Trust Formation for Reliable Cooperation

A dynamical-system and algebraic preliminary discussion

Wanhong HUANG
huangwanhong@serendip.ngo

Abstract

Trust formation presents a practical circularity: actors may need cooperative experience to judge a counterpart, while the encounters producing that experience already require a tolerable basis for participation. This paper examines how such beginnings can change the relation, its participants, and the conditions of later reliance. Drawing on prior Generative Relational work, it treats beings as historically extended relational configurations and investigates constitutive encounters, anticipatory commitment, continuity, compositional meaning, and emergent normative standing. Formation can produce capacities and reasons without making their emergence sufficient for justified trust. The main formal investigation connects a history-bearing spacetime representation to an interacting group-domain field and a boundary-changing algebra. Typed contractions generate two-complex histories; a worked cubic interaction composes a new coupling; creation operators make changes of boundary sector explicit. A regulated ensemble distinguishes stationary fields from correlations. Exact elimination of a mediating field induces a higher-order effective interaction, showing how mediation can disappear from retained variables while its enabling contribution remains. Spin-foam labels require additional representational commitments. Complementary finite models examine prerequisites, observation-based policies, protected-trial diagnosticity, and cumulative exposure; fixed-carrier linear response and finite observable algebras remain limited appendix models. These are several possible constructions, with stated interpretation and preservation obligations. Neither nonlinear mathematics nor algebraic compatibility establishes trust, normative legitimacy, or empirical social quantum dynamics. The enabling role of protected cooperation, historical constitution, and principal GR formal architectures are acknowledged as inherited. The account distinguishes learning about a capacity from generating its relational conditions, while retaining trust, protected reliance, warranted distrust, and responsible termination as possible developments.

Keywords: trust formation; bounded exposure; shared history; mediation; relational capacity; Generative Relational theory

Research Scope

This discussion draft studies the beginnings and development of trust relations. Its primary analytical object is a formation trajectory: a historically extended development that changes both the grounds on which participants may rely and the capacities available for subsequent cooperation. Its GR interpretation treats beings as spacetimes and develops two-complex, operator, and field representations alongside finite operational models. It develops the author's selected second paper in a compact trust research programme. The broader design of reliable cooperation despite incomplete trust remains a companion problem.

The conceptual argument is a proposal within the open Generative Relational research programme. The formal propositions establish consequences of stated assumptions, and the computations check deliberately small illustrative systems. Neither provides a calibrated model of social trust. A configuration, a capability label, an information set, or a likelihood ratio does not stand for the total reality of a relationship. The paper preserves heterogeneous local actors and history-sensitive interpretation without asserting a common ontological substrate.

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Notices

Status. This working draft is prepared for the author's review. It records an evolving research argument; definitions, section structure, and conclusions remain subject to revision. Completion of the draft does not imply publication, acceptance, or final author approval.

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Statement on the use of language models. The author selected the research problem, working title, and paper queue and provided the exploratory discussion and corrections. OpenAI Codex assisted with source retrieval, conceptual analysis, formal reconstruction, drafting, computational examples, and technical checks. This completed working draft awaits the author's review; its new formulations and arguments have not yet received final author approval. Authorship credit remains with the human author. This disclosure should be reviewed before external circulation.

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1 Trust-Formation Trajectories

This section identifies the problem, the analytical object, and the paper's contribution boundary. The inquiry concerns encounters that can help constitute a relation in which later trust becomes intelligible. Its method combines conceptual distinctions, conditional formal constructions, and hypothetical comparisons. The initial task is to explain why a familiar circularity requires a differentiated account of formation.

Two organizations considering a joint project have little experience of each other. Each would prefer evidence of the other's competence, responsiveness, and willingness to honor an undertaking. Much of that evidence can arise only if some work is attempted together. Yet a substantial joint undertaking may expose either organization to losses it cannot accept on the available evidence. A demand for sufficient prior trust can therefore exclude the interactions through which more adequate grounds for trust might develop. Conversely, a demand to proceed immediately can transfer the costs of learning to the party least able to bear them.

This circularity is familiar. Existing research examines high initial trust in new relationships, institutional supports for cooperation, and the production of later grounds for trust through experience [20, 2]. Prior GR work already distinguishes protected cooperation from trust and describes their possible historical connection [12]. The present paper does not claim to discover that cooperation can precede trust. It asks how the dependencies inside that possibility should be specified, and how a formation claim can be distinguished from an attractive retrospective story.

1.1 The formation object

A *formation trajectory* is a historically extended development that alters the conditions of subsequent reliance. It can be studied through situated encounters, including concurrent and differently interpreted histories. Those conditions include available evidence, practical capacities, commitments, interpretive interfaces, exposure, and possibilities of refusal or revision. An encounter may reveal a capacity, create one, damage one, or change what participants understand the capacity to be. Formation consequently concerns changes to the relation and its participants, as well as changes in an observer's estimate of an otherwise stable counterpart.

The trajectory is the analytical object because the significance of a first encounter depends partly on what it enables next. A small cooperative exercise can be successful yet produce no usable basis for a larger undertaking. Participants may learn that a mediator can prevent failure without learning whether direct coordination is possible. They may generate a shared record that one party cannot access. They may discover goodwill while discovering an inability to perform the intended task. These differences disappear if every successful encounter is counted as another increment of trust.

The term does not designate an inevitable developmental sequence. A trajectory can end in warranted distrust, a narrow arrangement of protected reliance, an informed decision to stop, or a relation in which trust becomes more defensible. Its phases can overlap and recur. An apparently new relation may inherit institutions, reputations, professional practices, or collective memories produced elsewhere. The absence of sufficient shared dyadic history is therefore a specific initial condition; it is not an absence of history altogether.

1.2 Formation criteria

The argument separates four questions. First, *entry*: what makes a limited encounter practically possible and acceptably bounded for those involved? Second, *diagnosticity*: what can its observations distinguish about the relevant explanations of conduct? Third, *capacity production*: what abilities, records, commitments, or interfaces become available because the encounter occurred? Fourth, *continuation*: under which supporting conditions do the resulting grounds and capacities remain usable?

These questions are coupled without being interchangeable. Protection may make entry possible while changing the evidential meaning of a successful outcome. A mediator may create an effective

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translation practice that remains dependent on mediation. A repair can disclose responsiveness while consuming a participant's remaining capacity to continue. A well-designed observation can be diagnostic even when the observed relation should be terminated. The existence of coupling is therefore not itself a warrant for optimism.

The title's reference to reliable cooperation specifies a practical purpose. Here reliability concerns a delimited undertaking and its relevant failure modes. It does not mean perfect success, equal benefit, moral legitimacy, or high interpersonal trust. Those predicates require separate arguments. A formation trajectory is of interest when it changes the grounds or capacities for such an undertaking; it need not establish an unrestricted disposition to cooperate.

1.3 Inherited foundations

Three bodies of prior work set important limits. Philosophical and organizational accounts disagree about the attitude, relation, commitment, or willingness constitutive of trust. Those disagreements constrain what a behavioral model can claim to explain. Research on institutions already explains how cooperation can proceed without strong interpersonal trust. GR accounts already emphasize recursive development, endogenous evidence, and historically changing possibilities [11]. None of these general insights becomes new merely by appearing together here.

The proposed contribution is a philosophical and formal reconstruction of their interaction at the point of formation. Its philosophical center concerns historically extended beings whose encounters can constitute capacities, commitments, and conditions of recognition. Its main formal investigation connects formation spacetimes, an interacting group-domain field, and a boundary-changing operator algebra. A cubic interaction specifies how compatible couplings can compose; its contractions generate history complexes. Eliminating a mediating field produces a higher-order effective interaction, rather than demonstrating independent endpoint capacity. Several finite operational models supply dependency tests, distinguish paths from implementable policies, and examine diagnosticity and cumulative exposure. The principal architectures are inherited and the derivations are scoped constructions; no mathematical priority or comprehensive novelty claim is made.

An additional task is internal clarification. An earlier GR discussion proposes that enforced compliance leaves the partner unobserved [15]. This paper treats that proposition as requiring a specified observation channel. A protected final outcome may indeed be uninformative under some conditions, while records of attempts, warnings, or repairs remain informative. Developing this qualification is part of preserving an open research programme: inheritance permits criticism rather than requiring the strongest earlier wording to become doctrine.

1.4 Argument structure

Sections 2–7 establish the conceptual account, including historical relational being, anticipatory commitment, shared meaning, and normative formation. Sections 8–10 develop the spacetime, generative field, and boundary-algebra investigations. Sections 11–15 retain several complementary finite constructions for prerequisites, observation-based policies, diagnosticity, and cumulative constraints. Sections 16–18 examine burdens, comparative sequences, and empirical identification. Sections 19 and 20 state limitations and implications. Appendices A and B preserve the fixed-carrier field reduction and finite observable models. The representations are linked by explicit interpretive roles and preservation requirements, not by an assertion that every model is equivalent.

2 Conceptual Foundations

This section fixes the distinctions needed to interpret a formation trajectory. It considers trust, reliance, justification, trustworthiness, and cooperative reliability, then explains why their disagreement matters for the models that follow. The objective is a usable analytical vocabulary without imposing a synthetic definition that erases substantive differences among trust theories.

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2.1 Trust attribution

Different accounts identify different features of trust. Jones characterizes an affective orientation involving optimism about goodwill and competence; Holton emphasizes a participant stance and distinguishes deciding to trust from deciding to believe someone trustworthy. Hawley's commitment account assigns a different role to the commitments on which reliance turns [17, 6, 5]. These are alternative analyses with overlapping concerns, not interchangeable descriptions of one already settled psychological variable.

Organizational research supplies another distinction useful here. Mayer, Davis, and Schoorman separate willingness to accept vulnerability from the behavior through which risk is actually taken, and distinguish perceived ability, benevolence, and integrity [19]. An observed decision to participate does not by itself reveal willingness, its basis, or the relevant perception of the counterpart. The same act may reflect optimism, obligation, limited alternatives, institutional protection, or a mixture.

This paper therefore uses *trust attribution* for an interpretation that a participant trusts another concerning a specified undertaking. The interpretation must name the relevant sense of trust and the evidence for attributing it. The paper itself primarily analyzes conditions that may enable or obstruct formation. It does not infer the presence of trust merely because a formal goal state has been reached.

This restraint does not make trust analytically idle. It identifies the additional interpretive work needed to connect observable trajectories to trust. If a participant learns that the other responds to being counted on, an affective account may regard that history differently from an account focused on commitments. If a participant deliberately adopts a trusting stance before acquiring stronger evidence, belief updating alone will miss the relevant change. A model can clarify the practical situation while leaving such philosophical judgments open.

2.2 Reliance justification

Reliance concerns the way one actor's undertaking depends on another's conduct, an institution, or an arrangement. It can be protected by monitoring and recourse without an attribution of interpersonal trust. A participant who proceeds because an escrow agent controls release of a resource may rely on a distributed arrangement. Identifying only the most visible counterpart misdescribes where the dependency lies.

Justification is a further question: whether the participant has adequate grounds for the particular attitude or action. A trusting attitude can be poorly grounded. A protected cooperative action can be defensible without a corresponding judgment of the counterpart's general trustworthiness. Evidence can also justify restricting trust to a narrower domain. The formation question must therefore retain the object and scope of the proposed reliance.

Möllering's distinction among expectation, interpretation, and suspension is relevant because trust cannot simply be equated with eliminating uncertainty [21]. This paper does not translate suspension into a numerical confidence threshold. Its narrower point is that practical participation can involve an orientation toward an uncertain future that is not exhausted by the evidence currently available. An account of formation must investigate how such participation becomes possible without treating the remaining uncertainty as either a defect to be abolished or a moral demand for courage.

2.3 Task-specific trustworthiness

Trustworthiness is indexed to what someone is entrusted to do and to the standards relevant to that undertaking. A willing counterpart may lack resources or skill. A competent counterpart may reject the commitment being attributed to it. A participant may act reliably within one arrangement because the arrangement supplies capacities unavailable elsewhere. Formation can consequently change the conditions that make trustworthy performance possible, rather than merely disclose a preexisting quality.

Consider two teams that cannot initially coordinate a sensitive data transfer. An encounter may establish a shared validation procedure, assign responsibility for anomalies, and provide a channel for suspending a transfer. Even if neither team's private motives change, the relation now contains an ability

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Table 1: Formation predicates

Predicate	Question addressed	Insufficient evidence on its own
Participation	Did an actor enter an encounter?	A signature as proof of free consent
Reliance	Which conduct or support sustains the undertaking?	A successful outcome as proof of personal trust
Trust attribution	Which trusting attitude or relation is present?	A larger transfer or longer interaction alone
Justified continuation	Are grounds adequate for this next step?	A favorable general reputation alone
Acquired capacity	What can now be done that was previously unavailable?	Confidence without a changed ability or interface
Reliability	Which requirements hold in which conditions?	Isolated success or internal benefit alone

it previously lacked. Calling this only an increase in information loses the practical production involved. Calling it proof of mutual trust goes too far. The appropriate intermediate description is an acquired cooperative capacity with a specified dependence on people, procedures, and resources.

The distinction is especially important when interpreting learning. Evidence that a team now performs competently need not support the counterfactual claim that it would have performed competently before training or without the new interface. A history can change the object about which evidence is obtained. The current paper treats that possibility as inherited from relational accounts of endogenous evidence, rather than as a refutation of every approach that uses belief updating.

2.4 Cooperative reliability

Cooperation denotes a practical coordination of contributions toward an undertaking recognized, at least sufficiently, by its participants. Their purposes need not coincide fully. Reliability concerns whether the arrangement sustains specified performance requirements under a stated range of conditions. Those requirements may include competent failure reporting and orderly interruption, rather than uninterrupted task completion.

A reliable harmful arrangement is possible. So is an arrangement that benefits its participants while imposing unacceptable costs on outsiders. Reliability thus cannot supply its own moral justification. The paper's later admissibility conditions record selected constraints; they do not derive an entire ethics from successful coordination. Nor does mutual benefit imply equal power. A participant can prefer an arrangement to its available alternatives while those alternatives are themselves unjustly restricted.

Trust and distrust may also concern different aspects of the same relation. A multidimensional approach permits their coexistence [18]. A team can trust a counterpart's technical competence while distrusting its account of commercial incentives. An appropriately limited formation trajectory may clarify this differentiated structure rather than resolve it into a single favorable assessment.

2.5 Analytical comparison

Table 1 records the principal distinctions used throughout the paper. Its purpose is to prevent a change in one dimension from silently counting as a change in every other dimension. The distinctions are analytical: concrete histories frequently affect several dimensions together.

The remaining sections use this vocabulary to resist a common compression: entry produces success, success demonstrates trustworthiness, and demonstrated trustworthiness necessarily produces trust. Each transition requires independent premises. The formation trajectory becomes analytically useful when those premises are made visible.

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3 Historical Relational Being

This section establishes the GR interpretation of the formation object. Its task is to explain why a beginning can change the beings involved, the relation they inhabit, and the conditions of their later intelligibility. It develops that argument through historical constitution, relational reality, anticipation, continuity, and normative address. The mathematical representations introduced later answer selected questions within this account; their state descriptions do not define the full philosophical object.

3.1 Historically extended objects

The GR starting point adopted here treats a being as a historically extended relational configuration: a spacetime whose organization includes the encounters, continuations, and transformations through which it becomes operative. A present description is a situated access to that organization. It may be sufficient for a local purpose, but sufficiency must be established relative to that purpose. The history is not merely an auxiliary list attached to an otherwise complete instantaneous object.

This orientation has a specific consequence for trust formation. Suppose an organization learns to recognize a partner's warning, acquires a procedure for responding, and accepts responsibility for making the response. Those developments partly constitute the organization that can now be relied upon in this relation. To hold the organization fixed while treating the encounters solely as evidence about it would omit a possible change in the object of reliance. The same organization can remain legally continuous while its effective relational capacities change.

The prior account of relational reality distinguishes historically operative participation from a manifestation's occurrence and from stronger claims about ultimate modes of being [10]. A reassuring message can occur without the promised capacity existing. Conversely, a capacity can become operative before participants adequately recognize it. Trust formation must therefore investigate at least two trajectories: what the relation comes to sustain and how that development becomes available to interpretation. Their possible divergence is substantive, not simply measurement error.

Describing the object as a spacetime does not require a preassigned physical manifold or one universal clock. The relevant extent can include overlapping institutional processes, remembered undertakings, asynchronous responses, and material channels. A two-complex can represent selected continuations and rearrangements within that extent. Representation labels and amplitudes can further instantiate a spin-foam model. These are distinct commitments: taking historical extension seriously does not make every graph a spin foam, and specifying a spin foam does not settle the ultimate ontology of the relation.

The point is also stronger than saying that past events influence present decisions. A history can constitute what presently counts as a participant, a response, an obligation, or a feasible action. GR's background-independence programme permits these effective coordinates to change rather than protecting them as a fixed stage [9]. For this paper, the relevant question is local: which coordinates of a prospective cooperative relation are themselves produced through its attempted beginning?

3.2 Constitutive encounters

An encounter can have epistemic and constitutive consequences together. Consider two teams that initially lack a usable translation between their warning categories. A joint exercise can reveal that one team uses a category differently. That is an epistemic consequence. If the teams then establish a translation, assign responsibility for checking it, and rehearse a suspension procedure, the encounter also changes what they can do together. The translation is an acquired relation, not a newly discovered intrinsic property of either team considered separately.

Three premises support the distinction. First, successful future coordination may depend on a practice unavailable before the encounter. Second, that practice can require contributions from both participants and supporting institutions. Third, its operation can alter later actions even when participants' general confidence remains unchanged. Under those conditions, the encounter has produced a relational capacity. Evidence of the capacity is evidence about a changed configuration.

This argument neither requires that every property be relationally constituted nor denies that preexisting competence can be discovered. The same exercise may reveal one skill, produce another, and expose a limitation that remains. Its analytical value lies in preventing revelation from silently standing in for production. The prior trust paper already treats interaction as generating evidence and changing the possibilities to which evidence refers [11]. The formation inquiry specifies where this inherited insight changes the evaluation of a beginning.

Constitutive effects also reach the participant's practical identity. A team can become an accountable collaborator by acquiring a recognized role, a response channel, and a history of answering for its actions. That role is neither a hidden essence waiting to be measured nor an arbitrary name that creates all its own conditions. It depends on conduct, recognition, institutional arrangements, and the continuing possibility of contesting what the role requires. Formation can fail at any of these points.

3.3 Anticipatory commitment

Trust concerns a future that cannot be made wholly present as evidence. A participant may decide to count on another, address it as responsible, or accept vulnerability before the relevant conduct occurs. Philosophical accounts of the participant stance and suspension identify aspects of this orientation [6, 21]. The GR question is how such anticipation enters the relation's subsequent history.

When a participant entrusts a task, it can change the addressee's situation. The addressee may acquire a reason to respond, an opportunity to develop competence, or a burden it did not choose. A commitment can organize later conduct precisely because it was made before the outcome was secured. This temporal structure helps explain why formation cannot be reconstructed only as retrospective classification of completed performances.

The generative possibility is conditional. Address does not automatically confer ability, impose a valid obligation, or guarantee uptake. A demand framed as trust can be coercive when refusal is treated as betrayal despite the absence of a freely accepted undertaking. Similarly, declaring confidence in an under-resourced colleague can transfer responsibility while withholding the means of performance. The possibility that trust helps constitute its own conditions therefore supplies no general argument for demanding greater trust.

A defensible formation account distinguishes an invitation, its uptake, the resources for answering it, and the authority of the resulting expectation. These distinctions permit trust to be productive without romanticizing exposure. A protected beginning can preserve the future orientation of an undertaking while bounding the costs of learning whether its addressee can and will respond. Protection need not eliminate the interpersonal significance of being counted on.

3.4 Continuity criteria

If encounters can change their participants, later reliance raises a continuity problem. On what basis is the later organization the bearer of an earlier undertaking? Numerical sameness of every component is too demanding for institutions that replace staff, procedures, or infrastructure. Continuity of a name is too weak: the same label can conceal a transformed authority or an abandoned practice.

The invariance-oriented GR account directs attention to specified continuities across transformation [9]. For a cooperative undertaking, relevant continuities might include responsibility for an archive, authority to correct an earlier decision, retention of a translation practice, or acceptance of an outstanding obligation. Which of these matters depends on the claim being made. Legal succession, practical competence, remembered familiarity, and moral responsibility can have different continuity conditions.

Formation consequently does not culminate in an unrestricted assertion that the partner has become trustworthy. It can establish a narrower historical entitlement: this continuing arrangement, with these retained capacities and responsibilities, has supplied grounds for this kind of reliance. If the continuity conditions change, the entitlement needs reconsideration. The earlier history remains real, but its current bearing becomes an additional question.

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This also clarifies why a relation can survive transformation without preserving every earlier expectation. Continuity can include a mutually recognized revision. Conversely, apparent stability can conceal the loss of a capacity essential to the undertaking. The two-complex presentation later records identifications and changes explicitly; it cannot decide their normative authority merely by drawing a continuous face.

3.5 Emergent normative standing

Cooperation can generate new objects of evaluation. Before two teams establish a reporting practice, a delayed message may be merely inconvenient. After they assign responsibility, define a response interval, and organize downstream work around it, a comparable delay may become a failure to answer an undertaking. The change concerns the relational structure through which the act becomes normatively addressable.

This is a reconstruction of one possible process, not a universal derivation of obligation from coordination. The agreement may be unauthorized, the response interval unreasonable, or the distribution of labor unjust. Explaining how a norm becomes effective leaves open whether it should govern. The background-independence programme expressly distinguishes the generation of evaluative possibilities from their vindication [9].

The distinction blocks two opposing shortcuts. One treats every effective expectation as authoritative because the relation has stabilized around it. The other regards historically generated expectations as incapable of having authority because their conditions can be traced. Neither follows from the account of formation. A history can supply reasons, commitments, evidence of harm, or grounds for criticism without making genealogy alone the final test of validity.

Recognition matters here because having a grievance and being able to make it count are different conditions. A participant may acquire an obligation to report failures without acquiring standing to challenge the classification of its own report. Formation can then expand cooperative capacity while narrowing the practical space of criticism. An inquiry concerned only with throughput would miss that change.

This paper therefore treats answerability as a proposed evaluative consideration: participants should be able to identify who can respond to an undertaking and how its terms can be questioned. That consideration is argued for by the risks of imposed vulnerability and unilateral interpretation. It is not deduced from a field equation or presented as GR's completed moral theory.

3.6 Formation judgments

The philosophical object now has a more demanding shape. An initial encounter may create a coupling, make a counterpart newly addressable, alter a possibility space, or change what a later observation means. It may also consolidate dependence or confer a misleading appearance of mutual understanding. These effects can occur together. Calling the sequence a formation trajectory identifies the object of inquiry without deciding its success.

