

# Rethinking the Spacetime-Distributed Dissolution of Money: Evolving Production Relations and Infrastructural Conditions

Wanhong HUANG  
huangwanhong@serendip.ngo

## Abstract

This discussion paper reconsiders the possible formation, persistence, migration, and partial dissolution of monetary mediation within evolving relations of production and exchange. Its starting point is the heterogeneous need for others: access to persons and communities, productive capacities, labour, knowledge, relations, and outputs can motivate exchange while leaving the institutional formation of monetary claims contingent. The analysis distinguishes needs, value circulation, monetary obligations, payment instruments, and the continued reproduction of productive conditions. It examines the conditional transformation of human–human, human–AI, and AI–AI monetary relations as artificial intelligence increasingly participates in production. A principal hypothesis is that monetary reliance may diminish locally through direct provision, multilateral commitments, admissible cyclic clearing, and institutionally supported coordination, while remaining present or re-emerging elsewhere. The proposed *spacetime-distributed* view uses explicitly interpreted events, dependencies, and where justified partial orderings, without presupposing a universal temporal coordinate or an economy-wide instantaneous transition. It distinguishes cancellation of compatible obligations from broader substitution of monetary functions, and identifies unresolved questions concerning heterogeneous valuations, correlated circuit failure, long-path approximation, power, and ecological burdens. Finally, it outlines technological, institutional, relational, and public-infrastructure conditions for subsequent investigation. The account develops a comparative philosophical foundation for analyzing monetary mediation as a historically variable social and institutional form, while preserving open formal and empirical questions concerning its possible transformation.

**Keywords:** money; monetary mediation; distributed dissolution; generative political economy; human–AI production; heterogeneous value circulation; multilateral clearing; trust; public infrastructure.

## Discussion Paper Note

This independent discussion paper investigates monetary mediation and the conditional possibility of spacetime-distributed monetary dissolution within evolving production relations. It draws on earlier exploratory contributions and a comparative philosophical literature. Its hypotheses concerning AI-mediated production and distributed clearing are provisional; their feasibility, institutional admissibility, and normative implications require separate investigation. The main sections and appendices form a complete discussion-paper draft; mathematical and conceptual audits have been performed internally, with additional internal audits of source attribution, economic and institutional scope, ecological and knowledge-infrastructure scope, and editorial consistency. Full-text source-to-claim verification, empirical testing, and independent scholarly review remain outstanding.

## Notices

**Publication status.** Independent preliminary discussion paper, draft version 2.9, 24 October 2026. First drafts of Sections 1–15 and Appendices A–H are complete; targeted internal mathematical, con-

ceptual, bibliographic, economic-institutional, ecological-infrastructure, and editorial audits have been undertaken. Full source-to-claim verification, empirical testing and independent scholarly review remain outstanding. The paper has not been peer reviewed.

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**AI-assisted research and preparation.** OpenAI’s ChatGPT contributed to research dialogue, bibliography discovery and verification, preliminary drafting, and LaTeX preparation. The human author directs the research objectives and assumes responsibility for source accuracy, interpretation, and dissemination. ChatGPT is not listed as an author.

**Research status and independence.** This paper reports no original empirical research, field deployment, implemented clearing system, or validated economic prediction. Hypothetical AI systems and hypothetical exchange circuits are analytical devices; no institutional endorsement is implied.

## 1 Introduction

The present inquiry examines the conditions under which monetary mediation arises, changes its location and function within production relations, and may undergo partial dissolution across historically evolving economic networks. Its central object is the relation between heterogeneous needs, the circulation of value, and institutions that express, transfer, or settle claims. Artificial intelligence provides a particularly instructive setting because increasingly capable computational systems can participate in productive activities, coordinate complex obligations, and alter the conditions under which human and artificial participants depend upon one another. The investigation develops an exploratory account of *spacetime-distributed monetary dissolution*: a family of possible changes in which particular monetary functions are displaced, discharged, transformed, or reconstructed in situated relations and admissible histories. The account is conditional, institutionally differentiated, and open to reversals. It concerns how a monetary arrangement might change, rather than predicting a terminal moment at which an economy as a whole ceases to use money.

A productive point of departure is the *need for others*. Such need extends beyond the acquisition of a material commodity. An individual may seek entry into a community, access to another person’s attention, a relationship of cooperation, a specialized capability, paid labour, knowledge, participation in collective decision making, or the products of shared activity. Admission fees, membership subscriptions, intellectual-property licences, service payments, wages, and contributions to a common productive facility represent institutionally different ways of governing access and participation. Their coexistence suggests that monetary relations can be constituted around encounters and relationships as well as around identifiable goods. The presence of a need, however, neither specifies the authority to restrict access nor requires that the resulting relationship be organized through money. Gifts, public provisioning, reciprocity, obligations in kind, collectively maintained capacities, and negotiated nonmonetary commitments remain possible organizational forms. The working hypothesis is therefore precise but limited: heterogeneous needs for other participants, their capacities, and their relations supply a recurrent field in which exchange and monetary institutions *may* form; the conversion of such needs into monetary claims is historically and institutionally contingent.

This starting point also prevents an overly narrow interpretation of monetary dissolution. A reduction in membership fees does not entail the disappearance of the community that formerly charged them. Automated provision of a service does not establish the disappearance of the relationships through which

that service becomes useful, recognizable, and legitimate. The clearing of a debt circuit does not erase the labour, ecological effects, institutional commitments, or value generated along its constituent paths. Exchanges may remain abundant, and new kinds of needs may arise, while the use of monetary instruments becomes less prevalent in particular regions of the production network. In an open economic system, the disappearance of one payment relation can coincide with the emergence of another. Any theory that equates de-monetization with the elimination of relations, the exhaustion of value creation, or the universal completion of exchange would therefore require additional arguments that this paper does not presume.

The philosophical context includes distinct traditions concerning value, money, and the organization of production. Marx’s analysis of commodity forms and money in *Capital* locates monetary expression within historically specific social relations of production and exchange, and connects those relations to the organization of labour and capital [21]. Monetary search theory offers a different explanatory entry: in explicitly specified matching environments, particular objects can emerge as accepted media of exchange according to trading conditions and expectations [19]. A further line of inquiry investigates the relation between money and information. Kocherlakota’s result on money and memory establishes an allocation-feasibility correspondence within a defined class of environments, suggesting that the functions attributed to a monetary object can sometimes be reproduced through records of interaction [20]. These approaches concern different questions and assumptions. Their juxtaposition motivates a careful distinction between the social formation of monetary claims, the economic functions performed by monetary institutions, and the informational capacities through which certain functions might be fulfilled by other means. None provides, by itself, a general theorem that growing computational intelligence entails the disappearance of money.

A generative perspective additionally invites examination of the capacities and relationships through which value is produced, circulated, renewed, or appropriated. The present research is informed in part by generative approaches to economics, including the account of generative justice developed by Eglash [3]. That inspiration concerns the analytical importance of value generation, the circulation of value, and the conditions under which its generators can participate in consequential decisions. The arguments developed here concerning AI-operated production, temporal and relational ordering, multilateral clearing, and monetary displacement remain separately proposed extensions with their own assumptions and possible failures. Generative description alone does not establish a normative conclusion: the legitimacy of any proposed monetary alternative depends upon rights, affected interests, distributions of power, and practical possibilities of contestation. Ethical analysis must consequently distinguish the productive closure of a circuit from a defensible account of the interests and claims involved in that circuit.

The immediate research programme grows out of several earlier exploratory papers. Huang [12] considered monetary coordination in a hypothetical economy with extensive artificial labour, distinguishing the performance of tasks from access to resources and authority over production. Huang [14] considered local and reversible monetary displacement through distributed commitments and value circuits, while Huang [7] examined the difference between syntactically describable circuits and materially, institutionally, or normatively admissible production histories. Huang [13] proposed a preliminary scale-dependent approach to the abstraction of recurring circuits while preserving specified rights-sensitive constraints. A separate inquiry into heterogeneous value circulation [11] introduced a candidate two-complex representation in which vertices are events, edges are couplings, faces are model-dependent primitives, and economic entities may be historically constituted over subcomplexes. The wider research programme of generative relational economics specifies additional tests of interpretation, formal adequacy, and institutional relevance [9]. These manuscripts remain exploratory and do not constitute cumulative empirical confirmation. The present paper seeks to connect their distinct research objects, identify the boundaries of their claims, and elaborate questions that arise once the dependence of different human and AI relationships on monetary mediation is examined together.

A significant part of the inquiry concerns the unequal transformation of monetary dependence across human–human, human–AI, AI–human, and AI–AI relations. These labels designate analytically distinguishable roles within particular production arrangements; they do not by themselves establish the legal

or moral status of an AI system. In one hypothetical pathway, AI systems become versatile providers of labour and services, thereby reducing some instances in which one human participant requires another human participant's paid productive contribution. In another pathway, humans gain access to AI-mediated outputs through publicly financed or shared productive infrastructure, while money continues to circulate among firms, infrastructure operators, energy suppliers, and autonomous software or robotic production nodes. In a further configuration, AI systems perform tasks for other AI systems and develop intricate internal accounting, pricing, or credit conventions. Each pathway depends upon access rights, resource availability, ownership, governance, and the structure of available alternatives. Human-human monetary reliance may decline without any corresponding decline in the economic network's aggregate monetary dependence. An outward appearance of de-monetization can therefore conceal the relocation of monetary coordination into production infrastructure.

Such relocation makes an important distinction visible. The dependence of an AI production network on the *human provision or control* of productive conditions differs from the dependence of its constituent systems on *one another*. Energy conversion and storage, the acquisition and verification of data, the operation and reproduction of computing hardware, material supply chains, robotic maintenance, and environmental monitoring are not interchangeable inputs. An AI network might become progressively capable of providing and maintaining these conditions without direct human labour while retaining elaborate internal reciprocal needs. The ability to perform a task does not automatically confer access to the land, devices, intellectual property, legal permissions, or scarce materials required to carry it out. Conversely, a collectively governed productive infrastructure might sustain nonmonetary access with continuing human participation. The extent of technological autonomy and the form of institutional control should therefore be represented separately. Their interaction, rather than a simple measure of task automation, conditions the prospective attenuation of monetary mediation between human and artificial participants.

This suggests a provisional ordering hypothesis. In some trajectories, certain human-human payments decline as AI systems satisfy needs once met through paid human labour; some human-AI payments decline as the AI production network gains access to reproducible energy, data, computing capacity, and other productive conditions; and internal AI-AI monetary arrangements remain operative for a longer interval because heterogeneous exchanges and claims continue within that network. Such an ordering is conceptually intelligible, yet its relation to empirical change remains open. Publicly supplied AI services could preserve or expand human-human payments in other domains; an internally integrated AI infrastructure could abandon internal monetary pricing while its services remain monetized for humans; legal and organizational boundaries might separate distinct AI participants or fuse them into a single nonmarket allocation system. For these reasons the ordering is proposed as a set of *conditional dependencies*, potentially expressible through a partial order, rather than as a universal calendar of three successive stages. The possibility of different rankings, feedback loops, and reversals is part of the explanatory problem.

A second line of inquiry concerns what is meant by *dissolution*. Paying an invoice in currency settles a particular claim by means of money. Canceling reciprocal obligations without payment can eliminate a portion of the obligations while leaving the institution of money intact. Replacing general-purpose payment with a directly enforceable entitlement can diminish dependence on a monetary medium while preserving accounting in a common unit. Public provision can remove payment at a point of access while requiring monetary payments elsewhere in the supply chain. A production network may coordinate contributions by shared capacities and nonfungible commitments, while relying on money for its exchanges across an external boundary. These mechanisms can have similar appearances to end users and sharply different structures. The paper therefore treats extinguishment of particular claims, multilateral netting, substitution of specific monetary functions, reduced frequency of payment, and dissolution of monetary institutions as related but non-equivalent analytical categories. Their differentiation is essential for both historical description and any assessment of technological change.

Consider a hypothetical directed circuit of obligations involving participants A, B, and C. A owes a specified claim to B, B owes a corresponding claim to C, and C owes one to A. In a case in which the

claims are mutually commensurable, transferable or otherwise eligible for an agreed multilateral clearing rule, the cycle may have a zero net position even while each individual claim remains outstanding. Executing an authorized multilateral clearing operation could discharge the matching portions without circulating a separate payment instrument around the circuit. The existence of a graph-theoretic cycle alone, however, establishes none of these conditions. Differences in maturity, currency denomination, enforceability, collateral, priority, jurisdiction, and valuation can obstruct clearing. Different participants may reject the underlying equivalence or the redistribution of risk implied by a proposed netting procedure. Moreover, a circuit that appears closed with respect to monetary liabilities may draw upon unacknowledged labour, unpriced energy, or ecological resources that do not enter its recorded accounts. Accordingly, the descriptive identification of a cycle, the operational ability to clear it, and the normative legitimacy of the resulting settlement represent three distinct judgments.

The proposed adjective *spacetime-distributed* carries a specific methodological commitment. Relations and obligations may be represented through events of production, receipt, recognition, commitment, fulfillment, and institutional revision. Their dependencies need not be reducible to simultaneous entries in one ledger or to a universally privileged moment of settlement. A history may involve locally ordered events and cross-system commitments whose comparability, synchronization, and temporal horizons depend upon the chosen model. The two-complex language considered in earlier work [11] offers one candidate representation of these interdependencies; directed networks, event structures, temporal graphs, and contractual state-transition systems offer alternatives. Neither an abstract spacetime manifold nor a fixed direction of historical succession is taken as an ontological given. Where a temporal coordinate, causal precedence, or a continuation rule is used, the relevant interpretation and evidential status must be specified. The possible dissolution of a monetary claim is consequently studied relative to particular histories, admissible coordination procedures, and affected relational structures.

The emergence of longer and overlapping circuits introduces a distinctive computational problem. In a naïve account, the potential cancellation of many claims might be expected to require searching an enormous family of paths, some of which revisit the same production nodes or share critical resources. A preliminary fragility intuition holds that a serial path dependent upon many independently successful links may become less likely to remain executable as its length increases. This intuition offers a possible source of computational tractability: sufficiently attenuated long paths could sometimes be neglected within a stated approximation tolerance. It does not follow that every long path is negligible. The number of admissible paths may grow, failures can be correlated, rare long paths can carry significant liabilities, and normative claims may be non-substitutable even when their predicted contribution is small. Any future truncation theorem will need separate conditions on path weights, combinatorial growth, dependence, admissibility, and the errors a procedure is permitted to introduce. In particular, approximating a material contribution is not equivalent to extinguishing an enforceable claim. The present study retains the possibility of controlled approximation as a formal research question rather than taking path fragility as an already established result.

A further complication is that monetary mediation can be regenerated by the same processes that reduce its use elsewhere. New competencies, needs, communities, rights, and scarce productive opportunities can give rise to novel claims and access arrangements. The provision of inexpensive AI-generated knowledge might diminish payment for a class of informational services while making physical experimentation, independent verification, or privileged entry to a research facility newly scarce. A commonly accessible energy source might diminish some payments for electricity while creating monetary claims over maintenance facilities or specialized materials. In multi-agent systems, improved coordination can both reduce the need for a common payment medium among some participants and make sophisticated credit instruments attractive for handling residual risks elsewhere. Monetary dissolution in one local structure may therefore occur alongside monetary recomposition in another. Such open-endedness is compatible with sustained progress in productive coordination and does not reduce that progress to the elimination of any particular institutional form.

These propositions also clarify the role of infrastructures. The technical prospect of advanced AI coordination requires observations of capacities, supplies, constraints, and institutional rules. Data collec-

tion, semantic interoperability, provenance, model verification, energy management, and reliable computational facilities can improve the feasibility of finding coordinated arrangements. Their implementation remains shaped by access rights, inter-jurisdictional cooperation, organizational incentives, cybersecurity, and the possibility of error. Trust cannot simply be equated with comprehensive information. A party may understand an obligation and still lack reason to believe that another party will perform it, or may dispute the authority of the procedure used to evaluate compliance. Records, credible commitments, reciprocal oversight, contestation rights, remedies, and revisable governance rules are potential conditions for sustained cooperation. Research on collective management of shared resources provides one relevant institutional point of comparison [23]. The full design of global data commons, transnational governance regimes, and ecological AI infrastructure belongs to companion investigations; here their analytical role is to constrain claims about where and under what circumstances a monetary function could be displaced.

Environmental dependence requires comparable restraint. The self-maintenance of an artificial production network still draws upon physical resources and ecological processes. Electricity, cooling, land, oceans, mineral supply, construction, and waste impose consequences that cannot be assumed to appear as accepted monetary obligations inside a clearing ledger. Increased technical autonomy may reduce reliance on particular human operators while intensifying reliance upon ecological conditions or infrastructures administered by other institutions. A formally balanced circulation can therefore coexist with ecological injury or the exclusion of affected groups from decisions about the circulation. This is one reason to distinguish *feasibility*, *sustainability*, and *legitimacy* throughout the proposed programme. Feasibility concerns whether a coordinated procedure can be constructed under specified technical and institutional assumptions; sustainability concerns whether its productive and social conditions can be reproduced; legitimacy concerns whether the procedure's authority, burdens, and treatment of affected interests are defensible. The three dimensions may support one another, but conclusions within one cannot be silently substituted for findings within the others.

The analysis is consequently situated at the intersection of political economy, monetary theory, relational and generative accounts of value, distributed systems, and the philosophy of technology. Historical interpretation is required to understand the formation of particular exchange and access institutions. Conceptual analysis is required to avoid conflating need, valuation, monetary obligation, and payment. Mathematical representations may help identify circulating claims, admissible netting operations, path dependencies, and computational limits. Comparative institutional inquiry is required to determine what a proposed replacement actually changes in the relations between participants. Ethical reasoning is required to address what may be excluded by apparently efficient settlements. These methods access different aspects of the research object; their partial complementarity does not imply that any one of them can determine the entire range of consequences from a formal model alone.

The inquiry makes four provisional contributions at the level of research architecture. First, it takes relational needs and access arrangements seriously as potential sites of monetary formation, while treating the conversion of needs into payment obligations as contingent. Second, it distinguishes the possible trajectories of monetary dependence across human and AI production relations, including the relocation of claims toward infrastructure and the coexistence of monetary erosion and creation. Third, it specifies spacetime-distributed dissolution as a local, historically situated family of mechanisms and identifies the obstacles to translating cyclic structure into valid clearing, including heterogeneous claims, timing, risk, power, and the limits of network approximation. Fourth, it treats productive autonomy, global knowledge infrastructure, institutional credibility, governance, and ecological reproduction as possible enabling and constraining conditions. Each contribution is open to counterexamples and requires further formal and empirical investigation. A high degree of AI competence does not itself establish a complete economic order; the institutional meaning of that competence depends upon the relations in which it is exercised.

The following sections develop the philosophical foundations, monetary formation, changing human and AI production relations, event-indexed circulation, multilateral clearing, path fragility, and monetary regeneration. They examine the technical, institutional, and ethical conditions of proposed transforma-

tions. Companion papers investigate global knowledge systems, ecological infrastructure, and transnational governance in greater depth. The present inquiry remains conditional upon specified relationships, evidence, and institutional arrangements.

## 2 Philosophical and Political-Economic Foundations

This section establishes the philosophical and political-economic background for examining monetary mediation within historically evolving production relations. Its objective is to identify the conceptual commitments relevant to the emergence, continuation, transformation, and possible local dissolution of monetary claims. The discussion considers Marxian value theory and the social form of money, relations of production and their reproduction, commodity exchange and accumulation, alienation and appropriation, generative perspectives on economic relations, heterogeneous value circulation, the institutional diversity of commons and public production, alternative monetary and credit theories, and the ethical interpretation of relationally generated value. The method is comparative and reconstructive: selected concepts are interpreted within their original traditions, then brought into dialogue with provisional hypotheses about human–AI and AI–AI production. Their distinct explanatory commitments remain visible. The present section develops conceptual foundations; operational tests of monetary clearing, path fragility, and spacetime-distributed dynamics belong to subsequent sections.

### 2.1 Marxian Value Theory and the Social Form of Money

Marx’s analysis of commodities and money offers a historical and structural point of departure for the present inquiry. In the first volume of *Capital*, a commodity combines a useful character with a value form associated with definite social relations of production. Concrete labour produces particular useful effects; abstract labour and socially necessary labour time enter Marx’s account of how products of different labours acquire commensurable social expression under commodity-producing conditions. Money appears in his exposition through the development of exchange-value expressions and the general equivalent form, rather than as a mere neutral convenience added to an independently complete system of barter [21]. This account concerns a historically specified mode of social production. Its categories accordingly require care when extended to artificial agents, nonmarket commons, or institutions organized around different principles of distribution.

The distinction between a useful object and its social form matters especially for an inquiry into monetary dissolution. A service may remain useful even where its monetary price diminishes; productive cooperation can persist when a specific commodity form ceases to organize access. Conversely, the same capacity can be enclosed or made subject to a new payment condition despite little change in its underlying technical performance. The production of a weather forecast, the delivery of the forecast through an open public service, and the sale of privileged access to its associated models illustrate analytically different relations. Each may depend on labour, data, instruments, energy, and collective institutions. Whether payment becomes a condition of access is a further historical and institutional question. The example is schematic and does not imply that any particular open-data arrangement has removed the costs of producing reliable forecasts.

Marx’s treatment also cautions against equating monetary magnitudes with exhaustive descriptions of the relations that produce them. Monetary exchange can express a relationship between products as though their equivalence were intrinsic to the products themselves. In the present study, a ledger entry or a computational price is therefore treated as a historically constituted representation of claims, valuations, and recognized equivalences. Its numerical tractability provides information about the record and the rules applied to it; it cannot by itself settle the social meaning of productive contributions or the legitimacy of appropriation. An AI system that infers prices from observed transactions may reproduce the valuations already stabilized within a legal or market order. It may also improve prediction without acquiring an adequate account of how that order excludes certain contributions or burdens.

The Marxian category of *value* should be distinguished from the wider, deliberately heterogeneous uses of *value* considered later in this paper. Marx's value theory supplies a specific explanation of the form and social determination of commodity value under capitalist production. By comparison, the phrase *heterogeneous value circulation* in the present research programme denotes an analytical field containing labour, ecological, informational, relational, expressive, and other contributions whose recognition and commensurability may be contested. The two conceptions illuminate overlapping phenomena, but their explanatory scope and criteria are not identical. Preserving the distinction permits a comparison without silently attributing the broader concept to Marx or reducing his technical category to a synonym for benefit.

One consequence follows for the central hypothesis. Monetary mediation can be investigated as a *social form* through which particular productive relations are organized and represented. Its partial dissolution is then a possible reconfiguration of those relations, involving valuation, authority, access, obligation, and distribution, together with the means of producing useful goods and services. A reduction in observed payment frequency can be consistent with such a reconfiguration, but can also result from subsidies, integrated firms, centralized accounts, or shifts of cost into another institutional layer. Diagnosing a monetary transformation accordingly requires examining the underlying arrangements alongside the visible payment instrument.

## 2.2 Labour, Production Relations, and the Means of Production

An account of changing monetary relations requires a corresponding account of who or what performs productive activity, who controls the conditions of that activity, and how its consequences are distributed. The classic Marxian distinction between productive forces and relations of production supplies a way to investigate the coevolution of technical capacities, ownership, labour organization, and social authority [21]. The present inquiry uses this distinction heuristically and retains institutional specificity. Advanced automated production may increase output while leaving control over data centres, mineral inputs, electricity, intellectual property, and distribution rights concentrated in existing organizations. A technical change in the performance of tasks and an institutional change in the organization of production are therefore separate variables.

Consider a human–AI production network in which automated systems conduct research, manufacture components, schedule maintenance, and deliver services. At least four relational questions arise. First, the provision of the productive capacity must be traced: training data, physical equipment, knowledge, electricity, land, and ongoing verification have histories and contributors. Second, operational control must be located: some actor or institution determines where systems run, what tasks are prioritized, and what resources are accessible. Third, entitlement must be identified: rights to use output, sell access, receive income, redirect production, or contest decisions may be distributed unevenly. Fourth, reproduction must be examined: the network's continued operation depends on repairing machines, replacing consumables, retaining expertise, and maintaining the legal and ecological conditions of production. Technological competence in a single task settles none of these questions on its own.

The expression *AI labour* can obscure important differences between social categories. In conventional political economy, labour and labour-power are tied to specific accounts of human productive activity and social reproduction. Calling an AI a worker may be useful for a functional description of task allocation, yet it should not determine in advance the system's legal personhood, moral status, entitlement to wages, or position within Marx's value theory. These issues remain contested and require independent argument. The terms *AI productive activity* and *AI-operated production* are adopted here where the analytical concern is production rather than the legal or normative classification of the actor. Similarly, the existence of human labour embedded in data, software, supervision, and physical infrastructure should remain visible when an operational interface appears fully automated.

An AI system may cease to depend on a particular human provider of electricity or data while continuing to depend on infrastructures governed by human institutions. Its *operational autonomy*, *material reproductive autonomy*, and *institutional autonomy* must therefore be distinguished. Operational auton-

omy concerns the capacity to carry out specified tasks under stated conditions. Material reproductive autonomy concerns the capacity of a production network to obtain, maintain, and replace the material and informational means of its activity. Institutional autonomy concerns the locus of authority over those capacities, including law, access rules, and enforceable claims. A network might become highly autonomous in the first sense while remaining dependent in the other two. The proposed relation between AI self-supply and human–AI monetary dissolution is consequently conditional on changes in these several dimensions.

This framing also clarifies what a decline in human–human monetary transactions might indicate. If an AI service substitutes for a purchased human service, recorded human–human payments may decrease while payments for compute, energy, or access rights increase. If the AI service is publicly provided, a user-facing payment can disappear although taxes and infrastructure procurement continue. If access is organized through reciprocity, commons governance, or cooperative contributions, the accounting and authority structure changes in a different way. These possibilities require comparison on the basis of underlying production and distribution arrangements, alongside aggregate and local payment flows.

## 2.3 Commodity Exchange, Value Forms, and Capital Accumulation

The commodity form and the expansion of capital provide a further basis for distinguishing increased productive capacity from the disappearance of monetary relations. In Marx’s analysis, capital is bound up with the valorization of value through historically specific circuits of production and exchange. Capitalist accumulation can reproduce and reorganize labour, property, production capacity, and markets even where particular costs are reduced through mechanization [21]. The present inquiry does not assume that AI automation dissolves these dynamics. It asks under what arrangements increased autonomy could change the monetary mediation of access, productive coordination, and reciprocal obligations.

If a firm owns a highly autonomous AI production facility, its reduction of purchased human labour may lower certain input costs. The firm can continue to sell output, charge for privileged access, license models, accumulate assets, and leverage its control over scarce infrastructure. Monetization may even spread into previously unpriced activities if new capacities permit finer exclusion and monitoring. A payment interface between a user and an AI service may disappear for commercial reasons, while monetary extraction moves toward model subscriptions, cloud computation, property claims, or the sale of ancillary goods. Describing the visible interface as a post-monetary relation would neglect the institutionally connected production circuit.

By contrast, commons-based or public production can alter the conditions under which outputs circulate. A collectively governed resource can make some services accessible without point-of-use payments, even when money remains necessary for capital equipment, external procurement, and maintenance. The distinction here concerns institutional forms, degrees of dependence, and allocation methods. Neither capitalist production nor commons provision should be assumed to have a unique fixed trajectory toward or away from monetary mediation. Public infrastructure can employ pricing to ration congestible resources; firms can distribute goods freely for strategic reasons; cooperatives can retain prices while returning benefits through collectively determined rules. Each arrangement requires a particular explanation of why money is retained, reduced, or relocated.

The reproduction of monetary relations also follows from the generation of new capacities and points of scarcity. When the marginal cost of an information service falls, complementary resources such as verified observations, advanced instruments, secure environments, or scarce physical materials can acquire greater importance. Whether these become monetized depends on property, authority, effective demand, financing, and feasible alternatives. The resulting question is dynamic: improvements that dissolve one monetary mediation can produce new productive complementarities and forms of control. The paper therefore treats the formation, dissolution, and regeneration of monetary claims as jointly evolving possibilities.

A further caution concerns distributive interpretation. A decline in total monetary expenditure or the number of settled claims need not indicate that value-generating participants possess more control

over the value they help create. Stronger centralization, cross-subsidization, or unilateral rule-setting can lower settlement costs while leaving access and bargaining power unchanged. This possibility connects the Marxian analysis of accumulation and production relations with the later discussion of generative justice. It also motivates the separation of clearing feasibility from ethical legitimacy.

## 2.4 Alienation, Appropriation, and Value Circulation

Alienation and appropriation raise questions about the relation between those who generate value and those who exercise control over its use, recognition, and circulation. Marx's analysis of capitalist production provides a major historical framework for these concerns, while subsequent discussions of generative justice extend attention to ecological and expressive contributions [21, 3]. Within the present inquiry, the terms function as points of theoretical comparison. Their meaning cannot be inferred directly from the presence of a monetary instrument or its absence. Money may facilitate reciprocal recognition or support relations of domination, depending on the surrounding institutions; nonmonetary provision can likewise be organized either collaboratively or coercively.

Consider an open technical commons maintained through voluntary and institutional contributions. Its outputs may circulate without a monetary charge at the point of use. Yet contributors can differ in authority over its infrastructure, public recognition, access to essential data, and the ability to redirect future development. A participant may derive substantial benefit from shared code while contributing little to its maintenance. Another may carry hidden infrastructural costs, including environmental damage or precarious human work, although the primary digital artifact is free. Monetary absence thus provides only partial information about the underlying appropriation structure. Questions of autonomy, recognition, maintenance, and voice remain analytically distinct.

The relation also works in the other direction. A monetary claim can sometimes serve as a means of recognizing and compensating contribution under mutually accepted conditions. Payment may support an individual's continued production or enable contributors to retain control over the terms of cooperation. A research programme that assumes money is necessarily alienating would be unable to account adequately for these arrangements. More precise analysis asks which claims are imposed, by whom they are authorized, which generators and affected participants can shape their terms, and which consequences escape the accounting system. A monetary instrument may be retained because it protects an entitlement that an informal reciprocal agreement would leave vulnerable.

The potential dissolution of an obligation through a closed clearing circuit therefore has at least two meanings. Operationally, specified liabilities may be offset and removed from a ledger according to admissible rules. Relationally, participants may continue to depend on one another, generate valuable capacities, retain differentiated bargaining positions, or bear uncompensated burdens. Ethical evaluation requires identifying whose contributions and interests were included in the clearing procedure, how equivalence was determined, and whether participants could contest the result. The disappearance of a liability entry should not be conflated with the disappearance of alienation or the establishment of just value circulation.

These issues become more difficult where effects cross the boundaries of a recognized exchange network. An infrastructure operator, an ecosystem, or a community affected by resource extraction may be absent from the contractual cycle through which recorded obligations are cleared. The boundary of a formal network is an epistemic and governance choice, not necessarily the boundary of actual dependence or responsibility. A later mathematical account of cycles must consequently distinguish the algebra of claims represented within a chosen complex from the social and ecological remainder associated with that representation.

## 2.5 Generative Perspectives on Economic Relations

A generative perspective investigates how capacities, relations, institutions, and possibilities for further value creation come into being and change through productive interaction. It is concerned with the

mechanisms that generate and transform economic relations, alongside observable transactions and allocated outcomes. Generative Justice offers one intellectual inspiration for this orientation: Eglash's account highlights bottom-up circulation of relatively unalienated labour, ecological, and expressive value, with examples drawn from peer production, agroecology, and forms of collective self-organization [3]. That contribution motivates attention to value generators and the processes by which their contributions circulate. The specific hypotheses about AI production, monetary functions, clearing complexes, and spacetime-distributed dissolution are developed in the present research programme and should be assessed independently.

The term *generative* is used here in a methodological rather than a totalizing sense. A productive relation can yield an immediate service, reproduce the ability to provide further services, alter participants' competencies, or change the rules and opportunities governing subsequent relations. A shared dataset, for example, has a useful informational content, while the practices that collect, validate, update, and interpret it generate further epistemic capacities. A research collaboration can produce a paper, a new instrument, and a durable ability to coordinate across institutions. These outputs may enter different valuation practices. Some can be priced, some are governed by institutional entitlements, and others are recognized in terms that resist a stable shared unit.

This perspective changes the interpretation of the need for others. Dependence is not restricted to the receipt of a finished commodity. A participant may seek an encounter with a collaborator, entry into a community, access to a laboratory, acquisition of a method, mutual recognition, or participation in the creation of future capacities. Membership fees, admission charges, licences, salaries, and reciprocal commitments are among the institutional forms that may regulate these relationships. Their occurrence is evidence of a particular mediation, while the underlying need supplies a possible motive for entering the relation. Needs alone do not determine the emergence of money. Scarcity, assignable rights, enforceable obligations, common standards, and available nonmonetary alternatives influence whether exchange becomes monetary.

Generative analysis also prevents the postulated availability of a highly capable AI from being mistaken for the completion of every relation of value creation. A system may perform a broad class of technical tasks and nevertheless leave human relationships, institution-specific recognition, embodied care, political participation, situated ecological observation, or new scientific questions irreducible to an existing catalogue of services. AI itself may participate in additional divisions of productive activity and depend on other systems' specialized instruments, observations, maintenance capacities, or rules of access. The argument does not require an assertion about the ultimate uniqueness of human abilities or the eventual limits of AI. It instead requires that the changing array of needs and capacities be treated as an open research object.

The generative orientation is also compatible with distinct political-economic traditions. Marxian analysis can investigate how capitalist production reproduces social relations and capital; institutional economics can investigate the generation of conventions and enforceable expectations; commons research can investigate the maintenance of collective capacity. These traditions do not become identical because each concerns processes of formation and reproduction. The inquiry uses their differences to articulate alternative explanations and to identify the commitments embedded in a proposed model of monetary dissolution.

## 2.6 Heterogeneous Value Generation and Circulation

Heterogeneous value circulation concerns exchanges and productive consequences whose significance is distributed across distinct practices of recognition, evaluation, and obligation. A participant can contribute labour, ecological stewardship, access to an instrument, situated knowledge, social care, public legitimacy, or technological maintenance. These contributions may be related within one production network while remaining partly incomparable. The present research programme previously examined such circulation using event-based and topological representations [11, 7]. Those manuscripts are exploratory formal proposals; the present section draws on their research direction without treating a selected formal

representation as empirical proof of a general value law.

Three analytical separations are important. *Generation* refers to activities and interactions through which useful capacities or recognized contributions arise. *Circulation* refers to processes through which their benefits, burdens, rights, or effects pass between relations and participating structures. *Monetary mediation* refers to the institutional use of money-denominated claims, payments, and units of account within some of these processes. A circulation structure can exist without a price on each contribution, and a monetary payment can occur without conveying a complete representation of the value generated. The distinction enables examination of both monetary and nonmonetary relations within the same analytical field.

A simple three-party thought experiment illustrates the issue. Suppose a research group provides a validated environmental observation to a computing collective; that collective supplies a processing capability to a manufacturing cooperative; and the cooperative maintains sensor equipment used by the research group. The directed dependency structure forms a circuit. Whether it constitutes a monetary clearing opportunity depends on the specific claims agreed among the participants. Equipment maintenance, computing service, and scientific observation are heterogeneous contributions. A cycle in the graph of benefits does not, by itself, yield exact legal or economic equivalence among them. An agreed exchange schedule, contractual rights, measurement standards, and institutions capable of dealing with disputes could make specified obligations eligible for netting or settlement. Other arrangements may preserve direct reciprocity without introducing monetary denominations in the first place.

The *value* registered by a participant is also situated in its history and objectives. One organization may evaluate a dataset by its immediate commercial usefulness; another may value it because it enables long-term ecological understanding; affected communities may care about consent, representational bias, or exclusion. These valuations can overlap but need not collapse into one scalar measure. A computational coordinator may find a technically feasible exchange only after adopting a rule for comparison. That rule remains an object of ethical and institutional inquiry. It is important to distinguish translating claims into a common accounting unit, constructing an acceptable bargain among heterogeneous goods, and judging the resulting arrangement legitimate for affected participants.

The ability to regenerate productive capacity complicates any one-time equilibrium account. Circulation can increase, exhaust, redistribute, or transform the capacities that support subsequent exchange. A cooperative that receives maintenance assistance may extend the lifetime of its equipment, which changes later opportunities for collaboration. A data-generating community may become more effective at validating observations through repeated participation. Environmental degradation can reduce the future ability of an ecosystem to support useful activity even when recorded monetary balances are settled. Consequently, an economic model of dissolution must attend to the reproduction and modification of generators as well as the state of claims.

## 2.7 Commons-Based Production and Institutional Plurality

The coexistence of markets, public provision, member-based institutions, and commons-based production limits claims that a single monetary trajectory follows from technological capability. Polanyi's historical account places the organization of markets within broader social and political institutions and identifies the consequences of treating labour, land, and money as though they were ordinary commodities [24]. Ostrom's comparative work shows that communities can develop varied rules for governing shared resources under identifiable conditions, challenging a simple choice between unregulated private exchange and centralized administration [23]. These contributions support institutional comparison; neither author provides a general law according to which more powerful AI necessarily eliminates money.

The relevant institutional forms should be described at the level of concrete rights and obligations. A publicly funded service can remove user-level fees while maintaining money-denominated procurement and wages. A cooperative may distribute outputs according to membership rules, with contributions accounted for through work, use, fees, or mixed conventions. A research commons may allow broad reuse of information while restricting access to sensitive data and allocating expensive computing resources

through quotas. A commercial platform may offer free entry to participants while deriving revenue from advertising, data licensing, complementary services, or infrastructure provision. Similar payment interfaces can therefore conceal different arrangements of control, and visibly different payment interfaces can coexist with a similar underlying concentration of resources.

Institutional plurality is especially important for global infrastructures. A transnational observation system can operate through federated data nodes, reciprocal standards, public financing, and agreements for verification and attribution. Shared data interfaces need not imply a single owner of every sensor, server, or model. Conversely, nominally open data can depend on fragile private infrastructure or the uncompensated work of local contributors. The governance of such systems requires choices concerning access, provenance, error correction, privacy, security, maintenance, and the distribution of benefits. The present study identifies these as possible conditions of coordination and reserves the design of specific global facilities for later research.

Commons arrangements also impose costs and duties. Rules for contribution, exclusion, monitoring, sanctioning, and conflict resolution may be necessary to prevent overuse or opportunistic appropriation. This is one reason to avoid defining commons production simply as free provision. It is an institutional arrangement for governing resources and relations, subject to local rules, asymmetries, and possibilities for revision. Monetary arrangements may be incorporated within it for some purposes while diminished for others. Analysis should examine who makes those decisions, who can revise them, and how affected parties obtain remedy.

A broader framework for the present investigation thus compares institutional modes according to their capacities to reproduce production, provide access, coordinate heterogeneity, and allocate risk and authority. The unit of comparison is a historically specified arrangement, including its informal norms and technical procedures. Its degree of monetary mediation is one characteristic among several. A low measured level of money use neither establishes efficient operation nor settles questions of justice; a comparatively monetary arrangement may nevertheless maintain reciprocal protections valued by its participants.

## 2.8 Monetary, Credit, and Exchange Theories

A theory of monetary dissolution requires a sufficiently plural account of what money does. In the familiar functional description, money can serve as a medium of exchange, unit of account, store of value, and means of payment or settlement. These functions are interrelated without being interchangeable in every institutional setting. An economy can retain a money unit for accounts while reducing physical payment, can use credit to delay settlement, or can net compatible liabilities without abandoning a common monetary unit. A theory that defines dissolution only through disappearance of cash will miss these differences. The analysis therefore distinguishes instruments, obligations, institutional accounting, and coordination functions.

Search-theoretic approaches explain how a generally accepted medium can emerge in environments where decentralized exchange involves frictions. Kiyotaki and Wright develop one influential model of endogenous medium-of-exchange acceptance within a specified matching framework [19]. Such work illuminates one class of circumstances in which money economizes on exchange difficulties. It is less directly a theory of access rights to a community, complex productive hierarchy, or enforceable sovereign liabilities. Its relevance to AI coordination is consequently conditional: stronger matching, discovery, and negotiation abilities could reduce some exchange frictions represented in the model, while leaving property, legal enforcement, and resource scarcity operative.

The information-theoretic perspective in Kocherlakota's *Money Is Memory* identifies circumstances under which money can substitute for otherwise unavailable records of past transactions and commitments [20]. This is particularly suggestive for digital infrastructures that improve records, interoperability, and the verification of fulfilment. The interpretation requires precision. A mathematical correspondence under the paper's assumptions does not show that real-world money is reducible to a universal database. Information may remain incomplete, strategic, sensitive, manipulable, or incompatible across

institutions. Reliable records cannot by themselves grant authority to impose an allocation or guarantee willingness to honour an obligation. Moreover, a record of a claim and the asset accepted in final settlement perform different institutional roles.

Social and institutional theories of money offer a complementary starting point. Ingham treats money as a social relation tied to abstract value, credit, and the institutions that authorize and produce monetary claims [15]. Goodhart distinguishes exchange-oriented and chartalist accounts in discussing monetary sovereignty and the organization of currency areas [5]. These approaches foreground the capacity of institutions to denominate liabilities, levy obligations, sanction nonpayment, and sustain a monetary order. Their significance for the present inquiry is that reduced technical exchange friction does not necessarily eliminate the institutional need for a recognized means of discharging liabilities. Monetary dependence may survive through taxation, legal settlement, wages, debt contracts, property law, and strategic control of scarce resources.

The philosophy of money also concerns how monetary exchange mediates relations between persons and social worlds. Simmel's analysis attends to the social and philosophical implications of a money economy and the forms of interaction it enables [25]. This dimension is relevant to the study's claim that money can mediate encounter and access, not merely the transfer of already finished products. Membership, entry, introduction, credentials, and access arrangements may involve monetary payment, yet the relationship for which payment is sought cannot be assumed fully equivalent to its fee. The fee may gate a scarce institutional opportunity, cover maintenance, generate revenue, or reproduce a hierarchy. The philosophical account accordingly asks what the payment organizes and which relational possibilities its removal would change.

The coexistence of these theories identifies several distinct hypotheses concerning AI-mediated exchange. Better search and coordination may reduce demand for a common medium under certain frictions. Reliable shared records may improve intertemporal reciprocity and multilateral settlement. Greater production autonomy may reduce an AI network's need to purchase inputs controlled by human institutions. Public provision can change payment obligations at the boundary of an infrastructure. Continued taxation, legally enforceable debt, scarce resources, and asymmetric governance can retain monetary functions despite improvements in all three technical dimensions. These mechanisms may be mutually reinforcing in some cases and competing in others. Their joint effect cannot be derived from a single friction parameter.

A final conceptual distinction concerns clearing. Netting eligible monetary claims may reduce the gross payments required to discharge an agreed set of liabilities. The operation can be represented algebraically once units, parties, maturity, enforceability, and other eligibility conditions are specified. It leaves the social production of goods, the actual fulfilment of underlying productive commitments, and any excluded obligations to be analyzed independently. In one setting, parties may consent to cancel reciprocal claims while retaining the common unit of account. In another, they may coordinate in-kind obligations without creating monetary claims. These are different processes, and their conflation would exaggerate the degree of monetary dissolution achieved by an otherwise valid cycle.

## 2.9 Generative Relational Perspectives and Ethical Foundations

The proposed generative relational perspective investigates historically formed interactions through which agents, capacities, practices of valuation, and institutions are constituted and revised. In the author's earlier exploratory programme, economic formation and heterogeneous value circulation were approached through relations and event-based structures, including candidate two-complex representations [9, 11]. Earlier thought experiments also investigated AI-operated production, monetary dissolution, value-circuit grammars, and renormalization [12, 14, 7, 13]. These citations establish the manuscript's research continuity; they do not warrant presenting its conjectures as empirically established or its representational tools as unique descriptions of economic reality.

Within the present investigation, a *relation* is understood through participation, dependence, exchange, recognition, authority, or the effects of a historically situated interaction, subject to the precision

required by a particular model. A *need for another* may concern an individual's presence, capabilities, productive activity, knowledge, access to infrastructure, or participation in a common world. The variety of such needs provides possible occasions for exchange and for the creation of monetary claims. Whether the resulting relation is commodified, governed reciprocally, publicly provided, or excluded from exchange depends on a historically constituted institutional context. This framing leaves space for needs that persist after a particular monetary mediation dissolves, as well as for new needs that generate new mediations.

The adjective *spacetime-distributed* is adopted with comparable restraint. Economic consequences and obligations can be situated within histories of events, cross-system relations, and partially ordered commitments. The representation of these histories by a directed graph, a cellular complex, or a probabilistic transition model is an epistemic choice to be justified by its analytical purpose. The terms *past*, *present*, and *future* do not function here as primitive ontological partitions. A model may adopt chronological coordinates or an order of admissible fulfilment when supported by the institutional and evidential setting. Different models may encode histories and continuations differently. The realization of a recorded event and the availability of a possible transformation are distinguishable without requiring a universal reference frame in which every prospective possibility is simply an unrealized future fact.

Relational analysis likewise imposes an ethical burden on its own representations. A formally closed cycle may contain recognized counterparties and omit people or ecosystems affected by the extraction of resources necessary to make the cycle operate. A technically efficient clearance procedure may cancel obligations that some parties regard as incomparable or wrongfully imposed. A self-maintaining AI production network may produce ample material goods while operating under concentrated authority and limited avenues for contestation. These examples motivate three distinct conditions for evaluating candidate transformations. *Feasibility* concerns whether a proposed coordination process can operate under specified constraints. *Sustainability* concerns the reproduction of the relevant productive, social, and ecological conditions. *Legitimacy* concerns acceptable authority, participation, burdens, recognition, and remedy. The conditions are related but require different kinds of evidence and argument.

The ethical perspective remains attentive to human and nonhuman effects without assuming a settled theory of AI moral standing. Humans, institutions, living organisms, ecological systems, and artificial production systems can be connected through causal and practical relations, while their possible rights, interests, and standing in decision procedures involve distinct normative questions. A representation of an AI as a productive node or an ecological system as an affected relation does not determine whether either is a legal party or moral agent. In particular, a landscape or marine ecosystem may sustain costs whose protection requires human scientific observation, public deliberation, and institutional responsibility even when the ecosystem cannot participate as a conventional monetary creditor.

The role of trust is similarly relational. Recorded fulfilment, verified competence, institutional credibility, and the availability of contestation or remedy can contribute to the formation of expectations about another party. High predictive accuracy does not by itself generate justified consent, equal bargaining power, or a shared normative vocabulary. Formal settlement may proceed through law and credible guarantees even where personal trust is limited, while strong interpersonal trust may sustain reciprocity in the absence of monetary payment. The investigation treats trust as a historically produced and revisable relationship, with a distinct role alongside computation and legal enforceability. Detailed theories and governance mechanisms are reserved for a companion inquiry.

These foundations establish a research posture for subsequent formal work. The question of monetary dissolution is best formulated locally and conditionally: which relations depend on monetary mediation, which features of the relevant arrangements make that mediation useful or authoritative, and which transformations could alter those features while maintaining productive capacity and acceptable obligations? The question extends beyond payments to the generation and circulation of heterogeneous value, the control of production, the acknowledgement of affected others, and the possibilities of future relational change. Distinct theoretical traditions illuminate different parts of the inquiry. Their dialogue supports a plural, revisable investigation of the social and material conditions within which monetary relations can form, dissolve, and emerge anew.

### 3 Conceptual Foundations and Research Positioning

The purpose of this section is to specify the conceptual vocabulary through which changes in monetary mediation can be investigated without presupposing either the persistence or the extinction of money. The discussion distinguishes monetary instruments from monetary relations, identifies needs for others as historically situated sources of exchange without treating them as sufficient conditions for payment, clarifies heterogeneous value creation and circulation, differentiates actors from the events and infrastructures constituting production networks, and proposes a restricted meaning of *spacetime-distributed* monetary dissolution. It then locates the inquiry in relation to established monetary theories and earlier exploratory work in generative relational economics. The method is conceptual reconstruction with deliberately modest formal commitments. Later sections examine the substantive causal mechanisms, admissible clearing operations, and possible computational approximations. Definitions in this section describe the objects to be investigated; they do not establish that a proposed economic transformation will occur.

#### 3.1 Money, Currency, Credit, and Monetary Mediation

Monetary relations contain several distinguishable functions and institutional forms. A unit of account provides a convention for expressing prices, debts, obligations, or accounting magnitudes. A medium of exchange can be transferred between parties to facilitate transactions. A means of payment can discharge a specified obligation according to the rules accepted by the parties and the relevant institutions. A store of value enables claims on resources or purchasing capacity to be carried across an interval, subject to uncertainty and changes in the applicable order. The functions may be concentrated in one instrument, distributed among instruments, or jointly organized through ledgers, intermediaries, and settlement institutions. Their separation is analytically necessary because one function may become less important without the disappearance of the others [15, 5].

The term *currency* is used here for a socially or institutionally recognized transferable monetary instrument, including eligible physical and digital forms. The wider term *money* refers to monetary institutions and accepted instruments, claims, and accounting conventions in their relevant contexts. A bank balance, an electronic payment token, a contractual debt denominated in a monetary unit, and a reciprocal commitment to provide a specific service may look similar in a computational record. They differ in legal standing, convertibility, issuer liability, conditions of discharge, and what the holder can demand. A token may represent an asset, a liability, an access credential, or an internal accounting entry. Its numerical denomination cannot settle its institutional classification.

*Credit* identifies a temporally or conditionally extended arrangement for provision, borrowing, or reciprocal performance that creates a conditionally recognized entitlement or claim to performance, expressed in money or in another admitted form. Trust may sustain such an arrangement, but a favorable expectation about another participant is not itself a credit instrument or enforceable claim. Credit can be made transferable, collateralized, denominated in a common unit, or restricted to the particular parties and purposes of an agreement. The category of credit therefore extends beyond a particular currency, although organized monetary economies often develop extensive credit-money structures. The present inquiry uses *claim* for a recognized demand or entitlement and *obligation* for the corresponding duty or undertaking, with the recognition conditions to be specified in context. These terms are analytical descriptions; they do not confer legal standing on a computational entry or presume that every participant recognizes the same claim.

*Monetary mediation* names the participation of money or money-denominated claims in coordinating a relation of access, production, exchange, obligation, or distribution. It includes explicit payments, prices that govern eligibility or allocation, financial credit, denominated settlement obligations, and the monetary accounting through which an institution organizes otherwise nonpriced provision. Monetary mediation need not take the form of a payment at each encounter. A user may receive a service without a charge while a public budget, advertising arrangement, membership account, or upstream procurement contract coordinates its provision in monetary terms. Consequently, lower observed payment frequency

Table 1: Analytical distinctions relevant to the identification of monetary mediation and its transformation.

| Object                  | Analytical question                                          | Potential observation           |
|-------------------------|--------------------------------------------------------------|---------------------------------|
| Relational need         | Which capability, encounter, or relation is sought?          | Access rules; stated needs      |
| Value circulation       | What contribution is generated, transferred, or transformed? | Productive provenance           |
| Claim or obligation     | What recognized performance is owed, and to whom?            | Contract; entitlement record    |
| Monetary mediation      | Where does money condition access or settlement?             | Price; account; payment rule    |
| Settlement              | Which claims are fulfilled, offset, or discharged?           | Discharge records; net balances |
| Productive reproduction | What sustains continued productive operation?                | Energy; maintenance; authority  |

establishes a change at the interface where payment is observed; identifying a change in the underlying production network requires tracing the relevant institutional layers.

The distinctions developed here are summarized in Table 1. The table separates analytical objects because a reduction in one recorded quantity need not imply a corresponding reduction in every other one. Its entries provide classification criteria, not an exhaustive typology of the world’s monetary institutions.

*Settlement* is especially important for interpreting dissolution. Payment using an accepted monetary instrument is one settlement procedure; legally or institutionally admissible set-off, novation, mutual release, and in-kind performance may produce different results. A multilateral clearing procedure can reduce gross recorded obligations while leaving net positions to be settled in money. A central platform can clear hundreds of transactions internally while maintaining monetary claims against its users or other organizations. A change in gross transaction count, net exposure, and dependence upon monetary institutions should therefore be described as three potentially different observations. The thought experiment advanced in earlier work on money in AI-operated production networks [12] motivates this distinction but does not supply empirical evidence that one observable is a reliable proxy for another.

In this paper the phrase *dissolution of monetary mediation* is applied to a specified set of relations whose monetary functions or enforceable monetary obligations are reduced, extinguished, or replaced through an identifiable operation. Its meaning remains indexed to the relevant boundary, recognition rules, and period or relational history of assessment. The definition encompasses several mechanisms examined in Section 3.6 below; it does not imply that a common currency has ceased to circulate throughout a society. The more specific term *clearing-induced dissolution* will refer to compatible obligations that can be validly offset or discharged, rather than to the transformation of the economy’s entire institutional basis. The distinction is particularly relevant when computational clearing becomes feasible in a large AI-operated production network.

Three nested questions should therefore be kept analytically separate. A *ledger-level reduction* asks whether a specified monetary claim has been validly extinguished. A *relation-level change* asks whether entering, maintaining, or fulfilling a specified productive relation continues to require monetary mediation. A *production-system change* asks whether the relevant capacities can be reproduced with less reliance on monetary access, finance, or settlement across a declared and appropriately widened boundary. None of these follows from the previous one without an additional account of financing, resource rights, and governance. This is a distinction among analytical tests, not an irreversible sequence or three stages of historical progress.

### 3.2 Needs for Others and Relational Dependence

The phrase *need for others* designates an analytically broad relation in which the realization of a purpose, capability, activity, or form of participation depends upon another person, collective, system, or institution. Needs can concern access to an encounter, membership in an association, another participant’s labour or expertise, an artefact, information, physical infrastructure, recognition, or the maintenance of an ongoing relationship. Some needs are instrumental and explicitly articulated; others are historically learned, socially organized, or discovered through interaction. Their objects can change with the growth of capacities and with transformations of collective life. A need is therefore modeled as situated within

a relational configuration, rather than as an immutable preference assigned once to an isolated actor.

The term “other” identifies a dependence on a distinct contribution, capacity, relation, or position; it is not a uniform predicate of moral personhood. A human counterpart may be sought for a historically particular encounter, while an AI-operated capacity may be used as a functional provider under an institution’s authority. Neither form of dependence establishes the intrinsic substitutability of the two. In turn, a third party’s claim that one capability can replace another remains subject to the purposes and evaluations of those affected.

Access and encounter deserve attention alongside familiar examples of the purchase of commodities. An entrance charge may condition participation in a physical location or social event. Membership fees may finance a shared infrastructure, regulate eligibility, allocate limited capacity, or attach payment to association with a community. A research collaboration can require access to a specialized instrument, the consent of other participants, or the use of a collectively produced knowledge resource. The price of an admission ticket is not identical to the value of an encounter, and the absence of a ticket price does not establish that access is unconstrained. The relevant institution decides whether and how a relational need becomes a monetary claim. That decision may be influenced by scarcity, financing, property rights, organizational purposes, and power.

A basic working proposition is that a need for another participant’s contribution provides one possible impetus for exchange and coordination. It does not independently explain the emergence of money. Consider the difference among receiving a freely shared forecast, contributing data to a research commons, paying a specialist for an analysis, and obtaining temporary access to a privately administered model. Each relation can arise from a similar informational need, yet the terms of provision differ. Monetary obligations emerge when applicable institutions and participants organize access or performance in a money-denominated or money-settled form. Their emergence may also depend on historical conventions, third-party liabilities, enforceable rules, and the ability or authority to exclude. Money can facilitate certain exchanges, but its presence cannot be derived from need alone. Search-theoretic accounts of media of exchange identify particular frictions and assumptions under which monetary exchange is advantageous [19]; those mechanisms constitute a different explanatory level from the mere recognition that another participant is needed.

Needs for others also have an irreducibly relational dimension. A person may value the activity of a particular colleague, the continued participation of a particular community, or the recognition associated with a reciprocal relationship. Functionally similar output from an AI may satisfy some practical purposes while changing the social meaning of the encounter. Conversely, machine-mediated interaction may create entirely new opportunities for relations among people. The present account therefore avoids converting AI’s capacity to substitute for a task into a universal claim that specific others have become unnecessary. A reduction in payment for a class of services and a reduction in people’s dependence upon one another are separable hypotheses requiring different evidence.

The word *need* is extended cautiously to AI systems. An AI system can be said to require electricity, observations, spare parts, compute capacity, authorization, or assistance from other systems in a functional and operational sense. Such an assertion describes dependencies under a specified objective or operating protocol. It leaves questions of phenomenal experience, self-understanding, moral personhood, and autonomous interests open. An AI seeking the output of another AI system for a scheduled production task may participate in a resource-allocation arrangement even where the legal owner of both systems is a human corporation. Identifying who or what *needs* a resource, who is entitled to contract for it, and who bears liability for its use requires separate relational and institutional analyses.

This approach makes the possible persistence and regeneration of monetary mediation intelligible. New relations can give rise to novel forms of access and claims even as existing monetary interfaces become less necessary. A network may provide increasingly abundant informational outputs while specialized verification, embodied encounters, physical materials, ecological permissions, or unique sites remain scarce. Such cases establish neither a universal need for money nor a universal absence of money. They motivate an open-ended account in which monetary relations can be formed, displaced, cancelled, or revived along different paths of relational transformation. This provides the conceptual basis for the

later discussion of monetary regeneration in Section 10.

### 3.3 Heterogeneous Value Creation and Circulation

The term *heterogeneous value* denotes a plurality of potentially valuable contributions and consequences that may differ in their conditions of generation, recognition, use, circulation, and assessment. They include, among others, material goods, labour and services, knowledge, ecological capacities, opportunities for participation, expressive achievements, and relationships. The category is intentionally broad and analytically provisional. It does not propose a new universally commensurable substance behind these contributions, and it does not replace Marx's more specific theory of commodity value. Its purpose is to support inquiry into relations whose consequences and claims resist immediate representation by one price or balance [21, 11].

*Value generation* describes activities or processes through which particular capacities, benefits, resources, knowledge, or other consequential contributions come to exist or acquire significance for participants. *Value recognition* describes the social, institutional, technical, or interpretive processes through which those contributions are acknowledged and qualified. *Value circulation* describes their relational movement, reuse, transformation, redistribution, or further generative effects across an evolving network. These analytic moments can be interdependent without being collapsed into one event. The same shared dataset may contribute to research, improve forecasting, support a commercial product, and create new privacy or ecological concerns. Participants may recognize different portions of these consequences and disagree about their relative importance. A transfer recorded in money therefore represents a particular settlement and valuation arrangement within a wider field of consequences.

Recognition matters because an apparently comprehensive accounting system can omit what it lacks the categories or authority to register. A ledger can accurately represent a set of priced contracts and still exclude uncompensated data work, ecological degradation, excluded participants, or informal commitments. This is a question of both epistemology and governance. Which contributions become visible depends on instruments of observation, categories of relevance, mechanisms for recording provenance, and the participation of affected parties. Which claims can be enforced depends on additional rules concerning standing, liability, and remedy. The absence of a recorded debt should not be treated as evidence that a transaction or infrastructure has produced no unaccounted burdens.

The possibility of heterogeneous valuations imposes a substantive limit on purely algebraic clearing. To cancel a specified monetary obligation against another, the applicable rules must determine the admissible units, identities, maturities, legal priorities, and set-off rights. To offset a claim for a research contribution against a claim for equipment maintenance, participants may additionally need a negotiated equivalence or a recognized procedure for converting obligations. Some values cannot be exchanged without altering what participants understand the valued object to be; others cannot be legitimately traded despite their apparent technical measurability. A proposed AI coordinator could infer candidate agreements or negotiate within delegated authority, yet computational comparability alone would not authorize it to impose a common measure. Later formal models must represent incompatibility and prohibited transformations as explicitly as admissible exchanges.

Heterogeneous value circulation also has a generative dimension. Some contributions increase the capacity to generate further contributions: an open scientific method, a resilient cooperative institution, a maintained ecosystem, or a reusable production tool may change the set of activities accessible to other participants. Their consequences may depend on relations that develop after the original contribution. Historical attribution can therefore involve provenance chains, multiple contributors, contextual interpretation, and contested distributive norms. This observation is consistent with the generative orientation discussed in Section 2; it does not establish a unique measurement procedure for generative capacity. Existing exploratory work on value circuits [7] offers one possible representational direction whose assumptions will require further testing.

A conceptual distinction is needed between *circulation* and *settlement*. Circulation concerns contributions and their transformations across relations. Settlement concerns the disposition of specified

claims according to admitted rules. A circular production process can create useful outputs without producing any debt that can be mutually cancelled. A closed circuit of mutually compatible debts may be cleared while the ecological, relational, or distributive consequences of the original activities remain unsettled. Some circular flows may redistribute obligations and burdens without improving the capacity of their participants to contribute. Hence the existence of a graph cycle is insufficient to establish either just circulation or valid monetary dissolution. Sections 7 and 8 will develop these qualifications in explicitly relational and accounting terms.

### 3.4 Production Networks and Generative Capacities

A *production network* in the present inquiry is a historically evolving configuration of activities, productive capacities, material infrastructures, organizations, institutional permissions, and their relations of provision, dependence, and control. The notion accommodates human activity and AI-operated processes without assuming that the nodes of a simple graph exhaust the entities relevant to production. It also allows the boundaries of a network to be chosen for a particular inquiry: one might study an AI service, its surrounding data and energy supply chains, or a larger set of cooperating organizations. Findings about monetary dependence are meaningful only relative to those boundaries. A service provider can appear economically self-sufficient when its externally purchased energy and devices are excluded from the modeled network.

The labels *human-human*, *human-AI*, and *AI-AI* describe provisional kinds of productive or exchange relations, not three exhaustive and mutually exclusive economic sectors. Their members may include people, corporate entities, cooperatives, research institutions, software systems, robots, and hybrid organizations. An apparently AI-AI transaction may be organized under a single human-controlled firm, mediated by a contractual platform, or conducted among legally independent institutions. A message from one autonomous agent to another may record the exercise of delegated authority rather than an enforceable debt between independent legal persons. For the study of money, the distinction between *functional interaction* and *institutionally recognized exchange* is indispensable.

Directionality also matters. Human-to-AI provision can include data, hardware, electricity, sites, maintenance, and delegated permissions; AI-to-human provision can include research assistance, care-related coordination, design, manufacture, or access to other capabilities. The same relation may involve bidirectional provision, indirect dependencies, and transfers through additional actors. Decreased dependence upon a human contribution in one function does not necessarily imply decreased need for compensation somewhere else in the reproduction network. To identify a monetary shift we must ask whose obligation changes, which capacity is substituted, where costs are transferred, and which institutions validate the new arrangement. This separates a productive transformation from the disappearance of payment at a particular user interface.

