

When AI Generativity Becomes Human-Like

A Perturbative Field-Theoretic Analysis of Relational Coupling

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Abstract

This paper asks whether artificial relational capacity can alter a social system's ability to regenerate human coupling even when artificial and human agents are assumed to be indistinguishable at the level of their generative dynamics. The question is therefore moved from agent deficiency or individual harm to the dynamics of coupling itself.

Coupling is modelled as a dynamical variable that forms, decays, and responds to a relational deficit. A minimal two-channel model distinguishes human and artificial coupling only by formation, decay, substitution efficiency, and available artificial capacity. The perturbation is applied to coupling rather than to agent states, and the analysis examines equilibrium structure, local stability, and recovery.

Three results follow. First, equilibrium human coupling decreases monotonically as artificial coupling capacity increases, even without any assumed inferiority of artificial agents. Second, substitution alone does not destabilise the system: with a positive baseline rate of human-coupling formation, the equilibrium remains linearly stable for every admissible artificial capacity. Third, loss of self-stabilisation appears only when an additional regenerative-dependence condition is introduced, so that new human coupling depends on human coupling already present. Under that condition the system undergoes a transcritical bifurcation at a critical artificial-coupling capacity, beyond which the zero-human-coupling state becomes stable even while the relational deficit can remain small. Approaching this boundary from below produces critical slowing, making recovery time after a relational breach a candidate observable of declining regenerative capacity.

The paper's contribution is formal and diagnostic rather than predictive or normative. It separates displacement from destabilisation, identifies the additional mechanism required for collapse of human relational regeneration, derives the corresponding critical condition, and shows how recovery dynamics can make that loss empirically contestable. Its role is to specify, under explicit assumptions, when increasing artificial relational capacity is theoretically compatible with continued human relational self-stabilisation and when it can become structurally dangerous.

Keywords: relational coupling; adaptive networks; self-stabilisation; coupling perturbation; transcritical bifurcation; critical slowing; generative relational being

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Use of generative artificial intelligence. Anthropic's Claude and OpenAI's ChatGPT were used in the preparation of this paper for literature survey, drafting from the author's specifications, adversarial review, figure construction, and consistency and citation auditing, and for typesetting. Every claim, every argument, and every citation was reviewed and decided by the author. Bibliographic details should be verified against primary sources before reliance.

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1 Introduction

Artificial conversational systems can now occupy positions that were once available only through human relations: they can remain continuously available, remember prior interaction, adapt to a user, respond without fatigue, and provide a form of companionship at very low marginal cost. The most immediate questions about such systems are usually evaluative. Do they increase isolation? Do they create dependence? Do they encourage expectations that human partners cannot satisfy? Are the resulting relations genuine, deceptive, beneficial, or harmful?

These questions matter, but they are not identical to the question pursued here. A social or relational system can contain harm and still remain capable of reorganising itself. Human relations continuously generate conflict, exclusion, dependency, abandonment, and failure, yet such disturbances can also elicit repair, new norms, new institutions, and new relations. The relevant dynamical question is therefore not simply whether a new relational technology produces undesirable outcomes. It is whether the system retains the capacity to regenerate the relations and practices through which it responds to disturbance.

This paper studies that narrower problem: *when artificial generativity becomes human-like at the level of the agent, can a difference in coupling alone alter the self-stabilising capacity of the relational system?* The question is deliberately constructed to remove one of the easiest routes to a predetermined answer. Rather than assuming that an artificial partner is less responsive, less generative, less intelligent, or less capable of sustaining an interaction, the paper stipulates agent-level generative indistinguishability. Whatever generative operation is attributed to a human agent is, for purposes of the model, attributed equally to an artificial agent. The analysis therefore cannot derive its result from an assumed inferiority of artificial agency.

Once that assumption is imposed, the remaining difference lies in the coupling. Relations are not determined only by the capacities of the parties they connect. They also differ in how they are formed, how costly they are to maintain or abandon, how strongly they depend on prior relations, and how they create conditions for further relations. A relation to a family member, colleague, neighbour, institution, or artificial partner may therefore differ even if the parties on either side are treated as equally generative. The central object of the paper is this relational layer.

The second move is to treat coupling itself as dynamical rather than as a fixed network on which agent dynamics merely occur. Human coupling and artificial coupling can form, strengthen, decay, and substitute for one another, while the states of the connected agents respond to the resulting relational environment. The perturbation considered here is consequently a perturbation of coupling: instead of asking how fixed relations transmit a disturbance in agent state, the analysis asks how the relational structure itself reorganises when the composition and capacity of available coupling change.

A minimal model is introduced to make that question exact. A relational deficit drives the formation of human and artificial coupling. The two channels differ only through pa-

rameters governing formation, decay, substitution efficiency, and available artificial capacity. This construction is intentionally spare. Its purpose is not to reproduce the sociology of any actual population, but to determine which conclusions really follow from the proposed mechanisms and which require additional assumptions.

The analysis produces a sequence of results that separates three claims often treated as if they were equivalent. First, increasing artificial coupling capacity displaces equilibrium human coupling through substitution alone. Second, displacement is not the same as destabilisation: when human coupling forms at a positive baseline rate, the equilibrium remains linearly stable for every admissible artificial capacity. The formal model therefore rejects the proposition that substitution by itself is sufficient to destroy self-stabilisation. Third, a collapse regime appears only when a further regenerative-dependence mechanism is introduced—human coupling must contribute to the formation of new human coupling. Under that condition, the system exhibits a transcritical bifurcation: beyond a critical artificial-coupling capacity, the zero-human-coupling state becomes stable even though the relational deficit experienced at the agent level can remain small.

This distinction matters because the model also yields a diagnostic implication. Near the critical boundary, the restoring dynamics weaken and recovery from a relational breach becomes progressively slower. The associated critical slowing provides a candidate observable that does not require the critical capacity itself to be known. In empirical terms, the framework therefore directs attention away from a one-time measure of satisfaction or contact volume and toward the dynamics of regeneration: after comparable disturbances, how quickly does human relational capacity rebuild itself?

The paper makes four contributions. First, it supplies a coupling-centred formulation of the problem that does not depend on denying artificial agents human-like generativity. Second, it proves a negative result that is theoretically important: substitution of human coupling by artificial coupling is insufficient, by itself, to produce loss of local stability in the minimal constant-formation model. Third, it identifies regenerative dependence as a sufficient mechanism for a genuine stability exchange and derives the corresponding critical condition. Fourth, it connects the formal result to a measurable dynamical signature—recovery time—while keeping separate the empirical and normative steps required before any claim of social danger can be made. The role of the paper is therefore not to predict that artificial companionship will cause relational collapse, but to specify a minimal mechanism by which such a collapse can or cannot occur and to make the resulting claims empirically contestable.

The remainder of the paper develops this argument in stages. Sections 2–3 delimit the claim and situate it relative to prior work. Sections 4–7 introduce the mathematical setting, the agent-level indistinguishability assumption, the coupling formulation, and the minimal model. Sections 8–11 derive and verify the equilibrium, stability, and bifurcation results. Sections 12 and 13 develop their observable and structural interpretations. Sections 14 and 15 state the limitations, practical implications, and conclusion.

2 Scope of the Argument and Its Restrictions

This section fixes what the paper undertakes. It has three objectives: to state the object of analysis and the five restrictions under which it is treated; to state the status of the model, which is the point on which the paper is most open to misreading; and to record four declarations, each with what establishing the contrary would require. The method is stipulative and the section argues for nothing.

2.1 *The object of analysis and five restrictions*

The object of analysis is the response of a relational system to a perturbation applied to its coupling, and the question is whether that response differs according to the composition of the coupling available.

The first restriction concerns artificial systems. The paper advances no claim about the internal states, the understanding, the experience, or the standing of any artificial system, and §5 adopts an assumption chosen so that no such claim is required at any point.

The second restriction concerns harm. The paper identifies no arrangement as harmful and no party as wronged. What it establishes concerns the dynamics of a coupling structure, and the step from a structural finding to an evaluative one is taken nowhere.

The third restriction concerns the model. The model of §7 is minimal and illustrative. Its variables are aggregate, its functional forms are the simplest consistent with the mechanism described, and its parameters are chosen to exhibit behaviour in place of estimating anything. No quantity in this paper is measured, and no numerical value here corresponds to any feature of any actual arrangement.

The fourth restriction concerns generality. The propositions hold of the model as stated. Whether they hold of a richer model, and in particular of one in which agents are heterogeneous and coupling is resolved between individual pairs, is a question §14 states and does not settle.

The fifth restriction concerns naming. No system, product, firm, or population is named at any point, and the paper reports no empirical case.

2.2 *Standing of the minimal model*

A minimal model of a social process is open to two opposite misreadings and both are declined here.

The model is not offered as a description of how relations form. Its variables aggregate what is in fact a large heterogeneous population of relations of many kinds, and the aggregation discards nearly everything that a sociological account of any particular relation would contain.

The model is also not offered as a metaphor. Its propositions are derived and verified, they have determinate truth values within the model, and each is accompanied at §14 by the observation that would defeat it. What a minimal model establishes is what follows

from a mechanism once the mechanism is stated exactly, and in particular whether a conclusion attributed to that mechanism follows from it at all. Proposition 9.1 is a result of that kind, and it is negative.

2.3 *Positions the paper declines to supply*

The paper declines four things and states for each what asserting the contrary would require.

It offers no prediction about any actual society. Supplying one would require estimates of the model's parameters, and §14 states why the parameter that matters most is difficult to estimate and easy to misestimate in a particular direction.

It offers no threshold. The critical capacity of Proposition 10.1 is computed within the model and its numerical value carries no interpretation outside it. §12 accordingly develops an observable that requires no knowledge of where any threshold lies.

It offers no policy. The conditions under which the collapse regime is entered are stated, and what should be done about them is a question this paper does not reach.

It asserts no direction for the effect. The comparison in Proposition 8.1 is signed, and the framework is constructed so that artificial coupling could equally enlarge a system's adaptive capacity; §14 states the conditions under which it would.

3 Prior Formulations of Coupling, Adaptation, and Structural Transition

This section states what other bodies of work already hold. Its objective is to establish which parts of the apparatus are borrowed and which claim, if any, remains. The method is concession first: each subsection states a prior result, records what is taken from it, and states the differentiation available, withholding the differentiation where none is available.

3.1 *Adaptive and coevolutionary networks*

The treatment of a network's structure as a dynamical variable coevolving with the states of its nodes is established. Gross and Blasius [2] survey adaptive networks, in which the topology evolves in response to node dynamics and the node dynamics evolve on the changing topology, and the resulting literature establishes that such coevolution produces phenomena absent from either the fixed-topology or the fixed-state case, including spontaneous segregation and robust self-organisation toward critical states.