A favorable judgment needs several kinds of support. There must be evidence for the capacity or orientation attributed to the relation. There must be a defensible connection between past encounters and the present undertaking. The undertaking's demands must remain open to normative assessment. Finally, a claim that trust itself has emerged must explain which trusting relation or attitude is present. No single formal observable supplies all four.

The investigations below distribute these tasks across representations. Spacetime composition addresses historical constitution; interacting fields specify the generation of coupling structures; boundary algebras describe transformations of the resulting possibilities and access. Finite policies address what a specified protocol can safely select. Their combination deepens the formation analysis because their inferential roles remain explicit.

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3.7 Generative intelligibility

The distinction between learning and formation requires a further philosophical step. An encounter can reveal an ability already operative in a relation. It can also establish the practice through which the ability first becomes possible. A warning that formerly had no recognized recipient can become an accountable request after a response channel is constituted. Describing both processes as an increase in confidence treats a transformation of the grounds of judgment as though it were only a revised judgment.

The relevant possibility is situated. Before the channel forms, an investigator may describe the hypothetical response perfectly well. The participants may nevertheless lack the practical attachments, standing, or shared classifications needed to perform it. Its conceptual availability to the investigator is not its availability within the relation. GR's generative account concerns how the latter availability is produced and sustained.

This does not require a metaphysical claim that a new possibility appears from nothing. Formation inherits languages, resources, earlier couplings, and unresolved conflicts. Its novelty can consist in a composition that changes what these inherited elements can do together. The prior distinction between representation and generative dynamics is useful here: recording a new configuration and specifying a process that can produce it are different explanatory achievements [16]. Neither achievement alone establishes that the process occurred in the case at hand.

The formal investigation therefore needs a kernel that actually composes couplings and an account of the history it generates. Adding nonlinearity to an otherwise unchanged actor model would not by itself answer the constitutive question. Equally, displaying a field action without identifying its couplings would leave the target problem untouched. The models below are assessed by what they make intelligible about the formation of a relation's capacities and conditions of recognition.

4 Initial Participation

This section examines the first admissible encounter. It distinguishes insufficient shared experience from the absence of all background support, then analyzes the apparent circularity of trust formation. The argument is constructive and conditional: a beginning becomes explicable when the supports for a limited encounter are distinguished from the capacities expected to emerge later.

4.1 Inherited backgrounds

Participants who have never met still arrive with histories. They may recognize a professional role, an institutional procedure, a common acquaintance, a publicly recorded commitment, or a material arrangement that limits the consequences of failure. Such conditions can support an initial action without supplying adequate grounds for a broad trusting attitude. Their existence must be investigated rather than stipulated away to create an artificially pure beginning.

Research on initial organizational trust challenges the assumption that new relationships uniformly start from a low level that rises only through dyadic experience [20]. The relevance here is limited but consequential: lack of shared experience does not by itself identify the initial attitude. Participants may bring strong expectations, whether well founded or poorly founded. The question is what supports those expectations and what later encounters can revise.

Several backgrounds can coexist. One participant may rely on a professional credential; another may rely on a deposit; a third may participate because a mediator has promised to remain present. Their reasons need not become identical before the encounter begins. They must, however, be sufficiently compatible with the proposed action. A participant who interprets a preliminary exchange as an enforceable commitment cannot safely be paired with one who understands it as an informal exploration unless that divergence is addressed.

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4.2 The circularity problem

The strongest apparent circularity asserts that trust is required for cooperation and cooperation is required for trust. As a universal claim, the first premise is too strong for the account developed here. Limited cooperation can be organized through supports that reduce the need for interpersonal trust. Cook, Hardin, and Levi explicitly recognize that institutions can enable encounters in which people learn about one another and may later develop trust [2]. The basic escape from the circle is therefore inherited territory.

The more difficult question concerns a specific dependency structure. A demanding undertaking may require a capacity that only an earlier encounter can create. That earlier encounter may require a different and less demanding set of capacities already available. There is no logical circle if these prerequisites are differentiated. For example, negotiating a shared error-reporting protocol may require an accessible communication channel and authority to pause, while deploying a joint system may additionally require the protocol, a tested interface, and agreement about escalation.

There can nevertheless be a genuine obstruction. If every relevant encounter requires a capability that none of the participants possesses and none can produce without that same capability, renaming the sequence "bootstrapping" explains nothing. A seed condition must come from somewhere: an inherited practice, a third party, a smaller undertaking, a unilateral preparation, or a newly accessible resource. Section 11 formalizes this elementary dependency point. The result identifies an obstruction within a specified rule system, not a metaphysical impossibility of beginning.

4.3 Minimal undertakings

A minimal undertaking is minimal relative to the question it can answer or the capacity it can create. It need not be trivial. A jointly drafted definition of a reportable error can expose whether the parties can recognize disagreement without treating it as disloyalty. A small technical test can reveal whether anomaly reporting is prompt enough for the intended coordination. An exchange of non-sensitive samples can establish a format while preserving the possibility of declining a larger transfer.

Smallness alone is inadequate. A short encounter may expose an identity, disclose a location, create a precedent, or confer public legitimacy that cannot be withdrawn. A small monetary amount may carry a serious symbolic implication. An undertaking qualifies as a defensible entry step only after its relevant consequences have been specified for the people concerned. The word "bounded" should identify those limits, not provide a reassuring substitute for their investigation.

The first encounter can also have multiple legitimate purposes. It may aim to establish whether further engagement is possible, rather than to produce favorable evidence. A process that reliably identifies a disqualifying incompatibility can fulfill that purpose. Requiring every initial encounter to count as trust building can pressure participants to reinterpret warranted refusal as a failure of the process.

4.4 Initial undertakings

An invitation to cooperate does more than transmit information. It proposes a relation of address, names an undertaking, and may create expectations about the response. The practical force of the invitation depends on who can issue it, who can refuse, and what refusal is taken to mean. An institutionally powerful actor may describe participation as voluntary while controlling access to opportunities that make refusal feasible.

The earlier GR typology discusses an opening commitment that cannot be demanded from another participant [15]. The present account adopts the narrow normative caution: explaining how someone could take an initial step does not establish that the person owes that step. A formation mechanism can be available without its use being obligatory. In particular, the more vulnerable actor need not bear the first unprotected exposure simply because the relation otherwise remains undeveloped.

Commitments should also be distinguished by their object. A promise to attend one exploratory meeting does not imply consent to the project that the meeting discusses. A promise to respond does not imply a promise to agree. Preserving these boundaries can itself generate evidence about how the

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other treats limited undertakings. It supplies a possible path toward trust without treating expansion as the default interpretation of continued contact.

4.5 Entry assessment

The relevant assessment identifies the initial supports, the proposed encounter, the exposed interests, the record it can produce, and the possible next steps. It also identifies who has authority to stop and whether that authority can be exercised in time. These are practical questions about a particular relation. They cannot be answered by an aggregate indication that a population is generally trusting.

The outcome of an entry assessment may be that no admissible beginning is currently available. That finding should be retained. It can direct attention to missing resources or unequal conditions without recoding the absence of participation as a deficient disposition in those who decline. The formation problem includes the conditions of possible beginnings and the reasons some beginnings should remain unavailable.

5 Bounded Encounters

This section examines encounters designed to make initial participation tolerable. It identifies several meanings of bounded exposure, explains their relation to evidential access, and considers repair as an early formation event. The objective is to specify what protection contributes without treating either protection or exposure as intrinsically productive of trust.

5.1 Exposure specification

Exposure concerns interests whose condition can be affected by another's conduct or by the joint arrangement. Relevant interests may include money, physical safety, confidential information, professional standing, interpretive authority, and the ability to withdraw. Their forms differ. Some losses can be compensated, some effects can be interrupted, and some disclosures cannot be undone. A single expected-loss number would obscure distinctions on which admissibility may depend.

An exposure statement should therefore name the affected participant, the interest, the possible event, the period during which intervention remains effective, and the applicable limit. A limit on transferred funds does not limit subsequent reputational effects. An access restriction does not establish that already disclosed information can be recalled. A right to leave does not establish that leaving is affordable. These distinctions are part of the substantive account, not implementation details added after trust has been discussed.

The relevant boundary is also cumulative. Ten individually affordable encounters may produce an unaffordable sequence, especially when each creates monitoring work or irreversible disclosure. A formation account that treats every encounter as a fresh beginning systematically underdescribes this problem. Section 15 formalizes the distinction between stage limits and trajectory constraints without aggregating unlike interests into one measure.

5.2 Protection mechanisms

Protection can operate at different points in a causal sequence. A mechanism may prevent an unauthorized action, detect an action before its consequences spread, compensate a loss after it occurs, or preserve a route to contest the interpretation of an event. These operations have different effects on what participants can observe. Preventing an act removes the resulting behavioral event; compensation can leave a record of the original act while changing its final consequences.

The familiar institutional role is to make limited cooperation feasible despite uncertainty about a counterpart. The prior public-trust account already articulates this role through bounded commitments, verification, staged implementation, and withdrawal procedures [12]. The present question concerns the

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formation consequences of choosing among such mechanisms. A successful protected interaction establishes, at minimum, something about the performance of the arrangement in the observed circumstances. The further inference to a counterpart's unsupported performance needs additional evidence.

Protection need not be externally imposed. Participants can jointly construct reporting procedures or sanctions. Experimental work on common-pool settings has examined communication and sanctioning arrangements in relation to self-governance [22]. That literature supports treating institutional form as variable rather than assuming a single external guarantor. It does not establish that the same arrangements will be legitimate or effective in every formation setting.

5.3 Evidential targets

An encounter is diagnostic only relative to a target distinction. A test may distinguish whether an interface is compatible while supplying little evidence about motives. A completed delivery may distinguish operational capacity under supervision while leaving behavior without supervision unresolved. A prompt warning can reveal responsiveness even when the underlying task fails. The evidential target should be specified before the outcome is interpreted as a success or failure of trust formation.

This point changes how one designs a protected trial. If the target concerns unsupported task performance, automatic correction of every mistake can obscure the relevant outcome. If the target concerns willingness to report an error, a record of reporting behavior can remain informative while automatic correction protects the recipient. The two channels must not be conflated. Section 14 derives this distinction within a deliberately narrow probabilistic example.

The example supplies no argument for removing safeguards to obtain more authentic behavior. A protection can be modified to preserve a relevant process record, or the investigator can accept that the protected setting answers a narrower question. In some cases the desired evidence cannot be ethically obtained. That is a limit of the inquiry, not a reason to place participants in an otherwise unacceptable encounter.

5.4 Responsive failure

A failed first attempt can contribute to formation when it creates a usable account of what happened and a credible response to it. A counterpart may acknowledge the failure, limit its effects, correct a procedure, and respect the other's decision to suspend further work. Each feature concerns a different part of the relation. The technical task failed; the process of response may nevertheless provide grounds for a particular future undertaking.

Repair literature already treats the restoration of relationships as a multidimensional problem rather than a single return of confidence [4]. The specific formation issue here is that a relation may acquire its first shared account of accountability through an early failure. The paper does not prescribe engineering failures to create such an opportunity. It asks how unplanned failures should be interpreted without allowing either the original harm or the subsequent response to erase the other.

Successful repair is not a universal credit that can be spent elsewhere. Replacing a damaged item does not settle whether private information was exposed. An apology may matter for an interpersonal relation while leaving a defective system uncorrected. A record should preserve these distinctions so that a later assessment can identify what was repaired, what remains unresolved, and which responsibilities were accepted.

5.5 Encounter termination

A bounded encounter needs a practicable end. Participants should be able to identify completion of the agreed task, expiry of the invitation, exhaustion of a resource, or emergence of a disqualifying condition. Without such boundaries, a limited trial can become an indefinite demand to continue demonstrating trustworthiness. The possibility of stopping is therefore part of the encounter's structure.

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Termination can itself be informative. A counterpart that honors an agreed refusal supplies evidence about respect for the undertaking's limits. A counterpart that converts a refusal into retaliation reveals a different relational condition. This observation does not imply that people must risk retaliation to acquire information about it. It identifies why actual termination practices belong in the record when a trial has been undertaken.

Protection, diagnosticity, and termination together define a more exact entry question: whether an encounter can produce relevant evidence or capacity within constraints that remain effective throughout its execution. A favorable answer concerns this encounter. Extending it to a trajectory requires the historical analysis developed next.

6 Shared Relational History

This section examines what an encounter leaves behind. It distinguishes a jointly produced history from a shared interpretation, then considers provenance, memory, capacity formation, and the transfer of experience to new tasks. The aim is to explain why repeated contact is an inadequate proxy for the development relevant to trust.

6.1 Historical records

A shared relational history consists of encounters in which the participants' activities became practically connected. It need not exist as one agreed narrative. Participants may preserve different records, remember different aspects, or dispute who had authority to define success. The history is shared in the sense of co-involvement; its availability and meaning remain separate questions.

A useful record identifies what was proposed, which supports were active, what each party could observe, which actions occurred, and what corrections followed. The point is not exhaustive recording. Some records would violate privacy or impose disproportionate burdens. The point is that the evidential use of an encounter depends on selected contextual information. Removing that information can transform a record of protected performance into an apparently general certificate of trustworthiness.

Attribution is particularly important. If a mediator quietly repaired a counterpart's errors, the final result should not be attributed wholly to the counterpart. If a participant prevented harm by stopping a defective process, a simple count of interrupted tasks can misclassify protective conduct as unreliability. A formation record needs enough structure to preserve these distinctions, including unresolved disagreement about them.

6.2 Interpretive plurality

Participants can disagree about an encounter without either lacking access to its visible outcome. One may regard an exception as a competent adaptation; another may regard it as an unauthorized departure. The resulting problem concerns the applicable standard and the authority to interpret it. More detailed observation alone may not resolve that disagreement.

The relational approach treats interpretation as an activity within the history. A meeting to clarify the status of an exception is itself an encounter that can alter expectations and commitments. It may produce a workable local convention without eliminating deeper differences. Such a convention can support a limited undertaking while leaving other parts of the relationship contested.

This is compatible with heterogeneous local models. Different actors may have distinct state descriptions, inputs, and interpretive rules; communication requires an explicit interface rather than an assumed common semantic space [8]. The finite public observations introduced later represent a selected protocol that the model assumes participants can use. They do not establish universal interpretive agreement.

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6.3 Constitutive effects

An encounter can change the relation through several routes. It can create a procedural capability, allocate a responsibility, establish a precedent, or generate an obligation that did not previously exist. It can also produce fatigue, resentment, dependence, or a new opportunity for exploitation. Formation names the general possibility of such changes; favorable development is only one outcome.

Prior GR work already makes evidence endogenous to interaction and treats future possibilities as history dependent [11]. The present analysis uses that inheritance to distinguish revelation from production. A calibration exercise can reveal an existing mismatch and also create a translation that removes it. A person can display a capacity to respond and simultaneously acquire the specific knowledge needed to respond better next time. These causal roles can coexist within one event.

The distinction matters for counterfactuals. Suppose a pair of teams succeeds after developing a shared troubleshooting practice. The result supports a claim about their present arrangement. It does not establish that they would succeed with an arbitrary other team, before the practice existed, or after the personnel who sustain it have left. The history is part of the ability being evaluated.

6.4 Historical composition

The history relevant to formation includes an organization of significance, not only an event inventory. The prior trust analysis calls certain historically situated, nonadditive configurations *poetic relational composition*: heterogeneous cues acquire significance through their arrangement, timing, and relation to an accumulated history [11]. The term is useful here only where that composition explains something a fixed contribution from each isolated cue cannot explain.

Consider a participant who discloses a limitation before accepting an undertaking. Compare a participant who discloses the same limitation only after its effects become unavoidable. The propositional content can match while the disclosure's bearing on responsiveness differs. Its position in the relation changes what it makes available: an opportunity for informed refusal in the first case, and an explanation after exposure in the second.

The inference requires context. Early disclosure can be strategic, and later disclosure can follow genuinely delayed discovery. Composition supplies a question about the relation among events, not a shortcut to motive attribution. The research burden is to identify the arrangement and compare rival explanations of its significance.

Later encounters can revise the interpretation of an earlier act. A small warning initially dismissed as irrelevant may become recognizable as an attempt to prevent harm after a subsequent failure. The earlier event has not physically changed. What changes is its place in the interpreted history and the reasons it now supplies. Event occurrence, later interpretation, and present practical consequence must therefore be distinguished.

A beginning can consequently acquire significance retrospectively. Participants may discover that what seemed like routine assistance established a pattern of answerability. They may instead discover that apparent openness helped produce dependency. A completed outcome should not be projected backward as if its meaning had been fully available at the start.

The operator example in Section B represents a limited order effect between practice and certification. It does not exhaust this semantic process. Its role is to show exactly how one historically significant distinction disappears under aggregation, while the philosophical account retains forms of interpretation that the finite example does not represent.

6.5 Normative formation

Shared history can change whose response is due and what counts as an adequate response. A jointly accepted reporting practice can make a warning attributable, create an expectation of acknowledgment, and supply grounds for contesting silence. These changes concern the relation's practical organization of reasons. They exceed an increase in knowledge about a fixed set of private dispositions.

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The argument requires uptake and justification. One participant's expectation alone does not establish that the other accepted the duty, and acceptance alone does not justify every imposed term. A relation may make an obligation effective through dependency or exclusion. Explaining that process is compatible with criticizing its result.

The resulting distinction is useful for the formation problem. Some encounters create only a predictable response; others also establish an accountable undertaking. Participants can then ask for reasons, contest a classification, or seek correction by invoking the history. Whether they possess that standing is a separate question from whether a response channel technically exists. A procedure controlled by one party can be highly reliable while denying the other any effective authority over its meaning.

The proposed normative test is consequently relational and revisable: inspect how the undertaking was established, who bears its burdens, who can challenge its interpretation, and how changes in capacity affect what can reasonably be demanded. This is a reasoned orientation for evaluating formation. It is not a theorem that every jointly generated practice is legitimate.

6.6 Memory maintenance

A record can become unusable even when it remains physically stored. Participants may lose the ability to interpret an obsolete format, reconstruct why an exception was granted, or distinguish a preliminary statement from an accepted commitment. Historical availability therefore requires some capacity for retrieval and interpretation. It may depend on people whose labor is easily omitted from descriptions of a technically successful system.

Preserving every detail is neither possible nor desirable. The practical issue is selective retention: which distinctions must survive for the next relevant decisions? A record adequate for repeating a small transaction may be inadequate for expanding access to sensitive resources. Section 12 treats finite summaries as conditional abstractions whose adequacy depends on precisely such preservation requirements.

Memory also distributes authority. Whoever controls the official account can influence what later participants regard as established history. An excluded party may be able to remember a failure without being able to make that memory consequential. A formation account should therefore examine access, correction, and standing alongside storage. Otherwise, the shared-history language can conceal unilateral control of the record.

6.7 Generalization boundaries

Experience transfers when the features relevant to the next undertaking are sufficiently preserved or appropriately reconstructed. A shared vocabulary for one technical task may carry over to a related task; competence with one class of data may not carry over to another. A counterpart's responsiveness during low pressure may have uncertain relevance when deadlines or incentives change. No numerical count of successful encounters resolves these questions automatically.

A generalization claim should state what remains fixed, what changes, and why the observed features bear on the new undertaking. If a mediator remains essential, the claim should retain that dependence. If a newly acquired practice is portable, its portability needs evidence. If the same people use a familiar term in a revised institutional role, identity of vocabulary does not establish identity of commitment.

The formation process thus produces a differentiated historical object. Some parts support renewed participation, some justify restrictions, and some remain uninterpretable. Trust can emerge from that history, but the history's analytical value does not depend on telling a uniformly progressive story about it.

7 Mediated Formation

This section investigates third-party support for a developing relation. It distinguishes access, translation, attestation, protection, and repair, then examines the conditions of eventual handoff. The central claim

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is conditional: mediation can enable a formation trajectory while remaining a constitutive part of the capacity that the trajectory produces.

7.1 Mediating functions

A mediator can connect parties that otherwise lack an accessible channel. It can translate between local vocabularies, attest to the origin of a message, hold resources until a condition is met, or provide a procedure for disputed outcomes. These functions need not reside in one person or institution. A formation analysis should identify the particular function rather than infer a general trusted intermediary from the fact that communication occurred.

The functions also have different evidential consequences. Attestation can establish that a statement came from a specified source without establishing that it is true. Translation can make an instruction usable while changing its framing. Protection can reduce the consequences of error without revealing how a counterpart would behave without it. Repair can restore an arrangement while hiding the original source of the difficulty if the record is incomplete.

Mediation is therefore an intervention in the relation. Even a mediator committed to neutrality selects channels, categories, timings, and procedures. The relevant question is how those selections are accountable to participants and compatible with the limited purpose of the encounter. Neutrality cannot simply be assumed from the mediator's third-party position.

7.2 Distributed grounds

Suppose one party trusts a mediator to identify qualified partners, while another trusts it only to hold a deposit. Their participation can be jointly possible even though the grounds differ. The mediator may provide separate guarantees that fit together operationally. This does not entail that either participant already trusts the other, or that trust in the mediator transfers transitively.

The failure of automatic transitivity is straightforward. Confidence in a mediator's integrity does not establish the competence of everyone it introduces. Confidence in its ability to enforce a limited undertaking does not establish the counterpart's behavior outside that undertaking. Even a correct attestation has a defined object. Expanding its scope requires another premise.

Distributed grounds can nevertheless be productive. Parties may learn about each other during a mediated encounter, develop a common procedure, or create direct channels for later clarification. The mediator can lower the entry requirement while preserving opportunities for such developments. The outcome depends on the form of mediation and on how its contribution is recorded.

7.3 Capacity dependence

Some capacities remain inseparable from mediation. A three-party arrangement may reliably coordinate a task that no pair can execute directly. Describing the mediator as a temporary scaffold in every case imposes a developmental destination that the relation may neither need nor support. Continued mediation can be a stable and legitimate form of cooperation.

The distinction between a temporary and a constitutive support is empirical and task specific. If the mediator only supplies an initial introduction, withdrawal may leave the subsequent capacity intact. If it continually translates, verifies, finances, or adjudicates, withdrawal changes the operating system of the relation. Apparent independence can also depend on unrecorded labor performed by former mediating personnel.

There is a risk of attributing the arrangement's reliability to the participants and then removing the arrangement that makes their conduct reliable. This is not merely a statistical inference error. The intervention changes what the participants can do. Section 15 formulates a restrictive transport condition for policies across support regimes and constructs a simple counterexample to automatic handoff.