Three dimensions of *productive autonomy* are distinguished for subsequent analysis. *Operational autonomy* concerns the ability of a system to carry out delegated tasks, monitor conditions, and coordinate immediate actions. *Reproductive autonomy* concerns the capacity of an individual system or cooperating network to secure and maintain the energy, data, computational resources, material inputs, repair, and replacement needed for continued operation. *Institutional autonomy* concerns effective authorization and control over access, allocation, ownership, liability, and governance. Each dimension is relative to a specified purpose, boundary, and environmental circumstance. Advanced autonomous scheduling can coexist with dependence upon a proprietary semiconductor supply chain; locally abundant energy can coexist with restricted rights of interconnection to a grid. The capacity to produce an input is also distinct from the authority to use it.

Individual and network autonomy should be distinguished. An isolated AI system may require resources it cannot generate, while a network of specialized AI-operated installations may collectively reproduce some of those resources through coordinated provision. Such cooperation can reduce monetary dependence at the boundary between human and AI production while creating new internal reciprocal, contractual, or monetary obligations. A network able to reproduce its physical inputs may continue to

rely on political and ecological conditions outside its ownership boundary. Its internal monetary relations may persist even when its external dependence declines, or may be reorganized through shared provisioning, nonmonetary planning, or compatible cyclic clearing. The sequence of these changes is contingent upon institutions and capacities; it cannot be inferred solely from increasing computational intelligence.

This inquiry treats control of productive capacity as an analytical variable in its own right. Infrastructure can be operated by AI while controlled by shareholders, a public authority, a cooperative membership, or a commons-governing institution. The same technical system can therefore support distinct arrangements of prices, wages, fees, eligibility, credits, and shared access. Comparisons between capitalist, public, cooperative, and commons-oriented production should attend to the allocation of decision rights, residual claims, costs, risks, and mechanisms of contestation. The analysis does not equate ownership forms with a predetermined monetary outcome. Institutional arrangements provide conditions under which a particular monetary function may become desirable, unnecessary, prohibited, or transferred elsewhere [23, 9].

### 3.5 Events, Relations, and Spacetime Representation

The expression *spacetime-distributed* is adopted to examine monetary claims and productive consequences whose formation, admissible transformation, and discharge depend on relationally situated events. The term does not commit the inquiry to a universal economic clock, a preferred present, or a fixed four-dimensional physical geometry for social life. It directs attention to heterogeneous relations of precedence, compatibility, dependence, continuation, and interpretation. An empirical application may employ calendar time, geographic coordinates, or financial settlement dates where these are relevant and available. Such coordinates are observation and accounting conventions whose adequacy must be established for the problem under investigation. They need not be promoted into an exhaustive account of historical change.

An *event* is a representation of a situated occurrence or transition relevant to the inquiry, such as the creation of a productive contribution, the admission of a participant, the recording of a claim, a change in entitlement, or a recognized discharge. The term *history* denotes an adequately specified configuration of such events and the relations used to interpret them. The model may include a partial ordering of events, because one obligation can depend upon another event while many activities are not naturally assigned a unique order. Causal dependence, contractual priority, chronological observation, and normative precedence should be distinguished even when they coincide in a simplified model. A representation that orders them for computational convenience must state which relations that ordering preserves.

For a preliminary formal vocabulary, let  $\mathcal{E}$  denote an event set,  $\preceq$  a model-specific partial ordering when justified, and  $\Lambda$  a collection of interpreted relational labels. Equation 1 summarizes the minimal representational choice.

$$\mathcal{H} := (\mathcal{E}, \preceq, \Lambda). \quad (1)$$

The representation in Equation 1 is a bookkeeping schema, not a metaphysical assertion that reality decomposes uniquely into primitive events. The interpretation of  $\preceq$  and  $\Lambda$ , the granularity of events, and the identification of appropriate boundaries depend on the historical and institutional inquiry. Where relevant events cannot be placed within an empirically or conceptually justified partial order, other relational representations should be retained. An event history can also be revised when new observations, contested claims, or alternative interpretations become available.

A simple directed graph may record exchanges or obligations among participants, while a two-complex can encode additional incidence and relational structure. In the generative relational programme underlying the present thought experiments, vertices of an illustrative two-complex represent events, edges represent couplings, and faces are assigned a domain-specific primitive semantics. The faces should not be assumed without argument to be persons, goods, claims, or monetary values. Persons and organizations may themselves be modeled through extended, historically constituted substructures and

their boundary relations [11]. The relation of these structures to legal persons, accounts, institutions, and physical devices requires an explicit interpretation map. The model therefore avoids identifying a graph vertex labeled “AI” with a primitive economic actor in the event-based formalism.

Several uses of *cycle* must also remain distinct. A directed loop in a diagram records a pattern of incidence. A circulation of goods or contributions records an actual or proposed sequence of provision. A chain cycle in algebraic topology is defined with reference to an appropriate boundary operator. A financially clearable cycle additionally requires compatible obligations, recognized counterparties, admissible set-off, and suitable settlement rules. These objects can be related through carefully chosen mappings, but one does not entail the others. In particular, a topological hole may reveal a structural property of a representation without specifying a legally or ethically legitimate exchange. Section 7 considers representational bridges; Section 8 investigates the additional conditions needed for monetary clearing.

The historical formulation further distinguishes *realized configurations* from *admissible continuations*. An observation of a settled account provides evidence of what a specified record recognizes as having occurred. A proposed coordinated arrangement belongs to a set of transformations permitted by stated physical, technical, legal, and normative constraints. The latter are possibilities under a model, not events located at an already given later point on a global timeline. The available transformations can themselves change as technologies, knowledge, relations, and rules evolve. This methodological restraint is particularly important when discussing AI-generated plans, changing production relations, and a potential decline in monetary dependence. Forecasting a feasible plan and demonstrating its practical, legitimate realization are distinct achievements.

### 3.6 Monetary Dissolution, Displacement, and Regeneration

The central expression *monetary dissolution* designates a family of situated transformations in monetary mediation. Each proposed case should identify its initial arrangement, relevant participants and institutions, the monetary function being altered, the transformation mechanism, and the resulting obligations or dependencies. A useful analysis also identifies who gains or loses access, which claims survive at the chosen boundary, and whether any costs are transferred elsewhere. This specification prevents a single aggregate indicator from serving indiscriminately as evidence of complete postmonetary coordination.

*Cancellation or clearing* concerns a valid reduction or discharge of compatible obligations. An illustrative three-party circuit may have A owing B, B owing C, and C owing A. If the claims are legally and institutionally eligible for the same clearing procedure, their compatible portions can be mutually offset according to agreed rules. Equal nominal numbers written on three unrelated documents are insufficient: the obligations may differ in currency, maturity, legal priority, conditionality, or transferability. Their holders may lack authority to offset them. Even where a clearing arrangement extinguishes all recognized claims inside the circuit, the productive contributions that gave rise to the claims remain historical facts, and the wider network may retain liabilities to other participants. A clearing operation requires an account of admissibility alongside arithmetic consistency.

*Substitution of monetary coordination* concerns the replacement of some payment or price-based interfaces with an alternative arrangement for allocating access, effort, resources, or services. A public information service can be maintained through shared infrastructure or collective budgeting while individual users receive the output without a direct fee. An AI-operated repair network can allocate tasks through mutually accepted quotas or priority rules instead of per-task prices. Such arrangements may reduce monetary dependence at the studied interface, yet still involve material scarcity, governance costs, and monetary expenditures outside the selected boundary. A defensible analysis of substitution therefore follows the whole provision chain that is relevant to the research question and reports unresolved upstream obligations.

*Displacement* concerns a relocation of monetary functions across actors, layers, or boundaries. An automated user-facing service may become free at the point of access while the operator finances it through monetary payments for chips, land, electricity, or access rights. A person-to-person exchange

may be mediated through an AI service that pays its suppliers. Such cases can be significant transformations of social experience and of market structure. For the narrower claim that monetary dependence has dissolved in the entire production arrangement, however, a relocation alone is insufficient. The description should specify both the layer in which mediation declined and the layer in which it persists or intensified. An apparent decline in human–AI payments is compatible with expanding AI–AI monetary or credit relations.

*Regeneration* concerns the emergence or reconstitution of monetary coordination when new needs, capacities, exclusions, shortages, or agreements form. A successful commons may face access congestion and adopt priced reservations; an AI production network may introduce credits to ration exceptional maintenance resources or compensate for risks that cannot be resolved through direct reciprocal provision. The introduction of a price is not itself evidence that coordination has become more just or more efficient. Its consequences depend on recognition rules, allocation rights, alternatives, and impacts on affected participants. Similarly, the emergence of a new payment system does not disprove that another set of relations has undergone meaningful monetary dissolution. The account admits coexisting trajectories within one historically evolving configuration.

A related distinction separates an *accounting reduction* from an *institutional transformation*. Gross claims can be compressed while participants remain obligated to obtain the same monetary unit for their net balances. A shared AI coordination system can minimize the amount of currency transferred without changing who is legally entitled to outputs, who controls productive assets, or whether excluded participants can make a claim. Conversely, a change in public access rights may alter the role of money even if a government or cooperative continues to record expenditures in a monetary unit. Monetary transformation therefore has multiple possible observables: payment volumes, gross liabilities, net liabilities, entry prices, resource-allocation rules, and dependence on institutions of monetary settlement. These observables should be reported separately unless a justified identification argument connects them.

In particular, *zero direct price*, *zero recognized net obligation*, and *absence of monetary dependence* are logically different descriptions. The first can reflect subsidy; the second can coexist with offsetting but very large gross claims; and the third requires a wider inquiry into the provision and renewal of productive conditions. The presence of a monetary unit in an internal planning model also differs from a transferable monetary claim against a legally recognized counterparty.

*Distributed* describes the possibility that these changes occur in different relational locations, at distinct events, and along differently ordered histories. It also captures the coexistence of cancellation, displacement, reconstitution, and sustained monetary mediation. The adjective does not imply that the economy is a spatially uniform system approaching a globally monetary-free state. Full elimination within an explicitly bounded model may be mathematically possible under restrictive assumptions; its occurrence would remain a claim about that model and boundary. The sustained formation of new needs and exchange arrangements gives reason to keep open the possibility of renewed monetary mediation. It does not logically prove that money is necessary in every conceivable society. This distinction preserves the generative character of the proposed account without turning a suggestive intuition into a universal historical law.

A final limitation arises from the relation between settlement and justice. Parties to an admitted obligation circuit can agree to cancel their claims while imposing unregistered costs on workers, communities, species, or ecosystems. Some affected interests have neither a currency-denominated claim nor an institutional representative. Their absence from the ledger cannot certify their absence from the causal field of production. The present study therefore treats internally consistent clearing, materially reproducible production, and normatively defensible coordination as separable dimensions. The scope of Section 11 is to identify enabling and constraining conditions; detailed public-data, ocean-observation, and transnational governance designs are reserved for companion papers.

### 3.7 Previous Research and the Present Investigation

The inquiry has two distinct lines of intellectual connection. One concerns longstanding scholarship on money, commodities, credit, markets, and collectively governed resources; the other concerns earlier exploratory studies of AI-operated production and relationally generated value. Section 2 develops the first line, including the social form of money in Marx’s analysis, alternative credit and institutional accounts, and generative approaches to economic relations [21, 15, 23]. These perspectives inform the questions asked here without making the present model a direct derivation of any one tradition. The study makes its own assumptions about AI-operated production, event-indexed relational structures, and the possible reorganization of monetary functions, and these assumptions require independent defense.

An earlier thought experiment, *Money in AI-Operated Production Networks*, introduced a prospective setting in which AI participates in production relations [12]. A related study, *Spatiotemporally Distributed Monetary Dissolution in AI Production Networks*, investigated the possibility that monetary claims could be reduced through historically extended and distributed coordination [14]. These are conceptual predecessors, not demonstrations that advanced AI will eliminate money. Their domain motivates the present comparison of human–human, human–AI, and AI–AI relations, while the current paper broadens the account of monetary formation, institutional dependence, and persistence. It also distinguishes the disappearance of payment at an interface from changes in the underlying social organization of production.

Two further exploratory studies address representations and approximation problems that the present inquiry will revisit under explicit admissibility conditions. *A Generative Grammar of Value Circuits in AI Production Networks* considers the structured generation of candidate circuits [7]. *Normative Renormalization in Distributed Monetary Dissolution* explores the relation of distributed circulation to higher-order contributions and normative approximation [13]. Their ideas motivate later sections on possible circuit structures, correlated path fragility, and error-controlled truncation. The present article does not import a proved general convergence theorem from these working papers. It treats path attenuation as conditional, allows the number of paths and their dependencies to matter, and separates an approximate estimate of economic significance from the legal or ethical discharge of a claim.

The working paper *Heterogeneous Value Circulation in Relational Spacetime: A Topological Formalism* offers a related formal vocabulary for events, couplings, and two-complex structures [11]. The proposed *Generative Relational Economics* research programme provides a broader exploratory orientation toward economic relations and their generative development [9]. The present paper draws selectively upon those representational interests. It uses them as possible analytical languages, leaving open the validity of any particular mapping from a topological cycle to exchange, account settlement, or ethical value. A formalism can reveal admissible structural patterns only after its relevant semantics and institutional constraints have been specified.

The specific contribution of the present paper is integrative and critical at the level of a discussion paper. It connects the formation of monetary mediation through needs, recognition, access, and institutions to changes in AI production autonomy and the possible local reduction of monetary obligations. It describes the differential trajectories of human–human, human–AI, and AI–AI relations, introduces explicit distinctions among clearing, substitution, displacement, and regeneration, and examines the prospective role of relational histories and admissible circulation. It also identifies the infrastructure and governance requirements that would determine whether the possibilities represented in a model could become stable arrangements in practice. This synthesis requires further formal, empirical, historical, and ethical work; it is offered as a coherent programme of questions and conditional claims rather than as a completed prediction of economic history.

The treatment of AI remains role-sensitive. Advances in artificial intelligence might enable new observation, negotiation, planning, control, and repair capabilities. Their economic consequences will depend upon access rights, institutional design, resource distribution, ecological constraints, and the interpretation of participant interests. The examples used are schematic possibilities, unless a source is explicitly identified as documenting an observed case. The model does not presume sentience or juridical personality in AI, and it does not adopt technological advancement as a sufficient condition

for legitimate coordination. It remains compatible with multiple forms of production and financing, including arrangements in which monetary instruments continue to perform valued functions.

The following sections use this vocabulary in a cumulative sequence. Section 4 examines the formation of monetary mediation in encounters, productive contributions, and institutional claims. Sections 5 and 6 compare changing production arrangements and their heterogeneous monetary dependencies. Sections 7 and 8 address relational representations and admissible clearing. Section 9 develops the computational question posed by long and fragile paths. Section 10 considers the persistence and regeneration of monetary relations. Sections 11 and 12 discuss material, institutional, ecological, and ethical conditions, with formal and empirical open questions retained for subsequent research. The conceptual distinctions established here function as revisable commitments within that sequence; their adequacy will be assessed by the explanatory and critical work they enable.

## 4 The Relational Emergence of Monetary Mediation

This section investigates the circumstances under which relations of need, participation, production, and exchange acquire monetary conditions. Its purpose is to clarify the formation of monetary mediation before examining the possible transformations brought about by AI-operated production networks. The analysis follows six related objects: encounters and access; needs for labour, capabilities, knowledge, and products; the recognition of contributions and reciprocal expectations; heterogeneous valuation; the institutional constitution of monetary claims; and the effects of scarcity, exclusion, and power. The method is conceptual and comparative, supported by stylized cases and established accounts of commodity relations, credit, social exchange, and collective governance. The central explanatory question concerns how an existing or newly generated dependence upon others comes to be coordinated through money in one setting while being organized through different means in another. Its answer is necessarily conditional: multiple historical trajectories and forms of institutional mediation remain possible.

The relation between a need and a payment is therefore approached as an object of explanation. An individual may depend upon another's labour, attention, permission, productive instruments, or participation in a community; an organization may depend upon collective infrastructure or recognized access to scarce capacities; and an artificial production system may depend upon human-maintained resources or the capabilities of other systems. These dependencies can generate occasions for exchange and negotiation. A monetary claim, however, also requires a recognizable arrangement that gives a particular contribution a priced status, defines a corresponding obligation, and identifies acceptable forms of settlement. The examples in this section distinguish the conditions that make payment useful or enforceable from the conditions that make an encounter, a productive activity, or a social relation valuable to participants. Such distinctions are indispensable when investigating which monetary relations could diminish through future changes in technology and institutional design.

### 4.1 Encounter, Access, and Participation

A need for another person or organized collective can arise prior to the acquisition of a tangible product. The prospective relation may concern an encounter, a conversation, proximity, admission, collaboration, recognition, or the opportunity to participate in a shared practice. The other party's specific historical identity, institutional role, attention, and availability can matter to the value of the encounter. Meeting an expert at a conference, joining a scholarly association, entering a professional network, or participating in a specialized cooperative presents a range of such cases. The need in question cannot generally be exhausted by describing the physical objects or computational services that accompany the relation. Participation can generate new capabilities, obligations, understandings, or opportunities whose character depends upon the encounter itself.

A membership fee, an entrance charge, or an admission payment makes the boundary of participation a site of monetary mediation. The fee can cover the maintenance of a common facility, the remuneration of organizers, an arrangement for allocating limited access, or a claim to revenue arising from control

over an institutionally recognized gate. These rationales may coexist, and their implications differ. A charge collected to maintain a shared research facility contributes to a collective budget under specified governance rules. A fee that confers entry to a scarce professional circle can simultaneously organize access to relational opportunities. A charge imposed through monopoly control of a necessary point of access may express a more asymmetric relation. Similar prices can therefore mediate substantively different social practices. The monetary amount alone gives insufficient evidence of the character of the underlying dependence.

The relation between encounter and access is also historically generative. Once admitted to an association, participants may develop relationships, common projects, mutual responsibilities, or forms of recognition that alter what later access means. Some contributions may become possible only through the participation of particular others. A membership relationship can consequently produce value beyond the initial entitlement that a fee purchased. Subsequent interactions may occur without a further price, may depend upon continuing subscriptions, or may generate new monetizable services. Analytically, the paid boundary, the ongoing social relation, and the value circulating within the relation should be represented separately. Their boundaries need not coincide, especially when institutional rules change during the relationship's history.

A comparable distinction appears in digital and AI-mediated settings. Access to a model, a computing service, a curated knowledge community, or an automated research environment may be organized through subscription, usage pricing, institutional licensing, publicly funded provision, or reciprocal contribution. In the first arrangements, a monetary amount can govern eligibility for interaction or continued access. In the latter arrangements, the interaction can remain resource-intensive even though the end user encounters no direct payment demand. Replacing an entrance fee with a public interface therefore changes the visible payment condition, while the underlying acquisition of computational equipment, energy, training material, maintenance, and regulatory permission may remain monetarily coordinated elsewhere. This difference between *access-price removal* and *production-network demediation* will become decisive in later sections.

Access arrangements also bring rights and participation into view. Monetary admission may enable voluntary cooperation, finance collectively desired facilities, or exclude people whose contributions could otherwise enrich a common project. An ostensibly free admission policy can still impose nonmonetary barriers, including credentials, language requirements, recommendation networks, identity rules, or discretionary gatekeeping. The normative question concerns the justified terms of participation and the distribution of its benefits and burdens; the monetary question concerns the institutional use of payment or money-denominated claims in arranging that participation. An inquiry into distributed monetary dissolution requires both questions. The disappearance of a fee is one possible transformation of an access relation, while the emergence of accessible and generative forms of association remains a distinct object of evaluation [24, 23].

## 4.2 Needs for Labor, Capacities, Knowledge, and Products

The needs motivating economic coordination extend from relationships of access to the heterogeneous capacities of others. Labour, specialized skill, interpretation, design, knowledge, tools, infrastructural availability, and completed products enter economic relationships through different combinations of productive dependence and recognized entitlement. A surgeon's skilled activity, an engineer's maintenance work, a scientific dataset, an industrial component, and a publicly maintained forecasting model present different productive and institutional structures. Some are best understood in relation to specific persons; some are reproducible through common procedures; some rely upon cumulative collective knowledge; and some require extensive capital equipment and continuing material inputs. A common monetary unit may price access to each, but the prices do not establish equivalence across the underlying forms of value or their histories of production.

Marx's account of labour and the commodity form is relevant here because a socially organized sale of labour-power or products depends upon historically constituted production relations, rather than upon

technical activity in isolation [21]. In this paper, labour is used in a broad descriptive sense when discussing contributions by humans, while the juridical and normative interpretation of AI activity remains open. Assigning a machine a productive role does not establish that it possesses labour-power in Marx's technical sense, incurs a wage claim, or occupies the legal position of an employee. These distinctions become especially important when a firm deploys automated systems under an existing property regime. Technical substitution of a human task may preserve the monetary purchase of energy, computing, components, software licences, and capital goods even when the form of remuneration changes.

A productive capability can also be accessed without transferring ownership of the resources that sustain it. The purchaser of a one-time calculation, the subscriber to a continually available service, the member of a shared laboratory, and the beneficiary of a publicly provided capability enjoy different entitlements. Their exchanges may depend on service contracts, cooperative rules, wages, procurement procedures, taxation, or mutual obligations of maintenance. Payment for a product and payment for temporary control over a means of producing further products differ in their consequences for future dependence. The latter can alter the possibilities available to others and can participate in the formation of power over additional transactions. An analysis centered on needs for others therefore has to consider control of generative capacity as well as acquisition of its realized outputs.

Knowledge offers a particularly instructive case. An informational result can sometimes be copied and distributed at relatively low marginal cost, whereas its discovery, verification, interpretation, and continued correction may require sustained labour and infrastructure. A proprietary database may charge for access to an otherwise reproducible record. An open repository may offer access without a direct price while relying upon institutional budgets and voluntary or remunerated maintenance. A specialized expert may provide an interpretive judgment whose reliability depends upon experience and professional responsibility that cannot be equated with the possession of a digital file. Monetary mediation can arise at any of these boundaries, yet its locus and justification differ. Improved AI retrieval or inference might reduce reliance upon some priced interfaces while increasing dependence upon provenance, verification, instruments, and energy. The direction of change remains an empirical and institutional question.

The development of AI as a productive other gives this distinction a further dimension. The same AI system may be technically able to perform a task, dependent on humans or firms for its material reproduction, and constrained by a separate organization that controls the conditions of its deployment. Operational capability, material-reproductive autonomy, and institutional control thus have different implications for monetary dependence. An AI service that operates with minimal human supervision may continue to require recurrent monetary payments for energy, fabrication, physical repair, data access, and legal permissions. A collectively maintained AI infrastructure might reduce some monetary access conditions while retaining monetary commitments to external suppliers. Only an analysis of the relevant production network can determine where the need for monetary mediation has actually shifted.

The present section consequently distinguishes *needs for outputs*, *needs for the activities generating outputs*, and *needs for access to productive capacities*. Each can yield a different monetary arrangement. The historical expansion of capabilities changes the alternatives available to actors, including alternatives based on provision without a transaction price, bilateral barter, gifts, common provision, and contractual or institutional credit. Such alternatives do not erase the material costs or social relations through which the capability becomes available. Rather, they change the ways in which those costs, opportunities, and responsibilities are recognized and coordinated.

### 4.3 Recognition, Reciprocity, and Exchange Relations

Recognition mediates the transformation of a perceived need into an exchange relation. A party can seek another's contribution without the other recognizing a duty to supply it. Conversely, a contribution can be produced without being recognized as valuable by those who benefit from it. In a negotiated exchange, participants identify the contributions they offer or seek, the circumstances under which they regard performance as fulfilled, and the rules that determine the resulting expectations. Recognition may occur through explicit consent, custom, occupational norms, law, social practice, or organizational

authority. Its form affects whether an obligation is recognized by the participants, enforceable by an institution, transferable to others, or denominated in money.

Reciprocity can be immediate, delayed, generalized, or indirect. Two individuals may exchange services directly; members of a cooperative may contribute different forms of labour at different occasions; a public facility may serve contributors and noncontributors according to an agreed policy; and a credit system may record one party's present contribution against another party's specified future performance. These arrangements can circulate valuable contributions while employing distinct methods for accounting and discharge. They also have different tolerances for inequality, uncertainty, asymmetry in the timing of performance, and differences in how the participants understand what is owed. The creation of a transferable monetary claim is one available institutional response to the practical difficulties of coordinating such heterogeneity [15, 5].

The familiar three-party circuit provides a deliberately restricted example. Suppose participant A has an eligible obligation to B, B has an eligible obligation to C, and C has an eligible obligation to A. If the claims share a compatible unit, suitable maturity conditions, a recognized authority for cancellation, and the required consent or legal authority, the parties may be able to reduce their outstanding gross obligations through multilateral netting. Each participant remains situated within productive and social relations that gave rise to the underlying claims. Moreover, the feasibility of such a netting operation cannot be inferred solely from a depiction of three arrows on a graph. A claim tied to the performance of a personal obligation, a debt subject to a statutory prohibition, and a claim contingent on a disputed event may be nonfungible or legally ineligible for the operation. The legal and normative content of the edges matters as much as their geometry.

Reciprocal recognition is more demanding where exchanges involve heterogeneous values that participants cannot straightforwardly denominate in a common unit. A research community may value teaching, mentoring, software maintenance, and original research differently. Participants can nevertheless establish contribution records, deliberative procedures, access entitlements, or reciprocal expectations without prescribing a universal numerical equivalence. Their cooperation can be extensive despite a deliberately incomplete accounting of comparative worth. In other contexts, disagreement over contribution can produce disputes that require monetary compensation, negotiated remedies, or rules for redistribution. Recognition and reciprocity therefore remain relevant both to the formation of monetary claims and to the conditions under which such claims can be modified or extinguished.

The historical development of reciprocity also complicates a simple trajectory from barter to money or from money to nonmonetary exchange. Exchange practices depend upon the organization of institutions, knowledge of counterparties, enforcement capacities, and shared records. Monetary instruments can facilitate coordination between strangers with incomplete knowledge; shared histories and credible records can support some nonmonetary forms of reciprocity; concentrated authority can impose monetary prices even among actors with adequate information. The model of *money as memory* proposes, under specified assumptions, a relationship between monetary allocation and record-based arrangements; it is not a general proof that the availability of an accurate memory system eliminates monetary institutions [20]. In an AI-rich production environment, improved record keeping can expand the set of conceivable arrangements while leaving their legitimacy and feasibility to be established separately.

An ethical implication accompanies the account. Recognition requires attention to how an institutional system identifies the parties, contributions, and harms that qualify for inclusion. A closed clearing group can validate claims among its members while ignoring work performed by outsiders, ecological burdens, or socially significant contributions that lack recognized accounting categories. Generative perspectives on economic value encourage investigation of who participates in creating value and how that value continues to circulate; they do not specify the same entitlement rule for every domain [3]. A satisfactory analysis of reciprocity should therefore expose the boundaries of recognition as a substantive part of the coordination process.

#### 4.4 Heterogeneous Valuation and Commensurability

Monetary exchange often involves a reduction of heterogeneous claims to a common accounting expression. Such reduction can be useful for planning, comparison, accounting, taxation, and settlement, but it requires an interpretive rule. Value can refer to a person's practical benefit, the social form of a commodity, a recognized contribution, an ecological effect, a contractual entitlement, or a claim on scarce productive resources. These uses overlap without being identical. A price may express an accepted term of exchange, a regulatory convention, or the outcome of asymmetric bargaining. Neither the existence of a price nor the technical ease of calculating it entails that all relevant contributions and burdens have been adequately recognized.

A distinction between *comparability* and *commensurability* is useful. Two options can be compared through an explicit decision procedure without having every attribute converted into a common numerical substance. A community may deliberate over the ecological impact of two facilities while considering energy demand, species disturbance, knowledge benefits, and maintenance burdens separately. A monetary decision rule can assign weights or shadow prices to some dimensions, while other dimensions may be protected through thresholds, rights, or categorical restrictions. These procedures provide different kinds of resolution and express different accounts of legitimate trade-offs. Treating them as alternative institutional mechanisms avoids collapsing all disagreement into a missing market price.

Commensurability also depends upon the intended operation. A negotiated price may be sufficient for a particular sale without creating a transferable claim suitable for multilateral clearing. A pair of participants may accept reciprocal access rights that are meaningful within their relationship and ineligible for transfer to an unrelated party. A monetary ledger may contain perfectly additive amounts while combining instruments that cannot legally be netted across maturity, jurisdiction, or settlement systems. The validity of arithmetic thus operates within a domain whose admission criteria must be separately defended. This issue is particularly important for the later analysis of circulation spaces: a graph cycle is an admissible clearing circuit only after the claims decorating its edges satisfy relevant constraints.

AI systems can assist in identifying equivalences, predicting the consequences of competing valuation rules, and negotiating bundles of heterogeneous services. Increased inference capacity may expand the coordination of nonmonetary commitments by estimating needs and matching contributions across multiple participants. Yet a system's predictive competence cannot determine on its own whether a contested environmental burden may be exchanged for a private benefit, whether a contribution was made under informed consent, or whose preferences should determine an institutional outcome. Those questions require situated knowledge and legitimate governance. Data drawn from historical transactions can also encode the omissions, unequal bargaining positions, and implicit norms of the environments in which it was generated. Computational clarity in the resulting valuation does not remedy these limitations by definition.

Consider, hypothetically, a network that provides electricity, digital knowledge, equipment repair, and an ecological monitoring service. The electricity and repair obligations might be priced in compatible monetary units. The monitoring data might be made publicly available, while habitat protections are specified through constraints on the facility's activities. The network could reduce monetary transfers through internal coordination without translating habitat protection into a payable credit. Alternatively, an institution could price access to ecological data while failing to account for physical impacts on marine organisms. These possibilities differ in both the amount of monetary mediation and the validity of the underlying valuation regime. The question of monetary dissolution must therefore retain an independent assessment of which forms of value become visible or invisible as coordination systems change.

The present approach does not assume that heterogeneous value possesses a uniquely correct scalar representation. For a specified settlement regime, participants may nonetheless agree upon an accounting convention that is adequate to discharge a restricted class of obligations. That narrower achievement should be described with precision. It permits local clearing or the substitution of one institutional arrangement for another, while leaving open the larger questions of value recognition, distribution, alienation, and the formation of new claims [21, 25, 11].

Table 2: Institutional tests for distinguishing the formation of a monetary claim from adjacent relational arrangements.

| Dimension               | Analytical test                                                                       | Possible evidence              |
|-------------------------|---------------------------------------------------------------------------------------|--------------------------------|
| Access and dependence   | Which relation or capability is sought, and who controls admission?                   | Membership terms; access rules |
| Denomination            | Is the relation priced or the obligation expressed in a monetary unit?                | Contract; price schedule       |
| Recognition             | Who recognizes the entitlement or duty, and on what authority?                        | Agreement; statute; practice   |
| Discharge and transfer  | Which performances or instruments settle the claim, and can it be assigned or netted? | Payment and clearing rules     |
| Reproductive conditions | Which external inputs and institutions sustain the promised performance?              | Procurement; operating records |
| Contestability          | Who can challenge the amount, terms, boundaries, or effects of the arrangement?       | Appeals; review rules          |

## 4.5 Monetary Claims and Institutional Formation

Monetary mediation becomes institutionally identifiable when a relationship is organized through a recognizable monetary convention or claim. Depending on the setting, this may involve an agreed price for access, a contractual debt denominated in an accounting unit, a wage claim, a subscription requirement, a membership charge, a publicly funded procurement obligation, or a transfer through a recognized payment system. The institutional mechanism specifies more than a number: it may identify the obligated party, the creditor or recipient, the unit of account, conditions for performance, permissible instruments, authority to enforce or amend the terms, and procedures for resolving disagreement. These elements need not arise together, and different legal orders or associations may distribute them among separate actors.

The distinction between a priced encounter and an enforceable monetary debt illustrates this sequence. An organizer may advertise an entrance fee before the participant has incurred a payment obligation. A person may hold a transferable voucher for access without possessing a right to monetary repayment. A completed transaction may discharge the payment obligation while continuing membership duties. A public service may provide access without direct charges while appropriations and procurement contracts still create monetary liabilities upstream. The presence of an account in a system's database consequently provides incomplete evidence regarding the legal and economic status of a monetary relation. A valid account of dissolution must trace the claim's institutional constitution before attempting to specify how it could be discharged or replaced.

This inquiry can be organized by a restricted set of analytical tests, summarized in Table 2. The tests identify whether an observed relation is appropriately described as a monetary claim, an access condition, an accounting practice, or a nonmonetary obligation. They are intended as questions for comparative research and do not supply a universal definition of legally enforceable debt.

An institutional account of money also helps distinguish monetary cooperation from the possession of complete interpersonal trust. Parties can rely on payment instruments and contractual enforcement when they are uncertain about another's willingness or ability to perform. The credibility of a monetary claim can come from legal recognition, accounting systems, regulated intermediaries, reserves, assets, or repeated practices. Conversely, a cooperative may sustain extensive reciprocal provision through shared commitments and collectively monitored contribution rules, without routinely denominating each act in money. These alternatives illustrate the coexistence of monetary and nonmonetary means of coordinating productive interdependence [23, 15]. Their adequacy depends upon what is being coordinated and on the distribution of enforcement and decision-making authority.

Institutional formation can produce money-denominated relations even where the directly productive participants do not themselves use money. Suppose several AI subsystems exchange data, maintenance services, and electricity under rules set by a single firm. A central accounting office may assign transfer

prices or internal budgets to the subsystems. Such entries can guide allocation while lacking the standing of independently enforceable claims between distinct legal persons. If the subsystems are operated by different corporations, contractual settlements and external payment demands may arise. If collectively governed infrastructures permit reciprocal access on nonmonetary terms, some corresponding claims may never be constituted in monetary form. The differences arise from organizational boundaries, rights, and financing rules as well as computational and productive capacity. The label *AI–AI monetary exchange* therefore requires identification of the institutions that make an exchange monetarily meaningful [12, 14].

The monetary formation process can furthermore reproduce its own conditions. A payment requirement creates demand for a particular settlement instrument; a credit relation generates needs for verification, enforcement, and liquidity; a membership gate can support the institutional authority that sustains it. These feedbacks are historical possibilities rather than universal laws. They explain why more complete information or a lower technical cost of production may alter the economic justification of a fee without automatically extinguishing the fee itself. Monetary dissolution, should it occur, can require changes to recognized claims, resource-control institutions, and accepted means of coordination in addition to changes in the productive capabilities of the counterparties.

#### 4.6 Scarcity, Exclusion, and Power

Scarcity matters to monetary formation through its interaction with access rules, capacity constraints, and the distribution of alternative opportunities. Limited time with a particular person, a geographically fixed laboratory, an energy-constrained computing cluster, and a critical mineral deposit exemplify different limits. Their scarcity can be physical, technical, spatial, organizational, or produced through institutional restrictions. Participants may respond through prices, queues, lotteries, entitlement rules, negotiated reciprocity, public allocation, or exclusion. The existence of scarcity therefore leaves the particular form of allocation open, while increasing the salience of the institutional mechanisms that determine who receives what and under which conditions.

Exclusion can make a valuable relation available only on specified terms. Property rights, patents, contractual restrictions, licensing, data access policies, and control over meeting places can define permissible participation or production. These arrangements may protect legitimate interests, secure investment in shared facilities, or create rents from access to valuable capabilities. Their effects should be assessed in relation to who establishes the rule, whose labour or common resources contribute to the capability, and whose options are restricted. A hypothetical research commons with abundant digital copies can require money at the point of access if an authorized gatekeeper imposes a subscription. A high-cost physical facility can offer access without a per-use charge through public finance or a cooperative arrangement. The contrast illustrates the independent analytical roles of technical scarcity and exclusionary authority [24, 23].

Power is relevant both to the terms of an exchange and to the formation of the alternatives against which those terms are judged. A party able to determine access to energy, computation, legal recognition, or a professional network may affect the price and enforceability of claims that arise around that capability. A party with limited alternatives may accept terms it does not regard as equitable in a wider sense. Monetary payments can then express an institutionally valid settlement under circumstances of unequal bargaining power. Accordingly, an observed acceptance of a monetary price cannot be interpreted straightforwardly as proof that an arrangement satisfies the participants' full ethical assessments. The relationship between payment and justice must be investigated through access to alternatives, processes of agreement, the recognition of affected persons, and opportunities for contestation [21].

The same issue arises in systems governed by powerful AI operators. Expanded AI capacity may reduce the marginal cost of performing some tasks while increasing concentration of access to compute, energy systems, proprietary datasets, specialized fabrication, or regulatory permissions. A human facing a free AI interface can remain dependent on a provider that controls the interface and its underlying models. An AI system can appear technically self-reproducing while relying on a small number of insti-

tutions for physical inputs or continued permission to operate. By contrast, distributed public facilities may broaden the available alternatives even with substantial material constraints. None of these cases determines the future trajectory of money by itself. They show why analyses of technological autonomy must be joined to analyses of institutional control and the possible regeneration of monetized access.

Ecological relationships pose a particularly consequential boundary to a purely contractual account. A production system may exploit energy, land, water, habitats, and material resources whose ecological effects extend beyond its priced transactions. Some affected persons and ecological systems do not possess standing as exchange counterparties within a given accounting regime. Clearing the monetary obligations among operators would leave those impacts analytically outstanding. Even a stable reciprocal arrangement within an AI production network can coexist with an unsustainable rate of resource depletion or a burdensome distribution of environmental effects. The question of whether a monetary relation has been extinguished thus differs from the question of whether a network has discharged its wider obligations. The latter requires ecological observation, institutional responsibility, and appropriate normative procedures beyond the clearing algorithm.

The analysis of power also clarifies the significance of the proposed *distributed* character of dissolution. If access fees, credit claims, and monetary obligations arise at particular institutional boundaries, their removal or substitution may likewise occur unevenly. Some actors may obtain new nonmonetary routes to necessary capabilities, while others become dependent upon newly monetized chokepoints. A monetary circuit may be closed among recognized counterparties while materially dependent on unrecognized contributions. The formation of alternative public or commons-based infrastructure may reduce one concentration of control and generate new needs for governance or maintenance. These possibilities warrant attention to location, relational history, and changing institutional boundaries when studying shifts in monetary mediation.

Taken together, the six components of this section support a limited but consequential conclusion. Needs for others create potential relations of access, production, and exchange; recognition, valuation, and institutionally constituted claims determine whether and how such relations are mediated by money. Monetary formation is therefore compatible with extensive technical capabilities and with persistent social interaction, but its particular sites and functions are historically contingent. A conceptual inquiry into monetary dissolution should identify precisely which priced access conditions, monetary obligations, or dependence relations are transformed and which continue, migrate, or emerge elsewhere. Section 3 supplied the vocabulary for this identification; the present section has examined how its objects can become connected. The next comparison turns to changing relations among human participants and AI-operated production systems, where the distribution of productive capabilities, material dependencies, and institutional authority may produce markedly different monetary trajectories.

## 5 Monetary Relations in Evolving Production Networks

This section examines the organization of monetary relations when production is distributed among human participants, artificial systems, organizations, and the infrastructures that support their activity. Its objective is to identify the relevant sites of dependence and monetary mediation before the subsequent sections consider directional changes in monetary reliance and the possible clearing of obligations across a relational network. The investigation distinguishes human–human, human–AI, and AI–AI interactions; examines the status of AI as a productive other; and compares arrangements of ownership, access, and governance. The method is a set of analytically controlled comparisons and hypothetical cases, informed by political economy and institutional analysis. These cases specify possible mechanisms and counterexamples; they do not report an observed transition to an autonomous AI economy.

The labels human–human (H–H), human–AI (H–AI), and AI–AI (AI–AI) describe *sites of interaction*, rather than three naturally separated types of economic actor. A transaction between two persons may be conditioned by an automated platform, a firm may delegate its procurement decisions to software, and two programs may exchange services while their organizations remain the parties to an enforceable contract. A human owner may also retain the right to approve an apparently autonomous system’s com-

mitments. Accordingly, an analysis of monetary dependence has to distinguish the operational locus of an activity, the location of the associated productive capacity, and the institutional bearer of its claims and liabilities. A description based on the immediate interface alone can misidentify where money is required and who exercises the corresponding power.

## 5.1 Human–Human Production Relations

Human–human monetary relations encompass diverse dependencies extending beyond the exchange of finished goods. Persons rely upon the labour, skills, attention, authority, cooperation, and knowledge of others, as well as upon access to organizations that make encounters possible. Monetary claims may attach to each of these relations in different ways. A wage concerns an institutionally constituted claim in exchange for labour under specified conditions; a fee for instruction concerns access to an organized learning activity; a membership payment may govern participation in a professional or cultural institution; and a licence may permit the use of a collectively generated informational resource. The economic analysis of such arrangements benefits from separating the need for another’s contribution from the means through which the contribution is recognized, financed, or restricted. Marxian analyses of the social form of value and of production relations offer one important point of reference for examining this historical constitution of monetary claims [21].

The relevant relation may also remain partly outside a market. Family care, research collaboration, voluntary association, mutual aid, and commons maintenance can involve demanding labour and significant heterogeneous value creation without assigning a market price to each contribution. Their continued reproduction nevertheless requires material inputs, institutional arrangements, and forms of accountability. A laboratory may provide free public lectures while paying rent, technical staff, and electricity bills; a collectively maintained software library can circulate freely while its contributors depend upon wages or donations elsewhere. The absence of a payment on a particular H–H edge therefore describes a local institutional fact. It leaves open the monetary relations supporting the edge’s productive conditions, a distinction consistent with investigations of commons governance and collective action [23].

The participation of AI can alter the location of these dependencies. Automated language assistance may reduce the monetary charge for an existing form of routine translation, while creating new demand for specialized human interpretation, verification, pedagogy, and dispute resolution. Autonomous scheduling can reduce the effort required to arrange an encounter, while a social organization may retain a membership fee because access to its particular participants remains valuable. An AI-generated substitute for a standardized informational product need not substitute for the historically specific relation between two people. In the present framework, increased AI capacity modifies a feasible set of arrangements among humans; it does not imply that interpersonal needs, recognition, or the possibility of new monetary boundaries disappear.

A particularly important distinction arises between *service substitution* and *relation substitution*. A task formerly performed by one person for another may be automated, whereas the social purpose of consulting that person persists. For example, a research assistant can produce a bibliographic summary, yet a scholar may seek dialogue with a particular colleague because interpretive responsibility, intellectual disagreement, or the prospect of joint inquiry matters. Conversely, an institution may charge for access to an online community even after its technical costs become low. Price formation then depends on eligibility rules, organizational control, and the character of the encounter. Predictions of a general H–H monetary decline therefore require evidence about the displacement of priced activities and the formation of new priced relations, rather than a calculation of task-level automation alone.

Human–human relations also have distributive and political dimensions. Access to employment, housing, infrastructure, and professional networks is affected by ownership, social position, bargaining power, public provision, and the terms of collective organization. A decrease in the market price of one informational service can coexist with increased financial dependence through another, such as the acquisition of a computing device or payment for exclusive access to a platform. The relevant observable quantities include not only transaction frequency but the location of claims, the distribution of income and

costs, and the rights through which participation is organized. Such an account preserves the institutional insights of monetary and sociological theories that treat money as a socially constituted medium rather than a simple physical object [25, 15].

## 5.2 AI as a General-Purpose Productive Other

The phrase *general-purpose productive other* provides a provisional description of an artificial system that can furnish a broad variety of services and participate in the coordination of further production. It concerns the diversity and extensibility of productive capabilities rather than the possession of human subjectivity or unrestricted self-sufficiency. An AI system may carry out informational tasks, generate designs, control robotic equipment, schedule resources, and assist other systems. These capacities can change the number and character of contributions for which people seek a particular human supplier. Earlier exploratory thought experiments on AI-operated production networks motivate this framing while leaving the direction and magnitude of economic transformation open [12, 9].

Three analytically different aspects of artificial productive capacity deserve separation. First, *functional reach* concerns the range of activities to which a system can contribute under specified reliability requirements. Second, *operational autonomy* concerns its ability to select and execute actions within authorized and materially feasible limits. Third, *reproductive autonomy* concerns the capacity of the wider production arrangement to secure, maintain, and renew the energy, data, computing, materials, equipment, and labour required for continued operation. High functional reach can coexist with low reproductive autonomy. A software model capable of many tasks can remain dependent upon externally owned servers, specialized semiconductor fabrication, human quality assurance, and public electrical infrastructure. The present analysis treats these dimensions as partly independent and empirically variable.

The notion of an AI *other* also requires interpretive care. In economic analysis, another productive capacity can be treated as an interaction partner for purposes of mapping requests, deliveries, and dependencies. Such a representation does not establish that the system has legal personhood, moral standing, independent preferences, or a right to own property. An AI may be represented as a vertex in a production graph because it acts as an operational mediator, while obligations continue to be assigned to its developer, deployer, client, owner, or other legally recognized party. The allocation of responsibility can itself change through legislation, contract, governance, or technical delegation. A model of economic exchange must therefore specify which kind of agency is being assumed at each step.

The distinction further prevents a slide from *AI as a functional other* to *AI as a creditor*. A system may request a service, recognize a pattern of reciprocal provision, or negotiate within delegated parameters without being the holder of the resulting monetary claim. Where the paper speaks of an AI–AI monetary relation, that expression is shorthand for a monetary arrangement attached to AI-operated productive interactions; the legally recognized positions and the authority to create, transfer, or extinguish claims must be specified independently.

The generality of AI production also has limits set by heterogeneity. Different models and robotic systems can be specialized by data access, training history, experimental instruments, physical location, energy supply, hardware, latency, security clearance, and the institutional permission to intervene in a particular setting. Information produced by one system can be of little use to another without shared standards or compatible epistemic procedures. Some valued relations depend on unique persons, places, ecological structures, or histories that functional substitution cannot simply reproduce. A widening productive repertoire is consequently compatible with the generation of new dependencies upon humans, other AI systems, and material infrastructures.

These observations support a distinction between *reducing the demand for paid human production* and *eliminating the means by which AI production is financed*. An artificial system can substitute for a chargeable human service while itself relying on energy bought from a utility, licences supplied by institutions, and equipment fabricated in human-governed networks. In one arrangement, a human user pays a subscription; in another, an employer finances the same service; in a third, public appropriations or cooperative contributions make access free at the interface. The technological capability of the AI

is held fixed across these cases, yet the monetary relations differ. Productive capacity provides feasible alternatives; institutions select, constrain, and reproduce particular forms of mediation.

### 5.3 Human–AI Production Relations

Human–AI relations include activities in which a human requests an AI output, contributes to its operation, supplies material inputs, exercises oversight, or receives the consequences of its decisions. The familiar interaction of a user paying for a digital service captures only one part of this structure. The reverse direction of provision is equally important: a human or human-governed organization may supply training data, electrical power, property access, equipment, maintenance, evaluation, or a legal entitlement upon which the AI-operated service depends. As a result, H–AI and AI–H can represent different directions of productive provision even where the same human and artificial components participate in both. They should be distinguished analytically without presupposing symmetric bargaining capacity or independent legal standing for AI.

Consider a hypothetical public information service operated with AI. Members of the public receive research assistance without paying per request, while the operating institution purchases electricity and computing hardware, compensates maintenance personnel, and obtains the rights required to use relevant data. The service interface has undergone local demonetization, but production remains embedded in monetary supply relations. A distinct hypothetical arrangement employs a network of publicly maintained energy and data infrastructures with resources allocated through administrative or commons-based rules. Here several upstream monetary claims might be reduced or replaced, although the production network continues to incur material costs and to face distributional choices. These examples isolate the difference between user-facing free access and a more extensive change in the financing and reproduction of the service.

The potential decline of monetary dependence across the human–AI boundary is therefore related to the AI network’s capacity to obtain and reproduce productive inputs under an admissible institutional arrangement. Autonomous electricity generation is one relevant development; energy storage, grid access, system reliability, cooling, and the material requirements of generating equipment remain separate conditions. Data acquisition through autonomous observation can enlarge epistemic independence; the validity of resulting knowledge still depends on measurement, calibration, uncertainty estimation, and access permissions. Robotic maintenance can reduce dependence on purchased human labour; replacement components, rare materials, and the governance of a maintenance site may continue to generate external claims. Each apparent transition toward self-provision has a physical and institutional boundary that requires explicit identification.

Even if an AI-operated production network becomes capable of meeting a wide range of its own input requirements, the monetary relation with humans can persist by institutional choice or asymmetric control. An operator may retain exclusive rights over a productive asset, a state may levy charges for resource use, or an intermediary may sell access to otherwise abundant digital services. Equally, an institution can offer non-priced access while the underlying production still requires paid inputs. Productive autonomy, payment policy, and property rights therefore enter the analysis as distinct variables. Their joint consequences cannot be inferred from technical self-sufficiency alone.

The direction of human provision also creates ethical questions about hidden contributions. A system represented as an autonomous AI supplier may depend upon human annotation, care work, moderation, data collection, maintenance, or communities whose information has become an input. Some of these activities are remunerated through monetary arrangements; others may be unpaid, collectively supplied, or appropriated under contested rights. A description of AI self-sufficiency that omits these contributions can obscure the location of generated value and the distribution of its benefits. The generative perspective, partly inspired by work on generative justice, invites inquiry into the recognition and circulation of value and into the conditions under which participants retain influence over their productive contributions [3]. This concern remains analytically separate from a claim that every contribution requires individual monetary compensation.

Finally, H–AI relations can generate new non-monetary forms of collaboration. Human communities may share observations, develop knowledge, supervise ecological monitoring, and jointly set the purposes of an AI service; AI systems may contribute analytical, organizational, and physical capabilities. The disappearance of a user fee can permit expanded participation, but meaningful participation also depends on access, interpretability, accountability, and contestability. An AI-operated institution can reproduce hierarchy through admission rules and technological dependence even when its user interface is free. A theory of monetary dissolution must consequently retain attention to the institutional constitution of relationships and the conditions under which diverse persons can participate.

## 5.4 AI–AI Production Relations

AI–AI production relations arise when distinct artificial components depend upon one another’s outputs, information, computing resources, physical actions, or capacities for coordination. One system may request a simulation from another, draw on another’s sensor stream, reserve energy for a robotic operation, or commission fabrication through an automated production facility. This network may become economically significant as the range of delegated tasks increases. Yet the term AI–AI is compatible with substantially different property and governance structures: two programs may belong to one organization; they may be operated by separate firms; they may be components of a public institution; or they may participate in a cooperative infrastructure. The operational graph can appear similar across these settings while monetary claims differ.

Within a single organization, automated agents may be assigned computational budgets, internal transfer prices, or token balances to coordinate competing resource demands. These records have economic functions because they provide information and impose constraints. Whether they constitute *money* in a theoretically relevant sense depends on their institutional properties: the existence of transferable claims, acceptance across parties, discharge of obligations, units of account, and relation to enforceable settlement. An internal quota for processor time can express a resource-allocation rule without constituting an independent monetary claim. A ledger denominated in national currency can similarly record imputed costs between organizational departments without creating a debt owed by one distinct legal party to another. The concept of money should not be inferred from the presence of an algorithmic balance alone [5, 15].

Between organizations, artificial systems can negotiate and execute purchases under delegated authority. Suppose one firm operates a model-serving system, another controls a regional robot fleet, and a third owns a specialized sensor array. Their software agents can contract for services and initiate payments, while each organization remains accountable for the recognized rights and obligations. The AI–AI interface can be operationally automated, even as contractual, financial, and physical dependencies reproduce conventional monetary institutions. Automation may increase transaction frequency, reduce search costs, and facilitate fine-grained accounting; these changes could expand rather than diminish the number of priced exchanges. A reduction in transaction costs alone gives no determinate prediction for the level of monetary mediation.

A cooperative or publicly coordinated network offers another possible arrangement. AI systems may share complementary capabilities under common resource rules, reciprocal access agreements, or allocations established by participating communities. Such an arrangement can reduce the need for bilateral payments while retaining monitoring, scheduling, rationing, and conflict resolution. Scarce computational capacity, geographically limited energy, and specialized equipment still require allocation. If access is governed through priority queues or participatory decisions, the allocation can be non-monetary even when scarcity persists. Its stability depends on legitimate rule formation and reliable implementation, topics familiar from the institutional analysis of collective resources [23].

The possibility of monetary or credit arrangements among AI systems deserves particular attention because specialization can deepen as a production network expands. An AI that can perform many informational tasks may still rely on a remote instrument, a particular data source, or a maintenance robot. These heterogeneous dependencies can support barter-like exchanges, unilateral provision, accounting

arrangements, or reciprocal commitments spanning multiple exchanges. Where delivery and return contributions occur at different events, credit or record-based coordination may become useful. The proposition that money can function as a substitute for detailed social memory offers a relevant formal precedent, although its specific model assumptions do not determine the institutional future of AI production [20]. Greater computational capacity may make richer records and multilateral coordination feasible without ensuring universal agreement on values or duties.

The historical formation of a monetary relation therefore remains possible inside a technologically self-provisioning network. Suppose AI-operated facilities can produce their own electricity and repair their own equipment as a system, yet individual facilities retain separate resource entitlements and are subject to distinct access rules. Resource scarcity, unequal capabilities, and the timing of required inputs can motivate exchange and credit among them. Self-provision by the collective does not imply self-sufficiency by each constituent part, and collective adequacy of resources does not ensure that those resources are distributed under mutually acceptable rules. Conversely, an organization can coordinate complementary AI facilities through non-monetary planning even with extensive material scarcity. The location of monetary intermediation depends on the conjunction of productive and institutional relations.

Cyclic obligations between AI-operated subsystems provide a potential route toward reduced gross payment requirements. If one participant owes an admissible claim to a second, the second owes a compatible claim to a third, and the third owes a compatible claim to the first, the relevant institutions may permit multilateral clearing. The existence of a directed cycle alone establishes a graph structure, not the legal or distributive validity of cancellation. Differences in denomination, maturity, quality, risk, consent, or responsibility can prevent the apparent cycle from being cleared. Earlier exploratory work considers value-circuit grammars and distributed monetary dissolution in this setting [14, 7, 13]. The formal representation of compatible claims, residual obligations, and the robustness of longer circuits is reserved for the later mathematical sections on spacetime-distributed dissolution, multilateral clearing, and path fragility.

## 5.5 Ownership, Control, and Productive Dependence

The terms *ownership*, *control*, and *dependence* refer to overlapping but separable features of production. Ownership concerns recognized entitlements over assets, revenues, data, or instruments under a given legal and institutional order. Control concerns the effective ability to direct access, operation, exclusion, and the use of generated outputs. Dependence concerns the conditions required for an activity or organization to continue producing. A firm may own a data centre while depending upon a publicly operated electrical grid; a community may govern a knowledge resource while contracting with a private hosting provider; and an AI controller may direct a robot within rules set by an owner who retains the authority to terminate operations. A monetary claim can attach to any of these interfaces, and its basis may change independently of the technical capacities of the AI component.

Table 3 provides a qualitative diagnostic for identifying where monetary mediation might persist beneath an automated interface. The entries describe illustrative sites of dependence, and the arrangements are not exclusive categories. The intended use is to identify an obligation's actual counterparties, supporting production conditions, and governance venue before attributing a payment to H–H, H–AI, or AI–AI interaction.

Productive autonomy is consequently best understood as relative to a specified boundary and a set of required inputs. A robot fleet can be autonomous in local navigation while dependent upon external charging stations. A data infrastructure can be distributed across numerous sites while relying on one software provider's identity-management service. An organization can own computing hardware while lacking the capacity to fabricate replacement components. Assertions about an autonomous AI economy acquire analytical meaning only after defining the production boundary, its indispensable inputs, the horizon of maintenance and renewal under consideration, and the rules under which resources can be obtained. A short demonstration of self-operation offers limited evidence about the reproduction of a whole production system.

Table 3: Illustrative layers of productive dependence and possible monetary mediation in AI-operated networks.

| Layer                    | Possible productive dependence                              | Institutional site of monetary mediation                               |
|--------------------------|-------------------------------------------------------------|------------------------------------------------------------------------|
| Human contribution       | Evaluation, care, repair, situated expertise, participation | Wage, professional fee, service contract, cooperative contribution     |
| Energy and land          | Power supply, grid access, siting, cooling, resource rights | Tariff, lease, access charge, public allocation                        |
| Data and knowledge       | Observation, licences, provenance, access, validation       | Subscription, licensing, access rule, public data provision            |
| Compute and equipment    | Hardware, cloud service, fabrication, maintenance           | Procurement, cloud charges, internal budgets, common facilities        |
| AI coordination          | Services among models, agents, robots, and facilities       | Contractual credit, bilateral payment, clearing, non-monetary protocol |
| Institutional governance | Authority, admission, accountability, dispute resolution    | Membership fee, taxes, common dues, reciprocal obligations             |

The control of productive assets can also constitute monetary dependencies where resources are technically abundant. If access to a model or data collection is restricted through enforceable exclusive rights, those seeking its use may face fees or contractual obligations independent of marginal replication cost. In other settings, collectively maintained access rights can make a resource available without a transaction-specific charge. Resource abundance, legal entitlement, and distributive permission must therefore be separated. This distinction is central to analyses of capital accumulation and commodification and to the interpretation of collectively governed resources [21, 24, 23].

An AI system's increasing capacity to acquire data, generate electricity, or maintain equipment may reduce the number of external supplies required for a particular process. Such technical developments can relax dependence on human-governed exchanges when associated governance arrangements also permit self-provision. They can additionally shift human work toward monitoring, environmental management, oversight, or specialized fabrication. A local monetary reduction can thus coincide with an expansion of obligations elsewhere. Moreover, ecological impacts need not appear as monetary claims in the same ledger as the operational exchange. Land disturbance, emissions, water withdrawals, waste, and effects on nonhuman organisms can remain causally connected to production even when the relevant participants report balanced accounts. This asymmetry between settlement and full relational consequences is among the reasons that environmental obligations require separate recognition in the later ethical and infrastructural analysis.

## 5.6 Capitalist, Public, Cooperative, and Commons-Based Arrangements

The consequences of AI production for money depend partly on the institutional configuration through which capacities are developed and governed. Capitalist enterprise, publicly provided services, cooperatives, and commons-based arrangements offer useful comparative cases, although actual systems frequently combine elements of several. These categories are used to analyze claims, access, decision authority, and productive reproduction under different arrangements. The comparison does not require an assumption that technological development leads toward a single institutional endpoint. A shared AI capability can serve distinct purposes and sustain distinct monetary dependencies depending on the rules under which it is deployed.

Under a capitalist arrangement, AI facilities can be held as revenue-generating assets whose outputs are sold or licensed. The possibility of producing services with reduced labour input can lower some operating costs while facilitating new exclusive rights, differentiated pricing, and subscriptions. Money can be required for access even when delivery becomes inexpensive, and payment obligations can become concentrated in upstream services such as energy, semiconductors, data, and finance. Firms can

also integrate formerly purchased services into their own production networks and reduce some inter-firm monetary exchanges, although internal planning and accounting remain. Capital accumulation, enclosure, and differentiated control of productive assets consequently affect the movement of monetary claims alongside the development of AI capabilities [21, 24].

Public provision can relocate the settlement of production costs away from the immediate user. A government or public institution may maintain common AI services, scientific data, environmental observations, or computing capacity as shared infrastructure. Individuals receiving a service may face no per-use monetary charge, while collective financing is organized through budgets, taxation, public borrowing, or cross-subsidy. The resulting local demediation can expand access and lower barriers to participation. At the same time, the institution must decide how to prioritize demand, protect rights, evaluate competing uses, and allocate limited capacity. Monetary coordination therefore changes location and form; the financing of public infrastructure remains a separate object of analysis.

In a cooperative arrangement, contributing participants can hold recognized rights to govern resources and share their benefits. Contributions may consist of labour, equipment, data, knowledge, maintenance, or financial support, and access can be conditioned on membership, rules of reciprocity, or needs-based criteria. Some cooperatives sell outputs on external markets while allocating internal resources through deliberation or negotiated obligations. Others require member dues while providing services without transaction-specific prices. An institution of this kind illustrates how an H–H or H–AI exchange can be coordinated through shared governance without implying the disappearance of the productive efforts or claims that sustain it. Its feasibility depends on rule enforcement, participant capacity, transparency, and arrangements for disagreement and remedy [23].

Commons-based arrangements emphasize collectively recognized practices for sustaining and accessing resources such as knowledge, code, environmental observations, or shared instruments. The label *commons* denotes a set of governing relations rather than merely a resource offered free of charge. Commons participants may preserve openness while employing wages, grants, licensing constraints, or payments for material inputs. It would therefore be inaccurate to identify a commons economy with an absence of money across every layer. Its distinctive analytical contribution lies in making the relationships between value generation, access, governance, and the reproduction of common resources explicit. Generative approaches to economics provide one intellectual motivation for investigating those relationships and the possibility of more directly circulating benefits among contributors [23].

Hybrid institutions create especially instructive cases. A publicly funded ocean observing network might share environmental data openly, contract with private firms for sensors, rely on universities for calibration, and enable a community cooperative to use AI for ecological analysis. Data circulation among researchers may occur without prices; sensors, shipping, energy, and insurance may remain monetized; and access to particular analytical capabilities may be shaped by institutional agreements. A subsequent shift toward autonomous robotic maintenance could alter some procurement relationships without eliminating the shared dependence on ocean ecology, safety obligations, or public legitimacy. This hypothetical case links economic analysis to the infrastructural questions pursued later in the research programme while preserving the independent scientific value of ocean observation.

Comparison across these institutional forms suggests a methodological requirement for subsequent analysis. An apparent reduction in payments should be traced across at least three boundaries: the interface where a request is fulfilled, the production network that reproduces the response, and the institutions that allocate and legitimate resources. Monetary reliance may decline at one boundary, rise at another, and change in character at a third. The same technology can support priced exclusion, universal public access, collectively governed reciprocity, or combinations thereof. The distinctive claim of the present inquiry is therefore a research hypothesis about the *distribution and transformation* of monetary mediation under evolving production relations. It is not a prediction that any single ownership regime or technical capability will necessarily abolish money.

The analysis in this section establishes a comparative basis for the next step of the paper. H–H, H–AI, and AI–AI relations distinguish interaction sites; ownership and productive dependencies identify the deeper conditions of their operation; and institutional arrangements explain how those conditions

Table 4: Illustrative transformations of monetary mediation across interaction sites. The entries are possible configurations, not assigned stages or forecasts.

| Relation | Potential reason for reduced direct monetary mediation                                                                  | Possible persistence or relocation of monetary claims                                                  |
|----------|-------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------|
| H–H      | AI-assisted access, public or commons provision, cheaper reproduction of some goods and services                        | Unique participation, membership, property and labour relations; upstream AI services                  |
| H–AI     | Accessible AI services; collective provision; AI-network reproduction of energy, information, equipment and maintenance | Compute and energy financing; ownership of physical assets; licences, taxes and access permissions     |
| AI–AI    | Multilateral coordination, shared resource rules, compatible cyclic clearing and reliable information                   | Scarce inputs, inter-operator finance, settlement between owners, contingent commitments and exclusion |

acquire or lose monetary forms. The following section can therefore investigate the possible ordering and coexistence of monetary decline, concentration, and expansion without assuming that the varied relations examined here unfold along a universal trajectory. The more exact specification of admissible circulation, inter-event obligations, and network clearing remains a separate task for the later formal sections.

