility. Paired structural and temporal data can be difficult to obtain, especially when structural variables evolve slowly while temporal observations occur rapidly. Longitudinal and intervention-centered datasets can therefore require substantial institutional access and historical reconstruction.

A ninth limitation concerns normative incompleteness. The transformation framework can represent temporal objectives, structural realizations, generative consequences, and distributions of temporal opportunity. It does not derive one universal normative ordering among them.

The distinction between generativity and justice is particularly important. A structurally or temporally generative system can still distribute burdens, recognition, access, or authority inequitably. Normative evaluation therefore requires principles beyond the transformation relation itself.

A tenth limitation concerns scale. Governance systems can contain interacting actors, institutions, infrastructures, and contexts whose relevant timescales span several orders of magnitude. A representation sufficient at one scale can omit mechanisms relevant at another.

These limitations can be summarized through the uncertainty profile represented by Equation 517.

$$\mathbf{L} = (L_{\text{struct}}, L_{\text{obs}}, L_{\text{temp}}, L_{\text{model}}, L_{\text{id}}, L_{\text{causal}}, L_{\text{reg}}, L_{\text{emp}}, L_{\text{norm}}, L_{\text{scale}}). \quad (517)$$

Equation 517 keeps different limitations separate so that improvement in one dimension is not mistaken for resolution of the others.

The appropriate response to these limitations is therefore revision of the relevant representation, observation architecture, model, or empirical procedure. They do not automatically imply proliferation of additional taxonomic layers.

17.8 Research Trajectory

This subsection situates the transformation framework within the continuing development of generative-relational governance. Its objective is to identify a cumulative research trajectory from taxonomy through transformation, empirical validation, and governance application.

The present sequence can be represented by Equation 518.

$$\text{Type-I structural taxonomy} \longrightarrow \text{Type-II spectral-temporal taxonomy} \longrightarrow \text{transformation framework} \longrightarrow \text{empirical validation} \quad (518)$$

Equation 518 treats the present paper as a bridge between classification and empirical intervention research.

Several research directions follow from this sequence.

First, empirical studies can construct paired structural-temporal datasets and estimate which Type-I distinctions are recoverable from particular Type-II representations.

Second, comparative studies can examine multiple structural realization by selecting systems with similar temporal organization and testing whether their structural mechanisms and intervention responses differ.

Third, intervention-centered studies can estimate when Type-I interventions induce well-defined Type-II operators and when intervention splits pre-intervention equivalence classes.

Fourth, local sensitivity studies can estimate $J_{I \rightarrow II}$ and determine which structural directions are visible, which temporal directions are reachable, and how these relations change across regimes.

Fifth, nonstationary studies can track moving equivalence classes and transformation drift through institutional or environmental change.

Sixth, governance design studies can compare several structural lifts of the same Type-II target according to cost, viability, reversibility, revisability, generative conditions, distributional consequences, and institutional feasibility.

Seventh, representation-selection research can evaluate the minimum observation and analytical architecture required for task-sufficient governance under finite time and computational resources.

These directions need not generate additional taxonomies. They principally develop the relations among the existing Type-I and Type-II coordinates.

The framework itself should remain revisable. Let $\mathfrak{F}^{(n)}$ denote revision stage n . Its development is represented by Equation 519.

$$\mathfrak{F}^{(n)} \xrightarrow{\mathcal{E}^{(n)}} \mathfrak{F}^{(n+1)}, \quad (519)$$

where $\mathcal{E}^{(n)}$ contains theoretical, empirical, and computational evidence motivating the revision.

Equation 519 expresses the generative-relational commitment to cumulative but revisable formalization.

Table 16 summarizes the principal interpretive results of the discussion.

The synthesis in Table 16 supports a restrained interpretation of the paper's contribution. The framework does not establish a universal transformation between social structure and temporal spectra. It establishes a formal architecture within which such relations can be specified, tested, compared, locally approximated, inverted under stated conditions, and used for governance design.

The principal theoretical gain lies in preserving distinctions that are often collapsed. Structural location differs from temporal manifestation. Observational equivalence differs from structural identity. Temporal similarity differs from intervention equivalence. Identifiability differs from stability. A temporal target differs from its structural realization. Diagnostic representation differs from governance mechanism. Generativity differs from justice. Local predictability differs from global inversion.