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7.4 Handoff inquiry

A handoff should specify which functions cease, which persist elsewhere, and which are newly acquired by participants. Relevant evidence concerns the resulting arrangement, not only past performance. A test conducted with a mediator silently correcting every error cannot establish that the correction function has been transferred. A documented joint repair performed without that assistance may bear more directly on the claim, subject to its exposure limits.

Gradual withdrawal can sometimes make these dependencies visible, but gradualness is not a guarantee. A small reduction can remove a crucial function. Conversely, a large visible change can leave the relevant support intact through another channel. What matters is the structure of the dependency, not the proportion of mediation nominally removed.

The inquiry also needs a return option. If a trial of direct coordination reveals a missing capacity, restoration of support may be more appropriate than interpreting the result as a betrayal. This option must be materially available; a mediator whose funding has ended cannot be restored by a clause in a plan. Formation therefore links epistemic questions about what has been learned to practical questions about which supports remain accessible.

7.5 Mediation accountability

Dependence can create power. A mediator may control admission, certify the official history, or determine which errors become visible. The participants may have unequal ability to challenge its interpretation. A favorable record generated within such an arrangement does not establish that the record is complete or that participation remains voluntary.

The formation-specific issue is whether participants can acquire capacities that make their own contributions and objections consequential. That acquisition need not remove mediation. It can alter the terms on which mediation operates: shared access to records, explicit correction procedures, replaceable technical functions, or authority to stop a particular encounter. The paper does not claim a general institutional design solution; it identifies the dependencies that a formation claim must address.

Mediated formation can consequently lead to several defensible outcomes: direct trust concerning a narrow task, enduring protected reliance, a plural arrangement with distributed trust, or withdrawal after a better understanding of the relation. The conceptual account now turns to formal constructions that test which such trajectories are possible under specified assumptions.

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8 Formation Spacetimes

This section represents formation through the history of a relational organization. It first fixes the incidence convention inherited from the heterogeneous GR framework, then defines composition and a possible spin-foam instantiation. Its purpose is to make changes in the participants' available couplings addressable. The construction represents selected spacetime structure, with declared identifications and boundaries; it supplies no dynamics until an evolution rule or history weight is added.

8.1 Boundary configurations

Following the prior framework, a boundary configuration has primitive elements, couplings, and an incidence assignment [8]. Let P and C be finite sets. Here *primitive* means undecomposed by this representation, rather than metaphysically elementary. A coupling joins at least two elements; every element meets one or two couplings in the graph sector. The resulting boundary has the following data.

$$\Gamma = (P, C, \iota), \qquad \iota : C \longrightarrow \mathcal{P}(P), \qquad |\iota(c)| \geq 2, \quad 1 \leq |\{c : p \in \iota(c)\}| \leq 2. \tag{1}$$

In Equation (1), $\mathcal{P}(P)$ is the power set. The boundary graph has couplings as vertices and elements as edges or half-edges. This convention matters: drawing actors as vertices and interpersonal relations as

edges would define a different graph. Such a graph may be useful, but its interpretation cannot silently replace the inherited convention.

For formation, a selected element might be an undertaking, a role, a report, or a resource; a coupling might join a report to an undertaking through an attribution of responsibility. An actor can occupy several roles and can itself require a subcomplex at a finer scale. Its name at a boundary does not erase its spacetime. A design that tracks only one role must state that limitation.

The incidence bound is a sector restriction. If an element participates in more than two couplings, this presentation needs a refinement or a more general incidence structure. Introducing auxiliary elements or couplings changes the presentation and creates a preservation obligation. Ease of drawing does not establish that the refinement retains all relevant distinctions.

8.2 Historical cells

Let F have vertices V, oriented edges E, and faces Φ, with incidence and boundary data. Vertices represent rearrangement events; edges represent coupling histories; faces represent histories of primitive elements. A face's continuation uses a declared identification across events. For the nonbranching sector, this identification is a partial injection between successive element sets. Introduction and termination occur where an identification has no predecessor or successor.

The finite combinatorial representation of the selected formation spacetime is the following tuple.

$$\mathcal{S} = (F, \Gamma_{\text{in}}, \Gamma_{\text{out}}, \tau, \jmath, \preceq), \qquad F = (V, E, \Phi). \tag{2}$$

Equation (2) supplements the two-complex with incoming and outgoing boundaries, a type assignment τ into declared role and event type sets, continuation identifications $\jmath$, and a causal partial order $\preceq$ on events. These are authored representational choices. A consistent oriented acyclic history induces an event order; metric duration requires further calibration. Not every possible two-complex admits the causal interpretation used here.

At an event v, its incident histories define incoming and outgoing coupling sets C_v^- and C_v^+. After placing identified persistent couplings in a common identifier set, the event's coupling residues are defined as follows.

$$C_v^{\text{create}} = C_v^+ \setminus C_v^-, \qquad C_v^{\text{terminate}} = C_v^- \setminus C_v^+, \qquad C_v^{\text{persist}} = C_v^+ \cap C_v^-. \tag{3}$$

Equation (3) adapts the prior history model's event residues. Without the identification convention, a set difference could mistake a renamed coupling for a newly formed relation.

A calibration encounter can create a coupling between previously incompatible warning practices. A rehearsal can preserve that coupling while changing its labels or response parameters. A withdrawal can terminate it. These are different represented changes even if all three events receive the coarse label *meeting*. A count of meetings cannot recover the residues without additional information.

This formal addressability does not establish that a proposed coupling exists socially. Evidence must connect the cells to conduct, interpretation, resources, or commitments. The benefit is that a claim about formation states what is alleged to have changed and where its history is represented.

8.3 Historical composition

Two histories can compose when the outgoing boundary of the first matches the incoming boundary of the second through an explicit type- and incidence-preserving isomorphism α. Use fresh interior identifiers so that the common boundary is the only intended overlap. The composite is the identified union.

$$F_2 \circ_\alpha F_1 = (F_1 \sqcup F_2)/\!\sim_\alpha. \tag{4}$$

Here $\sqcup$ denotes disjoint union before identifying boundary cells. Equation (4) is a gluing operation, not a claim that participants agree about the identification. A disagreement about institutional succession can be represented by alternative α maps, or by the absence of a warranted map.

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Proposition 8.1 (Compositional continuity)**.** *Suppose three histories have compatible boundary maps and pairwise fresh interiors. The two bracketings of their gluing produce canonically isomorphic cell structures with the same inherited types and continuation data.*

Proof. Start with the disjoint union of the three cell sets in each dimension. Generate one equivalence relation from both prescribed boundary identifications. Either bracketing identifies exactly the pairs in this relation: performing the first quotient changes intermediate class names but adds no identification outside it. Send each class in one bracketing to the class containing the same original cells in the other. This is a bijection preserving incidence, boundary, types, and continuation data because the boundary maps preserve them. Fresh interiors exclude unintended additional identifications. □

Proposition 8.1 restates a structural property of the inherited composition discipline. It warrants comparison of bracketings, not reordering of events. An attribution established before a trial may enable evidence that an attribution established afterward cannot recover. Associativity does not imply commutativity.

The philosophical implication concerns continuity of a formation account. A longer history must explain how later commitments attach to earlier participants and records. It cannot merely place episodes side by side and call the result shared history. The gluing map also does not prove that every carried commitment remains legitimate. Formal continuity and normative justification have separate roles.

8.4 Spin-foam instantiation

A spin foam is one possible richer instantiation of a labelled two-complex. The author's prior papers supply both the representation/intertwiner architecture and the distinction between history structure and its weights [14, 13]. The following finite specification states the additional data. It is an optional model class, not a required interpretation of every formation spacetime.

Choose a compact group G, a finite set J of finite-dimensional unitary irreducible representation labels, and a representation space V_j for each $j \in J$. Assign $j_f \in J$ to each face f. For an edge e, orientation determines whether the adjacent representation space or its dual enters the tensor product. The edge label belongs to the invariant subspace.

$$j_f \in J, \qquad \iota_e \in \mathrm{Inv}_G\left(\bigotimes_{f \supset e} V_{j_f}^{\epsilon_{ef}}\right), \qquad V_j^{+} = V_j, \quad V_j^{-} = V_j^{*}. \tag{5}$$

In Equation (5), $\epsilon_{ef} \in \{+, -\}$ records orientation, Inv_G denotes invariant vectors under the tensor-product group action, and V_j^* is the dual space. Choose an orthonormal basis in each nonzero intertwiner space when summing labels. A zero invariant space excludes that assignment. This introduces representation-theoretic constraints rather than merely renaming a sequence a spin foam.

Given face weights A_f, edge weights A_e, and vertex contractions A_v compatible with these labels, a fixed-complex amplitude with boundary labels b has the following form.

$$Z_F(b) = \sum_{\substack{\{j_f, \iota_e\}_{\text{internal}} \\ \text{compatible with } b}} \prod_f A_f \prod_e A_e \prod_v A_v. \tag{6}$$

The finite cutoff makes the sum in Equation (6) finite for a fixed finite complex and finite intertwiner bases. Local factorization is an assumption. Boundary factors need a convention that counts each shared factor once on gluing. With that convention, composition contracts common boundary indices. It does not amount to multiplying arbitrary normalized probabilities.

For a trust application, the group must express justified transformations of target labels, the intertwiners must express a defensible compatibility condition, and amplitudes need an observation rule. Complex amplitudes do not become probabilities merely by dividing by their sum. Positivity, normalization, and

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probabilistic interpretation are additional obligations. The present paper does not select an empirical spin variable for trust or claim that social processes obey quantum-gravitational dynamics.

This leaves a substantive distinction. The two-complex represents which histories compose. A spin-foam instantiation can additionally restrict compatible labels and weight alternative compositions. A group-field generator can generate complexes through specified interaction kernels, as developed in the prior spacetime paper [13]. Section 9 develops this generative level through a regulated group-domain field, a concrete strand contraction, and its induced effective interaction. The construction makes the creation of the carrier addressable before studying response on a selected carrier.

8.5 Representation interfaces

The paper now has several scales of description. A history spacetime records continuations and rearrangements. A boundary field records selected distributed degrees of freedom. An encounter sequence records one ordering of selected events. A finite protocol state retains only distinctions relevant to a stated decision problem. Each can serve an analytical purpose.

Moving between them requires a map and a preservation claim. Histories with the same initial and final boundaries can differ in intermediate coupling creation, responsibility, and exposure. Boundary agreement alone cannot justify identifying them for a future intervention. Conversely, a policy calculation need not retain every face when its summary provably preserves the policy problem.

Section 10 represents admissible boundary replacements and explains why an occupation vector alone loses incidence. Section 12 states preservation requirements for finite summaries. Appendices A and B examine fixed-carrier response and finite accessible distinctions. Their limited calculations remain useful without occupying the generative role of the main field construction.

9 Generative Relational Fields

This section investigates the production of relational structure. It specifies an interacting field whose contractions generate families of history complexes, develops a compositional example, and derives an effective interaction after eliminating a mediating field. The construction follows the generative architecture of the author's prior group-field and spacetime work [7, 13]. Its contribution here is a bounded realization for analyzing formation, with explicit assumptions and interpretive obligations.

The distinction between generation and reduction is central. The prior architectural study assigns representation, generative dynamics, historical criteria, and observation different analytical tasks [16]. A field on an already selected graph can describe distributed response; it does not by itself explain how the graph's couplings become available. An interacting group-domain field instead specifies how elementary coupling data can compose into complexes with different incidences and boundaries. A generated complex still needs a target interpretation. The field does not supply that interpretation through its mathematical name.

9.1 Coupling fields

Fix a finite group G with normalized counting measure. This finite choice supplies a regulator and an exactly defined integration domain; a compact-group limit is not taken. Let $\mathcal{T}$ be a finite set of coupling types, and let $d_t \geq 2$ be the number of ordered ports for type t. A port is an attachment position for an element history. Its group argument represents a possible label transformation, subject to a proposed realization in the target relation. It is not the position of a pre-existing actor.

For each type, impose simultaneous right invariance. The one-coupling space and its orthogonal projector are as follows.

$$\begin{aligned} \mathfrak{h}_t &= \{f \in L^2(G^{d_t}) : f(g_1 h, \dots, g_{d_t} h) = f(g_1, \dots, g_{d_t})\}, \\ (\mathsf{P}_t f)(\mathbf{g}) &= |G|^{-1} \sum_{h \in G} f(g_1 h, \dots, g_{d_t} h), \qquad \mathfrak{h} = \bigoplus_{t \in \mathcal{T}} \mathfrak{h}_t. \end{aligned} \tag{7}$$

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The inner product is conjugate-linear in its first argument. In Equation (7), averaging twice equals averaging once because multiplication permutes G; replacing h by h^{-1} shows self-adjointness. Thus P_t is indeed an orthogonal projector. The invariance is a model constraint. Its social meaning requires a justified redundancy of label choice; it is not an established gauge symmetry of trust. Port valence d_t is not automatically a physical dimension.

Choose an orthonormal basis $\{e_a\}_{a=1}^{M}$ of the finite-dimensional space $\mathfrak{h}$. A field φ has coordinates z_a, so $\varphi = \sum_a z_a e_a$. Its components concern possible local couplings. A person, organization, or established practice generally requires many couplings and their histories. No coordinate z_a is introduced as the amount of that person's trust.

Let $K : \mathfrak{h} \to \mathfrak{h}$ be positive definite and Hermitian. We use the convention that K is the kinetic operator and K^{-1} the free covariance, following the later spacetime formulation. This convention differs from the earlier linguistics manuscript's notation for its propagator [7, 13]. The distinction prevents an inverse from disappearing through inherited notation.

9.2 Interaction structure

An interaction type β specifies a finite collection of incoming and outgoing coupling slots, their port pairings, their types, and a contraction tensor V_β. Pairings identify the corresponding group arguments and sum them with normalized measure. A typed polynomial $B_\beta(z, \bar{z})$ results. Unlike a coefficient multiplying an existing graph edge, this polynomial specifies an elementary event through which coupling histories can join, split, terminate, or be introduced.

Choose finitely many such polynomials of degree at most m, complex coefficients λ_β, a positive constant ϵ, and an integer p with $2p > m$. Write $r^2 = \sum_a |z_a|^2$. A regulated Euclidean action is defined by the following expression.

$$S(z, \bar{z}) = \bar{z} K z + 2\,\mathrm{Re} \sum_\beta \lambda_\beta B_\beta(z, \bar{z}) + \epsilon r^{2p}. \tag{8}$$

All terms in Equation (8) are dimensionless in this finite representation. The last term is an explicit stabilizing interaction; it also contributes vertices if the whole action is expanded diagrammatically. It is not silently interpreted as a measured social force. Since each polynomial is bounded by a constant times $1 + r^m$, the highest positive term makes S coercive and bounds it below.

For a source $J \in \mathbb{C}^M$, use Lebesgue measure on the real and imaginary components. The generating integral is the following finite-dimensional integral.

$$Z(J, \bar{J}) = \int_{\mathbb{C}^M} \exp\bigl[-S(z, \bar{z}) + \bar{J} z + \bar{z} J\bigr]\; d^{2M} z. \tag{9}$$

The coercive bound implies that Equation (9) is finite for every finite J and that polynomial moments exist. At zero source it defines a positive formal ensemble of field configurations after division by $Z(0, 0)$. This mathematical probability is not an empirical distribution of cooperative histories. The distribution is on field coordinates; diagram weights require a separate interpretation and need not be nonnegative.

This regulator leaves important background choices in place: G, the type vocabulary, admissible interactions, and their coefficients. The construction generates a carrier relative to these choices. It does not prove absolute background independence, a continuum limit, or the emergence of every possible social distinction.

9.3 Generated history complexes

The free measure, obtained by retaining $\bar{z} K z$, has covariance $\langle z_a \bar{z}_b \rangle_0 = (K^{-1})_{ab}$. Expanding each interaction exponential as a formal power series produces products of field factors. Gaussian integration pairs each unbarred factor with a barred factor. Each pairing gives a propagator; the arguments carried through the interaction tensors trace strands.

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With a port-preserving propagator and declared port-pairing tensors, the resulting diagrams have precisely the three incidence levels required here. Interaction vertices are rearrangement events. Propagator edges are coupling histories. Strand cycles or open strand paths bound faces representing element histories. Uncontracted ports supply the boundary. The inverse operation of reading off the cells does not introduce a preselected adjacency graph: each admissible contraction pattern generates one.

For a boundary insertion $\mathcal{O}_\partial$, a schematic organization of these contractions is the following formal series.

$$Z_\partial \sim \sum_{F \in \mathfrak{C}_\partial} \frac{\prod_\beta (-\lambda_\beta)^{n_\beta(F)}}{|\operatorname{Aut}_\partial F|} \mathcal{A}_F(K^{-1}, V_\beta). \tag{10}$$

In Equation (10), conjugate and stabilizing interaction types are included in the index β. The family $\mathfrak{C}_\partial$ consists of stranded diagrams with the chosen boundary insertion; n_β counts vertices of each type; $\operatorname{Aut}_\partial F$ fixes that boundary; and $\mathcal{A}_F$ is the remaining finite index contraction. Coupling normalizations are chosen consistently with the factorials in the interaction expansion. The symbol $\sim$ denotes formal perturbative organization. Finiteness of the regulated integral does not prove convergence of its expansion about the free theory.

The expansion is not yet a chronological history model. Generic diagrams can have cycles, disconnected parts, or orientations incompatible with an intended event order. To obtain a formation spacetime of Equation (2), select diagrams admitting the declared orientation and continuation data, and apply a realization map that preserves the required boundary incidence. This selection changes the ensemble; it cannot be added while claiming to retain the original integral unchanged. A causal model must encode the restriction in its kernels or define the restricted sum separately.

Nor is every stranded diagram a spin foam in the stronger representation-theoretic sense. Decomposing group arguments into representation labels yields the label and intertwiner organization only with the requisite invariant contractions. Section 8 states those additional data. The present construction explains the generative role of the field architecture without treating the phrase *spin foam* as a certificate of social adequacy.

An actor's spacetime is thus represented by an interpreted history subcomplex, not by one field quantum. Formation can change its boundary, introduce a new route for addressing it, or alter the continuities by which a later undertaking attaches to an earlier one. This is the respect in which structure is generated. The common space of all admissible diagrams is still a mathematical representation chosen by the investigator.

9.4 Compositional channel formation

A small rank-two sector makes the generative event explicit. Take three coupling types with fields $U, V, W \in \mathfrak{h}_2$, where $\mathfrak{h}_2$ is Equation (7) with two ports. The group entries $i, j, k \in G$ are internal labels. Define the normalized composition and field inner product as follows.

$$(U \star V)_{ik} = |G|^{-1} \sum_{j \in G} U_{ij} V_{jk}, \qquad \langle W, U \star V \rangle = |G|^{-3} \sum_{i,j,k \in G} \bar{W}_{ik} U_{ij} V_{jk}. \tag{11}$$

Equation (11) is a cubic strand contraction. The i strand joins U to W, the k strand joins V to W, and the j strand joins the two incoming couplings. It is an elementary two-to-one boundary move. Reversing its port orientation supplies a one-to-two move. Rank two is a limited sector whose diagrams have ribbon structure; no four-dimensional spacetime interpretation is inferred.

Figure 1 separates a change of coupling incidence from a change of values on an unchanged actor graph. The persisting external ports also show what the move preserves. Introducing a different attachment rule would define a different model.

Composition preserves simultaneous right invariance. Indeed, in $(U \star V)(ih, kh)$ substitute $j = j'h$; invariance of U and V removes h from both factors. Thus the interaction remains within the declared field spaces. Writing $n = |G|$ and using the Frobenius matrix norm gives $\|U\| = \|U\|_{\mathrm{F}}/n$ and $\|U \star V\| \leq \|U\| \, \|V\|$. This norm estimate will control the effective action below.

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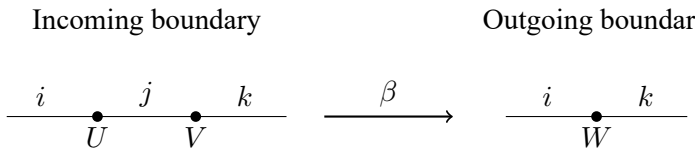

Figure 1: Boundary projection of the cubic composition. Vertices denote couplings, while edges and half-edges denote elements. The event β composes the two input couplings through their shared j port and creates the output coupling. In the event complex, U, V, W label coupling-history legs and their paired strands trace the element histories. The figure shows the selected incidence move, not the whole field ensemble or a pair of actors.

Consider a proposed social realization. U represents a warning's attachment to an undertaking; V represents an undertaking's attachment to an accountable response. Their shared port can support a composite warning-to-response channel W. The contraction requires a compatible intermediate attachment. Mere occurrence of both practices does not suffice. An agent who sends a warning through one classification scheme and another who responds through an incompatible scheme need not have formed the composite channel.

This interpretation is hypothetical. Group summation encodes the allowed compatibility relation; it does not discover the meaning of a warning. Field coefficients describe an assumed generative organization, not numerical sincerity. The example also does not equate the composite channel with trust. It identifies one capacity that a later trust attribution might need.

For positive k_U, k_V, k_W, ϵ and a complex g, a stable interacting action in this sector is the following polynomial.

$$\begin{aligned} S_{\mathrm{c}}(U,V,W) &= k_U\|U\|^2 + k_V\|V\|^2 + k_W\|W\|^2 - 2\,\mathrm{Re}(g\langle W, U \star V\rangle) + \epsilon R^4, \\ R^2 &= \|U\|^2 + \|V\|^2 + \|W\|^2. \end{aligned} \tag{12}$$

The quartic term in Equation (12) dominates the cubic interaction at large norm. It is therefore a special case of the regulated construction, with $p = 2$. Holding U, V as conditioned boundary data, variation with respect to $\bar{W}$ in orthonormal coordinates yields the following stationarity equation.

$$\left(k_W + 2\epsilon R^2\right) W = g(U \star V). \tag{13}$$

Equation (13) is nonlinear and involves a newly composed coupling field. Its significance does not lie in nonlinearity alone: its interaction has the port structure that generates the elementary boundary move.

Proposition 9.1 (Conditional composite channel)**.** *For fixed U, V, Equation* (12) *has a unique minimizing W. This minimizer is nonzero exactly when $g(U \star V) \neq 0$.*

Proof. The W-dependent part is a positive quadratic, a linear term, and $\epsilon(c + \|W\|^2)^2$, with $c = \|U\|^2 + \|V\|^2 \geq 0$. It is strictly convex on the real coordinates of W because $k_W > 0$, and it tends to infinity with $\|W\|$. Hence it has a unique minimizer, characterized by Equation (13). Its coefficient $k_W + 2\epsilon R^2$ is strictly positive. The equation therefore gives $W = 0$ exactly when its right-hand side vanishes. □

Proposition 9.1 concerns a conditioned stationary field, not the probability that a community acquires a capacity. It gives an explicit obstruction: incompatible incoming fields can have vanishing composition even when each field is nonzero. It also separates a pre-existing ability from a composite ability supported by their interaction.