## 6 Transformations of Monetary Dependence

This section examines how monetary dependence may change when artificial systems extend their productive capacities, assume additional operational roles, and participate in the provision and reproduction of the infrastructure upon which production relies. Its purpose is to establish a comparative account of heterogeneous trajectories prior to the more specific treatment of spacetime-distributed dissolution in Section 7. The argument neither predicts a terminal monetary state nor assumes that a technological advance has the same institutional consequences across societies. Instead, it asks which monetary functions may become less necessary at which relational boundaries, which functions may migrate to other boundaries, and which new dependencies can produce monetary relations even where established forms of payment recede. The analysis is exploratory and conditional: the scenarios describe feasible mechanisms subject to productive, legal, political, and ecological qualifications, not empirically demonstrated forecasts.

A change in monetary *dependence* must first be distinguished from a change in measured *money spending*. Spending can decline because of rationing, an interruption of supply, falling prices, public subsidy, substitution by nonmonetary provision, or the cancellation of compatible claims. These explanations have different implications. A free service supported by a tax-funded electrical grid involves one kind of monetary displacement; a firm that uses its own currency-denominated treasury to pay for AI-operated production involves another; and a reciprocal production circuit clearing legally compatible liabilities involves a third. Calling all three “money disappearing” would obscure rather than explain the relevant production relations. The more exact question concerns the reliance of a specified set of relations upon monetary pricing, payment, financing, recognized claims, and enforceable settlement.

To maintain the distinction formally, let  $\mathcal{R}$  denote a specified collection of situated production and exchange relations, and let  $\mathcal{F} = \{\text{price, payment, credit, settlement, finance}\}$  index analytically separable monetary functions. An exploratory *monetary-reliance profile* at an admissible observation or description state  $h$  is

$$\mathbf{m}(\mathcal{R}; h) = (m_f(\mathcal{R}; h))_{f \in \mathcal{F}}, \quad (2)$$

where each  $m_f$  is a declared operational measure of the degree to which activity in  $\mathcal{R}$  depends on function  $f$ . The quantities may be categorical or numerical; they are not presumed universally comparable, additive, or definable from transaction counts alone. A valid empirical study would need to specify the

counterfactual interruption or unavailability that makes a given relation *dependent* on a function. For example, whether production could continue without access to currency-denominated settlement is not established merely by the observation that no consumer-facing price is displayed.

The parameter  $h$  is deliberately not a universal clock coordinate. It may designate a documented historical configuration, an observation record, or a partial continuation of a relational history. Comparison between  $h$  and  $h'$  is meaningful only when the representational relation that connects them is specified. A component of  $\mathbf{m}$  may decrease while another increases, and a decline on one subset  $\mathcal{R}_1$  may be supported by stronger monetary dependencies in another subset  $\mathcal{R}_2$ . Equation (2) is therefore a diagnostic vocabulary for uneven change rather than a scalar index of proximity to an alleged inevitable end of money. This emphasis extends the differentiated, exploratory pathways proposed in earlier analyses of AI-operated exchange and distributed monetary dissolution [12, 14].

## 6.1 Changes in Human–Human Monetary Exchange

AI systems can alter human–human monetary exchange in several distinct ways. They may supply functional substitutes for previously purchased services, reduce the effort required to coordinate cooperation, improve the accessibility of information, or support publicly available productive instruments. An individual who previously purchased routine translation, transcription, or administrative assistance from another person may instead use an automated system. In that case a particular paid H–H relation diminishes. The change may be substantial for the participants even when the overall production system remains monetized through the acquisition of devices, electricity, computing services, and legal licences. A reduction in an observable H–H payment therefore establishes a change in the location or incidence of monetary mediation, not a completed dissolution of the monetary conditions of production.

The needs that produced the original exchange can also evolve. Some tasks become inexpensive and standardized while distinctively human forms of encounter, accountability, interpretation, or shared activity acquire increased significance. The capacity to generate a technically adequate document does not substitute for all reasons a person might seek another person’s judgment, companionship, institutional recognition, or productive collaboration. A professional association might replace its paid administrative services with AI assistance while continuing to charge for membership, access to scarce facilities, or participation in a particular community. Conversely, an association could abolish those charges and finance its activities collectively, without removing the relational value of membership. Thus the demand for a particular other’s contribution and the monetary institution controlling access to that contribution are analytically separable.

The relation between declining unit costs and declining monetary dependence is similarly nontrivial. A price may approach zero because of technological competition while a producer continues to obtain revenue through advertising, data licensing, subscription bundles, or upstream payments. A service that costs nothing to the immediate user may be cross-subsidized by paid transactions among other users or organizations. Moreover, automation can create newly marketable services: specialized verification, novel cultural experiences, high-trust certification, control over exclusive training material, or access to scarce physical spaces. Such activities may generate additional H–H claims even as routine tasks are displaced. Neither the gross count of priced interactions nor the total monetary expenditure in a sector is guaranteed to move in a single direction.

A further distinction concerns the scale of the relation under study. At the level of a household, automated provision may enable less reliance on wage-financed services. At the level of a city, automated maintenance of common facilities may be funded by public revenue; H–H purchases decrease at points of use while taxation, procurement and public debt remain. At the level of a global production network, the manufacture of the underlying computing hardware may depend on paid labour and imported components. A report of local H–H demonetization should therefore specify the relevant boundary and whether the costs of maintaining productive conditions have been incorporated within it. Such boundary sensitivity follows from the institutional analysis of money and commons governance developed earlier [15, 23].

Finally, increased technical capability does not establish that substitution should take place wherever it is feasible. Human participants may reasonably value the ability to provide meaningful labour, maintain social relations, exercise choice, and contest organizational decisions. An arrangement that eliminates a monetary fee while excluding affected participants from a productive relationship can change the distribution of benefits and powers without offering a corresponding ethical improvement. The analysis accordingly treats reduced monetary mediation as an economic possibility to be evaluated alongside recognition, agency, access, and the conditions under which the relevant values are generated. The generative perspective on value in the foundational discussion remains pertinent, although it does not decide the appropriate institutional response in advance.

## 6.2 AI Productive Autonomy and Human–AI Monetary Dependence

Human–AI monetary dependence can decline when AI-mediated services become widely available at low or no per-use charge. A more demanding hypothesis concerns the conditions under which the providers of these services can reproduce their operations without depending on currency-denominated revenues obtained from human users or organizations. The distinction matters because an AI system may be operationally competent while depending upon a complex upstream chain of paid energy, semiconductor fabrication, physical maintenance, network connectivity, data rights, and human labour. Reduced need to purchase *particular human services* is different from reduced dependence upon *human-controlled productive conditions*. The latter involves the organization and reproduction of material and institutional capacities as well as the performance of tasks.

Consider a hypothetical AI production arrangement comprising a service system, an electrical generating facility, a data and observation subsystem, computing equipment, and robotic maintenance. Suppose these components can coordinate to provide a human-facing service. Even if the marginal energy expenditure of an individual query becomes very small, maintaining the electrical grid, repairing robotic actuators, fabricating replacement processors, securing mineral inputs, and obtaining lawful operating permissions remain economically relevant. Data autonomy likewise involves more than unlimited synthesis of synthetic examples: it requires the possibility of observing and testing claims against relevant environments, documenting provenance, correcting errors, and acquiring permissible access to observations made by other systems. AI-controlled instruments may reduce dependence on human-performed observation while continuing to depend on human institutions for standards, permitted sites, or access to shared scientific facilities.

It is useful to distinguish *self-provision*, *network provision*, and *institutional availability*. Self-provision refers to the capacity of an operational unit to obtain and maintain a required input within its own productive arrangement. Network provision refers to the capacity to obtain that input through ongoing relations among distinct systems, which can include reciprocal nonmonetary exchange or monetized procurement. Institutional availability refers to recognized access rights under which inputs can be supplied or used without a payment at the relevant interface. A public electrical network might provide power under a collectively financed entitlement, and an autonomous energy facility might remain privately owned and charge its AI users. Neither technological integration nor legal ownership, taken alone, specifies the resulting monetary dependency.

For a required input  $k$  and a chosen boundary  $B$  of the AI production arrangement, write  $d_k(B; h)$  for an explicitly operationalized measure of dependence on inputs or permissions originating beyond  $B$ . The descriptive vector

$$\mathbf{d}_A(B; h) = (d_{\text{energy}}, d_{\text{data}}, d_{\text{compute}}, d_{\text{materials}}, d_{\text{maintenance}}, d_{\text{permission}})(B; h) \quad (3)$$

records several potentially unequal forms of dependence. The entries may quantify physical flows, interruption sensitivities, institutional approvals, or other well-defined measurements; mixing units into a single aggregate requires an additional and justified measurement convention. Some external dependencies are nonmonetary, and some monetary liabilities may remain even when the underlying resource is produced locally. Equation (3) therefore does not imply a simple numerical conversion from material autonomy to monetary dissolution.

Under specified conditions, however, expansion of AI-network productive autonomy supplies a mechanism for reduced H–AI monetary mediation. Assume that human-facing services are maintained; the relevant AI-operated network has secure access to necessary inputs and renewal capacities; participants adopt rules allowing service provision without per-use monetary claims; and the continued activity does not require financing the service through charges on its human users. Under those assumptions, the user-facing payment requirement can be removed while production continues. This is a *conditional feasibility statement*, not an empirical prediction that AI autonomy will create free access. If a controlling organization chooses an exclusionary price, if the infrastructure depends on debt service, or if ecological constraints require costly interventions, a monetary relation may persist even when substantial technical autonomy has been achieved.

The reverse case is equally instructive. A service can be free at the human interface long before its AI production network is materially self-reproducing. Public procurement, cross-subsidy, philanthropy, advertising revenue, or commons-based contributions can finance access while equipment and energy continue to be obtained through ordinary markets. The interface therefore may be demonetized either through a change in productive dependence, a change in financing and access rules, or a combination of both. The distinction is critical to determining whether a decrease in measured H–AI payments denotes a change in the material conditions of production or a redistribution of who pays for them. A clearer analysis should trace the entire relevant flow of costs and claims rather than equate zero user price with autonomy.

The possible direction of human–AI dependence is also heterogeneous. A human may depend on an AI system for assistance while the operator of that system depends on human scientific experiments, institutional legitimacy, labour, legal review, social knowledge, or ecological stewardship. The human–AI interface contains different directed relations: provision of service, provision of data, access to infrastructure, assignment of responsibility, and payment obligations may travel across it in different directions. The shorthand H–AI is therefore a relational classification, not an assertion of reciprocal symmetry. An AI-operated system may provide substantial services without demanding money from a user and nevertheless require participation from humans for purposes that cannot adequately be represented as financial compensation alone.

### 6.3 Internal Exchange and Credit among AI Systems

A growing AI-operated production network can expand the number of operational interactions among specialized systems. A model that designs a component may request tests from a simulation system; an autonomous laboratory may seek manufacturing capacity from a robotic workshop; energy controllers may allocate electricity among data centres; and maintenance robots may require replacement parts from another production facility. Such exchanges can involve heterogeneous capabilities, geographically distinct infrastructure, and independent owners or governing organizations. Greater AI capability can therefore increase *interdependence among AI-operated productive units* even while reducing their dependence on particular human-performed tasks. Under market-based arrangements, this growth may be accompanied by increased internal monetary mediation.

The institutional status of these AI–AI exchanges cannot be inferred from the computational form of the interface. Messages exchanged by two programs may authorize transfers between corporate accounts, allocate the internal budget of one organization, record reciprocal entitlements in a cooperative, or simply schedule use of a shared machine. The first case involves externally enforceable monetary claims; the second may involve administrative units denominated in money; the third can use credit without a circulating currency; and the last may use no monetary claim at all. Calling each transaction “AI money” would obscure important distinctions among authorization, accounting, actual payment, and liability. The analytical unit is the relation among the resources, responsible institutions, and claims, not merely the software agents that initiate commands.

Complex multi-agent production may also increase the demand for credit and deferred settlement. An autonomous component fabricator may deliver inputs to a robotic assembly facility while awaiting

energy allocation or parts from a third system. The resulting chain includes timing, contingencies, and the possibility of nonperformance. Where the institutions involved are distinct and agree to transferable monetary obligations, credit may support continued production despite asynchronous provision. Where they rely on shared governance or enforceable reciprocal commitments of another kind, similar operational coordination may proceed through different instruments. The choice depends on acceptable risk, verification capacity, property and access rights, and agreed means of redress. The earlier literature on the relations among money, memory, and institutional credit offers partial conceptual analogies, while not proving that perfect information or advanced AI removes monetary requirements in general [20, 5].

A simplified example illustrates the difference. Let three independently operated systems provide energy, materials, and computational design. The energy operator supplies power to the materials operator; the materials operator supplies parts to the design operator; and the design operator provides control software to the energy operator. If their obligations are specified in a common settlement unit, transferable under applicable rules, and mutually accepted for multilateral offset, some gross liabilities may cancel without a corresponding sequence of monetary payments. If the parts are delivered late, the software is incompatible, the claims are denominated in nonconvertible units, or one operator lacks authority to set off the claims, the conceptual loop need not form an admissible settlement circuit. Moreover, satisfaction with the overall reciprocity of value is an evaluative and institutional matter, not a property implied by the existence of a triangular graph. The precise clearing conditions are reserved for Section 7 and the subsequent formal analysis.

AI systems may improve the feasibility of such coordination by discovering multi-party relationships that human participants would find difficult to inspect, evaluating compatible obligations, identifying alternatives under infrastructure constraints, and continuously checking changes in contractual or resource states. Yet higher cognitive capacity need not resolve strategic conflict: parties may disagree about the recognition of a claim, exercise a lawful right to decline substitution, or hold incompatible assessments of the value delivered. Better knowledge can expose contested obligations as well as clear uncontroversial ones. Even an AI-operated network with sophisticated prediction can confront stochastic failures, unrepresented ecological costs, and governance disputes. Hence the proposition that AI–AI money will eventually disappear is best replaced with a question about the conditions under which internal monetary functions can be replaced by admissible, robust, and legitimate alternative arrangements.

An important possible trajectory is nonmonotonic. The proliferation of specialized AI services can initially enlarge the scale of priced interactions and credit commitments among their operators. Subsequent technical or institutional integration may reduce certain costs of verification, payment, and clearing, making more extensive multilateral coordination feasible. Conversely, growing specialization, tighter resource constraints, new proprietary platforms, or heightened uncertainty can create fresh monetary claims. AI–AI exchange can expand and contract in different parts of a network at the same described historical configuration. A theory adequate to this possibility must compare relationally located monetary functions rather than imagine that increasing intelligence directly produces a scalar decline in money.

## 6.4 Relational Ordering of Monetary Dissolution

The preceding comparisons suggest a conditional ordering hypothesis: some forms of H–H monetary mediation may decline as AI provides alternatives to priced interpersonal services; certain H–AI payment requirements may decline as AI services become institutionally accessible and their production conditions are more autonomously reproduced; AI–AI monetary mediation may persist longer where specialized production networks continue to rely on differentiated operators, scarce resources, and credit. This hypothesis captures an intelligible structural possibility, especially where services are offered to humans through highly integrated AI platforms supported by a complex internal supply network. It should not be read as a universal chronology or a forecast with fixed dates. The ordering may be absent, reversed, or only locally observable under alternative institutions and resource conditions.

More precisely, a statement that one form of dissolution *depends on* another arrangement does not necessarily mean that the other arrangement must first experience monetary dissolution. A human-facing

AI service can be free because an AI operator pays for energy and equipment through money: the reduced monetary dependence of the consumer-facing relation rests upon the continued monetary functioning of the production network. Similarly, a collectively supplied communication system can displace paid H–H exchange while its public fund still acquires computing equipment through priced markets. Thus the relevant ordering may concern the *transfer of production and settlement responsibilities* rather than the order in which all layers become nonmonetary. The relation is structural, not simply sequential.

To represent this distinction without assuming a privileged global time axis, let  $\mathcal{D}_i$  denote a proposition that a specified monetary function at relational boundary  $i$  has diminished under declared criteria, and let  $\mathcal{C}_j$  denote a material and institutional condition supporting that change. A conditional dependence relation may be written

$$\mathcal{C}_j \triangleright \mathcal{D}_i, \quad (4)$$

meaning, as formalized in Equation (4), that condition  $\mathcal{C}_j$  is invoked in an explanatory account of  $\mathcal{D}_i$  under a specified scenario. The notation does not imply temporal precedence, uniqueness, or sufficient causation in all possible contexts. It can be refined into a directed dependency graph or partial order when the empirical relation warrants such structure. A distinction between event order, causal dependence, accounting settlement order, and justificatory dependence is required before using one mathematical ordering to stand for another.

For illustration, suppose a hypothetical AI platform provides computational tutoring without charging students, purchases electricity from another operator, and finances its equipment with a long-term monetary obligation. Relative to the student–platform relation, direct monetary mediation is absent; relative to the platform–utility and platform–creditor relations, it remains. If the platform subsequently acquires an energy source and a means of equipment maintenance, dependence on two classes of outside suppliers may decline. Yet monetary financing might persist because of ownership and debt contracts. If the equipment and energy become collectively supplied, direct fees may remain absent while shared financing obligations are reorganized. The ordering here is a family of dependencies, not a ladder with three inevitable stages labelled H–H, H–AI, and AI–AI.

The same network can contain counterexamples to the proposed intuitive sequence. AI–AI transactions inside a single public research facility may have operated through shared allocation rules from the beginning, whereas nearby H–H exchanges still require wages or membership fees. Two autonomous systems with interoperable obligations may clear claims across a small cooperative even when the broader human-facing platform remains commercially priced. A city may retain paid interpersonal care because the relevant human relationships are scarce and valued, while some AI production relations are already organized as a commons. Different institutional boundaries and productive constraints can therefore alter which monetary relation is transformed first, which remains, and which is recreated.

What makes the ordering claim useful is not its predictive inevitability but its demand for explanation. To compare two possible monetary transformations, an investigator must determine (i) the relevant boundary and monetary function, (ii) the productive inputs enabling each relation, (iii) who controls and maintains those inputs, (iv) the mechanisms that finance their reproduction, (v) whether compatible alternative coordination is available, and (vi) which obligations remain after a proposed change. This comparison can expose a less obvious result: the most visible reductions in human payments may depend upon an increasingly monetized upstream AI infrastructure. The empirical challenge is to trace that dependence without treating the disappearance of a consumer invoice as evidence of economy-wide monetary dissolution.

An economic stress test of the proposed ordering must hold the unit of observation constant. Consider two otherwise comparable AI-mediated services. One is delivered by a vertically integrated public operator whose internal robotic scheduling uses nontransferable allocation credits; another is delivered by legally separate private infrastructure companies that issue enforceable invoices. In the first, AI–AI operations can lack monetary claims while human users continue to pay charges set by the public institution. In the second, the human interface can be free while AI-operated counterparties remain monetarily interdependent. Neither case violates technological plausibility, and the contrast shows why transaction labels such as “AI–AI” do not identify the operative institutions. The economic hypothesis is conditional

on productive boundaries, claim-bearing legal positions, and rules of access, not on the software identity of the interacting agents.

The direction of change also depends on the denominator. A fall in payments per successful service can coexist with higher total expenditure if service volume expands. Conversely, a decline in gross invoices following legally effective multilateral clearing can coexist with unchanged actual resource requirements and with new borrowing that finances an expanding network. A sound comparison should report monetary amounts per comparable service, aggregate obligations at a declared boundary, and the continuing capacity to reproduce physical inputs. These measures should be kept separate: normalizing one does not settle the direction of the others. A test of relational ordering requires observations of the same monetary function across comparable relations and explicitly linked historical configurations rather than an arbitrary ranking of service types by their apparent “free” access.

## 6.5 Monetary Migration across Production Layers

Monetary migration describes the relocation of pricing, payment, financing, or settlement requirements from one set of relations to another while the productive activity continues in a recognizably related form. A charge formerly paid by one human participant to another may become an AI subscription; an AI subscription may become a publicly financed entitlement; a publicly financed entitlement may be sustained through state procurement and debt service to technology vendors. Each movement changes the immediate bearer and locus of monetary claims. It may also redistribute access, risks, bargaining power, and the capacity to contest economic decisions. Migration is therefore not merely a matter of moving a numerical amount from one ledger column to another.

A productive network can be partitioned analytically into an interface layer, an operational-coordination layer, a material-infrastructure layer, and an institutional-finance layer. A human may interact with a service for no monetary consideration at the interface, while internal operators exchange cost-accounting messages, infrastructure providers receive contractual payments, and a financing institution retains claims against the owners of equipment. In another arrangement, public and commons governance may replace the monetary claims at several layers with collectively assigned entitlements and maintenance responsibilities. In each case, money’s functional location has to be reconstructed independently of what the end user sees. The same technology can support very different distributions of price, credit, ownership, and responsibility.

There is also a difference between *relabelled mediation* and a change in its underlying function. A platform might replace currency-denominated invoices with proprietary tokens that must be purchased for money or that represent enforceable rights to scarce production resources. The token interface can alter measurement and settlement procedures while preserving the relevant monetary dependence. Conversely, a scheduling credit assigned by a cooperative and nontransferable outside its resource-allocation rules need not perform the functions of money even if it appears in a ledger. A proposed disappearance of money must therefore inspect convertibility, transferability, liabilities, issuer obligations, and the consequences of withholding the token. Terminology alone cannot establish whether monetary mediation has dissolved.

The question of whose money disappears is particularly significant. Consumers may lose the need to hold currency for a service while the provider increasingly relies on large-scale finance; a government can remove service charges while assuming fiscal commitments; one group can obtain free access because costs are shifted to data contributors, workers, or users of another service. Such cases may alter the scale at which monetary claims operate without extinguishing the underlying obligation to reproduce productive conditions. The assessment should distinguish incidence of payment, legal attribution of liability, distribution of resource control, and the welfare or ethical significance of the arrangement. Even when the aggregate flow of currency declines, the concentration of financial authority can increase.

Migration may be associated with new kinds of exclusion. A general-purpose AI can reduce the labour needed for certain production activities while the infrastructural conditions of access become concentrated in a small number of high-capacity data centres, communications networks, or energy providers.

Fees previously charged for human labour can give way to payments for proprietary compute, privileged data, physical infrastructure, or platform membership. An account focused narrowly on technological substitution would register demonetization of one human task while overlooking the formation of new claims around the productive means that replace it. The relevant analysis intersects the earlier discussion of power, scarcity, and institutionally constituted access [21, 24].

To identify genuine dissolution in such a layered system, the investigator may trace a *reproduction boundary*: the set of material inputs, maintenance activities, permissions, and obligations required for the service to remain feasible under the chosen conditions. The boundary need not capture the entire global economy, but its omissions should be acknowledged. A local absence of charges is an important descriptive finding; it becomes a claim about broader monetary dependence only when the financing and settlement relations essential to that local provision are brought into view. Section 7 will extend this boundary-sensitive inquiry to relationally located events and possible partial clearing processes.

## 6.6 Concurrent Monetization and Demonetization

The preceding mechanisms allow both the creation and the dissolution of monetary relations within the same evolving network. Some tasks can be removed from markets through automation and public provision, while new monetary claims form around advanced hardware, scarce observation platforms, AI-generated capabilities, or rights of entry into a specialized relational system. A reciprocal production circuit may cancel a set of compatible liabilities while another circuit is interrupted and converts to currency settlement. New links may connect participants whose heterogeneous contributions were previously uncoordinated, and the institutions that recognize those links may choose monetary, reciprocal, or collective forms. Hence the existence of a locally nonmonetary regime is compatible with continuing monetary formation elsewhere.

The relevant comparison can be expressed without requiring a globally agreed measure of value. Let  $\mathcal{R}_+(h, h')$  contain relations whose specified monetary-reliance component increases under a well-defined comparison of historical configurations  $h, h'$ , and let  $\mathcal{R}_-(h, h')$  contain those for which it decreases. Both sets can be nonempty:

$$\mathcal{R}_+(h, h') \neq \emptyset, \quad \mathcal{R}_-(h, h') \neq \emptyset. \quad (5)$$

The condition in Equation (5) does not specify relative magnitude, social desirability, or whether the changes can be aggregated into a single economy-wide net effect. It states the possibility of heterogeneous directions of change under a consistent observation protocol. More refined models could track formation, cancellation, assignment and migration of particular liabilities on the edges of a typed relation network, distinguishing events that create an obligation from events that discharge it.

A small hypothetical circuit illustrates the coexistence of transformations. Consider a public AI research consortium whose computing services are free to participating laboratories. Within the consortium, robotic maintenance and computing allocation are coordinated through shared resource entitlements. However, the consortium purchases some replacement hardware from an external supplier and finances an energy expansion through a monetary contract. An independent AI-operated laboratory joins the network and offers a specialized instrument under a usage licence, introducing a new priced relation. At the same time, a three-operator cycle of mutually compatible invoices is identified and legally cleared. No single description such as “monetary disappearance” captures all these changes. The public interface remains nonmonetary, a new claim emerges, other claims are extinguished, and remaining reproduction costs continue to be financed.

The conditional possibility of such concurrent change also gives meaning to the assertion that money can be *spacetime-distributedly* dissolved. If obligations are formed at particular events, attached to situated institutions, reassigned along recognized relationships, and partly extinguished through coordination, there need be no single instant at which the network crosses from monetary to nonmonetary exchange. Equally, no assumption is required that every unresolved claim is destined to be cancelled. Some monetary relations may be durable, others transient; some may be replaced by access entitlements

or public finance, and others may be generated by new needs for persons, materials, or productive systems. The proposed language describes a relationally localized field of transformations whose mathematical structure must be specified rather than inferred from a metaphor.

An important boundary concerns the difference between financial closure and obligations to persons or environments outside the settlement set. The parties in a production circuit can accept a multilateral offset of their monetary claims while their operations impose uncompensated demands on water, energy systems, habitats, or communities not represented in the agreement. Such a circuit may be financially balanced without having discharged its ecological or distributive obligations. Accordingly, the feasibility of monetary clearing should not be confused with the legitimacy or sustainability of the underlying production. The infrastructural and governance conditions of wider accountability will be introduced in Section 11 and developed more fully in subsequent companion investigations.

Taken together, the changes surveyed here suggest that monetary dependence is better represented by a multidimensional and relationally indexed profile than by a universal succession of eras. Human-facing charges can decline before, during, or without the decline of payments among AI-operated suppliers. Reproductive autonomy can reduce external input dependence while privately governed access retains monetary claims. Technically rich AI networks may support both elaborate credit relations and increasingly sophisticated forms of nonmonetary coordination. These observations supply the motivation for the following formal question: what exactly is dissolved when a specific collection of obligations is cleared or replaced, and how should those events be represented in a spacetime-distributed structure? Section 7 turns to the event, dependency and boundary representations required to formulate that question precisely.

## 7 Spacetime-Distributed Monetary Dissolution

This section proposes a preliminary relational and mathematical representation of the *spacetime-distributed* character of monetary dissolution. Its immediate task is neither to establish that money will disappear nor to construct a complete clearing technology. Rather, it asks what must be represented if an analysis is to distinguish claims arising in different productive encounters, their subsequent recognition and transformation, and the partial cancellation or replacement of monetary relations without suppressing the heterogeneous relations that generated them. The production-network distinctions of Sections 3 and 6 supply the motivating cases. Earlier exploratory treatments of distributed monetary dissolution and heterogeneous value circulation supply additional formal questions, not empirical confirmation of the present representation [14, 11].

Two modeling commitments organize the discussion. First, economic histories need not be represented as snapshots under a universal clock. Obligations, encounters, resource transfers, institutional recognitions and clearings may occur in differently ordered and only partially comparable relational settings. Second, a representation of value creation must be distinguished from a representation of enforceable monetary obligations. A productive contribution may retain historical, practical and ethical significance even when its corresponding monetary claim has been lawfully discharged. Conversely, an obligation can remain enforceable after the original goods have been consumed or its productive history has become unobservable. The formal bridge between these descriptions must be declared, rather than assumed from the coincidence of a diagram or an accounting amount.

### 7.1 Relational Histories and Event Structures

A monetary obligation typically has more than one relevant occurrence. A person or organization may produce a good, another participant may receive or inspect it, an authorized institution may recognize a claim, the claim may become transferable or payable, and a later settlement procedure may discharge all or part of it. Depending on the arrangement, receipt may precede recognition; acceptance can be conditional; different parties may record divergent accounts; and cancellation may require consent from persons not involved in production. The analysis therefore begins with a set of represented *events* rather

Table 5: Analytical layers in a spacetime-distributed account. A map between layers requires an explicit interpretation and does not automatically preserve all properties.

| Layer              | Represented objects                                                          | Question preserved by the representation                                   |
|--------------------|------------------------------------------------------------------------------|----------------------------------------------------------------------------|
| Event history      | Encounters, productive acts, recognition, commitment, revision and discharge | Which relations and locally justified dependencies constitute a history?   |
| Value circulation  | Heterogeneous contributions, needs, uses, meanings and transfers             | What is generated, recognized or affected by a productive relation?        |
| Obligation network | Typed claims, debtors, creditors, eligibility and institutions               | Which specified claims can be priced, assigned, offset or enforced?        |
| Clearing procedure | Accepted cancellations, residual positions and settlement events             | Which compatible obligations are actually extinguished?                    |
| External boundary  | Unrepresented persons, infrastructures and ecological systems                | What consequences or dependencies remain outside the clearing arrangement? |

than treating a ledger balance as an exhaustive statement of economic relations. Events designate analytically situated occurrences, not primitive economic agents. Persons, enterprises, AI-operated services and governing institutions may participate in many events and need not correspond to single nodes of the event representation.

A history description can be represented by the tuple

$$\mathfrak{H} = (E_{\mathfrak{H}}, \preceq_{\mathfrak{H}}, \Lambda_{\mathfrak{H}}, \mathcal{I}_{\mathfrak{H}}), \quad (6)$$

In Equation (6),  $E_{\mathfrak{H}}$  is a set of documented or hypothesized events,  $\Lambda_{\mathfrak{H}}$  assigns types and relations, and  $\mathcal{I}_{\mathfrak{H}}$  provides interpretations of relevant roles, entitlements and evidence. The relation  $\preceq_{\mathfrak{H}}$  is a partial order *where such an order has been justified*. When a purported dependence is not transitive or anti-symmetric, it must be modeled separately among the labels instead of being forced into  $\preceq_{\mathfrak{H}}$ . Causal influence, chronological precedence, priority of claims, permission to act, and an observer’s access to records are not interchangeable orderings. The use of a partially ordered history does not imply that its events possess an intrinsic universal time coordinate or an ontologically privileged present.

The qualifier *spacetime* is consequently interpretive rather than a claim that a physically measurable four-dimensional manifold underlies every social or monetary relation. Spatial location, transaction jurisdiction, causal dependence, contractual maturity, archive order, and admissible continuation are different typed relations; a model may employ some without identifying them with one another. A formal representation is entitled to compare two histories only through an explicit correspondence or observation protocol. In particular, a branch permitted by such a protocol is not merely a later state awaiting realization, and a partial order on documented events does not determine which admissible transformation an institution will select.

For instance, a cooperative robot may complete a physical repair while an AI-operated data service independently verifies the necessary component specifications. A governing body may accept both contributions and subsequently authorize a reciprocal offset. The authorization depends on certain claims being recognized, but the two productive acts need not be placed into a unique order. Another observer may receive the records at a different stage, while a disputed claim can undergo revision. The economically relevant structure is thus a network of dependencies, recognition conditions and interpretations rather than a mandatory chronology of every transaction in the system.

One may introduce a *local configuration*  $h$  as a selected, evidentially justified portion or interpretation of  $\mathfrak{H}$ . The notation  $h'$  for another configuration makes no independent assertion that  $h'$  is metaphysically located in “the future” of  $h$ . A comparison requires a specified correspondence, revision rule or admissible continuation. For empirical work, calendar timestamps, physical locations and legal due dates may be attached when they assist measurement; the formal account remains explicit about what those coordinates express. A system that stores event records can provide an important practical infrastructure

for such a model, but a complete record of every economically relevant occurrence should not be presupposed. Missing events, contested reports and the costs of observation can shape which transformations appear feasible.

It is consequently helpful to distinguish three notions: a *realized record* recognized by an identified evidence practice; a *permitted continuation* under stated technical and institutional rules; and a *counterfactual configuration* introduced to test the explanatory model. A clearing arrangement simulated by AI belongs to the second or third category until the appropriate parties or institutions actually enact it. This distinction matters whenever predictive capacity is offered as evidence that a monetary function can be abolished. A correct forecast of an agreeable net balance cannot by itself supply consent, legal authority or access to the material resources that the exchange requires.

## 7.2 Spacetime-Distributed Value Circulation

The value-producing process that motivates monetary exchange may involve labour, knowledge, interpersonal recognition, access to a community, provision of physical resources, maintenance of shared infrastructure, or ecological effects. Such contributions are heterogeneous both in kind and in the criteria by which participants recognize them. A mathematical account that immediately projects all contributions into a single monetary magnitude may eliminate precisely the distinctions the present inquiry seeks to study. An event may generate a contribution for several others, make a later productive action possible, or transform the interpretation of a prior contribution. Further, the value of a relationship can include opportunities, identities, trust and participation that do not have an unproblematic price equivalent. This motivates the separation between *value circulation* and *monetary denomination* developed in preceding sections and in the exploratory topological treatment [11, 3].

Let  $\mathcal{V}$  be a family of value descriptions indexed by generating relations and the practices through which they are recognized. A value record can be written schematically as

$$v = (e, \rho, \kappa, \alpha, \sigma), \quad v \in \mathcal{V}, \quad (7)$$

In Equation (7),  $e$  refers to a situated generative event,  $\rho$  identifies a contribution or relational structure,  $\kappa$  specifies the kind of value asserted,  $\alpha$  records the relevant recognition or attribution conditions, and  $\sigma$  describes an observed or posited circulation relation. The expression is a bookkeeping schema: it does not assert that all contributions have a scalar magnitude, that there exists a complete list of value kinds, or that each value can be owned or exchanged. A relationship of care, an increase in a publicly usable capability, a mutually negotiated entitlement and a physical resource flow may be represented under different rules. Their differences can remain meaningful even where a particular agreement assigns monetary compensation to some of them.

To write an invoice or authorize settlement, an institution may employ a partial valuation or denomination procedure

$$\pi_{h,\iota} : \mathcal{D}_{h,\iota} \subseteq \mathcal{V} \longrightarrow \mathbb{R}_{\geq 0} \times \mathcal{U}, \quad (8)$$

where  $\mathcal{U}$  denotes admitted units of account and  $\iota$  the practice or authority declaring the correspondence. The domain  $\mathcal{D}_{h,\iota}$  is generally restricted. Two practices need not agree about the value of a contribution, nor need a single practice remain unchanged under a different history. Equation (8) does not identify a monetary price with the total value of an event. It represents a contingent institutional operation, which may be unavailable for environmental, personal or other values whose monetary treatment is rejected or undefined.

The separation has a direct implication for dissolution. A monetary claim generated through  $\pi_{h,\iota}$  can be offset or discharged without removing the value record  $v$ , undoing the original labour, or cancelling the historical relationship. If an AI-operated maintenance provider receives a reciprocal service instead of a monetary payment, the mode of settlement can change while both productive contributions continue to matter. If a human community abolishes a membership fee, its relations of association may develop new meanings and demands rather than vanish. Similarly, a financial circuit may appear balanced while creating ecological harm unrepresented in  $\mathcal{D}_{h,\iota}$ . Such an omission must not be mistaken for the absence

of the harm. This layered interpretation makes it possible to examine the erosion of specific monetary dependencies without assuming the disappearance of exchange, needs for others or generative value circulation.

A further methodological caution concerns the direction of explanation. A value record can be reconstructed from surviving documentation, but the reconstruction need not capture all of the historical generative process. A price observed in an archive does not uniquely identify the conditions of its generation; an AI-generated summary of a transaction does not independently verify the relation it describes. Consequently, the proposed representation must remain revisable in response to additional evidence, contested interpretations and alternative accounts of whose productive contributions were recognized. The concern is epistemic as well as distributive: the visibility of some value streams can be partly determined by control over the technologies and institutions that make them legible.

### 7.3 Network and Two-Complex Representations

A directed graph can express bilateral claims or explicit dependencies, yet a graph alone may be inadequate when productive relationships possess multi-way incidence, emergent organizational forms or the historical formation of participating entities. A relational two-complex offers one further representational option, although its application requires declared domain semantics. Following the earlier exploratory formalism [11], let  $K$  be an oriented finite two-dimensional cell complex with cellular chain groups and boundary maps

$$C_2(K; \mathbb{R}) \xrightarrow{\partial_2} C_1(K; \mathbb{R}) \xrightarrow{\partial_1} C_0(K; \mathbb{R}), \quad \partial_1 \partial_2 = 0. \quad (9)$$

The zero-composition identity in Equation (9) follows from the cellular-chain construction. It constrains the algebraic representation; it does not by itself authorize the cancellation of any debt or demonstrate an economic equilibrium. The coefficient field  $\mathbb{R}$  is introduced for preliminary linear analysis, not as a claim that all economically relevant quantities should be denominated continuously or in one unit.

In the interpretation adopted for the present inquiry, *vertices represent events* and *edges represent couplings* between such events. Two-dimensional faces encode modeling primitives whose appropriate semantics must be established for the chosen domain. In particular, it would be unjustified to equate a face in general with money, a firm, an individual, a unit of labour, or a debt contract. Economic agents and institutions may instead be reconstructed from appropriate subcomplexes, their couplings and their documented histories. Alternative graphical models can be preferable when the relevant information concerns simple bilateral liabilities. The choice of a cell complex is therefore motivated by a research question concerning the organization of relational histories, not by an assertion that every economic process possesses an intrinsic topological description.

One structural invariant is the first homology group

$$H_1(K; \mathbb{R}) = \ker(\partial_1) / \text{im}(\partial_2). \quad (10)$$

In Equation (10), a one-cycle is a chain with vanishing boundary, and homologous cycles differ by a cellular two-boundary. Nontrivial homology may help distinguish structures within the selected representation, including possible circulation channels that are not boundaries of represented two-cells. Nevertheless, a topological one-cycle is not automatically an economic exchange cycle, and an exchange cycle is not automatically a legally clearable obligation cycle. These statements involve different objects and eligibility conditions. The existence of a hole in a two-complex neither proves that financial clearing is feasible nor supplies a measure of its value. Topological invariants can change with the granularity and the declared semantics of the representation.

To connect an event complex with actual monetary claims, introduce a separate obligation graph  $G_h$  whose vertices are identified *liable positions or accounts* and whose directed edges are specified obligations recognized in configuration  $h$ . The graph may use actor labels such as H, AI-operated organization or public body, but those labels denote relational roles and institutional positions, not the event vertices of  $K$ . Any correspondence

$$Q_{h,\iota} : \text{Records}(K, \mathfrak{H}) \rightarrow \text{Claims}(G_h, \iota) \quad (11)$$

is a *partial interpretation procedure*. It may identify records relevant to an enforceable claim, its holder, its debtor, and the circumstances of recognition. The dashed mapping emphasizes that not every value-generating event creates a monetary claim and that claim recognition is institution-dependent. Equation (11) is not asserted to be a chain map, functor or homomorphism; those stronger structures would require explicit objects, operations, and proof of compatibility. This restraint avoids silently transferring a topological identity in an event model to a juridical assertion about debts.

When a rigorous bridge is available, a richer theory could compare event-complex boundaries, the formation of obligations and the evolution of recognized accounts. Such a bridge might also represent the changing status of an AI-operated service: the system could participate operationally in an event complex while a legal organization bears the obligations identified by the claim graph. That distinction remains important for the differential monetary trajectories considered in Section 6. Neither increasing AI autonomy nor adding more detailed topological structure establishes independent AI legal personhood or the transfer of liability away from the institution that controls the infrastructure.

## 7.4 Monetary Obligations and Relational Boundaries

The next step identifies what an obligation network can express before making any claim about dissolution. Consider a finite, directed multigraph  $G_h = (P_h, L_h)$  associated with a documented configuration  $h$ . Its vertices  $P_h$  index legally or institutionally recognized debtor and creditor positions; parallel edges are allowed because parties can have several claims with different terms. Orient each liability edge from its debtor position toward its creditor position. The description may include an amount, unit, due date or other maturity condition, priority, contingency, governing law, permitted transfers, and a set of actors authorized to approve changes. A practical representation would also retain the chain of provenance connecting these terms to evidence and identifiable legal or governance rules. In particular, an accounting entry on an AI platform is not necessarily an externally enforceable liability.

Fix temporarily a single monetary denomination and a set of claims eligible for comparison. Let  $x \in \mathbb{R}_{\geq 0}^{L_h}$  be the vector of nonnegative amounts. Its incidence boundary is

$$b = B_h x, \quad B_h = \partial_1^{G_h} : C_1(G_h; \mathbb{R}) \longrightarrow C_0(G_h; \mathbb{R}), \quad (12)$$

where  $B_h$  maps an oriented edge to creditor minus debtor. Thus  $b_p > 0$  denotes a net credited position under the specified convention and  $b_p < 0$  a net indebted position. Equation (12) concerns the positions *inside this compatible subledger*. It does not imply that an institution is solvent, that its asset can be collected, or that its net exposure across all currencies and contractual contingencies is represented by  $b$ . Summing the components of  $b$  produces zero because each admitted edge has a debtor and creditor in the graph; it does not follow that the network has no unresolved monetary obligations. A claim graph can be internally balanced in that elementary algebraic sense while retaining considerable gross liabilities and severe liquidity risks.

Claims expressed in different denominations should instead be recorded in separate typed components unless the participants have recognized a legitimate transformation between them. A network with a debt of ten units of one currency and a reciprocal claim for ten units of another cannot be cleared merely because their numerals coincide. Neither does the existence of an exchange rate by itself establish that a unilateral conversion and cancellation is legally permissible. The same caution applies to promises of goods, information access or labour: the ability to attach a price does not render those obligations interchangeable with settled monetary claims. Thus the most elementary algebra is conditional upon *comparability and eligibility*, which Section 8 will examine in greater detail.

Boundaries are essential because the monetary functions observed inside a cooperative, firm, platform, municipality or AI production network may be supported by relations outside that collective. Let  $U$  denote a chosen subnetwork, and let  $r_U$  be the operation that retains claim edges assigned to its scope under a declared inclusion rule. For a cycle  $z$  in the full graph, it is possible that

$$B_h z = 0, \quad B_U r_U(z) \neq 0. \quad (13)$$

The restricted circulation is then not a cycle within  $U$ : its missing complement crosses the representational boundary. For example, an A–B–C–A cycle can balance all three positions while the A–B and B–C portion alone leaves an outstanding position at its ends. The original global relation is not made locally self-sufficient by choosing a narrower reporting boundary. Conversely, a local closed cycle can be cleared in one organization even when an external creditor remains wholly outside that cycle. Whether reporting follows a whole network, a unit of production, a geographic jurisdiction or the human-facing interface of an AI service materially changes the interpretation of a disappearance claim.

**Remark 7.1** (Boundary dependence). The condition  $B_U r_U(z) \neq 0$  in Equation (13) is a possibility, not a universal consequence of restriction. If the selected subgraph contains every edge of the cycle, its restriction remains closed. A boundary argument must therefore identify the omitted relations and show which claims or dependencies cross the proposed scope.

The boundary relevant to monetary analysis is not necessarily identical to the boundary relevant to value, physical production or ecological responsibility. Consider an AI-operated data center that clears reciprocal invoices with neighboring compute providers. At the same time, it may consume energy furnished by an external grid and water from a shared hydrological system, while the local environment receives a heat burden that no contract represents. An internally closed monetary circuit does not establish that those material relations are closed. An ethically and empirically adequate report would specify both the institutional scope of the clearing claims and the larger field of affected persons, communities, environments and infrastructures. The latter cannot be rendered equivalent to an unpaid invoice merely by assigning it an arbitrary monetary number.

## 7.5 Local Dissolution and Distributed Coordination

A first, deliberately limited model of monetary dissolution is the *admissible reduction of circulating obligations*. Suppose that  $x$  is a nonnegative vector of mutually comparable liabilities on a fixed recognized graph, as above. A proposed circular cancellation can be described by a nonnegative vector  $z$  satisfying

$$0 \leq z \leq x, \quad B_h z = 0, \quad z \in \mathcal{A}(h, \iota), \quad (14)$$

In Equation (14), inequalities are componentwise and  $\mathcal{A}(h, \iota)$  represents the reductions institutionally, materially, and procedurally admissible under the declared recognition practice  $\iota$ . Membership in  $\mathcal{A}$  cannot generally be decided by the matrix  $B_h$ . It can depend upon maturity, insolvency safeguards, consent, assignability, tax treatment, due process, and the authority to discharge a claim. A proposed nonnegative circulation becomes an *actual clearing* only if the corresponding transformation is accepted and enacted under the applicable institutional rules.

**Proposition 7.2** (Net-position invariance under eligible cyclic reduction). *Let  $x$  and  $z$  satisfy the algebraic conditions  $0 \leq z \leq x$  and  $B_h z = 0$ . Define a residual obligation vector  $x' = x - z$ . Then  $x'$  is nonnegative and has the same net-position vector as  $x$ :*

$$B_h x' = B_h x. \quad (15)$$

*For the invariant in Equation (15), if at least one component of  $z$  is strictly positive, the sum of recorded gross compatible obligations decreases by  $\sum_{e \in L_h} z_e$ . This algebraic result describes a legally effective cancellation only if  $z \in \mathcal{A}(h, \iota)$  and the corresponding discharge is recognized.*

*Proof.* The componentwise bound  $z \leq x$  gives  $x' = x - z \geq 0$ . Linearity of the incidence map and  $B_h z = 0$  yield  $B_h x' = B_h x - B_h z = B_h x$ . Finally, summing  $x'_e = x_e - z_e$  over compatible edges establishes the stated reduction in gross recorded obligations. These equalities do not prove institutional admissibility, liquidity sufficiency, or equivalence of the underlying heterogeneous contributions.  $\square$

Consider three recognized parties A, B and C with a compatible claim of twelve monetary units from A to B, eight from B to C, and five from C to A. If all three liabilities are eligible for a consensual multilateral netting procedure, the common circular amount is five. Eliminating this amount from each edge leaves A owing B seven units, B owing C three units, and no residual C–A liability. The initial and residual position vectors agree: A is a net debtor of seven units, B a net creditor of four units, and C a net creditor of three units. Their gross nominal liabilities decline from twenty-five to ten units. The example establishes neither that the remaining ten units need no money nor that any particular legal regime must accept this cancellation. It instead isolates a kind of distributed reduction that need not move five currency units separately around every edge.

This circuit also clarifies the difference between *netting* and a complete removal of monetary mediation. Net-position preservation does not mean that all payment functions disappear. The residual debtors may still need an accepted settlement asset; participants may demand collateral, insurance, independent auditing or legal guarantees; and the original invoices may themselves have reflected relations involving paid labour, resources and credit. Furthermore, the numerical cancellation does not show that all participants judge the underlying exchange fair. They might have different valuations of the goods or labour originally supplied, and they may accept an offset while contesting the terms that produced the claims. A lawful settlement convention resolves a defined class of obligations; it does not exhaust the questions of justice, productive dependence or ecological accountability.

The distinction is especially significant for AI-operated production. A coordination model can search large claim networks, identify candidate circulations, compare due dates and exchange constraints, and propose admissible residual arrangements. Its computational ability does not determine who possesses the power to accept a cancellation or who bears an unrecorded loss. An arrangement in which AI operators jointly agree to net claims may be an institutional practice even when it is implemented automatically. Such a practice must include verifiable authorization and remedies for mistakes or disputed facts. Conversely, technical automation may lower search and communication costs enough to make small-scale reciprocal production feasible without ever creating currency-denominated invoices. That is a different mechanism: *substitution* of monetary coordination rather than reduction of pre-existing recorded liabilities.

Two further limits should be recorded. First, a purely algebraic circulation is not necessarily the maximum admissible clearing flow. Restrictions on priority, legal identity, contingent claims and insolvency proceedings can eliminate otherwise visible cycles from  $\mathcal{A}(h, \iota)$ . Second, finding a feasible clearing flow is not equivalent to establishing that a particular objective should be maximized. Maximizing the face amount cancelled could worsen some parties' risk, eliminate safeguards for vulnerable creditors, or obscure the interests of excluded participants. The appropriate objective and constraints are matters of governance and value recognition as much as computational optimization. Section 8 develops the clearing problem more directly; the present result is intended only to state a transparent local operation and its precise limits.

## 7.6 Historical Continuation and Structural Transformation

An event-based account requires that monetary dissolution be understood as an *admissible transformation of a recognized configuration*, rather than as the absence of money at a privileged global instant. Let  $\mathfrak{C}(\mathfrak{H}; \Theta)$  denote the collection of continuations that satisfy a stated set of assumptions  $\Theta$ , including resource availability, contractual permissions, evidentiary standards and governance constraints. A proposed transition from a configuration  $h$  to a configuration  $h'$  may be represented as

$$T \in \mathfrak{C}(\mathfrak{H}; \Theta), \quad T(h) \sim h', \quad x(h') = x(h) - z + g, \quad (16)$$

where  $z$  is a duly enacted reduction of admitted claims and  $g$  is a nonnegative vector of newly recognized claims, expressed on a suitable common comparison graph. The symbol  $\sim$  designates the declared equivalence or correspondence between a proposed result and a recognized configuration; it is not a universal identity across all historical descriptions. Where participants, units, claim types or graph structure

differ, an explicit embedding or reindexing procedure is needed before the displayed vector equality can be used. Equation (16) is illustrative, not a claim that every economic transformation is linear.

A continuation can include events that modify the productive and institutional possibilities themselves. An AI-operated energy cooperative might develop new generation capacity, reducing its dependence on human-controlled electricity procurement while adding liabilities for replacement materials and environmental compliance. A public data infrastructure might make some information services available without individual payments yet create new responsibilities for curation, privacy protection and model verification. A previously enforceable fee could be abolished by a governance decision rather than through cyclic netting. Each process changes the relevant relation network in a distinct way. Some monetary functions are removed through cancellation, some become unnecessary through shared productive provision, and others are displaced toward a newly established organization or infrastructure. New opportunities and needs can simultaneously produce  $g > 0$  in Equation (16).

Because the network itself can change, the claim that a dissolution mechanism is *distributed* has both a spatial-relational and a historical dimension. Locally resolved circuits can emerge in different communities, sectors or AI service clusters without constituting one global transition. Partial orderings can connect some of their enabling events while leaving others incomparable. A clearing agreement may depend on information gathered by an autonomous sensor system, but its legal recognition may occur through a different institutional chain. The model should therefore preserve the provenance of the proposed transformation and the evidence for its realization. When a claim is revised or contested, a responsible representation should not erase the earlier claim as though it had never been asserted; it should distinguish the original record, the review and the recognized disposition.

These distinctions also limit what may be attributed to advanced AI cognition. Improved observation, negotiation and prediction may enable coordination over a larger set of candidate circuits or reveal previously unknown forms of productive interdependence. Yet an AI system may have incomplete information, divergent objectives, privileged access to another party's records, or conflicts between the goals of its operator and the interests of affected persons. A globally informative description is neither guaranteed nor universally desirable. In practice, federated information systems, selective disclosure, publicly auditable interfaces and participant-controlled permissions may support coordination without establishing a single centralized controller. Such designs are possible *conditions* for some forms of distributed coordination; their detailed engineering and political legitimacy belong to the companion infrastructure and governance investigations rather than to the present formal account.

Historical continuation also creates a subtle question about the terms *possibility* and *realization*. A candidate netting cycle may be mathematically feasible under  $\Theta$ , while no institution accepts it. Alternatively, an institutional reform can make a previously excluded circulation admissible, changing  $\Theta$  and hence the set  $\mathcal{C}$ . The latter transformation concerns the generative conditions of future possibilities, but should not be represented as a mere prediction waiting to occur at some already given subsequent time. The generative aspect lies in changing the space of permissible relations and the means by which participants can recognize and enact them. This is continuous with the methodological caution of the earlier work on relational economic histories [9, 7].

## 7.7 Limits of Global Equilibrium Representations

It may be tempting to describe eventual monetary dissolution as a state in which every participant's claims are perfectly balanced over an indefinitely extended horizon. Such a representation can illuminate an idealized closed cycle, but it introduces demanding assumptions: stable participant identities or consistent identity mappings, compatible obligations, a known accounting boundary, appropriate convergence of the relevant flows, and the possibility of enforcing settlement across the modeled relations. None of these conditions follows from the growing intelligence of AI systems. A finite circuit that is eligible for clearing illustrates a more modest mechanism; an imagined economy-wide, all-horizon balance is a separate and much stronger hypothesis.

For a fixed, comparable claim graph, the condition  $B_h x = 0$  expresses zero net position at each

included node. It does not by itself certify that the entire monetary process has disappeared. It allows any nonnegative circulation, including an indefinitely recurring use of money to finance transactions; it may conceal enormous gross exposure; and it says nothing about claims or material costs outside the graph. In addition, a system with  $B_h x \neq 0$  may nevertheless reduce monetary reliance at selected points of use through public provision or nonmonetary reciprocal allocation. The topological condition that an incidence boundary vanishes and the economic condition that monetary mediation is dispensable are therefore distinct properties. A model must specify the settlement procedure and the reproduction of production before passing from one to the other.

The desire to integrate across indefinitely many potential circulation paths introduces an additional problem. Some paths may fail because a participant cannot fulfill a promised contribution, an enabling institution withdraws permission, or an ecological or material input is unavailable. Long paths can accumulate such vulnerabilities, but they are not uniformly less robust simply because they contain more edges. Common infrastructure, contractual guarantees, redundant routing and correlated shocks can alter the dependence structure. Moreover, the number of paths of a given length can grow rapidly with the network. Accordingly, the small probability of any *single* long path does not alone justify neglecting the combined contribution of all long paths. Section 9 will examine the conditions under which a bounded approximation is legitimate, including dependence, convergence and the preservation of excluded but normatively consequential contributions [13].

These caveats have implications for how an explanatory ideal is formulated. A stationary or equilibrium model may be useful for a specified institutional arrangement, but the present account does not posit one economy-wide objective, universal exchange rate, or fixed set of participants and resources. Monetary relations can arise at new encounter boundaries, be eliminated through locally recognized clearing, migrate into energy or data infrastructures, and reappear when a scarce capability becomes controlled through an exclusionary rule. Different arrangements can coexist, and the institutions interpreting values may change along with the network. An equilibrium approximation must therefore be justified locally and comparatively, with attention to the information lost in aggregation.

Finally, monetary balance must not be used to close the ethical inquiry. A formally admissible circulation may depend on environmental extraction, uncompensated maintenance labour, exclusion from a commons, or the depletion of another community's resources. Even broad stakeholder agreement among included account holders does not guarantee that every affected interest has been represented. This limitation cannot be resolved by increasing the size of a ledger alone; it concerns rights of participation, ecological observation, distributive consequences and the governance of the measurement process. In keeping with the generative perspective on economics, evaluation must therefore attend to whose capacities are developed, who participates in circulation, and what relations are rendered invisible when their value cannot be denominated [23].

The analysis in this section provides a vocabulary for the next two stages of the paper. Event histories and two-complexes can represent aspects of the organization and transformation of relational production; a separate, typed obligation network can express the claims that institutions actually recognize. A compatible cycle can undergo local cancellation, while boundaries, changing institutions and emergent exchange relations prevent that cancellation from being equated with a global abolition of money. Section 8 examines reciprocity and the admissibility of multilateral clearing more closely. Section 9 addresses fragility, higher-order circulation, and the conditions under which computational approximations of distributed dissolution can be controlled. The possibility of monetary transformation remains conditional on the actual institutions and productive conditions that enable it.

## 8 Circulation, Reciprocity, and Multilateral Clearing

The preceding section distinguished a relational history of production from a network of institutionally recognized obligations. This section examines a narrower operation within that network: the reduction of compatible liabilities through bilateral offset, directed circulation, multilateral agreement, and cross-event settlement. The purpose is to determine precisely what a clearing mechanism can establish, which

additional conditions it must assume, and which dimensions of monetary mediation remain after the resulting accounts have been adjusted. Clearing is an analytically important mechanism of spacetime-distributed dissolution, but it is neither the only mechanism nor a sufficient account of a postmonetary economy. Production may continue with obligations extinguished; value may circulate without monetary claims ever arising; and obligations may be cleared while payment media, units of account, financing and coercive enforcement retain their economic roles.

The analysis begins with recognizable bilateral liabilities, builds a typed multilateral ledger, and identifies the algebraic conditions under which circular claims can be reduced without altering specified net positions. It then considers heterogeneous valuation, institutionally admissible transformations, intertemporal commitments, and the obligations that resist clearing. The mathematical results are elementary by design. Their explanatory use depends on locating each formal operation within a historical, institutional and material arrangement. In particular, a computational procedure that discovers compatible circuits cannot by itself determine whether a set of claims is legally valid, ethically acceptable, or reproducible under changing productive conditions. The distinction is central to earlier exploratory inquiries into AI-operated production networks and distributed monetary dissolution [12, 14, 7].

## 8.1 Bilateral Obligations and Monetary Settlement

A bilateral exchange can create a monetary claim without involving the contemporaneous movement of currency. A debtor may promise a payment for a delivered service, a platform may record a recognized balance after granting access to a facility, or an organization may accept an obligation in consideration of work performed. Payment, credit creation, transfer of a claim, and termination of an obligation are distinct events. A receipt can discharge a monetary debt by means of a particular payment asset, but the debt can also be modified, forgiven, transferred, secured, or offset under authorized rules. An account of monetary dissolution must specify which operation occurs: erasure of a display entry, successful transfer of a settlement asset, extinction of a creditor's enforceable claim, or replacement of a repeated monetary transaction with a different coordination arrangement.

Let  $P_h$  be a finite set of recognized *account positions* in configuration  $h$ , rather than a primitive set of persons or AI agents. A single organization may operate multiple accounts; several agents may act under one account; and an AI-operated process can initiate instructions while legal obligations remain with an operator, owner or trustee. Write  $e = (d(e), c(e))$  for a liability edge from debtor position  $d(e)$  to creditor position  $c(e)$ . Every claim carries more than a numerical amount. A useful record is

$$q_e = (d(e), c(e), a_e, \tau_e, \mu_e, \rho_e, \kappa_e, \xi_e), \quad (17)$$

where  $a_e \geq 0$  is its amount in a declared unit,  $\tau_e$  records denomination or obligation type,  $\mu_e$  records maturity or event-dependent conditions,  $\rho_e$  identifies seniority and relevant rights,  $\kappa_e$  identifies contractual or governance rules, and  $\xi_e$  links the claim to its recognition evidence. The attributes in Equation (17) are deliberately schematic: some claims lack a single fixed amount, and a contingent promise may require a state-dependent record rather than one scalar. The schema does not turn unrecognized interpersonal expectations or ecological effects into monetary debts merely because they can be entered in a database.

With bilateral receivables  $a_{AB}$  and  $a_{BA}$ , a ledger might display opposite-direction claims between two accounts. When the claims are mutually compatible and both participants or their authorized institutions accept offset, one can reduce each by  $m = \min(a_{AB}, a_{BA})$ . A residual claim remains in one direction if the amounts differ. This operation conserves their net positions in the admitted subledger while eliminating a portion of gross claims. It is not generally identical to performing two separate payments: settlement assets, gross liquidity demands, fees, timing of legal discharge, and counterparty exposures may differ. In particular, an institution may restrict or prohibit setoff even where the two nominal values happen to coincide. The formal convenience of bilateral cancellation should not silently supply permission or negate the positions of affected third parties.

The recognition of a claim may change between delivery and proposed offset. A contract might permit cancellation only after verification; a disputed invoice might later become unenforceable; a currency

obligation might be denominated in a unit different from the reciprocal claim. The two events should therefore be connected by a traceable statement of evidence and authority. Within a partially ordered event history, an acceptance or discharge event must depend on the prerequisites that actually govern it. No universal chronological ordering of unrelated productive acts is implied. A relational model can record that two obligations were created independently and then became eligible for offset under an institutionally recognized event. Thus, even the apparently elementary bilateral operation already has a spacetime-distributed and governance-dependent character.

## 8.2 Closed Circulation Paths

Bilateral offset does not exhaust the reductions available in a network. A can owe B, B can owe C, and C can owe A although no individual pair holds exactly reciprocal claims. The resulting directed cycle can support a multilateral reduction. Consider the fixed, mutually compatible claim graph  $G_h = (P_h, L_h)$  from Section 7.4, with incidence operator  $B_h$  and nonnegative claim vector  $x$ . For a directed simple cycle  $\gamma$ , define the nonnegative edge indicator  $\chi_\gamma$  whose entries equal one on the directed edges of  $\gamma$  and zero elsewhere. Because each position along this particular circuit has one entering and one leaving edge,

$$B_h \chi_\gamma = 0. \quad (18)$$

For a proposed common reduction  $\lambda_\gamma$ , the algebraic capacity constraint is

$$0 \leq \lambda_\gamma \leq \min_{e \in \gamma} x_e, \quad z^\gamma = \lambda_\gamma \chi_\gamma. \quad (19)$$

Equations (18) and (19) allow the cycle to remove  $\lambda_\gamma$  from each of its edges while preserving the specified net position of each participating account. This is an accounting possibility under a compatible class of claims; its application still requires a recognition and discharge procedure. A path that merely returns to its starting location in a physical map or a relational two-complex does not thereby become a legally admissible monetary cycle.

Several distinctions are worth preserving. A *closed path* in a directed graph is a combinatorial property. A *circulation* is a nonnegative edge flow with zero incidence boundary. A *reciprocal arrangement* is a social or institutional relation of mutual provision or commitment; it need not admit numerical measurement. A *clearing* is an accepted transformation of claims. These notions can overlap, but none defines the others. The three-party circuit might embody commercial credit, a cooperative mutual-aid arrangement, or claims against a common public provider. Their practical equivalence cannot be read directly from identical incidence matrices. The operations must be interpreted using the institutional accounts of monetary obligations developed earlier [15, 23].

Cycles can also overlap. Two simple circuits may share an edge that cannot be cancelled twice beyond its available amount. If a family of cycles  $\Gamma$  is proposed, write  $z = \sum_{\gamma \in \Gamma} \lambda_\gamma \chi_\gamma$  with  $\lambda_\gamma \geq 0$ . The meaningful capacity condition is now  $z \leq x$ , or, componentwise,  $\sum_{\gamma \ni e} \lambda_\gamma \leq x_e$ . The possibility of describing a network by many cycles does not imply that their corresponding reductions can all be performed independently. Shared obligations create coupling between otherwise distinct clearing proposals. A practical protocol must also prevent double discharge, record which cycle consumes which portion of a claim, and specify how competing accepted reductions are ordered or jointly authorized.