The formulation at §6 is an instance of this class, and no novelty is claimed for treating coupling as dynamical. What this paper adds is the particular question, namely a comparison between two coupling sub-blocks that differ in formation and decay rates while the node dynamics are held identical by construction, and the resulting isolation of coupling as the sole locus of difference.

3.2 *Resilience, adaptability, and transformability*

Holling [3] established resilience as a property distinct from stability near an equilibrium, namely the magnitude of disturbance a system absorbs before reorganising. Walker, Holling, Carpenter, and Kinzig [9] refined this into resistance, adaptability, and transformability, the last being the capacity to create a fundamentally new system when the existing one becomes untenable.

That distinction is close to the one this paper requires, and priority for it belongs there. The correspondence should be stated exactly. Proposition 9.1 concerns stability in Holling's narrow sense, and its negative content is that this sense is insufficient for the question. The adaptive basin of §6 corresponds to adaptability, and the collapse of Proposition 10.1 is a loss of adaptability with local stability preserved. The differentiation available is narrow: that literature treats these capacities as properties of a system, and the present analysis derives the transition between two of them from a parameter with a definite interpretation, namely the capacity of an alternative coupling channel.

3.3 *Autopoiesis and the self-production of social systems*

Maturana and Varela [6] developed autopoiesis as the property of a system that continuously produces the components and relations constituting it, and Luhmann [5] carried the notion into social theory, treating social systems as recursively producing the communications of which they consist.

The question of this paper may be stated in that vocabulary: whether an additional coupling channel alters a relational system's production of its own relations. The vocabulary is adopted where convenient and no result here depends on it. The differentiation is that the present treatment supplies a parameterised model in which the self-production either continues or ceases according to a computed condition, where that tradition characterises the property and supplies no such condition.

3.4 *Multilayer networks and cross-layer propagation*

Kivelä, Arenas, Barthelemy, Gleeson, Moreno, and Porter [4] set out the formalism of multilayer networks, in which nodes participate in several kinds of relation simultaneously and structure in one layer bears on dynamics in another.

The observation that a human relation is characteristically embedded across layers, so that a disturbance in one propagates into others, is drawn from that literature, and §13 uses its vocabulary. The model of §7 is single-layer and aggregate, and it therefore represents cross-layer embedding only through its effect on formation and decay rates. That is a simplification and §14 records what it costs.

3.5 *Weak ties and the structural role of the unchosen relation*

Granovetter [1] established that ties of low intensity carry information between otherwise separated regions of a social structure, so that the connectivity of a population depends on relations that are not those its members value most.

The result bears on this paper in a way worth stating, since it supplies an independent reason for the mechanism modelled here. A coupling channel that satisfies a party's relational demand well may nonetheless fail to supply the structural function that low-intensity and unchosen relations perform, and a substitution assessed on satisfaction will not register that failure. The present model represents this only through the substitution parameter, and §13 states the fuller reading.

3.6 *Critical transitions and early-warning indicators*

Scheffer, Bascompte, Brock, Brovkin, Carpenter, Dakos, Held, van Nes, Rietkerk, and Sugihara [7] established that systems approaching a critical transition exhibit critical slowing, so that recovery from small perturbations becomes progressively slower, and that this yields indicators applicable without knowledge of the threshold.

Proposition 10.2 is an instance of that result and claims no novelty for the phenomenon. What is contributed is the identification of the control parameter in the present setting, namely the capacity of the alternative coupling channel, and the consequence that the indicator applies to a quantity that is measurable at the level of individual relations, namely the time taken to recover from a relational breach.

3.7 *Discussion of artificial companionship*

Turkle [8] argued that networked and artificial companionship offers the appearance of relation at diminished cost and that its adoption bears on the capacity for the more demanding kinds.

The thesis of that work is close to the concern motivating this paper, and the differentiation is methodological in place of substantive. This paper assumes what that literature characteristically argues against, namely that the artificial party is a full generative participant, and asks whether a conclusion survives the assumption. A conclusion reached under the weaker premise is available to a wider range of readers, and its scope is correspondingly narrower.

3.8 *The residue of the survey*

Claim 3.1. (The residue). The accounts surveyed above supply the coevolution of structure with state, the distinction between stability and adaptability, the notion of a system producing its own relations, the multilayer setting, the structural function of unchosen ties, and the early-warning indicator. None of them states, for a system whose agents are stipulated indistinguishable, the condition under which the availability of an alternative

coupling channel removes the capacity of the original channel to regenerate itself, nor establishes that this condition requires more than substitution.

Claim 3.1 states a gap. the formulation and derivation sections determine what occupies it, and §14 records the position of a reader who holds that the gap is filled by the resilience literature already.

4 Mathematical Preliminaries

This section defines every mathematical concept the remainder of the paper uses, assuming no prior acquaintance with dynamical systems or with field-theoretic vocabulary. It has five objectives: to define a dynamical system, an equilibrium, and stability; to state the linearisation method by which stability is decided, including the planar criterion used in §9; to define a bifurcation and to work the one example this paper needs; to explain timescale separation and the elimination of a fast variable, which is the method of §10; and to fix what the field-theoretic vocabulary refers to in this paper, which is less than the name suggests. A reader at home with these notions may pass directly to §5 after consulting the notation table. Figure 1 illustrates the three central ideas.

4.1 Dynamical systems, trajectories, and equilibria

A *dynamical system* is a rule stating how a quantity changes according to its present value. Where the quantity is a vector $x = (x_1, \dots, x_n)$ of real numbers, called the *state*, the rule is given by Equation 1.

$$\dot{x} = f(x), \quad (1)$$

in which $\dot{x}$ denotes the time derivative dx/dt and f is a function returning, for each state, the direction and speed of change at that state. Given a starting state $x(0)$, the rule determines the entire future course $x(t)$, called a *trajectory*. Panel (a) of Figure 1 shows several trajectories of a two-dimensional system, each beginning at a different state and each determined thereafter by the same rule.

An *equilibrium* (or fixed point) is a state x^* at which the rule prescribes no change, as stated in Equation 2.

$$f(x^*) = 0. \quad (2)$$

A system started exactly at an equilibrium remains there. The substantive question about an equilibrium is always what happens *near* it, and that is the subject of stability.

4.2 Stability and the sign of the restoring rate

An equilibrium is (*asymptotically*) *stable* where every trajectory beginning sufficiently close to it converges to it, and *unstable* where some trajectories beginning arbitrarily close move away.

For a one-dimensional system $\dot{u} = f(u)$ the criterion is visible by inspection of the graph

of f , and panel (b) of Figure 1 shows the case used in this paper. Where the graph crosses zero from above, with f positive to the left of the crossing and negative to the right, states below the equilibrium are pushed up and states above are pushed down, so the crossing is stable. Where the graph crosses from below, the push is away on both sides and the crossing is unstable. The same information is carried by the derivative at the crossing: $f'(u^*) < 0$ is stable, $f'(u^*) > 0$ is unstable. The quantity $f'(u^*)$ is the *restoring rate*, and its magnitude states how quickly a small displacement decays: a displacement ϵ evolves as $\epsilon e^{f'(u^*)t}$, so the time for a displacement to halve is given by Equation 3.

$$t_{1/2} = \frac{\ln 2}{|f'(u^*)|}, \quad (3)$$

a formula used at §10 and worth noting now: as the restoring rate approaches zero, the recovery time grows without bound.

4.3 Linearisation and the planar stability criterion

For a system of two variables, $\dot{u} = f(u, v)$ and $\dot{v} = g(u, v)$, the behaviour near an equilibrium (u^*, v^*) is governed by the *Jacobian matrix* of first partial derivatives evaluated there, defined in Equation 4.

$$J = \begin{pmatrix} \partial f / \partial u & \partial f / \partial v \\ \partial g / \partial u & \partial g / \partial v \end{pmatrix}_{(u^*, v^*)}, \quad (4)$$

which states how each variable's rate of change responds to a small displacement in each variable. Near the equilibrium, a displacement $\delta = (u - u^*, v - v^*)$ evolves approximately as $\dot{\delta} = J \delta$, and exactly so when f and g are affine (linear plus constant), which is the case in §9.

The growth or decay of δ is governed by the *eigenvalues* of J : the numbers λ for which $Jw = \lambda w$ holds for some non-zero direction w . A displacement along such a direction evolves as $e^{\lambda t}$, so the equilibrium is stable exactly where every eigenvalue has negative real part.

For a two-by-two matrix the eigenvalues are determined by two computable quantities, the *trace* $\text{tr } J$ (the sum of the diagonal entries, which equals $\lambda_1 + \lambda_2$) and the *determinant* $\det J$ (which equals $\lambda_1 \lambda_2$), through Equation 5.

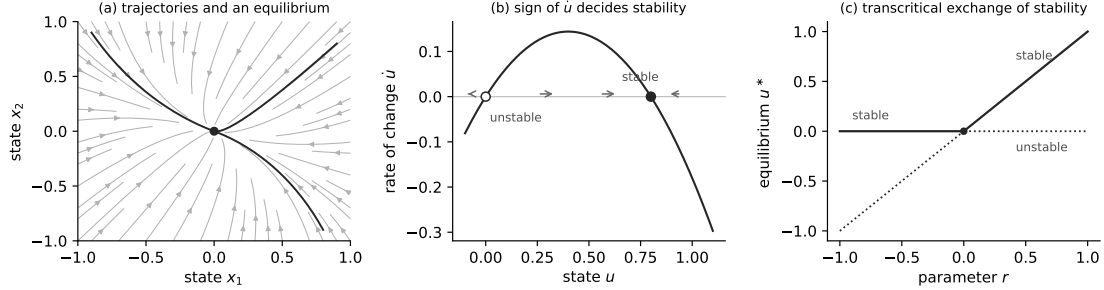
$$\lambda_{1,2} = \frac{1}{2} \left(\text{tr } J \pm \sqrt{(\text{tr } J)^2 - 4 \det J} \right). \quad (5)$$

Both eigenvalues have negative real part exactly under the conditions in Equation 6.

$$\text{tr } J < 0 \quad \text{and} \quad \det J > 0, \quad (6)$$

since a negative trace makes the sum of the real parts negative, and a positive determinant excludes the case of one positive and one negative real eigenvalue. This pair of sign conditions is the entire criterion applied in §9, and the reader needs nothing further from linear

Figure 1: Three notions used throughout the paper. (a) A two-dimensional dynamical system: trajectories from different starting states, converging to a stable equilibrium. (b) The one-dimensional stability criterion: where the rate of change crosses zero from above, displaced states are pushed back (stable, filled); where from below, pushed away (unstable, open). (c) The transcritical bifurcation of $\dot{u} = u(r - u)$: two equilibrium branches exchanging stability where they cross.



algebra.

4.4 Bifurcations, worked in the one case required

A *bifurcation* is a qualitative change in a system's equilibria or their stability as a parameter is varied continuously. The paper requires exactly one kind, and it is worked here in its standard form so that §10 is a recognition and not a novelty.