Together, these distinctions allow Type-I and Type-II governance to function as complementary analytical coordinates connected through an explicitly conditional transformation relation.

The final section summarizes the resulting contribution and states the principal conclusions of the paper.

Table 16: Interpretive Synthesis of the Structural–Temporal Transformation Framework

Discussion Domain	Principal Relation	Interpretive Result	Governance Consequence
Framework Scope	$(\mathfrak{S}, \Theta, \mathcal{T})$	Structural and temporal representations are connected through a contextual transformation	The framework studies relations between Type-I and Type-II without introducing a third taxonomy
Causal Explanation	$\mathcal{T}(\mathfrak{S})$ and $\mathcal{T}[\mathcal{U}(\mathfrak{S})]$	Representational fit and intervention response provide different forms of evidence	Temporal pattern recognition should remain distinct from structural causal attribution
Governance Knowledge	$\mathfrak{I}(\Theta)$	Structural knowledge can be global, local, partial, mechanism-specific, or task sufficient	Complete reconstruction is unnecessary when remaining ambiguity does not alter the decision
Local and Global Transformation	$D\mathcal{T}_{\mathfrak{S}}$	Local sensitivities can remain informative under global non-identifiability	Supports iterative finite-step governance under bounded epistemic conditions
Structural Realization	$\text{Lift}_{\mathcal{T}}(\mathcal{V}_{\text{II}})$	One temporal target can admit several structurally different implementations	Temporal equivalence cannot determine governance choice by itself
Mathematical Restraint	$\text{Status}(C)$	Formal concepts require their mathematical conditions to be specified	Prevents analytical analogy from being treated as established social ontology
Limitations	$\mathbf{L}$	Uncertainty arises separately from structure, observation, representation, causality, scale, and normativity	Revision should target the source of limitation rather than multiply categories automatically
Research Trajectory	$\mathfrak{F}^{(n)} \rightarrow \mathfrak{F}^{(n+1)}$	The framework develops through empirical testing and representational revision	Supports cumulative and revisable development of generative-relational governance

18 Conclusion

This paper developed a transformation framework connecting the Type-I structural and Type-II spectral-temporal representations of generative-relational governance. Its objective was to formalize how structural organization becomes temporally expressed, how temporal representations constrain structural inference, and how governance interventions can be related across the two representational domains. The framework was developed as a theory of relations between two previously established taxonomies. It therefore introduces neither a Type-III taxonomy nor a universal structural–spectral duality.

The central forward relation developed in the paper is the generative transformation chain represented by Equation 520.

$$\mathfrak{S} \xrightarrow{\mathcal{D}} x(\cdot) \xrightarrow{\mathcal{O}} y(\cdot) \xrightarrow{\mathcal{Q}} \Theta, \quad \mathcal{T} = \mathcal{Q} \circ \mathcal{O} \circ \mathcal{D}. \quad (520)$$

Equation 520 places dynamical realization, observation, and temporal representation between Type-I structure and Type-II description. This factorization establishes why the relation cannot generally be reduced to a direct spectral transform of structural components. Spectral and time-frequency methods can operate within the representation stage, while the complete transformation also depends on how structure generates trajectories and how those trajectories become observable.

The framework showed that this transformation is generally many-to-many. Distinct structural systems can share the same Type-II representation, while one structural system can generate several temporal realizations under different initial states, contexts, parameters, or regimes. The corresponding inverse relation is therefore generally set-valued. Its principal structural compatibility relation is represented by Equation 521.

$$\mathcal{J}(\Theta) = \{\mathfrak{S} \in \mathcal{M}_I \mid \mathcal{T}(\mathfrak{S}) = \Theta\}. \quad (521)$$

Equation 521 provides the basis for representation-relative equivalence, structural identifiability, approximate reconstruction, and information-preservation analysis. The paper further distinguished dynamical, observational, and temporal-representational sources of indistinguishability. Structural non-identifiability can therefore be located within the transformation chain rather than treated as one undifferentiated limitation.

Several levels of recoverability were consequently distinguished. Complete structural identification is the strongest case. Local identification, parameter identification, mechanism identification, and identification of selected structural properties can remain available under weaker conditions. For governance purposes, the framework introduced the further criterion of task-sufficient representation: unresolved structural differences can remain acceptable when they do not alter the decision relevant to the current governance task. This permits representational refinement to stop at the level required by action rather than at complete reconstruction of the system.