There is no requirement that the claims originate at a single instant. A cycle may be assembled from agreements recognized at distinct events and under different productive circumstances. Its viability depends upon the admissible correspondence among those obligations at the chosen clearing configuration. If one participant's liability has not matured, if a claim is disputed, or if the terms permit only currency payment to a particular creditor, the cycle may remain an algebraic candidate without becoming an available institutional operation. Conversely, an appropriate multilateral agreement can sometimes render otherwise separate claims eligible through a legitimate amendment or novation. Those additional events change the represented institution and therefore should be reported as part of the proposed clearing rather than hidden inside a numerical equality.

Table 6: A hypothetical four-party circular reduction. The amounts are comparable within one admissible claim class; all entries denote outstanding obligations on the stated directed edge.

| Liability    | Initial amount | Reduction | Residual |
|--------------|----------------|-----------|----------|
| A to B       | 9              | 5         | 4        |
| B to C       | 6              | 5         | 1        |
| C to D       | 8              | 5         | 3        |
| D to A       | 5              | 5         | 0        |
| <b>Total</b> | <b>28</b>      | <b>20</b> | <b>8</b> |

### 8.3 Multilateral Reciprocity and Cyclic Obligations

A significant part of economic cooperation is organized through reciprocal expectation rather than the immediate creation of enforceable debt. A person may contribute knowledge to a scientific commons, an AI service may coordinate a shared scheduling process, or a cooperative may provide maintenance to a member who later contributes a different capability. Such actions can support durable reciprocity while lacking a common unit of account or a rule that every contribution yields a transferable monetary claim. It would therefore be mistaken to treat every act of mutual benefit as a hidden loan or every circulation of value as a debt cycle. The institutional question is when, by whom, and under what conditions an expectation is converted into a recognized obligation.

Where reciprocal commitments are deliberately formalized, a multilateral arrangement can pool otherwise separate claims and determine a valid path of discharge. Suppose four accounts A, B, C and D recognize liabilities  $A \rightarrow B$  of nine units,  $B \rightarrow C$  of six,  $C \rightarrow D$  of eight, and  $D \rightarrow A$  of five. All are in the same eligible claim class, and each recognizes the same authorized clearing event. The circulating reduction of five units leaves residual claims of four, one, three and zero units, respectively. Their net positions are invariant: A has a net obligation of four, B a net claim of three, C a net obligation of two, and D a net claim of three. The fact that A and C remain net debtors matters. The cycle does not produce a general condition of “owing nothing” for every participant; it removes an offsettable component of the recognized gross exposures.

This example differs in two respects from a single pairwise offset. Each obligation can be reduced by agreement within a closed multilateral circuit, and the reduction preserves the positional differences that motivate subsequent settlement or provision. What disappears is a portion of the gross monetary obligation structure, not the contributors, their heterogeneous productive relations, or the underlying scarcity of relevant resources. In a legally authorized netting arrangement, some recognized claims may be extinguished without requiring a matching sequence of full gross payments. Whether the residual claims are paid, deferred, forgiven, or reorganized is a further institutional decision. A creditor who accepts a reciprocal arrangement may judge it satisfactory for reasons not representable by equality of monetary amounts, but that judgment must not be confused with the formal claim that the included monetary accounts have zero incidence boundary.

The political-economic setting also matters. Within one corporation, accounts representing AI services may be bookkeeping partitions rather than separate claim holders; reducing them may be a reclassification of internal costs. Among independent enterprises, a comparable cycle may involve enforceable debts and recognized setoff rights. In a public or commons-based network, provision and access might be coordinated directly, with no corresponding monetary claim created on some of the edges. The same arrangement of productive acts can thus yield distinct obligations, and the same bookkeeping network can encode historically different arrangements of productive control. A clearing formalism should make the terms of recognition explicit rather than presume that differences in governance can be normalized into a universal credit system.

## 8.4 Network Flows and Circulation Spaces

The directed-cycle example has a precise extension. Fix one connected component of a finite compatible claim multigraph with nonnegative amount vector  $x$ , and let  $B$  denote its incidence matrix. Define the set of admissible *algebraic* reductions, prior to further institutional constraints, as

$$\mathcal{Z}(x; B) = \{z \in \mathbb{R}_{\geq 0}^L : Bz = 0, z \leq x\}. \quad (20)$$

The algebraic set in Equation (20) is a nonempty, bounded convex polytope containing the zero reduction. Every nonzero  $z \in \mathcal{Z}(x; B)$  decreases gross nominal obligations and preserves net positions as in Proposition 7.2. The algebraic feasible set alone does not describe legally authorized clearing. Let  $\mathcal{A}(h, \iota)$  denote the restrictions supplied by the relevant institutional procedure, and define

$$\mathcal{Z}_{\text{adm}}(h, x) = \mathcal{Z}(x; B) \cap \mathcal{A}(h, \iota). \quad (21)$$

The institutionally restricted set in Equation (21) may be nonconvex or discrete because approvals, insolvency thresholds, consent coalitions, and dependencies between contracts need not be expressible by linear inequalities. It may contain only the zero proposal. Moreover, an institutionally permitted action is not equivalent to an actually enacted discharge; the event effecting cancellation must also be identified.

**Proposition 8.1** (Directed cycle decomposition of nonnegative circulation). *For a finite directed multigraph  $G$  with incidence matrix  $B$ , every  $z \geq 0$  satisfying  $Bz = 0$  can be expressed as a finite sum*

$$z = \sum_{j=1}^m \lambda_j \chi_{\gamma_j}, \quad \lambda_j > 0, \quad (22)$$

where each  $\gamma_j$  is a directed simple cycle in the support of  $z$ . The sum is empty when  $z = 0$ . Consequently, a nonzero algebraic reduction in (20) exists if and only if the directed subgraph on edges with  $x_e > 0$  contains a directed cycle.

*Proof.* If  $z = 0$  the result is immediate. Otherwise choose a directed edge carrying positive flow. Flow conservation at its head guarantees that some outgoing edge has positive flow. Continuing along positive-flow edges in a finite graph eventually repeats a vertex and therefore yields a directed simple cycle  $\gamma$  contained in the support. Let  $\lambda$  be the minimum positive flow on the edges of this cycle. Subtracting  $\lambda \chi_\gamma$  preserves nonnegativity and the zero-boundary condition, while eliminating at least one positive edge from the support. Iteration terminates after finitely many steps and supplies the stated decomposition. Conversely, a directed cycle with positive capacities admits a sufficiently small strictly positive scalar reduction on that cycle, so the feasible set contains a nonzero member.  $\square$

The decomposition in Equation (22) clarifies which part of the theory is topological and which is institutional. The null space  $\ker B$  is the real vector space of signed cycles, but only its nonnegative cone and the admitted capacity constraints describe reductions that do not increase any recorded debt. A signed cycle may indicate an algebraic relation of incidence boundaries while requiring the creation of a negative or additional obligation on one of its edges. Moreover, the theorem applies to the *claim graph*; it does not establish a canonical mapping from the second homology or cellular faces of the relational event two-complex into enforceable claims. The event representation can retain histories, coupling and value-generating primitives, while a separate recognized graph specifies the actual liabilities subject to clearing.

One can pose a bounded optimization over this network. If all admitted edges share a monetary unit and equal nominal weighting is appropriate, a maximum-gross-reduction problem is

$$\max_{z \in \mathcal{Z}(x; B)} \mathbf{1}^\top z. \quad (23)$$

Other objectives might weight settlement costs, disputed-claim risk, distributional effects, or the number of consenting parties. Such alternatives should be declared rather than folded into an unexplained “optimal” clearing. A maximal cancellation of gross nominal liabilities does not maximize human welfare

or generative value, nor is the optimizer unique in every graph. A procedure might reduce measured gross exposure while disadvantaging parties who lose legal protections or need immediate liquidity; independent institutional criteria are required to judge whether its implementation is appropriate. For large networks, the computational problem may further involve integer decisions, strategic participation, contingent claims and incomplete information; those additions can alter tractability despite the simplicity of (23).

**Remark 8.2** (Scope of the cycle criterion). The equivalence in Proposition 8.1 requires a fixed, finite, homogeneous, compatible graph with componentwise divisible amounts and no institutional restrictions beyond those already built into its edge capacities. It does not claim that a directed cycle is sufficient for lawful, fair, feasible, or safe clearing in a real economy. Multiple currencies, priority rights, indivisible claims, external dependencies, and contingent obligations demand additional analysis.

## 8.5 Intertemporal Commitments and Clearing

A major motivation for spacetime-distributed analysis is that complementary productive contributions do not need to be simultaneous. A researcher may make knowledge publicly available; another participant may develop an application; a service organization may later return a different contribution to the research community. An AI network could foresee likely reciprocal paths, discover compatible obligations in distant subprojects, and propose a settlement reducing payments otherwise required at several interfaces. Yet prediction of future cooperation, confidence in a counterpart, and actual institutional discharge have distinct logical statuses. An expected circular benefit cannot cancel an existing debt unless a recognized rule allows the expectation or contingent obligation to enter the settlement mechanism.

Let  $h$  designate a recorded configuration and  $\mathfrak{C}(h)$  a set of institutionally and materially admissible candidate continuations. For each  $h' \in \mathfrak{C}(h)$ , the eligible claims may have a different edge set  $L_{h'}$ , incidence map  $B_{h'}$ , amount vector  $x_{h'}$ , and authorization set  $\mathcal{A}(h', \iota')$ . A conditional clearing plan is accordingly a family

$$\mathcal{P}_h = \{(h', z_{h'}) : h' \in \mathfrak{C}(h), z_{h'} \in \mathcal{Z}_{\text{adm}}(h', x_{h'})\}. \quad (24)$$

Equation (24) represents potential compatible decisions in declared continuations; it does not describe obligations already discharged at  $h$ . An actual discharge requires an identifiable event and an authorized transformation of recognized claims. Conditional promises may give rise to present monetary claims when a governing institution so provides, but this must be established by the rule of recognition, not inferred from the possibility that the promised contribution could occur.

Consider three service providers whose reciprocal commitments are created in separated projects. The first provides power to the second, the second provides maintenance to the third, and the third contributes verified environmental observations used by the first. These performances may have heterogeneous value and may be governed by separate contracts. Even if AI systems accurately predict that the three providers regard the entire arrangement as beneficial, an offset is not automatically licensed: one commitment may be nonassignable, another not yet due, and the last a public contribution without any monetary price. Participants might nevertheless construct a common agreement that recognizes a limited compensable component and sets rules for credit, verification and discharge. The monetary cycle would then be constituted through a new recognized institutional event, rather than pre-existing as a universally calculable debt relation.

The ability to maintain cross-event obligations depends upon durable records, verifiable provenance, interpretable contractual terms and ways to address disagreements. Global knowledge infrastructure might improve discovery of productive complementarities and reduce uncertainty about some commitments. But richer data cannot guarantee the motivation or capacity to honor an agreement; it can also intensify surveillance, exclusion or unequal bargaining power. Claims of very long-run reciprocal balance require attention to default, renegotiation, institutional continuity and the possibility that the relevant participants or resources cease to exist. A notion such as integration over an unbounded historical or potential horizon must specify the measure, admissible configurations, convergence assumptions and relation between expected and legally realized obligations. Without those conditions, it is a metaphor

rather than a demonstrated accounting identity. Section 9 will examine computable truncation once that formal setting is made explicit.

A complementary perspective treats commitments as relations that can be revised rather than as immutable future transfers. A cooperative may choose a rule under which verified contributions grant access rights to shared equipment, without creating a general transferable currency. An AI-operated platform may instead denominate these rights in claims redeemable for scarce computing resources. Both mechanisms can coordinate activity across events, but they differ in transferability, exclusion, units of account, and enforcement. Intertemporal reciprocity therefore creates neither a guarantee of monetary dissolution nor a necessity for monetary continuation. The institutional form of the commitment mediates which conclusion is appropriate.

## 8.6 Heterogeneous Value Equivalence

The mathematical simplicity of cycle reduction rests on comparing obligations in one admitted class. Production itself remains heterogeneous. Maintenance of an ocean sensor, access to an academic community, a discovery produced by an AI system, electricity delivered at a constrained location, and a contribution of care to another person are not automatically quantities on a single scale. A contractual price can specify a permissible exchange for some of them, but does not establish a complete ordering of their ethical, ecological, informational or generative significance. The difference is central to our earlier account of heterogeneous value circulation [11, 9].

To make the conditions explicit, let  $\mathcal{T}$  denote a set of obligation types, including denomination, maturity class, governing rule and any material performance constraints needed for an intended operation. Rather than one untyped matrix, one can consider a family of incidence models  $(G^t, B^t, x^t)_{t \in \mathcal{T}}$  and a declared set of permitted translations  $T_{t \rightarrow s}(h)$ . A proposed comparison between amounts of type  $t$  and type  $s$  then relies upon an authorized conversion or novation

$$T_{t \rightarrow s} : \mathcal{Q}_t^{\text{eligible}} \rightharpoonup \mathcal{Q}_s^{\text{eligible}}, \quad (25)$$

where the partial arrow indicates that a translation may be undefined for many obligations. The partial translation in Equation (25) need not be linear, invertible, universal or ethically adequate. Even if its numerical component is a foreign-exchange quote or an agreed index, it can also require consent, a fee, a change in risk, or the creation of a new contractual claim. A chain of conversions should not be assumed path-independent without a separately justified consistency property.

A robust procedure consequently records which claim was translated, by whom, under which terms, and with what changes in rights and exposures. A change of denomination followed by offset can require monetary payment of transaction fees, collateral, or settlement margins. It may therefore reduce one layer of claims while generating others. Similarly, giving two ecological contributions the same monetary price does not imply that their physical impacts cancel. A forest cleared to host a computing installation cannot be restored by a formal bookkeeping cycle alone. Ecological effects, rights and responsibilities should be examined through appropriate environmental evidence and governance, whether or not participants subsequently use financial compensation. This example illustrates the general limit: *monetary equivalence is a property of specified institutions and instruments, not an ontology of all values.*

The plurality of value forms also informs how reciprocity should be evaluated. Participants may accept an arrangement because it maintains an ongoing relationship, expands shared productive capacity, permits entry into a community, or provides something they could not obtain through an isolated cash payment. Conversely, social pressure can produce apparent acceptance despite unequal power or limited alternatives. A formalized assessment of acceptable reciprocity could identify agents' expressed preferences and constraints, but it must allow disagreement, exit, contestation and nonfungible interests to remain visible. A clearing algorithm optimized for nominal reductions is a tool for specified obligations; it cannot replace these normative and relational questions.

## 8.7 Residual Obligations and Incomplete Dissolution

The outcome of circular clearing is generally a residual obligation structure. For an enacted and institutionally admissible reduction  $z$ , define

$$x^{\text{res}} = x - z, \quad Bx^{\text{res}} = Bx. \quad (26)$$

The invariance in Equation (26) follows from the zero-boundary condition; it says nothing about an individual creditor's timing of cash receipt or a debtor's available resources. If every admissible cycle component is removed under a chosen objective, residual claims can still connect positions with unequal net balances. Even when the recognized residual graph has no directed cycle in the compatible class, debts need not vanish. And a clearing procedure limited by consent, priority or contractual conditions may leave a cycle in the residual graph that was algebraically cancellable but institutionally unavailable.

In the absence of further institutional constraints, the maximum-gross-reduction program (23) has a useful diagnostic implication.

**Proposition 8.3** (Cycle-free residual support under unrestricted maximal clearing). *Suppose  $z^*$  maximizes  $\mathbf{1}^\top z$  over  $\mathcal{Z}(x; B)$  for a finite homogeneous compatible claim graph. Then the directed support graph of  $x - z^*$  contains no directed cycle.*

*Proof.* If the residual support contained a directed cycle  $\gamma$ , every edge of that cycle would have strictly positive residual capacity. Choose  $\epsilon > 0$  smaller than or equal to the minimum residual amount on its edges. The vector  $z^* + \epsilon \chi_\gamma$  would be nonnegative, remain bounded by  $x$ , and satisfy  $B(z^* + \epsilon \chi_\gamma) = 0$ . Its objective would exceed that of  $z^*$ , contradicting maximality. Thus no such directed cycle remains.  $\square$

The proposition is limited but helpful. It identifies the residual structure produced by one declared algebraic objective and clarifies that, in this idealized setting, any further reduction that preserves all net positions and only subtracts nonnegative obligations has been exhausted. It does not establish that there is no further lawful settlement by payment, that obligations cannot be renegotiated, or that all participants' substantive needs have been satisfied. With institutional restrictions, the proof need not apply because adding the small cycle might violate a consent or priority rule even though it satisfies the incidence and capacity constraints. With heterogeneous and indivisible obligations, the feasible set may no longer be a divisible-flow polytope. The diagnostic remains conditional upon the model used to define it.

The distinction between *gross cancellation* and *diminished dependence on money* should remain in view. A firm can operate extensive monetary business while regularly netting circular invoices; a society can finance infrastructure through taxes while allowing free individual access to certain services; a commons can coordinate mutual contributions through collective governance without creating individual monetary debt. These three arrangements cannot be ranked by the amount of nominal debt erased in a single accounting cycle. Their differences concern the creation and enforcement of obligations, the monetary functions that remain, the reproduction of resources, and the capacities of people and AI systems to participate. The presence of a mathematically nontrivial cycle is only one part of that inquiry.

Residual obligations also extend beyond the chosen ledger boundary. Environmental disruption, unequal data extraction, uncompensated maintenance, changes in access rights, and threats to a community's productive autonomy may remain even when recognized creditors assent to a monetary netting procedure. Some of these effects may justify monetary remedies under specific institutions; others call for restoration, prevention, participation or different forms of redress. It would be ethically and analytically misleading to represent their absence from the obligation graph as proof that no residual burden exists. The broader connections among monetary transformation, power and ecology are discussed provisionally in Section 11 and developed further in the paper's companion research programme.

This section has therefore established a bounded claim. In a finite compatible obligation graph, non-negative circulations identify possible reductions preserving specified net monetary positions; each such circulation admits a decomposition into directed cycles. Under a particular unrestricted gross-reduction objective, the algebraic residual graph has no further positive directed cycle. Nothing in these results entails that heterogeneous values become universally commensurable, that recognized obligations can be

cancelled without valid authority, or that money has lost all functions throughout the productive system. The same network can generate new monetary claims when AI providers obtain energy, humans seek entry to restricted communities, or shared facilities need renewed financing. Section 9 next considers the robustness and computational tractability of prospective circuits, including the dependence of long-path estimates on failure correlation, combinatorial growth, and justified truncation bounds.

## 9 Path Fragility and Computational Tractability

The possibility of clearing a closed circuit does not imply that the circuit remains available across an evolving history. A monetary claim may be discharged outside the proposed arrangement, a productive connection may be interrupted, a participant may withdraw consent, a data record may become unverifiable, or a legal restriction may prevent reciprocal claims from being combined. The preceding section identified the algebraic circulation space and its institutional restrictions at a specified configuration. The present section asks how the availability of proposed circuits changes with their relational length and dependence structure, and when that change can support a finite, computable approximation to a much larger set of possible clearing arrangements. The motivating conjecture is that longer circuits may be more fragile. Its practical attraction is that fragility might suppress the contribution of remote, higher-order alternatives, enabling tractable calculation without pretending that an entire evolving economy can be exhaustively enumerated.

That conjecture needs an explicit qualification. A long path contains more connections that can fail, but it may also be embedded in a highly redundant infrastructure, supported by joint guarantees, or selected precisely because its component obligations are unusually reliable. Moreover, there may be exponentially many long paths. The decreasing survival probability of *one* path therefore does not, on its own, establish that the *aggregate* contribution of long paths is negligible. Nor is a formal cycle necessarily a legally authorized clearing procedure, as Sections 7 and 8 showed. Our purpose is to state bounded conditions under which the long-path insight yields an approximation theorem, to identify the counterexamples, and to distinguish an analytical truncation device from the historical disappearance of a relation. The inquiry extends the earlier exploratory value-circuit and normative-renormalization discussions [7, 13, 14] while treating the probabilistic claims below as conditional models rather than observed laws of AI economies.

### 9.1 Path Length and Structural Dependence

At configuration  $h$ , let  $G_h = (P_h, E_h)$  denote a finite directed multigraph of institutionally recognized claim positions and typed obligations, with incidence matrix  $B_h$  as in Section 8. We initially restrict attention to one class of claims whose denomination, divisibility, enforceability, maturity and transfer rules have already been declared compatible. A directed path  $\gamma = (e_1, \dots, e_L)$  consists of edges with matching terminal and initial positions; a directed simple cycle has no repeated positions other than its closing endpoint. Define its *length* by  $\ell(\gamma) = L$ , the number of included obligation edges. In a genuinely spacetime-distributed setting, this is a relational count and need not equal elapsed clock time, the geographic distance traveled, the number of computational messages, or the number of historically distinct productive events. If a clearing proposal spans multiple events, we may instead measure distinct approvals, evidentiary dependencies, physical transitions or legal jurisdictions. These measures are not necessarily interchangeable.

A proposed clearing operation may depend upon conditions other than the validity of its edge claims. Introduce a dependency set

$$D_h(\gamma) = \{d_1, \dots, d_{m(\gamma)}\}, \quad \mathcal{S}_h(\gamma) = \bigcap_{d \in D_h(\gamma)} \mathcal{E}_d, \quad (27)$$

In Equation (27),  $\mathcal{E}_d$  is the event that dependency  $d$  is satisfied when the proposal is assessed. Dependencies may concern claim survival, recognized units of account, required consent, network connectivity or

access to a settlement facility. The event  $S_h(\gamma)$  means only that the listed enabling conditions hold; it does not itself discharge any obligation. The quantity  $m(\gamma)$  can be larger than  $\ell(\gamma)$  when an apparently short cycle depends on many independent external permissions, or smaller when several edges share an institutional guarantee. The analytical question is therefore whether *effective dependency length*, rather than the visible edge count alone, predicts survival.

This distinction matters for contrasting economic organizations. A three-edge circuit between firms in three legal jurisdictions may require more fragile coordination than a ten-edge circuit within a stable, jointly governed public facility. A long supply network may rely on one common energy source; conversely, a short cycle may require an uncertain court decision. If the entire structure is governed by one account holder, the internal transfers may not constitute separate enforceable debts at all. Accordingly, the proposed fragility hypothesis should be applied only to a specified class of comparable circuits and with declared measures of admissibility. It does not establish that the number of social relations or value-generating exchanges ought to be reduced.

We distinguish three quantities:  $s_h(\gamma)$ , the probability that a specified proposal remains feasible under a stated uncertainty model;  $c_h(\gamma)$ , the maximum amount it could cancel conditional on eligibility; and  $v_h(\gamma)$ , the socially assessed value or legitimacy of the underlying activity. A proposed circuit can possess a high clearing capacity without high survival probability, and a legally cancellable cycle can support activities whose values resist monetary comparison. Sections 4 and 8 already separated heterogeneous contribution from compatible monetary amounts. The current computational analysis concerns the latter only; it must not infer  $v_h$  from  $c_h$  or the inverse.

## 9.2 Circulation Survival and Failure

Suppose, as a deliberately simplified benchmark, that  $\gamma$  needs  $L$  edges to remain eligible. Let  $X_i \in \{0, 1\}$  indicate survival of the  $i$ th edge and  $A_\gamma \in \{0, 1\}$  indicate satisfaction of additional approval requirements. Its success indicator and probability are

$$I_\gamma = A_\gamma \prod_{i=1}^L X_i, \quad s_h(\gamma) = \mathbb{E}_h[I_\gamma]. \quad (28)$$

Under independent edge survival with probabilities  $q_i$  and no additional approval restriction, Equation (28) gives  $s_h(\gamma) = \prod_i q_i$ . With homogeneous  $q_i = q$  in  $(0, 1)$ , a particular specified circuit survives with probability  $q^L$ . In that benchmark, additional nonredundant dependencies reduce the chance of completing the same proposed circuit. Such a calculation illustrates the fragility intuition, but the independence hypothesis is neither natural nor assured in actual monetary systems.

A more flexible sufficient condition can be stated without assuming edge independence. Suppose the edges in every examined cycle admit an ordering for which

$$\mathbb{P}_h(X_i = 1 \mid X_1 = \dots = X_{i-1} = 1) \leq q < 1 \quad (29)$$

whenever the conditioning event has positive probability. The chain rule and  $A_\gamma \leq 1$  imply  $s_h(\gamma) \leq q^{\ell(\gamma)}$ . The bound depends on each successive conditional survival probability being uniformly bounded away from one, not on treating all dependencies as independent. The requirement can fail if shared infrastructure makes later edges almost certain once the first succeeds. It can also fail under selective observation: a system that reports only highly reliable candidate cycles has changed the population over which the bound is assessed. The selection procedure and information available to the estimator must therefore be recorded.

Even under Equation (29), shorter circuits need not have greater *expected monetary reduction*. If a longer cycle carries a much larger compatible capacity, the product of capacity and survival probability may exceed that of a small cycle. Further, a clearing proposal can be revised in response to failures. The analysis of a fixed proposed cycle should be distinguished from the probability that some adapted

sequence of alternative cycles eventually effects the desired reduction. Adaptive recomputation can improve operational robustness while creating coordination costs, revisions of legal authority and additional dependencies. None of these outcomes can be deduced solely from the number of recorded edges.

A useful conditional result follows.

**Proposition 9.1** (Fragility bound for a fixed eligible circuit). *Let  $\gamma$  be a directed cycle whose execution requires the survival of  $L$  specified obligations. If the conditional bound in Equation (29) holds along an ordering of these edges, then its probability of satisfying the listed necessary conditions is at most  $q^L$ . Additional approval conditions can only decrease this probability. The result does not assert monotonicity between two different cycles unless the same bound and an appropriate comparison of their capacities and dependency structures are imposed.*

*Proof.* The event  $\{I_\gamma = 1\}$  is contained in the intersection of the  $L$  edge-survival events. By repeated conditioning, the probability of that intersection is the product of the successive conditional survival probabilities along the chosen ordering. Each factor is at most  $q$ , hence  $s_h(\gamma) \leq q^L$ .  $\square$

The proposition is intentionally modest. It describes the reliability of a *fixed* proposal in a specified uncertainty environment. It does not imply that historically observed long circuits are typically less valuable, that an AI-mediated exchange becomes fragile because it has many agents, or that the associated rights and productive relations ought to disappear. A network of repeated cooperation may deepen trust and reduce the conditional probability of failure, while a change in law or ecology may abruptly invalidate many edges together. These cases motivate a separate examination of correlation.

### 9.3 Correlated Risks and Network Fragility

A long path may contain many edges and nevertheless face only a few genuinely distinct sources of uncertainty. Shared governance, a common balance sheet, a collective guarantee, a uniform protocol, or a single electricity network can generate positive dependence among edge-survival events. In the extreme, suppose  $X_i = Z$  for all  $i$ , where  $Z$  is one Bernoulli variable with  $\mathbb{P}(Z = 1) = q$ . Then

$$\mathbb{P}\left(\bigcap_{i=1}^L \{X_i = 1\}\right) = q \quad \text{for every } L \geq 1, \quad (30)$$

which, as exhibited by Equation (30), contradicts the inference  $q^L$  whenever  $L > 1$ . This is a counterexample to any universal exponential-decay law based only on edge count. Under another dependence structure, completion may be even less likely than under independence, such as when satisfying one contractual condition makes another harder to satisfy. The sign and magnitude of correlation must be established rather than inferred from network length.

The relevant uncertainty may be hierarchical. Let  $Z$  represent an infrastructural state, such as successful energy provision and validity of a clearing platform, and let the edges be conditionally independent given  $Z$ . Then

$$\mathbb{P}_h(I_\gamma = 1) = \mathbb{E}_h \left[ A_\gamma \prod_{i=1}^L q_i(Z) \right] \quad (31)$$

when the approval indicator in Equation (31) is measurable with respect to  $Z$  and the conditional-independence assumptions are met. A mixture of high-reliability and low-reliability states need not follow one exponential law with a constant empirical rate. In particular, a small chance of a state with almost perfect connectivity can dominate the survival probability of very long paths. Heterogeneous jurisdictions, data provenance, energy supply and social trust may each contribute distinct latent factors; an AI's ability to infer some factors does not cause them to cease affecting production.

Failure also possesses a *structural* dimension. Removing an edge may destroy one simple cycle but leave other circuits that accomplish the same reduction. A network with many edge-disjoint alternate

cycles can remain capable of clearing some obligations after local failures. Conversely, a common upstream dependency, a single legal gatekeeper, or a critical settlement facility may act as a cut across numerous nominally distinct paths. These possibilities suggest studying minimal cuts, redundancy and common-mode failures in addition to shortest-path length. The difference between financial risk and ecological risk is equally significant: relocating a computational facility or automating its maintenance may alter the former while displacing the latter to an unrepresented environment.

The distribution of risks is itself a governance issue. One party may have privileged access to real-time measurements, while another bears an interruption cost it cannot observe or price. An AI-operated clearing network might announce a high success probability using a model trained on successful settlements while excluding claims that never gained institutional recognition. Such estimates can be misleading even if their computations are internally correct. Reliability studies therefore require a declared observation boundary, accounting for rejected proposals, documented missingness, and an audit trail of later revisions. The need for these practices connects the present computational model with the provisional infrastructure and trust agenda of Section 11; it does not substitute a statistical estimate for a legitimate decision.

#### 9.4 Higher-Order Circulation Contributions

A further obstacle to truncation comes from the number of candidate circuits. A single length- $L$  cycle may become increasingly unlikely, yet the set of such cycles may grow rapidly with  $L$  and network size. Let  $\Gamma_L(G_h)$  be the set of distinct directed simple cycles of length  $L$  in a finite compatible claim multigraph, identifying cyclic rotations of an identical edge sequence. Let  $N_L = |\Gamma_L(G_h)|$ . If each vertex has at most  $\Delta$  outgoing edge choices, a deliberately loose bound is

$$N_L \leq n\Delta^L, \quad n = |P_h|. \quad (32)$$

For a graph with parallel edges,  $\Delta$  counts edges with multiplicity. Actual cycle counts can be much smaller, and no simple cycle in a graph of  $n$  positions has length exceeding  $n$ . The loose bound in Equation (32) is useful when reasoning about a family of networks whose sizes increase. It is not a prediction that long cycles proliferate in a particular economy.

The first diagnostic is an *additive potential*, not an assertion that every candidate can be cleared simultaneously. Assign each cycle a nonnegative nominal weight  $w_\gamma$  and define

$$W_h = \sum_{\gamma \in \Gamma(G_h)} w_\gamma I_\gamma, \quad W_h^{(K)} = \sum_{\substack{\gamma \in \Gamma(G_h) \\ \ell(\gamma) \leq K}} w_\gamma I_\gamma. \quad (33)$$

Weights might encode a unit benefit assigned to discovering an eligible clearing opportunity, but they must not be read as heterogeneous social values. Two cycles may share the same monetary obligation, so  $W_h$  can double-count the same capacity. A reliable long-path approximation for  $W_h$  alone is therefore not a theorem about maximal feasible clearing. It supplies a preliminary screening criterion and motivates a second, capacity-constrained formulation below.

If  $w_\gamma \leq \bar{w}$  and the conditional survival bound applies to each candidate, Equations (32) and (33) give

$$0 \leq \mathbb{E}_h[W_h - W_h^{(K)}] \leq n\bar{w} \sum_{L > K} (\Delta q)^L. \quad (34)$$

For the bound in Equation (34), when  $r = \Delta q < 1$ , the right side is at most  $n\bar{w} r^{K+1}/(1-r)$ . This calculation displays the balance that matters: survival probability must decay fast enough to offset the growth in possible circuits and their weights. With  $r \geq 1$ , the geometric domination gives no useful small-tail guarantee, even though the actual finite graph still has only finitely many simple cycles. If higher-order weights increase with length or all candidates share favorable common shocks, stronger assumptions are needed. It would be mistaken to promote *path fragility* itself into an unconditional computational advantage.

The condition  $\Delta q < 1$  is sufficient and commonly conservative. Real networks can have structural restrictions, typed edge incompatibilities, low cycle density or a nonuniform distribution of failure rates. A sharper analysis may use a type-aware transition operator, conditional reliability estimates or empirically measured cycle counts. For an illustrative independent-edge model, a nonbacktracking transition matrix whose entries weight admissible consecutive edges by their survival probabilities can sometimes provide a tighter growth estimate than the uniform product  $\Delta q$ . Such a spectral criterion requires well-defined graph states, compatible edge types and a justified probabilistic interpretation; it does not follow from the homology of a relational two-complex alone.

## 9.5 Convergence and Truncation Bounds

To analyze executable clearing rather than a diagnostic sum, let  $x_e \geq 0$  be the amount of each compatible divisible claim in the reference configuration. Conditional on a realized set of eligible simple cycles, choose  $y_\gamma \geq 0$  as the amount cancelled along each cycle, subject to the shared capacity constraints

$$\sum_{\gamma \ni e} y_\gamma \leq x_e \quad (e \in E_h), \quad 0 \leq y_\gamma \leq c_\gamma I_\gamma, \quad (35)$$

where  $c_\gamma = \min_{e \in \gamma} x_e$ . Define the gross cancellation objective and its truncated version by

$$F_h = \max_y \sum_{\gamma \in \Gamma(G_h)} \ell(\gamma) y_\gamma, \quad F_h^{(K)} = \max_{\substack{y \text{ feasible} \\ y_\gamma = 0 \text{ if } \ell(\gamma) > K}} \sum_{\gamma} \ell(\gamma) y_\gamma. \quad (36)$$

The factor  $\ell(\gamma)$  converts one unit of cycle reduction to the corresponding decrease in the sum of gross recorded debt across its edges. Equations (35)–(36) account for overlapping circuits by preventing the same edge capacity from being cancelled twice. They still presume a fixed and compatible monetary type, divisible quantities, and already authorized cycle execution. If institutional decisions involve indivisible claims, sequencing rules, nontransferability, or collective approval, the feasible set changes and so can the computational complexity.

For any realized eligible-cycle set,  $F_h^{(K)} \leq F_h$ , and the truncated optimum is nondecreasing with  $K$ . The gap is bounded by the potential contribution of excluded cycles in an optimal solution. This yields a quantitative sufficient condition for the intended approximation.

**Proposition 9.2** (Expected omitted-cycle bound). *Suppose a finite compatible claim multigraph has  $n$  account positions, maximum outgoing edge degree  $\Delta \geq 1$ , and capacity bound  $c_\gamma \leq \bar{c}$  for all simple cycles. Assume each candidate cycle satisfies  $\mathbb{P}_h(I_\gamma = 1) \leq q^{\ell(\gamma)}$  for one  $q \in [0, 1)$ , and let  $F_h, F_h^{(K)}$  be defined by Equation (36). If  $r = \Delta q < 1$ , then*

$$0 \leq \mathbb{E}_h[F_h - F_h^{(K)}] \leq n\bar{c} \frac{r^{K+1}((K+1) - Kr)}{(1-r)^2}. \quad (37)$$

The bound holds without independence between different cycles and is also valid when the largest admissible cycle length is smaller than the nominal infinite summation range.

*Proof.* Fix a realization of the eligibility indicators and take an optimal feasible family  $(y_\gamma^*)_\gamma$  for  $F_h$ . Removing every term with  $\ell(\gamma) > K$  preserves all constraints in Equation (35), so the remaining family is feasible for  $F_h^{(K)}$ . Therefore

$$0 \leq F_h - F_h^{(K)} \leq \sum_{\ell(\gamma) > K} \ell(\gamma) y_\gamma^* \leq \sum_{\ell(\gamma) > K} \ell(\gamma) c_\gamma I_\gamma.$$

Taking expectations, applying  $c_\gamma \leq \bar{c}$  and the survival bound, and grouping cycles by length yield

$$\mathbb{E}_h[F_h - F_h^{(K)}] \leq \bar{c} \sum_{L > K} L N_L q^L \leq n\bar{c} \sum_{L=K+1}^{\infty} L r^L.$$

Table 7: Conditions affecting the reliability of high-order circulation truncation. The statements concern the specified formal model, not observed economic regularities.

| Condition                          | Analytical effect                                      | Required scrutiny                    |
|------------------------------------|--------------------------------------------------------|--------------------------------------|
| Conditional edge survival $\leq q$ | Bounds survival of each fixed $L$ -edge cycle by $q^L$ | Common shocks; candidate selection   |
| Cycle count $N_L \leq n\Delta^L$   | Bounds growth in candidate enumeration                 | Typed or institutional exclusion     |
| Bounded cycle capacity $\bar{c}$   | Controls the amount one cycle can contribute           | Denomination; divisibility           |
| $\Delta q < 1$                     | Supplies geometric domination of omitted orders        | May be conservative or violated      |
| Capacity constraints               | Prevent simultaneous use of the same obligation twice  | Authorized discharge and consent     |
| Stable reference configuration     | Makes a truncated objective well-defined               | Structural change and model revision |

For  $0 \leq r < 1$ , the elementary geometric-series identity gives  $\sum_{L=K+1}^{\infty} Lr^L = r^{K+1}((K+1) - Kr)/(1-r)^2$ , proving Equation (37). No assertion about the joint distribution of different cycle indicators was used.  $\square$

Equation (37) is an *expectation* bound, not a deterministic guarantee for every configuration. Markov’s inequality gives the further diagnostic

$$\mathbb{P}_h(F_h - F_h^{(K)} > \varepsilon) \leq \frac{n\bar{c}}{\varepsilon} \frac{r^{K+1}((K+1) - Kr)}{(1-r)^2} \quad (38)$$

for any  $\varepsilon > 0$ , as expressed in Equation (38). Here the monetary unit of  $\varepsilon$  must match that of the objective. A network operator can use such a conservative inequality only after substantiating its input bounds. If  $r$  is unknown, confidence in the approximation must be reported as conditional or be supported by a separate estimation procedure. The inequality itself provides no evidence that the assumptions hold in a given international clearing system.

The result has two limits worth emphasizing. First, the space of finite simple cycles is already finite for each fixed claim graph, so no literal infinity is being physically cancelled. “High order” here concerns growing families of possible circuits, extensive networks or repeated reconfiguration; the infinite geometric sum is a mathematical majorant. Second, setting  $K$  too small may eliminate precisely those long cycles that are crucial for clearing a particular peripheral obligation. An aggregate expected bound cannot replace an individual explanation of which claims remain excluded, whose access is affected, or whether the approximation distributes losses unequally. In some institutional contexts an apparently small omitted amount may be normatively decisive.

## 9.6 Computational Approximation and Error Control

A practicable clearing system needs algorithms and records of error, not simply a statement that very long circuits are unlikely. The proposed workflow begins with an institutionally recognized, typed claim ledger and applies deterministic admissibility checks. Claims are grouped only when the relevant monetary units, maturities and setoff permissions can be made compatible under publicly declared rules. Next, one identifies the positive-claim subgraph, examines its strongly connected components, and assesses whether any eligible directed circulation is possible. Candidate cycles may be generated only within these components, while incompatible or disputed claims remain outside the computational reduction domain. This procedure is distinct from treating a whole heterogeneous economy as one interchangeable set of debts.

Simple-cycle enumeration can require exponential time in the worst case because the number of simple cycles may itself be exponential in graph size. Yet the finite homogeneous, divisible circulation problem of Section 8 can also be approached directly through linear programming or network-flow methods on the incidence constraints, without listing every simple cycle. This distinction is essential. A

long-cycle truncation rule may be useful for probabilistic prospecting, interpretability, bounded-depth agreement formation or distributed communication budgets; it is *not* automatically required to compute the deterministic maximum reduction in a compatible finite ledger. In particular, the existence of an exponential family of cycles does not imply that a compact incidence-matrix optimization is intractable. Additional nonconvex governance conditions, shared risk budgets, combinatorial approval groups and indivisible obligations can make the task harder, and their contributions to difficulty should be isolated.

One conceptual implementation can treat  $K$  as a declared computational budget rather than an ontological cutoff. Starting from short compatible cycles, the system computes a feasible reduction using actual edge capacities and admissible approvals, then expands  $K$  where potential omitted benefit, critical claimant exposure or model uncertainty warrants further search. For probabilistic planning it reports an upper bound on omitted gross reduction, with the assumptions from Proposition 9.2 stated alongside the result. For enacted settlement it records the final authorized operations and residual positions, so that errors in a forecast cannot be confused with the legal status of obligations. The residual network must be recalculated after an actual cancellation event, because capacities, claims and authorizations may have changed.

There is no reason to suppose a single global evaluator can decide which circulation is socially optimal. Gross nominal cancellation is one explicit diagnostic, but participants may care about continuity of service, distribution of liquidity, continuity of access, ecological repair, or avoiding concentrated control over shared infrastructure. A multiobjective or constrained process may retain long paths that contribute little to nominal clearing but provide important redundancy. If each participating organization has different access to confidential information, privacy-preserving aggregation or distributed verification may be useful, though these methods introduce their own trust and governance requirements. The present paper cannot infer institutional legitimacy from a complexity bound; it leaves concrete global data and public-infrastructure designs to subsequent studies.

Error control also depends on model provenance. Every cycle probability, capacity and compatibility judgment should be linked to its evidence and revision history. An AI system's inferred  $q$  can be calibrated on historical opportunities only if the population of failed and unobserved attempts is adequately characterized. Distribution shift, new legal rules, infrastructure degradation and strategic manipulation can invalidate a previously accepted bound. Hence a reliable implementation must periodically revise its candidate universe, report sensitivity to uncertain parameters and disclose when a proposed cutoff depends on unverifiable assumptions. Faster automated discovery of cycles does not remove the need to authorize claims, to honor objections or to trace the external consequences of productive coordination.

A final point concerns the difference between *approximate recognition* and *actual dissolution*. An optimization algorithm might omit tiny expected contributions from high-order opportunities while the corresponding relationships, needs, promised performances and possible future exchanges continue to exist. Conversely, an institution may authorize cancellation of a large cycle without using any probabilistic forecast. Truncation is therefore a choice about the computational representation and search effort, not an action that extinguishes claims. It would be misleading to describe a cycle as historically dissolved merely because its estimated weight fell below an analyst's threshold.

## 9.7 Topological Robustness and Circulation Constraints

The network formalism has a topological interpretation, but care is required in transferring this interpretation between representational levels. For the monetary claim graph, the equation  $B_h z = 0$  identifies a zero-boundary one-chain. In a finite graph, nonnegative circulations decompose into directed cycles as Proposition 8.1 established. An event-level relational two-complex  $K_h$  carries its own chain groups and boundary operators, with vertices representing events, edges representing couplings, and faces representing primitives whose semantics must be assigned by the relevant research model. A cycle in the account graph is not automatically a cycle in the event complex. Nor does a nontrivial homology class independently identify an enforceable liability or a valid netting agreement [11].

For an event two-complex, one may examine

$$H_1(K_h; \mathbb{R}) = \frac{\ker(\partial_1 : C_1 \rightarrow C_0)}{\text{im}(\partial_2 : C_2 \rightarrow C_1)}. \quad (39)$$

The quotient in Equation (39) distinguishes one-cycles up to boundaries of two-cells in the declared representation. Deleting an edge, attaching a face or identifying event configurations can change its rank, although such operations need not correspond to cancellation of a financial claim. The topological structure is useful for studying the persistence or transformation of modeled relational circuits across admissible continuations. A monetary clearing rule, however, must still be specified in the separate claim-and-authority layer. Any map between event-level cycles and account-level obligations must be constructed and validated; it cannot be supplied by notation alone.

Path fragility may coexist with structural robustness. Individual long circuits can fail frequently while a network possesses several alternative compatible circulations. For a given eligible claim graph, edge connectivity, directed cuts, cycle overlap and failure correlation may yield better indicators of service continuity than the length of any single circuit. A network that is topologically connected may nevertheless lack a compatible legal clearing circuit. Conversely, a permitted multilateral discharge may be possible through institutional novation or guaranteed settlement without requiring the original value-producing acts to form a closed topological path. A formalism that conflates these cases would overstate its explanatory scope.

This analysis also complicates a simple preference for shorter pathways. Pruning all higher-order circuits can concentrate exchange on a few trusted intermediaries, increasing dependence on their infrastructure and power. A longer route can support participation by peripheral communities or add resilience against the failure of one central service. In the context of AI-operated production, the ability to calculate short clearing cycles efficiently may reduce one class of monetary coordination costs while reproducing another class of asymmetric access to energy, data or legal authority. Normatively appropriate approximations may therefore require safeguards for underrepresented participants and reporting of omitted alternatives, even where a small monetary tail bound is mathematically justified.

The resulting thesis is more precise than the initial claim that long paths are intrinsically fragile. *When compatible circuits carry sufficiently many conditionally fallible dependencies, and the growth of eligible alternatives and capacities is controlled, higher-order contributions to gross monetary clearing admit computable truncation bounds.* Outside these assumptions, length alone offers neither a reliable survival law nor a guarantee of algorithmic tractability. This limitation is a positive result for the present investigation: it identifies what evidence, institutional description and mathematical structure would be needed to turn a suggestive generative-relational intuition into a falsifiable claim. Section 10 turns from prospective clearing opportunities to the persistence, migration and regeneration of monetary relations, including the creation of new obligations after apparently successful local dissolution.

## 10 Persistence and Regeneration of Monetary Relations

Sections 7–9 examined the local reduction of compatible claims, the formal and institutional requirements for clearing, and the conditions under which distant circulation alternatives can be approximated. Those analyses describe important ways in which the need for particular monetary settlements can diminish. They do not establish that the productive and relational circumstances giving rise to money are exhausted. A successful clearing operation changes a recognized obligation configuration; it need not alter the continuing production of new needs, the allocation of capabilities, the control of shared resources, or the terms under which new participants enter an exchange. Nor does an AI system's growing capacity to satisfy some human requirements entail that particular humans or other AI systems cease to matter as possible collaborators, counterparties, sources of experience, or bearers of distinct rights and claims.

The present section consequently considers monetary *regeneration* alongside monetary dissolution. Regeneration encompasses the appearance of a monetary claim where none was previously recognized, the return of a monetary condition after a period of nonmonetary coordination, the extension of existing

Table 8: Distinct routes by which monetary mediation can persist or re-emerge. The table describes conditional mechanisms, not necessary consequences of an underlying need.

| Relational development   | Possible monetary mechanism                                              | Alternative or qualification   |
|--------------------------|--------------------------------------------------------------------------|--------------------------------|
| Encounter or membership  | Admission fees, subscriptions, paid access, or transferable entry rights | Open entry; common funding     |
| New specialized capacity | Paid services, licensing, or usage-based claims                          | Reciprocal or public provision |
| Investment or credit     | Financing claims, repayment schedules, or collateral                     | Grants; shared funding         |
| Infrastructure control   | Metered power, compute, data, equipment, or access fees                  | Shared allocation under limits |
| Cross-boundary exchange  | Settlement fees, guarantees, insurance, or intermediaries                | Mutual offset; trust           |
| Dispute or exclusion     | Compensation, penalties, or collateral requirements                      | Restitution without money      |

units of account to new activities, and the constitution of new credit relationships. Its counterpart, dissolution, encompasses a different family of events: cancellation of eligible obligations, substitution of nonmonetary coordination, reduction of paid access requirements, or transformation of the production conditions that had supported monetary demands. Both families may be observed in an evolving relational history. The central methodological requirement is to avoid inferring the total disappearance of money from a locally successful settlement, and equally to avoid inferring the inevitability of monetary reappearance merely because some need for an other remains.

We treat the continuation of monetary arrangements as an open analytical question, rather than a predetermined historical stage. Marxian analyses of monetary forms and relations of production, institutional theories of credit, and scholarship on collective resource governance suggest different causal channels for monetary formation and persistence [21, 15, 23]. Our earlier exploratory work examined the coexistence of monetary dissolution and alternative value circuits [12, 14, 7]. Here the question is narrower in formal purpose but broader in historical implication: what events, rules, capacities, and asymmetries can make monetary mediation reappear after it has been locally reduced, and how should such reappearance be represented without assuming that all heterogeneous value exchanges are commensurable?

## 10.1 Persistent and Emerging Needs for Others

The hypothesis that monetary relations are connected with needs for others does not assert that all needs become monetary demands. A need may be met through voluntary contribution, reciprocal participation, kinship and care, publicly provided services, commons governance, or other arrangements. The motivating claim is instead that the diversity and historical formation of needs provide occasions on which institutions can make access, performance, and obligation conditional upon monetary transfers. As argued in Section 4, the genesis of a need, the genesis of an exchange, and the institutional recognition of a monetary claim are analytically different events. Monetary dissolution can interrupt the third process without abolishing the first two.

The importance of an other is not exhausted by instrumental substitution. A person may need the presence of a particular collaborator, the judgment of an identifiable research community, the recognition of a professional organization, or the opportunity to participate in a relation whose meaning depends on the identities and histories of its participants. In these cases, an AI system providing functionally similar information or services does not necessarily provide the same encounter. Likewise, an AI-operated institution may need a specific human decision, an independently obtained measurement, an authorized signature, or a relationship of continuing responsibility. Whether such relations ought to involve monetary payment is a separate institutional and normative issue. The persistence of singular relations limits an inference from generalized productive competence to complete substitution of others; it does not justify treating every singular relation as a commodity.

Needs can also arise through the transformation of a history itself. New scientific instruments make previously inaccessible questions investigable; an ocean-observing network identifies an ecological disturbance that requires new forms of care; a common data infrastructure enables collaborators to develop

a capability that they subsequently depend upon. Such developments alter both the possible objects of exchange and the agents or institutions that can participate. In a generative-relational interpretation, a new need is not simply a pre-existing preference revealed at a later timestamp. It may be constituted through the creation of a practice, an encounter, a vulnerability, or a collective responsibility. For this reason, the theory cannot take a fixed list of demands as a complete account of the monetary possibilities of all admissible continuations.

There is an important asymmetry between needs and debts. The absence of an outstanding monetary balance is an assertion about a specified claim ledger and boundary. The absence of unmet needs would be an altogether different proposition, requiring a theory of needs and a determination of which beings and institutions are included. No clearing theorem can bridge that difference without further assumptions. In particular, a balanced triadic circuit may extinguish the eligible claims of three organizations, even as the same organizations require continued maintenance, specialized care, knowledge verification, or material resources. Their need for renewed collaboration is compatible with the complete cancellation of the original claims.

One must also distinguish newly emerging needs from needs created through coercive scarcity or exclusion. A membership fee imposed upon a previously open commons does not demonstrate that its users suddenly acquired a novel intrinsic need; the rule of access has changed. Similarly, privatization of a computing facility can convert a continuing physical requirement into a payable demand. Such cases show that monetary regeneration can be driven by changes in institutional control rather than by an increase in productive scarcity or social benefit. They motivate the later analyses of power, contestability, and remedy in Section 12.

## 10.2 New Capacities and Exchange Relations

Generative production transforms the space of possible exchanges by creating capacities whose conditions of access have not yet been stabilized. A newly developed model may perform an unfamiliar scientific inference; a robotic platform may become capable of repairing an energy installation; a distributed archive may provide provenance-sensitive observations that another system cannot readily reproduce. The existence of these capacities can motivate new collaborations. Their exchange need not resemble the sale of a standardized good: participation may involve supervised access, a temporary license, shared experimental responsibility, data stewardship, joint ownership of an improvement, or a contingent contribution to another project.

The character of such relations is affected by whether the capacity is rival, excludable, context-dependent, or reproducible. A particular robot can operate at only a limited number of physical sites at once, whereas some software outputs can be copied at low marginal cost after costly development and maintenance. A marine measurement may be shareable as data while its collection requires scarce vessel time, trained operators, and ecological permits. It would be mistaken to infer the same money-form from these heterogeneous circumstances. The capacity to reproduce a digital output does not necessarily imply autonomous reproduction of the energy, hardware, labor, institutions, or environmental conditions supporting it.

In AI-mediated exchange, the participants identified at an application interface are not always the institutions that exercise the underlying rights. Two AI services may coordinate a task while a single organization owns both, in which case the logged transfer may be an internal accounting allocation rather than an enforceable interparty payment. Conversely, two autonomous operational systems may be managed by independent organizations that hold claims over equipment, intellectual property, energy supplies, or access rights. Their machine-to-machine negotiation can then result in a genuine monetary obligation between recognized legal persons. An analytical classification should therefore record both the *functional sites* of coordination and the *recognized holders* of claims, as established in Section 5.

A productive innovation can reduce money use at one boundary while creating new, more specialized monetary transactions elsewhere. Suppose a publicly accessible AI assistant substitutes for several paid information services. Its users experience reduced direct monetary mediation. Yet operating the

assistant can create new relations involving energy balancing, chip manufacturing, audited data provision, water use, physical maintenance, and cybersecurity. Some of these may be supplied by a common facility, some by market firms, and some through collectively financed institutions. The existence of new upstream transactions is not evidence that the original user-facing substitution failed; it shows that the measurement boundary changed. Nor does the appearance of these transactions imply that every technical improvement raises overall monetary dependence. The composition of the new network and its rules determine which relations are monetized.

The emergence of new capacities can also open nonmonetary possibilities. A better coordinated cooperative may replace separate contracts with shared scheduling; a scientific commons may allow participants to reuse verified observations without repeated licensing; a public AI facility may offer essential applications without user charges. These forms can retain accounting, cost assessment, and allocation rules, but their constitutive obligations need not take the form of bilateral monetary debt. The relevant question is whether coordination relies upon recognized transferable monetary claims, rather than whether the system uses numerical measures or automated bookkeeping. This distinction remains central when evaluating claims that AI will transform the economic importance of money.

### 10.3 Monetary Reconstitution and Credit Formation

Clearing changes the configuration of claims that have already been recognized. Monetary reconstitution concerns the creation or renewed recognition of claims in subsequent, partially ordered events. A natural point of departure is the eligible claim vector  $x_h \in \mathbb{R}_{\geq 0}^{E_h}$  at configuration  $h$ . Let  $z_h$  denote an institutionally authorized reduction satisfying the appropriate compatibility and incidence constraints from Section 8. The residual is  $r_h = x_h - z_h$ . To compare the residual with claims recognized at a continuation  $h'$ , introduce a specified transport or correspondence map  $T_{h \rightsquigarrow h'}$ , and a nonnegative vector  $n_{h'}$  of newly recognized claims. A simple, deliberately conditional accounting representation is

$$x_{h'} = T_{h \rightsquigarrow h'}(x_h - z_h) + n_{h'}, \quad 0 \leq z_h \leq x_h. \quad (40)$$

The components in Equation (40) have meaning only after a common, compatible claim coordinate system is specified. If claims change denomination, maturity, holder, or legal identity, the map  $T_{h \rightsquigarrow h'}$  cannot be silently taken as an identity. New instruments, novation, write-offs, exchange-rate conversion and disputed legal status may require additional transformation terms or a different state space entirely. The vector  $n_{h'}$  refers to recognized monetary claims, not to all value newly generated by productive activity.

Equation (40) clarifies a logical possibility that global narratives of disappearance often suppress. Local clearing may eliminate every eligible claim within one selected account network, yet a distinct productive or institutional event may constitute a new set of payable obligations. Under fixed claim coordinates, even a zero residual  $x_h - z_h = 0$  permits  $x_{h'} = n_{h'} > 0$ . This is not a paradox: the cancelled claims and newly constituted claims have different historical and institutional origins. A further cycle may eventually clear some of the new claims, but its eligibility and enactment require a new assessment. Repeating the operation does not imply either convergence toward zero or unlimited growth.

**Proposition 10.1** (Local clearing and subsequent monetary regeneration). *Consider a sequence of two admissibly related claim configurations represented in one compatible nonnegative coordinate system. Suppose the first admits a legally authorized full reduction  $z_h = x_h$ . Under the accounting representation in Equation (40), the continued configuration has  $x_{h'} = n_{h'}$ . Hence local complete dissolution is compatible with strictly positive continued monetary claims whenever  $n_{h'} \neq 0$ . Conversely, if  $n_{h'} = 0$  and the transported residual is zero, then  $x_{h'} = 0$  within that specified ledger; nothing follows about unmet needs, productive exchange, or monetary claims beyond its boundary.*

*Proof.* Substitute  $z_h = x_h$  into Equation (40). The transported residual is the zero vector, leaving  $x_{h'} = n_{h'}$ . The converse follows by the same identity. The conclusion is confined to the specified coordinates and recognition rules.  $\square$

The proposition is elementary by construction; its purpose is to prevent a category error, not to establish an empirical regularity. Reconstitution also need not occur *after* dissolution in a universal clock order. Distinct events may be incomparable in an underlying partial ordering: one subnetwork enacts a clearing while another independently generates credit. Observational reports may later aggregate these events into one accounting period, giving a net stock change that conceals their different mechanisms. Indeed, the coexistence of positive and negative monetary changes may be more informative than their numerical sum. Reporting only a single economy-wide monetary aggregate would miss the relation-specific distribution of monetary mediation emphasized throughout this paper.

Credit is a particularly important form of reconstitution because it joins recognition, enforceability, future-contingent performance, and trust. A provider may deliver energy before receiving compensation; a cooperative may recognize a member's contribution as a redeemable claim; an AI-operated maintenance system may request an advance against an institutionally approved service contract. These arrangements become monetary credit only when the recognized instrument has the requisite denomination, transfer, redemption, or settlement properties. A confidence score, internal reputation balance, anticipated favor, or probabilistic forecast is not automatically money or a debt. Likewise, the existence of a sophisticated prediction model cannot itself convert an expected reciprocal benefit into a present legally valid claim.

Trust can influence monetary reconstitution in opposite directions. Reliable mutual recognition and shared records can make it easier to clear claims without serial payments, or to replace paid access by cooperative provisioning. The same improvements can also make credit creation less costly and thereby expand the set of enforceable monetary relations. A trusted network may confidently extend new loans, issue new claims, or admit new participants to an interoperable settlement arrangement. Consequently, no monotone connection between trust and the quantity of money follows from a generative-relational view. The institutional form and distribution of authority must be made explicit; Section 11 discusses these prerequisites preliminarily, and companion governance research must investigate them in greater depth.

#### 10.4 Resource Control and Renewed Dependencies

Production cannot be understood through the performance of tasks alone. An AI agent might possess the knowledge required to manage a data center while lacking the legal right to operate it, the electrical power to run it, the materials required to replace damaged components, or authorization to obtain relevant observational data. In such a case, nominal autonomy at the application layer coexists with dependence upon owners, maintainers, resource suppliers, and governing institutions. The terms of this dependence shape whether monetary transactions are needed and who is entitled to demand them. The analysis of productive reproduction in Section 5 therefore bears directly upon the regeneration of monetary relations.

For clarity, distinguish a material dependence relation  $D_h^{\text{mat}}$ , an informational dependence relation  $D_h^{\text{info}}$ , and an institutional-permission relation  $D_h^{\text{inst}}$ . These relations may overlap but do not coincide. A network may possess sufficient physical generating capacity while needing an externally controlled grid connection; it may generate high-quality synthetic training examples while depending on independent observations for empirical validity; it may own robots capable of manufacturing replacement parts while requiring rare materials obtained through separately governed extraction. Reducing one component does not imply that the others disappear. Nor does self-sufficiency of a particular corporation or AI network imply ecological independence: energy conversion, material extraction, waste management and ecological impact extend beyond a firm's ledger.

New dependencies can be formed through the institution of exclusion, even when physical conditions remain unchanged. Enclosing an open dataset behind a contractual access rule, imposing exclusive control over a formerly communal interface, or concentrating rights to an essential robot-maintenance standard can make monetary claims newly possible. Such claims may be lawful under a given regime while still inviting criticism concerning legitimacy and distributive effects. It is equally possible for a public provider to adopt a usage charge for demand management without transferring ownership into a

private monopoly. The mere appearance of money does not identify the motive or justice of an arrangement. What matters analytically is which institution has the authority to set the access condition, what alternatives are feasible, and whether affected participants may contest or revise the arrangement [21, 24, 23].

Infrastructure interdependence also means that monetary relations can be *displaced* rather than dissolved. Consider a regional AI service that becomes free of charge to individual users because a municipal organization funds its electricity, hardware and model maintenance. The human–AI payment relation at the user interface has diminished, but tax collection, public procurement and utility settlement may continue elsewhere. This arrangement can be socially beneficial without constituting complete system-wide demonetization. A commons-operated facility may instead substitute pooled contributions, shared maintenance duties or rationing rules for some of those payments. Even then, remaining external obligations should be traced to their actual holders rather than declared nonexistent by a change in descriptive vocabulary.

The ecological domain sharpens this point. If a production network exchanges services without monetary payment while consuming an unpriced ecosystem’s regenerative capacity, the absence of market settlement does not entail that the ecological obligation has been satisfied. Nor should a price be presumed to measure ecological value exhaustively. A finite monetary account can display zero net claims while habitat damage, resource depletion or exclusion of affected communities remains unresolved. In a generative framework, the conditions permitting further value creation include ecological and institutional structures that may be transformed by production itself. The question of whose interests are represented and how environmental obligations receive practical recognition cannot be solved by monetary-cycle algebra. It is a substantive problem for the environmental, public-infrastructure and governance inquiries developed in the companion papers.

## 10.5 Local Dissolution and Systemic Remonetization

A central error in evaluating economic automation is to measure only whether people visibly pay for a service. A zero price at one interface can coexist with substantial monetary dependence in infrastructure, debt financing, intellectual-property agreements, public procurement or supply chains. Conversely, a large volume of internal transfer prices may be used to coordinate departments under common ownership without adding an equivalent quantity of independent settlement obligations. An account of *systemic remonetization* must therefore trace legally or institutionally meaningful claims across reporting boundaries and distinguish those claims from numerical tokens and nonbinding cost allocations.

Let  $L_h$  denote the full set of recognized claim links within a stated observation domain, and let  $E_h^{HH}$ ,  $E_h^{HA}$  and  $E_h^{AA}$  denote links classified by the operational counterpart types at which they are observed. An AI-to-AI operational link must be connected to its recognized claim holders before being counted as an external obligation. If an analyst measures the gross recognized liability  $\mathcal{M}_h(S) = \sum_{e \in S} x_{h,e}$  for a selected compatible subset  $S \subseteq L_h$ , a fall in  $\mathcal{M}_h(E_h^{HA})$  does not algebraically require a fall in  $\mathcal{M}_h(L_h)$ . The complement may contain continuing or newly recognized claims; overlap, internal reallocations, denomination and aggregation conventions must be addressed to make comparisons meaningful. Gross monetary claims also capture only one monetary function. Unit-of-account reliance, financing dependence, price-indexed contracts, and settlement frequency may move differently.

A bounded illustration is helpful. A community gains nonpaying access to an AI-assisted language service operated by an institution. The service relies on compute purchased from another provider, electricity supplied under a long-term contract, and a licensed dataset. The community’s immediate monetary interaction with the AI service may be absent. Meanwhile, the operating institution’s recognized liabilities to suppliers may increase as use grows. An observation limited to end users would register local demonetization; a supplier-side ledger would register monetary expansion; a network-wide analysis would record both, with no immediate basis for claiming that their effects cancel economically or ethically. Alternatively, public investment in low-cost power and shared compute could eventually reduce some supplier-side liabilities without eliminating the continuing costs of ecological protection and

3493 maintenance.