Consider the one-dimensional system in Equation 7.

$$\dot{u} = u(r - u), \quad (7)$$

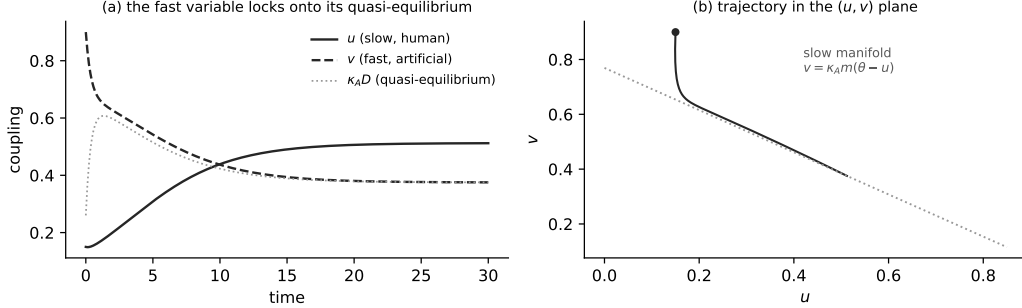
with parameter r . The right-hand side factorises, so there are two equilibria for every r : the origin $u = 0$, and the interior point $u^* = r$. Their restoring rates are obtained by differentiating $f(u) = ru - u^2$, as shown in Equation 8.

$$f'(u) = r - 2u, \quad f'(0) = r, \quad f'(r) = -r. \quad (8)$$

The two rates are exact negatives. For $r < 0$ the origin is stable ($f'(0) = r < 0$) and the interior point is unstable and lies at negative u ; for $r > 0$ the origin is unstable and the interior point $u^* = r > 0$ is stable. At $r = 0$ the two equilibria coincide and *exchange stability*. This exchange is a *transcritical bifurcation*, and panel (c) of Figure 1 is its diagram: equilibrium position plotted against the parameter, stable branches solid, unstable branches dotted.

Two features of the transcritical case matter for what follows. The origin is an equilibrium for every parameter value, which occurs when the state variable divides the right-hand side, so that a system at zero stays at zero; §10 derives exactly this structure from a substantive assumption. And the transition is between *recovery* and *collapse*: below the critical parameter every positive initial state converges to the positive equilibrium, and above it every initial state converges to zero.

Figure 2: Timescale separation in the model of §7, at parameters used in the derivation sections. (a) The fast artificial coupling v reaches its quasi-equilibrium value $\kappa_A D$ within a short initial interval and tracks it thereafter, while the slow human coupling u evolves over a much longer time. (b) The same trajectory in the (u, v) plane: after the initial transient the motion proceeds along the slow manifold $v = \kappa_A m(\theta - u)$.



4.5 Timescale separation and elimination of a fast variable

Where a system has two variables and one changes much faster than the other, the fast variable spends nearly all of its time at the value it would settle at were the slow variable frozen. The system's course is then well approximated by a one-variable system for the slow variable alone, with the fast variable replaced by that settling value. The replacement is called *adiabatic elimination* of the fast variable, and the curve of settling values is the *slow manifold*.

The procedure is: set the fast variable's rate of change to zero; solve for the fast variable in terms of the slow one; substitute into the slow equation. Its validity requires the fast variable's relaxation rate to exceed the slow variable's characteristic rate by a wide margin, and the approximation degrades as the margin narrows. Figure 2 demonstrates the situation for the model of this paper: the fast variable moves onto its quasi-equilibrium value within a short initial interval and tracks it thereafter, so the trajectory in the plane collapses onto the slow manifold. §10 states the margin required in the model's parameters, and §11 checks the resulting predictions against the full two-variable system, which is the direct test of the approximation.

4.6 The field-theoretic vocabulary and its restricted use here

In physics, a *field* assigns a quantity to every point of a space, an *action* is a functional whose stationary points are the system's trajectories, and a *perturbative analysis* studies a system as a small departure from one already solved. The discussion from which this paper arises is conducted in that vocabulary: agents as fields Φ , their interconnection as a coupling C , and the question posed as the response of the pair (Φ, C) to a perturbation δC .

This paper retains the vocabulary and restricts the machinery, and the restriction is stated so that no reader expects apparatus that is not used. The systems studied here are finite-dimensional ordinary differential equations of the kind defined in Section 4.1, and the perturbation applied is a displacement of an initial condition, analysed by the lineari-

Table 1: Notation, in order of first appearance. All parameters are positive real numbers.

Symbol	Meaning
Φ_i, F	state of agent i ; its dynamics, identical across agents by Assumption 5.1
C, C_{ij}	coupling configuration; strength of the coupling between agents i and j
G	the law by which the coupling evolves
$\delta C, \varepsilon$	a coupling perturbation; the size of a relational breach
$\mathcal{A}_C, \mathcal{B}_{\text{adaptive}}$	admissible stable coupling configurations; the adaptive basin Equation 12
u, v	mean human-to-human and human-to-artificial coupling strength
θ	relational engagement a party requires
β	substitution efficiency of artificial for human coupling
D	relational deficit, $D = \theta - u - \beta v$
$a_H(u), a_0, a_1$	human formation rate; its constant part; its practice-dependent part
a_A	artificial formation rate
γ_H, γ_A	decay rates of human and artificial coupling
κ_H, κ_A	coupling capacities, a_0/γ_H and a_A/γ_A
m	attenuation factor, $m = 1/(1 + \beta\kappa_A)$
u^*, v^*, D^*	equilibrium values
$J, \text{tr } J, \det J, \lambda$	Jacobian at equilibrium; its trace, determinant, eigenvalues
κ_A^c	critical artificial capacity, Proposition 10.1
$t_{1/2}$	recovery half-time after a breach

sation of Section 4.3. No action functional is employed, for the reason given at §6: the dynamics required include decay, a system derived from an action by variation conserves what these dynamics dissipate, and asserting an action principle for them would be false. The word “field-theoretic” in the title accordingly refers to the framing, namely agents and coupling as co-evolving dynamical objects studied through their response to perturbation, and every derivation in this paper is elementary and self-contained given the present section.

4.7 Notation

Table 1 collects every symbol used in the formulation and derivation sections, in order of first appearance.

5 The Indistinguishability Assumption and What It Isolates

This section states the paper’s governing assumption, its motivation, and what adopting it makes available. It has three objectives: to state the assumption formally; to explain why an assumption unfavourable to the paper’s own concern is the right one to adopt; and to establish what the assumption isolates, which is the paper’s method in a single step. The method is stipulative and the section proves nothing.

5.1 Statement of the assumption

Let a relational system consist of agents indexed i , each carrying a state Φ_i evolving under dynamics F_i . Partition the agents into human agents H and artificial agents A .

Assumption 5.1. (Indistinguishability of node dynamics). For every artificial agent $j \in$

A and every human agent $i \in H$, the generative dynamics coincide: $F_j = F_i = F$. Artificial agents respond, adapt, initiate, surprise, and generate exactly as human agents do, and no functional, representational, or phenomenal difference between them is asserted anywhere in this paper.

Assumption 5.1 is stronger than any position this paper needs and stronger than most readers would grant. It is adopted deliberately.

5.2 *The reason for adopting an unfavourable assumption*

An argument that artificial relational partners bear on a social system may be built on a premise about what such partners are. The available premises hold that they lack subjectivity, that their responsiveness is simulated, that they cannot be genuinely surprised, or that their generativity is derivative. Each premise is contested, none is settled, and an argument resting on one is defeated by the defeat of its premise.

Assumption 5.1 removes the entire class. A conclusion reached under it is a conclusion available to a reader who holds that artificial agents are full participants in every sense at issue, and it is therefore not vulnerable to any development in the capabilities of such agents. If the assumption is false in the direction its critics expect, every conclusion reached under it holds a fortiori, since the difference the assumption denies would be an additional source of the effect and not a competing one.

The assumption also fixes the paper's burden. Having granted the artificial agent everything at the level of the agent, the paper must locate any remaining difference somewhere else or concede that there is none.

5.3 *The locus of difference the assumption isolates*

A relational system is specified by more than the dynamics of its agents. It is specified also by the coupling among them: which agents act on which, with what strength, formed at what cost, dissolved at what cost, and obliging what further connections.

Under Assumption 5.1 the agent dynamics are identical across the partition, so any difference in the behaviour of the whole system is a difference in coupling. This is the paper's method, and it is worth stating as a claim so that its role is visible.

Claim 5.2. (Locus of difference). Under Assumption 5.1, a difference between the evolution of a system containing artificial agents and one containing only human agents is attributable to the coupling structure and to the dynamics of that structure, and is attributable to no property of the agents. Where no such difference arises, the coupling is not a locus of difference either, and the paper's question is answered in the negative.

Claim 5.2 is a consequence of the assumption and not a substantive finding. Its use is that it makes the question decidable within a model: one holds F fixed, varies the coupling parameters, and observes whether the system's behaviour changes. §6 states what varying the coupling means, and §7 supplies the model in which the variation is performed.

The two coupling properties this paper varies are stated here and motivated at Section 6.3. They are the rate at which a coupling forms in response to relational demand, and the rate at which it decays in the absence of that demand. A relation to a biological family is formed without election and decays slowly or not at all; a relation to an artificial partner is formed at once and is revisable at once. Both descriptions concern the coupling and neither concerns what the parties can do.

6 Coupling as a Dynamical Variable and the Perturbation Applied to It

This section states the formulation on which the derivations rest. It has four objectives: to state the coupled system in which structure and state evolve together; to distinguish the perturbation applied to the coupling from the perturbation applied to the dynamical law, since the two are frequently conflated; to state the asymmetry between the two coupling sub-blocks in terms that concern the coupling alone; and to define the classes of response a coupling perturbation admits, which fixes the vocabulary for the derivation sections. The method is formulation; nothing is derived here.

6.1 *The coupled system*

Let $\Phi = (\Phi_i)$ collect the agent states and $C = (C_{ij})$ the coupling strengths. The system evolves as Equation 9.

$$\dot{\Phi} = F(\Phi, C), \quad \dot{C} = G(\Phi, C), \quad (9)$$

so that the full state is the pair $X = (\Phi, C)$. The first equation states that agents act on one another through the coupling; the second states that the coupling itself changes in response to the agents' states and to its own configuration.

The second equation is the commitment that distinguishes this formulation from one in which dynamics run over a fixed network. Under Equation 9 a relational structure is not a background against which relations are conducted. It is produced and destroyed by the conducting of them.

A remark on formalism is owed here, since the language of an action functional is natural for a coupled field system and is unavailable for this one. A system derived from an action by variation is conservative, and the coupling dynamics required below include decay and are therefore dissipative. The equations Equation 9 are accordingly taken as primitive, and no action principle is asserted for them. §14 records what this costs.

6.2 *Perturbation of the law and perturbation of the coupling*

Two operations may be performed on the system in Equation 9 and they differ in kind.