For example, take $G = \mathbb{Z}_2$. Let $I_{ij} = 2\delta_{ij}$ and $P_{ij} = 2\delta_{j,i+1}$, with addition modulo two. Both are invariant under simultaneous translation, and $I \star P = P$. Thus conditioning on $U = I, V = P$ produces a nonzero W when $g \neq 0$. In contrast, the constant kernel $U_{ij} = 1$ and the difference kernel $V_{ij} = (-1)^{i-j}$ are separately nonzero but have $U \star V = 0$. In a target realization this would represent a specific compatibility failure, rather than insufficient quantities of trust.

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9.5 Field correlations

The stationary equation is only one analytical level of the interacting field theory. Its relation to an ensemble requires care. Integration by parts in the finite, coercive measure has no boundary contribution. For a polynomial observable $\mathcal{O}$ and mode a, the resulting identity is as follows.

$$\left\langle \frac{\partial \mathcal{O}}{\partial \bar{z}_a} \right\rangle_J = \left\langle \mathcal{O} \left(\frac{\partial S}{\partial \bar{z}_a} - J_a \right) \right\rangle_J . \tag{14}$$

In Equation (14), $\partial/\partial \bar{z}_a$ is the Wirtinger derivative and $\langle \cdot \rangle_J$ denotes normalization by Equation (9). Taking $\mathcal{O} = 1$ and zero source yields a mean field equation involving higher correlations. In the cubic sector, the W component has the following form.

$$k_W \langle W \rangle + 2\epsilon \langle R^2 W \rangle = g \langle U \star V \rangle. \tag{15}$$

Equation (15) does not generally close on $\langle U \rangle, \langle V \rangle, \langle W \rangle$. Replacing the right side by $g\langle U \rangle \star \langle V \rangle$ would discard cross-correlations; replacing $\langle R^2 W \rangle$ by a product would discard others. These are additional approximations, not identities of GR.

Symmetry makes the distinction concrete. At zero source the action is invariant under $U \mapsto e^{i\alpha} U$, $V \mapsto e^{i\beta} V$, $W \mapsto e^{i(\alpha+\beta)} W$. The finite ensemble therefore has zero one-field means, including $\langle W \rangle$, by invariance under arbitrary phases. Nevertheless, the invariant composite $\langle \bar{W}_{ik} U_{ij} V_{jk} \rangle$ is permitted and need not vanish. A nonzero conditioned minimizer does not establish spontaneous symmetry breaking in this finite system. Boundary conditioning, invariant correlations, and an infinite-limit phase claim are distinct constructions.

For formation, the corresponding methodological point is that averaged manifestations can conceal compositional organization. That interpretation still requires a target observation map. The algebra does not prove that any particular instance of silent cooperation possesses a hidden relational capacity.

9.6 Effective formation interactions

Eliminating a mediating field can change the order of an effective interaction. To derive this exactly, choose a related sector that is Gaussian in W but interacting in U, V. Put $s^2 = \|U\|^2 + \|V\|^2$, let $\mu, k > 0$, and take $\eta > |g|^2/(4k)$. The action is the following expression.

$$S_{\mathrm{m}} = \mu s^2 + \eta s^4 + k\|W\|^2 - 2\,\mathrm{Re}(g\langle W, U \star V \rangle)\,. \tag{16}$$

Equation (16) is a separately specified model, not the result of deleting the W-dependent stabilizer from an otherwise unchanged approximation. Its retained quartic term is chosen to keep the full integral finite. Completing the square gives the following identity.

$$S_{\mathrm{m}} = k \left\| W - \frac{g}{k}(U \star V) \right\|^2 + \mu s^2 + \eta s^4 - \frac{|g|^2}{k} \|U \star V\|^2. \tag{17}$$

The norm bound after Equation (11), together with $\|U\|^2 \|V\|^2 \le s^4/4$, proves that the last three terms in Equation (17) are bounded below by $\mu s^2 + (\eta - |g|^2/(4k)) s^4$. Integration over the shifted W coordinates is therefore legitimate.

Proposition 9.2 (Induced composite interaction)**.** *For Equation* (16)*, integration over W gives, up to a constant independent of U, V, the exact effective action below.*

$$S_{\mathrm{eff}}(U, V) = \mu s^2 + \eta s^4 - \frac{|g|^2}{k} \|U \star V\|^2. \tag{18}$$

Proof. In an orthonormal complex basis of $\mathfrak{h}_2$, translate W by $(g/k)(U \star V)$. Translation preserves Lebesgue measure. If $M_W = \dim \mathfrak{h}_2$, the integral of the first term's exponential in Equation (17) is $(\pi/k)^{M_W}$. The remaining factor is $\exp(-S_{\mathrm{eff}})$. The lower bound just established makes the retained integral finite and justifies the elimination. □

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Proposition 9.2 yields a quartic contraction of retained fields. In strand language, joining two cubic vertices through an internal W propagator creates a four-leg effective vertex. There is no general reduction to a linear endpoint conductance. Gaussianity of an eliminated field does not imply Gaussianity of the retained theory, because the eliminated field couples to a composite rather than to a linear coordinate.

Equation (18) also specifies the philosophical limit of an effective description. A mediator can disappear from the retained variables while its enabling interaction remains. Physically removing the mediating mechanism corresponds to a different intervention, such as setting $g = 0$ and reassessing all other terms. It does not follow from integrating over W. A composite response recorded at the endpoints therefore need not be a capacity acquired independently by those endpoints.

This argument concerns a relation's constitution. A translation practice may operate so routinely that accounts of success mention only the parties who use it. Its disappearance from their description does not establish its disappearance from the conditions of performance. Conversely, if the interaction becomes reproducible through another organization, that transfer needs a new realization and continuity argument. The effective action records a dependence; it does not decide whether dependence is acceptable or how its burdens should be distributed.

9.7 Formation interpretation

The field model makes three distinct formation claims available for examination. First, a new coupling can be composed through a specified interaction rather than merely inferred from repeated successful observations. Second, the generated histories can have different coupling counts and attachment patterns. Third, suppressing internal fields in an account can preserve higher-order dependencies that its visible boundary no longer names.

These distinctions strengthen the earlier philosophical argument without completing it. The existence of a composed warning channel does not establish that warnings receive honest attention. A capacity can also be imposed, appropriated, or accessible only through an authority that controls its classification. Trust formation requires investigating whether the resulting addressability supports the relevant reasons, responsiveness, and voluntary vulnerability. The field action represents one possible mechanism of capacity formation; it cannot make its output morally authoritative.

Nor does the model claim that established dynamical-systems methods are incapable of describing generation. A sufficiently enlarged state space can encode whole complexes and their transformations. The difference is the retained explanatory structure: the incidence data, interaction rules, generated boundaries, and their target meanings must remain visible. An ordinary nonlinear flow on preassigned actor variables does not acquire these features by being nonlinear.

The resulting hierarchy gives the earlier calculations a proper domain. A chosen generated carrier, calibrated coordinates, a stable background configuration, and a justified small-variation regime can support a quadratic response model. Appendix A studies such a fixed-carrier model in its own right. No automatic derivation of its particular matrix from the present action is claimed. The next section develops operators that act on the boundary structures generated here.

10 Formation Algebras

This section supplies an algebraic account of changes to relational boundaries. It first retains incidence data explicitly, then gives a creation–annihilation realization of the cubic field interaction, and finally examines the observation and responsibility claims that such changes can support. The algebra organizes possible transformations of coupling structures. It is more than a reformulation of a fixed list of actor states, but remains a proposed representation rather than an ontology theorem.

10.1 Boundary sectors

Let $\mathfrak{B}$ be the countable collection of finite, typed, finitely labelled boundary configurations satisfying Equation (1), considered up to a declared incidence-preserving isomorphism. Port labels and attachment

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pairings are retained. If a historical attribution is needed at the boundary, include its finite record as additional data. A bare configuration is not assumed to summarize its whole past.

Use orthonormal basis vectors $|\Gamma\rangle$ for these combinatorial presentations. The boundary carrier and its finite-support domain are as follows.

$$\mathscr{K} = \ell^2(\mathfrak{B}) = \bigoplus_{n\geq 0} \mathscr{K}_n, \qquad \mathscr{K}_n = \overline{\text{span}}\{|\Gamma\rangle : |C_\Gamma| = n\}, \qquad \mathscr{D} = \text{span}_{\text{finite}}\{|\Gamma\rangle\}. \tag{19}$$

Equation (19) permits different numbers of couplings. Its inner product is a convenient representation of discrete alternatives; it supplies no empirical Born rule. The zero-coupling sector is empty only relative to this selected description. It is not a claim that actors can begin without any inherited relations.

For an interaction type β, let $\text{Emb}_\beta(\Gamma)$ be a finite set of admissible occurrences of its input boundary inside Γ. Each occurrence includes port identifications and side conditions. Replacement produces $\Gamma \circ_e \beta$ with fresh identifiers, followed by the chosen isomorphism convention. The elementary operator is defined by the following rule.

$$R_\beta|\Gamma\rangle = \sum_{e\in \text{Emb}_\beta(\Gamma)} w_\beta(e,\Gamma)\,|\Gamma \circ_e \beta\rangle. \tag{20}$$

In Equation (20), weights belong to $\mathbb{C}$ and are prescribed by the representation. If distinct occurrences yield the same output, their coefficients add. This counting convention is explicit; it is not the symmetry-factor convention of an unlabelled path integral unless the weights are chosen to implement that convention.

Assume inverse incidence is also locally finite: a boundary has finitely many weighted predecessors for each move. Then the algebraic adjoint maps $\mathscr{D}$ into itself. Products, finite linear combinations, and adjoints of the R_β, together with bounded diagonal observables of boundary data, generate a star-algebra on $\mathscr{D}$. The generally unbounded replacements are not declared elements of a C-star algebra of bounded operators. Their domains are part of the specification.

10.2 Historical products

The product $R_{\beta_2}R_{\beta_1}|\Gamma\rangle$ sums first over occurrences of β_1, then over occurrences of β_2 in the resulting boundaries. The second set of admissible occurrences can depend on what the first move creates. The product therefore represents more than two responses to an unchanged input graph.

To retain the complete spacetime, annotate each term with its attached elementary event complex. Composing terms uses the boundary gluing of Equation (4); Proposition 8.1 guarantees associativity up to the prescribed isomorphism. If these annotations are forgotten, distinct histories ending at the same boundary contribute to the same vector. That forgetful map loses historical attribution. An endpoint matrix element alone cannot recover which party incurred an exposure or which event created an obligation.

The distinction has practical force. Establishing an appeal route before a joint undertaking can enable a different encounter from promising the same route afterward. Even if both processes end with the same institutional chart, their histories differ in opportunities for refusal and correction. A formation algebra can retain that order and its admissibility conditions; a single final adjacency matrix need not.

An algebraic adjoint also has a limited meaning. It reverses matrix elements and conjugates weights. It does not undo an exposure, restore lost time, obtain consent, or vindicate the reversal of a commitment. A physical rollback must be modeled as an admissible operation with its own costs and conditions. Treating every adjoint as a practically available inverse would erase the very histories the construction is meant to preserve.

10.3 Creation operators

The author's earlier group-field formulation supplies a Fock realization of variable coupling number [7]. With the finite-dimensional one-coupling space $\mathfrak{h}$ of Equation (7), use its bosonic symmetric Fock space

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and finite-particle domain.

$$\mathscr{F} = \bigoplus_{n=0}^{\infty} \mathrm{Sym}^n \mathfrak{h}, \qquad \mathscr{D}_{\mathrm{F}} = \bigoplus_{n=0}^{\mathrm{alg}} \mathrm{Sym}^n \mathfrak{h}. \tag{21}$$

The superscript alg in Equation (21) means that each vector has finitely many nonzero particle sectors. For $f \in \mathfrak{h}$, creation adds a coupling mode and symmetrizes the tensor factors. Annihilation is its adjoint on this domain.

$$\begin{aligned} a^\dagger(f)\psi_n &= \sqrt{n+1}\,\mathrm{Sym}(f \otimes \psi_n), \\ [a(f), a^\dagger(h)] = \langle f, h\rangle I, &\qquad [a^\dagger(f), a^\dagger(h)] = [a(f), a(h)] = 0. \end{aligned} \tag{22}$$

Equation (22) holds on $\mathscr{D}_{\mathrm{F}}$. The annihilation map is conjugate-linear in its argument. Bosonic symmetrization treats indistinguishable presentations of the same selected coupling mode alike; it does not assert that social actors are bosons. A finite occupation cutoff alters the canonical commutator at the cutoff boundary, so an exact identity on the infinite finite-particle domain must not be tested by silently imposing the identity on that boundary.

Fock occupation numbers alone are insufficient to identify a relational boundary. They count local modes but do not record which ports attach to which other modes. Explicit contraction tensors and port labels must accompany a boundary construction. Even then, different presentations can map to the same vector because of linear identities. Thus $\mathscr{F}$ is not identified with $\mathscr{K}$, and no faithful representation of every combinatorial boundary is asserted. The incidence-complete algebra above retains the information that this local realization can lose.

For an interaction tensor V_β with p incoming and q outgoing slots, its normal-ordered realization has the following form in the chosen mode basis.

$$\widehat{V}_\beta = \sum_{\mathbf{b},\mathbf{a}} (V_\beta)_{\mathbf{b};\mathbf{a}}\, a^\dagger_{b_1} \cdots a^\dagger_{b_q} a_{a_p} \cdots a_{a_1}. \tag{23}$$

The indices in Equation (23) range over the finite mode basis; $a_a = a(e_a)$. The tensor supplies the same port contraction as its field polynomial. This is a specified correspondence between polynomial kernels and operators, not a proof that the Euclidean ensemble is a causal quantum evolution of a social process.

10.4 Composite coupling algebra

Restrict the cubic contraction of Equation (11) to three orthonormal typed modes u, v, w with nonzero contraction coefficient, absorbing that coefficient into the coupling constant. The elementary gluing operator and number operators are the following polynomials.

$$T = a^\dagger_w a_v a_u, \qquad N_\ell = a^\dagger_\ell a_\ell \quad (\ell = u, v, w), \qquad N = N_u + N_v + N_w. \tag{24}$$

Equation (24) removes two incoming coupling quanta and creates one composite quantum. The interaction term $-gT - \bar{g}T^\dagger$ corresponds to the cubic term in Equation (12). Its use as an actual generator of time evolution would require a time interpretation and operator-domain control beyond this polynomial construction.

On normalized occupation vectors, the elementary move has the following exact action.

$$T|n_u, n_v, n_w\rangle = \sqrt{n_u n_v (n_w + 1)}\,|n_u - 1, n_v - 1, n_w + 1\rangle. \tag{25}$$

Equation (25) is zero if either incoming occupation is zero. It records an enabling dependency built into the interaction. The square root is a normalization coefficient of the bosonic representation, not an empirical rate of social formation.

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Proposition 10.1 (Composite-sector identities)**.** *On $\mathscr{D}_{\mathrm{F}}$, the elementary move satisfies the following identities.*

$$\begin{aligned} [N, T] &= -T, \qquad [N_w, T] = T, \\ T^\dagger T &= N_u N_v (N_w + 1), \\ T T^\dagger &= (N_u + 1)(N_v + 1) N_w. \end{aligned} \tag{26}$$

Proof. Equation (25) decreases total occupation by one and increases w occupation by one, proving the first line on a basis. Applying the adjoint after T gives the squared coefficient $n_u n_v (n_w + 1)$, proving the second line. Applying the adjoint first creates one u and one v while removing one w; its squared coefficient is $(n_u + 1)(n_v + 1)n_w$, proving the third line. Linearity extends the identities to every finite-particle vector. □

Proposition 10.1, through Equation (26), makes the change of sector explicit. It also proves that $T^\dagger$ is not an inverse of T. For example, some boundary modes are annihilated and the nonzero normalization factors depend on occupation. The representation already rejects automatic invertibility before the further practical irreversibility of a target history is considered.

There are finite invariant sectors for the pair $T, T^\dagger$: both preserve $Q_u = N_u + N_w$ and $Q_v = N_v + N_w$. At fixed nonnegative integer values q_u, q_v, n_w ranges from zero to $\min(q_u, q_v)$, with $n_u = q_u - n_w$ and $n_v = q_v - n_w$. This gives a finite matrix realization without an artificial occupation cutoff inside the sector. Matrix calculations in these sectors test the identities exactly; they do not demonstrate an empirical trust process.

The counts Q_u, Q_v express the attachment bookkeeping of this selected kernel. They are not conserved persons, values, or amounts of trust. A more general formation model may include introduction, termination, or reassignment kernels that do not preserve them. The point of stating the conservation here is to expose the model's restriction, not to turn it into a universal GR law.

10.5 Changing observation domains

A boundary-changing move can make a previously unavailable observation meaningful. A warning archive becomes a possible source of evidence only when a practice creates and retains one. A public appeal route changes who can make a refusal count. Treating all future records as already available to the original participants would confuse the investigator's comprehensive description with the participants' situated capacities.

Mathematically, partition $\mathfrak{B}$ into sets $\mathfrak{B}_\sigma$ indexed by a declared coarse boundary type σ. Put $\mathscr{K}_\sigma = \ell^2(\mathfrak{B}_\sigma)$; each sector retains its varying labels and records. Let $\mathcal{A}_\sigma \subseteq \mathcal{B}(\mathscr{K}_\sigma)$ be a specified algebra of accessible bounded observables. For a bounded block $R : \mathscr{K}_\sigma \to \mathscr{K}_{\sigma'}$ of a formation operator, an output effect E with $0 \le E \le I$ pulls back to $R^\dagger E R$. Positivity follows directly from the quadratic form.

$$\langle \psi, R^\dagger E R \psi \rangle = \langle R\psi, E R \psi \rangle \ge 0. \tag{27}$$

Equation (27) does not imply that the pullback lies in $\mathcal{A}_\sigma$. It also need not be bounded by I unless R is a contraction. A normalized probabilistic instrument requires further operators and a normalization condition. These missing assumptions cannot be supplied by calling E an observable.

The algebraic issue is the possible failure of accessible closure. A later effect may depend on distinctions absent from the initial interface. More accurate measurement within that interface then need not resolve the difference. Appendix B establishes the corresponding factorization criterion in a finite commutative sector. That criterion is useful after a boundary and observation vocabulary have been selected; it is not a replacement for their generation.

At the target level, new access can change the meaning of an earlier success. An archive may reveal that a mediator repeatedly corrected an apparently autonomous team. An appeal procedure may make previously unrecorded refusals visible. Formation then changes both capacities and the conditions under which capacities can be attributed. A history of success is not self-interpreting merely because its events have been preserved.

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10.6 Generative scope

The field and algebra constructions now perform connected tasks. Interaction tensors specify local composition; their contractions generate complex histories; boundary operators compose admissible rearrangements; accessible algebras delimit what a situated interface can distinguish. None of these tasks is exhausted by assigning an evolving trust coordinate to a fixed actor.

Nevertheless, the formation of an admissible operator vocabulary remains a further problem. A new social practice may require a coupling type absent from $\mathcal{T}$. The present construction can compare an expanded vocabulary with its predecessor through a declared embedding or partial translation. It does not derive every future vocabulary from its original action. The prior action-level discussions identify this more demanding generative problem [7, 9]; solving it would require explicit transformations of kernels and types, together with preservation or failure criteria.

For the current paper, the bounded result is sufficient to sharpen the formation object. The question becomes which couplings a history produces, which subsequent operations they enable, which participants can recognize those changes, and which responsibilities persist through them. The normative assessment still asks whether the resulting vulnerabilities and commitments are defensible. The remaining finite models address selected operational questions once the required vocabulary and closure assumptions have been made explicit.

11 Formation Prerequisites

This section formalizes the dependency problem identified in Section 4. It uses a finite construction system to distinguish a supported beginning from an unseeded cycle. The representation concerns the availability of specified capabilities during a limited construction window. It does not model an amount of trust, predict voluntary participation, or establish that every available construction should be performed.

11.1 Primitive capabilities

Let C be a finite set of capability labels. Each label identifies a concrete, locally interpreted ability or support, such as an accessible communication channel, authority to stop a trial, or an agreed procedure for reporting an anomaly. A label can include its holder and scope. The set does not place heterogeneous persons in one common state space; it lists predicates selected for this particular dependency inquiry.

Let $S \subseteq C$ be the initially available capabilities. Let R be a finite set of construction rules. Each rule $r \in R$ has a prerequisite set $A_r \subseteq C$ and an effect set $B_r \subseteq C$. The intended reading is that an encounter or preparation described by r can make the effects available when the prerequisites are available. If actual participation is contingent, the rule records a possible construction, not a guarantee that the people concerned will undertake it.

The construction window is assumed to preserve acquired capabilities. Rules consume no capability, and there are no mutually exclusive effects or unrepresented resource constraints within this abstraction. These are strong assumptions. They make the first investigation a test of prerequisite structure; later sections relax its adequacy by recording exposure, refusal, and changing capacities in a transition system.

For a set $X \subseteq C$ of available capabilities, define the expansion map $F : \mathcal{P}(C) \to \mathcal{P}(C)$, where $\mathcal{P}(C)$ denotes the set of all subsets of C. The map retains existing capabilities and adds the effects of enabled rules.

$$F(X) = X \cup \bigcup_{\{r \in R : A_r \subseteq X\}} B_r. \tag{28}$$

Equation (28) makes persistence explicit. Starting from $C_0 = S$, successive sets and their union are defined by repeated expansion.

$$C_{n+1} = F(C_n), \qquad C_\infty = \bigcup_{n \geq 0} C_n. \tag{29}$$

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Equation (29) uses an integer construction stage n, not a physical time or a measure of psychological development. Several independent constructions can appear in one stage.

11.2 Constructive closure

The following proposition records exactly what the finite abstraction establishes. A *finite witness* is an ordered list of rule applications in which each rule's prerequisites are present when it is applied and each effect is retained.

Proposition 11.1 (Prerequisite closure)**.** *The sequence in Equation* (29) *stabilizes after at most* $|C \setminus S|$ *strict expansions. Its limiting set is the least fixed point of* F *containing* S*. A capability lies in* C_∞ *exactly when it has a finite witness from* S *under the stated persistence assumptions.*

Proof. The union with X in Equation (28) gives $X \subseteq F(X)$. If $X \subseteq X'$, every rule enabled at X is enabled at X', so $F(X) \subseteq F(X')$. The iteration is therefore increasing. Every strict expansion adds at least one element of $C \setminus S$, of which there are finitely many. At the first non-strict stage $C_{n+1} = C_n$, so the sequence remains fixed and its union equals that set.