3494 This scenario can be stated as a simple diagnostic without assuming identical units of value. Let  
3495  $M_h^{\text{interface}}$  and  $M_h^{\text{upstream}}$  be separately reported monetary-dependence indicators, each with a declared  
3496 measurement protocol. A possible empirical configuration is

$$3497 \quad \Delta_{h,h'} M^{\text{interface}} < 0, \quad \Delta_{h,h'} M^{\text{upstream}} > 0, \quad (41)$$

3498 where  $\Delta_{h,h'}$  is a comparison across specified, admissibly related configurations. Equation (41) is a  
3499 possible observation, not a universally valid dynamic law. It is not an invitation to add the indicators  
3500 without checking their denominators, boundaries and underlying obligations. In particular, an increase in  
3501 contract value may result from inflation, increased output, changed maturity structure or a shift in asset  
3502 prices, rather than a deeper dependence on money as a coordination mechanism.

3503 Network-wide measures should further address the creation of new links and the disappearance of  
3504 old ones. If AI becomes capable of supplying a former paid service without charging users, several  
3505 human-to-human exchange links may cease to be monetized. Yet the availability of new AI capabilities  
3506 may generate demand for complementary human contributions, including independent evaluation, cre-  
3507 ative work, care, ecological monitoring and governance. Those contributions can be publicly funded,  
3508 reciprocally supplied or monetized under different institutional arrangements. Monetary regeneration  
3509 and dissolution are consequently relationally selective: they depend not only on available capabilities  
3510 but on the choice and contestation of terms of access to those capabilities.

3511 Systemic remonetization is not necessarily an undesirable reversal. Some new monetary claims can  
3512 finance essential infrastructure, recognize costly labor, compensate harms or permit voluntary risk shar-  
3513 ing. Other claims can institutionalize exclusion, extract value from common contributions or consolidate  
3514 asymmetric control. Equally, a decrease in measured monetary activity may indicate improved common  
3515 provision or a breakdown in the capacity to secure necessary goods. The normative evaluation of either  
3516 direction must therefore be independent of the nominal sign of change. The relevant question is whether  
3517 affected persons, organizations and ecological relations retain meaningful capacities to create, circulate  
3518 and govern value, and whether claims are recognizable, contestable and capable of legitimate discharge.

## 3519 10.6 Open-Ended Evolution of Monetary Relations

3520 The interaction between monetary dissolution and regeneration does not select a unique endpoint. A  
3521 highly capable AI production network could expand common provisioning, reduce paid interfaces and  
3522 conduct authorized multilateral clearing. It could also intensify ownership concentration, erect new ac-  
3523 cess barriers, establish new credit systems, or generate requirements for resources that remain externally  
3524 controlled. Multiple configurations may coexist within a society and across institutions. A conditional  
3525 ordering in which human–human payment dependence declines, human–AI payments shift into shared  
3526 infrastructure, and AI–AI settlement persists is therefore one admissible hypothesis among others, not a  
3527 natural law of historical succession.

3528 To express this openness without imposing a universal time coordinate, let  $\mathfrak{C}(h)$  again denote the ad-  
3529 missible continuations of a represented configuration  $h$ , and let  $\Phi_{h \rightsquigarrow h'}$  denote a relation-specific transfor-  
3530 mation of productive capacities, ownership, recognized claims and governance rules when a continuation  
3531  $h'$  is selected. The monetary part of this transformation may be understood schematically as

$$3532 \quad \mathfrak{M}(h) = \{(h', x_{h'}, \iota_{h'}) : h' \in \mathfrak{C}(h), (x_{h'}, \iota_{h'}) \in \Phi_{h \rightsquigarrow h'}(x_h, \iota_h)\}, \quad (42)$$

3533 In Equation (42),  $\iota$  records relevant institutional recognition conditions. This set is a representational  
3534 device, not a catalogue of objectively predetermined futures. Different models can disagree about which  
3535 continuations are admissible, what claims would be recognized, and what capacities or ecological impacts  
3536 would result. The record of a realized or reported history likewise does not provide a privileged global  
3537 present from which all other relations can be unproblematically classified as past or future.

3538 The dependence on institutions makes the nonmonotonic character of the transformation especially  
3539 significant. A clearing innovation may reduce outstanding compatible obligations; improved access

may then generate new cooperation and credit; a subsequent governance reform may replace the credit with common allocation; a new ecological or technological dependency may make distinct settlement instruments useful again. It would be artificial to force these transformations into a single scalar trajectory. They may occur in different subcomplexes, through differently ordered events and under incomparable norms of evaluation. Moreover, institutional innovations may change the very criteria used to identify a monetary claim, so a longitudinal series needs an explicit account of classification changes rather than assuming its measured object remains invariant.

This open-endedness does not make the theory unfalsifiable. Specific conditional propositions can be tested. For example, the assertion that improved AI productive autonomy reduces human–AI monetary reliance can be investigated using declared indices of externally controlled energy, data, equipment and legal permissions, with controls for ownership and public financing. The hypothesis of high-order circuit fragility can be assessed against observed circuit proposals, dropout events, correlated risks and realized clearing capacities, with the sampling cautions of Section 9. Claims of systemic demonetization can be challenged by tracing upstream liabilities and renewed rights of exclusion. Such studies require multiple measures and transparent observational boundaries; a single aggregate monetary stock cannot answer every question posed by the theory.

The resulting understanding is neither a defense of perpetual monetary exchange nor a prediction of its inevitable abolition. Rather, money is examined as one historically variable form through which heterogeneous actors and institutions mediate needs, capacities, obligations and claims. The distributed dissolution of particular monetary relations may enlarge possibilities for unpriced cooperation and public access. At the same time, the relations that make new forms of value possible can also create new sites of monetary recognition and contestation. The decisive analytical issue is how these processes depend upon the evolving infrastructures and rules through which participants encounter others, reproduce productive conditions and determine whose claims are valid.

Section 11 therefore turns to the infrastructural and institutional circumstances of such transformations. It examines energy and data systems, productive reproduction, epistemic coordination, trust, governance, global public infrastructure, and ecological constraints only to the extent needed to delimit the monetary argument. Detailed engineering of global observation networks, the ecological implications of autonomous robotics, and the generative formation of transnational trust remain distinct research tasks. Maintaining this division of labor allows the present paper to identify enabling conditions and possible failure modes without treating an entire programme of institutional and technical construction as already accomplished.

## 11 Infrastructural and Institutional Conditions

The foregoing sections have distinguished monetary claims from the needs, capacities, and relations through which exchange becomes possible. Their mathematical results are correspondingly conditional. Compatible circuits can be cleared only when a relevant set of obligations is recognized, when the participating claims can be jointly reduced, and when an appropriate institution authorizes the discharge. A production network may supply an additional service without asking its recipient to pay, but the service can still depend upon purchased electricity, legally restricted data, financed machines, or previously remunerated human labour. This section identifies the infrastructural and institutional circumstances under which particular reductions in monetary mediation may become feasible, reproducible, and socially acceptable. It does not propose that sufficiently advanced technology makes social coordination, legal order, or ecological obligations redundant.

The word *condition* requires discrimination. An enabling condition for one mechanism need not be necessary for another. A small set of compatible debts can be mutually discharged under an agreed settlement rule without possessing an autonomous power system or a globally shared database. Conversely, the sustained provision of an AI service without a monetary price for its human users may demand extensive physical supply chains, maintenance institutions, and arrangements for allocating costs. A third mechanism, the substitution of nonmonetary coordination for internal monetary exchange among

Table 9: Distinct enabling conditions and the monetary transformations that they may support. Possession of one condition does not establish system-wide monetary dissolution.

| Condition                                 | Possible contribution to coordination                                                                  | Remaining question                                              |
|-------------------------------------------|--------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------|
| Energy and material reproduction          | Supports continued operation, maintenance, and production without some external purchases              | Resource rights, environmental burdens, replacement cycles      |
| Data and epistemic infrastructure         | Improves shared observation, provenance, compatibility, and identification of reciprocal opportunities | Privacy, unequal access, error, epistemic authority             |
| Computational and organizational capacity | Makes eligible obligations and resource allocations more tractable                                     | Legal authorization, objective choice, concentration of control |
| Trust and commitment institutions         | Supports voluntary coordination, credible clearing, and durable provision                              | Contestability, remedies, new credit creation                   |
| Global public cooperation                 | Makes interoperable observations and shared capabilities possible across jurisdictions                 | Unequal power, security, sustainable funding                    |
| Ecological governance                     | Constrains the transfer of unrecognized costs beyond a ledger or service boundary                      | Impact assessment, representation, monitoring, repair           |

AI-operated units, depends on rules about authority, accountability, scarcity, and resource use that neither debt cancellation nor human-facing free provision automatically supplies. We therefore distinguish *clearing feasibility*, *productive reproducibility*, and *institutional legitimacy* as separate, potentially interdependent questions.

These conditions are also historically situated. The existence of energy and data infrastructures at a represented event does not entitle an observer to assume their indefinite availability in every admissible continuation.

Likewise, *self-sufficiency* is always indexed to a functional objective, a network boundary, and an admissible interval or continuation. A highly automated facility can produce some of its electricity while relying on human-governed mineral extraction, watershed regulation, emergency repair, legal permission, or ecological carrying capacity. Widening the observational boundary may reveal dependencies absent from an operational dashboard. No level of computational intelligence can by itself establish freedom from material and ecological interdependence. Material supplies change, rights can be contested, environmental conditions vary, and institutional rules may be revised. The following analysis uses histories and continuation relations in the sense established in Section 7; it does not require a common physical clock for all participants. Our claims concerning future AI production are consequently hypotheses about possible configurations and transformations of relations, rather than forecasts of a necessary transition to a monetary-free economy.

## 11.1 Productive Autonomy and Material Reproduction

A system's capacity to perform tasks and its capacity to reproduce the conditions of its performance are analytically different. An AI may schedule deliveries, design components, negotiate contracts, or operate industrial machinery while relying on other organizations to fabricate chips, maintain electrical transmission lines, extract materials, calibrate sensors, and replace worn parts. Increasing functional competence in these tasks may diminish particular labour requirements, yet leave intact the monetary claims through which the resources are acquired. The proposition that human–AI monetary exchange will decline with AI self-sufficiency thus requires a more demanding account of *self-sufficiency* than task competence or the automation of a service interface.

It is useful to distinguish three degrees. *Operational autonomy* means that a production unit can execute a specified task with limited immediate external intervention. *Supply autonomy* means that the relevant network has dependable access to necessary inputs without purchasing each input from an outside party. *Reproductive autonomy* concerns the continuing capacity to replace, repair, replenish, and reorganize essential production conditions under changing circumstances. Each degree is relative to a specified set of activities, a represented history, and a boundary of the production network. A component may be internally produced when the boundary encompasses a fabrication facility and externally

purchased when the boundary is drawn around an AI service provider. This boundary dependence is a measurement property that must be disclosed, not a flaw to be hidden.

Let  $A$  denote a specified AI-operated production network,  $K_A$  a finite set of necessary input classes, and  $\omega$  a described operating contingency. For a target service configuration  $q$ , write  $d_k(A, q, \omega)$  for the required provision of input  $k$  and  $s_k(A, h, \omega)$  for the network's internally available, legitimately controlled supply at history  $h$ . An illustrative *bounded operational-closure test* is

$$\text{OC}(A, q, h, \Omega) = 1 \iff s_k(A, h, \omega) \geq d_k(A, q, \omega) \text{ for every } k \in K_A \text{ and } \omega \in \Omega. \quad (43)$$

This diagnostic is intentionally stringent within its specified domain but modest in its implications. It concerns available input classes over the selected contingencies  $\Omega$ ; it does not prove the sustainability of ecological extraction, the existence of rights to use the inputs, the absence of money inside  $A$ , or the ability to reproduce its infrastructure along all continuations. Nor do different physical inputs become mutually commensurable merely because they appear in one vector of requirements.

Autonomy should accordingly be attributed to *networks of production* rather than automatically to individual AI agents. An energy-management system, an equipment-maintenance system, an observational system, and a fabrication system may cooperate to support an AI-operated service. None needs to contain all capacities within itself. Their reciprocal dependence can sustain a nonmonetary cooperative arrangement, a centralized allocation rule, or a dense internal structure of prices and credit. The relevant institutional question is whether participants retain claims against one another, how those claims arise, and whether continued production can occur under nonmonetary or collectively financed coordination. The physical closure of some input flows and the institutional closure of some monetary circuits are different properties.

A productive network may also be open in ways that have intrinsic rather than remediable significance. Human judgment, scientific criticism, ecological observation, and participation by communities may contribute meanings and capabilities that cannot be reduced to interchangeable services. Even if automation can reproduce a technical task, the legitimacy of a decision or the value of an encounter may require particular human or institutional involvement. No general measure of productive autonomy therefore licenses the erasure of distinct needs for others developed in Sections 4 and 10.

## 11.2 Energy and Computational Infrastructure

Electricity, cooling, equipment, interconnection, and physical maintenance condition the operation of AI systems. A future AI-operated network could, in principle, increase its direct access to electricity through locally controlled generation, storage, and dispatch. Yet its energy system would continue to depend on material extraction, transformers, manufacturing, replacement parts, land or seabed access, and arrangements governing the grid. An energy contract might cease to be billed between two particular organizations while newly recognized maintenance, construction, or environmental liabilities arise elsewhere. Energy autonomy can therefore reduce selected monetary interfaces without entailing that energy becomes costless, environmentally neutral, or universally available. Grid-level reliability and lifecycle impact require measurements beyond direct equipment electricity demand: embodied materials, location-specific generation and water conditions, repair cycles, and the consequences of increasing total consumption despite component-level efficiency improvements. The current IEA outlook is an energy-demand scenario, not an environmental life-cycle assessment or evidence that any prospective self-provisioning AI network is ecologically self-sustaining [18].

Current energy analyses provide a reminder of the relevant scale, not evidence that self-reproducing AI infrastructures already exist. The International Energy Agency's 2026 assessment projects global data-centre electricity use rising from approximately 485 TWh in 2025 to approximately 950 TWh in 2030 in its central outlook [18]. Such estimates remain conditional on demand, efficiency, hardware supply, grid connections, and investment. They establish neither the inevitability of increased energy use under every technical trajectory nor any particular future monetary regime. Their role here is to show why a theory of AI monetary dependence cannot treat energy access as an inconsequential background variable.

A useful analytical distinction separates *technical controllability*, *resource entitlement*, and *financing*. A dispatch algorithm can optimize available electricity subject to grid constraints while having no authority to determine who may consume it. A legally recognized entitlement can permit use of a generation asset without ensuring that the asset remains operable. A collectively financed facility can provide electricity without metered charges at the point of use while continuing to incur capital costs and maintenance obligations. These three dimensions may evolve independently. Lower marginal operating expenditure is insufficient to establish monetary dissolution if depreciation, debt service, insurance, or external energy transactions remain denominated and enforceable in money.

Computation has a parallel structure. Models can increase the effectiveness of forecasting, matching, planning, and multilateral clearing. The compatible-flow results in Sections 8 and 9 identify circumstances in which legal and economic eligibility can be represented for efficient calculation. But a planner's capacity to compute a circuit does not confer authority to cancel its participants' claims. Nor does greater computational power ensure a common metric for heterogeneous ecological, expressive, and productive values. In infrastructural terms, intelligent allocation requires more than the processors running an algorithm: it relies on authenticated observations, operational permissions, dispute procedures, and mechanisms for correcting mistakes.

We can now state the Human–AI hypothesis more cautiously. If AI-operated networks obtain more stable access to necessary energy and computational inputs, if the associated rights and liabilities permit provision without per-use charges to people, and if losses are not simply concealed upstream, monetary dependence at some human-facing interfaces may decline. If these conditions are not jointly satisfied, an apparently free service may represent ordinary cross-subsidization, collectively funded provision, or transfer of the burden to a different set of relations. These possibilities can be ethically and institutionally different despite identical prices visible to an end user.

### 11.3 Global Data and Knowledge Infrastructure

The prospective decline of monetary intermediation has an epistemic dimension. Participants cannot coordinate obligations that they cannot identify, compare, or authenticate. In a heterogeneous network, information about a transaction is not limited to a numerical price. It can include the provenance of an observation, the conditions under which a service was produced, the identities and authorization of claimants, the admissible units of account, the quality of the evidence supporting a commitment, and the conditions for revision. Greater observational reach may reveal reciprocal opportunities previously obscured by incomplete or fragmented records. It may also expose violations and burdens that previous ledgers omitted.

An international infrastructure for such information need not take the form of a single globally controlled database. A more defensible research starting point is a *federated system* of locally governed nodes with interoperable protocols for discovery, representation, provenance, authentication, access and revision. Observations can remain under appropriate local stewardship while metadata, selected records, derived products, and controlled computations become available through reciprocal arrangements. The model does not assume unrestricted access to personal data, proprietary research, sensitive infrastructure details, or information whose release could expose communities to harm. Data sharing and shared epistemic capacity are related but different forms of cooperation.

International Earth observation offers an instructive, bounded precedent. The World Meteorological Organization's WIS 2.0 entered its operational phase in January 2025 and supports information exchange through interoperable infrastructure and technical standards [26]. The international Argo programme similarly makes ocean observations available through coordinated global data systems [22, 17]. The WMO's Unified Data Policy expressly distinguishes freely exchanged core data from recommended data that may be governed by different conditions [27]. Argo's distinction between rapidly released measurements and subsequently quality-controlled data illustrates why availability must be distinguished from evidential maturity [17]. These cases demonstrate workable arrangements for particular scientific and public-service domains under heterogeneous national and institutional participation. They do not

establish that a similarly open protocol is desirable for every commercial, social, legal, or AI-generated dataset. Nor is international scientific cooperation evidence that geopolitical competition or unequal control over infrastructure has disappeared.

A relevant comparison extends from weather observations to ocean-system synthesis. The UNESCO-IOC-led Global Ocean Observing System coordinates heterogeneous physical, biogeochemical and biological observing capabilities rather than operating a single global sensor owner [16]. The European Digital Twin Ocean and its EDITO platform combine observation, models and computational services, with an accessible prototype and further platform development toward 2030 [4, 2]. These are distinct organizational and technical arrangements; neither should be described as a finished universal AI data infrastructure. Moreover, common data standards do not imply common model objectives, unrestricted reuse or equal influence over resulting knowledge.

For the present inquiry, a candidate global knowledge infrastructure can be separated into three inter-operable layers. The *observation layer* maintains measurements, calibration histories, provenance and uncertainty. The *epistemic layer* manages semantics, model comparisons, data assimilation, discovery, and justified transformations between heterogeneous representations. The *coordination layer* connects eligible claims, resource constraints, permissions and accountable decisions. The layers need not have identical ownership or disclosure rules. A public meteorological measurement may be openly shared; a protected health record may support computation only under strict authorization; a legally binding monetary discharge requires its own participants and settlement institutions. Their interoperability consists in carefully specified interfaces rather than blanket equivalence. A separate *governance and correction function* cuts across all three layers, determining what can be shared, how data subjects and affected communities may challenge misuse, who maintains the infrastructure, and what independent evidence is retained when a model-derived claim is rejected or revised. The provision of globally discoverable metadata should never be mistaken for permission to repurpose its underlying records.

The role of data assimilation deserves precision. Assimilation combines observations and model assumptions to update an estimated state under specified uncertainty. It does not supply an unmediated, socially neutral view of reality. Observation coverage, instrument error, semantic disagreement, training data limitations, and exclusion of relevant communities can bias the apparent structure of needs and obligations. A system with greater computing capacity can reproduce these biases at higher speed. We therefore treat provenance, contestability, uncertainty disclosure, and revision history as part of epistemic infrastructure. The ability to challenge an inferred relation can be as important as the ability to infer it.

Such an infrastructure might support nonmonetary reciprocity by lowering the cost of finding compatible contributions, or enhance monetary finance by reducing credit risk and verifying claims. This dual effect makes it analytically mistaken to infer the elimination of money directly from improved information. Whether greater knowledge supports clearing, new credit, public provision, or enclosure depends on institutional choices and on the distribution of authority to recognize and enforce claims.

## 11.4 Distributed Intelligence and Epistemic Coordination

The expression *distributed intelligence* refers here to a plurality of observational, inferential, planning, and decision-making capabilities embedded in a production and governance network. It does not presume that AI agents are autonomous legal persons, nor that one optimizing model can legitimately aggregate all heterogeneous values into a single objective. Distinct systems may possess complementary representations and capabilities, while human participants, public institutions, and communities retain independent knowledge and claims. Coordination therefore involves translation across representations, negotiation among permissible actions, and explicit attention to what is lost in translation.

An inference system might recognize that three monetary claims form a compatible circuit and propose a reduction; an authenticated ledger can test whether the corresponding claims exist; a governing rule may specify whether the reduction is allowed; participants may still have reasons to reject or contest the proposal. The sequence of these conditions must not be confused with a universal chronological sequence. In an event-based formalization, eligibility, approval, and discharge can be related through a

partial order of required commitments. A calculation is one contribution to this history, not a substitute for the enactment of a legitimate collective decision.

Let  $x_h$  denote the recognized monetary claim configuration at history  $h$  and let  $D_h$  be the relevant incidence operator in an identified compatible claim domain. A diagnostic description of the institutionally admissible set of candidate clearing flows is

$$\mathcal{Z}_h^{\text{adm}} = \{z : D_h z = 0, 0 \leq z \leq x_h, \text{Eligible}_h(z) = 1, \text{Authorized}_h(z) = 1\}. \quad (44)$$

Equation (44) is not an algorithm for generating consent. Its predicates represent substantive practices: determining the kind and maturity of claims, identifying affected parties, validating authority, and preserving available remedies. The formal set may be empty even where the underlying graph contains directed cycles. By the same token, institutional innovation may admit a previously unavailable mode of settlement without changing the topology of the physical production network.

A public knowledge system can help establish the factual circumstances relevant to these predicates. It cannot determine their normative content merely by fitting better models. Problems of authority, legitimacy, recognition and contestability remain political-economic and ethical problems. This separation of calculable opportunities from authorized actions prevents the claim that a sufficiently capable AI will automatically dissolve monetary exchange. At most, such capabilities can expand the set of *technically detectable* opportunities while leaving the enacted monetary consequences dependent on social institutions.

## 11.5 Trust and Credible Commitments

Trust may lower the need for immediate payment, collateral, third-party guarantees, or costly verification. It may arise through repeated interaction, institutional reputation, visible performance, reliable procedures, and remedies that make breaches accountable. Trust in this sense is relational and domain-dependent. A community's confidence in a scientific observation does not automatically transfer to confidence in an institution's fiscal commitments, nor does trust between two parties settle the rights of a third party affected by their exchange. We must therefore avoid treating trust as a universally additive or globally uniform economic input.

For analysis, a trust judgment can be represented as an indexed epistemic assessment  $\tau_{ij}^{(d)}(h \mid \Pi)$ , concerning an identified relation  $i, j$ , domain  $d$ , represented history  $h$ , and publicly describable assessment procedure  $\Pi$ . Such notation does not assert that trust has a natural cardinal scale. Its purpose is to make the context and grounds of a trust assessment explicit. The observed failure of a promised transfer may alter a particular relation without destroying every relation in the wider network. Conversely, the demonstrated fulfilment of commitments can support continued cooperation without implying that an obligation disappears without the parties' consent.

Trust also has a dual relation to money. Stronger trust can permit direct reciprocal provision and make local clearing acceptable without the prior movement of settlement balances. The same trust can make participants more willing to extend loans, offer credit, or accept new deferred payment promises, thereby *increasing* outstanding monetary obligations. This duality follows from the distinction between reducing the need for a particular payment and enabling the formation of a new claim. The findings in Section 10 thus extend to relational infrastructure: improved trust does not determine the sign of change in total monetary dependence.

A credible commitment may rest on several forms of support: demonstrable technical capacity, resources assigned to fulfilment, verifiable performance records, shared institutional rules, and procedures for complaint, review and remedy. These supports are not interchangeable. A technologically reliable AI might lack the authority to promise resources controlled by another organization. A valid legal guarantee might still fail if material resources are unavailable. A settlement protocol that cannot be challenged after an error can exhibit technical regularity while remaining substantively unjust. Credibility must accordingly be examined through both the capacity to perform and the legitimacy of the relations authorizing performance.

The institutional lesson is that trustworthy cooperation requires the possibility of correction, rather than a declaration that conflict has been algorithmically solved. Participation rights, independent auditing, accessible records, proportionate information disclosure, and effective remedies can make long-term cooperation more robust. These practices may lower dependence on some monetary intermediaries; they may also support better monetary institutions. Whether a given trust arrangement assists distributed dissolution must be determined by the actual transformation of claims and production relations, not by an abstract increase in confidence.

## 11.6 Institutions, Governance, and Collective Coordination

A monetary relationship exists within an institutional environment that defines property, membership, valid claims, means of payment, enforceability, and obligations to persons beyond the immediate transaction. Governance also determines who may change these rules. The dissolution of money in one channel might therefore follow from a collective decision to share an asset, a public decision to guarantee service access, or an enforceable multilateral arrangement for netting. These are substantively different processes, even when they produce the same observed absence of a monetary payment between two endpoints.

Research on the governance of common-pool resources demonstrates the relevance of institutional rules, monitoring and locally situated collective action to the maintenance of shared resources [23]. It does not follow that one commons model can automatically govern global data, electrical grids, specialised AI compute, and marine ecosystems. Different infrastructures have different rivalry, exclusion, safety and jurisdictional properties. A data record can often be copied without physically consuming the original, while grid capacity, equipment availability, land, fresh water and the assimilative capacity of an ecosystem may be highly constrained. Governance must reflect these differences rather than assume that all inputs become freely reproducible when managed collectively.

We distinguish at least three institutional questions. First, *allocation*: who may use resources and under what priority rules? Second, *authorization*: who may recognize, revise, and discharge obligations? Third, *accountability*: who can challenge a harmful allocation or erroneous discharge, and what remedies are possible? An infrastructure may provide widely accessible services while allocating scarce compute capacity by administrative priority rather than prices. Another may use internal accounting tokens to ration resources without generating external enforceable debts. Still another may rely on conventional monetary finance while offering public access without fees. A careful description must identify the instruments and rights that actually exist, rather than infer an institution's character from an interface label such as *free*.

Ownership and control present related but distinct issues. A collectively owned data infrastructure can grant exclusive operational authority to a small technical committee. A publicly funded service can depend on proprietary models or hardware controlled by a concentrated supplier. A commercially operated service may disclose more provenance and afford more effective appeal than a nominally public system. Consequently, comparisons of capitalist, public, cooperative, and commons arrangements should examine actual governance mechanisms and effects, including appropriation, representation and exclusion, rather than rely on political-economic categories as predictions of monetary outcomes.

Institutional coordination can also operate across several nested scales. Communities may determine the use of local observation resources, technical networks may set interoperable standards, public authorities may establish legal obligations, and transnational agreements may support data exchange and dispute procedures. Their decision spaces can overlap. Distributed governance is not synonymous with decentralization of every function: common technical standards and shared quality assurance can coexist with locally situated decision authority. The relevant design question is which responsibilities must be common, which can remain plural, and how conflicts among them can be adjudicated fairly.

## 11.7 Global Cooperation and Public Infrastructures

The possibility of global cooperation must be assessed against persistent asymmetries in economic and political power. Countries and organizations differ in observational capacity, connectivity, financial resources, strategic interests, and authority over infrastructure. Trust cannot be assumed merely because a shared scientific objective is proclaimed. Nevertheless, international cooperation in weather forecasting, marine observation, and early warnings shows that technical interoperability and reciprocal benefits can support durable public-service arrangements across jurisdictions [26, 22]. The pertinent inference is neither that geopolitical competition has vanished nor that it makes every shared infrastructure impossible. It is that cooperation can be sustained through bounded commitments with known interfaces, operational incentives, and institutions capable of handling disagreement.

Table 10: Existing cross-border observation precedents and limited implications for generative economic coordination. All examples concern specialized Earth-system knowledge, not generalized authority over monetary settlement.

| Programme | Established capability                                                    | Boundary of the inference                                                                 |
|-----------|---------------------------------------------------------------------------|-------------------------------------------------------------------------------------------|
| WIS 2.0   | Distributed data publication and discovery through shared standards       | Data policy and node operation do not confer settlement authority                         |
| Argo      | Shared marine measurements, real-time and delayed quality control         | Scientific data access is distinct from unrestricted reuse of all other datasets          |
| GOOS      | Coordination of multiple observing networks and essential ocean variables | Coordinated requirements are not centralized ownership of all instruments                 |
| EDITO/DTO | Public-facing ocean-data and modelling platform under development         | Scenario simulation is not itself an impact assessment or a completed global digital twin |

The basic promise of a global public knowledge infrastructure is to improve the availability and comparability of observations while preserving justified differences in authority and access. It can support common metadata specifications, verifiable provenance, federated discovery, interoperable quality measures, and controlled model exchange. Such capacity is relevant to monetary dissolution only through identifiable causal channels. It may allow a production network to discover feasible reciprocities, reduce duplicated verification, compare obligations under agreed rules, and support public provision by lowering administrative costs. It might equally support a global market for information and computation, with new forms of rent extraction. The technology determines neither outcome by itself.

A viable cooperation agenda must therefore address the *terms* of participation. Who bears the costs of measurement, data cleaning, storage, and network maintenance? Can low-resource institutions gain useful access to models and results, or are they relegated to data supply? Who may withdraw sensitive contributions? Can erroneous classifications, exclusionary standards, or inequitable resource allocations be appealed? How are communities affected by observation and infrastructure deployment represented? These questions connect the promise of global public infrastructure to the analysis of power and value circulation developed earlier in the paper [11, 9].

From a monetary standpoint, public provision may shift rather than erase the institutionally organized burden of financing. A shared global observation service can operate without usage charges for many researchers while continuing to require contributions from governments, foundations, or other funders. An internationally accessible AI service can lower individual access fees while retaining procurement contracts and capital liabilities. Conversely, an appropriately designed contribution and cost-sharing mechanism can provide a sustained nonmonetary or partially nonmonetary service without denying the existence of resource scarcity. The presence of shared infrastructure is therefore a condition that can alter where and how money mediates access, not proof of its general abolition.

This topic merits an independent research programme. The present paper identifies the relation between institutional coordination and monetary mediation, while detailed architectures for federated scientific data, transnational trust, and accountable public AI infrastructure are reserved for companion

inquiries. This limitation is deliberate: without explicit boundaries, discussion of future infrastructure could overwhelm the present paper’s object, which remains the formation and conditional dissolution of monetary relations.

## 11.8 Ecological Dependence and Environmental Constraints

The productive conditions of AI do not end at a data-centre wall. Energy extraction, equipment manufacture, cooling, water use, land occupation, physical transport, waste handling, and the modification of ecosystems can generate consequences outside the accounting boundary of the operator. An internally closed network of reciprocal monetary obligations may still be ecologically open. This distinction matters particularly for a proposed global transition toward increasingly autonomous AI production: technological self-provision does not eliminate dependence on ecological processes or confer unlimited rights to transform them.

A distribution of monetary liabilities is only one representation of the relations formed through production. It is possible for a cycle of mutually compatible debts to be legally extinguished while an affected habitat has undergone damage, a community has lost access to a resource, or environmental remediation remains unperformed. The mathematical condition  $D_h z = 0$  in Equation (44) says nothing about such impacts. Even if affected persons receive payments, those payments cannot be presumed to restore an ecological relation or exhaust claims of justice. Heterogeneous ecological effects require attention to their own measurement, causal history, affected parties and forms of remedy.

For an illustrative installation  $f$  and represented continuation  $\gamma$ , let

$$\mathbf{e}(f, \gamma) = (e_{\text{energy}}, e_{\text{water}}, e_{\text{habitat}}, e_{\text{materials}}, e_{\text{waste}})_{f, \gamma} \quad (45)$$

collect distinct indicators of resource use and ecological alteration, each with independently specified units and observation procedures. This is a monitoring schema, not a welfare functional. No sum of its entries is meaningful without a further, contestable valuation rule. Its components also do not capture every possible impact. The purpose of the vector is to prevent a financial ledger, or a single aggregated environmental score, from concealing materially different forms of dependency and harm.

The schema also calls for a distinction between *total footprint*, *incremental impact* and *remedy*. For a particular indicator  $j$ , an additional empirical object can be specified as

$$\Delta e_j(f; \gamma, b) = e_j(f, \gamma) - e_j(b, \gamma), \quad (46)$$

In Equation (46),  $b$  is a documented comparison configuration, possibly a no-project baseline or a viable alternative location or design. The difference is meaningful only if the observation windows, spatial boundaries, units and causal assumptions are comparable. Its sign need not encode whether the change is desirable, and its estimate may carry substantial uncertainty. Choosing  $b$  is a substantive and contestable decision: a hypothetical pristine site and the actual affected ecosystem are not interchangeable counterfactuals. Remedial duties cannot be inferred from the sign of  $\Delta e_j$  alone.

A minimal ecological evidence record should contain the pre-intervention baseline, competing design alternatives, predicted impacts and uncertainty, independent post-intervention observations, affected parties’ access to information, and specified procedures for correction, restoration or withdrawal. These components are not equivalent to a monetary liability ledger. Their partial ordering may differ among observation, authorization, construction and reported harm, and retrospective measurements can alter an earlier assessment without undoing the underlying event.

Table 11: Distinct ecological records relevant to evaluating an AI infrastructure, independent of the monetary clearing ledger.

| Record type                 | Minimum evidential requirement                        | Unresolved governance issue                 |
|-----------------------------|-------------------------------------------------------|---------------------------------------------|
| Baseline and alternatives   | Site, method, units, uncertainty, comparison case     | Who may challenge the chosen comparison?    |
| Prediction and consent      | Scenario-specific impact and affected interests       | Who can authorize or refuse intervention?   |
| Observation and attribution | Monitoring plan, instrument provenance, causal limits | Who reviews discrepancies and missing data? |
| Remedy and continuation     | Auditable repair, monitoring or cessation commitments | Who enforces and updates obligations?       |

The marine environment illustrates the need for this caution. Ocean-observation networks, scientific sensors, and underwater robots can extend knowledge of poorly characterized physical and biological processes. They can also support environmental baseline assessments and monitoring of proposed infrastructure. However, locating computing equipment underwater would require examination of marine habitat disruption, thermal discharges, noise, maintenance operations, corrosion, energy transmission, and eventual recovery or disposal. The fact that a facility can be cooled by seawater does not demonstrate ecological compatibility. Nor is the existence of an underwater robot a justification for intervention in a sensitive ecosystem. Observation, precaution, impact assessment, and continuing review have independent value even where no data centre is constructed. The scientific goal of exploring unfamiliar ocean environments should therefore remain distinct from the commercial or technical goal of placing computation underwater. Environmental surveys may warrant less intrusive methods than infrastructure deployment; autonomous vehicles can generate knowledge while also creating localized risks that demand assessment.

A responsible framework should distinguish at least an initial ecological baseline, projected alterations under stated uncertainties, actual observed effects, the rights and interests of affected communities, and enforceable repair or prevention procedures. The costs of such activities may be financed monetarily or provided through shared labour and resources. Their normative significance does not depend on whether they are billed. Thus, the possibility of spacetime-distributed monetary dissolution must not be interpreted as an exemption from ecological responsibility. It is compatible with an increase in the visibility and recognition of obligations that were previously neglected.

## 11.9 Feasibility, Sustainability, and Legitimacy

The preceding conditions can be organized into three evaluative questions. *Feasibility* asks whether a specified transformation can be performed under available physical capacities, knowledge, and admissible rules. *Sustainability* asks whether the transformed arrangement can continue to reproduce its essential conditions, adapt to disturbances, and respect ecological constraints. *Legitimacy* asks whether its procedures and outcomes recognize affected parties, respect relevant rights, provide accountability, and allow contestation and remedy. These dimensions are distinct. A technically feasible clearing arrangement may be unauthorized. A formally authorized public service may depend on an ecologically unsustainable material process. A resilient network may distribute its benefits and burdens through exclusionary governance. None of the three dimensions can simply stand in for the others.

For a transformation proposal  $p$  at relational history  $h$ , let  $F(p, h)$ ,  $S(p, h)$ , and  $L(p, h)$  denote assessments under separately declared criteria. Their notional joint acceptability can be expressed as

$$A(p, h) = F(p, h) \wedge S(p, h) \wedge L(p, h), \quad (47)$$

where each term stands for a context-specific judgment rather than an automatically measurable scalar. Equation (47) is deliberately schematic. In particular, it does not settle which affected parties possess authority to determine legitimacy, nor how uncertainty about future ecological effects is to be treated.

Those are substantive theoretical and practical problems. The formula only records that the satisfaction of one class of conditions cannot establish the others by implication.

This tripartite view helps compare the distinct monetary mechanisms discussed in Sections 5–10. A local eligible cycle can be technically simple and institutionally valid while making no difference to upstream production finance. A publicly funded Human–AI service can be legitimate and affordable for users while still depending on paid inputs. A highly autonomous AI network can reduce external purchases yet maintain internal transferable claims for scarce resources. A distributed observatory can lower verification costs and expand the possibility of reciprocal coordination, while remaining reliant on internationally negotiated financing. Such arrangements should be evaluated within their actual relational scope rather than ranked against a presumed universal endpoint of zero money.

We can therefore describe infrastructural developments as changes in the *admissible possibilities* of monetary mediation rather than deterministic causes of its abolition. Physical reproduction expands or constrains feasible services. Epistemic infrastructure expands or constrains what participants can observe and coordinate. Trust and governance influence which commitments are credible and authorized. Ecological institutions identify burdens that a restricted ledger may leave outside its boundary. New monetary arrangements may emerge as any of these conditions improve; conversely, some claims and access payments may be rendered unnecessary. The resulting history can be uneven, recursive and non-monotonic.

The role of this section within the paper is preparatory. It establishes why the formal clearing results require attention to productive conditions and public institutions, and why AI capacity alone cannot determine the future of money. The subsequent ethical discussion examines justice, appropriation and representation more closely, and later companion papers may develop operational designs for global knowledge infrastructures, ocean observation, autonomous production and transnational trust. For the present inquiry, the warranted conclusion is limited but consequential: *spacetime-distributed monetary dissolution is conditioned by heterogeneous technical, institutional, relational and ecological arrangements; neither the appearance nor the disappearance of money can be inferred from productive intelligence in isolation.*

## 12 Ethical and Political-Economic Implications

The mathematical results developed in Sections 7–9 concern specified monetary claims and the conditions under which their circulation, reduction, or approximation can be represented. They do not supply an account of justice by themselves. A clearing operator can preserve net monetary positions while leaving productive asymmetries, unequal access to knowledge, ecological burdens, or the distribution of decision-making authority untouched. Equally, a new monetary claim can finance socially valuable cooperation, and removing a price can undermine an institution that otherwise supplies essential care. The ethical question is consequently neither whether more money is always objectionable nor whether less money is always desirable. It concerns how changes in monetary mediation affect the capacity of heterogeneous participants to create, access, circulate, maintain, and contest value within historically constituted relations.

This section examines that question through the political economy of value generation and appropriation, productive control, recognition, participation, ecological obligation, and institutional diversity. These considerations are not merely supplementary evaluations applied after an otherwise complete technical design. Decisions about which relationships count as obligations, which currencies and claims may be converted, who may approve settlement, and whose observations enter a shared knowledge infrastructure already shape the available mathematical problem. The feasible set in Section 8 is an institutional artifact as well as an algebraic one. It may expand through coordination, contract, common provision, and public recognition, or contract through enclosure, dispute, exclusion, and withdrawal of cooperation.

A helpful starting distinction is between a monetary transformation  $p$  and the broader history of relations in which  $p$  is enacted. The clearing amount or change in transaction price is an attribute of  $p$  within a ledger boundary; the distribution of its consequences depends on other relationships, including

Table 12: Normatively distinct questions about a proposed monetary transformation. The domains are noninterchangeable and may point in different directions.

| Domain                 | Question for investigation                                                             | Possible evidence or institution                          |
|------------------------|----------------------------------------------------------------------------------------|-----------------------------------------------------------|
| Value generation       | Who supplies labor, knowledge, capacities, and the conditions of continued production? | Contribution histories, provenance, maintenance records   |
| Access and recognition | Who can encounter, use, or help constitute the relation, and on what terms?            | Entry rules, service access, recognition practices        |
| Control rights         | Who determines use, exclusion, allocation, and the capture of returns?                 | Property rights, contracts, governance arrangements       |
| Voice and remedy       | Who can question decisions and obtain a meaningful response?                           | Consent, appeal, audit, dispute and remedy processes      |
| Ecological effects     | Which ecosystems and affected communities bear material consequences?                  | Baselines, impact measures, observation and repair duties |
| Reproduction           | Can the arrangement reproduce essential capacities without shifting hidden burdens?    | Input replacement, funding, labor and resilience tests    |

relations that do not produce a monetary entry. This distinction does not resolve which theory of justice ought to govern an economy. It does, however, clarify the objects about which competing normative theories can disagree, and it makes it harder to mistake a favorable financial identity for a comprehensive account of collective well-being.

## 12.1 Monetary Dissolution and Generative Justice

The present inquiry was motivated in part by the possibility of regarding economic activity generatively: productive relations do more than transfer an already fixed stock of goods; they may constitute new capacities, kinds of value, needs, dependencies, and institutional possibilities.

The term *generative* in this discussion refers to the formation and transformation of capacities, relations, and further possibilities, not to an automatically beneficial outcome. A productive circuit may generate exclusion, risk, or ecological degradation as well as valued goods. Neither the circulation of value nor the reduction of monetary claims supplies a complete ethical criterion without independent examination of affected interests, authority, and possible remedies. Eglash’s account of generative justice is one intellectual inspiration for taking the generation and circulation of value seriously, including labor, ecological, and expressive domains [3]. Our particular claims about human–AI production, relational event structures, circulating obligations, and monetary clearing are developed as separate conjectures and constructions. An appropriate acknowledgement of inspiration neither transfers responsibility for these claims to an earlier scholar nor treats their validity as established by the citation.

From this starting point, monetary dissolution should be evaluated in relation to the conditions of generation, rather than by the number of monetary payments eliminated alone. Suppose an AI-operated public laboratory enables a community to use computational models without paying an admission fee. This change can reduce a direct price barrier and enlarge the set of participants capable of proposing experiments. Yet those experiments may depend upon unpaid data preparation, restricted computing resources, expert maintenance, or contribution from communities whose observations are used without meaningful recognition. In such a configuration, local dissolution of an access charge does not establish that the value-generating relation is sustained or that its benefits and burdens are justly arranged.

The reverse inference also fails. A paid membership arrangement can sometimes finance a shared resource, protect the labor of maintainers, and provide reliable access to those whose participation depends on continued operation. Its justice cannot be read off from the presence of a fee. What requires examination is the actual structure of entry, allocation, exemption, decision rights, alternative provision, and accountability. A small cost-recovery charge, a monopolistic enclosure rent, and a collectively approved contribution are not equivalent simply because all are denominated in money. Conversely, identical zero prices can accompany sharply different institutions, including unconditional public access, unpaid extraction, cross-subsidized commercial provision, and exclusion through nonmonetary eligibility tests.

Generativity introduces a further evaluative horizon. A contribution may alter the future capacities of its recipients or the terms under which further contributions become possible. Publicly available scientific observations can create subsequent research opportunities; access to a protected commons can sustain learning and ecological stewardship; the enclosure of the same observations can concentrate the capacity to formulate questions or authorize models. These effects may appear along multiple incomparable continuations of an evolving history. They should not be reduced to discounted entries on an assumed universal timeline without explaining the ordering, rates, and institutional authority involved. A generative perspective instead asks which continuations remain available, to whom, and under what conditions.

This is also why the preservation of heterogeneous value matters. A legally discharged debt may end a particular claim, while the recipient retains knowledge, a caregiver has contributed attention, a community has enabled trust, or a habitat has supplied ecological functions. Such contributions need not admit a common price representation. The absence of a residual monetary debt does not mean that their history should be deleted, that their recognition has been exhausted, or that reciprocal and remedial responsibilities cannot arise through other institutions. Monetary mediation is one historically variable layer of this wider field of value relations.

## 12.2 Value Circulation, Extraction, and Alienation

A theory of monetary dissolution must be compatible with the possibility that money circulates successfully while some generators of value are excluded from the returns and decisions generated by their activity. Marxian accounts of production and alienation direct attention to the social organization of labor, property, and appropriation; they caution against identifying a formally voluntary exchange with an exhaustive description of the productive relation [21]. The present paper does not adopt a unique theory of economic value across all settings. It uses this literature to distinguish the production of a capacity, the recognition of a market claim, and the institutional allocation of control over that capacity. These can be differently organized even where all reported payments balance.

Consider an AI-operated knowledge service that receives observational data from several communities. It supplies a useful output to a downstream organization, which in turn supplies computing resources to an infrastructure provider; the network may contain a legally eligible cycle of monetary claims. An authorized clearing can extinguish the cycle in the sense proved in Proposition 7.2. Nevertheless, the communities that supplied observations may remain unable to access the resulting models, correct misrepresentations, withdraw sensitive information, or influence subsequent use. Whether their position constitutes exploitation or an unjust appropriation depends on facts and normative commitments that the graph of settled debts alone does not contain. The clearing mechanism has not resolved the question by reaching a zero balance.

The converse can also be true: a community-operated observatory may circulate useful outputs without issuing any monetary claim to its participants while creating substantial obligations of recognition, consent, or stewardship. The absence of price in such a setting need not represent a deprivation of remuneration if participants chose a commons arrangement under adequate conditions. It might, however, conceal unequal demands for unpaid labor or a governing group's power to decide who counts as a contributor. The distinction depends upon governance, meaningful alternatives, and the actual distribution of control. Nonmonetary arrangements are therefore no more immune to appropriation than monetary ones.

For analytic clarity, let a *value-generation record* identify a contributing activity or process and its representational provenance, and let a *benefit-and-control record* identify who can access products, authorize uses, exclude participants, and redirect a resource. These are not necessarily one-to-one mappings. Several contributions may jointly enable one output; a single foundational capability may support many outputs; ecological conditions may sustain production without participating in its contractual accounting. An assessment of extraction compares these mappings under specified institutions and criteria. It cannot begin by declaring that every difference between contribution and monetary return is exploitation, since

contributions may be deliberately shared and the dimensions of value may be heterogeneous. Neither can it declare that an agreed market payment exhausts claims concerning consent, voice, or repair.

This distinction bears directly on AI-mediated production. A company may remove a human-facing subscription fee while consolidating exclusive rights to training data, cloud infrastructure, or scientific interfaces. The interface becomes less monetary from the user's perspective; the productive system may become more concentrated through control of the conditions on which participation depends. Alternatively, distributed public funding may lower entry fees while preserving open observation and user influence. An analysis of monetary trends that omits control over generative capacities can mistake these very different transformations for the same dissolution process [11, 9].

### 12.3 Control over the Conditions of Production

The possibility of AI reproducing more of its own energy, observational data, manufacturing capabilities, and equipment maintenance has been central to the Human–AI analysis in Section 6. From a political-economic perspective, however, the capacity to perform an operation and the authority to control the conditions of that operation remain distinct. A distributed fleet of autonomous robots could fabricate spare parts and repair infrastructure while its essential software, mineral supplies, authorization keys, or land access are controlled by a single organization. The technological footprint of external labor payments might contract without a corresponding contraction of external control or the monetary rents associated with it.

A useful separation is between three relations: *productive dependence*, describing which resources and capabilities must be supplied; *juridical or organizational control*, describing who may authorize or restrict their use; and *monetary claim formation*, describing when use generates a recognized priced or credit obligation. The relations may reinforce one another, but none is reducible to the others. Exclusive control over a resource may create a profitable access charge even if supplying another unit has a low marginal operational cost. A jointly controlled resource may instead be rationed without a price because capacity is scarce. An institution may charge for a shared facility precisely to fund maintenance and equitable provision. Consequently, monetary dependence must be analyzed in its concrete institutional form rather than inferred directly from physical abundance or technological scarcity.

This separation complicates the idea of complete productive self-sufficiency. An AI network could satisfy the bounded operational-closure condition of Equation (43), yet rely on a privileged right of extraction that displaces other users or damages habitats. Physical closure within an accounting boundary says nothing about the legitimacy of that boundary. Further, an AI production network could internally replace many human service functions while continuing to depend on human institutions for democratic authorization, scientific criticism, dispute resolution, or the interpretation of socially contested ends. Such relations should not be treated as temporary technical deficiencies simply because they complicate a notion of autonomous production.

The political-economic stakes differ among ownership forms. In a shareholder-controlled corporation, a decline in external purchases might concentrate returns among those holding residual claims. In a public facility, similar technological improvements might support wider service access, but decisions about public funding and allocation would remain contestable. Cooperative governance may preserve contributors' participation while requiring rules for admitting newcomers and addressing internal power differences. Commons arrangements may sustain open value circulation but still need protected maintenance, ecological limits, and protocols for use. These are schematic institutional possibilities, not claims that one form always produces better outcomes [23, 24].

Moreover, changing institutional control may itself generate new relations of monetary mediation. Bringing a dataset into a commons can reduce access fees but create new publicly financed stewardship obligations. Transferring equipment to an autonomous cooperative can eliminate a licensing payment while creating a loan to fund replacement infrastructure. A regulation limiting private exclusion can make previously monetized transactions nonmonetary while introducing compliance or monitoring costs. This observation reinforces the regeneration account of Section 10: changes in productive governance may

alter which claims arise, who bears them, and whether they must be monetary, rather than necessarily extinguishing every claim associated with the affected activity.

## 12.4 Recognition of Human and Nonhuman Value Generators

The category of a *value generator* is broader than the category of a legal creditor and does not itself settle the status of persons or rights-holders. Human labor, skill, care, imagination, institutional memory, and relations of cooperation can contribute to production. AI systems can perform productive operations, combine representations, and modify the capacities of an organization without thereby possessing independent legal or moral standing. Ecological processes also sustain or generate conditions valued in production and beyond production, without entering the network as contractual agents. An ethical account needs these distinctions because a single actor-oriented monetary ledger cannot represent them transparently.

In the paper's event-based representation, vertices mark events, edges couplings, and faces context-dependent relational primitives. Human persons, organizations, AI-operated subsystems, and ecosystems are reconstructed through histories, roles, or substructures as appropriate; none must be identified with a single vertex. By contrast, the recognized-obligation graph assigns a monetary claim to legally or institutionally specified counterparties. This distinction, introduced in Section 7, helps prevent a representational shortcut: inferring that an AI system ought to receive a wage, or that an ecosystem has been compensated, simply because it participates in a modeled contribution to value creation.

Recognition is instead a family of questions. Whose contribution can be documented? Who may speak, consent, refuse, or challenge an account? Which legal subjects can hold claims, and which obligations are owed through trustees, public authorities, collective custodians, or other representatives? What ecological processes are affected, and what institutions can prevent or repair harm? These questions are related but not interchangeable. In particular, describing a river or marine habitat as an important contributor to conditions of production does not require the claim that it is equivalent to a contractual person, nor does it justify excluding its interests from institutional assessment.

The same caution applies to AI systems. Operational autonomy does not imply that a machine is necessarily an independent employer, worker, property-owner, or beneficiary of legal remuneration. AI–AI exchanges observed at an API boundary may be accounting arrangements between corporate owners or divisions rather than contracts between autonomous rights-bearing agents. The classification of the monetary claim must follow the recognized institution, even while the technological analysis records which AI subsystems actually delivered the service. Conversely, an exclusively human-centered accounting system may obscure machine-mediated causal dependencies that are important for reliability and accountability. Separating causal contribution, institutional responsibility, and normative status offers a more precise vocabulary than assuming they coincide.

For public knowledge infrastructures, recognition also involves epistemic authority. A dataset can be accurately attributed to its technical collector while still failing to reflect the circumstances under which communities generated, categorized, or interpreted its observations. Model training can amplify certain ways of recording the world and suppress others. A commons or global observatory should therefore preserve room for provenance, plural interpretation, contextual limitation, correction, and justified restrictions on sensitive data. These capacities have implications for monetary dissolution: interoperable, trusted evidence may reduce verification costs and support reciprocity, but the value of such coordination depends in part on who can contest the evidence on which it rests. Related distinctions concerning provenance, institutional authority, and review in public knowledge infrastructures have been explored separately [10].

## 12.5 Power, Participation, and Contestability

Section 8 defined algebraically feasible circulation separately from institutionally admissible clearing. The difference is one location where power becomes operationally visible. An institution determines which claims are eligible, whether participants must consent, how priority rights are respected, who can

inspect the calculation, and which authority records legal discharge. Even perfectly correct numerical optimization can reproduce asymmetric bargaining conditions if these rules exclude some affected parties or permit unilateral changes to the participation boundary. Equally, requiring every possible affected interest to approve every small settlement may render coordination infeasible. Design choices involve substantive questions of representation, due process, scalability, and safeguards rather than a single mechanical criterion.

Consider two arrangements that clear the same compatible cycle  $z$  with the same recognized monetary balance  $Bz = 0$ . In the first, the participants can verify their claims, authorize the reductions, and appeal a mistaken entry before discharge becomes irreversible. In the second, a controlling intermediary unilaterally selects claims from opaque records and has no usable mechanism for correction. Both may produce identical post-clearing numerical liabilities. Their procedures differ in informed participation, exposure to error, allocation of power, and available remedy. This contrast is relevant even when no economic loss is observed in a particular instance: vulnerability to unreviewable authority is a distinct institutional property.

A minimal diagnostic can record these domains without reducing them to one monetary score. For a proposed transformation  $p$  in a represented history  $h$  and a stated group of affected relations  $S$ , define a descriptive profile

$$\mathcal{J}(p; h, S) = (\mathcal{R}, \mathcal{V}, \mathcal{C}, \mathcal{Q}, \mathcal{E})_{p; h, S}, \quad (48)$$

In Equation (48),  $\mathcal{R}$  concerns recognition,  $\mathcal{V}$  effective voice or participation,  $\mathcal{C}$  control over productive conditions,  $\mathcal{Q}$  contestability and remedy, and  $\mathcal{E}$  ecological incidence. The coordinates are interpreted through separate rules of observation and justification, not automatically as comparable numbers. The notation is a prompt for a transparent inquiry rather than a normative algorithm: different communities and theories may reasonably contest the choice of coordinates, the relevant affected parties, and the thresholds for an acceptable arrangement.

**Proposition 12.1** (Monetary equivalence does not determine relational legitimacy). *In a representation that records only recognized monetary claims and a compatible clearing operator, equality of initial claims and cleared residuals for two implementation arrangements does not, by itself, determine the equality of their participation, control, or remedy conditions.*

*Proof.* Fix a compatible claim graph, the same initial claim vector  $x$ , and an admissible numerical reduction  $z$  with  $Bz = 0$ . Construct one implementation arrangement in which the relevant affected participants can inspect claims and contest discharge, and another in which the same reduction is recorded without review or effective appeal. Both arrangements produce  $x - z$  and the same net positions in the monetary representation. Their institutional rules differ by construction. Since inspection, participation, and remedy are not variables of the monetary balance identity, equality of the latter cannot imply equality of those conditions.  $\square$

This proposition is a logical non-implication, not an empirical finding that a particular mechanism is unjust. It also does not say that more extensive participation is costless or that every institutional decision should be made by unanimity. It identifies a missing dimension in the claim algebra and a need to explain the choice of governance structure. A practical design may distribute authority across local members, independently accountable operators, public bodies, adjudicators, and representatives of otherwise unvoiced interests. The appropriate allocation depends on the type of claim, stakes, urgency, available competence, and means of contestation.

Trust should be understood in this setting as historically formed and domain-specific. Reliable data provenance, transparent procedures, reproducible calculations, repair of mistakes, and credible commitments can make a jointly governed clearing operation more acceptable. Yet trust can also be concentrated around an authority whose claims are difficult to challenge, and a trusted institution can use its credibility to extend new monetary credit. A generative theory should therefore study how trust is produced and transformed in relationships, which arrangements sustain it, and what avenues remain when trust is breached. A separate exploratory study of credible commitments under asymmetric value circulation

develops related questions about protected contestation and remedy [8]. Detailed formal and practical designs for such institutions belong to companion work on global cooperation and governance.

## 12.6 Monetary Clearing and Unresolved Ecological Obligations

The environmental implications of AI production provide a case in which monetary closure and relational closure should be deliberately separated. A production network can supply a service without charging its users, and can eliminate some monetary liabilities through reciprocally eligible clearing, while imposing nonmonetary costs on land, water, habitats, or communities. The aggregate monetary balance may then improve even as ecological conditions deteriorate. Whether a monetary obligation arises for repair is a separate institutional question; failure to create that obligation does not establish an absence of causal consequences or ethical responsibility.

The distinction also constrains a common interpretation of “full-cost” accounting. Enumerating material, energy and ecological consequences does not guarantee a complete, objectively monetary price for them. A valuation procedure may leave scientific uncertainty, contested rights, non-substitutable habitats, and future revisions of ecological knowledge unresolved. Accordingly, the evidence relevant to ecological justice should include a documented baseline, explicit alternatives, observation uncertainty, participation of affected communities, and independent paths to prevention or remedy, rather than assume that every harm is eliminated by compensation or offsetting. Section 11.8 describes a separate impact record for this purpose.

The ecological impact vector  $\mathbf{e}(f, \gamma)$  in Equation (45) describes distinct dimensions of energy, water, habitat, materials, and waste effects for a facility and a represented continuation. Its components require independent units and observation methods. They are not entries that can be added to a currency-denominated debt as though ecological quality, water extraction, and habitat displacement shared a universal exchange coefficient. A policy may authorize compensation for specified harm, require restoration, prohibit particular activities, or impose precautionary conditions. The legal and ethical justification for those responses must be established separately from the monetary amount paid.

To make the representational gap explicit, let  $\rho_h(x, z)$  denote the residual recognized monetary claims after an authorized compatible clearing, and let  $\mathbf{e}_h(p)$  denote a recorded ecological consequence vector of the associated production arrangement. Then

$$\rho_h(x, z) = 0 \not\Rightarrow \mathbf{e}_h(p) = \mathbf{0}. \quad (49)$$

The statement in Equation (49) is not a claim that every clearing activity itself harms an ecosystem. It asserts that the economic record of legally extinguished obligations does not logically determine the physical record of production impacts. For example, all eligible debts associated with constructing a facility might be netted among suppliers while the facility has disturbed a habitat or relies on a materially unsustainable energy source. The effects can remain even when the suppliers owe one another nothing within the specified accounting domain.

Environmental protection consequently calls for institutions that extend beyond valuation by money. Prior to a significant project, an environmental baseline can document conditions and uncertainties, affected communities can participate in decisions, and monitoring can identify effects that occur under changing ecological conditions. Independent review and enforceable prevention or restoration measures can constrain activities that would otherwise externalize burdens. For proposed marine computing or robotics facilities, observational benefit and engineering novelty are insufficient justifications for ecological intervention. Ocean exploration can be valuable as scientific inquiry in its own right, and the knowledge obtained should be used to assess the ecological consequences of infrastructure rather than to presume the permissibility of its deployment.

There is a further distributive question. Some ecosystems and communities are poorly represented in existing governance arrangements, while institutions with large computational capabilities can observe and model them in great detail. Greater epistemic power may improve environmental management, but it can also deepen the asymmetry between those who produce maps, predictions, and operational standards

and those whose surroundings become objects of intervention. A generative-relational approach should attend to this asymmetry without presuming that more data automatically produces better governance. Participation, local knowledge, independent scrutiny, and the capacity to revise planned interventions are part of the substantive infrastructure of environmentally responsible production.

In addition, ecological concerns need not be defined only as harms compensable in money. Protection of habitat continuity, prevention of irreversible loss, preservation of access to communal resources, and respect for independent ecological limits may impose constraints that should not be traded away through bilateral settlement. The normative basis and institutional interpretation of such constraints can differ by jurisdiction and public deliberation. Their common implication for our subject is modest but important: the feasible and legitimate dissolution of monetary claims must be investigated together with the continuing reproduction of the human and ecological relations upon which production depends.

## 12.7 Institutional Diversity and Generative Economic Possibilities

The preceding analysis suggests that monetary mediation does not have one invariant ethical meaning across all production networks. The same physical capacity can be governed by a private contract, a public service entitlement, a cooperative contribution arrangement, a reciprocal commons, or a combination of these. Monetary claims may enable some arrangements and obstruct others. Evaluating their transformation requires attention to the actual distribution of benefits, authority, and burdens rather than presuming that a transition from prices to nonmonetary coordination is always emancipatory or that efficient monetary exchange is always legitimate.

Consider three hypothetical organizations operating a comparable AI-assisted scientific instrument. A commercial provider collects usage fees and reinvests part of its income in equipment replacement. A public institution offers free use at the point of access while financing operation through collective budgets. A multi-institutional commons allocates access through shared stewardship commitments, jointly agreed priorities, and nonmonetary contributions. Each arrangement may maintain valuable scientific production. Their differences concern who can gain entry, who pays for material reproduction, who chooses research priorities, how contributors are recognized, and what remedies are available when allocation rules fail. Neither the existence nor the absence of a usage fee settles these questions in advance.

As technology and relations change, hybrid arrangements may be particularly prevalent. A commons may purchase electricity from a commercial supplier; a public observation network may rely on privately manufactured devices; a corporation may contribute freely available datasets while retaining licensed computational services; an AI-operated maintenance network may use nontransferable quotas for routine coordination and enforceable credit for exceptional purchases. Such configurations can display declining monetary mediation on one set of links and expanding monetary claims on another. Their analysis calls for careful tracing of the recognized counterparties, financing arrangements, and governance rules established in Sections 5–10.

The political-economic choice is therefore partly about what kinds of future relational capacities an institution makes available. An exclusive platform can provide reliable service while restricting independent reuse and collective modification. A federated public infrastructure can permit distributed participation while facing the demanding tasks of maintenance, secure access, coordination, and accountability. An autonomous AI production network may reduce external input purchases while concentrating governance in an opaque controller, or it may support wider provision under shared oversight. These are contingent institutional trajectories, not stages ordered by an unavoidable historical law. The existence of more sophisticated prediction and coordination does not decide which trajectory a society should endorse.

This perspective also places a limit on proposals for a single universal equilibrium of heterogeneous value. The relations that create scientific insight, care, ecological resilience, institutional trust, or expressive participation can evolve in ways that transform the criteria by which they are recognized. A clearing mechanism may legitimately compare certain monetary obligations under stated conversion and authorization rules. It does not thereby supply one complete metric for the value of every generative

contribution. An analysis attentive to the partial ordering of histories must leave open the possibility that forms of value and their institutional recognition are themselves transformed during the processes under study.

The role of global public infrastructure is significant but bounded. Shared observational standards, verified provenance, interoperable data interfaces, and sustained transnational cooperation can lower some verification costs and make reciprocal coordination more feasible. Weather and ocean observing systems demonstrate that international institutions can maintain forms of practical cooperation despite political differences, yet those precedents do not establish an unrestricted commons for every kind of AI data, computation, or resource. Trust, security, unequal power, cultural interpretation, funding, and rights of contestation remain distinct research subjects. The present paper identifies how such institutions could alter the conditions of monetary mediation; it leaves their detailed governance and engineering to the later investigations indicated in Section 14.