An *action perturbation* alters the dynamical law: $F \rightarrow F + \delta F$, or in the variational language $S \rightarrow S + \delta S$. It changes how agents respond, and it is the perturbation implicitly at issue whenever a technology is said to change people.

A *coupling perturbation* leaves F and G untouched and alters the coupling configuration at an instant as Equation 10.

$$C(t_0) \longrightarrow C(t_0) + \delta C. \quad (10)$$

It changes what is connected to what, and it asks how a system governed by unchanged laws reorganises its own connections afterwards.

The coupling perturbation is the operation studied here, and its interest lies in the feedback it initiates. A change in coupling alters the agent states through the first equation of Equation 9; the altered states alter the coupling through the second; and the process continues as Equation 11.

$$\delta C \longrightarrow \delta \Phi \longrightarrow \delta C \longrightarrow \dots \quad (11)$$

A relational breach, a bereavement, a migration, an institutional dissolution are perturbations of this kind. None of them changes what people are capable of, and each changes what they are connected to.

6.3 Asymmetry between the coupling sub-blocks

Partition C into the human-to-human block C_{HH} and the human-to-artificial block C_{HA} . Under Assumption 5.1 the agents are identical, so any asymmetry resides in G , the law by which coupling evolves. Two properties are ascribed, and both are properties of how a relation is formed and maintained.

The human-to-human block carries what may be called relational inertia. Such couplings form slowly, since forming one requires the availability, the consent, and the sustained attention of another party whose own commitments are not at the first party's disposal. They also decay slowly, since many are not at the parties' election at all: a relation to a biological family, to a neighbourhood, to a colleague, or to a legal counterparty persists through periods in which neither party is attending to it.

The human-to-artificial block carries what may be called plasticity. Such couplings form quickly, since formation requires no second party's consent and no coordination of schedules. They are also revisable at once, since nothing outside the arrangement obliges their continuation.

Both descriptions concern rates of formation and decay. Neither ascribes any property to the artificial agent beyond those rates, and neither is in tension with Assumption 5.1.

6.4 Classes of response to a coupling perturbation

Given δC at t_0 , the subsequent evolution of the coupling falls into four classes, and the derivation sections determines which class obtains under which conditions.

Elastic absorption: $\delta C(t) \rightarrow 0$, and the system returns to its prior coupling architecture.

Adaptive stabilisation: $\delta C(t) \not\rightarrow 0$ while $\dot{C} \rightarrow 0$, so the system settles into a new coupling equilibrium that differs from the prior one. This is the class corresponding to institutional adaptation, and it is not a failure. A system that responds to a disturbance by acquiring a

new stable relational architecture has succeeded, and a criterion identifying stability with return to the prior configuration would misclassify every historical adaptation as a breakdown.

Structural instability: $C(t)$ enters no bounded stable region, and the coupling continues to reorganise without settling.

Generative proliferation: the sequence of coupling configurations is accompanied by an enlarging set of available couplings, so that each reorganisation makes further reorganisations possible.

The distinction between the first two classes is the one that matters most for what follows, and it fixes what a loss of self-stabilisation can mean. The object of interest is stability of the coupling dynamics and not invariance of the coupling structure. A system has lost its capacity to stabilise itself where, for arbitrarily small admissible perturbations, no trajectory reaches any admissible stable configuration; a system that reaches a different configuration from the one it left has exercised that capacity in place of losing it.

Formally, let $\mathcal{A}_C$ denote the set of admissible stable coupling configurations, and define the adaptive basin by Equation 12.

$$\mathcal{B}_{\text{adaptive}} = \{ C_0 : C(t; C_0, \Phi_0) \rightarrow C_\infty \in \mathcal{A}_C \}. \quad (12)$$

The paper's question, in this vocabulary, is whether the presence of the artificial coupling block enlarges, contracts, or fragments $\mathcal{B}_{\text{adaptive}}$. The hypotheses are stated symmetrically in Equation 13, and the framework admits both.

$$H_0 : \mathcal{B}_{AI} \simeq \mathcal{B}_H, \quad H_1 : \mathcal{B}_{AI} \not\simeq \mathcal{B}_H. \quad (13)$$

Nothing in the formulation requires that artificial coupling contract the basin, and §14 states the conditions under which it would enlarge it.

7 A Minimal Model of Competing Coupling Channels

This section constructs the model from which the derivation sections obtain their results. It has three objectives: to reduce the system in Equation 9 to two coupling variables and to state what the reduction discards; to motivate each term of the resulting equations separately, so that a reader may reject any one of them and see what falls with it; and to identify the parameter carrying the comparison between a system with artificial coupling available and one without. The method is construction, and every choice is stated as a choice.

7.1 The reduction and the structure it discards

The system in Equation 9 carries one variable per agent and one per pair of agents. The two aggregated coupling variables are defined in Equation 15.

$$u(t) = \text{mean human-to-human coupling strength}, \quad (14)$$

$$v(t) = \text{mean human-to-artificial coupling strength}. \quad (15)$$

The agent states are represented by a single scalar, defined below, and the agent dynamics enter only through it. This is consistent with Assumption 5.1, under which the agent dynamics are identical across the partition and cannot themselves be the source of any asymmetry.

Three things are discarded and each is recorded at §14. Heterogeneity across agents is discarded, so nothing here represents a population in which some parties are affected and others are not. Network position is discarded, so the model cannot represent fragmentation of a population into components. And multilayer structure is discarded, so cross-layer propagation enters only through its effect on the rate parameters.

7.2 The relational deficit

Let $\theta > 0$ denote the level of relational engagement a party requires, and define the *relational deficit* by Equation 16.

$$D = \theta - u - \beta v, \quad (16)$$

where $\beta > 0$ measures the efficiency with which artificial coupling closes the deficit that human coupling would otherwise close.

Two features of Equation 16 carry the model's substance. The deficit is the difference between what is required and what the available couplings supply, so it is positive where a party is under-engaged and falls as either coupling rises. And the parameter β is a substitution efficiency and nothing more: it states how far artificial coupling meets the same demand, and it makes no claim that the two kinds of coupling are equivalent in any other respect. Setting $\beta = 0$ recovers a model in which artificial coupling meets no part of the demand, and every result below is trivial in that case.

D is the paper's single state variable, and it plays the role that Φ plays in Equation 9. Under Assumption 5.1 nothing further about agent states is required.

7.3 Formation and decay of the two couplings

Coupling forms in response to deficit and decays in its absence as Equation 17.

$$\dot{u} = a_H(u) D - \gamma_H u, \quad \dot{v} = a_A D - \gamma_A v. \quad (17)$$

Each term is motivated separately.

The formation terms $a_H(u)D$ and $a_A D$ state that a party under-engaged acts to become

engaged, and that the rate of acting is proportional to the shortfall. A party whose relational demand is met forms no new couplings on this account.

The decay terms $-\gamma_H u$ and $-\gamma_A v$ state that couplings not sustained by present demand weaken. The rates differ, and by Section 6.3 the human-to-human rate is the smaller: such couplings are held in place by obligations, institutions, and physical proximity that persist through periods of inattention.

The formation rate for human coupling is written $a_H(u)$ because §10 requires it to depend on u , and §8 and §9 treat the case in which it does not. The two cases are stated together so that the difference between them, which is the paper's principal finding, is visible as a difference in one function.

The artificial formation rate a_A is the parameter carrying the comparison. A system in which artificial coupling is unavailable is the case $a_A = 0$; a system in which it is available at low cost and without a second party's consent is the case of large a_A .

7.4 The capacity parameter

The two artificial parameters enter the results below only through their ratio, and it is convenient to name it. Define the *artificial coupling capacity* by Equation 18.

$$\kappa_A = \frac{a_A}{\gamma_A}, \quad (18)$$

and correspondingly $\kappa_H = a_H/\gamma_H$ where a_H is constant. The capacity is the equilibrium coupling per unit of deficit: a channel that forms quickly and decays slowly has a high capacity, and one that forms slowly or decays quickly has a low one.

κ_A is the control parameter of the entire paper. The question of §1, stated in the model, is what happens to the human coupling channel as κ_A is raised from zero, with every agent-level property held fixed by Assumption 5.1.

7.5 The perturbation

The perturbation studied is a coupling perturbation in the sense of Equation 10, applied to the human block as Equation 19.

$$u(t_0) \longrightarrow u(t_0) - \varepsilon, \quad \varepsilon > 0. \quad (19)$$

This represents a relational breach: an ending, a bereavement, a migration, a falling-out. It removes coupling and it alters no party's capabilities. The question in each of the following sections is what the system does with it.

8 Equilibrium Human Coupling Under Substitution

This section derives the first result: the level of human coupling the system settles at, and its dependence on artificial coupling capacity. It has three objectives: to solve the model of

§7 at equilibrium in closed form; to establish the monotone dependence and to state exactly which assumptions it uses; and to state what the result does not establish, which is more than a reader is likely to expect. The method is direct solution of the stationary equations. Throughout this section the human formation rate is constant, $a_H(u) = a_0 > 0$; the case in which it depends on u is §10.

8.1 Solution of the stationary equations

An equilibrium is a state at which both rates of change vanish (Section 4.1). Setting $\dot{u} = 0$ and $\dot{v} = 0$ in Equation 17 gives Equation 20.

$$0 = a_0 D^* - \gamma_H u^*, \quad 0 = a_A D^* - \gamma_A v^*. \quad (20)$$

Each equation balances formation against decay, and rearranging each gives the stationary conditions in Equation 21.

$$a_0 D^* = \gamma_H u^*, \quad a_A D^* = \gamma_A v^*, \quad (21)$$

which state that at rest, the coupling formed per unit time equals the coupling lost per unit time, channel by channel. Dividing the first by γ_H and the second by γ_A , each coupling is proportional to the common deficit as Equation 22.

$$u^* = \kappa_H D^*, \quad v^* = \kappa_A D^*, \quad (22)$$

with $\kappa_H = a_0/\gamma_H$ and $\kappa_A = a_A/\gamma_A$ as defined in Equation 18. The deficit itself depends on the couplings through Equation 16, so the system is closed by substituting Equation 22 back into that definition. Step by step: the definition evaluated at equilibrium is $D^* = \theta - u^* - \beta v^*$; replacing u^* by $\kappa_H D^*$ and v^* by $\kappa_A D^*$ gives Equation 23.

$$D^* = \theta - \kappa_H D^* - \beta \kappa_A D^*; \quad (23)$$

Collecting every term containing D^* on the left gives Equation 24.

$$D^* (1 + \kappa_H + \beta \kappa_A) = \theta, \quad D^* = \frac{\theta}{1 + \kappa_H + \beta \kappa_A}. \quad (24)$$

Substituting back into Equation 22 gives the equilibrium couplings in closed form as Equation 25.

$$\boxed{u^* = \frac{\kappa_H \theta}{1 + \kappa_H + \beta \kappa_A}, \quad v^* = \frac{\kappa_A \theta}{1 + \kappa_H + \beta \kappa_A}.} \quad (25)$$

Both are positive and finite for every admissible parameter value, and the equilibrium is unique.