If D is any fixed point containing S, induction gives $C_0 \subseteq D$ and $C_{n+1} = F(C_n) \subseteq F(D) = D$. Hence $C_\infty \subseteq D$, proving leastness. To obtain a witness, consider a capability first added at stage $n+1$. It is an effect of a rule whose prerequisites lie in C_n. By induction those prerequisites have finite witnesses. Concatenate their witnesses and then apply the rule. Persistence ensures that earlier witnesses do not remove prerequisites needed later; duplicate rule applications can be omitted. Conversely, induction along any finite witness shows that every produced capability lies in the fixed point, because a rule with prerequisites there has its effects there as well. □

Proposition 11.1 does not derive behavioral cooperation from a capability inventory. It establishes a finite prerequisite witness conditional on the model's construction rules. If a proposed rule says that a participant will voluntarily disclose information whenever a channel exists, its substantive premise needs justification. Mathematics does not make that premise more credible.

11.3 Unseeded dependencies

A set of missing capabilities can form an obstruction even when each has a verbal explanation in terms of another. Let $U \subseteq C$ be nonempty and initially absent. Suppose every rule that produces a member of U already requires some member of U. The two conditions have the following set-theoretic form.

$$S \cap U = \varnothing, \qquad B_r \cap U \neq \varnothing \implies A_r \cap U \neq \varnothing \quad (r \in R). \tag{30}$$

Equation (30) quantifies over every producing rule. Merely finding a cycle in a diagram is insufficient if another rule can enter the cycle from outside.

Proposition 11.2 (Unseeded dependency obstruction)**.** *If Equation* (30) *holds, then* $C_\infty \cap U = \varnothing$*.*

Proof. The base set $C_0 = S$ contains no element of U. Suppose $C_n \cap U = \varnothing$. Any rule producing an element of U has a prerequisite in U, by Equation (30), and is therefore disabled at C_n. No such element enters C_{n+1}. Induction proves the result at every stage and hence at their union. □

Proposition 11.2 gives a precise interpretation of one bootstrapping failure. If establishing an authorized channel requires an already recognized protocol, while establishing the protocol requires that same channel, and no other route exists, their mutual dependence supplies no beginning. A mediator, inherited procedure, or narrower preparation can matter by supplying an actual entry route. It does not defeat logic; it changes the specified prerequisites.

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11.4 Illustrative construction

Consider five capability labels: c for an accessible channel, s for an effective stop procedure, r for an interpretable trial record, i for a tested interface, and o for readiness to undertake a specified operation. Let the initial set contain c and s. The arrows below denote three rule applications from prerequisite sets to effect sets.

$$\{c, s\} \longrightarrow \{r\}, \qquad \{c, r\} \longrightarrow \{i\}, \qquad \{r, i, s\} \longrightarrow \{o\}. \tag{31}$$

Equation (31) abstracts a bounded trial, a calibration based on its record, and an operational preparation. It deliberately includes neither trust nor trustworthiness as an automatically generated label. Iteration adds the record, the interface, and operational readiness in successive stages.

$$C_0 = \{c, s\}, \quad C_1 = \{c, s, r\}, \quad C_2 = \{c, s, r, i\}, \quad C_3 = \{c, s, r, i, o\}. \tag{32}$$

Equation (32) illustrates a beginning that requires less than its later operation. It also exposes what the verbal story assumes: an accessible channel and an effective stop procedure already exist. If the only rules capable of producing c and s require one another, neither is initially present, and there is no external route, the obstruction in Proposition 11.2 applies.

The example should not be read as an institutional recipe. A record may be false, an interface may decay, and a stop procedure may fail under pressure. Such possibilities violate the simple rule interpretation and must be added to a richer model. The closure construction is useful precisely because it identifies where these substantive claims enter.

11.5 Analytical limits

Monotone closure is inappropriate when a trial consumes its own prerequisites, when alternative commitments exclude one another, or when a capability depends on a continuing support that can disappear. It is also insensitive to the time needed to assemble the witness. A theoretically available construction can arrive too late for the intended operation. These limits connect the present formation problem to the earlier maintenance problem without making criticality an assumed mechanism here.

The closure test consequently supplies a necessary discipline for a certain kind of explanation, rather than a complete theory of formation. It asks whether the proposed stages actually have a beginning under their stated prerequisites. The next constructions ask whether participants can identify and execute an admissible sequence while the relation changes and observation remains incomplete.

12 History-Sensitive Formation Models

This section defines the primitives for the policy investigation. It connects heterogeneous local actors to attributed encounter histories, then states the conditions required for a finite summary. The formal representation is deliberately local to an inquiry. It does not identify a finite configuration with the complete relational reality of its participants.

12.1 Local actor types

The inherited heterogeneous-network formalism represents actor i by local state, input, and emission sets, denoted S_i, A_i, and E_i. Its local update and emission maps are $\phi_i : S_i \times A_i \to S_i$ and $\epsilon_i : S_i \times A_i \to E_i \cup \{\perp\}$, where the additional symbol $\perp \notin E_i$ denotes no emission. These components define the local object n_i.

$$n_i = (S_i, A_i, E_i, \phi_i, \epsilon_i). \tag{33}$$

Equation (33) follows the typed structure of the prior framework [8], with a distinct no-emission symbol to avoid confusion with the empty set. No common metric, alphabet, or state type is assumed. A local state may carry history or a history-sensitive summary; it is not necessarily an instantaneous point stripped of its provenance.

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A coupling from i to j uses a partial translation $\Pi_{ij} : E_i \rightharpoonup A_j$. Let $s_i \in S_i$ and $s_j \in S_j$ be local states, and let $a_i \in A_i$ be an input. When an emission $e = \epsilon_i(s_i, a_i)$ is present and the translation is defined, the receiver's updated state $s'_j \in S_j$ follows its local update map.

$$s'_j = \phi_j\big(s_j, \Pi_{ij}(e)\big). \tag{34}$$

Equation (34) does not imply identical meaning at sender and receiver. Undefined translation, no emission, or an unavailable receiver response is a modeled possibility, not an automatically successful communication. If an encounter changes the translation rule itself, the model must record that change rather than retaining a fixed interface by assumption.

12.2 Attributed encounter histories

Let H be a prefix-closed set of finite encounter histories. A history records the participants, local contributions, supporting arrangements, available observations, and the subsequent changes selected as relevant. Prefix closure means that the record up to any earlier encounter is also an admissible historical prefix. Empty or inherited initial records are permitted; they are not claims that the participants themselves lack prior history.

An encounter record can include hidden facts from an analyst's scenario as well as observations available to participants. The distinction must be explicit. A participant's observational history is a projection of the modeled history and may identify several histories as indistinguishable. The full record is therefore not an omniscient database presumed available to the people acting.

Section 8 represents element histories, coupling histories, and rearrangement events through the inherited two-complex. The present investigation selects attributed encounter sequences for a different local task: the worked construction has one ordered protocol. A sequence is one possible representation of selected history, not the GR object in its entirety. Concurrent histories require additional causal and boundary information before such a reduction can be justified. No automatic equivalence to the field or spin-foam model is assumed.

12.3 Finite summaries

Let Q be a finite set of scenario configurations selected for the investigation, and let $\pi : H \to Q$ map histories to their summaries. A configuration can name a structured collection of typed local descriptions, interfaces, capacities, support states, and exposure records. It is not a common state space in which the actors' own states have been identified. The finite set is an analyst's list of distinctions relevant to a bounded problem.

Let $\mathcal{A}$ be a finite set of protocol moves. A move can represent a joint proposal, a trial, a repair, or an authorized interruption. Let $\mathrm{En}(h) \subseteq \mathcal{A}$ be the moves available after history h, and let $T(h, a) \subseteq H$ be the nonempty set of possible next histories when a is available. Unavailable moves have no successors. Refusal, error, and opportunistic responses must be included among successors whenever the application regards them as possible.

A finite summary is sufficient only if it preserves the distinctions needed for the task. Any histories h, h' with $\pi(h) = \pi(h')$ must satisfy the following equalities.

$$\begin{aligned} \mathrm{En}(h) &= \mathrm{En}(h'), \\ \{\pi(k) : k \in T(h, a)\} &= \{\pi(k) : k \in T(h', a)\} \quad (a \in \mathrm{En}(h)). \end{aligned} \tag{35}$$

Equation (35) requires equality of enabled moves and equality of successor classes. Additionally, histories with the same summary must agree on safety, the specified operational target, and the current public observation. These requirements are assumptions to be checked, not automatic benefits of naming a finite state.

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The induced transition map $\Delta : Q \times \mathcal{A} \to \mathcal{P}(Q)$ assigns the successor summaries represented by the corresponding next histories.

$$\Delta(\pi(h), a) = \{\pi(k) : k \in T(h, a)\}. \tag{36}$$

Equation (36) is well defined because Equation (35) makes the right-hand side independent of the chosen representative history. We restrict Q to the summaries represented by histories. Unavailable moves have empty successor sets.

Proposition 12.1 (History-summary preservation)**.** *Under the stated summary conditions, every finite history path projects to a configuration path with the same observations, safety classifications, and target classifications. Conversely, every configuration path starting at $\pi(h)$ can be lifted, step by step, to a history path extending that particular history h.*

Proof. For projection, each history transition contributes its successor summary to Equation (36). The assumed observation and predicate agreement preserves the classifications at every stage. For lifting, suppose a configuration step takes $q = \pi(h)$ to $q' \in \Delta(q, a)$. Equation (36) supplies a next history $k \in T(h, a)$ with $\pi(k) = q'$. The equality for all representative histories is what permits the lift from this particular h, rather than only from some history in its class. Repeat the argument along the finite configuration path. Each step preserves the classifications by the same predicate agreement. □

Proposition 12.1 establishes a finite-path correspondence under explicit assumptions. It does not guarantee that those assumptions hold in a social application. If two histories have the same count of successes but different outstanding obligations, a success-count summary can fail enabledness or safety preservation. The proposition then supplies a reason to enrich the representation or abandon the finite reduction.

12.4 Public observations

Let Y be a finite set of public protocol observations and $o : Q \to Y$ the observation map. An observation may include an attributed trial result or a report of refusal. The shared protocol alphabet is an additional local achievement or assumption. It is not derived from the heterogeneous node types, and it does not imply that all private meanings coincide.

At a decision point, let $B \subseteq Q$ be a nonempty set of configurations still possible given the protocol record and the model. This is an *information set* in the present set-valued sense. It carries no probabilities and need not represent any participant's psychological belief. A protocol move $a \in \mathcal{A}$ usable without hidden-state knowledge must be enabled at every configuration in B. After observing $y \in Y$, the update $U(B, a, y) \subseteq Q$ retains precisely the compatible successors.

$$U(B, a, y) = \{q' \in Q : o(q') = y \text{ and } q' \in \Delta(q, a) \text{ for some } q \in B\}. \tag{37}$$

Equation (37) first allows every modeled successor and then retains only those compatible with the observation. Empty sets correspond to observations impossible under the current model and move. An actually observed event of this kind calls for model revision; it should not be forced into a nonempty set by pretending that its possibility had already been represented.

The construction assumes a coordinating protocol can use this public record. It does not solve decentralized decision-making with different private observations or strategic disagreement over the next move. Those problems can require richer games or distributed protocols. The modest purpose here is to distinguish a formation path that an informed analyst can see from one that a specified observation-based procedure can actually implement.

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13 Admissible Formation Policies

This section asks when an observation-based protocol can reach a specified formation target while respecting selected constraints. It derives a finite-horizon criterion, distinguishes a successful path from an implementable policy, and examines the consequences of refusal and lost observations. The analysis concerns operational readiness, not a guarantee that participants will come to trust.

13.1 Operational readiness

Use the finite configurations, moves, transitions, and observations defined in Section 12. Let $K \subseteq Q$ contain the configurations that satisfy the selected admissibility constraints. Calling these configurations safe is shorthand relative to those constraints; it does not assert complete safety or comprehensive moral acceptability. Let $G \subseteq K$ be a specified operational target, such as possession of an agreed reporting procedure and a tested interface for a bounded task.

For an information set B, let $E(B) \subseteq \mathcal{A}$ denote its common enabled moves and let $Y(B,a) \subseteq Y$ denote its possible observations after move a. The definitions use the primitive transition map and observation update.

$$\begin{aligned} E(B) &= \{a \in \mathcal{A} : \Delta(q,a) \neq \varnothing \text{ for every } q \in B\}, \\ Y(B,a) &= \{y \in Y : U(B,a,y) \neq \varnothing\}. \end{aligned} \tag{38}$$

Equation (38) prevents a protocol from choosing a move available only in an unknown favorable configuration. Nonempty successors also prevent a vacuous claim that an impossible move succeeds because it has no adverse outcomes.

A policy selects a move or a stopping decision from the observed protocol history. It achieves *certified operational readiness within n moves* when every modeled realization remains in K and the policy stops within n moves at an information set contained in G. Certification is part of this definition: the policy must be able to identify readiness from its information. Merely visiting G secretly at different times along different realizations is a weaker property and is not the target of the following result.

13.2 Finite-horizon recursion

Let $\mathcal{B}_K$ be the nonempty subsets of K. Define collections $W_n \subseteq \mathcal{B}_K$ recursively, starting with information sets that already certify readiness.

$$\begin{aligned} W_0 &= \{B \in \mathcal{B}_K : B \subseteq G\}, \\ W_{n+1} &= W_n \cup \big\{B \in \mathcal{B}_K : \exists a \in E(B) \text{ such that} \\ &\qquad U(B,a,y) \in W_n \text{ for every } y \in Y(B,a)\big\}. \end{aligned} \tag{39}$$

Equation (39) works backward from readiness. It retains information sets already manageable within the shorter horizon and adds those admitting one move whose every possible observation leaves a manageable continuation. Successors outside K cannot enter W_n, so the recursion enforces the selected constraints at every stage.

Proposition 13.1 (Certified formation policy)**.** *For the finite model and the certification objective just defined, $B \in W_n$ if and only if there exists an observation-based policy achieving certified operational readiness within at most n moves from initial information set B.*

Proof. At horizon zero, the policy can only stop. This is successful exactly when $B \subseteq G$, which is the definition of W_0. Suppose the equivalence holds at horizon n. If $B \in W_n$, reuse its existing policy. Otherwise membership in W_{n+1} supplies a common enabled move a. After its observation y, Equation (37) gives the complete set of successors compatible with that observation. Each nonempty successor set belongs to W_n, so the induction hypothesis supplies a continuation. Choosing that continuation after each observed y produces a policy using at most $n+1$ moves. Its configurations remain in K because all information sets used by the construction lie there.

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Conversely, take a successful policy with horizon $n+1$. If it stops immediately, its information set lies in W_0. Otherwise its first move must be enabled at every possible initial configuration; hence it lies in $E(B)$. For every possible observation, the residual policy succeeds from the corresponding set $U(B, a, y)$ within n moves. The induction hypothesis puts each such set in W_n, yielding membership in W_{n+1}. A successful first move cannot have an unsafe possible successor, since that successor would be realized along a policy branch and violate the objective. □

Proposition 13.1 is a model-relative existence criterion. It assumes the transition and observation descriptions are available to the designer, not that actors know the true configuration. It also assumes the proposed coordinating policy can be implemented. Its universal quantifier covers modeled successors, not unforeseen transformations omitted from Q.

13.3 A formation example

Consider configurations $u_0, v_0, u_1, v_1, g, d, b$. The initial possibilities u_0, v_0 have different interface requirements but the same public observation. A bounded trial t produces both a calibration record and a usable interface state, taking u_0 to u_1 and v_0 to v_1. The resulting records distinguish the cases. The configuration g represents readiness, d orderly termination, and b a prohibited failure. Move a is the appropriate next operation for u_1; move c is appropriate for v_1. An arrow labeled by a move denotes a singleton successor under Δ.

$$\begin{aligned} u_0 \xrightarrow{t} u_1 \xrightarrow{a} g, \qquad & v_0 \xrightarrow{t} v_1 \xrightarrow{c} g, \\ u_1 \xrightarrow{c} b, \qquad & v_1 \xrightarrow{a} b. \end{aligned} \tag{40}$$

Equation (40) also assumes that operations attempted before the trial lead to b, since the interface has not been established. A stop move leads to d from every nonterminal configuration. Let $K = Q \setminus \{b\}$ and $G = \{g\}$; stopping at d is admissible but is not operational readiness. Terminal configurations have no further moves. The trial is available only at the two initial configurations, and a, c are available at both initial and prepared configurations.

With observations distinguishing u_1 from v_1, the initial information set $\{u_0, v_0\}$ belongs to W_2. Choose t, then choose a after the u_1 record or c after the v_1 record. The trial creates a capacity and supplies information needed to use it. These are separate effects of the same encounter.

Now merge the two prepared-state observations and retain the same transitions. The post-trial information set is $\{u_1, v_1\}$. Neither a nor c is safe from both possibilities, and stopping reaches d, outside G. No successful policy exists at any horizon. Yet there is a safe path to g from each initial configuration if an analyst is told which configuration is actual. This difference is the substantive purpose of the example: individually available paths do not establish that participants can select them from the information they possess.

13.4 Observation refinement

The example suggests a limited monotonicity result. Hold configurations, moves, transitions, and target predicates fixed. Suppose a finer observation map $o_f : Q \to Y_f$ admits a coarsening map $\rho : Y_f \to Y_c$ such that $o_c = \rho \circ o_f$. Fine observations can then reproduce coarse observations without changing the underlying relation.

Proposition 13.2 (Fixed-model observation refinement)**.** *For a fixed initial information set and horizon, any policy achieving certified readiness with o_c can be implemented with o_f. Additional observational discrimination cannot destroy policy existence under these fixed-model assumptions.*

Proof. At each step, transform the fine observation using ρ and supply the resulting coarse observation to the original policy. The move sequence is exactly what that policy would choose along the corresponding coarse history. Every actual realization is among the original policy's modeled realizations, so its safety guarantee remains valid. When the coarse policy stops, all configurations compatible with its coarse

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history lie in G. Configurations compatible with the finer history form a subset and therefore also lie in G. The horizon is unchanged. □

Proposition 13.2 does not justify unlimited monitoring. Acquiring finer records can change incentives, participation, privacy, costs, or the transition structure itself. Those changes violate the fixed-model premise. In a formation setting, additional observation is a relational intervention whose effects require examination rather than a free input to the theorem.

13.5 Voluntary participation

If the trial may end in an admissible refusal state d, the universal operational-readiness guarantee disappears. Add d to both trial successor sets. There is then a modeled branch that never reaches g, and Proposition 13.1 correctly rejects guaranteed readiness. The theorem cannot turn an optional encounter into compulsory cooperation.

A designer can instead specify a different target, $G' = \{g, d\}$, representing certified readiness or an orderly decision to end. That target can be attainable even when readiness alone is not. The change must be stated openly: a policy for responsible resolution is not a proof that trust or cooperation will form. Conditional claims are also possible, such as readiness on branches in which specified voluntary contributions occur. Their conditions must not disappear from the conclusion.

The example and its variants are reproduced computationally. The computation checks the backward recursion against explicit policy-tree search on small models; it does not supply empirical evidence about actual trust formation. The formal result remains a tool for exposing what a proposed sequence requires of observation, capacity, and participation.

14 Protected-Trial Diagnosticity

This section isolates the informational effect of protection. It constructs a small probabilistic model, derives the information carried by final outcomes, and then adds a process record. The purpose is to qualify the claim that guaranteed compliance always leaves the partner unobserved. The analysis concerns specified performance hypotheses during a fixed window; it is not a probabilistic definition of trust.

14.1 Primitive experiment

Let $\theta \in \{H, L\}$ denote two hypotheses about a counterpart's attempt-level performance in one specified task. These are model labels, not moral types or permanent character categories. For the duration of the experiment, an attempt indicator $X \in \{0, 1\}$ has success probability p_θ, with $0 < p_L < p_H < 1$. A rescue indicator $R \in \{0, 1\}$ is independent of X and θ, with probability $\alpha \in [0, 1]$ of being available. Rescue converts an unsuccessful attempt into a successful final outcome.

The final outcome indicator Z equals one for either an initially successful attempt or a successfully rescued failure. Its success probability is denoted $q_\theta(\alpha)$.

$$Z = X + (1 - X)R, \qquad q_\theta(\alpha) = \Pr(Z = 1 \mid \theta) = p_\theta + (1 - p_\theta)\alpha. \tag{41}$$

Equation (41) follows by separating successful attempts from unsuccessful attempts followed by rescue. Protection changes the final outcome channel while the model holds attempt behavior fixed. This invariance is a strong assumption. If participants change effort because rescue is expected, the attempt probabilities must be indexed to the support regime and the following transport interpretation no longer follows.

Subtracting the two final success probabilities isolates the effect of rescue on their difference.

$$q_H(\alpha) - q_L(\alpha) = (1 - \alpha)(p_H - p_L). \tag{42}$$

Equation (42) identifies the exact circumstance in which the final binary outcome loses this particular discriminatory capacity: complete rescue removes the difference. It does not say that protection eliminates every possible observation about the counterpart.

14.2 Outcome likelihoods

Suppose repeated trials are conditionally independent given a fixed θ, with the same p_θ and α. Let n be the number of trials and k the number of final successes in a specified sequence. For $\alpha < 1$, define $\Lambda_Z(k, n; \alpha)$ as the sequence's likelihood ratio in favor of H over L. Conditional independence gives the product expression.

$$\Lambda_Z(k, n; \alpha) = \left(\frac{q_H(\alpha)}{q_L(\alpha)}\right)^k \left(\frac{1 - q_H(\alpha)}{1 - q_L(\alpha)}\right)^{n-k}. \tag{43}$$

Equation (43) is a ratio of sequence probabilities; if only the count is observed, the binomial coefficient cancels and the same ratio results. A positive prior odds ratio is multiplied by this expression to obtain posterior odds within the experiment. No prior is supplied as a social fact, and posterior odds are not renamed a trust score.

Proposition 14.1 (Outcome masking)**.** *In the primitive experiment with distinct attempt probabilities, the final-outcome distributions under H and L coincide exactly when $\alpha = 1$. For a sequence of $n \geq 1$ final successes, the likelihood ratio decreases strictly toward one as α increases to one.*

Proof. A binary distribution is determined by its success probability. Equation (42) is zero exactly at $\alpha = 1$ because $p_H - p_L > 0$. At that endpoint every final outcome is a success under both hypotheses, so the observed all-success sequence has likelihood ratio one; sequences containing failure have zero probability under both and supply no defined posterior update in this model.