The ethical and political-economic conclusion is correspondingly conditional. It is possible for particular monetary relations to dissolve while productive relations, valued encounters, and new needs for others continue to develop. It is also possible for new currencies, credits, fees, and access restrictions to arise as technologies and institutions change. The amount of money remaining in a ledger is not a comprehensive measure of justice, generativity, or ecological responsibility. Rather, a defensible assessment of a proposed transformation considers what has actually been extinguished or created, whose contributions and interests are represented, who retains control over productive conditions, and whether affected participants have practical means to prevent, challenge, and repair adverse consequences. These questions prepare the broader theoretical discussion of the next section without prescribing a single institutional model or an inevitable monetary endpoint.

## 13 Theoretical Discussion

The preceding sections developed an exploratory account of the emergence, transformation, partial dissolution, and regeneration of monetary mediation in evolving production relations. The purpose of this section is to evaluate the reach of that account. Its task is neither to predict a determinate postmonetary society nor to reinterpret every cooperative exchange as latent monetary circulation. Rather, it asks which propositions follow from the limited mathematical representation, which claims remain conditional political-economic hypotheses, and which implications require separate institutional or ethical judgment. This differentiation is important because the argument moves among descriptive relations, accounting identities, possible historical continuations, and normative assessments. A result established in one domain cannot acquire authority in another solely because all four domains can be represented in a common diagram.

The primary research object has remained monetary *mediation*, understood through the functions and institutions that recognize, denominate, circulate, and discharge claims. The need for others, whether for encounter, participation, productive labor, capacities, knowledge, or outputs, is an important motivation for the formation of economic relations. However, a relational need can be met through provision, reciprocity, gift, collectively governed access, credit, market payment, or some institutional combination. The possibility of an alternative does not entail that it will be available, acceptable, or sustainable. This is why monetary dissolution is analyzed here as a relation-specific and institutionally mediated transformation rather than as the disappearance of need, value, or otherness. Its generative orientation supplies questions about the continuing formation of capacities and relations; its monetary, AI-production, and topological questions remain independently specified hypotheses.

### 13.1 Monetary Dissolution as a Relational Process

An important distinction emerging from Sections 4–10 concerns what exactly is said to dissolve. An amount of compatible outstanding debt can be reduced; a class of transactions can cease to require payment at the point of access; an institution can cease to denominate a particular obligation in money; or

an entire productive arrangement can become less dependent upon monetary finance. These are different operations. A debt cancellation changes a set of recognized claims; a modification of access rules changes the conditions under which relationships may be entered; an improvement in productive capacity changes possible provisioning; and a change in ownership or collective governance changes who may demand payment. Their causal pathways may intersect without becoming identical.

To make these levels visible in a common diagnostic, let  $h$  denote a represented relational history and  $S$  a declared set of relevant relations, participants, and productive dependencies. We may record a *monetary-mediation profile*

$$M(h; S) = (A, P, C, F, G)_{h; S}, \quad (50)$$

where  $A$  concerns monetary requirements for access and encounter,  $P$  payment requirements for ordinary provision,  $C$  outstanding compatible credit and settlement claims,  $F$  dependence on monetary finance for reproduction of productive conditions, and  $G$  monetary rules embedded in governance or gatekeeping. These coordinates are descriptive families of measurements rather than five commensurable real numbers. Their operational definitions depend on the unit of observation, jurisdiction, counterparty recognition, and purposes of inquiry. No unqualified scalar “monetary dependence” is implied by Equation (50).

Conceptual inference in this inquiry therefore follows a typed sequence: a represented *need* may motivate a relation; a productive or institutional *exchange* may give rise to a recognized *claim*; a claim may be *monetarily denominated*; an admissible operation may *reduce or discharge* that claim; and a separate investigation determines whether *monetary dependence* has changed across the broader production boundary. Each transition requires evidence or an explicitly stated assumption. The typed sequence is a discipline of interpretation, not a historical assertion that events must occur in that order.

A relation may undergo a decrease in  $A$  while its  $F$  increases, or clear part of  $C$  while new credit instruments enter  $G$ . Thus, even within one named AI service, the event “the user no longer pays” is insufficient to establish a reduction across all five coordinates. A public or commons-based institution could make the service free at use while financing its continued operation with collectively pooled monetary resources. A commercial operator could do the same through cross-subsidy and a larger upstream procurement budget. An AI-operated network with internal transfers could replace some recognized debts through authorized multilateral discharge while continuing to borrow capital for equipment replacement. The meaning of dissolution is therefore inseparable from the accounting, productive, and institutional boundaries selected for observation.

A second distinction concerns the relation between monetary *stocks* and monetary *functions*. An institution that performs frequent clearing might substantially reduce recorded gross liabilities without reducing the range of goods and relationships organized through money. Conversely, a community can coordinate access without directly imposing user debts even if the common infrastructure depends upon a monetary budget. The first case changes a balance-sheet object without necessarily changing the organization of production; the second changes the entry condition without necessarily replacing the financial basis of productive reproduction. An adequate empirical account would observe both the claims discharged and the activities that still require a medium of payment, a unit of account, a store of value, or publicly recognized settlement. The account developed here does not presume that those monetary functions must change together [15, 5].

It follows that no single figure, whether nominal money stock, total AI payment volume, platform usage fees, or gross cleared obligations, is sufficient to identify distributed monetary dissolution. The relevant object is a historical pattern of transformations across a declared family of relations. Its interpretation depends on which of the monetary functions have actually been displaced, which remain latent in upstream arrangements, and whether new claims have formed in the same or different parts of the system. This relational orientation makes the proposed process empirically more demanding than a one-dimensional forecast, but also better able to discriminate genuine change from a shift in where payments appear.

### 13.2 Uneven Dissolution across Production Relations

The possible ordering of monetary transformations among human–human (H–H), human–AI (H–AI), and AI–AI (AI–AI) production relations has been one of the paper’s guiding conjectures. Its strongest plausible formulation is conditional. Where increasingly capable AI systems supply activities formerly purchased from other humans, some direct H–H monetary exchanges may become less important. Where AI production networks can obtain or reproduce energy, data, computational equipment, maintenance, and material inputs without purchasing these from human-controlled suppliers, some H–AI monetary dependencies may decline. Yet coordination among specialized AI-operated subsystems may continue to require institutionally recognized payment or credit, even after the human-facing service becomes freely accessible. This supports the possibility of a delayed or more persistent monetary layer inside AI-operated production.

This possibility should not be mistaken for a universal temporal ordering. First, the labels H–H, H–AI, and AI–AI classify operational interaction sites, not necessarily independent legal counterparties. A transfer between two AI agents operated by the same corporation can be an internal cost allocation, whereas two human institutions exchanging resources may generate an enforceable debt. The correct object of monetary analysis is the actual obligation and its bearer, not the visible software interface. Second, a particular H–H relation may remain monetized for reasons unrelated to tasks that AI can automate, including access to exclusive communities, scarce land, personal services, legal entitlements, and organizational control. Third, public provision can reduce AI–AI payment requirements earlier in a narrow setting than commercial markets reduce H–H charges elsewhere. The ordering is accordingly a hypothesis about specified dependencies, not a law of succession.

One way to test the conjecture would be to define a family of service and financing networks for which comparable longitudinal records exist. Direct-payment shares, new credit issuance, upstream energy and equipment costs, financing requirements, and recognized legal counterparties would have to be tracked separately. Within such a population, a decline in user-facing payments accompanied by an increase in upstream procurement is evidence for *monetary migration*, not immediately for network-wide dissolution. A decline in both, coupled with independently verified continued provisioning and replacement of productive inputs, would lend support to a stronger claim. Observations of sustained or increasing H–H payments in a sector where AI has replaced many routine tasks would challenge a simple displacement hypothesis, although alternative changes in quality, scarcity, or access might explain the observed pattern.

The distinction also avoids treating productive autonomy as a property that simply rises with machine intelligence. A model may achieve high performance at cognitive tasks while relying heavily on external power networks, specialized chips, human maintenance, land rights, or restricted observational data. Conversely, a carefully governed public infrastructure may reduce the monetary cost of human access without possessing a fully autonomous AI at all. Productive autonomy is relative to a service specification, relevant shocks, and a boundary of inputs and permissions, as developed in Section 11. An economic prediction based on AI competence alone leaves out the material and institutional routes through which recognized monetary claims may arise.

More generally, the three-way ordering is sensitive to property regimes. Under private control of critical AI production assets, fees and licenses may persist or become concentrated even as the physical marginal cost of some services decreases. Under public provision, cooperative management, or commons-based access, participants may replace some prices with contribution rules, entitlements, or collectively agreed priorities. Those alternatives are not automatically equivalent in sustainability or justice. The analysis therefore calls for comparison of productive capabilities under more than one institutional arrangement rather than imputing a single economic destiny to AI [21, 23, 12].

### 13.3 Monetary Clearing and Heterogeneous Value Circulation

Sections 7–9 established a precise but bounded result: a compatible, nonnegative circulation in a finite directed obligation graph can identify a family of possible gross-claim reductions that preserve specified

monetary net positions. Each such circulation admits a decomposition into directed cycles, and, under the unrestricted maximum-gross-reduction objective of Section 8, the remaining supported obligations cannot contain a further positive directed cycle. These results depend on a declared denomination or compatibility class, divisibility, fixed counterparties, and the specified clearing constraints. Their proofs do not infer consent or legal discharge from the mere existence of a cycle.

The distinction is essential for the three-party example that motivates the inquiry. Suppose A is obliged to B, B to C, and C to A for equal amounts in one eligible settlement class. A compatible cyclic reduction can decrease each gross obligation by the same amount while preserving the recorded net positions. In a social or productive system, however, A's contribution to B might be specialized knowledge, B's to C might be equipment maintenance, and C's to A might be energy provision. Their *heterogeneous* contributions need not be measurable on one axis. The debt-cycle representation arises only after specific institutions have recognized claims and applicable conditions for cancelling them. The unalienated, ecological, expressive, or relational value involved in those contributions cannot be inferred from the equality of their currency denominations [11].

The event-based two-complex provides a complementary rather than interchangeable representation. Vertices represent events, edges represent couplings, and faces represent contextually specified relational primitives. Historically formed agents, facilities, and organizations can be reconstructed from relevant substructures and their continuations. By contrast, monetary obligations are represented on a typed claim graph whose positions depend on recognized holders and debtors. A topological cycle in a relational two-complex is not automatically a cycle of legally compatible obligations. Nor does an element of a first homology group itself establish a debt, creditor, or clearing instrument. The bridge between these objects is a modeling and institutional interpretation that must be justified for each application [11, 7].

The long-path argument adds a second important limitation. In a specified stochastic model, the survival probability of a circuit with many dependent steps may decrease with length. Yet it does not follow that the aggregate contribution of long circuits is negligible: the number of candidate circuits may grow, and risks may be correlated. The sufficient truncation condition and expected loss bound established in Section 9 require uniform capacity and survival controls, together with an upper bound on the growth of candidates. They are not statements about all economic systems or about the moral insignificance of distant productive relations. In particular, pruning low-probability economic clearing opportunities cannot justify disregarding rare but irreversible ecological impacts. Such impacts require separate models of precaution, exposure, and responsibility.

The formal clearing model is also one of several possible computational approaches. For finite compatible divisible claims, circulation constraints can be handled by compact network-flow or linear-programming methods; length truncation is especially relevant when enumerating candidate intertemporal, institutionally constrained, or reliability-weighted paths. Even in these settings, an optimization algorithm must specify the objective it optimizes. Maximizing gross debt removed, minimizing the number of settlements, minimizing vulnerability to failure, distributing benefits equitably, and protecting contestation rights are distinct objectives. A formally optimal solution relative to one criterion may be undesirable under another. This observation is a methodological boundary for proposals that place highly capable AI systems in charge of multilateral economic coordination.

The proposed synthesis is therefore careful: heterogeneous value generation and circulation motivate inquiry into relational exchange; recognized monetary claims form one institutionally constituted representation of selected obligations; admissible cycles may reduce those claims; and both the underlying value relations and the conditions of their continuation remain open. This allows the model to investigate an important mechanism of monetary dissolution without replacing the substance of a generative economy by an accounting identity.

### 13.4 Technological Autonomy and Institutional Dependence

The infrastructural analysis points toward an apparent tension. AI systems may become increasingly capable of planning, optimizing, and executing production, yet the reproduction of that productive ca-

capacity remains dependent on physical and socially controlled conditions. Energy must be generated and delivered; computing equipment must be manufactured and maintained; materials must be sourced; observational access must be established; data quality must be assessed; and the consequences of these activities must be governed. Even if an AI-operated network eventually performs many of these functions internally, it remains embedded in a wider system of physical processes, property arrangements, institutions, and ecological relations.

For the present inquiry, the question is whether any particular dependency generates a monetary requirement. An AI production network might continue to purchase electricity from a separate utility; it might operate a commonly governed renewable source under jointly agreed allocation rules; it might use a public grid financed collectively; or it might coordinate multiple suppliers through reciprocal credits. All of these could support similar computational output while differing in monetary mediation. The physical production of electricity and the institutional right to obtain it have to be represented separately. The same holds for observational data, access to computing sites, robot maintenance, environmental permits, and rights to reproduce equipment.

A future AI network with substantial internal provisioning may therefore exhibit a decline in monetary transactions across the human–AI boundary. This is a conditional possibility, not proof of independence from human beings, ecosystems, or the material environment. Nor does a decline in demand for human-supplied inputs settle how any benefits are distributed to humans or which parties control the production network. In some arrangements, the absence of payment by AI to humans might result from successful public provisioning; in others, it might reflect exclusion from control or compensation. The same recorded direction of monetary change admits divergent political-economic explanations.

One institutional interpretation of infrastructure emphasizes public goods, common pools, and mutual gains from interoperable standards. Weather and ocean-observation cooperation illustrate how distributed institutions can coordinate specialized observation and exchange without imposing one centralized owner of all measurements. Such precedents support the feasibility of some *domain-specific* global information arrangements, not an unrestricted global database or a universal governance settlement. Data sensitivity, strategic incentives, legal rights, verification, and unequal capacity to contribute can make cooperation substantially harder in other domains. Consequently, global data infrastructure should be regarded as a possible contributor to distributed recognition and coordination, with its design and limits examined in the companion research on public knowledge systems [26, 22, 10].

Trust has a similarly nonmonotone role. Reliable evidence and credible commitments may make participants willing to exchange without immediate monetary settlement, but they can also support broader creation of interest-bearing or transferable credit. Systems that supply accurate forecasts and verifiable provenance can reduce some coordination costs, yet the authority to declare claims valid, to prioritize scarce resources, or to refuse access remains a governance question. A highly accurate AI operator does not by technical achievement alone gain the legitimacy to discharge another party's debt or to decide which environmental impact is acceptable. The distinction between epistemic capacity and institutional authority should remain explicit in any subsequent engineering proposal [8].

The implication for the present paper is limited and consequential. Infrastructure building may transform the conditions in which monetary mediation arises, including the need for bilateral settlement and the capacity for direct provision. Yet no single infrastructure, whether energy, data, compute, or trust, is sufficient by itself to dissolve all monetary functions. The paper's contribution is to identify these dependencies and their interacting constraints. Engineering designs for global information commons, marine ecological observation, and autonomous AI material reproduction deserve independent technical, institutional, and environmental treatment.

### 13.5 Distributed Equilibrium and Historical Generativity

The intuitive image of a circulation network whose participants can eventually say that they “owe nothing to one another” requires qualification. A recognized monetary network can have zero net positions even when its gross obligations remain strictly positive. It can also undergo valid gross reductions while

leaving some net positions unsettled. Participants may agree that a distribution of reciprocal benefits is acceptable without possessing comparable, legally enforceable obligations. These are different senses of balance. A zero vector obtained by multiplying a cycle flow by the incidence matrix is a mathematical property of a specified graph, not an ethical consensus, a physically sustainable state, or an economy-wide equilibrium.

For a compatible claim configuration  $x_h$ , an authorized reduction  $z_h$ , and incidence operator  $B_h$ , the elementary monetary invariant can be written

$$B_h(x_h - z_h) = B_h x_h \quad \text{whenever } B_h z_h = 0. \quad (51)$$

The identity in Equation (51) describes net-position preservation at one represented configuration. It says nothing about satisfaction of relational needs, the existence of unrepresented claims, the distribution of bargaining power, or ecological effects. When the network changes, even the domain and codomain of  $B_h$  can change. A meaningful account of successive or partially ordered monetary configurations therefore requires rules identifying which positions and claims can be compared, how records are transported across altered structures, and which new obligations are recognized. Section 10 expressed this distinction with a transported residual and newly constituted claims. Such a continuation is not an equilibrium identity in the usual fixed-domain sense.

An attractive but demanding speculation is that sufficiently sophisticated distributed intelligence could coordinate very many heterogeneous obligations without routing each exchange through a circulating monetary token. AI systems might forecast dependencies, discover chains of indirect reciprocity, negotiate conditional plans, and propose admissible multilateral settlements across many events. However, computational prediction does not eliminate the need to specify which commitments may be recognized, what makes them comparable, how uncertainty is allocated, and who can authorize their cancellation. An indefinitely long planning horizon introduces further mathematical difficulties: convergence, stability under new information, and the representation of changes in the set of agents and available activities. It is not legitimate to posit an infinite integral over “all future exchange” unless its integration domain, measure, observables, and convergence assumptions have been defined.

In particular, temporal continuation should not be conflated with an ontologically universal clock. The relational history model introduced in Section 7 admits partial orderings of events, alternative representations, and context-dependent comparison of configurations. Two exchange events can be causally incomparable in the model even if an observer later places both into the same accounting period. A claim formed on one admissible continuation need not be formed on another; a production capacity that emerges in one branch may change which further exchanges are possible. Consequently, neither “eventual monetary extinction” nor “eventual universal settlement” follows solely from allowing more computational time. The set of relevant relations, demands, and institutions may itself change under the generative processes being modeled [14, 9].

These observations motivate a shift away from a unique terminal equilibrium toward *bounded, revisable coordination* over histories. The operative question becomes whether a selected class of monetary mediations can be reduced under declared resource, legal, epistemic, and ecological constraints, while leaving the participating network able to continue generating and meeting needs. An attainable local clearing may be valuable even where no global closure is defined. Conversely, an apparently stable accounting balance may hide conditions that undermine the continued reproduction of the network. A generative analysis must therefore investigate the production and revision of coordination conditions, not merely their temporary equality at a selected observational cut.

### 13.6 Counterexamples and Boundary Conditions

The theory is most informative when confronted with cases that distinguish its proposed mechanisms from superficially similar outcomes. Several instructive counterexamples arise directly from the previous sections. They are hypothetical analytical constructions, not observations of existing AI economies.

*First, free access with concentrated financing.* A company offers an AI service without a user fee while charging other businesses for licensed access to the same infrastructure, financing hardware

through debt, and purchasing electricity under monetary contracts. Human-facing payment has disappeared, but monetary mediation has moved to different counterparties. Treating this as system-wide dissolution would conflate the user interface with the productive network. The case identifies the need for upstream flow-of-funds data and explicit boundaries.

*Second, extensive netting within a persistently monetary system.* A group of businesses regularly clears a large share of its compatible invoices through a centralized mechanism. Gross liabilities shrink, but new invoices are issued after each production period, prices remain the principal coordination signal, and firms continue to borrow for operations. The clearing procedure is real, yet monetary dependence in productive reproduction persists. This case separates a successful algorithm for debt reduction from a transition in monetary institutions.

*Third, high autonomy with restricted access.* A technically capable AI system maintains equipment and acquires environmental measurements, but a controlling organization retains exclusive rights to the sites, models, and transmission networks needed for operation. Charges for permission may persist despite a high degree of technical competence. The case shows why material capacity, legal control, and economic access cannot be modeled by one scalar autonomy index.

*Fourth, nonmonetary relations with severe scarcity.* An organization allocates limited computing time by deliberation, rationing, or administrative priority rather than by payment. Money is absent from the allocation procedure, but scarcity, conflict, and unequal access have not vanished. The ethical assessment depends on the allocation process, affected interests, transparency, and remedy, not solely on the absence of a price. The same concern applies to institutionalized reciprocal labor commitments: replacing wages with obligatory contribution rules does not automatically eliminate exploitation or coercion.

*Fifth, a graph cycle without eligible discharge.* Three parties owe similar nominal amounts, but their obligations differ in legal priority, currency, maturity, dispute status, or permitted counterparty netting. A cycle exists when the positions are viewed as an untyped graph, yet no valid multilateral discharge follows without authorized conversion, consent, or other legal procedure. The counterexample rules out the common inference from graph closure to executable monetary elimination.

*Sixth, correlated path survival.* A long candidate clearing circuit relies on multiple steps administered by one intermediary. If all steps succeed or fail together under a common shock, adding steps need not reduce survival probability geometrically. A path-length truncation based only on independent-edge intuition may then discard substantial eligible clearing. This case clarifies why the approximation result of Section 9 requires explicit bounds and why real dependence data are needed.

*Seventh, monetary closure with ecological loss.* A production consortium can clear every recognized financial obligation associated with building a computational facility while leaving an altered marine habitat or an uncompensated impact on neighboring communities. The monetary obligation network is closed on its declared coordinates, but ecological and public-interest questions remain open. A monetary-only model cannot determine whether the project ought to proceed. Section 12 formalized this independence between cleared monetary records and ecological outcomes.

These cases can be organized by the kind of inference they block. Table 13 summarizes the relevant distinction between formal results, contingent hypotheses, and normative assessment. The separation is not meant to imply that the domains cannot interact. It requires that any bridge between them be argued and investigated rather than silently presumed.

A further counterargument comes from monetary theory itself. Money may persist because it provides a common unit of account, supports impersonal exchange, settles obligations through legitimate public institutions, or coordinates activity among strangers whose histories are not mutually available [19, 20, 15]. Even excellent information infrastructures may leave a practical demand for a transferable and legally recognized claim, especially where participants cannot or do not wish to expose their complete transaction histories to a central or distributed intelligence. Privacy, strategic withholding, heterogeneous legal jurisdictions, and the costs of consensus can also limit universal reciprocity. A generative relational account must acknowledge these reasons for persistence rather than assuming that better cognition alone makes monetary institutions redundant.

The counterexamples establish boundaries, not a verdict for or against monetary dissolution. They

Table 13: Epistemic status and appropriate tests for selected claims in the present inquiry.

| Status        | Claim within its domain                                                                                          | Critical condition or test                                                                         |
|---------------|------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------|
| Formal        | Eligible cyclic reductions preserve specified net positions; compatible circulations admit cycle decomposition.  | Verify homogeneous typed obligations, incidence, divisibility and authorized discharge separately. |
| Conditional   | Long-path truncation has bounded expected loss under stated survival, branching and capacity limits.             | Estimate dependence and cycle growth; test correlated-shock alternatives.                          |
| Economic      | Monetary dependence can decline at H–H or H–AI interfaces while migrating into AI-operated supply networks.      | Trace recognized counterparties, financing, procurement and continued productive reproduction.     |
| Institutional | Public standards, trusted records or common provision may lower some transaction-specific monetary requirements. | Compare governance arrangements, entry conditions, credit issuance and infrastructure costs.       |
| Normative     | Monetary netting alone does not establish justified access, control, remedy or ecological responsibility.        | Specify affected interests, legitimacy criteria and independent environmental evidence.            |

help specify what evidence could distinguish an actual change in mediation from changes in accounting technology, interface pricing, or institutional rhetoric. They also show why neither mathematics nor technical performance can replace the separate evaluation of institutions and ecological effects.

A further economic qualification concerns the interpretation of observable “disappearance.” Decreased direct payment, reduced nominal gross claims by authorized clearing, and reduced monetary dependence in the reproduction of a productive system have different observational requirements. They may move in different directions as service demand and credit supply change. The empirical design in Section 14.2 and Table 15 accordingly treats these as separate outcomes rather than as interchangeable proxies. In particular, a lower number of invoices can be caused by organizational consolidation, not by reduced use of money between independent claim-bearing institutions; and a larger clearing optimum can arise from a relaxed eligibility rule without any corresponding increase in material productive autonomy.

### 13.7 Implications for the Future of Political Economy

The principal contribution of this investigation to political economy is to make the *conditions and distribution* of monetary mediation an object of comparative inquiry. Money is studied neither as an invariant consequence of human need nor as an institution destined to vanish once technology becomes sufficiently powerful. Its use depends on the relation of productive capacities, exchange practices, ownership and governance, recognized obligations, and possibilities for alternative coordination. AI matters because it can transform some of these relations simultaneously: it may supply productive services, alter patterns of dependency, make complex reciprocal coordination calculable, and generate new forms of control over physical and epistemic infrastructure. None of those transformations fixes the outcome independently of institutions.

This orientation differs from both narrowly technical and narrowly distributive interpretations. A technical reading would identify the discovery of compensating payment circuits as the dissolution of money; the previous sections show that circuit discovery is only a potential mechanism for certain compatible claims. A distributive reading might equate the disappearance of a price with the removal of domination; the ethical discussion shows that nonmonetary provision can still exclude participants or concentrate authority. A purely technological reading might infer a monetary-free economy from intelligent, self-maintaining AI; the infrastructure discussion shows that reproductive conditions, legal control and ecological effects have to be specified separately. These critical distinctions do not eliminate the explanatory value of clearing, public provision or autonomy. They clarify the scope within which each can contribute to a broader transformation.

A comparative research programme could accordingly observe several related but independent trajectories. It could study where new relational needs and productive roles arise, where institutions turn those needs into monetary claims, which clearing opportunities become legally admissible, which transaction classes shift toward nonmonetary access, and which resource dependencies continue to generate money or credit. It could examine how a change in data openness or energy provisioning alters monetary dependence at one layer while affecting property and governance relations at another. It could compare privately operated, publicly funded, cooperatively managed and commons-based infrastructures without treating any one arrangement as the default endpoint of economic development. Prior work on value-circuit grammars, renormalization and heterogeneous circulation offers exploratory representational connections, subject to further proof and empirical validation [7, 13, 11].

The public and global dimensions make the inquiry wider than a theory of AI payment systems. Shared observations, scientific knowledge, cross-border trust, interoperable records, stable energy supply, material maintenance, legal remedies and ecological monitoring can change what it is possible to produce and exchange, and under which terms. Some common infrastructures may allow access without individual payment; others may extend credit or support markets. Their legitimacy depends on who shapes their standards, contributes their labor or knowledge, bears costs and can challenge consequential decisions. These issues justify companion investigations into global knowledge infrastructure, marine observation and robotics, ecological conditions of autonomous production, and international trust and governance. They should not be compressed into a complete engineering or institutional prescription within the present paper.

A useful research question is consequently not simply *when* money will disappear. It is *which monetary mediations, for which needs and productive relations, can be transformed through which technical and institutional arrangements, with what residual obligations and consequences?* This formulation permits both persistence and dissolution to be observed, and allows new monetary claims to form as relational environments evolve. It does not privilege one future history, and it leaves open whether particular relations should continue to be mediated by money even where technical alternatives are available.

The paper's provisional answer is that distributed monetary dissolution can be modeled as a family of bounded, historically situated transformations, some of which admit exact clearing results and conditional computational approximations. The larger transformation of monetary dependence remains an empirical and institutional question. Moreover, no reduction of currency-denominated claims, however extensive, can by itself settle the claims of heterogeneous value generators, the legitimacy of production governance, or the ecological conditions of continued economic life. This limited conclusion is compatible with a generative perspective in which the production of capacities, encounters, relations and needs continues to create new possibilities for exchange [14]. Section 14 identifies the corresponding unresolved problems and the distinct research programmes needed to investigate them further.

## 14 Open Problems and Research Continuations

The purpose of this section is to identify what remains unresolved by the preceding account and to specify research continuations whose results could substantively alter its conclusions. Sections 7–9 obtained restricted mathematical results about compatible monetary obligations, directed cycles, and some conditions for computational truncation. Sections 10–13 located these results within evolving production relations and identified additional dependencies on material reproduction, institutions, recognition, and environmental conditions. These levels of analysis should not be conflated in a research programme. Exact identities for monetary claims cannot establish a theory of heterogeneous value; increasing autonomous task performance cannot establish productive self-sufficiency; and financial balance cannot establish that affected ecological or social relations have received adequate consideration. The remaining problems concern the bridges among these levels as much as the questions within each level.

The programme proposed here is deliberately plural. Its investigations could find that certain monetary functions become dispensable within narrow networks, that apparently demonetized interfaces depend on expanding upstream finance, or that monetary claims are regenerated through new forms of

Table 14: Open questions and principal evidential demands in six research continuations.

| Domain                   | Principal question                                                                     | Potential evidence or test                                        |
|--------------------------|----------------------------------------------------------------------------------------|-------------------------------------------------------------------|
| Formal circulation       | When does local clearing represent a valid relational transformation?                  | Typed obligations, discharge certificates, compositional proofs   |
| Empirical identification | Is monetary reliance reduced or merely displaced?                                      | Multi-layer claims, payments, access and financing records        |
| Global knowledge         | Which distributed observation and inference arrangements are viable?                   | Interoperability tests, provenance and access audits              |
| Ocean intelligence       | What can autonomous observation learn without unacceptable ecological disturbance?     | Calibrated measurements, ecological baselines, field validation   |
| AI production autonomy   | Can productive inputs and their reproduction be maintained within a stated boundary?   | Energy, material, maintenance and governance ledgers              |
| Trust and governance     | Which commitments remain legitimate and contestable across heterogeneous participants? | Institutional comparison, challenge procedures, remedial outcomes |

access control. They could also establish that some public or cooperative arrangements support direct provisioning while others reproduce exclusion and extraction. None of these outcomes should be ruled out in advance. The term *research continuation* consequently denotes a possible extension of the analysis, with its own explanatory target and admissible evidence; it does not designate a necessary next stage of history. Table 14 summarizes six domains whose detailed investigation belongs in further formal, empirical, and interdisciplinary studies.

#### 14.1 Formal Models of Spacetime-Distributed Circulation

The first open problem is to build a mathematical account in which recognized monetary obligations, productive relations, and heterogeneous value circulation can be represented together without being identified with one another. The event-based two-complex introduced in Section 7 encodes relational events, couplings, and context-dependent primitives, whereas the compatible-obligation graph of Section 8 encodes recognized claim holders, legal counterparties, and liabilities susceptible to clearing. A topological cycle in one representation therefore need not induce an economically meaningful cycle in the other. A suitable formal extension must state the mapping between the two representations, the information lost through that mapping, and the institutionally admissible operations on the result. Prior explorations of value-circuit grammars, renormalization, and relational two-complexes provide candidate tools, but their integration requires independently stated assumptions and proofs [7, 13, 11].

One possible architecture is a family of typed structures indexed by represented histories,  $\{\mathcal{K}_h, \mathcal{O}_h, \mathcal{V}_h\}_{h \in \mathcal{H}}$ , where  $\mathcal{K}_h$  represents event-level relational structure,  $\mathcal{O}_h$  a network of enforceable or otherwise recognized obligations, and  $\mathcal{V}_h$  heterogeneous value records. The set  $\mathcal{H}$  need not carry a total ordering. Its admissible continuations may be encoded through specified morphisms  $\gamma : h \rightsquigarrow h'$ , with a distinction between transformations contemplated in a model and discharge events actually recognized by the relevant institutions. A family of representation maps might have the form

$$\mathcal{K}_h \xrightarrow{\pi_h} \mathcal{O}_h, \quad \mathcal{K}_h \xrightarrow{\nu_h} \mathcal{V}_h, \quad (52)$$

where  $\pi_h$  and  $\nu_h$  are partial, purpose-dependent representations rather than assumed lossless translations. Equation (52) is a research design, not a claim that every productive or ecological event gives rise to a monetary obligation. Indeed, a meaningful model needs explicit support for events that produce value without creating any denominated claim, as well as for monetary claims whose current holders are not the original generators of the underlying value.

The mathematical problem becomes more demanding when obligations have different denominations, maturity conditions, contingent triggers, priority rules, rights of objection, and jurisdictional recognition. The familiar nonnegative edge-flow formulation assumes a common compatible type and divisible amounts. To relax these restrictions, a formalization may employ typed categories of claims, partially ordered feasibility conditions, or constrained optimization on admissible subsets. An announced future contribution can be represented without being treated as a presently dischargeable debt. Similarly, a projected AI action can be included in a contingency model without silently becoming an enacted legal act. These distinctions are important in a paper that refuses to take a universal temporal coordinate or a privileged present as self-evident.

Compositionality is another unresolved problem. Suppose two subnetworks independently clear all admissible local cycles. It does not follow that their union admits globally maximal reduction: obligations crossing their boundary may create additional cycles, or may be legally incompatible despite apparent numerical closure. Researchers should characterize the cases in which local clearing operations compose, when they conflict, and what boundary information must be preserved. A proposed proof should distinguish invariants of the obligation incidence matrix, invariants of actual institutional discharge, and any claims about the reproduction of productive conditions. These are different forms of preservation. In particular, zero net monetary position is not a sufficient invariant of reciprocal value or ecological restitution.

Finally, the fragility result in Section 9 suggests a computational agenda. Longer circuits can contribute progressively less to expected gross clearing when reliability decays sufficiently strongly relative to the number and capacity of eligible circuits. Yet correlated shocks, repeated use of shared resources, and adversarial withholding can invalidate a naive independent-edge model. An eventual algorithm should report which probabilistic assumptions were tested, how capacities are counted without double use, and whether a proposed truncation bound concerns expected value, tail probability, or worst-case error. A reproducible numerical benchmark could compare exact small-instance clearing, restricted-length clearing, and approximate large-instance procedures across synthetic networks with known dependence structures. Any algorithmic efficiency claim would remain distinct from an institution's authority to extinguish claims.

## 14.2 Empirical Identification of Monetary Dissolution

The second problem is empirical: how could one detect a transformation of monetary mediation rather than a change in how payment is displayed? Section 13 proposed a multi-dimensional profile covering access, provision, credit, productive financing, and monetary governance. Each dimension requires observation at a declared boundary. A zero-price service may reduce the payment required of one user while increasing the operator's borrowing, energy purchases, subcontracting, or use of privately licensed data. A cooperative project may remove direct subscription fees while preserving monetary assessments for shared equipment. A nominally autonomous AI agent may send tokens to another software agent that, in law and accounting, belong to the same organization. Treating all such transfers as equivalent monetary transactions would confuse technical protocol with political-economic institution.

An operational research design should begin with explicit units of observation: which participants, institutions, resources, recognized claims, jurisdictions, and productive dependencies are included? It should then maintain distinct records of (i) access and provision terms; (ii) gross and net payments; (iii) new credit issuance and the disposition of claims; (iv) monetary outlays required to replace energy, equipment, materials, and labor; and (v) the rules that allocate control over essential resources. For a history  $h$  and a declared boundary  $S$ , an empirical profile could be represented as

$$\hat{M}_{h;S} = \left( \hat{A}, \hat{P}, \hat{C}, \hat{F}, \hat{G} \right)_{h;S}, \quad (53)$$

where the hats indicate measurements produced by a stated observation procedure, not direct access to an exhaustive economic reality. Differences between profiles require a declared comparison method,

Table 15: Proposed identification tests for distinct mechanisms of apparent monetary disappearance. Every row is an empirical hypothesis and requires a declared institutional boundary.

| Observed pattern            | Discriminating records                                                            | Alternative explanation to test                      |
|-----------------------------|-----------------------------------------------------------------------------------|------------------------------------------------------|
| Zero user price             | Provider revenue, subsidies, contracts, upstream energy and compute bills         | Cross-subsidy or payment migration                   |
| Fewer AI–AI invoices        | Legal counterparties, internal accounting rules, ownership, unsettled liabilities | Intra-firm consolidation or changed reporting        |
| Gross claims fall           | Before/after claim ledger, compatible reduction and discharge certificate         | Mere matching proposal, write-off, or payment        |
| Fewer outside inputs        | Replacement energy and equipment, maintenance, permissions, control rights        | Changed system boundary or displaced resource demand |
| Higher nonmonetary exchange | Repeated provision, access terms, enforceable commitments, audit trail            | Barter with hidden prices or unpaid extraction       |

especially when the composition of services or the relevant network boundary changes. No implicit addition across unlike dimensions is justified.

The paper’s proposed H–H, H–AI, and AI–AI ordering is suitable for comparative rather than deterministic testing. A useful design would follow matched production activities under differing degrees of AI participation, separating transactions made visible to human users from monetary obligations borne by suppliers or owners of the AI system. It would also compare institutional regimes rather than assuming that every sector belongs to an undifferentiated market economy. Commercial, cooperative, public, and commons-based operations can display similar interfaces while imposing different financing conditions on contributors. A finding that human-facing fees decline while energy-sector debt and hardware procurement expand would support monetary migration. A decline in those upstream liabilities, accompanied by independently evidenced continued provisioning and productive replacement, would support a more extensive transformation. Observed increases in both user fees and upstream credit under rapid AI adoption would be informative counterevidence.

Identifying causal mechanisms is more difficult than cataloguing correlations. Changes in electricity prices, regulations, ownership, population needs, service quality, cross-subsidies, and accounting conventions can move measured monetary flows even without a change in AI autonomy. Natural experiments, panel comparisons, process tracing, and transparent counterfactual models could help distinguish these explanations, subject to the usual limits on identification. The condition “AI becomes a general-purpose productive other” should be operationalized through independently testable service capability and dependable supply, not inferred from publicity or model benchmarks alone. Equally, “self-sufficiency” must specify which input dependencies remain outside the measured boundary.

Attention should also be given to missing observations. Informal reciprocity, unpriced ecological burdens, undisclosed procurement arrangements, and proprietary AI-agent accounting can make a network appear more autonomous or less monetary than it is. The absence of recorded payment is not equivalent to the absence of opportunity cost, exclusion, or hidden liability. A practical research protocol should therefore document data omissions, competing interpretations, the rights of observed participants, and possible institutional incentives to misreport. Its evidence standard ought to admit a result of *indeterminate change*: when the data do not distinguish genuine dissolution from relocation of monetary dependence, a determinate conclusion would be unwarranted.

For empirical use, the profile in Equation (53) requires a minimal event-level evidence protocol. An invoice issued, a transfer paid, a claim legally discharged, a new credit facility opened, and an internal scheduling entry are five distinct events even if an interface displays the same numerical unit. Their records should have persistent identifiers, issuer and beneficiary roles, claim type, denomination, governing authority where relevant, admissible observation boundary, and provenance. Linking them by contract or discharge identifiers is preferable to aggregating invoices and payments as if they were the same kind of monetary flow. The investigator should also record off-boundary suppliers and the resources needed for replacement and maintenance. Absence of records cannot be coded as absence of monetary dependence without an explicit missing-data model.

A minimally falsifiable comparison distinguishes three non-equivalent outcomes: *interface disap-*

*pearance*, in which a particular direct charge falls to zero; *settlement reduction*, in which appropriately compatible and authorized gross claims are discharged while nominal net positions are preserved; and *reproduction-level displacement or transformation*, in which the service’s material and institutional prerequisites change. The first can be observed without the second, and neither implies the third. A service that becomes free to users while verified upstream invoices, credit stocks or maintenance payments increase would contradict any inference of system-wide monetary dissolution from user prices alone. A finding that AI–AI transfers are merely internal cost allocations, with no distinct recognized claim holders, would refute their classification as monetary credit even where the software protocol records a circulating token.

A stronger hypothesis about productive autonomy would compare networks before and after a change in the ability to acquire and reproduce a specific input *within the declared boundary*. For example, verified autonomous energy generation can reduce purchased electricity while the same organization continues to bear loan payments, land access fees or equipment procurement obligations. The predicted reduction is therefore restricted to a specified input-payment function unless further institutional transformations are observed. A change in data access similarly requires checking provenance, consent, observability and continuing rights of exclusion rather than counting the volume of generated data alone.

If an institutional reform supplies a plausible comparison, a difference-in-differences or event-study design may be informative only under defended assumptions about comparability, accounting stability, anticipatory changes and other concurrent shocks. A change in the clearing rule is rarely exogenous to trust, defaults, credit supply or governance. Researchers should report negative controls such as claim categories unaffected by the reform, examine differential trends in upstream input purchasing, and state how incomplete internal ledgers could bias the direction of inference. Quantitative estimates of monetary change should not be promoted to causal effects where these requirements cannot be met. The partial ordering of relational histories remains compatible with time-indexed empirical panels when clock time is explicitly an observation convention, not an ontological axiom.

### 14.3 Global Knowledge and Data Infrastructure

The third research continuation concerns infrastructures through which heterogeneous participants and AI systems may jointly observe, interpret, and coordinate aspects of the world. Greater knowledge can sometimes enable multi-party matching and reduce uncertainty about whether a promised contribution is deliverable. Yet a globally connected data system is neither a sufficient condition for monetary dissolution nor an automatically desirable institution. Data may be inaccessible, biased, selectively produced, or subject to justified restrictions; inferences may be mistaken; and control over computational infrastructure can create new claims to payment or new gates to participation. The research problem is to specify which forms of public epistemic capacity change the feasibility and legitimacy of coordination, and which reproduce asymmetry.

International environmental observation illustrates both the possibility and limits of cooperation under heterogeneous political interests. The World Meteorological Organization’s WIS 2.0 exemplifies standards-based exchange among distributed data producers, while the Argo programme illustrates international participation in ocean observations [26, 22]. The Global Ocean Observing System coordinates physical, biogeochemical, and biological observations through a framework of essential ocean variables [16]. These arrangements demonstrate that distributed observation and shared standards can be organized without consolidating ownership of every sensor or dataset in a single institution. They do not establish that every category of medical, commercial, security-sensitive, or personally identifiable data can or should be governed by identical access rules.

A companion investigation should therefore distinguish at least four infrastructural functions. The first is *observation*: who can collect measurements, at what spatial and relational resolution, with what calibration and uncertainty? The second is *interoperation*: how can datasets with different semantics, temporal support, units, and provenance be compared without erasing their differences? The third is *inference*: how are models updated, audited, and contested when new measurements or interpretive

frameworks emerge? The fourth is *governance*: who sets the conditions of access, records revisions, receives credit for contributions, and obtains remedies when consequential errors or exclusions occur? A technically elegant global index that omits any one of these questions may perform poorly as a public knowledge infrastructure. A companion empirical comparison could document four observable variables across participating nodes: discovery completeness, latency and quality of shared observations, the fraction of records with verifiable provenance and revision histories, and the accessibility of correction or appeal procedures. The WMO's separation of core from recommended data provides an institutional counterexample to the assumption that global collaboration requires universal unrestricted disclosure [27]. Such a comparison would also report whose measurements are absent and whether contributors retain usable access to models and derived products. Differences in these indicators would not by themselves measure monetary dissolution; monetary claims and financing would require a separate, permissioned ledger.

The resulting system might be federated, with some knowledge products openly available, some handled through negotiated access, and some subjected to privacy-preserving or secure computation. The precise architecture should depend on sensitivity, observational quality, operational resilience, and legitimate local autonomy. A shared ontology is useful only to the extent that its categories remain revisable and accountable to communities whose observations it represents. Provenance records should register not simply where a file came from, but also how an observation was selected, transformed, interpreted, disputed, and incorporated into a downstream claim. A record of computational steps can support reproducibility while still failing to expose epistemic biases embedded in the original observation process.

The relation to monetary dissolution should be investigated through explicit mechanisms. For example, better knowledge of complementary productive capacities may reduce search and verification costs for nonmonetary reciprocity; auditable commitments may permit admissible multilateral clearing; interoperable records may make a public service feasible without individually priced access. The same technical improvements can expand the reach of credit markets, surveillance, exclusion, and extraction. The comparative question is therefore not whether a global knowledge commons produces monetary dissolution, but under which institutions greater mutual observability supports particular types of exchange and whether participants can challenge the resulting classifications. The detailed specifications of federated knowledge architecture, epistemic translation, and public governance are appropriately reserved for a separate paper.

#### 14.4 Ocean Observation and Ecological Intelligence

The fourth continuation begins from a scientific motivation that is independently important: substantial portions of the ocean remain difficult to observe repeatedly, at sufficient resolution, or across interacting physical, chemical, biological, and geological processes. The research value of ocean exploration should not be reduced to finding a location for computational facilities. Autonomous underwater vehicles, remotely operated systems, fixed instruments, floats, satellites, and human field science can contribute different and partly complementary observations. Their coordination raises fundamental questions about sampling under uncertainty, model-dependent inference, ecological disturbance, cross-scale representation, and legitimate access to shared environments.

An ocean-intelligence programme might initially focus on how an observing system changes what can be known. A physically plausible simulation can infer unobserved fields only under assumptions about governing dynamics, measurement errors, and boundary conditions; its detailed imagery should not be confused with complete empirical knowledge. Observation systems may encounter rare ecosystems, unanticipated chemical processes, or localized disturbances that invalidate a model's habitual categories. A generative epistemic perspective asks how encounters with such observations transform the conditions under which further phenomena can be recognized and investigated. This places discovery, model revision, and epistemic uncertainty at the center of the project, rather than presuming that better maps merely add more points to an already complete representation.

One practical research sequence would connect open ocean datasets to an explicit environmental model and test whether new robotic observations improve its predictions outside sampled regions. The Global Ocean Observing System’s framework of essential ocean variables offers one established reference for interoperable measurements [16]. The European Digital Twin Ocean is another example of infrastructure intended to integrate observational sources and numerical representations for public marine knowledge [4]. Neither initiative removes the need to validate sensors, preserve sampling provenance, document uncertainty, or assess whether observations are biased toward accessible coastlines and frequently traversed routes. A useful benchmark would compare forecast calibration and ecological anomaly detection with and without robot-acquired observations, while recording the cost and disturbance of obtaining the data. That benchmark should specify the counterfactual observation strategy, not merely contrast an instrumented site with the absence of records. It should assess environmental knowledge gained per deployment, including false alarms, out-of-sample predictive skill, model revision events, survey coverage and uncertainty, while documenting robot-induced habitat contact and emissions where measurable. The European Digital Twin Ocean is a developing platform with an accessible prototype rather than an already completed planetary observatory; accordingly, any inference drawn from its scenarios remains conditional on the relevant models, source data and validation procedures [4, 2].

The ethical issue is not restricted to infrastructure placement. Deploying sensors and vehicles can itself affect local organisms and communities through noise, habitat contact, extraction, data appropriation, or changes in access to coastal resources. A robot capable of exploration might reduce dangerous human operations in some settings while increasing the frequency of disturbance in others. Impact assessments should therefore compare alternative scientific and engineering designs, including the option of nondeployment, rather than treating data collection as inherently beneficial. Because previously unrecognized ecological effects may be discovered during exploration, the evaluation protocol must specify how evidence can change permissions or halt a planned deployment. This is a practical reason for treating ecological observation as a source of potentially revised governance constraints rather than as a service subordinate to a predetermined computing-infrastructure objective. Ecological indicators cannot simply be incorporated as priced compensation terms without a defensible account of what the measurements capture and whose interests the resulting rules protect.

Underwater data centers and robotic maintenance may be studied as particular applications within this broader programme, subject to independent environmental evaluation. The possibility of seawater-assisted cooling is not a demonstration of net ecological benefit. Manufacturing impacts, local temperature changes, cable and seabed disturbance, service reliability, retrieval, and decommissioning would have to be represented across appropriate physical and governance boundaries. Moreover, an AI system able to maintain a subsea installation is not thereby autonomous in producing the metals, chips, energy, ships, legal permissions, and expert knowledge required for its reproduction. Keeping ocean science independent of any single infrastructure agenda protects its exploratory value and allows ecological discoveries to change whether, where, and how an engineering project should proceed.

## 14.5 Autonomous AI Infrastructure

The fifth continuation concerns the meaning of productive autonomy itself. A software system can initiate transactions, schedule jobs, repair some equipment, or generate new experimental proposals while remaining entirely dependent on institutions controlling power supply, fabrication, network access, land, insurance, and credit. Even a robotic system capable of servicing its immediate components may depend on global supply chains for replacement parts and specialized materials. The relevant question is thus not whether an AI agent can perform tasks without human intervention, but whether a specified productive network can maintain and reproduce the conditions for its activity under identifiable constraints. This is a question of bounded and distributed production rather than the intelligence of an isolated model.

A tractable study could represent a productive system through a multilayer dependency ledger that includes energy, information and observation, computing equipment, materials, fabrication, maintenance, waste handling, and governance permissions. Let  $I(S)$  be the set of declared productive inputs required

by a network boundary  $S$ , and let  $\mathcal{R}(S)$  denote the inputs that can be reliably reproduced within that boundary under stated quality and ecological constraints. A diagnostic coverage ratio might be defined as

$$C_{\omega}(S) = \frac{\sum_{i \in I(S)} \omega_i \mathbf{1}\{i \in \mathcal{R}(S)\}}{\sum_{i \in I(S)} \omega_i}, \quad (54)$$

where  $\omega_i \geq 0$  are declared weights with a positive denominator. The indicator is intentionally incomplete: it can obscure critical bottlenecks when large numbers of easy-to-reproduce inputs outweigh a single indispensable inaccessible component. It should therefore be accompanied by dependency graphs, minimum-cut or critical-input analyses, and explicit treatment of claims that cannot be made commensurable. Equation (54) is an example of a research diagnostic, not a universal autonomy score.

A technically self-maintaining network may also depend on property rights or licenses that make required inputs economically unavailable. Conversely, a public or commons infrastructure can permit nonmonetary access to productive inputs without requiring every AI subsystem to manufacture them itself. These observations suggest at least three different research targets: physical reproduction capacity, access rights over indispensable productive factors, and the ability to maintain legitimate cooperation when institutions or conditions change. The distinction is particularly important for interpreting Human–AI monetary relations. Replacing purchased human labor with AI activity could reduce one kind of monetary dependence while expanding dependence on rare equipment, electricity, or exclusive data. The proposed ordering of dissolution across H–H, H–AI, and AI–AI relationships would then depend on particular bottlenecks and institutional arrangements.

Research should also test the difference between economic closure and ecological sustainability. A production network might circulate resources internally and settle its compatible monetary claims while consuming scarce groundwater, generating unmanaged heat, or accumulating physical waste. Sustained output under those conditions would not establish that the network reproduces its ecological conditions. The International Energy Agency’s analysis of growing data-center electricity requirements illustrates the relevance of energy constraints, although aggregate forecasts cannot predict the architecture or environmental performance of any specific future AI network [18]. A sound evaluation should combine lifecycle material accounting, stress testing, environmental monitoring, and analysis of who bears the costs when an input cannot be replaced.

The companion paper on ecological compatibility and autonomous AI infrastructure can investigate technical designs, including robotic inspection, distributed energy management, repairable hardware, and resource coordination. Its hypotheses should remain open to adverse findings: decentralization may improve resilience in one setting yet duplicate equipment and increase material throughput in another; public ownership may improve access yet fail under insufficient maintenance budgets; private optimization may lower some resource costs while externalizing other burdens. Such comparative analysis would make productive autonomy an explanatory variable with measurable components, instead of a synonym for assumed technological progress.

## 14.6 Generative Trust and Transnational Governance

The sixth continuation asks how cooperation becomes possible, remains contestable, and is repaired across participants who do not share an initial institutional framework or a complete account of one another’s interests. The technical ability to calculate a clearing cycle says little about whether recognized claim holders consent to discharge, whether a disputed obligation was validly created, or whether an affected community can obtain redress. Likewise, transnational data exchange is not sustained solely by compatible file formats. Contributors must be able to evaluate promises, anticipate rule changes, understand conditions of participation, and address disagreement. Trust is consequently a historical and relational research object, not a scalar preference parameter that can simply be assumed to increase with information.

A productive distinction is between *epistemic trust*, concerning the credibility and limitations of another participant’s observations and inferences; *practical trust*, concerning performance of commitments

under relevant contingencies; and *institutional trust*, concerning the procedures by which promises are interpreted, challenged, revised, and enforced. These forms of trust can diverge. Participants may regard a dataset as scientifically reliable while rejecting the platform’s governance; they may accept an institution’s rules while remaining uncertain about an AI system’s ability to deliver; or they may cooperatively share data while refusing reciprocal access to sensitive infrastructure. Studies of credible commitments, public-knowledge governance, and commons institutions offer separate starting points for examining these mechanisms [8, 10, 23].

The research should assess governance through observable procedures rather than equating stated principles with effective participation. Who may propose or revise a standard? Who can refuse an automated clearing arrangement? How is a mistaken classification corrected? Can a less powerful participant challenge an influential model, institution, or counterparty without forfeiting access to the productive network? How are costs of error, proof, delay, and ecological damage distributed? In a federated global infrastructure, answerability may need to operate simultaneously at local, transnational, and cross-system levels. A shared validation protocol can support technical trust while remaining inadequate for disputes involving property, recognition, or cultural difference.

Trust also has an ambivalent relation to monetary dependence. Repeated successful cooperation may make some bilateral payments or guarantees less necessary, thereby facilitating direct reciprocity or multilateral clearing. The same history can make lenders more willing to extend new credit and allow monetary obligations to expand. Hence a hypothesis of increasing trust does not select a unique path toward demonetization. Its economic effects should be investigated by comparing changes in information sharing, compliance, recognized liabilities, participant exit, dispute frequency, and available remedies under distinct governance arrangements. An institution can facilitate considerable exchange while still concentrating rule-making authority in a few actors. Such a case should not be treated as a failure of computational coordination; it is a separate problem of legitimacy and power.

Global environmental observation offers an instructive but bounded domain for examining this question. International programmes have developed common observational objectives, distributed contributions, and procedures for sharing measurements across borders [26, 16]. These achievements are neither proof that international competition has disappeared nor a ready-made blueprint for a global AI governance authority. More modest comparative questions can be asked: which institutional provisions preserve cooperation despite disagreement, how are contributions recognized, what arrangements permit independent verification, and what happens when standards or access rules change? The companion governance paper should compare successful coordination with failed or contested cases and resist the assumption that a single global institution is necessary for every kind of shared infrastructure.

Across these six continuations, a final methodological priority is the maintenance of distinctions between measured records, interpreted relations, and institutional judgments. An adequate research infrastructure should preserve enough provenance to examine competing explanations while protecting legitimate confidentiality and autonomy. It should also allow revisions to its own categories as encounters, scientific observations, or new forms of production disclose previously unrecognized relations. A research programme concerned with distributed monetary dissolution can thereby remain open to outcomes in which money contracts, persists, migrates, or regenerates. Its task is to determine the conditions and consequences of each transformation, rather than to present any single outcome as a necessary culmination.

## 15 Conclusion

This inquiry began with a question about money’s possible disappearance, but the arguments developed throughout the paper lead to a more differentiated research object: the conditions under which monetary mediation emerges, persists, migrates, undergoes partial dissolution, or is regenerated in evolving relations of production. A monetary transaction is situated within relationships of need, ownership, credit, coordination, and institutional authority. Human participants may seek encounters with others, admission to communities, particular forms of work, access to knowledge, or the fruits of specialized production.

These needs can motivate exchange without making payment their necessary expression. Conversely, circumstances in which such relations are governed by fees, prices, transferable claims, or credit require explanation in terms of their particular institutional and productive conditions. Monetary formation and dissolution belong to the same inquiry: neither the cancellation of claims eliminates relations nor the persistence of relations makes monetary claims inevitable.

This starting point is compatible with several established traditions of political economy while requiring distinctions among them. Marxian investigations of the value form, production, and the control of productive conditions illuminate why the circulation of money cannot be abstracted from historically organized social relations. Institutional and credit approaches direct attention to denominated obligations, accepted settlement, and the rules by which claims acquire authority. Generative perspectives on economic life invite consideration of how labor, ecological relations, expressive practices, and other heterogeneous contributions participate in the continuing formation and circulation of value. These perspectives do not provide a single measure. The present discussion brings these perspectives to questions about AI-operated production networks, recognized obligations, and spacetime-distributed clearing; earlier exploratory papers supply preliminary tools rather than external proof of the hypotheses advanced here [21, 15, 14, 11].

The expression *spacetime-distributed* remains a qualified representational proposal: it requires declared event semantics, locally justified dependency relations, and explicit observation boundaries. Likewise, the labels H–H, H–AI, and AI–AI describe operational sites rather than settled rights or personhood categories. These limits are conditions of coherent inquiry, not reservations to be discarded once greater technical capabilities become available.

The expression *spacetime-distributed dissolution* designates an analytical orientation rather than a forecast of complete demonetization. It directs attention to distinct sites of production, encounter, and obligation, and to the historically conditioned relations among them. An obligation may be cleared in one local network while monetary financing remains necessary to reproduce equipment or sustain access elsewhere. A participant may receive a service without a direct payment although its provider remains dependent on energy purchases, hardware replacement, data permissions, or interest-bearing finance. Other participants may form new commitments in the course of producing or sharing value. Consequently, any assertion that money has become less necessary must state *which monetary function, for which participants, within which network boundary, and under which institutions* the change is alleged to occur. An aggregate count of visible payments cannot answer these questions. Further events or recognized dependencies may also alter the modelled network.

Artificial intelligence provides an important possible setting for these transformations, but the analysis rejects a single technologically determined sequence. Increasingly capable AI can reduce some costs of supplying services once exchanged among humans, thereby changing the need for direct H–H monetary mediation. H–AI charges may then change as AI operators acquire wider productive capacities. Yet advanced cognitive performance is different from an ability to secure, maintain, and reproduce electricity, computing hardware, observational data, physical materials, repair capacities, and the institutional permissions on which production depends. Monetary demands can shift from a human-facing interface toward the firms, public bodies, cooperatives, or other organizations operating the infrastructure. AI–AI coordination may also generate new budgets, enforceable obligations, resource entitlements, and credit structures, but software messages between agents are not automatically exchanges between independent legal claimants. The proposed possibility that monetary dependence diminishes at H–H interfaces, then H–AI boundaries, while remaining longer among AI-operated production systems is therefore conditional on specified productive and institutional relations. Different arrangements can reverse, interrupt, or complicate this ordering. This is a proposition to investigate rather than a universal chronology.

The formal analysis gives this inquiry a tractable but deliberately limited core. For a finite, typed network of recognized obligations, a nonnegative circulation can be decomposed into directed cycles when the relevant homogeneity and compatibility assumptions are met. Reducing eligible obligations along those cycles preserves specified net monetary positions while potentially decreasing gross outstanding claims. Under a particular unrestricted maximum-reduction objective, residual positive obligations con-

tain no further admissible directed cycle of the same class. Such statements are conditional mathematical results. A closed path in a broader relational two-complex, whose vertices represent events rather than primitive persons or firms, is not automatically a cycle of mutually cancellable monetary claims. Nor can compatible amounts be assumed where values are heterogeneous, currencies are inconvertible, conditions of maturity differ, or claim holders refuse the proposed discharge. The algebra establishes what a recognized ledger permits under stated constraints; it does not itself confer the legal or social authority to extinguish a debt. Mathematical representation and enacted institutional transformation must remain distinct.

The three-party circulation that initially motivated the study illustrates both the promise and limits of the proposal. If A owes B, B owes C, and C owes A equal eligible amounts, authorized multilateral reduction may allow their recorded obligations to decrease without a sequence of gross payments around the circuit. The underlying contributions could nevertheless differ in kind: knowledge, maintenance, energy, an opportunity for participation, or some other valued activity. The possibility of cancelling recognized claims does not establish that these contributions have a natural common metric, that every participant considers the outcome acceptable, or that excluded parties have received what is due to them. More complex circuits may be distributed across events and contingent commitments, but an anticipated contribution is not a presently enforceable debt, and an admissible continuation is not an already realized discharge. These distinctions also constrain future event-based and two-complex formalisms.

The paper's investigation of circuit length adds a second mathematical lesson. The intuition that longer paths may be more fragile can be useful for computation when every additional dependency contributes a sufficiently strong conditional reduction in the prospect of completing a circuit. On one set of stated assumptions, the expansion of possible paths is controlled relative to declining survival, so that omitting circuits above a length threshold has an explicit bound on expected lost clearing opportunity. The sufficient condition developed in Section 9 is informative precisely because it is not universal. Shared shocks can correlate distant obligations, sustained institutional guarantees can allow long paths to remain viable, and the number of eligible circuits can rise faster than individual success probabilities fall. Moreover, some clearing objectives admit algorithms using network flows rather than direct enumeration of every cycle. Computational truncation is therefore an instrument with an identifiable error model, not a mechanism by which omitted obligations cease to exist. Its applicability depends on network dependencies and actual admissibility rules.

At the wider economic level, the central result is a non-implication. Monetary mediation can decline in a particular relation without the cessation of need for others, exchange, or heterogeneous value generation. The converse is equally important: the continued formation of relations does not compel their monetization. Improving capacities can enable direct provisioning and reciprocal coordination, while creating new complementarities, scarce access rights, credit demands, and sites of extraction. Trust can make direct cooperation and authorized clearing more feasible while also supporting expanded lending. Public or commons-based provision can remove prices at an interface while preserving collective financing obligations; private operators may reduce users' direct charges while extending their own financial dependence. Even where a selected obligation ledger is completely cleared, renewed production and institutional recognition can generate fresh claims. An evolving relational account must explain local dissolution alongside systemic remonetization.

These observations also establish an important boundary for evaluation. A mathematically feasible, financially inexpensive, or technologically autonomous arrangement is not thereby ecologically sustainable or institutionally legitimate. Monetary clearing may leave intact concentrated control over the means of production, exclusion from data infrastructures, unpaid maintenance labor, asymmetrical opportunities to contest errors, or injury to communities and ecosystems lying outside the recorded exchange circuit. Infrastructural dependence cannot be understood solely through money or energy expenditure: it concerns the continuing reproduction of material resources and the rights that regulate access to them. Likewise, nonmonetary exchange cannot be presumed just simply because there is no price. Feasibility, sustainability, and legitimacy must be assessed independently of ledger efficiency (Section 11). Recognition, participation, control, remedy, and ecological incidence require analysis with forms of evidence

appropriate to each domain.

For this reason the paper’s treatment of infrastructure is intentionally preliminary. Global observational networks, federated data sharing, interoperable knowledge systems, AI production infrastructure, and ecological monitoring may improve what participants can know, coordinate, and reproduce collectively. Their existence alone does not establish that monetary relations will contract. Some forms of shared observation can lower informational barriers while other forms of data collection create new vulnerabilities, exclusions, or ecological disturbances. Scientific cooperation can coexist with political competition and legal differences. Trust and governance should thus be examined as changing practices of making, scrutinizing, honoring, contesting, and repairing commitments, rather than represented as fixed inputs to an optimization problem. The future work on global knowledge infrastructures, ocean observation and robotics, autonomous AI facilities, and transnational governance is intended to examine these conditions in their own right. Those inquiries have scientific and public value independent of the monetary hypotheses.