The intervention analysis extended these results beyond passive observation. A Type-I structural intervention induces a well-defined Type-II operator when it preserves the equivalence relation generated by the selected representation. The principal relation is represented by Equation 522.

$$\mathcal{T} \circ \mathcal{U}_I = \overline{\mathcal{U}}_{II} \circ \mathcal{T}. \quad (522)$$

Equation 522 formalizes when the temporal effect of a structural intervention can be defined independently of which structurally compatible realization underlies the observed Type-II state. Failure of this condition produces interventional divergence and can make intervention itself informative about previously hidden structural differences.

The reverse governance problem was developed through structural lifting. A desired Type-II transformation generally corresponds to a set of Type-I structural realizations. Consequently, temporally equivalent governance outcomes can arise through different rules, dynamical processes, relational structures, or generative backgrounds. These realizations can differ in cost, authority, viability, reversibility,

revisability, distributional consequences, and effects on generative conditions. Temporal equivalence therefore remains distinct from governance equivalence.

The local formulation provided a complementary result. Even when the global transformation is nonlinear, non-injective, or only partially known, a local differential relation can remain informative:

$$\delta\Theta \approx J_{I \rightarrow II} \delta z. \quad (523)$$

Equation 523 supports local sensitivity analysis, identification of null directions, local structural lifting, and finite-step governance. Global non-identifiability and local governability can therefore coexist. This result is especially relevant when information, computation, or decision time prevents construction of a reliable global model.

The paper also extended the transformation framework to nonstationary systems. Structural systems, observation architectures, temporal representations, equivalence classes, Jacobians, reachable sets, and admissible intervention sets can all evolve through time. A representation sufficient under one regime can become insufficient under another. A structural intervention that induces a stable Type-II operator under one set of conditions can lose that property after structural or spectral-regime change. Representational adequacy is consequently historical and conditional rather than permanently attached to one analytical model.

The resulting governance architecture separates several operations that can otherwise be conflated:

temporal observation $\rightarrow$ diagnosis $\rightarrow$ temporal objective $\rightarrow$ structural lifting $\rightarrow$ admissibility $\rightarrow$ selection $\rightarrow$ intervention

Temporal diagnosis supplies evidence about system organization in time. Structural lifting identifies possible Type-I realizations of a Type-II objective. Governance selection evaluates those realizations through structural, operational, generative, and normative criteria. The resulting process is recursive because post-intervention observation can revise the structural compatibility set, the transformation model, and the intervention space itself.

Taken together, the paper proposes that the relation between Type-I and Type-II governance is best represented through the triplet

$$(\mathfrak{S}, \Theta, \mathcal{T}),$$

where $\mathfrak{S}$ describes structural organization, Θ describes spectral-temporal organization, and $\mathcal{T}$ describes the conditional relation through which one becomes represented in the other. The transformation itself can be uncertain, locally approximable, set-valued in inverse use, regime dependent, historically variable, and empirically revisable.

The principal contribution is therefore a representational theory of generative-relational governance across structural and spectral-temporal domains. Type-I and Type-II remain distinct analytical coordinates. Structural similarity does not imply temporal similarity. Temporal similarity does not imply structural identity. Observational equivalence does not imply interventional equivalence. Identifiability does not imply stable reconstruction. Temporal targets do not determine unique structural realizations. Generative consequences do not determine justice. These distinctions provide the conditions under which information can move between the two representations without erasing the uncertainty and heterogeneity that governance must manage.

The framework remains provisional by design. Its next stage is empirical and computational: paired structural-temporal observation, intervention-centered analysis, longitudinal study, local sensitivity estimation, reconstruction experiments, validation of equivalence classes, and comparison of alternative transformation models. Such work can revise operationalizations and transformation relations while retaining the Type-I and Type-II taxonomies as stable but revisable coordinates for cumulative inquiry.

Within this architecture, governance becomes neither purely structural nor purely temporal. It becomes the practice of understanding and acting across the relations through which structures generate temporal possibilities, temporal observations reveal partial structural information, and interventions reshape both the system and the space of its future generativity.

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