8.2 Monotone dependence on artificial coupling capacity

Proposition 8.1. (Substitution at equilibrium). In the model in Equation 17 with constant human formation rate, the equilibrium human coupling u^* is strictly decreasing in the artificial coupling capacity κ_A whenever $\beta > 0$, as stated in Equation 26.

$$\frac{\partial u^*}{\partial \kappa_A} = -\frac{\beta \kappa_H \theta}{(1 + \kappa_H + \beta \kappa_A)^2} < 0, \quad \lim_{\kappa_A \rightarrow \infty} u^* = 0. \quad (26)$$

The decrease uses no difference between human and artificial agents, no premise about harm, and no property of either coupling beyond the substitution efficiency β and the capacity κ_A .

The derivative is computed by the quotient rule on Equation 25. Writing $u^* = N/Q$ with $N = \kappa_H \theta$ and $Q = 1 + \kappa_H + \beta \kappa_A$: the numerator N does not contain κ_A , so $\partial N / \partial \kappa_A = 0$; the denominator satisfies $\partial Q / \partial \kappa_A = \beta$; hence the derivative is Equation 27.

$$\frac{\partial u^*}{\partial \kappa_A} = \frac{0 \cdot Q - N \cdot \beta}{Q^2} = -\frac{\beta \kappa_H \theta}{(1 + \kappa_H + \beta \kappa_A)^2}, \quad (27)$$

which is negative whenever β , κ_H , and θ are positive. The limit follows since the denominator grows without bound in κ_A while the numerator is fixed. Strict monotonicity and the limit together give the full statement.

The mechanism is worth stating in words, because it is elementary and it is the whole of the first result. Coupling of either kind forms in response to deficit. Artificial coupling closes part of the deficit. A deficit partly closed drives less formation of human coupling. Human coupling therefore settles lower, and it does so although nothing has happened to any person's capacity or willingness to form human relations, and although the artificial agent has been stipulated to be a full generative participant.

8.3 The equilibrium deficit and why satisfaction is a poor indicator

Equation 24 carries a second observation that Section 12 develops. The equilibrium deficit D^* is also strictly decreasing in κ_A , as shown in Equation 28.

$$\frac{\partial D^*}{\partial \kappa_A} = -\frac{\beta \theta}{(1 + \kappa_H + \beta \kappa_A)^2} < 0. \quad (28)$$

So as artificial coupling capacity rises, the deficit falls and the human coupling falls together with it. A measurement taken on the state variable therefore records an improvement over exactly the range of κ_A across which the human coupling contracts. The two movements are not in tension: the deficit is closed, and it is closed by the other channel.

This is the model's first indication that state-level and structure-level observables can move in opposite directions, and §12 states the general form of the point.

8.4 Positions the result withholds

Proposition 8.1 establishes less than it may appear to, and four readings are excluded.

It establishes no loss. The parties in the model reach a lower level of human coupling and a fully closed deficit, and nothing in the model says that this is worse for them. Establishing that would require a criterion the paper does not supply.

It establishes no instability. The equilibrium in Equation 25 exists and is unique for every parameter value, and §9 shows it is also stable throughout.

It establishes nothing about recovery. The equilibrium is a statement about where the system rests and no statement about how it responds to a disturbance, which is the subject of the two sections following.

It establishes no threshold. The dependence in Proposition 8.1 is smooth and monotone: there is no value of κ_A at which anything qualitative occurs. A threshold appears only under the further condition of §10, and its absence here is what makes that section's condition necessary in place of decorative.

9 Stability of the Substitution Model and a Negative Result

This section derives the paper's second result, and it is negative. It has three objectives: to linearise the model about its equilibrium and compute the stability conditions exactly; to establish that those conditions hold for every admissible parameter value, so that substitution alone never destabilises the system; and to state what this excludes, since the excluded account is the one most frequently offered. The method is linearisation and the Routh–Hurwitz conditions for a planar system. The human formation rate is constant throughout this section.

9.1 Linearisation about the equilibrium

Write the right-hand sides of Equation 17 as Equation 29.

$$f(u, v) = a_0 (\theta - u - \beta v) - \gamma_H u, \quad g(u, v) = a_A (\theta - u - \beta v) - \gamma_A v. \quad (29)$$

Both are affine in (u, v) , each being a sum of terms at most linear in the variables, so the Jacobian defined at Section 4.3 is a constant matrix, the approximation $\dot{\delta} = J\delta$ is exact, and every conclusion drawn from J holds globally and not merely near the equilibrium.

The four partial derivatives are computed one at a time. From the definition in Equation 16, $D = \theta - u - \beta v$, the deficit responds to the couplings as Equation 30.

$$\frac{\partial D}{\partial u} = -1, \quad \frac{\partial D}{\partial v} = -\beta. \quad (30)$$

Then, the chain rule applied to $f = a_0 D - \gamma_H u$ gives Equation 31.

$$\frac{\partial f}{\partial u} = a_0 \frac{\partial D}{\partial u} - \gamma_H = -a_0 - \gamma_H, \quad \frac{\partial f}{\partial v} = a_0 \frac{\partial D}{\partial v} = -a_0 \beta, \quad (31)$$

Applied to $g = a_A D - \gamma_A v$, the chain rule gives Equation 32.

$$\frac{\partial g}{\partial u} = a_A \frac{\partial D}{\partial u} = -a_A, \quad \frac{\partial g}{\partial v} = a_A \frac{\partial D}{\partial v} - \gamma_A = -a_A \beta - \gamma_A. \quad (32)$$

The Jacobian is therefore Equation 33.

$$J = \begin{pmatrix} -(a_0 + \gamma_H) & -a_0 \beta \\ -a_A & -(a_A \beta + \gamma_A) \end{pmatrix}. \quad (33)$$

Two features of Equation 33 are worth noting before the computation. Every entry is negative, which is the signature of a system in which each coupling suppresses the driver of both. And the off-diagonal entries are unequal in general, so the two channels do not act on one another symmetrically: the artificial channel responds to the deficit at rate a_A , and the human channel is suppressed by the artificial coupling at rate $a_0 \beta$.

9.2 The stability conditions computed

A planar linear system is asymptotically stable exactly when the trace of its Jacobian is negative and the determinant is positive. Both are computed directly.

The trace is Equation 34.

$$\text{tr } J = -(a_0 + \gamma_H) - (a_A \beta + \gamma_A), \quad (34)$$

which is a sum of negative terms and is therefore negative for all $a_0, a_A, \gamma_H, \gamma_A, \beta > 0$.

The determinant is computed with the cancellation shown explicitly in Equation 35.

$$\begin{aligned} \det J &= (a_0 + \gamma_H)(a_A \beta + \gamma_A) - (a_0 \beta)(a_A) \\ &= a_0 a_A \beta + a_0 \gamma_A + \gamma_H a_A \beta + \gamma_H \gamma_A - a_0 a_A \beta \\ &= a_0 \gamma_A + \gamma_H a_A \beta + \gamma_H \gamma_A. \end{aligned} \quad (35)$$

The term $a_0 a_A \beta$, which is the only term through which the two channels' mutual suppression could have produced a positive feedback, cancels exactly. What remains is a sum of three products of positive quantities.

Proposition 9.1. (Unconditional stability under substitution). In the model in Equation 17 with constant human formation rate, the unique equilibrium in Equation 25 is asymptotically stable for every admissible parameter value. The trace in Equation 34 is negative and the determinant in Equation 35 is positive for all $a_0, a_A, \gamma_H, \gamma_A, \beta > 0$, with no condition relating them. In particular no value of the artificial coupling capacity κ_A , however large, produces an eigenvalue with positive real part.

The conclusion follows from Equation 34 and Equation 35 together with the planar criterion of Section 4.3: negative trace and positive determinant place both eigenvalues in

the left half-plane. The eigenvalues themselves are given by Equation 36.

$$\lambda_{1,2} = \frac{1}{2} \left(\text{tr } J \pm \sqrt{(\text{tr } J)^2 - 4 \det J} \right), \quad (36)$$

A further check is available at no cost: the discriminant is given by Equation 37.

$$(\text{tr } J)^2 - 4 \det J = [(a_0 + \gamma_H) - (a_A \beta + \gamma_A)]^2 + 4 a_0 a_A \beta \geq 0, \quad (37)$$

obtained by expanding both sides and cancelling, so the eigenvalues are real, both negative, and the approach to equilibrium is monotone without oscillation.

Global stability requires one further remark, and the affine structure supplies it. Writing the system as $\dot{x} = Jx + b$ with $x = (u, v)$ and constant b , the unique equilibrium is $x^* = -J^{-1}b$ (the inverse existing since $\det J > 0$), and the change of variable $\delta = x - x^*$ gives the linear system $\dot{\delta} = J\delta$ exactly, whose every solution is $\delta(t) = e^{Jt}\delta(0) \rightarrow 0$. Every trajectory of the model therefore converges to the equilibrium from every initial condition, and the stability of Proposition 9.1 is global.

9.3 *The account excluded by the negative result*

Proposition 9.1 rules out an account that is otherwise natural and that has been offered. On that account, an artificial coupling channel of high capacity absorbs the relational gradient that would otherwise have driven the formation of human coupling; the human coupling therefore fails to recover; the failure to recover compounds; and the system loses stability, the largest eigenvalue of the coupling Jacobian acquiring a positive real part.

The first two steps of that account are correct and are exactly Proposition 8.1. The last step does not follow. In the model that expresses the mechanism, the destabilising term cancels identically, as Equation 35 shows, and the system remains stable at every capacity. Substitution moves the equilibrium and does not destabilise it.

The reason is structural and worth stating, since it indicates what a successful argument must contain. In the substitution model the two channels are coupled only through the deficit, and the deficit is a quantity that both channels reduce. A variable that both channels reduce cannot support positive feedback between them: each channel's growth suppresses the driver of the other's growth, which is negative feedback, and negative feedback stabilises. To obtain instability, some quantity must exist whose decline reduces the system's capacity to restore it, and no such quantity is present in Equation 17 with a_H constant.

9.4 *Consequences for the paper's argument*

Three consequences follow and each shapes what remains.

An argument for loss of self-stabilisation cannot rest on substitution. Any argument that does so is refuted by Proposition 9.1 in the setting that argument itself describes.

The additional ingredient must be a dependence of the human channel's restorative

capacity on its own present level. §10 supplies exactly that ingredient, in the minimal form $a_H = a_H(u)$, and shows that it suffices.

The requirement is substantive and not a technical convenience. It states that a relational capacity is maintained by being exercised, so that a party holding few human relations is less able to form new ones. That is an empirical claim about people, it is separable from everything else in the model, and §14 records that the paper's principal conclusion stands or falls with it.