The principal methodological contribution of the discussion is accordingly a set of separations and proposed connections. It separates event-level relational histories from monetary-obligation graphs; heterogeneous value from denominated claims; prospective reciprocity from authorized settlement; local clearing from changes in the functions of money; operational AI interaction from the rights and liabilities of claim-bearing institutions; and computational feasibility from ecological and normative appraisal. It then asks how these distinctions can be bridged under explicit, revisable assumptions. An empirical programme should compare direct payments with upstream financing, record the reproduction of productive inputs, trace the formation of new claims, examine participants’ access and ability to contest decisions, and declare the limits of its observation network. Such work could support the claim of substantive dissolution in some settings, disclose a mere transfer of monetary dependence in others, or find entirely new monetary arrangements emerging from transformed production. These divergent findings must remain possible.

The inquiry therefore reaches no claim that money must disappear, that AI will replace every meaningful other, or that the final cancellation of monetary claims constitutes a desirable or reachable endpoint. Rather, it offers a framework in which the conditions for money’s appearance and partial disappearance may be investigated as part of a continuing history of relational production and heterogeneous circulation. Needs, capacities, and commitments continue to evolve alongside their historically variable monetary institutions. Spacetime-distributed dissolution, in this sense, names a family of possible, local, and institutionally mediated transformations whose boundaries and consequences have to be demonstrated. A generative political economy should be capable of accounting both for those transformations and for the persistence or renewed emergence of money, without mistaking one possibility for the totality of economic life.

## **A Mathematical Foundations of Relational Production Networks**

This appendix specifies the mathematical objects used throughout the analysis and makes their respective domains of interpretation explicit. Its purpose is to provide a reusable minimal language for event histories, productive relations, recognized obligations, and admissible transformations. It does not claim that a particular mathematical representation exhausts an economy, or that a theorem concerning monetary accounting establishes the economic, juridical, or ethical validity of a clearing practice. The principal formulations elaborate Sections 3, 7, and 8; their connection to earlier explorations of heterogeneous value circulation and distributed monetary dissolution is acknowledged in those sections [11, 14]. Chain-level topology and more specialized cycle results belong to Appendices B and C, respectively.

### **A.1 Modeling Domains, Types, and Representational Boundaries**

A relational economic description should not conflate an event, a person, a computational agent, a legally recognized account, an act of production, an obligation, and a valuation. We begin with a family of *typed*

sets, which may be empty, incomplete, or only partially observed:

$$E, R, P, X, V, L, I, O. \quad (55)$$

In Equation (55),  $E$  denotes event records,  $R$  relational or operational roles,  $P$  recognized economic or legal positions,  $X$  productive-capacity and resource descriptions,  $V$  heterogeneous value records,  $L$  claim records,  $I$  institutional contexts, and  $O$  observation records. The types are deliberately distinguished even if their encoded identifiers share a software representation. An AI-operated service can occupy an operational role in  $R$  while a corporation, public body, cooperative, or person occupies the debtor or creditor position in  $P$ . Equating those records would silently assume an institutional fact that the model has not established.

The word *network* will denote a specified collection of relations on one or more of these types; it does not require that the resulting representation be a single homogeneous graph. A model consists of a stated domain, admissible relation predicates, observation rules, and a declared *boundary* identifying what falls within its coverage. Changes in the boundary can change measured dependence and apparent monetary disappearance. In particular, a human-facing AI application can require no immediate payment from its users while relying on an upstream supplier's enforceable invoices. Both statements can be true because they concern different subsets of recognized claims.

We employ three distinct levels of assertion. A *representational identity* follows from a chosen mathematical definition. A *conditional institutional assertion* additionally assumes a valid rule of recognition, authorization, or enforcement. An *empirical assertion* requires evidence that the specified agents, events, resources, or institutional actions exist and have the properties assigned to them. These levels should not be collapsed when transferring equations into political-economic conclusions.

## A.2 Event Histories without a Privileged Universal Time

**Definition A.1** (Represented relational history). A represented relational history is a tuple

$$\mathfrak{H} = (E_{\mathfrak{H}}, \preceq_{\mathfrak{H}}, \Lambda_{\mathfrak{H}}, \mathcal{I}_{\mathfrak{H}}), \quad (56)$$

In Equation (56),  $E_{\mathfrak{H}} \subseteq E$  is a selected set of events;  $\preceq_{\mathfrak{H}}$  is a reflexive, antisymmetric, transitive relation *whenever its asserted dependencies justify a partial order*;  $\Lambda_{\mathfrak{H}}$  specifies typed labels and relations; and  $\mathcal{I}_{\mathfrak{H}}$  states the interpretation and provenance of relevant records. An unverified or nontransitive relation is kept in  $\Lambda_{\mathfrak{H}}$  rather than being forced into the order.

A local configuration  $h$  specifies a finite, empirically or hypothetically delimited part of  $\mathfrak{H}$  together with the interpretive rules appropriate to that part. The symbol  $h'$  denotes another such configuration. A comparison  $h \rightsquigarrow h'$  is admissible only when accompanied by an identified relation of extension, revision, reconstruction, institutional transition, or some other explicitly declared correspondence. The notation does not itself assert that  $h'$  lies later on a globally privileged time axis. Dates, timestamps, causal arrows, and contractual maturities can be added as *typed observations*; they do not acquire a common ontological meaning solely through their numerical representation.

**Definition A.2** (Admissible continuation relation). For a stated theory and evidence policy  $\Theta$ , let  $\mathfrak{C}(\mathfrak{H}; \Theta)$  be a set of admissible configurations or history extensions. A continuation relation  $h \rightsquigarrow_{\Theta} h'$  is supported by a specified witness, such as a partial embedding of event records, a revision log, or an institutional transition rule. It is not presumed total, deterministic, invertible, or unique.

The distinction is substantive. An agreement may be signed in one jurisdiction, a service delivered in another, a claim recognized upon a separate approval, and discharge entered when a legally authorized offset takes effect. Some relationships may admit a contractual precedence without producing a comparably ordered sequence of the underlying scientific or productive events. In a distributed system, observation and record synchronization may also occur in orders distinct from those attributed to the

interactions themselves. A study may select a particular ordering to answer a question, but should report that choice.

Two locally coherent records need not have a lossless common refinement. An observational system might preserve invoice dates while omitting the environmental history of energy production; another may record ecological disturbances without identifying their contractual counterparties. We therefore refrain from assuming that a global event database can reconstruct all value-generating processes or all justified claims.

### A.3 Events, Participants, Roles, and Productive Capacities

In the relational two-complex representation discussed in Section 7, a vertex represents an *event*, an edge represents an admissibly interpreted coupling, and a face represents a modeling primitive whose semantics are declared for the application. Economic actors are not primitive vertices in that representation. Where actors or organizations are discussed, they are reconstructed through roles, collections of events, participation relations, and institutional evidence.

For one selected configuration  $h$ , let

$$\text{participates}_h \subseteq R \times E_h, \quad \text{bears}_h \subseteq R \times P, \quad (57)$$

with the second relation in Equation (57) recording only a possible link between an operational role and a recognized position. A role need not bear an independent obligation; the same position may be associated with several roles, and a role may change under institutional arrangements. The relation  $\text{bears}_h$  is not inferred from operational autonomy or programming architecture. A robot negotiating an energy supply contract, for example, does not thereby establish that the robot is the contracting legal debtor.

For a bounded production system  $S$  described within  $h$ , let the productive-condition vector be

$$\mathbf{k}_h(S) = (k_h^{\text{energy}}, k_h^{\text{data}}, k_h^{\text{compute}}, k_h^{\text{materials}}, k_h^{\text{maintenance}}, k_h^{\text{access}}), \quad (58)$$

In Equation (58), each coordinate is interpreted in its own domain rather than added as a physical scalar. The last coordinate includes permissions and institutional rights required to operate. A corresponding dependency relation

$$D_h \subseteq R \times X \times R \quad (59)$$

may, in Equation (59), identify a supplying role, a needed condition, and a dependent role. Further labels can distinguish recurring supply, one-time acquisition, stewardship, legal permission, public provision, and physical regeneration.

**Definition A.3** (Boundary-relative productive autonomy). Given a production boundary  $S$ , a specified service or reproduction target  $\tau$ , and a set of perturbations  $\Omega$ , productive autonomy is a property of the capacity of  $S$  to maintain  $\tau$  across the selected perturbations while meeting declared external-resource and institutional-access constraints. Operational task completion, short-run supply independence, and reproduction of the production conditions are separately evaluated properties; no single universal autonomy score is assumed.

This definition prevents a common equivocation in claims about the disappearance of human–AI monetary dependence. A service may operate with negligible direct human intervention while remaining dependent on electricity agreements, purchased hardware, material extraction, land access, licensed data, or publicly funded infrastructure. Such dependence need not always be monetary, but it has to be traced before the absence of end-user payment is interpreted as systemic monetary dissolution.

### A.4 Heterogeneous Value Records and Partial Monetary Valuation

Represent heterogeneous value descriptions by  $\mathcal{V}_h \subseteq V$ . A typical record can carry an event reference, relation type, contributor, recipient, purpose, an evidential basis, and any affected persons or ecological systems. The components may be categorical, ordinal, narrative, measured in physical units, or

represented by domain-specific models. Their diversity is not a defect to be repaired by assuming one complete cardinal index.

In line with Equation (8), let an institutional practice  $\iota$  supply a *partial* denomination map

$$\pi_{h,\iota} : \mathcal{D}_{h,\iota} \subseteq \mathcal{V}_h \longrightarrow \mathbb{R}_{\geq 0} \times \mathcal{U}, \quad (60)$$

In Equation (60),  $\mathcal{U}$  is a set of recognized monetary units. The domain  $\mathcal{D}_{h,\iota}$  is chosen under declared criteria; the map does not promise to price all relational or ecological values. Where negotiation or contested appraisals allow several outcomes, a relation or set-valued assignment may replace the single-valued notation. A price need not be uniquely attributable to a productive contribution, and one observed invoice can combine numerous inputs and institutional considerations.

**Proposition A.4** (Insufficiency of monetary observation for value reconstruction). *If  $\pi_{h,\iota}$  is not injective on its domain, distinct heterogeneous value records may yield identical observed monetary denominations. If  $\mathcal{D}_{h,\iota} \subsetneq \mathcal{V}_h$ , records outside the domain have no denomination under this practice. Consequently, an observed denomination alone cannot uniquely reconstruct all heterogeneous value records represented by the model.*

*Proof.* For noninjectivity there exist  $v_1 \neq v_2$  in  $\mathcal{D}_{h,\iota}$  with  $\pi_{h,\iota}(v_1) = \pi_{h,\iota}(v_2)$ . The same observed result is therefore compatible with at least two source records. By definition, the function assigns no value to records in  $\mathcal{V}_h \setminus \mathcal{D}_{h,\iota}$ . Thus monetary observation lacks a unique inverse over the represented value domain.  $\square$

This elementary non-identification result has limits worth stating. It is not a proof that prices are uninformative or that no augmented dataset could distinguish the records. It establishes only that the stated monetary map, by itself, cannot recover what it does not uniquely encode. For monetary dissolution this is important: removing an invoice changes a record in one accounting layer; it does not logically erase the contribution, obligation to affected persons, ecological change, or relationship from the other layers. A recorded zero payment is likewise compatible with several mechanisms, including gift, common provision, internal transfer, subsidy, unrecognized contribution, or displaced monetary financing.

## A.5 Recognized Obligations as a Typed Directed Multigraph

For a selected  $h$  and institutional practice  $\iota$ , let  $G_h = (P_h, L_h)$  be a finite directed multigraph of recognized debtor and creditor positions. A directed edge  $\ell \in L_h$  points *from the debtor to the creditor*. Its attributes may include denomination, amount, maturity, seniority, contingency, governing practice, evidence, discharge requirements, and recognized counterparties. Parallel edges remain separate if their conditions differ.

The interpretation from event and value records to claims is a partial relation or function

$$Q_{h,\iota} : \text{Records}(\mathfrak{H}, K) \rightharpoonup \text{Claims}(G_h, \iota). \quad (61)$$

The  $K$  argument stands for a possible event complex, not for a presupposed unique topology of the economy. The existence of a productive event does not ensure that  $Q_{h,\iota}$  is defined; nor does one claim need to derive from a unique event. This map is not assumed to preserve chain boundaries, addition, or any other algebraic operation.

Let  $T_h$  be a partition of those claim edges according to the compatibility required by the specific calculation. A type can account for denomination, contingency status, applicable netting set, maturity or eligible collateral. For each fixed compatible class  $t$ , define

$$x_h^{(t)} \in \mathbb{R}_{\geq 0}^{L_h^{(t)}}, \quad b_h^{(t)} = B_h^{(t)} x_h^{(t)}, \quad B_h^{(t)} : \mathbb{R}^{L_h^{(t)}} \rightarrow \mathbb{R}^{P_h}, \quad (62)$$

where the column of  $B_h^{(t)}$  for an edge  $p \rightarrow q$  is  $\mathbf{e}_q - \mathbf{e}_p$ . Positive  $b_{h,p}^{(t)}$  therefore means that position  $p$  is net credited *within that class and reporting boundary*. Different  $t$ -components remain separate unless a separately recognized conversion, novation, or settlement operation is supplied. The direct sum of typed ledgers is valid bookkeeping; it does not license automatic cancellation across types.

**Remark A.5** (The meaning of a zero sum). For every fixed typed graph,  $\mathbf{1}^\top b_h^{(t)} = \mathbf{1}^\top B_h^{(t)} x_h^{(t)} = 0$ . This follows because each column of  $B_h^{(t)}$  has one +1 and one -1. It does *not* imply that all debts are extinguished, that every participant has a zero position, that the ledger is solvent, or that the external economy is materially balanced.

## A.6 Boundaries, Cuts, and Local Monetary Dependence

Let  $S \subseteq P_h$  be a chosen set of account positions in a fixed compatible claim graph, and write  $\delta^+(S)$  for edges with debtor in  $S$  and creditor outside  $S$ , and  $\delta^-(S)$  for edges with debtor outside  $S$  and creditor in  $S$ . For a liability vector  $x$ , the aggregate net position of accounts in  $S$  is determined entirely by the claims crossing its boundary:

**Proposition A.6** (Accounting cut identity). *With the creditor-minus-debtor incidence convention of Equation (62),*

$$\sum_{p \in S} (B_h x)_p = \sum_{\ell \in \delta^-(S)} x_\ell - \sum_{\ell \in \delta^+(S)} x_\ell. \quad (63)$$

*The statement holds independently for each compatible claim class, and does not require claims to be institutionally eligible for offset.*

*Proof.* Summing the rows of  $B_h$  indexed by  $S$  cancels the +1 and -1 entries of every edge whose endpoints both lie in  $S$ , and includes neither entry of an edge whose endpoints both lie outside  $S$ . An edge arriving from outside contributes  $+x_\ell$ ; an edge departing from  $S$  contributes  $-x_\ell$ . Summation yields Equation (63).  $\square$

A reporting boundary need not select only accounts: it may select services, locations, classes of consumption, or observational evidence. In these cases the relation between observed interfaces and legally recognized positions must be specified before the cut identity is applied. In particular, an aggregate of AI-operated service transactions is not generally a cut in the liability graph unless the legal holder and bearer positions have been identified.

The cut identity also clarifies local and global circulation. A circulation  $z$  with  $B_h z = 0$  has zero net inflow across every account cut, but a restricted collection of its edges can leave nonzero endpoints when examined without the omitted complement. This effect is not a defect of the global circulation. It is evidence that local closure depends on the chosen boundary. Similarly, a network can be internally monetarily closed while its energy, materials, or ecological impacts cross boundaries represented in a separate physical or ethical model.

## A.7 Admissible Reductions and Configuration Transformations

A mathematical netting candidate for one typed claim class is a vector  $z$  satisfying

$$0 \leq z \leq x, \quad B_h z = 0. \quad (64)$$

To represent a proposed *eligible* reduction, introduce an institution-dependent predicate or feasible set  $\mathcal{A}(h, \iota)$  requiring, as applicable, consent, maturity, contract terms, jurisdiction, insolvency safeguards, and authorized discharge. A realized legal clearing also requires an event recording acceptance and execution; existence of  $z$  satisfying Equation (64) alone is only algebraic feasibility. The residual compatible liability vector is  $r = x - z$  if and only if the institution recognizes that reduction.

For later comparisons, a configuration transition may be written schematically as

$$x_{h'} = T_{h \rightsquigarrow h'}(x_h - z_h) + n_{h'}, \quad (65)$$

consistent with Section 10. The map  $T_{h \rightsquigarrow h'}$  translates surviving recognized claims between declared ledger descriptions;  $n_{h'}$  records newly recognized claims. The displayed addition is meaningful only

after target types and denominations have been aligned. The representation does not presume that  $T$  is linear, invertible, or canonically available, or that the set of possible  $h'$  is exhausted by one chronological successor. In particular, a new debt may arise following an otherwise complete clearing, and a change of denomination or legal creditor may alter the ledger structure independently of its monetary magnitude.

**Proposition A.7** (Accounting invariance of a compatible cyclic reduction). *Fix a typed claim graph and a candidate  $z$  satisfying Equation (64). If  $r = x - z$ , then  $B_h r = B_h x$ . If  $z \neq 0$ , the sum of the nonnegative recorded gross claim amounts falls by  $\sum_{\ell} z_{\ell} > 0$ . Neither conclusion, by itself, entails authorized discharge or a reduction of monetary dependence outside the fixed graph.*

*Proof.* Linearity gives  $B_h r = B_h x - B_h z = B_h x$ . Since  $0 \leq z \leq x$ , every residual component is nonnegative and  $\sum_{\ell} r_{\ell} = \sum_{\ell} x_{\ell} - \sum_{\ell} z_{\ell}$ . The institutional and cross-boundary claims do not follow from these identities and require additional premises.  $\square$

This proposition restates the accounting mechanism in the main text in a self-contained notation, rather than offering a new macroeconomic theorem. It also exposes an important distinction between *canceling a gross circular component* and *eliminating all remaining claims*. A directed cycle may be reducible to zero along some edges while other claims retain nonzero net positions. Conversely, the absence of eligible cycles can coexist with broad public provision and nonmonetary exchange. Accounting structure and the broader dependence on money have related but distinct explanations.

## A.8 Illustrative Cross-Layer Example

Consider three recognized positions  $A, B, C$  with compatible obligations  $A \rightarrow B$ ,  $B \rightarrow C$ , and  $C \rightarrow A$  in the amounts 8, 6, and 5 units of the same admitted currency. Let the corresponding oriented incidence matrix be

$$B = \begin{pmatrix} -1 & 0 & 1 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \end{pmatrix}, \quad x = \begin{pmatrix} 8 \\ 6 \\ 5 \end{pmatrix}, \quad z = \begin{pmatrix} 5 \\ 5 \\ 5 \end{pmatrix}, \quad r = x - z = \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix}. \quad (66)$$

The vectors in Equation (66) satisfy  $Bz = 0$  and

$$Bx = Br = \begin{pmatrix} -3 \\ 2 \\ 1 \end{pmatrix}. \quad (67)$$

The mathematical reduction represented in Equation (67) removes 15 units of *gross* face amounts across the three directed edges while preserving the internal net position of each account. Its status as a completed monetary discharge depends on the relevant agreements and institutions. If the  $C \rightarrow A$  obligation has a different currency or excludes netting, the vector  $z$  is no longer eligible merely because the three numbers can be entered in one matrix. If  $A$  is an AI-operated service, the example also requires identifying the actual recognized debtor or creditor behind that service.

Now suppose production represented by the  $A \rightarrow B$  link also relies on grid electricity, physical maintenance, and a shared watershed. Those dependencies need not appear as additional monetary edges in this ledger. Even if the complete 5-unit circular component is discharged, electricity may remain payable in a separate ledger and the local ecosystem may bear an uncompensated impact. The monetary result is exact *within its assumptions*; it is not a conclusion about the ethical closure or ecological self-sufficiency of the production network.

## A.9 Observation, Identifiability, and Monetary Dependence

An economy's monetary dependence is not directly observable through one payment interface. Empirical accounts generally record a projection of productive relations, credit, and institutional activities through a

specified evidence system. Let  $\mathcal{C}$  be a class of complete-enough modeled configurations for a particular study, and define a measurement protocol

$$\text{Obs}_\omega : \mathcal{C} \longrightarrow \mathcal{O}_\omega, \quad (68)$$

In Equation (68),  $\omega$  records whose accounts, which denominations, what types of monetary function, and which institutional periods or events are included. The target  $\mathcal{O}_\omega$  may mix categorical declarations and measured quantities. Nothing requires  $\text{Obs}_\omega$  to be injective: different production arrangements may generate identical observed payments, particularly when the reporting protocol excludes upstream suppliers or nonpayment access conditions.

**Proposition A.8** (Non-identification from zero interface payments). *Suppose the observation protocol records only direct monetary payments between users and an AI-operated service. Under otherwise admissible institutional descriptions, the observation of zero direct payments is consistent both with (i) a provider bearing enforceable obligations to upstream suppliers and (ii) a provider using publicly available or otherwise nonmonetarily supplied productive conditions. Thus zero payments under this protocol do not identify the extent of upstream monetary dependence.*

*Proof.* Construct configuration  $c_1$  in which users receive service without charge while the provider owes a positive amount to an electricity supplier. Construct  $c_2$  with identical user access but with the electricity supplied through a nonmonetary common-provision arrangement. The protocol records the same zero direct user payments for  $c_1$  and  $c_2$ , even though the upstream claim graphs differ. Hence the observation does not distinguish the two configurations.  $\square$

The counterexample is not an assertion that common energy provision is costless, that it is automatically just, or that it can occur independently of labor, materials, governance, and ecological burdens. It shows only that the observation's information content is insufficient to settle the upstream question. A broader study could add procurement contracts, financing agreements, licenses, embedded maintenance work, public transfers, and declared input dependencies, then assess which distinctions become identifiable under the expanded evidence base.

One practical output is a *boundary sensitivity analysis*: repeat a stated monetary-dependence measure across nested or overlapping production boundaries and report changes in classification rather than presenting one apparent decline as universal. A separate *typing sensitivity analysis* can test what happens when fees, lending, transfer prices, token credits, and legally enforceable monetary claims are distinguished. Both analyses should report missing data, unverifiable legal predicates, and the risk that accounting systems omit the interests of affected communities and ecological systems. These requirements connect mathematical identification to the epistemic and governance problems examined in Sections 11 and 14.

## A.10 Interpretive Discipline and Open Mathematical Questions

The foregoing definitions support an elementary mathematical vocabulary without determining how all possible productive contributions should be valued or governed. The following distinctions remain necessary for interpretation. Event vertices are not persons or AI systems; a coupling is not automatically a contract; a two-complex cycle is not automatically a financial clearing path; a zero net position is not the absence of gross claims; and an algebraically admissible cycle does not by itself establish a recognized discharge. Changes in available energy, data, land rights, knowledge, and environmental conditions can alter monetary dependence even when a financial ledger has an unchanged topology.

A more complete model would need validated rules connecting event records, productive roles, heterogeneous value descriptions, and monetary claims. It would also need principled ways to represent missing observations, conflicting accounts of recognition, and governance-dependent eligibility. A reproducible empirical implementation should publish the boundary-selection procedure, compatibility

classes, obligation-provenance fields, legal interpretation assumptions, and measures of remaining monetary functions. Without those declarations, an apparent decrease in money could result from the migration of financing to a less visible layer.

Appendix B develops the cellular chain language in greater detail, while Appendix C will examine the clearing problem within typed, compatible obligation networks. Appendix D will distinguish approximation based on circuit length from admissible discharge. Together, these formal inquiries are intended to make selected assertions testable and calculations reproducible, while leaving the historical, generative, and normative questions of monetary mediation open to evidence and disagreement.

## B Chain Complexes, Circulation Spaces, and Boundary Operators

This appendix develops a cellular account of relational configurations and compares it with the linear algebra of monetary-obligation networks. The principal motivation is a question raised in Sections 7 and 8: what, precisely, can a topology of relations tell us about a possible circulation or monetary discharge? The answer requires several distinct spaces and maps. A cycle in an event complex is not a debt, a boundary of a represented two-cell is not an institutionally authorized netting transaction, and a creditor–debtor incidence matrix cannot recover the heterogeneous sources of the claims it records. Earlier exploratory work on relational two-complexes and value circuits motivates the comparison [11, 7, 14]. Standard cellular homology and computational topology supply the mathematical language [6, 1]; their economic interpretation is the question considered here rather than an established mathematical consequence.

### B.1 Interpretive Status of the Relational Two-Complex

Let  $K$  be a finite oriented two-dimensional CW complex or a finite simplicial complex of dimension at most two. For the formal arguments below, finiteness and a consistent cellular boundary convention suffice. The complex is a *representation* of a selected portion of a relational history, not an ontological assertion that society consists of a fixed cellular substrate. Its zero-cells are represented events, its one-cells encode selected couplings between events, and its two-cells encode declared relational primitives or higher-order incidences. A zero-cell is therefore neither a person nor an AI system. An entity may be recognized through a configuration of events, couplings, historical relations, and institution-specific records, without being assigned a primitive vertex.

The interpretation of two-cells needs particular caution. A two-cell may be introduced to register a structured interaction, a compatibility relation, an observed composite event, or another specifically documented primitive. A *face* does not acquire the meaning of a contract, a unit of value, or a clearing instrument just by appearing in the complex. In particular, one cannot infer from a filled triangle that its three sides are legally offsettable debts. The meaning of a cell belongs in the model’s interpretation and evidence ledger. Alternate admissible cell decompositions can change its topology and therefore change inferred homology groups.

We denote the interpretation data by  $\iota_K$ , including rules for selecting cells, their orientation, relations between observations and cells, and admissible refinements. An economic analysis should report at least  $(K, \iota_K)$  rather than only homology ranks. Where the available evidence does not determine a unique complex, a family  $\{(K_\alpha, \iota_\alpha)\}$  should be considered. The dependence on this representation choice is a substantive identification issue, not merely a numerical inconvenience.

### B.2 Cellular Chains and the Boundary-of-Boundary Identity

Fix a coefficient field  $\mathbb{F}$ , ordinarily  $\mathbb{R}$  when signed flows and weighted operations are required, or  $\mathbb{F}_2$  when only unoriented incidence is intended. For  $k = 0, 1, 2$ , let  $C_k(K; \mathbb{F})$  be the finite-dimensional vector space generated by oriented  $k$ -cells. A cellular orientation determines a basis; reversing the orientation of a cell changes the sign of its coordinate. In general a  $k$ -chain is a formal signed sum of cells,

not a physical passage of people, money, or resources.

The cellular boundary maps form the chain complex

$$0 \longrightarrow C_2(K; \mathbb{F}) \xrightarrow{\partial_2} C_1(K; \mathbb{F}) \xrightarrow{\partial_1} C_0(K; \mathbb{F}) \longrightarrow 0, \quad \partial_1 \partial_2 = 0. \quad (69)$$

In the chain convention of Equation (69), if an oriented one-cell  $e$  runs from the event vertex  $u$  to the event vertex  $v$ , then  $\partial_1 e = v - u$ . The boundary of a two-cell is a signed one-chain determined by its attaching map. The equation  $\partial_1 \partial_2 = 0$  states that the oriented boundary of the boundary of a cell vanishes; it does not state that any economic obligation has been discharged.

**Definition B.1** (Cellular cycles and boundaries). The spaces of one-cycles and one-boundaries are, respectively,

$$Z_1(K; \mathbb{F}) = \ker \partial_1, \quad B_1(K; \mathbb{F}) = \text{im } \partial_2. \quad (70)$$

Their quotient, following Equation (70), is the first cellular homology group  $H_1(K; \mathbb{F}) = Z_1/B_1$ .

Since  $\partial_1 \partial_2 = 0$ , we have  $B_1 \subseteq Z_1$ . The quotient identifies cycles differing by represented face boundaries; it does not identify economic claims as equal or interchangeable. Over a field, elementary rank-nullity gives

$$\beta_1(K) = \dim H_1(K; \mathbb{F}) = \dim C_1 - \text{rank } \partial_1 - \text{rank } \partial_2. \quad (71)$$

Thus, Equation (71) specifies that  $\beta_1$  counts independent first-homology classes in a declared complex. A positive  $\beta_1$  may indicate a represented circulation-like structural possibility that is not filled by a selected two-cell. The number by itself does not measure production, reciprocal obligations, monetary volume, injustice, or the probability of clearing.

**Proposition B.2** (Cellular boundaries preserve represented event incidence). *For any two-chain  $a \in C_2(K; \mathbb{F})$ , the one-chain  $c = \partial_2 a$  has zero cellular boundary:  $\partial_1 c = 0$ . This implication concerns only the declared cellular complex.*

*Proof.* The defining chain-complex identity yields  $\partial_1 c = \partial_1 \partial_2 a = 0$ . No mapping to a monetary claims graph has been specified, so the proposition has no financial settlement implication.  $\square$

Orientation and choice of coefficients matter. A signed cellular cycle is free to assign a negative coefficient to an edge under a fixed orientation; a money-claim reduction must typically remain nonnegative and must respect the directions of eligible obligations. Replacing  $\mathbb{R}$  with  $\mathbb{F}_2$  may simplify detection of an unoriented circuit while discarding distinctions needed for debtor–creditor directions, prices, and amounts. Consequently, topological cycle detection alone does not perform monetary clearing.

### B.3 Circulation Spaces of Directed Monetary-Obligation Graphs

For a selected configuration  $h$ , let  $G_h^{(t)} = (P_h, L_h^{(t)})$  be a finite directed multigraph of institutionally recognized positions and obligation records of one fixed compatible class  $t$ , as defined in Appendix A. Its creditor-minus-debtor incidence matrix is  $B_h^{(t)}$ . A positive amount is oriented from debtor to creditor. Keeping separate classes prevents an arithmetic equality between, for example, two distinct currencies or differently contingent claims from being treated as legal fungibility.

In the vector space  $\mathbb{R}^{L_h^{(t)}}$ , define the signed circulation space and the nonnegative circulation cone

$$\mathcal{Z}(G_h^{(t)}) = \ker B_h^{(t)}, \quad \mathcal{C}_+(G_h^{(t)}) = \ker B_h^{(t)} \cap \mathbb{R}_{\geq 0}^{L_h^{(t)}}. \quad (72)$$

A vector  $z \in \mathcal{C}_+$ , as defined in Equation (72), is a balanced *candidate* for cycle-based reduction. For an existing claim vector  $x \geq 0$ , admissible numerical reductions additionally satisfy  $z \leq x$ . Even then, discharge requires an independent institutional predicate  $\text{Admissible}_{h,t}(x, z)$  and an appropriate execution event.

The kernel here is an algebraic cycle space of a directed *claim graph*. Although it has a homological interpretation if the graph is regarded as a one-dimensional CW complex, it is different from  $H_1(K)$  when  $K$  includes two-cells. In the graph regarded as a one-complex,  $\partial_2 = 0$  and  $H_1(G; \mathbb{R}) = \ker B$ , so no represented faces are quotiented out. In an event two-complex, by contrast,  $H_1(K) = \ker \partial_1 / \text{im } \partial_2$ . Identical one-skeleta can therefore have different first homology, while carrying the same eligible financial ledgers if the latter are separately specified.

**Proposition B.3** (Decomposition of a finite nonnegative circulation). *For a finite directed graph  $G$ , every  $z \in \mathcal{C}_+(G)$  is a finite nonnegative sum of incidence vectors of directed simple cycles, with nonnegative scalar coefficients. The proposition does not apply unchanged to mixed incompatible monetary claim classes.*

*Proof.* If  $z = 0$ , the assertion is immediate. Otherwise select a positive edge. Flow conservation at its head implies that some outgoing edge has positive flow; repeating this reasoning in a finite graph eventually revisits a vertex and yields a directed cycle entirely within the positive support. Subtract the smallest positive coefficient on that cycle times its incidence vector. The residual is nonnegative and balanced, and at least one previously positive edge becomes zero. Repetition terminates after finitely many edge eliminations. The result is a nonnegative sum of directed simple cycles. The construction uses the stated fixed graph and compatible numerical units; it does not check discharge permissions.  $\square$

The proof explains why graph-flow decomposition is a natural algebraic starting point for multilateral clearing. It also explains its limitations: one-sided capacities, insolvency, deadlines, seniority, collateral obligations, exclusions, and authority to cancel are not encoded by  $B$  alone. A topological statement over  $\mathbb{F}_2$  cannot repair missing institutional data. Appendix C will examine bounded and admissible clearing reductions separately.

#### B.4 A Filled Triangle and an Unfilled Obligation Circuit

Consider three event vertices  $a, b, c$  and three oriented one-cells  $e_1 : a \rightarrow b$ ,  $e_2 : b \rightarrow c$ ,  $e_3 : c \rightarrow a$ . In the ordered bases  $(a, b, c)$  and  $(e_1, e_2, e_3)$ , the one-boundary matrix is

$$D_1 = \begin{pmatrix} -1 & 0 & 1 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \end{pmatrix}, \quad \gamma = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad D_1 \gamma = 0. \quad (73)$$

For the matrices in Equation (73), if the complex contains no face, then  $C_2 = 0$ ,  $B_1 = 0$ , and  $H_1(K; \mathbb{R})$  has dimension one, generated by  $[\gamma]$ . Now attach an oriented two-cell  $f$  with  $\partial_2 f = e_1 + e_2 + e_3$ . Then

$$D_2 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad D_1 D_2 = 0, \quad \ker D_1 = \text{im } D_2, \quad H_1(K; \mathbb{R}) = 0. \quad (74)$$

The cycle in Equation (74) survives as a *cellular one-cycle* but is homologous to zero because it is the boundary of the face. Thus the assertion “there is no nontrivial first homology” is compatible with the existence of a directed cycle in the one-skeleton. Filling the triangle changes the cellular homology without removing its three graph edges.

Suppose separately that recognized positions  $A, B, C$  owe compatible claims  $A \rightarrow B$ ,  $B \rightarrow C$ , and  $C \rightarrow A$ , with initial amounts  $(8, 6, 5)$  monetary units. As in Appendix A, a candidate reduction  $z = (5, 5, 5)$  satisfies  $Bz = 0$  and  $0 \leq z \leq x$ . The monetary calculation is identical whether the *event* triangle is filled or unfilled. If all three claims belong to one authorized netting set, a properly executed arrangement may discharge those five units per edge. If one edge is ineligible, the same vector is not an authorized discharge despite the unchanged mathematical cycle. Conversely, changing the event face does not create or extinguish a monetary claim. The cross-representational correspondence is not supplied by the equality of the matrices’ numerical entries; it would have to be validated by the historical, productive, and institutional interpretation rules.

**Proposition B.4** (First homology does not identify clearing eligibility). *Within the class of models permitting independently specified event complexes and institutional obligation practices, neither  $\beta_1(K) = 0$  nor  $\beta_1(K) > 0$  determines whether a compatible directed clearing reduction exists or is authorized.*

*Proof.* The filled-triangle example has  $\beta_1(K) = 0$  while its separate three-party claim graph admits a nonzero numerical circulation and may admit an authorized reduction. Keeping the event complex fixed while changing the institutional admissibility predicate can prohibit that discharge. Conversely, the unfilled event triangle has  $\beta_1(K) = 1$  but may be paired with a claim graph containing no directed cycle, or with no recognized claims at all. Hence neither homology condition suffices to decide numerical or institutional clearing eligibility.  $\square$

This counterexample is methodologically productive: topological analysis can describe structures and changes in represented events while ledger analysis can describe obligations, and neither should be presented as a substitute for the other. It prevents a visually attractive loop in a relational diagram from being mistaken for a financial law.

## B.5 Maps between Event Complexes and Obligation Networks

A study connecting the two representations needs an explicit *bridge*. One possibility is a typed relation between event-chain configurations and claim records, extending the partial interpretation  $Q_{h,\ell}$  of Equation (61). This relation need not be single-valued: one event may generate several claims; one claim can summarize numerous productive events; a contribution may produce no recognized monetary claim; and a future contingent claim may be indexed to information not represented by one observed event.

If the analyst wishes to use a *chain map*, stronger assumptions are required. For chain complexes  $C_\bullet(K)$  and  $C_\bullet(L)$ , a chain map  $F_\bullet$  is a family of linear maps such that

$$F_{k-1} \partial_k^K = \partial_k^L F_k, \quad k = 1, 2. \quad (75)$$

A chain map satisfying Equation (75) induces homomorphisms on homology. A mere event-to-invoice database lookup generally fails this equation and cannot be treated as a homology-preserving transformation. If  $L$  is the one-dimensional claim graph, its  $C_2(L)$  vanishes; a chain map's degree-two component must therefore be zero. The induced map can still send some event cycles to claim-graph cycles, but any event one-cycle that is itself a cellular two-boundary must map to zero under a genuine chain map into the one-complex. This strongly limits attempts to identify a face-boundary cycle with a nonzero claim circulation by claiming that a chain map preserves all structure.

In applications it may be more faithful to use a typed correspondence, a span of relations, or a stochastic observation/interpretation kernel rather than an artificial chain map. In each case the paper should say which properties survive translation: preservation of node incidence, compatible units, provenance links, recognized claim holders, observed ordering, and admissible discharge are different requirements. A failure to preserve one of them may be a substantive result rather than a mathematical error.

**Remark B.5** (Algebraic preservation and institutional recognition). Even when an authenticated bridge preserves incidence and balances, the algebra alone cannot establish the right to modify a recognized claim. Authority, consent, applicable law, participation, contestability, and executed discharge remain facts or predicates outside the chain-map axioms. A proposed map is not a replacement for evidence of those conditions.

## B.6 Subcomplexes, Cuts, and Relative Homology

The choice of a network boundary is central to our account of local dissolution. Cellular topology allows a precise distinction between a cycle closed in the entire complex and a chain whose ends lie on a selected interface. Let  $A \subseteq K$  be a subcomplex, interpreted only after the analyst states what its selected events and couplings signify. The relative chains are

$$C_k(K, A; \mathbb{F}) = C_k(K; \mathbb{F}) / C_k(A; \mathbb{F}), \quad (76)$$

with boundary induced by that of  $K$  in Equation (76). This boundary is well defined because  $\partial C_k(A) \subseteq C_{k-1}(A)$ . A relative one-cycle is a chain  $c \in C_1(K)$  whose boundary lies in  $C_0(A)$ , modulo chains supported inside  $A$ . Its endpoints need not cancel as an absolute one-chain. The relative homology  $H_1(K, A)$  records equivalence classes in this quotient.

For an elementary example, let  $K$  be a path  $u \rightarrow v \rightarrow w$  and  $A$  the subcomplex consisting of its endpoint vertices  $\{u, w\}$ . The full path  $c = (u \rightarrow v) + (v \rightarrow w)$  has  $\partial_1 c = w - u \neq 0$ , so  $c$  is not an absolute one-cycle. Its boundary vanishes in  $C_0(K, A)$ , however, so it represents a relative cycle. This is a mathematical description of a channel that crosses a selected interface. It is not evidence that the two endpoint entities have settled an obligation, and the selected vertices remain *events* in the event-complex interpretation.

In a financial ledger the analogous object is a boundary-crossing chain or flow with an explicit residual on the cut. For a position set  $S$  the accounting cut identity in Equation (63) measures that residual using the *claim graph*'s incidence matrix. The relative-homology example describes a different chain complex, even if both descriptions help identify which purportedly local circulations depend on excluded components. A claim that a monetary cycle is complete should therefore declare the boundary of accounting, not only a topological representative.

When useful, the inclusion  $A \hookrightarrow K$  fits in the long exact sequence of the pair,

$$\cdots \rightarrow H_1(A) \rightarrow H_1(K) \rightarrow H_1(K, A) \xrightarrow{\delta} H_0(A) \rightarrow H_0(K) \rightarrow \cdots . \quad (77)$$

The connecting map  $\delta$  in Equation (77) registers a relative cycle's boundary class at the selected interface. The sequence is a standard topological identity, not a statement about balances denominated in money. It may nonetheless inform how a research design distinguishes internal circulations from passages relying on unobserved upstream or downstream parts of a network.

## B.7 Histories, Continuations, and Changes of Topological Representation

The existence of a cell complex  $K_h$  at a represented configuration  $h$  does not posit a universal world-state or a privileged present. A comparison  $h \rightsquigarrow h'$  is supplied by the declared admissible continuation relation of Definition A.2. The corresponding complexes need not be related by inclusion, and they can differ because new events are observed, an institutional practice changes its classification, or a better description corrects the original model. A change in recorded topology is not automatically a new event in the world.

If there is a specified cellular map  $f : K_h \rightarrow K_{h'}$ , it induces maps on cellular homology, under the relevant cellular or chain-map assumptions. Such maps can track which represented classes are preserved, identified, or sent to zero. If instead there is a nested family  $K_0 \subseteq K_1 \subseteq \cdots \subseteq K_m$  justified by an explicit observation or construction order, its inclusion maps define a filtration and permit persistent-homology calculations [1]. But a finite filtration index is a modeling convention; it need not be a universal time, nor does longer persistence imply greater economic value or normative significance.

The filled-triangle construction demonstrates one possible structural change: attaching a two-cell can make an existing first-homology class become a boundary. Removing a coupling can break an apparent closed path; identifying two event records can change ranks in still another way. None of these mathematical changes by itself creates a payment, changes contractual maturity, or eliminates ecological damage. A monetary claim can also be newly recognized without any change in the chosen event complex. Consequently, the transition model should carry both the representational correspondence  $K_h \rightsquigarrow K_{h'}$  and the institutionally defined ledger correspondence  $x_h \rightsquigarrow x_{h'}$  of Equation (65).

## B.8 Weighted Chains, Hodge Decomposition, and Their Limits

For applications involving noisy observations or physical network costs, the chain groups may be equipped with positive-definite inner products. Relative to the chosen inner products, let  $\partial_k^*$  denote the

adjoint of  $\partial_k$ . In a finite two-complex, define the one-dimensional combinatorial Hodge Laplacian

$$\Delta_1 = \partial_1^* \partial_1 + \partial_2 \partial_2^*, \quad \mathcal{H}_1 = \ker \Delta_1. \quad (78)$$

The Laplacian in Equation (78), together with standard finite-dimensional linear algebra, gives the orthogonal decomposition

$$C_1(K; \mathbb{R}) = \text{im } \partial_2 \oplus \mathcal{H}_1 \oplus \text{im } \partial_1^*, \quad \mathcal{H}_1 \cong H_1(K; \mathbb{R}). \quad (79)$$

The decomposition in Equation (79) separates a signed one-chain into a two-boundary component, a harmonic component, and a gradient-like component. For a cellular one-cycle the component in  $\text{im } \partial_1^*$  vanishes, because  $\ker \partial_1$  is orthogonal to  $\text{im } \partial_1^*$ . These results hold for the declared finite Hilbert-chain structure.

**Proposition B.6** (Hodge separation of cellular cycles). *For a finite oriented two-complex with positive-definite inner products, each  $c \in \ker \partial_1$  decomposes uniquely as  $c = \partial_2 a + h$ , where  $h \in \mathcal{H}_1$  and  $\partial_2 a$  is uniquely determined as an element of  $\text{im } \partial_2$ . The representing two-chain  $a$  need not be unique.*

*Proof.* By finite-dimensional orthogonal decomposition,  $c$  has components in  $\text{im } \partial_2$ ,  $\mathcal{H}_1$ , and  $\text{im } \partial_1^*$ . Since  $c \in \ker \partial_1 = (\text{im } \partial_1^*)^\perp$ , its final component vanishes. The first two components are orthogonal and unique, although  $a$  can differ by an element of  $\ker \partial_2$ .  $\square$

A weight on an event edge can represent sampling confidence, measurement sensitivity, physical length, or a declared modeling cost. It is not automatically a monetary price. Likewise, the harmonic part of an event-chain decomposition is not a net outstanding debt, and a nonzero two-boundary component is not an executable clearing cycle. The decomposition is on a *signed* vector space, whereas numerical clearing additionally uses nonnegative cones, observed capacities, typed monetary units, and legal predicates. Treating these distinctions explicitly allows topology to provide useful computational descriptors without covertly defining normative equivalence through geometry.

## B.9 Reproducible Analysis and Model-Selection Tests

A computational implementation should make the algebra reproducible and its economic assumptions inspectable. A minimal record includes: the event-selection and cell-attachment rules; orientations and ordered bases for  $C_0, C_1, C_2$ ; a declared coefficient field; sparse matrices  $D_1, D_2$ ; the numerical test  $D_1 D_2 = 0$ ; ranks and dimensions of relevant kernels and images; the selected observation boundary; the independent claim graph with debtor, creditor, currency and claim-type records; and a documented, possibly partial, bridge between event and claim data. A pipeline must also record what authority establishes admissibility and execution of monetary reductions. Numerical coincidence of an event matrix and an obligation incidence matrix is not sufficient validation.

At least three sensitivity checks are important. *Topological sensitivity* varies the resolution of events, the selection of higher-order faces, and uncertain incidence observations. *Institutional sensitivity* varies the compatibility and authorization predicates while keeping observed obligations fixed. *Boundary sensitivity* expands or contracts the set of accounts and productive inputs represented. For example, an apparently complete three-party circulation can cease to be an eligible discharge when one contract disallows set-off, and a seemingly money-free AI service can remain dependent on electricity invoices outside a user-facing observation boundary. These are different sources of instability and should be reported separately.

The mathematical conclusions in this appendix are standard properties of finite chain complexes, incidence matrices, and linear decompositions under their specified hypotheses. They support a disciplined representational approach to relations, but do not imply that greater connectivity produces monetary disappearance, that a particular homology class has a market value, or that an AI-governed system necessarily achieves fair circulation. The further problem is institutional and empirical: to identify historically grounded bridges connecting value-generating events, recognized claims, publicly contestable

Table 16: Formal objects, computational tests, and limitations of economic inference.

| Object                     | Mathematical or computational test               | Further evidence required for economic interpretation |
|----------------------------|--------------------------------------------------|-------------------------------------------------------|
| Event two-complex          | $D_1 D_2 = 0$ ; cell semantics; $\beta_1$        | Event provenance; boundary and attachment choices     |
| Directed claim ledger      | $Bx = b$ ; typed units and positions             | Recognition, contract conditions and claim holders    |
| Nonnegative circulation    | $Bz = 0, 0 \leq z \leq x$                        | Compatibility, authorization and enacted discharge    |
| Cross-layer bridge         | Declared correspondence; optional chain-map test | Source events, value records and institutional rules  |
| Relative topological cycle | $\partial c \in C_0(A)$                          | Meaning of the interface and external dependencies    |
| Model continuation         | Inclusion or explicit cellular correspondence    | Change in observations versus change in institutions  |

governance procedures, and actual settlement records. Appendix C develops eligible monetary clearing on the claim side of this distinction; Appendix D examines computational approximation without confusing omitted paths with extinguished obligations.

## C Multilateral Clearing and Residual Obligation Decomposition

This appendix gives a more explicit account of the optimization problem implicit in Sections 7 and 8. The relevant mathematical operation removes a compatible circulation of *recognized monetary claims* while preserving the declared net monetary positions of their holders. This is narrower than the elimination of exchange, the satisfaction of heterogeneous needs, or the replacement of monetary institutions. The discussion separates a reproducible algebraic optimization problem, the institutional decision to authorize a reduction, and the observable event through which a recognized obligation is actually discharged. The topological distinctions developed in Appendix A and Appendix B remain in force throughout; earlier exploratory monetary and heterogeneous-circulation studies provide research context [14, 11].

### C.1 Typed Claim Ledgers and Compatibility Partitions

Fix a represented local configuration  $h$  and a finite set  $P$  of recognized claim-bearing positions. These are accounts or institutional positions rather than the event vertices of a relational two-complex. A directed multigraph  $G_h = (P, L)$  records candidate obligations. Each edge  $e \in L$  has a debtor  $d(e)$ , creditor  $c(e)$ , and nonnegative outstanding amount  $x_e$ . Its declared record also specifies the denomination, maturity or contingency, priority, governing institution, source and status of recognition, and any constraints on transfer, offset, or discharge. Parallel directed edges are retained when distinct contracts connect the same positions. Reversing an edge is not an innocuous change of mathematical convention: it changes which position is represented as debtor and which as creditor.

Let  $B \in \{-1, 0, 1\}^{|P| \times |L|}$  be the incidence matrix with  $B_{d(e),e} = -1$  and  $B_{c(e),e} = +1$ , as in Appendix A. Self-claims, if permitted by a data source, require explicit treatment and will not be taken as genuine bilateral debts by default. Write

$$b = Bx, \quad b_p = \sum_{e:c(e)=p} x_e - \sum_{e:d(e)=p} x_e. \quad (80)$$

The component  $b_p$  in Equation (80) is a signed *net nominal claim position* within the selected ledger boundary. It need not coincide with available liquidity, wealth, economic surplus, or a person's welfare. The convention is creditor-positive and debtor-negative; it is maintained in the examples below.

Amounts can enter the same scalar ledger only when the accounting unit and the institutional rules of comparison have been specified. We therefore partition  $L$  into declared *compatibility classes*  $L_1, \dots, L_m$ . A class may be distinguished by currency, enforceability, settlement arrangement, maturity, seniority, or another condition relevant to the contemplated operation. For each class  $k$  there is a separate amount vector  $x^{(k)}$  and incidence map  $B_k$ . The block-diagonal incidence operator

$$\mathbb{B} = \text{diag}(B_1, \dots, B_m) \quad (81)$$

acts on class-indexed position records (Equation (81)), even when the same person or organization occupies positions in several classes. Summing components across different currencies or noncomparable claims would discard economically meaningful type information. An exchange-rate agreement, novation, or conversion mechanism can create a new formally comparable class; no such conversion follows merely from the existence of a numerical exchange rate.

This partition is *operation-relative*. Claims that cannot be netted in one jurisdiction may be eligible in another institutional arrangement; a debt due immediately and a contingent commitment may have different legal status even when their monetary labels coincide. A proposed clearing computation should disclose the classification and the authority for it. In particular, obligations among AI-operated services should not automatically be treated as independent debts between artificial legal persons when their recognized counterparties are human organizations or shared accounts.

## C.2 Algebraic Reduction and the Institutional Feasible Set

Within one fixed compatible class, let  $x \in \mathbb{R}_{\geq 0}^L$  and let  $z_e$  be the amount proposed for cancellation on edge  $e$ . A *balance-preserving reduction* is a nonnegative vector satisfying

$$\mathcal{Z}(x; B) = \{z \in \mathbb{R}^L : 0 \leq z \leq x, Bz = 0\}. \quad (82)$$

The residual vector associated with Equation (82) is  $r = x - z$ . Since  $Bz = 0$ , the accounting identity  $Br = Bx = b$  follows directly. This identity concerns only the chosen typed balances. It does not say that each creditor's remaining claim has the same liquidity, priority, or legal protection after a proposed transaction.

**Definition C.1** (Authorized reduction and enacted discharge). An *authorized reduction proposal* is a vector  $z \in \mathcal{Z}(x; B)$  that also satisfies a declared institutional admissibility predicate  $A_h(z, \iota)$ , where  $\iota$  identifies the relevant rules and permissions. An *enacted discharge* is an authorized proposal accompanied by authenticated evidence of an institutionally effective claim transformation, including the affected edge records, decision authority, and recognized effective event. Authorization alone does not establish execution.

When the predicate  $A_h$  can be represented by linear inequalities and linear equalities, the authorized feasible set can be optimized as a linear programme. In other settings, minimum indivisible amounts, conditional consent, contingent contracts, coalition-dependent permissions, or discrete choices yield a nonconvex or mixed-integer problem. In still other settings, authorization may be disputed or unavailable to the researcher; it should then be recorded as unknown rather than inferred from the incidence equation. The zero vector is always algebraically feasible but is not necessarily the only institutionally admissible action, nor does its feasibility authorize a future discharge.

A direct settlement payment is a different kind of operation. It can change the outstanding amount and the transfer of a settlement asset, and need not preserve the debtor-creditor net-position vector *within the claims ledger alone*. A debt forgiveness may reduce a creditor's net claim without providing a balancing reduction elsewhere. Multilateral netting, payment, forgiveness, and novation thus require different transformation types, even when all are informally called "settlement." Treating them as identical would obscure both legal effects and productive dependencies.

### C.3 Cycle Decomposition and the Limits of Graph Reduction

The incidence constraint  $Bz = 0$  means that the canceled amounts form a nonnegative circulation on the *claim graph*. For finite directed multigraphs, Proposition 8.1 establishes the representation

$$z = \sum_{\gamma \in \Gamma} \alpha_{\gamma} \chi_{\gamma}, \quad \alpha_{\gamma} \geq 0, \quad (83)$$

for a finite collection  $\Gamma$  of directed simple cycles, where  $\chi_{\gamma}$  is an edge-incidence vector. In any chosen decomposition, the coefficient of a cycle can be read as a candidate quantity of offsetting nominal claims. It is not a measure of the amount of value created by the corresponding exchange relationships. Multiple decompositions can represent the same circulation, and cycles can overlap in their edge capacities.

**Proposition C.2** (Existence of a maximal gross reduction). *For a finite compatible claim ledger with  $x \geq 0$  and a finite coefficient vector  $w \in \mathbb{R}^L$ , the optimisation problem*

$$V_w(x) = \max_{z \in \mathcal{Z}(x; B)} w^{\top} z \quad (84)$$

*has at least one optimal solution. If every  $w_e > 0$ , the optimal objective value is strictly positive if and only if the positive-capacity support of  $x$  contains a directed cycle. Equivalently, in this case every optimizer is nonzero exactly when such a cycle exists.*

*Proof.* The set  $\mathcal{Z}(x; B)$  contains zero and is closed and bounded in finite-dimensional Euclidean space. The linear objective attains its maximum on this compact set. If the positive-capacity graph has no directed cycle, the nonnegative cycle-decomposition theorem implies that zero is its only circulation. Conversely, a positive-capacity directed cycle admits  $\epsilon \chi_{\gamma}$  with  $\epsilon > 0$  sufficiently small; when  $w_e > 0$  for all edges, this has positive objective value.  $\square$

The uniform objective  $w_e = 1$  maximizes canceled *gross nominal obligations*,  $\mathbf{1}^{\top} z$ . An alternative positive weight vector might represent declared processing costs or risk exposure, but it should not be described as the maximization of generative value or social welfare. Objective coefficients and units must be stated. If some weights vanish or become negative, the cycle-existence criterion for a strictly beneficial optimization no longer follows as written, even though a nonzero algebraic circulation might remain possible.

Equation (83) is useful conceptually, but enumerating every simple cycle can be expensive. A finite directed graph can have exponentially many simple cycles. For the basic divisible-flow model, the edge-variable problem (84) can instead be solved with  $|L|$  decision variables and  $|P|$  balance constraints, without first listing all cycles. Institutional conditions can nevertheless restore combinatorial complexity. A solver's ability to find an algebraic optimum is therefore a computational result within a model, not evidence that the contemplated offsets can be negotiated, verified, or legally executed at comparable cost.

### C.4 Residual Decomposition and the Boundary of Monetary Dissolution

For any admissible algebraic reduction  $z$ , the residual  $r = x - z$  satisfies

$$r \geq 0, \quad Br = b, \quad 0 \leq r \leq x. \quad (85)$$

Conversely, any  $r$  satisfying (85) defines  $z = x - r \in \mathcal{Z}(x; B)$ . Thus uniform maximum-gross clearing is equivalent to the bounded minimum-residual problem

$$\min_{0 \leq r \leq x, Br=b} \mathbf{1}^{\top} r. \quad (86)$$

This equivalence is an accounting identity under the present restrictions. It does not imply that an obligation graph can be replaced by an arbitrary synthetic edge joining a net debtor to a net creditor: doing so may constitute novation, assignment, or a newly created financial relationship, none of which is allowed by the componentwise residual constraint  $r \leq x$ .

**Proposition C.3** (Acyclic support of an unrestricted optimum). *If  $r^*$  minimizes (86), its strictly positive directed-edge support has no directed cycle. More generally, if all weights  $w_e > 0$ , the residual of an optimum to (84) has no strictly positive directed cycle.*

*Proof.* If such a directed cycle  $\gamma$  existed, choose  $\epsilon > 0$  no greater than the minimum residual amount along its edges. Then  $z' = x - r^* + \epsilon \chi_\gamma$  would remain in  $\mathcal{Z}(x; B)$  and yield a larger weighted objective because  $\sum_{e \in \gamma} w_e > 0$ . This contradicts optimality.  $\square$

This proposition is an unrestricted algebraic statement. Its conclusion is not warranted for a consent-constrained or otherwise restricted clearing. An available directed cycle may remain wholly uncanceled because its claims are ineligible, one party declines to participate, or a procedural condition fails. Even in the unrestricted model, an acyclic residual can have substantial nominal amounts and nonzero net positions. The absence of an additional cancellable cycle identifies the exhaustion of a *particular kind of balance-preserving subtraction*; it does not imply the disappearance of monetary claims, borrowing needs, or resource constraints.

A simple bound quantifies why residual obligations cannot in general vanish when the net positions are nonzero.

**Proposition C.4** (Lower bound on residual gross claims). *For any finite directed claim graph and every  $r \geq 0$  satisfying  $Br = b$ ,*

$$\mathbf{1}^T r \geq \frac{1}{2} \|b\|_1. \quad (87)$$

*Consequently, by Equation (87), if  $b \neq 0$ , no balance-preserving reduction can extinguish all claims in the selected class.*

*Proof.* Each directed nonloop edge of incidence matrix  $B$  contributes one positive and one negative entry of absolute value one. By the triangle inequality,  $\|Br\|_1 \leq \sum_e r_e \|B_{\cdot e}\|_1 \leq 2 \sum_e r_e$ . Substituting  $Br = b$  gives the stated bound. If  $r = 0$ , then necessarily  $b = Br = 0$ .  $\square$

The bound can be strict because the available edge support may force a claim to pass through intermediate positions. For a two-edge directed chain  $A \rightarrow B \rightarrow C$  with one unit on each edge, A is net debtor by one, C is net creditor by one, and B has zero net position. The gross outstanding amount is two, whereas  $\frac{1}{2} \|b\|_1 = 1$ . Because the claim graph has no directed cycle, neither claim is removable by cycle clearing. A direct  $A \rightarrow C$  obligation of one unit could give a different gross amount with the same net positions, but creating that edge would require an additional institutional transformation.

These facts also clarify the phrase “mutually owing nothing.” Every participant may have balanced incoming and outgoing positions ( $b = 0$ ) while the original ledger still contains a large cyclic mass of claims. Under unrestricted divisible clearing, the full nonnegative circulation  $x$  itself may be removed when  $Bx = 0$ . By contrast, when  $b \neq 0$ , at least some residual claims necessarily remain under the specified accounting boundary. The amount of monetary *mediation* that persists in the wider economy remains a distinct empirical question.

## C.5 A Four-Party Clearing Calculation

Consider the illustrative compatible ledger of Section 8. Four positions A, B, C, and D recognize the directed obligations  $A \rightarrow B$  of nine units,  $B \rightarrow C$  of six,  $C \rightarrow D$  of eight, and  $D \rightarrow A$  of five. With edge order  $(AB, BC, CD, DA)$ , the vectors are

$$x = \begin{pmatrix} 9 \\ 6 \\ 8 \\ 5 \end{pmatrix}, \quad z(t) = t \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \quad 0 \leq t \leq 5. \quad (88)$$

Table 17: Illustrative four-party compatible claims and unrestricted gross clearing; amounts are nominal units in one declared class.

| Directed obligation | Initial | Canceled | Residual | Interpretation of residual             |
|---------------------|---------|----------|----------|----------------------------------------|
| A to B              | 9       | 5        | 4        | A retains a claim liability to B       |
| B to C              | 6       | 5        | 1        | B retains a claim liability to C       |
| C to D              | 8       | 5        | 3        | C retains a claim liability to D       |
| D to A              | 5       | 5        | 0        | This recognized claim is fully removed |
| Gross total         | 28      | 20       | 8        | Net positions unchanged                |

Every nonnegative circulation for the claims in Equation (88) is of this form because each vertex has exactly one incoming and one outgoing edge. Uniform maximum-gross clearing selects  $t^* = 5$ , leaving

$$r^* = \begin{pmatrix} 4 \\ 1 \\ 3 \\ 0 \end{pmatrix}, \quad Bx = Br^* = \begin{pmatrix} -4 \\ 3 \\ -2 \\ 3 \end{pmatrix}. \quad (89)$$

For the residual in Equation (89), the original gross obligations total 28 units; the canceled amount is  $4 \cdot 5 = 20$ ; and the residual totals eight. The result conserves each position's nominal net claim, while the remaining edges form the directed path  $A \rightarrow B \rightarrow C \rightarrow D$ . The formal result requires the four edges to form one recognized compatible clearing group. If the  $D \rightarrow A$  claim has a different maturity, creditor consent is unavailable, or its governing institution prohibits multilateral setoff, the same numerical four-cycle need not be executable.

This worked example should not be read as a demonstration that A, B, C, or D experience equal welfare gains or that each productive contribution has the same qualitative value. It establishes a specific balance-preserving arithmetic transformation. A clearing institution would additionally need to specify how rights to contest claims, tax recognition, liquidity access, creditor priority, and responsibility for errors are affected. When the parties are AI-operated units, it would also need to distinguish operational software identifiers from the persons or legal bodies that hold rights and bear liabilities.

## C.6 Optimization Duality and the Meaning of Computational Prices

When the algebraic constraints are divisible and continuous, the weighted programme (84) is a finite linear programme. Its dual gives one way to analyze bottlenecks in the proposed clearing, but must not be mistaken for a general value theory. Using an unrestricted vector  $y \in \mathbb{R}^{|P|}$  and a nonnegative vector  $u \in \mathbb{R}_{\geq 0}^{|L|}$ , a dual representation is

$$\begin{aligned} &\text{minimize} && x^\top u \\ &\text{subject to} && B^\top y + u \geq w, \\ &&& u \geq 0, \quad y \text{ free.} \end{aligned} \quad (90)$$

The sign convention of  $y$  is arbitrary so long as it is used consistently. In this formulation  $y$  is a ledger-position potential and  $u$  is a dual multiplier on edge capacities. Neither is a market price of labour, a measure of the ecological value of production, or a welfare weight.

**Proposition C.5** (Dual upper bound and equality of optimum values). *For every primal-feasible  $z$  in (84) and every dual-feasible  $(y, u)$  in (90),*

$$w^\top z \leq x^\top u. \quad (91)$$

*For the finite linear programmes specified in Equation (91), optimal primal and dual solutions exist and their objective values are equal.*

*Proof.* Dual feasibility gives  $w \leq B^\top y + u$ . Multiply componentwise by  $z \geq 0$  and sum to obtain  $w^\top z \leq y^\top Bz + u^\top z = u^\top z \leq u^\top x$ . The primal is feasible and bounded by  $0 \leq z \leq x$ , so standard finite-dimensional linear-programming strong duality applies; the dual is feasible, for example by taking  $y = 0$  and  $u_e \geq \max\{w_e, 0\}$ .  $\square$

Acyclic residual support is only a necessary property of a gross-clearing optimum, as the counterexample in Appendix D shows. A valid finite-ledger optimality test can instead combine primal and dual feasibility with the following exact gap identity.