For $\alpha < 1$, write $r(\alpha) = q_H(\alpha)/q_L(\alpha)$. The quotient rule gives its derivative.

$$r'(\alpha) = \frac{(1 - p_H)q_L(\alpha) - (1 - p_L)q_H(\alpha)}{q_L(\alpha)^2} = \frac{p_L - p_H}{q_L(\alpha)^2} < 0. \tag{44}$$

The equality in Equation (44) follows by expanding both products: the terms containing α cancel, as do the equal cross-products in $p_H p_L$. For an all-success sequence, Equation (43) becomes $r(\alpha)^n$. Its derivative is negative because $n \geq 1$, $r > 0$, and $r' < 0$. Finally $q_H(1) = q_L(1) = 1$, so the limiting ratio is one. □

Proposition 14.1 concerns one evidential comparison and one family of observations. It does not imply that every particular observed sequence becomes less unfavorable as rescue increases: for example, an observed failure has its own conditional interpretation. The proposition also distinguishes exact non-diagnosticity from weak evidence. Almost complete protection can make a run of successes only weakly discriminating without making the two distributions identical.

For illustration, take $p_H = 0.9$, $p_L = 0.3$, and ten final successes. With no rescue the likelihood ratio is $3^{10} = 59049$. With $\alpha = 0.95$, the final success probabilities are 0.995 and 0.965, and the ratio is approximately 1.36. These are illustrative choices for inspecting the algebra; they are not estimates of any population. Figure 2 plots the corresponding log likelihood ratios and makes the approach to zero visible without suggesting a calibrated trust scale.

14.3 Process observations

Now suppose the attempt indicator X is recorded accurately before rescue, and this record is accessible to the participants entitled to use it. The joint observation is (X, Z). Rescue still protects the final outcome. For k recorded attempt successes in n independent trials, denote the attempt likelihood ratio by $\Lambda_X(k, n)$. Its factors use the attempt probabilities directly.

$$\Lambda_X(k, n) = \left(\frac{p_H}{p_L}\right)^k \left(\frac{1 - p_H}{1 - p_L}\right)^{n-k}. \tag{45}$$

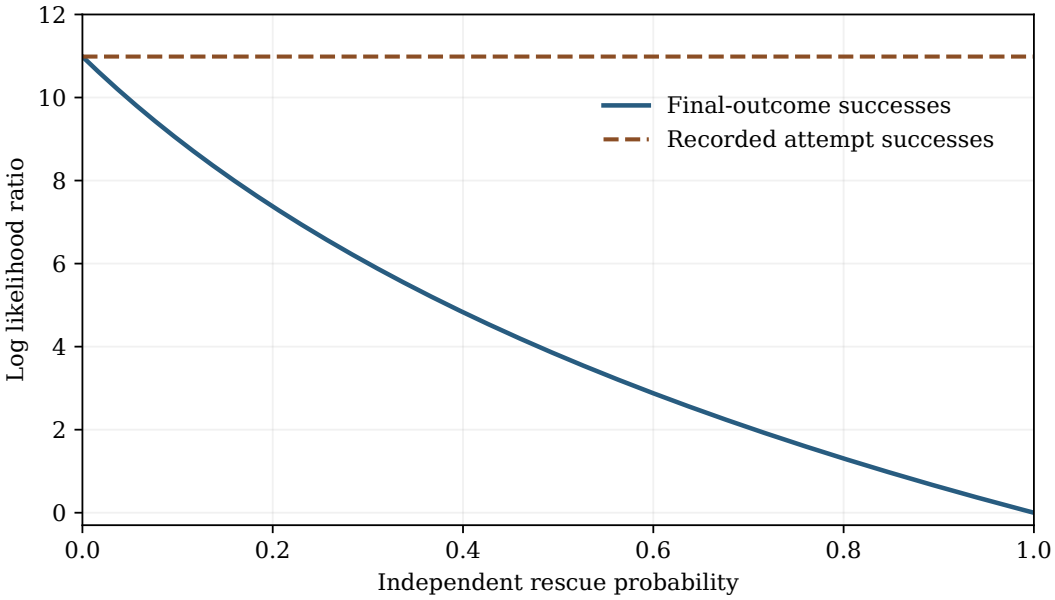

Figure 2: Illustrative evidence from ten successes in each observation channel. Each curve conditions on ten successes in its own channel; final successes do not imply ten successful attempts. Independent rescue increasingly masks final outcomes. The attempt-record comparison remains unchanged only because attempt behavior and recording quality are fixed. The vertical axis compares two task-performance hypotheses; it does not measure trust.

Equation (45) contains no rescue parameter because the model makes the attempt process independent of rescue. It should not be used after that assumption has been relaxed without deriving a new likelihood.

Proposition 14.2 (Protected process discrimination)**.** *Under accurate attempt recording and the invariance assumptions of the primitive experiment, observing* (X, Z) *preserves the likelihood discrimination supplied by* X*, including at complete rescue. Complete protection of the final outcome therefore does not imply non-diagnosticity of the full record.*

Proof. The joint probability factors as $\Pr(X = x \mid \theta) \Pr(Z = z \mid X = x)$. The second factor depends on the rescue mechanism and x, but not on θ. For every joint event with positive probability it cancels in the likelihood ratio, leaving the ratio of the attempt probabilities. Products over independent trials give Equation (45). At $\alpha = 1$, Z is always one while X still has success probabilities p_H and p_L, which differ. Thus the final outcome is uninformative about the two hypotheses but the recorded attempt is not. □

Proposition 14.2 provides a counterexample to an unrestricted claim that protection necessarily removes observations of the partner. It does not show that every enforcement system preserves useful process evidence. The model concerns rescue after an attempt, whereas coercive deterrence can change the attempt itself. The earlier typology's informational claim must therefore be assessed by specifying both the intervention and the observation channel [15]. A claim about an unobserved disposition outside the enforced setting requires still further assumptions.

14.4 Diagnostic-model limits

Several variations defeat easy transfer of the result. Participants may strategically alter effort when protected. A record may be incomplete or controlled by an interested intermediary. Recorded attempts may measure a narrow technical ability while missing responsiveness to being trusted. Training during the sequence may change the relevant performance probabilities, invalidating the fixed-type assumption. A process may also become less acceptable because recording creates surveillance or exposure.

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These variations are reasons to specify the mechanism, not reasons to abandon formal analysis. A model with regime-dependent probabilities can investigate changed behavior, and a model with a noisy record can investigate lost discrimination. Neither extension would establish a universal relation between enforcement and trust. The philosophical lesson of the counterexample is narrower: claims about what cooperation teaches must identify the target distinction, the support regime, and the observations actually available.

15 Cumulative Exposure

This section adds two constraints that the earlier abstractions leave open: the accumulation of exposure and the withdrawal of enabling support. It derives a simple resource-augmented representation and a sufficient condition for transporting a policy between support regimes. Both investigations concern the continued admissibility of a formation sequence, not the optimization of trust.

15.1 Typed accumulation

Let I be a finite set of affected participants. For each participant i, let D_i be a finite set of cost types that can meaningfully be accumulated within the chosen period, such as hours of monitoring or units of an explicitly specified material resource. For every modeled transition (q, a, q'), assign a nonnegative vector $\ell_i(q, a, q') \in \mathbb{R}^{D_i}_{\geq 0}$. Coordinates retain their own units. No sum across participants or unlike types is defined.

For a path $q_0, a_0, q_1, \ldots, a_{n-1}, q_n$, let $L_i(n)$ be its cumulative cost vector and let $b_i \in \mathbb{R}^{D_i}_{\geq 0}$ be a specified budget. Admissibility requires a componentwise comparison of the accumulated costs with that budget.

$$L_i(n) = \sum_{t=0}^{n-1} \ell_i(q_t, a_t, q_{t+1}), \qquad L_i(n) \leq b_i \quad (i \in I), \tag{46}$$

The relation $\leq$ in Equation (46) means coordinatewise comparison. The budget requires independent justification; the equation does not derive an acceptable burden from the model.

For disclosures that are tracked by occurrence rather than additive magnitude, let $\mathcal{J}_i$ be a finite set of protected items and let $J_i(q, a, q') \subseteq \mathcal{J}_i$ list items newly exposed along a transition. Their union defines the cumulative disclosure record $J_i^{\text{cum}}(n)$.

$$J_i^{\text{cum}}(n) = \bigcup_{t=0}^{n-1} J_i(q_t, a_t, q_{t+1}). \tag{47}$$

In Equation (47), $J_i^{\text{cum}}(n)$ denotes the set of items disclosed by stage n. Admissibility can be specified by an allowed family of disclosure sets, for example by excluding any set containing a protected identity. Counting an item once avoids pretending that repeated disclosure creates a new independent item each time. The representation still omits inferences from combined disclosures unless the model includes them explicitly.

15.2 Stage bounds

Proposition 15.1 (Cumulative cost bound)**.** *Let $c_i \in \mathbb{R}^{D_i}_{\geq 0}$ be a vector of stage limits. For a path of length at most n, componentwise bounds $\ell_i(q, a, q') \leq c_i$ on every transition imply $L_i \leq nc_i$. They do not by themselves imply $L_i \leq b_i$ for a separately chosen budget. A uniform per-stage bound permits an unbounded cumulative cost over unrestricted horizons whenever a repeatable cycle remains admissible under the stage constraints and incurs a strictly positive cost in some coordinate.*

Proof. Sum the componentwise inequalities over the path; nonnegative costs also make every shorter prefix no more costly than the complete path. This proves the bound nc_i. If a coordinate of nc_i exceeds

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the corresponding budget, a path incurring c_i in that coordinate at each stage is a counterexample to the claimed budget guarantee. For the final statement, repeating a positive-cost cycle m times produces m times its positive total in that coordinate. This exceeds any finite budget for sufficiently large m while preserving the same stage bounds. □

Proposition 15.1 is elementary, but it identifies a frequent omission in formation narratives. An apparently modest sequence of demonstrations can exhaust a participant through repetition. The cost need not be a payment to the counterpart; it may consist of repeatedly preparing evidence, remaining available, or maintaining a suspended alternative.

15.3 Augmented configurations

Exposure can be incorporated into the earlier policy problem by extending a configuration with its accumulated costs and disclosure sets. A transition updates each cost vector by adding ℓ_i and each disclosure set by union with J_i. Define the new admissible set to require both the original condition $q \in K$ and every cumulative constraint. The target also retains these constraints; reaching operational readiness after an impermissible disclosure does not retrospectively erase the violation.

The augmented model is finite only under additional restrictions. A finite horizon yields finitely many possible accumulated values from finite primitive transitions; alternatively, bounded integer-valued resources can give a finite enumeration. General real-valued accumulation over unbounded time need not be finite. The recursion in Equation (39) applies directly only when its finite-model premise is satisfied. Exact computation should not silently discretize a morally significant boundary.

An illustration needs no scalar aggregation. Suppose one participant can contribute at most five hours of monitoring to a trial sequence, and each trial requires two hours from that participant. Each trial fits the individual encounter limit, but three trials violate the trajectory budget. A second participant's unused hours do not automatically compensate the first if the work cannot be transferred. The formal coordinates preserve this difference.

15.4 Support transport

Now compare a supported model with transition map Δ_s and a proposed withdrawn-support model with map Δ_w. Assume the same configuration labels have justified meanings in both, and keep the observation map, safety set, target, and initial information set fixed. This is already a substantive transport assumption. If support withdrawal changes what a configuration means, a separate mapping between models is needed instead.

A strong sufficient condition preserves every supported available move and permits no new successor under withdrawal. The following implication is imposed for every relevant configuration q and move a.

$$\Delta_s(q, a) \neq \varnothing \quad \Longrightarrow \quad \varnothing \neq \Delta_w(q, a) \subseteq \Delta_s(q, a). \tag{48}$$

Equation (48) is intentionally restrictive. It is not a description of typical mediator withdrawal. It states a condition under which an already verified universal guarantee can survive the change.

Proposition 15.2 (Policy transport)**.** *Under the shared-meaning assumptions and Equation* (48)*, every supported-model policy that achieves certified readiness from the specified initial information set also does so in the withdrawn-support model with the same horizon.*

Proof. Follow the supported policy using the same observation history. At each configuration reached in the withdrawn-support model, its chosen move remains available by Equation (48). Every possible withdrawn-support successor is a supported successor, so induction on path length shows that every withdrawn-support realization is a supported realization with the same observations. The supported guarantee therefore ensures safety and stopping within the horizon. At stopping, the actual remaining possibilities are a subset of those permitted by the supported history, all of which lie in G. Certification is retained. □

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Proposition 15.2 gives no license to assume its premise. In the example of Section 13, suppose the mediator performs the calibration that makes the trial safe. Withdrawal adds the bad state b as a possible trial successor. The subset condition fails, and the supported two-step guarantee fails with it. An unchanged record of past success cannot repair that logical gap.

The same issue arises when the mediator's departure removes an observation, changes a commitment, or consumes a resource. Each change requires a new model or a justified relation between models. Formation may produce a genuinely portable capacity, but portability is a substantive finding. It is not entailed by repetition, a favorable outcome record, or the aspiration that direct trust will eventually replace support.

16 Asymmetric Formation

This section examines the distribution of formation burdens. It asks who supplies the initial support, who bears the costs of demonstration, and who can make a refusal or correction consequential. Its method is normative and diagnostic. The formal existence of a path is treated as one input to evaluation, not as a reason that anyone must follow it.

16.1 The demonstration burden

A proposed trust-building process commonly asks someone to demonstrate reliability. The demand can sound symmetrical while assigning unequal work. One party may need to provide documents, remain available for review, or undertake repeated small exposures; another may only need to decide whether the demonstration is sufficient. The latter controls both the threshold and the pace of recognition.

This structure matters even when each requested act is individually reasonable. Repeated demands can accumulate into a burden that prevents the demonstrating party from pursuing alternatives. The evaluator may preserve its own optionality while the other waits. The cumulative model in Section 15 identifies one formal aspect of this asymmetry, but it does not determine which distribution is just. That judgment requires attention to responsibility, capacity, vulnerability, and the social conditions that made the demonstration necessary.

The burden can also be epistemic. A participant may be required to explain conduct in the evaluator's categories while receiving no comparable account of the evaluator's decisions. Evidence that cannot be expressed in the authorized format may fail to enter the official history. A formation trajectory can then appear to lack adequate evidence because the relation excludes the very observations that would make the evidence intelligible.

16.2 Admissibility authority

The formal set K records a selected boundary. Someone must decide which exposures it excludes, whose interests it represents, and how uncertainty about those interests is handled. Those decisions cannot be justified by the later discovery that a policy exists. A model can make an exploitative policy look admissible if the relevant harm has been omitted from its primitives.

There are at least three distinct questions. A participant may regard an encounter as preferable to its available alternatives. The arrangement may meet an institution's procedural standard. An independent normative assessment may still find the background conditions or burden distribution objectionable. Agreement among these assessments should not be presumed. The paper's use of admissibility leaves room for their divergence.

Practical refusal is equally demanding. A formal stop move may be available while its use leads to retaliation, loss of essential access, or an unaffordable transition. Those consequences belong among the modeled successors and costs if the move is to represent actual refusal. Naming an exit state without representing its conditions creates a spurious appearance of choice.

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16.3 Protected distrust

An injured or historically subordinated participant can have reasons to withhold trust even while entering a limited cooperative arrangement. The arrangement's success need not invalidate those reasons. It may show that specified safeguards work, that one counterpart has become responsive, or that one task is manageable. Broader historical judgments may remain unsettled.

The formation account should therefore preserve protected distrust as a possible continuing position. It should also permit differentiation: a participant may revise a judgment about one undertaking while maintaining caution about another. This is consistent with multidimensional trust and distrust, and it avoids treating a uniform favorable attitude as the criterion of successful cooperation.

Protected distrust can nevertheless become self-insulating when no possible response is allowed to count. The diagnosis requires care. Refusal to generalize from weak or irrelevant evidence is not self-insulation. The question is whether the stated grounds of judgment identify conditions under which revision could be warranted, and whether those conditions are accessible without imposing unjustifiable exposure. A demanding evidential standard may be appropriate to the stakes; an impossible or moving standard needs a different explanation.

16.4 Induced dependence

A formation process can create capacities and dependence at the same time. A small project may train participants in a proprietary system, produce records controlled by a central intermediary, or make continued participation necessary to preserve access to accumulated work. What began as a bounded encounter can alter the feasibility of later refusal.

The mechanism is historical. Early commitments change the alternatives available at later stages. Assessing each stage against its immediate alternatives can therefore miss the cumulative production of dependence. A proper account asks not only whether the next step is affordable now, but also whether earlier steps have closed alternatives that were supposed to remain open.

The reverse possibility also matters. A mediator can create independent capacities that make later refusal more feasible. Training, portable records, shared procedural knowledge, and alternative channels may reduce dependence. Such effects should be established concretely. Their presence is not guaranteed by labeling an institution empowering or a relation generative.

16.5 Responsibilities of stronger participants

Greater capacity can support a less coercive beginning. A participant with more resources may absorb preliminary costs, make its own procedures inspectable, or accept a smaller initial undertaking. Such actions can change the entry conditions without demanding an immediate change of attitude from the other. They may also provide relevant evidence about how power is exercised.

These possibilities do not establish a universal allocation rule. The stronger participant may itself face legitimate constraints, and a resource transfer can carry conditions that reproduce dependence. The normative inquiry must inspect the actual undertaking and its alternatives. Its formation-specific principle is that the need for a beginning does not automatically allocate the first risk to whoever is asked to trust.

The analysis stops short of the broader resource and power programme. A separate paper would require a substantial account of burden transfer, justice, and the maintenance of alternatives. Here those issues constrain what can responsibly be called an admissible formation trajectory. Their inclusion preserves the meaning of the object without turning every constraint into a new publication.

17 Comparative Formation Sequences

This section applies the distinctions to three hypothetical cases. Each case specifies initial supports, the encounter sequence, the observations produced, and the limits of subsequent inference. They are

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constructed comparisons, not empirical case studies. Their purpose is to expose different outcomes that a generic story of repeated successful cooperation would obscure.

17.1 Joint technical development

Two research teams intend to combine analytical tools for a public-interest project. They lack shared working history and use different conventions for recording uncertainty. Neither is prepared to transfer sensitive data or make its results depend on the other's untested pipeline. Both nevertheless have authority to conduct a limited exercise using non-sensitive material.

The initial support consists of a common communication channel, an agreed right to stop, and a restricted sample. The first encounter asks each team to process the sample and explain its uncertainty report. A mismatch appears. One team's report uses a category that the other initially reads as a stronger assurance than intended. The visible numerical result is compatible with both interpretations, so further data of the same kind would not resolve the problem.

The teams then create a translation rule and document the circumstances in which it may be used. A second exercise tests the rule and includes a deliberately specified opportunity to report an unresolved ambiguity. One team reports such an ambiguity promptly. The immediate task is delayed, but the reporting process provides evidence relevant to the future handling of uncertainty. The encounter also creates an escalation procedure that did not exist beforehand.

Several changes have occurred. The teams have learned something about the other's practice, created a local interface, and acquired a record of how a limited disagreement was handled. They may also develop interpersonal trust. That last attribution requires evidence about their attitudes and commitments; it is not settled by the interface test alone.

The proposed next undertaking should remain indexed to these developments. The record supports a limited claim about coordinating the specified tools with the new procedure. It does not establish competence for every dataset or a common interpretation of all uncertainty categories. Expansion to sensitive material adds exposure that the initial sample did not test. A defensible trajectory therefore includes a new assessment rather than treating the first two exercises as universal credits.

This sequence illustrates both parts of the formal example: the encounter produces a capacity, and its record helps select the appropriate next action. If the uncertainty reports are flattened into a binary success indicator, the capacity may exist while participants lack the information required to use it appropriately. The relation's history is functionally relevant, not merely commemorative.

17.2 Mediated service exchange

Two organizations consider exchanging a specialized service. A third party holds a deposit, checks delivery, and corrects minor technical defects before the recipient receives the result. Ten deliveries are completed without a visible failure. The organizations now consider removing the intermediary to reduce cost.

The formation claim depends on what has been observed. If the recipient sees only final completion, the record concerns a supported arrangement. It may provide little evidence about how frequently the provider needed correction. If the intermediary preserves an accurate defect and response record, the organizations can distinguish initial performance, willingness to respond, and the intermediary's contribution. Protection and diagnosticity then coexist, although the record may still omit behavior that would emerge without supervision.

The provider may also have learned from the corrections. If so, its current capacity differs from its initial capacity. A favorable current test could support a claim about acquired competence, but a fixed-type likelihood calculation across all ten deliveries would be inappropriate if performance changed through training. The causal question is whether the intermediary revealed, supplied, or helped produce the relevant ability.

A responsible handoff identifies each intermediary function. Deposit control may be replaced by staged exchange; technical correction may require new staff; dispute resolution may remain external. If

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a function is removed without replacement, the earlier success record does not establish that the new arrangement is reliable. If the function is genuinely no longer needed, that conclusion should be supported by evidence about the changed relation.

Continued mediation is also a legitimate outcome. The organizations may find that a distributed arrangement remains preferable, even after becoming more familiar with one another. Calling this arrested trust development would presuppose that direct interpersonal reliance is the proper destination. The present account makes no such presumption. Formation can clarify a durable division of responsibilities rather than eliminate institutional support.

17.3 Community representation

A public agency invites representatives of a community to participate in a pilot consultation. Previous encounters elsewhere have given community members reasons for caution. The agency regards the pilot as a modest opportunity to build trust. Participants regard it partly as a test of whether disagreement will be recorded accurately and whether participation will later be represented as consent.

The initial encounter is bounded by a narrow topic, an explicit statement that attendance does not endorse the eventual policy, and a procedure for correcting the meeting record. These limits matter more than the number of meetings. A successful meeting that misrepresents dissent may reduce rather than improve the grounds for trust, even if it remains orderly and produces a publicly usable document.

Suppose the agency accepts a correction, publishes the disagreement within the authorized record, and changes a later proposal in response to a documented concern. These actions may provide evidence about responsiveness and create a practical precedent for consequential participation. They do not erase prior harms or establish that the agency will act similarly when the stakes are higher. The scope of the evidence remains local.

Now suppose further participation requires repeated unpaid preparation while the agency repeatedly postpones its decision. Each request is described as minor, and participants can formally withdraw. Over time, however, the burden consumes their capacity to pursue other forms of representation. A per-meeting exposure assessment misses the cumulative effect. The formation process may preserve the agency’s options by narrowing the community’s.

An alternative trajectory ends after participants conclude that the agency will not make their objections consequential. If the record remains available and the refusal is respected, the process can have produced useful knowledge without producing trust. If withdrawal triggers retaliation or public allegations of unwillingness to cooperate, the claimed stop procedure was inadequately specified. The case illustrates why the semantics of consent, record authority, and cumulative burden belong inside formation analysis.