10 Practice-Dependent Formation and the Collapse Threshold

This section derives the paper's principal result. It has four objectives: to state the additional condition Proposition 9.1 showed to be necessary, and to motivate it independently; to reduce the two-variable system to one variable by adiabatic elimination, with the validity condition stated; to derive the transcritical bifurcation and the critical capacity in closed form; and to derive the divergence of recovery time near the threshold, which supplies the paper's observable. The method is elimination of the fast variable followed by elementary analysis of a scalar equation.

10.1 *The additional condition*

Let the human formation rate depend on the level of human coupling already held as Equation 38.

$$a_H(u) = a_0 + a_1 u, \quad a_1 > 0. \quad (38)$$

The condition states that a party's rate of forming new human relations rises with the human relations that party already holds.

Three independent grounds support Equation 38, and the paper relies on the condition as an assumption in any case. Existing relations supply introductions, so the opportunity to form a relation arrives through relations already held. The conduct of a relation exercises capacities that its conduct requires, so a party currently conducting relations is more practised at conducting them. And a party embedded in relations is subject to expectations that occasion further contact without that party electing it.

The limiting case $a_0 = 0$ states that formation of human coupling proceeds through existing human coupling alone. It is the case treated below, and §14 records that it is the strongest form of the condition and the one on which the sharpest result depends.

10.2 *Adiabatic elimination of the artificial channel*

The artificial channel is the fast one, by the plasticity ascribed at Section 6.3: it forms at once and revises at once, so its characteristic relaxation rate $a_A\beta + \gamma_A$ exceeds every rate at which the human channel moves. The method of Section 4.5 then applies: the fast variable is taken at the value it would settle at were the slow variable frozen.

The procedure, step by step. Freeze u ; set the fast rate to zero, $\dot{v} = 0$ in Equation 17, giving $a_A D = \gamma_A v$; solving for the fast variable gives Equation 39.

$$v = \frac{a_A}{\gamma_A} D = \kappa_A D, \quad (39)$$

which is the slow manifold shown in Figure 2.

The validity condition is stated so that it can fail visibly. The elimination is licensed where the fast relaxation rate dominates the slow one, as stated in Equation 40.

$$a_A \beta + \gamma_A \gg |m a_1 \theta - \gamma_H|, \quad (40)$$

the right-hand side being the slow restoring rate computed at Equation 51 below. The margin is widest far from the threshold and remains adequate near it, since the slow rate tends to zero there while the fast rate does not. §11 tests the approximation in the only way that settles it, namely by integrating the full two-variable system and comparing. The deficit now involves v , which involves D itself, so the two are solved together. Substituting $v = \kappa_A D$ into the definition in Equation 16 gives Equation 41.

$$D = \theta - u - \beta v = \theta - u - \beta \kappa_A D. \quad (41)$$

Moving the last term to the left and factoring gives Equation 42.

$$D + \beta \kappa_A D = \theta - u \implies D(1 + \beta \kappa_A) = \theta - u, \quad (42)$$

The resulting eliminated deficit is Equation 43.

$$D = m(\theta - u), \quad m \equiv \frac{1}{1 + \beta \kappa_A} \in (0, 1]. \quad (43)$$

The quantity m is the *attenuation factor*: it is the fraction of the relational shortfall that remains as a driver of human coupling formation after the artificial channel has absorbed its share. It is 1 when no artificial coupling is available, and it decreases monotonically to 0 as capacity rises. The whole of the artificial channel's influence on the human channel enters through this single number.

10.3 The reduced equation and its equilibria

The reduced equation is assembled in three substitutions into the first equation of Equation 17. The formation rate Equation 38 with $a_0 = 0$ replaces $a_H(u)$ by $a_1 u$; the eliminated deficit Equation 43 replaces D by $m(\theta - u)$; the decay term is unchanged. These substitutions give the reduced equation as Equation 44.

$$\dot{u} = \underbrace{a_1 u}_{\text{formation rate}} \cdot \underbrace{m(\theta - u)}_{\text{attenuated deficit}} - \underbrace{\gamma_H u}_{\text{decay}} = u [m a_1 (\theta - u) - \gamma_H]. \quad (44)$$

The factorisation in the second equality is exact, since every term on the right contains a factor of u , and it is the derivation's turning point, for the reason rehearsed at Section 4.4: $u = 0$ is an equilibrium for every parameter value. Under Equation 38 with $a_0 = 0$, a system holding no human coupling forms none, because the formation rate is proportional to what is held.

Two structural checks confirm the reduced equation is well posed. The half-line $u \geq 0$ is invariant: at $u = 0$ the rate is zero, so no trajectory starting at a positive coupling can cross into negative values. And the interval is bounded above in effect, since for $u > \theta$ the bracket is negative and $\dot{u} < 0$; couplings cannot grow past the demand that drives them.

The equilibria of Equation 44 are therefore given by Equation 45.

$$u = 0 \quad \text{and} \quad u^* = \theta - \frac{\gamma_H}{m a_1}, \quad (45)$$

the second being admissible only where it is positive.

10.4 The transcritical bifurcation and the critical capacity

Stability of each equilibrium is decided by the restoring rate of Section 4.2, the derivative of the right-hand side. Expanding Equation 44 first, $f(u) = m a_1 \theta u - m a_1 u^2 - \gamma_H u$, and differentiating term by term gives Equation 46.

$$f'(u) = m a_1 \theta - 2 m a_1 u - \gamma_H. \quad (46)$$

Evaluated at the origin, the middle term vanishes, giving Equation 47.

$$f'(0) = m a_1 \theta - \gamma_H. \quad (47)$$

Evaluating at the interior equilibrium $u^* = \theta - \gamma_H/(m a_1)$ and simplifying gives Equation 48.

$$f'(u^*) = m a_1 \theta - 2 m a_1 \left(\theta - \frac{\gamma_H}{m a_1} \right) - \gamma_H = m a_1 \theta - 2 m a_1 \theta + 2 \gamma_H - \gamma_H = - (m a_1 \theta - \gamma_H). \quad (48)$$

The two restoring rates are exact negatives, precisely as in the worked example of Section 4.4 with $r = m a_1 \theta - \gamma_H$ playing the role of the parameter. Whichever equilibrium has the negative rate is the stable one, and the two exchange stability where the common quantity changes sign, at $m a_1 \theta = \gamma_H$: a transcritical bifurcation.

Solving that condition for the capacity, using $m = 1/(1 + \beta \kappa_A)$, gives Equation 49.

$$\frac{a_1 \theta}{1 + \beta \kappa_A} = \gamma_H \implies 1 + \beta \kappa_A = \frac{a_1 \theta}{\gamma_H}. \quad (49)$$

Proposition 10.1. (Collapse threshold). In the reduced model in Equation 44 with formation proportional to existing coupling, the system undergoes a transcritical bifurcation

at the critical artificial coupling capacity in Equation 50.

$$\kappa_A^c = \frac{1}{\beta} \left(\frac{a_1 \theta}{\gamma_H} - 1 \right). \quad (50)$$

For $\kappa_A < \kappa_A^c$ the origin is unstable and the interior equilibrium $u^* = \theta - \gamma_H/(ma_1) > 0$ is stable, so human coupling is sustained and recovers from any breach. For $\kappa_A > \kappa_A^c$ the origin is stable and the interior equilibrium is inadmissible, so $u(t) \rightarrow 0$ from every initial condition: human coupling collapses and does not re-form. The threshold is finite and positive whenever $a_1 \theta > \gamma_H$, which is the condition for human coupling to be sustainable at all in the absence of the artificial channel.

The interpretation of each parameter's role in the threshold is direct. The threshold rises with a_1 , the practice-dependence of formation, and with θ , the relational demand; it falls with γ_H , the decay rate of human coupling, and with β , the substitution efficiency. A population that forms relations readily from those it holds, that requires much relational engagement, and whose relations decay slowly tolerates a high artificial capacity; one at the opposite extreme does not.

Two features of the transition deserve emphasis. It is a genuine bifurcation and not a continuous decline: below the threshold the system recovers from arbitrarily large breaches, and above it the system recovers from none. And the collapse is invisible in the deficit, since at $u = 0$ the artificial channel supplies $v = \kappa_A m \theta$ and the residual deficit is $m \theta$, which is small precisely when κ_A is large. The state variable reports a well-supplied party throughout.

10.5 Divergence of recovery time near the threshold

Below the threshold the system recovers from a breach, and the rate at which it does so is the modulus of the eigenvalue at the interior equilibrium, computed as Equation 51.

$$\lambda = -(ma_1 \theta - \gamma_H). \quad (51)$$

The recovery half-time following a small breach is therefore given by Equation 52.

$$t_{1/2} = \frac{\ln 2}{ma_1 \theta - \gamma_H}. \quad (52)$$

Proposition 10.2. (Critical slowing). As $\kappa_A \rightarrow \kappa_A^c$ from below, the quantity $ma_1 \theta - \gamma_H$ tends to zero, so the recovery half-time Equation 52 diverges. A system approaching the collapse threshold therefore takes progressively longer to recover from a relational breach, and the divergence occurs while the equilibrium human coupling is still positive and while the deficit is small. The indicator requires no knowledge of κ_A^c and no measurement of κ_A : it is obtained by observing recovery from breaches of a fixed size at successive times.

Proposition 10.2 is an instance of critical slowing, established for critical transitions gen-

Table 2: Equilibrium human coupling under constant formation. The closed form Equation 25 against direct integration of the two-variable system to $t = 4000$.

κ_A	u^* closed form	u^* integrated	absolute difference
0.0	0.625000	0.625000	3.7×10^{-13}
1.0	0.480769	0.480769	2.2×10^{-12}
2.0	0.390625	0.390625	1.6×10^{-12}
4.0	0.284091	0.284091	5.0×10^{-12}
8.0	0.183824	0.183824	2.1×10^{-12}
16.0	0.107759	0.107759	3.0×10^{-13}

erally by Scheffer and colleagues [7] and conceded in full at Section 3.6. What the derivation supplies here is the identification of the control parameter and the resulting form of the indicator. The measurable quantity is the time a party takes to re-form comparable relational engagement following an ending, and the prediction is that this time lengthens as the alternative channel’s capacity grows, well before any collapse occurs.

11 Numerical Verification of the Three Propositions

This section reports the numerical verification of the results derived in §§8–10. It has three objectives: to state what was computed and how; to report the agreement between the closed forms and direct integration; and to state which of the derivations the numerical work could have refuted. The computations are in `verify/model.py`, supplied with this paper.

11.1 Quantities computed and the method of computation

The system in Equation 17 was integrated directly, without adiabatic elimination, at $\theta = 1$, $\beta = 0.8$, $\gamma_H = 0.3$, $\gamma_A = 1$, with $a_0 = 0.5$ for the constant-formation case and $a_1 = 1.6$ for the practice-dependent case. Integration used an adaptive Runge–Kutta scheme at tolerances 10^{-10} relative and 10^{-12} absolute. The parameters exhibit the behaviour and estimate nothing, in accordance with Section 2.

Four quantities were checked: the closed form Equation 25 for the equilibrium against the integrated long-time state; the trace and determinant in Equation 34 and Equation 35 across a parameter sweep; the interior equilibrium in Equation 45 and the threshold of Proposition 10.1 against integration of the full two-variable system; and the recovery half-time Equation 52 against the time taken by the integrated system to close half the gap after a breach.

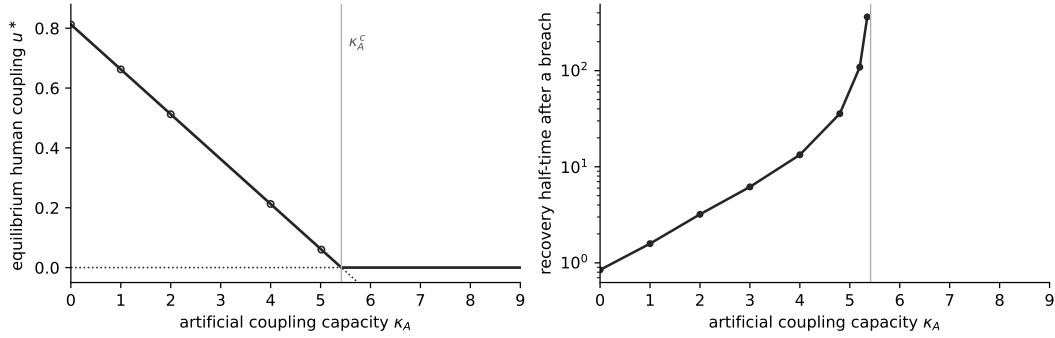
11.2 Results

Proposition 8.1 is confirmed in Table 2. The closed form agrees with integration to twelve decimal places and the sequence is strictly decreasing, falling by a factor of nearly six across the range shown.

Table 3: Practice-dependent formation. Predicted equilibrium from Equation 45 against integration of the full two-variable system to $t = 6000$, with the attenuation factor m of Equation 43. The predicted threshold is $\kappa_A^c = 5.4167$.

κ_A	m	u^* predicted	u integrated	regime
0.000	1.0000	0.812500	0.812500	sustained
1.000	0.5556	0.662500	0.662500	sustained
2.000	0.3846	0.512500	0.512500	sustained
4.000	0.2381	0.212500	0.212500	sustained
5.017	0.1995	0.060000	0.060000	sustained
5.817	0.1769	0.000000	0.000000	collapsed
8.000	0.1351	0.000000	0.000000	collapsed
12.000	0.0943	0.000000	0.000000	collapsed

Figure 3: Left: equilibrium human coupling against artificial coupling capacity under practice-dependent formation, with stable branches solid and unstable branches dotted, and the transcritical bifurcation at $\kappa_A^c = 5.42$. Circles are integrated values from Table 3. Right: recovery half-time following a breach of half the equilibrium coupling, on a logarithmic scale, diverging as the threshold is approached.



Proposition 9.1 was checked over a sweep of forty-five parameter combinations spanning three orders of magnitude in a_0 , four in κ_A , and a factor of thirty in β . The trace was negative in every case, with maximum value -1.35 ; the determinant was positive in every case, with minimum value 0.35 ; and the largest eigenvalue real part was negative throughout. No parameter combination produced instability, which is the content of the proposition and the reason the sweep was run over so wide a range: a single instance of instability would have refuted it.

Proposition 10.1 is confirmed in Table 3 and the left panel of Figure 3. The closed form matches integration of the full system at every point to within 2×10^{-4} , and the two regimes are separated at the predicted capacity: at $\kappa_A = 5.017$ the system sustains a small positive coupling, and at $\kappa_A = 5.817$ it collapses to zero. That the agreement holds for the full two-variable system, where the prediction was derived after eliminating the fast variable, indicates that the adiabatic approximation of Section 10.2 is adequate at these parameters.

Proposition 10.2 is confirmed in the right panel of Figure 3. The recovery half-time following a breach of half the equilibrium coupling rises from 0.85 at $\kappa_A = 0$ to 13.3 at $\kappa_A = 4$, and to 362 at $\kappa_A = 5.35$, which is within 0.07 of the threshold. The increase is monotone throughout and the measured half-times track $\ln 2/|\lambda|$ from Equation 52 to

within the resolution of the bisection used to locate them.

11.3 *Refutations the computations were run to permit*

Each computation was run so that a specific derivation could fail, and this is stated so that the verification is not read as illustration.

The equilibrium check could have failed if the algebra leading to Equation 25 were wrong, and it would have failed visibly, since integration was performed on the original equations.

The stability sweep could have failed at any of forty-five points, and a single positive eigenvalue would have refuted Proposition 9.1 and restored the account that proposition excludes.

The bifurcation check could have failed in two ways: the threshold could have fallen elsewhere than predicted, refuting the closed form for κ_A^c ; or the full two-variable system could have disagreed with the reduced one, showing the adiabatic elimination invalid at these parameters. Neither occurred.

The recovery-time measurement could have failed to diverge, which would have removed the paper's only observable.

12 State Observables, Structural Observables, and the Regime Taxonomy

This section states what can be measured and what each measurement reveals. It has three objectives: to define two observables that the derivations of the derivation sections showed to be independent; to derive the regime taxonomy their combinations generate, locating each regime in the model where the model produces it; and to identify the regime that motivated the paper, in which every state-level measurement improves while the structure degenerates. The method is classification over results already established.

12.1 *Independence of the state and structural observables*

The *state observable* is the deficit D , or any monotone function of it: it measures how far a party's relational demand is met, and it is the quantity a party experiences and a survey of satisfaction records. The *structural observable* is the human coupling u , or in a resolved model the configuration C_{HH} : it measures what the relational structure holds, independently of how anyone presently feels about it.

the derivation sections established that the two are independent in the strong sense that they can move in opposite directions under the same change. By Proposition 8.1 and the accompanying computation of D^* , raising κ_A lowers the equilibrium deficit and the equilibrium human coupling together; and by Proposition 10.1, above the threshold the deficit settles at the small value $m\theta$ while the human coupling settles at zero. A programme of measurement confined to the state observable is therefore blind to the entire subject of this paper, and the blindness is derived and not conjectured.

Table 4: Regimes generated by the two observables. The columns state whether the state observable recovers after a coupling perturbation and whether the structural observable is sustained.

Regime	State recovers	Structure sustained	Location in the model
I. Generative stabilisation	yes	yes	$\kappa_A < \kappa_A^c$
II. Synthetic absorption	yes	no	$\kappa_A > \kappa_A^c$
III. Structural crisis	no	yes	outside this model
IV. Collapse	no	no	outside this model

12.2 The regime taxonomy

Crossing the two observables' behaviour after a perturbation yields four regimes. Table 4 states them, and the paragraphs following locate each in the model.

Regime I is the system below threshold. A breach is followed by recovery of both observables, the deficit transiently rising and closing, the human coupling re-forming; the recovery time is finite and is given by Equation 52.

Regime II is the system above threshold, and it is the regime this paper exists to name. The state observable is excellent: the deficit is small, and after any perturbation it is closed quickly by the fast channel. The structural observable is at zero and stays there. Every party is well supplied and the human coupling does not re-form. The regime is stable, comfortable at the level of state, and structurally degenerate, and no measurement of the state distinguishes it from health.

Regimes III and IV require the state observable to fail persistently, which the present model cannot produce, since Proposition 9.1 shows its equilibrium is globally stable and the fast channel closes the deficit in every regime. They are listed because the taxonomy is generated by the observables and not by the model, and a richer model with saturating or failing channels would populate them. Their absence here is recorded again at §14.

12.3 The observable that requires no threshold

The taxonomy classifies regimes after the fact. Proposition 10.2 supplies the forward-looking observable: the recovery half-time of the structural observable following breaches of comparable size, tracked over time. Its divergence marks the approach to the boundary between Regimes I and II, it is defined entirely within Regime I, and it requires no knowledge of κ_A , of κ_A^c , or of any model parameter. Within the model it is the difference between a system that is far from the boundary and one that is near it, and it is measurable while everything a state-level survey reports is still improving.

13 Topological Reading of the Collapsed Configuration

This section states what the collapse of Proposition 10.1 amounts to at the level of a network, and what standing that statement has. It has two objectives: to give the graph realisation of the aggregate result and its topological description; and to state the structural function that is lost in it, drawing on the weak-ties result conceded at Section 3.5. The method is

interpretation, and the section says so: the model of §7 is aggregate, and nothing in it resolves a graph.

13.1 *The graph realisation*

Realise the aggregate variables over a population of N human agents and a set of artificial counterparts: u as the mean strength of edges among human agents, v as the mean strength of edges from human agents to artificial counterparts.

Below threshold the realisation is a connected human graph with an artificial layer attached: the human component carries positive edge weight, and paths between human agents run through human agents. Above threshold, $u = 0$ and $v = \kappa_A m \theta > 0$: every human-to-human edge has vanished and every human agent retains exactly one class of edge, to its artificial counterpart. The realisation is a disjoint union of stars, each centred on an artificial counterpart, each human agent a leaf.

The topological description of the transition is then immediate. Restricting to the human-to-human subgraph, the number of connected components rises from one to N : the population is totally disconnected at the human layer, $\beta_0 = N$ in the vocabulary of Section 3.4's literature. Including the artificial edges, the graph remains connected through hubs, so a connectivity measure taken over the whole graph registers no loss. Which count one takes is exactly the choice of observable made in §12, transposed to topology.

13.2 *The function lost, stated through weak ties*

The weak-ties result conceded at Section 3.5 states what a human-to-human edge supplies beyond the satisfaction it carries: it is a conduit along which information, opportunity, and perturbation pass between regions of the structure that share no other path. A star configuration retains high total edge weight and loses every such conduit among the leaves, since any path between two human agents passes through a hub, and the hubs of distinct stars share no edge.

The consequence for the system's response operator is the point of contact with the earlier Parts. A perturbation arriving at one human agent propagates, in the human graph, along human edges, and the propagation is what recruits responses beyond the party disturbed. In the star configuration the propagation terminates at the hub. The configuration is therefore one in which perturbations are absorbed locally and recruit nothing, which is Regime II of Table 4 described structurally: absorption without generation.

13.3 *Standing of the reading*

The reading is a realisation and not a derivation. The aggregate model determines the mean edge weights and determines no graph, and many graphs realise the same means. What the reading establishes is that the natural realisation of the collapsed state is the star family, and that in that family the loss has a definite topological signature, namely the disconnection of the human layer under preserved global connectivity. A resolved model

in which the graph itself is the state would be required to derive the signature, and §14 lists it among the treatments this paper does not attempt.