17.4 Comparative implications

Table 2 compares the cases by the object produced and the inference requiring restraint. The comparison shows why a single measure of interaction frequency would be inadequate. The technical case creates an interface; the service case distinguishes supported performance from transportable capacity; the consultation case tests whether participation and disagreement have the meanings participants were promised.

The cases also reveal an important commonality. In each, the meaning of later evidence depends on the earlier structure of the encounter. Who acted, what support operated, which obligations were accepted, and what alternatives remained available all affect the inference. A formation trajectory is therefore neither a list of outcomes nor an unrestricted history of everything that happened. It is a structured account of the dependencies relevant to the next undertaking.

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Table 2: Hypothetical formation sequences

Sequence	Potential formation result	Unsupported extension
Technical development	A tested translation and escalation practice	General competence or unrestricted data access
Service exchange	A differentiated record of attempts, correction, and acquired skill	Reliability after unexamined mediator withdrawal
Community representation	Consequential correction and a limited participation precedent	Reconciliation, policy consent, or unlimited further participation

18 Empirical Identification

This section specifies how an empirical investigation could discriminate among competing formation explanations. It separates observable cooperation from trust attribution, identifies evidence for capacity production, and considers selection, support dependence, and ethical constraints. No new empirical finding is reported. The aim is to make the conceptual proposals vulnerable to contrary evidence.

18.1 Competing explanations

An increase in cooperative activity is compatible with several explanations. Participants may have learned that a counterpart is capable. They may have acquired a new joint capacity. An institution may have absorbed more risk. Less confident participants may have left, changing the composition of those observed. Alternatives may have deteriorated, making participation more necessary. A trusted mediator may have become more involved without that involvement being recorded.

These mechanisms can coexist. An investigation should not force every case into one exclusive category merely for analytical convenience. It should identify which changes are evidenced and which causal claims remain unresolved. A before-and-after increase in participation does not, by itself, discriminate among them.

Experimental trust research offers a relevant methodological warning. Berg, Dickhaut, and McCabe study an investment setting and a social-history treatment; Cox develops a design intended to separate trust and reciprocity interpretations from some other-regarding preference explanations [1, 3]. Their relevance here is the need for identification, not a license to treat transferred resources as a transparent measure of trust. The philosophical and institutional meanings of a real formation setting require additional evidence.

18.2 Episode reconstruction

A qualitative investigation can begin with a limited sequence of encounters. For each episode it can reconstruct the proposed undertaking, available supports, observations, attributed responsibilities, accepted commitments, and subsequent changes. The reconstruction should preserve contemporaneous disagreement rather than writing the final interpretation backward into the earlier record.

Several sources can be compared: participant accounts, procedural documents, records of interventions, and evidence of actual use or refusal. None is automatically authoritative. An official record may omit an informal repair; a retrospective interview may reorganize memory around the eventual outcome. The objective is to understand how the available evidence supports competing accounts, including where reconstruction fails.

The inquiry should also record what was unavailable to participants at the time. An analyst's later access to the mediator's logs cannot be projected into the participant's earlier decision. This distinction corresponds to the difference between a full modeled history and its observable projection. It is central to evaluating whether a formation policy was practically available rather than merely reconstructible after the event.

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18.3 Capacity identification

Evidence for capacity production should identify a changed ability or interface and the process through which it arose. A new shared procedure, a functioning communication channel, a tested translation, or an accepted distribution of responsibility can supply such evidence. Mere confidence about an unchanged arrangement is insufficient, although confidence may itself be relevant to participation.

The crucial counterfactual concerns the task before and after the formation event under comparable supports. Would the earlier arrangement have lacked something the later arrangement now uses? A convincing answer may require technical testing, process tracing, or comparison with an otherwise similar sequence. Historical settings often prevent clean isolation, so causal conclusions should reflect the strength of the available comparison.

Evidence against a capacity-production interpretation includes the discovery that the supposedly new function was already supplied by an unrecorded support, that the function is never actually used, or that the apparent improvement disappears when the selected participants change. These findings need not make the encounter irrelevant. They change what the encounter can be said to have produced.

18.4 Support-sensitive measurement

The diagnosticity model suggests comparing outcome records with process records and explicit support histories. If completed tasks become uniformly successful after automatic correction is introduced, the investigator should examine whether the underlying attempt distribution changed, whether defects were merely repaired, or whether the recording channel changed. The same visible success series can arise through different mechanisms.

This comparison requires careful treatment of intervention. A process-recording system may itself change behavior. If participants work differently because they are monitored, the attempt probabilities cannot be assumed invariant across regimes. Likewise, removing protection to obtain a cleaner comparison may change the population willing to participate. Selection and treatment effects can then become entangled.

The paper proposes no universal experimental design. In some settings, existing procedural variation permits a responsible comparison. In others, protected retrospective records may be the only acceptable source. Where the counterfactual would require exposing people to unacceptable harm, it may remain unidentified. An explicit limit is preferable to presenting a protected outcome as evidence for an untested unsupported condition.

18.5 Trajectory sampling

The generative investigation first calls for a comparison of attachment structures. Evidence should identify the incoming practices, the intermediate compatibility condition, and the composite operation that becomes usable. A case in which both practices existed but failed to compose is especially informative: it can distinguish production of a channel from accumulation of successful episodes. Observing a new response without reconstructing its mediating attachments would not identify the proposed mechanism.

The appendix field and observable models add two narrower comparisons. First, repeat a boundary probe at different time scales. Agreement after equilibration can coexist with different short-interval responses, as Equation (62) demonstrates. Second, identify which observation functions remain usable after an intervention. A summary that predicts current records but fails Proposition B.2 is inadequate for that intervention even if its retrospective fit is excellent.

Neither comparison warrants an unrestricted experimental intervention. Existing timing variation, recorded handoffs, or protected rehearsals may permit partial tests. When the relevant fields cannot be calibrated, the field calculation remains a conditional mechanism model. When competing historical identifications cannot be adjudicated, a two-complex should preserve alternative presentations rather than manufacture a uniquely known history.

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Table 3: Formation evidence

Claim	Supporting observation	Evidential limitation
Entry support	Participation becomes feasible after a specified support appears	Participation is compelled or enabled by an omitted support
Diagnostic learning	Accessible records discriminate the stated alternatives	Outcomes are identical under the relevant alternatives
Capacity production	A new procedure or ability is used in later coordination	The apparent ability is supplied entirely by unchanged assistance
Historical sufficiency	Retained distinctions support later decisions accurately	Histories merged by the summary require different actions
Support portability	Relevant functions persist under a justified regime comparison	Withdrawal removes a needed function or observation
Cumulative admissibility	Each participant's trajectory remains within justified limits	Repetition exhausts resources or creates forbidden exposure

18.6 Historical sampling

Studying only relationships that persist creates a particular bias for formation research. Early refusals, failed entries, discontinued mediation, and abandoned projects may contain the evidence needed to understand the conditions of beginning. A sample of successful continuations can make formation look more automatic than it is.

A trajectory-oriented design should therefore identify its entry population and track exits with reasons where ethically possible. It should distinguish inability to begin, chosen refusal, operational failure, and loss of observation. These categories can overlap, and participants may disagree about their interpretation. Recording that uncertainty is part of the evidence rather than a defect to be removed by forced classification.

Timing also matters. A process may produce useful evidence only after an opportunity has expired. A capacity may become available after one participant's resources have been exhausted. The existence of eventual learning does not establish practical formation for the original undertaking. This temporal issue can occur without a critical transition and should first be investigated in its ordinary form.

18.7 Model confrontation

Table 3 connects each analytical distinction to evidence that could support or weaken it. The table is a research guide, not a checklist that mechanically validates a case. The interpretive and causal judgments require contextual argument.

The formal models can be confronted with evidence at their assumptions as well as their conclusions. An unmodeled refusal challenges the successor set. A disputed observation challenges the common protocol alphabet. A forgotten obligation challenges summary sufficiency. A changed task challenges the target predicate. Empirical inquiry should be allowed to revise these objects rather than merely assigning observed cases to a fixed formal vocabulary.

18.8 Research ethics

Formation research can change the relation it studies. Asking participants to demonstrate trust may expose them or imply that distrust requires justification to an investigator. Collecting process records can alter privacy and authority. Publishing a reconstructed history can close alternatives that the original encounter preserved.

These effects require explicit decisions about consent, access, minimization, and the consequences of interpretation. They cannot be solved by the paper's formal safety predicate alone. A participant's refusal to provide data can also be a legitimate boundary rather than evidence of untrustworthiness. The

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inquiry should preserve the distinction between an analytically desirable observation and an observation it is permissible to obtain.

19 Research Boundaries

This section examines objections to the account and identifies the conditions under which its formal representations cease to be adequate. The objective is to distinguish a useful investigation of formation conditions from a general theory of trust production. The discussion also locates the paper within the adopted research programme without expanding its scope into the subsequent cooperation-design paper.

19.1 The trust objection

The most direct objection is that the formal models concern preparation and reliable coordination rather than trust itself. The objection is correct about the models' immediate targets. A capability closure or a winning policy cannot establish an affective orientation, a participant stance, or a commitment-based trust relation. Treating operational readiness as a trust measure would contradict the distinctions established in Section 2.

The reply is that a theory of formation can investigate enabling and obstructing conditions without reducing the phenomenon to those conditions. An account of how a relation acquires a usable history, an interpretable undertaking, and an admissible next step can explain aspects of the setting in which trust becomes possible. To establish that trust actually emerges, the inquiry must add evidence and an account of what counts as trust. The paper therefore offers an analysis of formation trajectories rather than a sufficient-condition theorem for trust.

This boundary also prevents an overly instrumental interpretation. A participant need not experience the relation as a calculated series of tests. Trust can involve generosity, attachment, commitment, or an orientation that exceeds available evidence. The analytical reconstruction can still ask which practical capacities and exposures sustain that orientation. It should not redescribe every meaningful gesture as a covert experiment or a strategic investment.

19.2 The familiar-mechanisms objection

The mechanisms are familiar: institutions enable cooperation, interaction creates evidence, experience can change capacities, and mediation can sustain coordination. Existing work already joins several of these ideas. The paper's novelty cannot therefore rest on listing them or claiming that previous theories uniformly treated trust as a fixed individual property.

The proposed analytical gain is the explicit separation of dependencies and inference types. A formation claim is assessed for an actual beginning, a usable observation sequence, a created or revealed capacity, a cumulative exposure boundary, and a justified continuation regime. The formal counterexamples show that these conditions can come apart. The contribution remains a proposal for conceptual and methodological clarification; its distinctiveness relative to all adjacent work has not been demonstrated.

There is also no claim that the finite-horizon recursion, monotone closure, or likelihood calculation is a new mathematical device. Their derivations are included to expose assumptions and make the present application reproducible. Further specialist comparison with planning, game theory, control, and epistemic models could sharpen the account. It should be undertaken before attributing originality to any formal construction.

19.3 The endogenous-learning objection

One might object that sufficiently rich learning models already represent changing capabilities, interventions, and endogenous observations. That objection identifies an important limit to the contrast used here. The paper does not claim that all Bayesian or dynamical models assume a fixed counterpart. The

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distinction is between particular explanatory tasks, not between an inherently relational method and an inherently nonrelational one.

A model can include latent capacities that change through practice, evolving strategies, and observation processes affected by intervention. Such a model may represent the present concerns well if its states and transitions preserve the relevant distinctions. Conversely, a model labeled relational can erase those distinctions through unjustified aggregation. Adequacy depends on the construction and its use, not on disciplinary vocabulary.

The fixed-type diagnostic experiment is intentionally narrower. It isolates one masking effect under behavioral invariance. When formation changes attempt probabilities, the experiment must be revised. That limit is especially significant here because the paper's broader conceptual object expressly allows changes to the participant and relation being investigated.

19.4 The coordination objection

The policy model assumes a protocol that can choose joint moves from a common observation record. Establishing that protocol may itself require cooperation. If the model silently presupposed the very capacity it purported to produce, it would recreate the original circularity.

The dependency account supplies the required discipline. Initial protocol functions must be listed among the seed supports, or their construction must be represented as an earlier stage with its own prerequisites. A mediator can supply a limited coordinating function without establishing a fully shared world. If no such seed is available, the information-state model does not apply to the initial situation as written.

Even with a common record, participants may disagree about the appropriate move. The model's policy existence result is then only a result about a possible joint procedure, not a prediction of agreement to use it. Strategic bargaining, private information, and incompatible objectives require further structure. The paper deliberately includes refusal among possible outcomes rather than converting a proposed joint move into assumed compliance.

19.5 The representation objection

A finite configuration space can appear to conflict with the GR requirement to preserve heterogeneous local types. The apparent conflict is resolved only by specifying the representational level. The set Q is a list of scenario summaries used by an analyst for a bounded inquiry. It does not embed every actor's local state in one common semantic or metric space. The summary conditions must preserve the distinctions needed for the chosen task, and their adequacy remains open to challenge.

This answer has practical limits. If no finite summary preserves changing commitments, interpretations, or admissibility, the finite model should be abandoned or restricted. The two-complex in Section 8 preserves selected incidence history, but it also needs an adequacy argument. The interacting field generates candidate complexes, while the boundary algebra composes admissible rearrangements. The appendix models supply a finite observation-closure test and a fixed-carrier response calculation. These are specific inferential roles; they do not establish that every mathematical language must appear in every GR paper.

The resulting model family includes spacetime presentations, interacting group-domain fields, boundary algebras, discrete response fields, heterogeneous local maps, attributed histories, set-valued transitions, and conditional probability. Each addresses a selected problem. No unproved equivalence among them is assumed, and no one model supplies a complete ontology of the relation.

19.6 Strategic performance

A counterpart may perform well in small trials to obtain access to a larger opportunity. A formation trajectory that treats early success as unrestricted evidence can therefore create the conditions of its own exploitation. The present account does not eliminate that possibility. It limits the inference by indexing evidence to the task, incentives, support regime, and observation channel.

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A sequence of low-stakes successes may be genuinely informative about low-stakes performance while leaving a high-stakes divergence unresolved. Preserving safeguards during expansion can be appropriate for that reason. Yet safeguards can also be costly or alter the relation in ways participants reject. The paper supplies a framework for stating the tradeoff, not a universally optimal solution.

The same caution applies to apparently candid disclosures. An admission can provide evidence of responsiveness, but it can also be selected to create a favorable impression. Distinguishing these explanations may require a broader history, independent records, or evidence of subsequent conduct. Some uncertainty remains unavoidable. Trust formation cannot be made equivalent to the elimination of all strategically possible deception.

19.7 Normative scope

The target of reliable cooperation does not justify every relation that becomes more reliable. A formation trajectory can strengthen an exclusionary association, an exploitative dependency, or a harmful collective project. Nor does the participants' mutual benefit settle the effects on outsiders. The normative evaluation must inspect the undertaking and its wider consequences.

The paper's safeguards concern how formation is analyzed: distinguish willingness from obligation, preserve the possibility of refusal, identify burden distribution, and do not equate success with legitimacy. These are substantive methodological commitments, but they do not amount to a complete political morality. Conflicts among privacy, accountability, equality, and effective coordination can remain unresolved within a particular case.

19.8 Formal plurality

The GR investigations inherit substantial architecture from prior publications. Two-complex composition, group-domain fields, interaction kernels, Fock construction, accessible boundary algebras, historical order effects, and equilibrium reduction are not presented as newly discovered structures. Their application here asks which aspects of formation they clarify. Cubic field interactions, Gaussian elimination and creation–annihilation identities are established mathematical constructions. The paper's proposed contribution concerns their disciplined realization for this formation problem, not priority for the mathematical tools.

The generative model fixes a finite group, a type vocabulary and a family of kernels. It regulates the field integral, but does not prove convergence of the perturbative diagram expansion or derive a chronological social process from a Euclidean ensemble. Its conditioned channel result is not a finite-system theorem of spontaneous symmetry breaking. An occupation vector does not uniquely determine a boundary complex, and an operator adjoint does not undo a social history. These restrictions belong to the model's content.

The appendix field additionally assumes linear translation, positive mismatch weights and a selected relaxation law on a fixed carrier. It remains an effective-model alternative. The spin-foam realization requires justified group labels, intertwiners and an observation interpretation. Within the stated models, strand contraction, induced quartic coupling, boundary replacement, observable closure and field memory have exact consequences. Claiming that a particular social relation realizes any of them requires evidence and interpretation.

19.9 Computational verification

The accompanying source packet checks the illustrative constructions with an exact computational audit. For forty generated four-capability rule systems, it compares closure with explicit enumeration of sequentially reachable capability sets from every initial subset. For twenty-four generated three-configuration transition systems, it compares the information-set recursion with independently enumerated contingent policy trees at horizons zero, one, and two, checking every nonempty initial information set. The latter

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evaluation follows concrete hidden-state realizations rather than reusing the backward recursion as its own test.

Additional checks cover the worked formation example, erased observations, refusal, the distinct responsible-resolution target, and a support-withdrawal failure. Rational arithmetic checks the protected-outcome difference and process-likelihood cancellation on an illustrative parameter grid. Separate examples check cumulative cost and repeated disclosure. The fixed generation seed is recorded so these finite checks can be reproduced.

The retained finite and fixed-carrier investigations yield 1,413 and 83 passing assertions respectively. The latter include comparison of the memory identity with full-state spectral evolution using independent numerical quadrature. The generative construction adds checks of the invariant projector, typed convolution, compatible and vanishing-composition cases, the nonlinear stationary equation, the exact effective-action identity, and the creation–annihilation identities on finite invariant sectors. An independent quadrature checks the complex Gaussian elimination. These computations expose calculation errors; they do not validate the social adequacy of the primitives or establish a trust attribution. The analytic proofs carry the general conditional claims.

19.10 Programme interfaces

The earlier near-critical trust investigation concerns changes in relational and interpretive structures that can undermine maintenance and correction. Those processes can also interrupt formation. A newly created interface can become obsolete, or evidence can lose its practical value before participants can act on it. Such events do not require criticality to be intelligible. A criticality hypothesis would need an independently specified mechanism before it entered this paper's central explanation.

The next companion problem concerns reliable cooperation where substantial trust remains absent or incomplete. The present paper reaches that boundary whenever a formation trajectory stabilizes as protected reliance rather than developing interpersonal trust. A broader account could compare institutional arrangements across that range. The current argument does not attempt that synthesis; it examines how beginnings and subsequent encounters alter the relation's possibilities.

Finally, resource maintenance and unequal revisability remain possible extensions only where a substantial independent argument emerges. Their presence as constraints here does not automatically create another paper. The formation object remains integrated: its local mechanisms matter because they jointly determine whether a beginning can become a usable and defensible history.

20 Formation Prospects

This final section states the resulting account and its practical limits. Trust formation is approached through historically situated encounter sequences that change the grounds and capacities for subsequent reliance. The account retains a familiar insight—cooperation can help generate conditions for later trust—while making its internal dependencies more explicit.

The GR interpretation places those encounters within the spacetime of historically constituted beings. A first undertaking can alter what its participants can do, what they owe, and which aspects of their conduct become recognizable. The account therefore concerns the generation of relational possibilities as well as learning about a counterpart. Its normative claims remain distinct: a generated commitment can be questioned, and a stable capacity can serve an objectionable arrangement.

The spacetime, field, and algebraic representations make connected parts of that argument precise. Historical composition records the continuations through which later acts attach to earlier configurations. The interacting field supplies local gluing kernels whose contractions generate history complexes. Its cubic sector specifies the composition of a new channel; the boundary algebra represents the corresponding changes of sector and admissible operations. These are explicit mechanisms within selected representations, not a universal derivation of social structure.

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Eliminating a mediating field leaves a quartic effective interaction. An endpoint description can therefore retain a dependency that it no longer names as a mediator. Stationarity, field correlations, and practically available responses require separate interpretation. A formally generated channel does not by itself establish honest responsiveness or justified vulnerability. The model's philosophical use is to specify what may have been constituted and which additional judgments remain necessary.

These are several possible models with explicit domains. The optional spin-foam construction supplies additional label and amplitude structure without making it compulsory for every GR analysis. The finite protocol models remain useful local investigations when their preservation assumptions hold. The appendix models of linear field response and finite observable closure analyze selected effective arrangements. Model plurality requires stated interfaces and limits, rather than treating one convenient state space as the ontology of the relation.

A beginning requires actual support. It need not require the same capacities as a later undertaking, but the prerequisites of the initial encounter cannot all be deferred to its hoped-for results. Monotone closure makes that point precise within a limited construction system. The history-sensitive model then distinguishes the existence of a favorable sequence from a policy that participants can select through the observations available to them.

Protection has multiple formation effects. It can make an encounter admissible, supply part of its performance, alter incentives, and change the evidence produced. The diagnostic experiment shows why a protected final outcome and an informative process record must be distinguished. Neither a general hostility to safeguards nor an automatic inference from protected success follows. The proper inference depends on the specific intervention, target distinction, and accessible observations.

Formation can also produce a capacity that remains dependent on its supports. Shared history, mediation, and a tested interface may become constituents of a reliable arrangement. Their contribution should be retained in its description. A claim of portability requires evidence about the changed arrangement; repeated success alone does not establish it. Cumulative exposure likewise belongs to the trajectory, so a succession of individually bounded encounters cannot automatically be called bounded overall.

These results leave room for several outcomes. Participants may develop a trusting attitude, a justified narrow reliance, a durable mediated arrangement, a better-founded distrust, or an informed decision to end. A theory that recognizes only increasing trust would misclassify some of these achievements and obscure some of their costs. The formation perspective instead asks what the relation has made possible, what grounds support the next undertaking, and whose alternatives remain meaningful.

The paper's propositions establish conditional facts about explicit representations. Its comparative cases identify questions for investigation, and its empirical framework specifies evidence that could revise the account. The further claim that a particular trajectory has produced trust requires a defensible trust attribution and evidence from that relation. Preserving that requirement keeps the formal investigation connected to its philosophical object without pretending that the object has been exhausted.

A Fixed-Carrier Field Reduction

This appendix retains a fixed-carrier linear model of distributed mediation. It derives an equilibrium boundary response, eliminates an internal field without discarding its temporal effects, and compares two arrangements with the same equilibrium response. It analyzes response on an already selected relational carrier. The generation of that carrier is addressed separately in Section 9. Its coordinates describe calibrated coordination variables rather than levels of trust.

A.1 Field interpretation

The prior public-trust paper represents trust-related conditions through several coupled fields, including institutions, information, power, epistemic practices, and affective organization. Its action-like functional organizes interactions and interventions; it does not derive a universal physical law [12]. The

background-independence account likewise requires a field's domain, quantities, couplings, feedback, and interpretation to be stated [9]. These requirements guide the present construction.

A field is useful here when a local response depends on an extended arrangement. A warning can become actionable through a translator, a reporting practice, a recipient's calibration, and a channel for interruption. The relevant capacity is distributed across that organization. Altering a mediating condition can change several local responses even when no actor's general attitude changes.