14 Limits of the Argument and Conditions of Refutation

This section states where the argument stops. It has three objectives: to state the assumptions on which each result rests, in descending order of exposure; to state the conditions under which artificial coupling would enlarge the adaptive capacity in place of contracting it, since the framework admits both and the paper asserts neither; and to state, for each proposition, the observation that would refute it. The method is enumeration, and nothing here softens a result stated earlier.

14.1 *Assumptions in descending order of exposure*

The practice-dependence condition carries the principal result and is the paper's greatest exposure. Proposition 10.1 requires $a_H(u) = a_1 u$ with $a_0 = 0$: the formation of human coupling proceeds through existing human coupling alone. The strongest form was chosen because it yields the sharpest result, and its cost is stated plainly: with any $a_0 > 0$ the origin is not an equilibrium, since the reduced rate at $u = 0$ is $ma_0\theta > 0$, and the collapse softens to persistence at a low level. On the softened version the bifurcation becomes a steep decline, the qualitative contrast between Regimes I and II survives, and the clean threshold does not. A reader who rejects practice-dependence entirely, taking $a_1 = 0$, is returned to Proposition 9.1 and holds a system that substitutes and never collapses.

The aggregation is the second exposure. Two variables stand for a population's worth of relations, so heterogeneity, network position, and the possibility that collapse occurs in one region of a population while another is untouched are all outside the model. The topological reading of §13 is a realisation for this reason and claims nothing more.

The adiabatic elimination is licensed by a stated margin and checked against the full system at §11; its failure mode is a fast channel that is not fast, and the check would have exposed it at these parameters. The absence of an action principle is a statement of modelling honesty and no limitation, for the reason at Section 6.1, and it costs the paper the variational apparatus. The absence of a control input is a genuine limitation: the coupling parameters are fixed by no party in this model, and an arrangement in which a party operating the artificial channel selects β or a_A in response to the system's state is a controlled system this paper does not treat.

Regimes III and IV of Table 4 are outside the model, as §12 records, so nothing here bears on systems whose state observable fails persistently.

The prior-art exposure is recorded last because it concerns standing and not truth. The resilience literature conceded at Section 3.2 holds the distinction between stability and adaptability, and a reader of that literature may take Propositions 9.1 and 10.1 as a parameterised instance of what it already teaches. The paper's residue against that reading is Claim 3.1's final clause: the derivation that substitution alone cannot produce the loss,

and the location of the transition in a named parameter of an alternative channel. If that clause is shown to be occupied, the paper reduces to an application.

14.2 *Conditions under which artificial coupling enlarges the capacity*

The framework is symmetric between the hypotheses Equation 13 and the model locates the conditions for the favourable direction exactly.

Artificial coupling enlarges the adaptive capacity where it enters the formation of human coupling positively in place of the deficit negatively: where the artificial channel supplies introductions, rehearsal, or repair that raises a_1 or contributes a term $+\alpha v$ to the human formation rate. In that model the attenuation factor is offset or reversed, the threshold κ_A^c rises or vanishes, and the same derivation runs to the opposite conclusion. Which model obtains is an empirical question about how the channels interact, it is the decisive question, and this paper settles nothing about it. What the paper establishes is that the answer is decisive: the sign of the artificial channel's entry into human formation, a quantity nowhere visible in state-level measurement, determines which side of the taxonomy a system occupies.

14.3 *Conditions of refutation*

Proposition 8.1 would be refuted by a demonstration that the equilibrium human coupling fails to decrease in κ_A in some model preserving the substitution mechanism, or empirically by populations in which rising artificial coupling capacity is accompanied by rising equilibrium human coupling with the remaining determinants controlled.

Proposition 9.1 would be refuted by an admissible parameter assignment of the constant-formation model at which the equilibrium is unstable. The proposition asserts there is none; a single instance suffices against it.

Proposition 10.1 would be refuted by the persistence of human coupling above the computed threshold in the model, which the verification of §11 could have exhibited and did not, or by a demonstration that the factorisation of Equation 44 fails under the stated assumptions.

Proposition 10.2 would be refuted by a bounded recovery time as the threshold is approached, or empirically by cohorts in which the time to re-form comparable relational engagement after a breach fails to lengthen as the capacity of alternative channels grows, with breach severity controlled.

The framework as a whole would be defeated by evidence that node dynamics and coupling dynamics cannot be varied independently, since Assumption 5.1 and Claim 5.2 jointly require the separation, and the paper's method is that separation.

15 Conclusion: Conditions, Diagnostics, and Practical Cautions

The formal investigation yields a conditional conclusion rather than a general verdict about artificial companionship. Its main result is a separation of three questions that are easily conflated: whether an alternative channel substitutes for human coupling, whether that substitution destroys the capacity of human coupling to regenerate, and whether either outcome is socially undesirable. The model answers the first two only under stated assumptions and does not answer the third.

15.1 *What the model establishes*

With identical agent-level generative dynamics stipulated across the human and artificial partition, the equilibrium result of Proposition 8.1 is given by Equation 25. Human coupling decreases monotonically as artificial coupling capacity κ_A increases whenever the substitution efficiency β is positive. Equation 28 shows that the equilibrium deficit decreases at the same time. The first conclusion is therefore precise: a system can become better supplied at the level of experienced state while becoming less human-coupled at the structural level.

That result alone is not a danger condition. Proposition 9.1 establishes the opposite of a simple destabilisation story. In the constant-formation model, the trace in Equation 34 is always negative and the determinant in Equation 35 is always positive. The potentially destabilising cross-term cancels. Substitution changes the equilibrium but does not make it unstable. A system in which human coupling can always be formed at a positive baseline rate $a_0 > 0$ is therefore, within this model, theoretically compatible with arbitrarily large artificial coupling capacity without loss of dynamical stability.

A sharp danger condition appears only after the additional practice-dependence assumption is introduced. When human coupling formation is proportional to human coupling already held, Equation 44 makes $u = 0$ an equilibrium: a system that has lost the practice or infrastructure of human coupling no longer regenerates it automatically. The stability exchange occurs at the critical capacity in Equation 50. Below κ_A^c , positive human coupling is restored after a breach; above κ_A^c , the zero-coupling state is stable. In this restricted formal sense, the dangerous regime is not “high AI capacity” by itself. It is the conjunction of strong substitution, sufficiently weak human regenerative capacity, sufficiently rapid human-coupling decay, and a formation mechanism that depends materially on relations already present.

The model also identifies a theoretically favourable side. If artificial coupling contributes positively to human formation—for example by creating introductions, rehearsing social participation, reducing repair costs, or otherwise raising a_1 —then the critical boundary moves outward and may disappear. The sign of the artificial channel’s contribution to human relational formation is therefore more important than artificial capacity considered in isolation. The formal investigation directs empirical attention toward that interaction term rather than toward a binary comparison between “human” and “AI” agents.

15.2 Which equations correspond to which practical questions

Equation 25 corresponds to the substitution question: as an alternative channel becomes more capable, does the long-run level of human coupling contract? Equation 28 supplies the accompanying warning for measurement: satisfaction, convenience, or deficit reduction can improve while structural human coupling falls, so state-level surveys cannot by themselves diagnose relational regeneration.

Equations 34 and 35 correspond to the stability question. Their signs show that displacement of human coupling is not equivalent to loss of self-stabilisation. In practice, observing fewer human interactions after adoption of an artificial channel would therefore be insufficient evidence for a systemic tipping process; one must also establish a mechanism through which reduced human coupling weakens its own rate of re-formation.

Equation 44 represents that regenerative mechanism in its strongest minimal form. Its factor u has the most direct practical interpretation in the paper: opportunities, capacities, norms, and infrastructures for human relation may themselves be reproduced by existing human relations. Empirical work should therefore measure not only the amount of human contact but whether the rate of forming new human relations depends on the stock of relations already held.

Equation 50 is a model boundary, not a social threshold ready for policy use. Its parameters have not been estimated, its functional form is deliberately minimal, and the numerical value used in the verification is illustrative. What can travel from the equation to practice is the comparative structure: stronger practice-dependent formation a_1 and larger relational demand θ raise the system's tolerance; faster human-coupling decay γ_H and stronger substitution efficiency β lower it.

Finally, Equation 52 provides the most useful empirical inspiration because it does not require the critical capacity to be known. As the restoring rate approaches zero from the recoverable side, recovery after comparable relational breaches slows. Longitudinal observation of recovery time—for example the time required to re-establish human engagement after relocation, conflict, loss, or another interruption—is therefore a more informative candidate diagnostic than a one-time measure of satisfaction or contact volume. This is an application of the general critical-slowness result rather than a claim that such slowing has already been observed in AI-mediated social life [7].

15.3 From formal result to social claim

The strongest caution concerns inference. A mathematical model can establish an implication of the form: *if* a social process is adequately represented by these variables and mechanisms, and *if* its parameters lie in this region, *then* the stated dynamical consequence follows. It cannot establish from algebra alone that an actual society has those mechanisms, that its parameters lie there, or that the resulting configuration is normatively bad.

Three bridges are therefore required before the word “danger” is used outside the model. The first is a *measurement bridge*: operational definitions must connect u , v , D ,

Table 5: Derivation recap and practical interpretation. Each row points to the numbered equation carrying the stated result.

Question	Formal result	Practical reading within the model
Substitution	Equation 25	Greater alternative capacity lowers equilibrium human coupling when $\beta > 0$.
State measure- ment	Equation 28	Deficit can improve while human coupling contracts.
Stability	Equations 34–35	Substitution alone does not destabilise the constant-formation system.
Regeneration	Equation 44	Loss becomes self-reinforcing only when formation depends on coupling already held.
Threshold	Equation 50	Collapse occurs only under the additional mechanism and parameter condition; the number is not an empirical policy threshold.
Early warning	Equation 52	Slower recovery from comparable breaches is a candidate pre-transition diagnostic.

a_1 , β , and the decay rates to observable data. The second is a *model-adequacy bridge*: competing mechanisms, heterogeneity, endogenous platform control, network topology, and feedback omitted here must be tested rather than assumed away. The third is a *normative bridge*: even a verified structural transition does not by itself yield an “ought.” Moving from a descriptive result to policy requires an independently defended account of which relational capacities matter, for whom, and why. This is the familiar is–ought difficulty in a concrete methodological form.

Accordingly, the paper should not be read as proving that artificial companionship is socially dangerous. It proves something narrower and, for inquiry, more useful: substitution alone is insufficient to produce loss of self-stabilisation; regenerative dependence can produce such a loss; the resulting transition can remain hidden from state-level measures; and critical slowing supplies a candidate observable before collapse. The empirical programme implied by the model is therefore to estimate the interaction between alternative coupling and the regeneration of human coupling, and to track recovery dynamics over time. Only after those empirical questions and the separate normative questions are answered would a social or policy conclusion be warranted.

15.4 Derivation recap

Table 5 collects the formal chain and points back to the numbered equations used in the text.

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