For an exact tractable model, choose a fixed boundary graph from Section 8. A site is a coupling vertex; an internal link is an element incident to two such vertices. Use the connected sector with at least one retained site. At each site c, let V_c be a finite-dimensional real inner-product space of selected local field coordinates. At each internal link p, let W_p be a common comparison space, and let $R_{pc} : V_c \to W_p$ translate the selected coordinates at an incident site into that space. These maps are part of the model, not a claim that heterogeneous participants possess identical meanings.

Write $z = (z_c)_c$ for the field in $\mathcal{V} = \bigoplus_c V_c$. Orient each internal link only to define a signed difference. With $c_-(p)$ and $c_+(p)$ its endpoints, the restriction-difference operator is the following linear map.

$$D : \mathcal{V} \longrightarrow \bigoplus_p W_p, \qquad (Dz)_p = R_{p,c_+(p)} z_{c_+(p)} - R_{p,c_-(p)} z_{c_-(p)}. \tag{49}$$

Equation (49) measures a specified interface mismatch. A practical candidate would be a discrepancy between calibrated task representations, such as two encodings of a common reference signal. Measuring goodwill, dignity, or normative agreement by this difference would require an additional argument that the present construction does not supply.

Let W be a positive-definite block weighting operator on $\bigoplus_p W_p$. The quadratic mismatch functional and its associated field operator are defined as follows.

$$\mathcal{E}(z) = \frac{1}{2}\langle Dz, WDz\rangle, \qquad L = D^*WD, \qquad \nabla\mathcal{E}(z) = Lz. \tag{50}$$

The adjoint D^* in Equation (50) uses the declared inner products. The functional is nonnegative, but it is not a moral value or a universal social energy. Rescaling field coordinates requires corresponding changes in R, W, and any calibrated dynamics. In the illustrative calculations, coordinate and time units are fixed by nondimensionalization.

A.2 Local field dynamics

Choose identity mobility as an explicit constitutive assumption and let $u(t) \in \mathcal{V}$ be a continuous imposed drive. On an interval with fixed carrier and coefficients, the proposed relaxation law is the following.

$$\dot{z}(t) = -Lz(t) + u(t), \qquad z(0) = z_0. \tag{51}$$

Equation (51) defines a linear dynamical model of the selected mismatch response. It is not derived from GR alone. Nonlinear, asymmetric, stochastic, or actively strategic dynamics are alternatives when the case requires them.

For zero drive, the chain rule yields an exact energy identity.

$$\frac{d}{dt}\mathcal{E}(z(t)) = \langle Lz(t), \dot{z}(t)\rangle = -\|Lz(t)\|^2 \le 0. \tag{52}$$

Equation (52) proves decay of the selected mismatch functional. It does not prove increasing trust. Kernel modes can remain, and a low mismatch can describe coercive alignment or a jointly harmful practice. The philosophical distinction between generated coordination and justified relations therefore remains necessary even in a mathematically stable model.

Formation can change the carrier, the restriction maps, or the response coefficients. The present representation treats such changes as events between fixed-coefficient intervals. Each event supplies a new boundary graph and an explicit map transferring retained field coordinates or initializing newly created ones. It is a hybrid description of declared coupling events and continuous local relaxation, not an assumed law that practice automatically strengthens every coupling.

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A.3 Equilibrium boundary response

Partition the field coordinates into retained $x \in \mathbb{R}^n$ and internal $y \in \mathbb{R}^m$, using orthonormal coordinates in their respective spaces. The internal field can represent a mediator's selected coordination variables. In this subsection A, B, C denote matrix blocks, independently of the earlier coupling-set notation.

$$z = \begin{pmatrix} x \\ y \end{pmatrix}, \qquad L = \begin{pmatrix} A & B \\ B^* & C \end{pmatrix}, \qquad \mathcal{E}(x,y) = \frac{1}{2}x^*Ax + x^*By + \frac{1}{2}y^*Cy. \tag{53}$$

All coordinates are real, so $*$ is transpose in Equation (53). Assume C is positive definite. In the scalar connected graph sector, retaining a nonempty boundary ensures this property for its grounded internal Laplacian. For general restriction maps it is a separate assumption; an internally invisible mode can make it fail.

The next result is an adaptation of the exact boundary reduction already developed in the author's spacetime-of-desire paper [13]. Its formation interpretation is that an effective endpoint capacity can summarize continuing internal mediation.

Proposition A.1 (Equilibrium mediation)**.** *For fixed x and positive-definite C, the unique minimizing internal field and the retained quadratic response are given by the following expressions.*

$$y_*(x) = -C^{-1}B^*x, \qquad Q_{\mathrm{m}} = A - BC^{-1}B^*, \qquad \min_y \mathcal{E}(x,y) = \frac{1}{2}x^*Q_{\mathrm{m}}x. \tag{54}$$

Proof. Complete the square in Equation (53). The exact decomposition is the following.

$$\mathcal{E}(x,y) = \frac{1}{2}(y + C^{-1}B^*x)^*C(y + C^{-1}B^*x) + \frac{1}{2}x^*(A - BC^{-1}B^*)x. \tag{55}$$

The first term in Equation (55) is nonnegative and vanishes only at $y = -C^{-1}B^*x$. Substitution gives Equation (54). Since the original energy is nonnegative, so is the minimized quadratic form. □

Proposition A.1 preserves equilibrium boundary response. It neither removes the mediator physically nor demonstrates that the retained sites have learned to coordinate without it. The inverse C^{-1} inside Q_{m} is precisely where the internal response remains represented.

For a scalar path with retained sites a, b and internal site m, take link weights $g_a, g_b > 0$, identity restriction maps, and the energy stated below.

$$\mathcal{E} = \frac{1}{2}g_a(x_a - y)^2 + \frac{1}{2}g_b(y - x_b)^2, \qquad Q_{\mathrm{m}} = \frac{g_a g_b}{g_a + g_b}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}. \tag{56}$$

Equation (56) follows by taking $A = \mathrm{diag}(g_a, g_b)$, $B = (-g_a, -g_b)^*$, and $C = g_a + g_b$ in Equation (54). It is the inherited mediated-adjacency calculation applied to formation. No direct link between a and b was present in the carrier. If the internal site and both links are physically removed without replacement, the retained mismatch operator becomes zero. The nonzero reduced operator is therefore evidence of an effective mediated response, not of an independently acquired direct channel.

A.4 Dynamical memory

An internal field need not equilibrate before the boundary changes. Set $u(t) = (f(t), 0)$, where $f(t) \in \mathbb{R}^n$ drives the retained coordinates. Equation (51) then becomes the coupled system below.

$$\dot{x} = -Ax - By + f(t), \qquad \dot{y} = -B^*x - Cy, \qquad y(0) = y_0. \tag{57}$$

The variables in Equation (57) describe the same field sector as the equilibrium calculation. They add a specified temporal law, rather than obtaining time dependence from the static graph alone.

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Proposition A.2 (Retained field memory)**.** *For fixed finite matrices in Equation* (57) *and continuous drive, eliminating y yields an exact retained equation with an initial-history term and a memory kernel.*

$$\begin{aligned} \dot{x}(t) &= -Ax(t) - Be^{-Ct}y_0 + \int_0^t M(t-s)x(s)\,ds + f(t), \\ M(t) &= Be^{-Ct}B^*. \end{aligned} \tag{58}$$

Proof. Multiply the internal equation by e^{Ct} and integrate. The resulting internal trajectory is the following.

$$y(t) = e^{-Ct}y_0 - \int_0^t e^{-C(t-s)}B^*x(s)\,ds. \tag{59}$$

Differentiating Equation (59) recovers the internal equation and its initial condition. Substituting it into the retained equation in Equation (57) gives Equation (58) term by term. □

Proposition A.2 is an elementary elimination result for the declared linear model. The history dependence is causal: the integral runs over earlier boundary values. It is not retroactive physical causation. The term involving y_0 records a selected internal condition inherited at the beginning of the observation interval. That condition may summarize earlier interaction without exposing its full history.

The result distinguishes two analytical operations. Retaining only x as explicit coordinates can be exact if the memory term is retained. Replacing that term by an instantaneous response is an approximation requiring additional temporal assumptions. A relation that appears simple at its interface may therefore carry internal historical effects even within a classical linear field model.

A.5 Equilibration conditions

The difference between dynamic and instantaneous internal response can be bounded. Let $\lambda_{\min}(C) = \lambda > 0$ and suppose x is continuously differentiable on $[0, t]$. Define the internal tracking error relative to instantaneous equilibrium by $\eta(t) = y(t) + C^{-1}B^*x(t)$. Direct differentiation gives its evolution.

$$\dot{\eta} = -C\eta + C^{-1}B^*\dot{x}, \qquad \dot{x} = -Q_{\mathrm{m}}x - B\eta + f. \tag{60}$$

Equation (60) identifies the precise omitted term in an instantaneous reduction: it is $-B\eta$ in the retained dynamics.

Proposition A.3 (Equilibrium tracking bound)**.** *For the stated symmetric positive-definite internal operator, the tracking error satisfies the following estimate in the Euclidean norm.*

$$\|\eta(t)\| \le e^{-\lambda t}\|\eta(0)\| + \frac{\|C^{-1}B^*\|}{\lambda}(1 - e^{-\lambda t}) \sup_{0\le s\le t} \|\dot{x}(s)\|. \tag{61}$$

Proof. Variation of constants in Equation (60) gives $e^{-Ct}\eta(0)$ plus the convolution of $e^{-C(t-s)}C^{-1}B^*$ with $\dot{x}(s)$. Symmetric positivity implies $\|e^{-Cr}\| \le e^{-\lambda r}$ for $r \ge 0$. Apply the triangle inequality and the operator-norm inequality, bound the derivative by its supremum, and integrate $e^{-\lambda(t-s)}$ from zero to t. This integral is $(1 - e^{-\lambda t})/\lambda$, giving Equation (61). □

Proposition A.3 gives conditions for using equilibrium evidence in a changing encounter. Small initial error and a slowly varying boundary relative to the internal relaxation rate can make the approximation adequate for a stated tolerance. Rapidly changing demands or a poorly prepared internal state can defeat it. No criticality hypothesis is needed for that conclusion.

The bound also limits a common inference from rehearsal. Success after a long settling period does not establish the same response during rapid handoff. The relevant difference may be a temporal property of the mediating field, rather than a change in motivation. An empirical investigation should therefore vary timing as well as inspect final outcomes.

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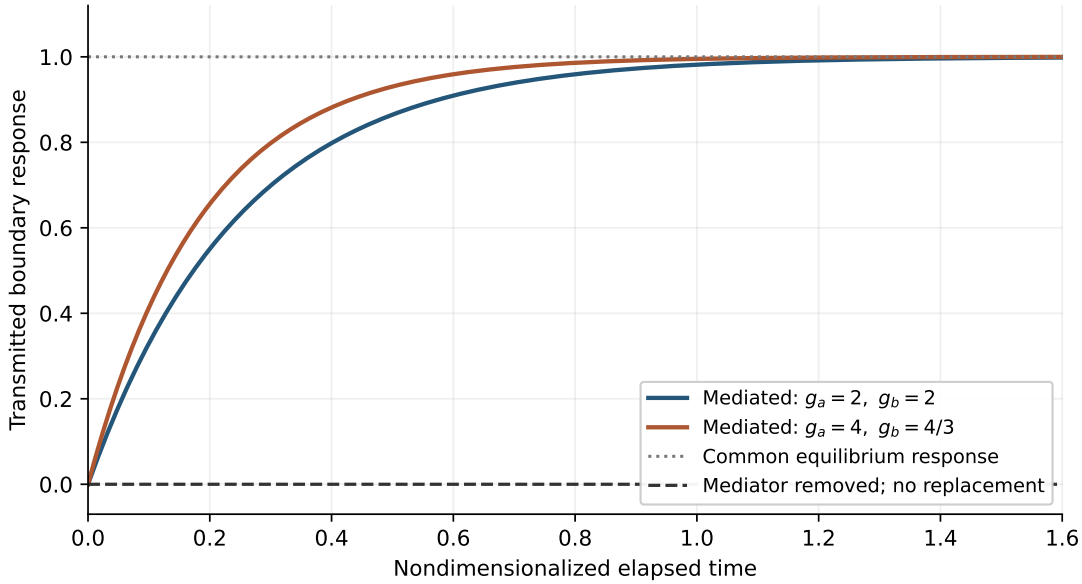

Figure 3: Mediated boundary response. Two uncalibrated illustrative fields share an equilibrium coefficient but respond at different rates. Physical removal of the mediator without a replacement channel gives zero transmitted response. Coordinates and time are nondimensionalized; the vertical axis is a task-field response, not a trust score.

A.6 Equivalent equilibrium responses

The scalar path permits a direct comparison. Hold $x_a = 1$, $x_b = 0$, and initialize $y(0) = 0$. The internal field obeys $\dot{y} = g_a - (g_a + g_b)y$. Define $J_b(t) = g_b(y(t) - x_b)$ as the transmitted boundary response at site b. Solving the scalar equation gives the following result.

$$y(t) = \frac{g_a}{g_a + g_b}\left(1 - e^{-(g_a+g_b)t}\right), \qquad J_b(t) = \frac{g_a g_b}{g_a + g_b}\left(1 - e^{-(g_a+g_b)t}\right). \tag{62}$$

Equation (62) describes a boundary probe with the endpoint values held fixed. Compare the weight pairs $(2, 2)$ and $(4, 4/3)$. Both have equilibrium coefficient $g_a g_b/(g_a + g_b) = 1$, but their relaxation rates are 4 and $16/3$. They agree on the equilibrium response and disagree on finite-time responses. Figure 3 displays this exact illustrative comparison.

The example does not show that either arrangement is generally better. Speed may have costs outside the model, and the calibrated variables may fail to capture the undertaking's important features. Its exact conclusion is narrower: a shared equilibrium operator does not determine the transient response, and neither reduced description establishes support independence.

A.7 Formation evidence

These calculations identify three different claims about an apparently improved relation. First, the existing mediated arrangement may have become better calibrated. Second, participants may have acquired a direct coupling that supplements the mediator. Third, an analyst may simply have replaced an explicit mediator with an effective boundary operator. Only the second claim establishes the creation of the specified direct channel, and even it does not guarantee that all other support dependencies have disappeared.

A formation investigation should accordingly identify the carrier before and after the encounter, the intervention that allegedly created the coupling, the boundary probes used to evaluate response, and the temporal regime in which those probes operate. Where support withdrawal is unacceptable, provenance and constrained perturbations may provide partial evidence. The model does not justify hazardous withdrawal merely to demonstrate independence.

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The field analysis thus returns to the philosophical account. A capacity belongs to a historically constituted arrangement. Its apparent localization at an endpoint can depend on how the arrangement is represented. Trust can be directed toward an actor while the practical grounds of reliance remain distributed across its spacetime. Adequate attribution should preserve that distribution whenever it matters for the next undertaking.

B Finite Observable Algebras

This appendix formalizes two questions within a fixed finite boundary vocabulary: which distinctions a participant can register, and how interventions change later possibilities of registration. It inherits the accessible-algebra distinction from prior GR work and the emphasis on compositional historicity from the background-independence programme [14, 9]. The construction supplies a limited observation-closure test. The generation of boundary structures is addressed separately in Section 10.

B.1 Accessible observables

Choose a nonempty finite set Ω of boundary configurations selected from the histories under investigation. Each configuration can summarize a structured boundary rather than a complete being. Let $\mathcal{H} = \ell^2(\Omega)$ be the complex vector space of functions on Ω, with inner product $\langle f, g\rangle = \sum_{q\in\Omega} \overline{f(q)}g(q)$. For a function $f : \Omega \to \mathbb{C}$, define the multiplication operator M_f by $(M_f\psi)(q) = f(q)\psi(q)$.

An observation map $o : \Omega \to Y$, with finite image $o(\Omega) \subseteq Y$, determines the accessible multiplication algebra. Here $\mathcal{B}(\mathcal{H})$ denotes the algebra of linear operators on the finite-dimensional space $\mathcal{H}$.

$$\mathfrak{A}_o = \{M_{g\circ o} : g : o(\Omega) \longrightarrow \mathbb{C}\} \subseteq \mathcal{B}(\mathcal{H}). \tag{63}$$

Equation (63) defines a unital algebra closed under adjoints, since products and complex conjugates of functions constant on observation fibres remain constant there. In finite dimension it is a C^*-algebra with the inherited operator norm. It is commutative. Its use here does not assert nonclassical probabilities or physical quantum coherence.

Let $p : \Omega \to [0, 1]$ have total mass one, and let I denote the identity operator. The mixture can describe an analyst's stated alternatives without representing a person's full belief. Its state on accessible multiplication operators is the following positive normalized functional.

$$\omega_p(M_f) = \sum_{q\in\Omega} p(q)f(q), \qquad \omega_p(M_f^* M_f) = \sum_q p(q)|f(q)|^2 \geq 0, \qquad \omega_p(I) = 1. \tag{64}$$

Equation (64) states positivity directly. No signed or complex history amplitude is presumed to define this probability mixture.

Proposition B.1 (Accessible equivalence)**.** *Two probability mixtures p and p' agree on every operator in $\mathfrak{A}_o$ if and only if they assign the same total probability to each observation fibre $o^{-1}(y)$.*

Proof. For necessity, use the indicator function of each observation value. Its expectation is precisely the mass of the corresponding fibre. For sufficiency, group the sum in Equation (64) by observation values. Every accessible function is constant on each fibre, so equality of fibre masses gives equality of all its expectations. □

Proposition B.1 gives a precise limit to a record. An interface that reports only successful delivery can make a competent attempt and a rescued failure indistinguishable. Adding an attributable attempt record changes the observation partition and can enlarge the algebra. It does not retroactively establish that the earlier observer possessed the missing distinction.

This is an epistemic limit relative to an interface. It neither identifies relational reality with observed manifestation nor proves that an inaccessible difference is irrelevant. A later intervention may make that difference consequential and observable.

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B.2 Intervention operators

For a deterministic local intervention a, let $T_a : \Omega \to \Omega$ be its declared configuration map. This subsection assumes total maps on a fixed finite carrier. Partial interventions and changing carriers require restricted domains or a family of spaces with maps between them. The pullback operator acts on observables.

$$(K_a f)(q) = f(T_a(q)), \qquad (K_a K_b f)(q) = f(T_b(T_a(q))). \tag{65}$$

Equation (65) fixes the order convention: $K_a K_b$ evaluates an observation after performing a and then b. The operator algebra generated by these maps lies in $\mathrm{End}(\mathcal{H})$. Its linear combinations are mathematical operations on functions; they need not themselves be available social interventions. Likewise, the adjoint of a pullback need not correspond to reversing the historical act.

An observation summary is dynamically adequate only if the future observable after an intervention depends on the present observation, rather than on a difference the summary discards. The exact deterministic condition follows.

Proposition B.2 (Observable closure)**.** *An intervention T_a admits a map $\overline{T}_a : o(\Omega) \to o(\Omega)$ satisfying $o \circ T_a = \overline{T}_a \circ o$ if and only if $o(T_a(q)) = o(T_a(q'))$ whenever $o(q) = o(q')$. Equivalently, pullback by K_a preserves the functions defining $\mathfrak{A}_o$.*

Proof. If $\overline{T}_a$ exists, equal present observations give equal future observations by substitution. Conversely, define $\overline{T}_a(y)$ using any q with $o(q) = y$. The fibre condition makes that definition independent of the representative. Every pulled-back function $g \circ o \circ T_a$ then factors through o. If the fibre condition fails, an indicator distinguishing the two future observation values pulls back to a function that differs within one present fibre, so preservation fails. □

Proposition B.2 is the operator counterpart of the summary conditions later used for policy analysis. It explains why an excellent current fit can be inadequate for formation: the encounter may expose distinctions that the current observation algebra suppresses. Increasing confidence within the old algebra cannot recover a missing distinction without a change of probes or representation.

B.3 Order-sensitive formation

Consider a local four-configuration example $\Omega = \{0,1\}^2$. Write $q = (c, r)$, where c records whether a specified coordination procedure has been acquired, and r records whether a designated test most recently certified that procedure. Both variables are operational indicators, not amounts of trust. A practice intervention T_P establishes the procedure. A witnessing test T_W accurately records its present availability.

$$T_P(c, r) = (1, r), \qquad T_W(c, r) = (c, c), \qquad f_r(c, r) = r. \tag{66}$$

Equation (66) declares a stylized intervention pair. It assumes successful practice and accurate testing so that the order effect can be isolated. It makes no claim that all training works or that real certification is infallible.

Starting from $q_0 = (0, 0)$, practice followed by witnessing reaches $(1, 1)$, whereas witnessing followed by practice reaches $(1, 0)$. Using the convention in Equation (65), the record observable has the following commutator value.

$$([K_P, K_W] f_r)(q_0) = (K_P K_W f_r)(q_0) - (K_W K_P f_r)(q_0) = 1 - 0 = 1. \tag{67}$$

Equation (67) makes one form of historical composition explicit. The same two interventions produce the same acquired procedure but different available grounds for attributing its acquisition. Counting interventions, recording only final capability, or treating testimony as independent of its temporal position loses this difference.

The observation $o(c, r) = r$ also illustrates Proposition B.2. Configurations $(0, 0)$ and $(1, 0)$ agree on the present record. Applying T_W distinguishes them. Their common record is therefore insufficient

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for predicting the effect of that test. By contrast, the full pair (c, r) closes this particular finite problem. It still does not describe an entire person or institution.

Temporal noncommutativity and compositional meaning already appear in the prior trust analysis [11]. The example supplies a narrow exact realization, not a claim that noncommutativity was previously absent from GR. Nor does its nonzero commutator establish uncertainty relations, entanglement, or other specifically quantum effects.

B.4 Interpretive change

Formation may alter the observation map itself. A new translation practice can make a previously unintelligible warning available as a reason for action. In the finite representation, this can refine o and enlarge $\mathfrak{A}_o$. A suppressive reporting regime can instead coarsen it. Neither direction by itself measures the quality of the relation: unrestricted observability can violate privacy, and some limits protect legitimate independence.

The philosophical distinction is between accumulating answers to an unchanged question and becoming able to ask a different question. A new capacity for recognition can belong to the relation's formation even if no posterior confidence rises on the old variables. Conversely, a richer algebra of observable distinctions does not guarantee that a participant has standing to use them. Record availability, interpretive competence, and practical authority remain different conditions.

This construction also identifies a boundary of the current model. If formation produces genuinely new effective configurations or categories, a fixed Ω may cease to suffice. One must then specify translations between successive carriers and observables. The two-complex account permits represented births and terminations; the present operator example studies a fixed local sector within that wider possibility.

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