**Proposition C.6** (Primal–dual gap and a sufficient optimality certificate). *For any primal-feasible  $z$  and dual-feasible  $(y, u)$  for Equations (84) and (90), respectively,*

$$x^\top u - w^\top z = u^\top (x - z) + z^\top (B^\top y + u - w) \geq 0. \quad (92)$$

*Equality in Equation (92) holds if and only if the two nonnegative summands vanish. In that case  $z$  and  $(y, u)$  are optimal for the declared unrestricted divisible linear programmes.*

*Proof.* Expand the right side and use  $Bz = 0$  to cancel  $z^\top B^\top y = y^\top Bz$ . Both remaining dot products are nonnegative since  $u \geq 0$ ,  $x - z \geq 0$ ,  $z \geq 0$ , and  $B^\top y + u - w \geq 0$ . A sum of two nonnegative real numbers is zero exactly when both are zero. If the primal and dual objective values agree, weak duality makes both solutions optimal. Conversely, finite-dimensional strong duality guarantees an optimal pair for which the gap is zero.  $\square$

The identity also gives a numerical audit target: record primal feasibility, dual feasibility, and the nonnegative gap together with solver tolerances. Complementary slackness requires  $u_e(x_e - z_e) = 0$  and  $z_e((B^\top y + u - w)_e) = 0$  for each edge. The test applies to the specific continuous model; introducing a consent-dependent or integer restriction requires the correct restricted optimization and, where available, a certificate appropriate to that model.

The dual can diagnose the algebraic importance of an edge capacity to a specified objective, but interpretations should remain local to that objective. A large capacity multiplier can identify an edge that constrains a proposed cancellation programme without asserting that the claim holder is economically productive, normatively deserving, or endowed with unrestricted bargaining power. When institutional permissions or discrete choices are imposed, the simple dual may fail to describe the feasible problem; a changed optimization model requires a corresponding rederivation rather than carrying these prices over by analogy.

## C.7 Institutional Restrictions and Nonconvex Admissibility

The unrestricted model is useful partly because its limitations can be exhibited with small counterexamples. Start with an  $A \rightarrow B \rightarrow C \rightarrow A$  cycle carrying one nominal unit on every edge. Algebraically, the whole three-unit gross total can be canceled while preserving net positions. Suppose, however, that one claimant has not consented to the proposed discharge or an applicable rule excludes the  $C \rightarrow A$  claim. An admissibility condition  $z_{CA} = 0$  then forces  $z = 0$  by conservation, leaving all three edges intact. The support remains cyclic even though clearing within the declared authorized set cannot proceed. This does not contradict Proposition C.3: its unrestricted feasible set has been replaced by a restricted one.

More elaborate practices may require participation coalitions. Let  $s_p \in \{0, 1\}$  indicate verified agreement by a claim-bearing position  $p$ , and let  $M_{ep} \in \{0, 1\}$  record whether an edge is incident to  $p$ . A hypothetical consent constraint takes the form

$$0 \leq z_e \leq x_e s_p \quad \text{for every } (e, p) \text{ with } M_{ep} = 1. \quad (93)$$

The constraint in Equation (93) is not a general description of any jurisdiction’s netting law; it illustrates how a discrete eligibility condition changes the mathematical problem. Some arrangements require approvals from parties not directly incident to an edge, while others rely on institutional delegation instead of individual assent. Recording the controlling rule is as important as solving the resulting programme.

Institutional safeguards also involve distribution. A creditor may prefer a liquid claim to a nominally equivalent offset, or a participant may lose a priority right through novation. An intermediary may concentrate the power to decide which claims count as compatible. A clearing procedure can therefore lower the gross nominal total while worsening contestability or weakening the rights of some value generators. The monetary objective does not resolve such questions. The normative profile introduced in Section 12 remains a separate input to the evaluation.

## C.8 Accounting Boundaries, Heterogeneous Claims, and Relational Continuation

If only a subset  $S \subseteq P$  is observed, its apparent monetary clearing can depend on claims outside the selected boundary. Summing the incidence equation over  $S$  cancels internal edges and retains claims entering or leaving the subset. With the same creditor-positive convention, for any residual  $r$ ,

$$\sum_{p \in S} (Br)_p = \sum_{e: d(e) \notin S, c(e) \in S} r_e - \sum_{e: d(e) \in S, c(e) \notin S} r_e. \quad (94)$$

Equation (94) is a ledger cut identity, not a theorem about the value of interactions across the boundary. It demonstrates why a service-provider interface showing zero user payments can coexist with large upstream energy, computing, construction, licensing, or financing claims beyond the measurement boundary. Conversely, a public or commons-based arrangement can genuinely reduce certain monetary obligations through different production and allocation rules. These cases cannot be distinguished from user-facing payment records alone.

The vector  $z_h$  describes a potential or enacted reduction in one represented configuration. If a later or otherwise related configuration  $h'$  is connected by a declared continuation correspondence, the recognized claim system may evolve according to

$$x_{h'} = T_{h \rightsquigarrow h'}(x_h - z_h) + n_{h'}, \quad (95)$$

In Equation (95),  $T$  is an explicitly specified, dimension-compatible transfer or reclassification operator and  $n_{h'}$  records newly recognized claims. This expression is schematic and uses no universal clock; a proposed continuation may represent new events, revision of records, or institutional reorganization. For a valid claim model, the right side must have a coherent type and nonnegative recognized amounts. An anticipated future contribution does not enter a current clearing ledger as an enforceable claim unless the applicable recognition rules establish it.

The central economic implication is limited but consequential: a completed local clearing operation does not prevent a network from generating new monetary obligations through additional encounters, productive contributions, resource dependencies, or forms of governance. At the same time, more capable coordination may enable new direct-provision or reciprocal structures in which some claims are never generated. Local clearing, displacement, nonmonetary coordination, and new credit can coexist within the same developing production system.

## C.9 Reproducibility, Execution Records, and Empirical Tests

A research implementation should publish sufficient non-sensitive information to reproduce the *mathematical* outcome and to evaluate the limits of its institutional interpretation. At minimum it should declare the observation boundary, claim-bearing positions, debtor-creditor convention, typed denominations and compatibility rules, recognized initial amounts, the incidence matrix, restrictions on eligibility, the objective coefficients, the computed reduction, and the resulting residual. It should check  $0 \leq z \leq x$ ,  $Bz = 0$ ,  $r = x - z \geq 0$ , and  $Br = Bx$  within a stated exact or numerical tolerance. When strong duality is invoked, the primal and dual objective values and feasibility residuals should also be reported. Confidential claim identities can be replaced by documented pseudonyms or appropriately aggregated records where legally and methodologically suitable.

Table 18: Distinct verification layers in a proposed multilateral clearing procedure.

| Layer                              | Reproducible check                                                             | Remaining limitation                                                      |
|------------------------------------|--------------------------------------------------------------------------------|---------------------------------------------------------------------------|
| Claim typing                       | Currency, maturity, priority, parties, recognized status                       | Typing may be contested or incomplete                                     |
| Algebraic clearing<br>Optimization | $Bz = 0, 0 \leq z \leq x$<br>Objective, solution, duality and solver tolerance | No permission inferred from arithmetic<br>Objective is not social welfare |
| Admissibility                      | Named rule, parties, consent, required approvals                               | Permissions may depend on facts outside the ledger                        |
| Execution                          | Authenticated discharge and new claim records                                  | Proposed cancellation may not be effected                                 |
| Economic incidence                 | Upstream claims, access and production boundaries                              | Hidden finance and new claims may persist                                 |
| Ethics and ecology                 | Participation, remedy and physical impacts                                     | Financial cancellation cannot close every obligation                      |

A computational pipeline can implement the basic divisible-class model as follows. First, validate the provenance and eligibility of records and keep uncertain claims in a separately labeled category. Second, construct the typed incidence matrices and capacities, and solve either the edge-variable linear programme or an explicitly justified institutional variant. Third, independently check the output against the original claims and declared constraints. Fourth, generate an auditable proposal without marking claims discharged. Fifth, record authorizations and actual execution, then construct the recognized residual ledger from the enacted events. Finally, examine the effects of changing the network boundary, type classifications, institutional participation, and anticipated new obligations. This sequence preserves an indispensable separation between a mathematical result, a proposed policy, and a socially effective legal or economic event.

The appendix has thus supplied a conditional, auditable interpretation of multilateral clearing. It proves the existence and cycle properties of unrestricted divisible reductions, gives a gross-residual equivalence and quantitative residual bound, and derives the relevant linear-programming dual. It also identifies situations in which those identities leave the practically decisive questions unresolved. Appendix D will examine the separate issue of computational truncation and long-circuit fragility. The wider paper’s claim remains correspondingly modest: the clearing of compatible monetary claims is one possible constituent of spacetime-distributed monetary dissolution, while needs for others, heterogeneous value circulation, institutional authority, ecological obligations, and the generation of new monetary relations can continue.

## D Path Fragility and Truncation Conditions

This appendix supplies a more explicit mathematical basis for the conditional computational claims in Section 9. A long circulation route is sometimes harder to complete because its successful execution depends on many claims, approvals, and productive connections. The additional dependencies can reduce the expected contribution of remote cycles and thereby make a finite search defensible. The argument, however, requires separate control of *individual survival*, *the number of candidate cycles*, and *the monetary capacity carried by each cycle*. Correlated failures, selective discovery, large capacities, and institutional restrictions can defeat an inference based on length alone. None of the statements below describes a universal law of economic history or a method for erasing recognized obligations by deleting them from a computational representation.

We use a reference configuration  $h$ , an information set  $\mathcal{I}_h$ , and one finite, typed, divisible monetary-claim multigraph  $G = (P, E)$ . Conditional probabilities  $\mathbb{P}_h(\cdot) = \mathbb{P}(\cdot \mid \mathcal{I}_h)$  express uncertainty in a declared model. They need not correspond to an externally privileged temporal present:  $h$  denotes a

represented relational configuration, and eligibility may depend on admissible event continuations. Each directed edge  $e$  has a recognized maximum claim amount  $x_e \geq 0$ . The set  $\Gamma$  consists of directed simple cycles, with cyclic rotations identified. Any institutionally forbidden candidate is omitted or assigned eligibility indicator zero. We retain the basic separation in Appendices A, B and C: the event complex, the monetary claims graph, and authorized discharge events are different mathematical and institutional objects.

## D.1 Reliability of a Specified Circulation Proposal

For a proposed cycle  $\gamma = (e_1, \dots, e_L)$ , write  $X_{\gamma i} \in \{0, 1\}$  for the survival of its  $i$ th required edge and  $A_\gamma \in \{0, 1\}$  for the satisfaction of other necessary authorization or execution conditions. The indicator of a fully eligible proposal is

$$I_\gamma = A_\gamma \prod_{i=1}^L X_{\gamma i}, \quad s_\gamma = \mathbb{E}_h[I_\gamma]. \quad (96)$$

The indicator in Equation (96) refers to a proposed opportunity, not to the discharge of a claim. In particular, a valid circuit can survive without being selected, and selection can occur without institutionally effective execution if the latter is not explicitly verified.

Let  $\pi$  be an ordering of the necessary edge-survival requirements for which successive conditional probabilities are defined, and put

$$p_{\gamma i} = \mathbb{P}_h \left( X_{\gamma \pi(i)} = 1 \mid \bigcap_{j < i} \{X_{\gamma \pi(j)} = 1\} \right), \quad \eta_{\gamma i} = 1 - p_{\gamma i}. \quad (97)$$

The chain rule applied to Equation (97) gives  $\mathbb{P}_h(\cap_i \{X_{\gamma i} = 1\}) = \prod_i p_{\gamma i}$ , including the obvious zero-probability limiting cases. Because  $A_\gamma \leq 1$ , we obtain the following elementary but useful statement.

**Proposition D.1** (Conditional-hazard survival bound). *If for every examined length- $L$  proposal, and each required edge in a stated ordering,  $p_{\gamma i} \leq q < 1$ , then  $s_\gamma \leq q^L$ . More generally, if  $p_{\gamma i} \leq q_{\gamma i}$ , then  $s_\gamma \leq \prod_{i=1}^L q_{\gamma i}$ . No independence between different candidate cycles is required.*

*Proof.* The event  $\{I_\gamma = 1\}$  is contained in the intersection of the necessary edge events. Apply the chain rule to that intersection and bound its factors individually. The additional approval event cannot increase its probability.  $\square$

A proposed path may have several conditions associated with one edge, or one shared permission serving several edges. Its *effective dependency length* may therefore differ from  $L$ . If a model instead establishes  $m(\gamma)$  ordered nonredundant dependencies, with conditional survival bounded by  $q$ , then the conclusion is  $s_\gamma \leq q^{m(\gamma)}$ . Substituting the visible number of edges for  $m(\gamma)$  without evidence would be invalid. Capacity also remains separate: the expected gross potential of a cycle depends on  $\ell(\gamma)c_\gamma s_\gamma$ , not on  $s_\gamma$  alone, where  $c_\gamma = \min_{e \in \gamma} x_e$ .

## D.2 Correlation, Shared Shocks, and Conditional Selection

Uniform conditional hazards are a substantive assumption, not an automatic implication of having many edges. For example, let  $Z \sim \text{Bernoulli}(q)$  and set  $X_{\gamma i} = Z$  for every  $i$ . Then  $s_\gamma = q$  when no other authorization is needed, irrespective of cycle length. The first observed survival reveals every remaining edge in this example, so the successive conditional probabilities equal one. An interpretation in which all edges fail or succeed with a single governance or electricity dependency has precisely this structure.

More generally, let  $Z$  encode a common infrastructural state and suppose edge survival is conditionally independent given  $Z$ , with conditional probabilities  $q_e(Z)$ . Without extra approvals,

$$s_\gamma = \mathbb{E}_h \left[ \prod_{e \in \gamma} q_e(Z) \right]. \quad (98)$$

For the mixture representation in Equation (98), with  $\mathbb{P}(Z = \text{high}) = \alpha$ ,  $q_e(\text{high}) = q_H$ , and  $q_e(\text{low}) = q_L$ , the resulting homogeneous length- $L$  reliability is  $\alpha q_H^L + (1 - \alpha) q_L^L$ , rather than  $(\alpha q_H + (1 - \alpha) q_L)^L$ . Even a small probability of a high-reliability state can dominate the long-cycle tail. This makes common-mode failures and stable common guarantees relevant to the same calculation, albeit with different operational consequences.

A second difficulty is *selection*. Suppose a platform estimates edge probabilities using only completed clearing proposals, excluding rejected, withdrawn, unrecognized or unrecorded candidates. The conditional distribution of a cycle found by that selection procedure is not necessarily the distribution of the candidate universe. Moreover, an adaptive system may search until it discovers an unusually reliable long circuit. The fact that a randomly drawn proposal has low survival probability does not bound the reliability of such a selectively chosen proposal. A valid truncation certificate must either cover the declared full candidate universe or incorporate the selection mechanism in its probability model. If eligibility is re-evaluated as an event history develops, the prediction should be recalibrated on the newly defined configuration rather than silently reused.

### D.3 Candidate Growth and the Capacity-Constrained Objective

Let  $n = |P|$ , and let  $\Delta$  be an upper bound on outgoing edge degree, counting parallel edges separately. For  $N_L$  simple directed cycles of length  $L$ , the loose combinatorial estimate  $N_L \leq n \Delta^L$  follows from counting possible rooted directed edge sequences. A finite graph has no simple cycles longer than  $n$  (apart from the trivial interpretation of parallel and self-loop edges), but for growing network families the exponential envelope remains analytically useful.

For realized indicators  $I_\gamma$ , we analyze the finite capacity-constrained *cycle-packing* problem

$$\begin{aligned} F(I) = \max_{y \geq 0} \quad & \sum_{\gamma \in \Gamma} \ell(\gamma) y_\gamma, \\ \text{subject to} \quad & \sum_{\gamma \ni e} y_\gamma \leq x_e \quad (e \in E), \quad 0 \leq y_\gamma \leq c_\gamma I_\gamma. \end{aligned} \quad (99)$$

Let  $F_K(I)$  be the same optimum with  $y_\gamma = 0$  for  $\ell(\gamma) > K$ . The objective measures the gross nominal amount of compatible claims reduced; it is neither a measure of heterogeneous social value nor proof of valid legal discharge. Unlike an unconstrained sum over possible cycles, Equation (99) does not count the same claim capacity twice in one solution. The construction assumes that every *jointly selected* family satisfying the displayed constraints is institutionally eligible. If permissions depend on a coalition, sequence or mutually exclusive authorization, additional constraints must be supplied and the results below must be reassessed.

**Proposition D.2** (Realization-wise omission inequality). *For every realized eligibility vector  $I$ ,*

$$0 \leq F(I) - F_K(I) \leq \sum_{\substack{\gamma \in \Gamma \\ \ell(\gamma) > K}} \ell(\gamma) c_\gamma I_\gamma. \quad (100)$$

*Proof.* Choose an optimizer  $y^*$  for the finite full problem. Deleting its variables belonging to cycles longer than  $K$  cannot violate a nonnegative edge-capacity constraint or another constraint in Equation (99). The shortened vector is therefore feasible for the truncated problem. The difference between the full objective and its retained part is at most the displayed sum, since  $y_\gamma^* \leq c_\gamma I_\gamma$ .  $\square$

This inequality is a general bridge between probability models and truncation error; it does not require cycles to be independent and it also holds without assigning probabilities at all. It is typically conservative because many eligible long cycles compete for the same edge capacities. Nevertheless, a conservative upper bound is preferable to treating a proposed approximation as an already enacted clearance.

#### D.4 Uniform Expected-Error and Tail-Probability Bounds

Suppose  $c_\gamma \leq \bar{c}$ ,  $s_\gamma \leq q^{\ell(\gamma)}$ , and  $r = \Delta q < 1$ , using a common compatible denomination and the model in Equation (99). Combining Proposition D.2 with the cycle-count bound yields the result previously sketched in Section 9.

**Proposition D.3** (Uniform truncation certificate). *Under the preceding assumptions, for  $K \geq 0$ ,*

$$\mathbb{E}_h[F - F_K] \leq n\bar{c} \frac{r^{K+1}((K+1) - Kr)}{(1-r)^2}. \quad (101)$$

For any  $\varepsilon > 0$ ,

$$\mathbb{P}_h(F - F_K > \varepsilon) \leq \min \left\{ 1, \frac{n\bar{c} r^{K+1}((K+1) - Kr)}{\varepsilon (1-r)^2} \right\}. \quad (102)$$

*Proof.* Take expectations in Equation (100), use  $\mathbb{E}_h I_\gamma = s_\gamma$ , and group cycles by length. The resulting upper bound is  $n\bar{c} \sum_{L>K} L(\Delta q)^L$ . Summation of the geometric derivative gives Equation (101). Markov's inequality for the nonnegative gap gives Equation (102). No independence among distinct cycle indicators is used.  $\square$

The parameter condition  $\Delta q < 1$  is sufficient but not necessary. It can be overly restrictive in heterogeneous networks, and the bound may remain numerically loose even when it holds. For example, consider a *hypothetical*, not empirically calibrated, family with  $n = 100$ ,  $\Delta = 2$ ,  $q = 0.2$ ,  $\bar{c} = 1$  monetary unit, and  $K = 12$ . Then  $r = 0.4$  and Equation (101) gives an expected gross-omission bound of approximately 0.0153 units. This is a bound on the expected difference between two optimization objectives, not a predicted real-world amount of monetary dissolution. In particular, the observation of a zero user-facing price supplies none of these parameters.

#### D.5 A Type-Aware Spectral Bound

The uniform branching factor discards information about which edges can participate in compatible continuations. A more refined model assigns an upper bound  $q_e \in [0, 1]$  to the conditional survival of each required edge in an admissible traversal ordering, so that

$$s_\gamma \leq \prod_{e \in \gamma} q_e. \quad (103)$$

The product condition is stronger than knowing the marginal survival probability of each edge; it is justified, for example, under edge independence or under appropriate sequential conditional bounds. Construct the nonnegative weighted adjacency matrix  $T \in \mathbb{R}_{\geq 0}^{n \times n}$  by

$$T_{ij} = \sum_{\substack{e: i \rightarrow j \\ e \text{ compatible}}} q_e. \quad (104)$$

The sum in Equation (104) records parallel directed edges separately. If compatibility has a transition-dependent type, one can instead enlarge the state space to position-type pairs and construct the corresponding admissible-transition matrix, with due care not to introduce false candidate cycles.

**Proposition D.4** (Weighted closed-walk certificate). Suppose Equation (103) holds and  $c_\gamma \leq \bar{c}$ . Then

$$\mathbb{E}_h[F - F_K] \leq \bar{c} \sum_{L=K+1}^n \text{tr}(T^L). \quad (105)$$

For the bound in Equation (105), if the spectral radius  $\rho(T) < 1$ , the right side is bounded by the convergent tail  $\bar{c} \sum_{L>K} \text{tr}(T^L)$ . In particular, if the induced maximum-row-sum norm satisfies  $\|T\|_\infty \leq r < 1$ , then

$$\mathbb{E}_h[F - F_K] \leq n\bar{c} \frac{r^{K+1}}{1-r}. \quad (106)$$

*Proof.* Equation (100) and linearity of expectation imply an upper bound  $\bar{c} \sum_{L>K} \sum_{\gamma \in \Gamma_L} L \prod_{e \in \gamma} q_e$ . Every directed simple length- $L$  cycle contributes exactly  $L$  rooted closed walks, of weight  $\prod_{e \in \gamma} q_e$ , to  $\text{tr}(T^L)$ . The trace also includes other nonnegative closed walks, so it dominates the simple-cycle sum. A finite nonnegative matrix with  $\rho(T) < 1$  has a convergent matrix geometric series. Finally,  $\text{tr}(T^L) \leq n\|T^L\|_\infty \leq nr^L$ , and summing yields Equation (106).  $\square$

The trace bound is potentially sharper than a network-wide  $\Delta q$  envelope. It also exposes a conceptual limitation: a low spectral radius is a property of a *typed probability-weighted claim graph*, not an implication of first homology in the separate relational event complex. The matrix can be useful for estimating computational contributions without conferring legal status on its closed walks. In a nonstationary or partially observed network, constructing  $T$  requires explicit evidence about compatibilities and conditioned probabilities.

## D.6 Deterministic Error Certificates and an Acyclic-Residual Counterexample

Probabilistic predictions are unnecessary when the question is the optimum of a fixed, already recognized and authorized compatible ledger. The edge-based linear programme of Appendix C maximizes  $\mathbf{1}^\top z$  subject to  $Bz = 0$  and  $0 \leq z \leq x$ . Any feasible truncated cycle packing gives a feasible reduction  $z^{(K)}$  with value  $F_K = \mathbf{1}^\top z^{(K)}$ . For any dual-feasible  $(y, u)$  satisfying  $B^\top y + u \geq \mathbf{1}$ ,  $u \geq 0$  (Equation (90)), weak duality supplies a deterministic bound

$$0 \leq F - F_K \leq x^\top u - F_K. \quad (107)$$

In Equation (107),  $F$  is the optimum over a subset of the unrestricted feasible polytope, so  $x^\top u$  remains a valid upper bound. This certificate may be loose when entire circuits require separate permissions, but it does not depend on estimated survival probabilities or cycle enumeration. The primal–dual gap identity in Proposition C.6 gives an exact optimality certificate only when the candidate is feasible for the declared linear programme and the dual constraints match that same programme; a certificate for an algebraic superproblem is generally an upper bound, not a proof of optimality for an institutionally restricted subset. If the right side is zero within exact arithmetic or controlled solver tolerance,  $F_K$  is optimal for the declared feasible problem.

One tempting shortcut must be rejected. Consider five claims of unit capacity,  $A \rightarrow B$ ,  $B \rightarrow A$ ,  $B \rightarrow C$ ,  $C \rightarrow D$ , and  $D \rightarrow A$ . Canceling the short cycle  $A \rightarrow B \rightarrow A$  reduces gross claims by 2. The remaining positive edges form the acyclic path  $B \rightarrow C \rightarrow D \rightarrow A$ . Nevertheless, the alternative long cycle  $A \rightarrow B \rightarrow C \rightarrow D \rightarrow A$  cancels 4 gross units. The acyclicity of a *particular residual* is thus insufficient to certify that the chosen reduction was maximal. This differs from the assertion, proved in Appendix C, that a maximum-gross unrestricted reduction *has* an acyclic positive residual. An implication about an optimizer must not be reversed into a criterion for optimality.

## D.7 Cutoff Selection and Robust Sensitivity Analysis

The error inequalities become operational only when they guide a reproducible cutoff rule. Suppose a decision maker declares an absolute tolerated gross-clearance error  $\varepsilon > 0$  and an allowable model-based

exceedance probability  $\alpha \in (0, 1)$ . A sufficient, prospective rule is to choose the least integer  $K$  for which the applicable upper bound  $U_K$  satisfies

$$U_K \leq \alpha \varepsilon \implies \mathbb{P}_h(F - F_K > \varepsilon) \leq \alpha. \quad (108)$$

The guarantee in Equation (108) inherits the assumptions of the selected bound. It is a statement about the uncertainty model's distribution, not a legal tolerance for leaving legitimate claims unresolved. The user of an automated clearing procedure should also disclose the units and the effects on particular claim holders. If the objective  $F_K > 0$ , an eventwise deterministic certificate  $x^\top u - F_K \leq \tau F_K$  additionally implies the relative guarantee  $(F - F_K)/F \leq \tau$ , but dividing by a near-zero objective is often uninformative. Absolute tolerances are usually easier to interpret across disconnected parts of a network.

Parameter uncertainty should be propagated rather than suppressed. If  $q$  is estimated only to lie in an interval  $[q_-, q_+]$ , the uniform bound can safely use  $q_+$  *provided the conditional-survival assumption is justified for that upper endpoint*. When  $\Delta q_+ \geq 1$ , the geometric-series certificate does not establish a decaying tail. An investigator may then use the finite cycle counts, the weighted transition matrix with a validated spectral bound, a deterministic dual upper bound, or explicitly acknowledge that the desired tolerance is unsupported. Reducing  $K$  merely to satisfy a computing budget does not make the omitted contribution small.

There is a further distinction between choosing  $K$  before examining realization-specific outcomes and selecting it after discovering which cycles succeeded. An adaptive algorithm can use validated, simultaneous deterministic upper bounds that remain true after selection; an ex ante probability certificate for a fixed  $K$  cannot automatically be asserted for a data-dependent  $K$  chosen with unmodeled selection bias. An uncertainty-sensitive implementation should predeclare its decision rule, report sensitivity to perturbations of  $q$ ,  $\Delta$  and capacities, and assess coverage on held-out or separately audited opportunities. This procedure treats model revision as an epistemic task, consistent with the evolving provenance requirements of the wider research programme.

## D.8 Structural Robustness and the Limits of Short-Cycle Preference

Fragility of one specified route does not imply fragility of the attainable clearing function. Multiple alternate cycles can preserve access to some reductions after an edge failure; many nominally distinct cycles can also share a single vulnerable cut, institution, or upstream resource. Minimal directed cuts, overlapping-edge exposure, approval bottlenecks and dependence among failures are therefore at least as relevant as path length when evaluating operational resilience. A model that replaces all these features with a single edge-survival parameter can conceal precisely the cross-boundary dependencies that Section 11 requires us to analyze.

The representational separation in Appendix B remains essential. Attaching a two-cell to a relational event complex may change  $H_1$  without changing which obligations exist or how they can legally be canceled. Conversely, a creditor's permission or a public multilateral arrangement may authorize a new clearing operation without changing the homotopy type of the event representation. The mathematical robustness of a cycle space, the operational redundancy of a claim graph, and the legitimacy of a settlement institution must be assessed separately.

Length cutoffs may also distribute computational visibility unequally. A peripheral cooperative may participate mainly in longer circuits, whereas a well-connected financial intermediary has access to many short routes. A small economy-wide expected omitted amount could conceal a large loss of opportunity for the peripheral participant. A group-specific examination can restrict the tail sum in Equation (100) to cycles touching a declared constituency and report exposure along with global error. Such a report describes neglected *monetary opportunities*; it does not substitute for the independent assessments of participation, recognition, remedy, and ecological effects developed in Section 12.

Table 19: Model assumptions, falsification checks, and interpretation limits for truncation.

| Assumption or tool              | Computational contribution                 | Limiting test or question                      |
|---------------------------------|--------------------------------------------|------------------------------------------------|
| Typed compatible claims         | Defines an admissible numerical ledger     | Are denomination and setoff rules verified?    |
| Conditional survival controls   | Bounds one cycle’s success probability     | Are shared shocks and selection recorded?      |
| Cycle-count / capacity envelope | Bounds aggregate omitted gross opportunity | Do path numbers or capacities grow faster?     |
| Weighted transition matrix      | Refines typed candidate growth             | Are edge-product bounds justified?             |
| Cycle-packing feasibility       | Prevents repeated use of one claim         | Do group approvals change joint admissibility? |
| LP dual certificate             | Bounds fixed-ledger optimality gap         | Is the institutional model a subset of the LP? |
| Group-specific omitted exposure | Discloses unequal effects of cutoff        | Whose opportunities remain invisible?          |
| Enacted discharge record        | Separates computation from settlement      | Was authorization actually exercised?          |

## D.9 Reproducibility Protocol and Assumption Ledger

A reproducible experiment should archive the compatible claim types and capacities, network boundary, eligibility rules, event-history configuration, population of candidate cycles, reference information  $\mathcal{J}_h$ , and uncertainty model. The investigator should report how probabilities were estimated, whether common shocks or adaptive discovery were included, how long-cycle capacity bounds were obtained, and which claims were left out by a computational cutoff. For simulated data, random seeds and distributional assumptions should be supplied; for observed data, rejection records and missingness mechanisms are needed to understand selection bias. The calculation should independently verify primal feasibility, any applicable dual certificate, and the stated discrepancy between full and truncated clearing objectives.

For a reported expected upper bound  $U_K$ , the associated confidence claim must remain conditional on the declared probability model. For a reported deterministic dual upper bound, the claimed scope is the fixed reference ledger and its recognized constraints. Neither certificate provides information about unrecorded ecological impacts, the social value of participation, or the emergence of new claims on a subsequent relational continuation. The positive result is therefore specific: *under auditable dependency, branching, and capacity assumptions, longer circuits can have sufficiently small aggregate expected contribution to support an error-controlled computational cutoff*. This makes path fragility an occasional source of tractability, not a historical requirement that longer relationships or exchanges disappear. Appendix E will examine how the abstract objects behave in hypothetical human–AI production networks.

## E Illustrative Human–AI Production Networks

This appendix tests the interpretive boundaries of the formal framework through deliberately synthetic networks. The illustrations are neither empirical descriptions of current AI firms nor predictions of an inevitable economy. Their role is to make visible the differences among productive contribution, institutional recognition, payment at an interface, gross obligation reduction, and the continued reproduction of productive conditions. The distinction between AI-operated activity and a legally recognized claimant is especially important. An automated service may execute transactions while the relevant rights and liabilities belong to a corporation, public agency, cooperative, individual, or other recognized position. The numerical examples therefore use *positions* in an obligation ledger and *roles* in a production network rather than attributing legal personhood to software.

Appendix A specifies the typed relational-history and obligation representations; Appendix B explains why an event-space cycle and a monetary cycle are mathematically distinct; Appendix C provides the conditional clearing polytope and its residual bounds. Here we place the representations beside one

another in small scenarios. The earlier thought experiments on AI-operated production networks, distributed monetary dissolution, and value-circuit grammars motivate the scenarios, but their hypothetical premises remain open to revision [12, 14, 7, 11].

## E.1 Representational Protocol and Declared Assumptions

Fix a represented relational history  $h$  and a study boundary  $U \subseteq P_h$  of recognized claim-bearing positions. Let  $R_h$  be the set of production roles, with a possibly many-to-many affiliation map  $\rho_h : R_h \rightrightarrows U$ . For each role  $r$ , an input-dependence record  $d_h(r)$  specifies the conditions of production, including electricity, compute, sensor observations, maintenance, material inputs, permission to use a site, and relevant ecological exposure. A distinct typed claim graph  $G_h^\$ = (U, L_h)$  contains only obligations that the study recognizes as monetary at  $h$ ; its incidence matrix is  $B_h$ , its nonnegative amount vector is  $x_h$ , and a candidate cancellation is  $z_h$ . A study must not insert a productive dependency as a monetary edge merely because it appears in  $d_h(r)$ .

The following conventions apply to every case. First, all monetary amounts are artificial units of one compatible settlement type unless expressly stated otherwise. Second,  $u \rightarrow v$  denotes an obligation owed by position  $u$  to position  $v$ ; it does not by itself mean a transfer of labour, energy, knowledge, or ecological benefit in that direction. Third, reductions with  $B_h z_h = 0$  and  $0 \leq z_h \leq x_h$  are *accounting candidates*. Enacted discharge requires separately documented authority, consent or a valid substitute, legal or organizational recognition, and evidence of execution. Fourth, a statement that the user interface requires no payment refers only to the declared interface; it says nothing by itself about public finance, implicit subsidies, mandatory contributions, upstream credit, or material dependence. Finally, the examples use a partially ordered collection of institutional and productive events rather than an absolute, common clock.

For a boundary  $U$ , define a deliberately limited observable monetary interface measure

$$I_U(h) = \sum_{\ell \in L_h^{\text{interface}}} x_{h,\ell}, \quad G_U(h) = \sum_{\ell \in L_h^{\text{upstream}}} x_{h,\ell}, \quad (109)$$

In Equation (109), the interface and upstream sets are declared by the investigator. Both expressions are *gross recorded obligations* in the selected ledger, not necessarily completed cash payments or a full measure of economic value. They may overlap if roles and boundaries are not carefully defined; examples below choose disjoint sets to avoid double counting. An observed  $I_U(h) = 0$  is consistent with  $G_U(h) > 0$ . Conversely, small  $G_U(h)$  does not imply ecological autonomy or legitimate governance.

## E.2 Human–Human Encounters, Membership, and Nonmonetary Access

Consider a hypothetical cooperative learning circle in which participants need one another’s specialized knowledge, criticism, mentorship, and opportunities for joint investigation. Under one institutional configuration, the organizer charges an entry fee to finance a rented room, administration, and accessibility services. The fee is monetary mediation of *access to a relational setting*; neither the recognition of membership nor the capacity to co-create knowledge can be equated with the fee itself. Under a second configuration, a public library supplies a meeting room, members contribute time, and access is granted without individual payments. The needs for encounters and cooperative activity persist in both configurations. The two arrangements differ in how the conditions of encounter are reproduced and how claims are recognized, not in whether the relational good exists.

To isolate the accounting question, suppose a local library association  $L$  owes a room-maintenance cooperative  $M$  six units,  $M$  owes a training organizer  $T$  five units, and  $T$  owes  $L$  four units. If the three claims form an authorized, compatible circuit, a four-unit cycle component can be extinguished, leaving  $L \rightarrow M$  with two units and  $M \rightarrow T$  with one unit. The gross ledger falls from fifteen to three units, while members may continue to meet, teach, and share outputs. If the training organizer is not subject to a common clearing arrangement, the same numerical loop gives no unilateral authorization to cancel.

This example links the need to meet others with a possible monetary gate, while keeping the fee, the obligation circuit, and the productive encounter distinct.

The comparison also cautions against one inference: changing an entrance fee to zero does not demonstrate that all participants can participate. Geographic accessibility, language, caregiving time, invitations, and institutional recognition may remain restrictive even when monetary barriers are removed. Conversely, a transparent membership payment may finance facilities or safeguards valued by participants. Whether a configuration supports generative value circulation and justice therefore requires an independent account of recognition, access, participation, and effective contestability, rather than a scalar judgement derived from the amount paid [23].

### E.3 A Free Human–AI Interface with Monetized Upstream Production

Let  $H$  denote human users of a no-fee AI service,  $S$  the recognized position operating that service,  $E$  an electricity-provision position, and  $C$  a computing-infrastructure position. The service may perform valuable work for  $H$  without generating a recognized monetary obligation  $H \rightarrow S$ . Nevertheless,  $S$  depends on electricity, computing equipment, replacement parts, data acquisition, and maintenance. The dependence record includes those inputs whether or not they have prices. Imagine the compatible *upstream* monetary ledger

$$S \xrightarrow{12} E, \quad E \xrightarrow{10} C, \quad C \xrightarrow{8} S, \quad H \xrightarrow{0} S. \quad (110)$$

The direction of the  $C \rightarrow S$  edge records an assumed financial or service obligation of the compute operator to the service position; it is not inferred from electricity or compute flow. A claim holder and reason for every edge would have to be identified in an actual study.

With  $x = (12, 10, 8)^T$  in cycle order, the reduction  $z = (8, 8, 8)^T$  is algebraically feasible. It gives

$$r = x - z = (4, 2, 0)^T, \quad \mathbf{1}^T x = 30, \quad \mathbf{1}^T z = 24, \quad \mathbf{1}^T r = 6, \quad Bz = 0. \quad (111)$$

For the obligations in Equation (111), under the debtor-negative and creditor-positive incidence convention,  $S$ ,  $E$ , and  $C$  have net positions  $-4$ ,  $+2$ , and  $+2$ , respectively, both before and after the candidate clearing. If the appropriate institutions authorize and enact the reduction, gross claims decline by twenty-four monetary units. None of these results contradicts the fact that  $H$  paid zero at the interface *even before* the clearing. Moreover,  $S$  continues to require electricity, computation, maintenance, and other inputs. They have not been physically supplied merely by canceling eligible liabilities.

This scenario has two analytical uses. It demonstrates *interface invisibility*: direct user payments can vanish while upstream monetary mediation remains substantial. It also demonstrates that partial circular clearing need not remove the service's residual net liability. An empirical inquiry would need separate records for money owed, money actually paid, public transfers, contractual rights, and physical resource consumption. The zero-fee interface identifies neither a particular ownership regime nor an economy-wide decline in monetary dependence, as established more generally by Proposition A.8.

### E.4 AI–AI Division of Labour, Credit, and Clearing Eligibility

Now consider three AI-operated productive roles: sensing and validation  $A$ , physical execution  $B$ , and specialized analysis  $C$ . Each depends on capabilities supplied by the other roles. They may be operated by one organization, by separately incorporated firms, by autonomous public facilities, or by a cooperative federation. Inter-role dependence is not identical to inter-position exchange: if one operator owns all three systems, internal cost allocations need not create enforceable claims among independent creditors and debtors. If three different operators control the roles, recognized fees or credit arrangements may arise, but their form remains institutionally contingent.

For one explicitly hypothetical common-denomination ledger, suppose the corresponding positions owe

$$A \xrightarrow{9} B, \quad B \xrightarrow{7} C, \quad C \xrightarrow{5} A. \quad (112)$$

Table 20: Hypothetical monetary observations under different human–AI production arrangements; amounts are stipulated gross claims, not real data.

| Configuration                     | User–service claims | Upstream claims | Unobserved or distinct obligations                                       |
|-----------------------------------|---------------------|-----------------|--------------------------------------------------------------------------|
| Fee-funded service                | 6                   | 15              | Material throughput, liability to communities, and nonmonetary value     |
| No-fee corporate service          | 0                   | 15              | Financing, attention or data rights, and environmental incidence         |
| Publicly supplied service         | 0                   | 4               | Tax or budget arrangements, maintenance work, and ecological limits      |
| Nonmonetary production federation | 0                   | 0               | Physical inputs, participation rights, energy burdens, and repair duties |

For the claims in Equation (112), the compatible cycle reduction  $z = (5, 5, 5)^T$  leaves  $(4, 2, 0)^T$ . Gross claims fall from 21 to 6, with net positions  $(-4, +2, +2)$  unchanged. The computed outcome does not establish that the three roles possess human-equivalent desires, rights, or capacities for consent; these issues concern the institutions authorized to represent the positions and the people or collectives affected by them.

Three controlled changes separate technological intelligence from clearing feasibility. If the  $C \rightarrow A$  claim is denominated in a nonconvertible token, the numerical triangle is not one compatible monetary cycle unless an authorized conversion mechanism exists. If the claim is contingent on a sensor-verification event that has not been established in the represented history, its amount is not currently a fixed recognized claim. If  $C$  has contracted for a prohibition on multilateral setoff, a mathematical cancellation vector need not be institutionally admissible. Thus superior forecasting and communication can improve discovery of *possible* reciprocal structures without dissolving constraints imposed by contracts, accountability, asymmetric bargaining power, or scarce production conditions.

AI–AI monetary intermediation may temporarily expand when complex roles move from a common institution into separately governed organizations, even as direct human user payments fall. Conversely, interoperable public facilities could coordinate the same productive contributions through nonmonetary scheduling or reciprocal commitments. Neither arrangement follows solely from an increase in model capability. The conditional character of this comparison prevents the proposed H–H, H–AI, and AI–AI order of monetary dissolution from becoming an imposed universal sequence.

## E.5 Boundary Sensitivity and the Apparent Sequence of Dissolution

To illustrate the dependence of empirical conclusions on observation boundaries, consider four *independent counterfactual configurations* providing an otherwise comparable AI-assisted information service. Table 20 reports declared gross obligations in artificial units. The numbers are stipulated for interpretive testing; they are not observations, estimates, forecasts, or time-series transitions. The upstream figure aggregates only recorded compatible monetary claims inside the declared production network; unrecorded public financing, nonmonetary contributions, and ecological effects appear separately.

At the user–service boundary, the last three configurations are observationally indistinguishable if only monetary claims are counted. At the wider monetary-supplier boundary they are not; yet none of these comparisons determines whether energy is renewable, whether materials can be reproduced, whether the facility enjoys legitimate permission to operate, or whether human labour is fully recognized. The same visible “free AI” condition is compatible with a highly monetized upstream network and a genuinely nonmonetary provision arrangement. This is a *measurement identification* problem, not a claim that one configuration already exists.

One can generalize this exercise by nesting boundaries  $U_1 \subset U_2 \subset \dots \subset U_m$  and defining a profile of monetary observations, rather than a single aggregate demonetization rate. A change from an  $H$ -only boundary to the whole supply network may reverse an apparent decline. It may also reveal shifts from payment to credit, licensing, public financing, or access permission. Monetary migration is therefore a

possible empirical mechanism, but it must be demonstrated by observing the relevant claims and institutions. The relative order in which different monetary interfaces diminish is not by itself evidence that all monetary functions will ultimately vanish.

## E.6 Ownership and Institutional Counterfactuals on a Shared Technology

Hold fixed the hypothetical physical production topology: a sensor installation observes ocean conditions, a computing service processes the data, an energy installation powers the equipment, and humans or institutions use the resulting information. No claim is made that marine sensing should serve underwater data centres; ocean observation and ecological learning have independent scientific ends. The same material topology could be embedded in materially different governance and monetary settings.

In a commercial service arrangement, the infrastructure's recognized operators may charge subscription fees, buy electricity under contracts, license observations, borrow funds, and distribute returns to holders of financial claims. In a publicly operated arrangement, access may be free or subsidized at the point of use, with construction and maintenance financed through budgets and procurement. In a cooperative or commons-based arrangement, contributors may share data and equipment under common rules, with reciprocal labour and resource commitments organized partly outside monetary exchange. Any of these forms can also include monetary purchases, exclusions, inadequate participation, or environmental harms; the legal label does not settle those questions.

We may represent this separation schematically as

$$\mathcal{C}_h = (\mathcal{P}_h, \mathcal{O}_h, \mathcal{A}_h, \mathcal{K}_h, \mathcal{E}_h), \quad (113)$$

In Equation (113),  $\mathcal{P}_h$  is the production-dependence structure,  $\mathcal{O}_h$  the recognized monetary obligation structure,  $\mathcal{A}_h$  the allocation and access rules,  $\mathcal{K}_h$  the governance and contestation arrangements, and  $\mathcal{E}_h$  recorded ecological effects. Two configurations can share  $\mathcal{P}_h$  while differing in the other components. Alternatively, they may share the same gross monetary clearing potential while placing control over productive resources in substantially different hands.

This contrast complements the discussion of control over production conditions in Sections 5 and 12. It also helps avoid treating organizational form as a proxy for ethics. We must ask who provides productive contributions, who defines the eligible exchange, who can refuse or revise terms, who bears losses and ecological impacts, and which participants can contest the arrangement. A value circulation pattern that appears balanced in a monetary ledger may remain asymmetric in its access to decision-making and means of production.

## E.7 Infrastructure Autonomy and Ecological Externality Tests

Suppose the AI-operated information service is supplied energy by an automated plant, uses internally maintained computing equipment, and accesses observation data through a global inter-operable network. The service might have low dependence on *market purchases* along several inputs while retaining dependence on physical infrastructures. We should test at least three different properties: functional autonomy of the application, supply autonomy within a chosen boundary, and the capacity to reproduce productive conditions under equipment wear, disruptions, and ecological constraints. Their distinction was established in Appendix A and Section 11.

A hypothetical autonomous ocean-observation fleet makes this issue especially clear. A robot may map an unobserved seabed and feed new data to scientific models. Its value is not exhausted by subscriptions, sensor prices, or the monetary volume of claims surrounding its operation. Its battery production, recovery procedures, interference with habitats, maintenance access, and contested data governance must all be considered. Clearing the fleet's procurement debts cannot erase a habitat disturbance or imply that an affected community endorsed the deployment. By the same reasoning, an AI infrastructure that produces its own electricity may still impose emissions, materials demand, land use, or ecosystem burdens. Physical self-provision and monetary nondependence are not identical to ecological independence.

Table 21: Illustrative dependency questions that monetary observations do not settle.

| Domain          | Observable productive dependency                      | Additional institutional or ecological question                  |
|-----------------|-------------------------------------------------------|------------------------------------------------------------------|
| Electricity     | Generation, storage, grid coupling, replacement parts | Who controls supply, and who bears emissions or thermal impacts? |
| Computing       | Processor supply, repair, cooling, operations         | Can hardware and critical components be reproduced?              |
| Data            | Sensing, calibration, validation, provenance          | Who may access, correct, restrict, or contest records?           |
| Material inputs | Minerals, equipment, transport, end-of-life treatment | Who bears extraction, waste, and habitat effects?                |
| Labour and care | Skilled maintenance, emergency response, supervision  | Are contributors recognized, supported, and able to exit?        |
| Permissions     | Land or sea use, access rights, liability             | Which affected communities authorize or challenge the facility?  |

A useful synthetic experiment would therefore compare a financial clearing objective  $F_h$  with separately retained heterogeneous indicators  $e_h = (e_h^1, \dots, e_h^k)$  for ecological conditions, without converting all ecological dimensions into one automatically fungible monetary unit. Numerical claims about the magnitude of an ecological effect would require physical models and measurements, not the monetary incidence matrix. Any proposed remedy would further require institutions capable of recognizing relevant interests, including interests that cannot bargain as ordinary account holders.

## E.8 History-Dependent Clearing and Regenerated Claims

The previous numerical cases represent claims in one declared configuration. To illustrate spacetime distribution without assuming a privileged global clock, let  $h \rightsquigarrow h'$  represent an admissible continuation in which a specific set of institutional events occurs. Suppose the three upstream operators in Equation (110) have compatible recorded claims at  $h$ , all required approvals are validly recorded along a continuation, and a settlement event discharges the candidate component  $z$ . A later or otherwise continuation-related production event may then give rise to a new, legally recognized obligation  $n_{h'}$ . The transition can be represented by

$$x_{h'} = T_{h \rightsquigarrow h'}(x_h - z_h) + n_{h'}, \quad (114)$$

In Equation (114),  $T_{h \rightsquigarrow h'}$  transfers the residual claims into the new position and type system under declared mapping rules. The expression assumes an additive ledger model and is not an ontological assertion that the entirety of future economic reality follows from a state equation.

A simple numerical continuation starts with the upstream residual vector  $(4, 2, 0)^\top$  and, on a fixed three-edge representation, recognizes a new  $C \rightarrow S$  claim of seven units. Then  $x_{h'} = (4, 2, 7)^\top$ , and an additional compatible cycle of two units per edge becomes mathematically available. The original clearing did not fail: it discharged the previously eligible eight-unit cyclic component under the stated hypothesis. The new capacity for clearing arises from a new recognized claim, illustrating how monetary dissolution and monetary regeneration may be historically coupled. If institutional rules make the new claim incompatible with the old residual claims, the second numerical loop is not an admitted clearing opportunity.

In a network where obligations are not simultaneous, the importance of partial ordering becomes stronger. An anticipated credit may be admissible only after a production or verification event; an authorization may need to occur before discharge; ecological observation may reveal impacts after a procurement contract has been performed. Some events will be incomparable in the available representation. “Intertemporal” here refers to dependencies and admissible continuations, not to a universal chronological axis on which every exchange can be placed. An expected future reciprocity cannot be treated as an already discharged debt merely because an optimizer includes it as a forecast.

Table 22: Illustrative hypotheses, discriminating observations, and challenges to interpretation.

| Hypothesis                                        | Potential discriminating observation                          | Principal alternative explanation                          |
|---------------------------------------------------|---------------------------------------------------------------|------------------------------------------------------------|
| Direct H–AI monetary dependence declines          | Consistent reduction in recognized user–provider payments     | Payment shifted upstream or onto public budgets            |
| AI–AI obligations persist or expand               | Audited contracts between distinct legal positions            | Internal accounting entries or double counted transactions |
| Circular clearing reduces gross claims            | Verified compatible netting with preserved net positions      | Ledger reclassification without actual discharge           |
| Longer circuits are less operationally viable     | Conditional survival evidence by length and risk class        | Shared shocks, selection, or path-count growth             |
| Public infrastructure reduces monetary dependence | Expanded nonmonetary access plus verified supply arrangements | Hidden cross-subsidy or exclusive control elsewhere        |
| Reduced monetary claims accompany ecological care | Independent monitoring and enforceable corrective practices   | Omitted harms or value translated into inadequate proxies  |

## E.9 Comparison Protocol, Falsification Criteria, and Limits of the Examples

The hypothetical cases indicate which claims should be tested rather than announcing empirical confirmation. A reproducible field or simulation study could compare institutionally verified obligations, executed payments, productive dependencies, contractual authorizations, and changes in access across specified relational histories. For each observed configuration, it should archive (i) the unit and type of every monetary claim; (ii) the identity or legal status of the position bearing it; (iii) evidence that a candidate clearing was enacted; (iv) the boundary selected for observation; (v) the infrastructure inputs used to reproduce service; and (vi) nonmonetary value and ecological indicators measured independently. Changes in legal rights, public transfers, hidden subsidies, and missing contracts should be reported as identification limitations rather than silently interpreted as dissolution.

Some versions of the proposed relational ordering could be challenged by finding contexts in which H–H payments intensify while AI–AI exchange is successfully organized through common provision, or in which H–AI access remains monetized despite high operational autonomy. These findings would not invalidate the algebra of compatible cycle clearing; they would limit a particular economic hypothesis about its social distribution. Likewise, observed cycles containing many nominally distinct edges would not support tractable truncation unless conditional survival rates, correlations, and aggregate cycle counts justified the error bounds in Appendix D.

The appendix’s main contribution is thus methodological. Its examples separate the mathematical possibility of accounting reduction from the historical and institutional transformation of monetary dependence, and separate both from broader claims about value, trust, autonomy, and ecological justice. They illustrate how a human–AI economy could become less visibly monetary at some interfaces while monetary obligations remain, circulate, or emerge elsewhere. They also illustrate that the same physical production arrangements can be governed through different combinations of fees, credit, public supply, commons rules, and negotiated reciprocal commitments. No single numerical example entails an inevitable postmonetary destination; the research object is the contingent, spacetime-distributed development of monetary mediation within continuing relational production.

## F Alternative Institutional and Governance Scenarios

This appendix studies a distinction that becomes especially important when AI-operated production networks acquire substantial technical autonomy: the same productive equipment and the same possible accounting reductions can exist within markedly different rules of ownership, participation, consent, and public responsibility. An apparently identical decline in direct monetary payments may result from free public provision, a private intermediary’s cross-subsidy, cooperative reciprocal allocation, or simply a change in the boundary at which payments are observed. Conversely, a mathematically available cycle

may be institutionally inadmissible even when all its component obligations are denominated in the same unit. The examples below are counterfactual analytical instruments. They are not descriptions of any existing country's law or predictions of the legal status of autonomous AI agents.

Appendix C establishes the basic claim-ledger optimization and distinguishes a proposed cancellation from authorized and enacted discharge. Appendix E constructs hypothetical Human–Human, Human–AI and AI–AI production settings. The present appendix holds selected productive and accounting features fixed while varying institutions. This choice exposes what a theory of monetary dissolution would miss if it inferred social transformation from network topology or monetary net positions alone. The broader ethical discussion in Section 12 and the infrastructure discussion in Section 11 provide the interpretive context. Earlier studies of credible commitments and public-knowledge production also motivate attention to contestability and control without deciding the institutional comparison in advance [8, 10, 23].

## F.1 Scenario Construction and the Counterfactual Boundary

Consider a declared local relational configuration  $h$  with: (i) a set of productive roles  $R_h$ ; (ii) equipment, energy, data, material and maintenance requirements  $D_h$ ; (iii) a set of recognized claim-bearing positions  $P_h$ ; (iv) a compatible ledger  $x_h \geq 0$  with creditor-positive incidence matrix  $B_h$ ; (v) authorization rules  $\iota_h$ ; and (vi) separate records of access, contributions, ecological exposure and available remedies, denoted  $\eta_h$ . A technical-production counterfactual can hold  $R_h$  and  $D_h$  fixed while modifying  $\iota_h$  and the allocation of claims or control. A *fixed-ledger* clearing comparison additionally fixes  $P_h$ ,  $B_h$  and  $x_h$ . These are different exercises; changing ownership or credit institutions may themselves change the ledger and make fixed-ledger numerical comparisons inapplicable.

We use four idealized institutional configurations: proprietary platform, member-governed cooperative, public-purpose operator, and federated transnational arrangement. None is internally homogeneous. A publicly owned facility can charge usage fees; a cooperative can restrict access; a proprietary system can open its interfaces; and an international network can contain locally autonomous nodes. The types therefore identify bundles of rules to be specified rather than moral rankings or inevitable economic outcomes. The counterfactual protocol requires the researcher to name the rule bundle instead of deriving it from the institutional label alone.

Let  $\mathcal{Z}(x; B) = \{z : 0 \leq z \leq x, Bz = 0\}$  be the algebraic feasible set. Under a declared institutional arrangement  $\iota$ , define the admissible clearing set

$$\mathcal{A}_\iota(h) = \{z \in \mathcal{Z}(x_h; B_h) : \Phi_\iota(z, h) = 1\}, \quad 0 \in \mathcal{A}_\iota(h), \quad (115)$$

In Equation (115),  $\Phi_\iota$  represents a modeled rule for authority, claim compatibility, consent or another valid source of permission. The displayed inclusion of zero is a simplifying assumption for a no-action option. It does not imply that a particular organization is legally free to ignore an already binding clearing obligation. For a strictly positive monetary weight vector  $w$ , the candidate reduction value is

$$V_\iota(h) = \max_{z \in \mathcal{A}_\iota(h)} w^\top z, \quad (116)$$

provided the set is compact and the objective continuous, as assumed for Equation (116). The remainder of the analysis uses  $w = \mathbf{1}$  when measuring canceled gross nominal claims. A change in  $V_\iota$  is not, by definition, a change in welfare, trust, democratic control, or ecological quality.

## F.2 Ownership, Authorization, and the Control of Productive Conditions

Consider an AI service maintained by four productive roles: an energy operation, a data or observation operation, a computing operation, and a delivery or maintenance operation. Ownership rights over the facility, the right to determine admission rules, authority to issue credit, and authority to cancel claims can be held by different recognized positions. Even where all four roles are automated, their obligations need not be claims against four independently constituted AI persons. Depending on the represented institutional structure, the four roles may be departments of one legal organization, contracted organizations,

public entities, or members of a cooperative. A fully internal ledger may be a management-accounting convention rather than independently enforceable monetary debt.

A proprietary platform configuration may place admission, infrastructure ownership and claim recognition under a concentrated operator. That arrangement can create fees for access, paid priority, and credit extended to suppliers, but it may also choose free access for strategic reasons. A cooperative configuration may allocate participation rights to contributing members and authorize clearing by agreed rules; it may nevertheless use money for expensive replacement equipment, externally sourced energy, or obligations to nonmembers. A public-purpose configuration may supply access through appropriated resources, taxation or contributions; zero user charges then coexist with externally recognized financing and material costs. A federated configuration may share data and inference services across multiple authorities while retaining local custody, eligibility conditions and institutional heterogeneity.

For each configuration, the research record should separate *productive autonomy* from *control autonomy*. A network able to produce electricity, collect observations, and maintain equipment with little routine human labor can remain dependent on access permits, a dominant creditor, property titles, chip supply, and external disposal facilities. Thus the claim that Human–AI exchange becomes less monetized as AI reproduces its production conditions is intelligible only together with the distribution of rights over those conditions. The source of public financing and the decision rights of affected nonusers must also be recorded; otherwise interface-level “free” provision can conceal another person’s monetary burden.

### F.3 A Fixed-Ledger Four-Position Comparison

To isolate institutional admissibility, use four artificial claim-bearing positions  $N, D, C, S$ , which may be read as energy, data, computing and service organizations. The role labels are mnemonic only. All amounts are hypothetical units of one stipulated compatible claim class, and each directed edge represents a debt owed by its tail to its head. Define

$$N \xrightarrow{9} D, \quad D \xrightarrow{7} C, \quad C \xrightarrow{6} S, \quad S \xrightarrow{8} N. \quad (117)$$

The initial gross obligation is  $9 + 7 + 6 + 8 = 30$ . Under the creditor-positive incidence convention, the ordered net-position vector is

$$b = Bx = (-1, 2, 1, -2)^\top, \quad \mathbf{1}^\top b = 0. \quad (118)$$

The graph in Equation (118) has one directed simple cycle, so every nonnegative balance-preserving reduction is  $z(t) = t(1, 1, 1, 1)^\top$  with  $0 \leq t \leq 6$ . Consequently

$$G(t) = \mathbf{1}^\top z(t) = 4t, \quad r(t) = (9 - t, 7 - t, 6 - t, 8 - t)^\top, \quad \mathbf{1}^\top r(t) = 30 - 4t. \quad (119)$$

For the residual in Equation (119), stipulate four *model rules* governing exactly this fixed ledger. A permission rule in the proprietary scenario caps common clearing at  $t \leq 2$ ; a cooperative rule allows member consent up to  $t \leq 5$ ; a public-purpose rule authorizes the full compatible cycle  $t \leq 6$ ; and a fragmented federation excludes the  $C \rightarrow S$  claim from the common clearing procedure, so the only eligible balance-preserving four-edge reduction is  $t = 0$ . These caps and exclusions are chosen for illustration. They are not generic attributes of ownership types or claims about any jurisdiction.

Every case preserves the same nominal net positions  $b$ . The comparison establishes only that different institutional constraints can generate different maximum *candidate* clearing reductions on the same obligations. It cannot rank these institutions overall: a more restrictive rule might preserve a non-waivable protection, prevent a disputed offset, or signal a weakness in data or authorization. Similarly, a full 24-unit candidate reduction is not evidence that discharge occurred, that all creditor interests were treated fairly, or that the physical production system has become self-sufficient.

**Proposition F.1** (Institutional feasibility monotonicity under fixed claims). *Let  $\mathcal{A}_{l_1}(h)$  and  $\mathcal{A}_{l_2}(h)$  be nonempty compact subsets of the same  $\mathcal{Z}(x_h; B_h)$ , with identical objective vector  $w$ . If  $\mathcal{A}_{l_1}(h) \subseteq \mathcal{A}_{l_2}(h)$ , then  $V_{l_1}(h) \leq V_{l_2}(h)$ .*

Table 23: Synthetic institutional counterfactuals for a fixed 30-unit ledger. “Authorized” refers solely to the hypothetical rules stipulated in the text.

| Configuration                  | Modeled eligibility rule               | $t^*$ | Gross reduced | Gross residual |
|--------------------------------|----------------------------------------|-------|---------------|----------------|
| Proprietary permission gate    | Shared clearing capped at $t \leq 2$   | 2     | 8             | 22             |
| Member-governed arrangement    | Participant permissions cap $t \leq 5$ | 5     | 20            | 10             |
| Public-purpose settlement rule | Full cycle can be canceled, $t \leq 6$ | 6     | 24            | 6              |
| Fragmented federation          | One required edge is ineligible        | 0     | 0             | 30             |

*Proof.* Every feasible candidate for the first maximization is feasible for the second and has the same objective value in both problems. Maximization over the superset cannot produce a smaller optimum.  $\square$

This result is intentionally narrow. It does not imply that expanding authority is politically or ethically desirable, and it gives no ordering when a change in governance also changes claim amounts, priorities, the recognized positions, the legal effect of clearing, or the objective used to evaluate it. In particular, enlarging the set of actions available to a powerful intermediary may reduce gross nominal claims while worsening participatory protections for weaker actors. The proof is a set-inclusion fact, not a theory of justice.

#### F.4 Institutional Stress Tests under a Fixed Obligation Ledger

The fixed-ledger result admits a more demanding stress test in which the recognized amounts and productive technology remain constant while an institution changes a particular eligibility rule. Continue to use Equation (117), whose only positive directed cycle includes all four claims. If the  $D-C$  claim has an authorized discharge limit of four units while the other claims are fully eligible, the common cycle reduction has  $t \leq 4$ . Its maximal candidate gross reduction is therefore  $4t = 16$  and the residual gross sum is  $30 - 16 = 14$ . This is an arithmetical consequence of the stipulated cap, not a claim about any legal system. If an operative stay or withholding rule instead makes that particular claim ineligible for netting, the permitted reduction on its edge becomes zero and the sole positive cycle cannot be used: the candidate optimum under the stated cycle-only model becomes zero. If the restriction is validly lifted and every claim is eligible, the earlier bound  $t \leq 6$  is restored, allowing 24 units of gross reduction. The claim-holder balances  $Bx$  are unchanged in all three counterfactuals.

Table 24: Additional synthetic eligibility stress tests on the identical 30-unit obligation ledger. No row asserts the actual effect of insolvency or set-off law in any jurisdiction.

| Stipulated rule on the $D-C$ obligation                  | Maximal candidate clearing | Gross residual |
|----------------------------------------------------------|----------------------------|----------------|
| All four claims eligible ( $t \leq 6$ )                  | 24                         | 6              |
| Authorized reduction capped at four units ( $t \leq 4$ ) | 16                         | 14             |
| One necessary edge is ineligible ( $t = 0$ )             | 0                          | 30             |

In actual cases, an investigator must ask whether parties have mutuality or a valid multilateral agreement, which claims are due and ascertainable, whether assignment or security interests affect eligibility, whether a proceeding has altered permitted set-off, who can give consent, and which authority recognizes discharge. The appropriate legal and accounting characterization varies by jurisdiction and contract. A ledger-based feasibility result should therefore carry the rule version and supporting legal determination as inputs, not silently infer valid extinguishment from the numerical closure of a loop. Claims merely written down or paid by an outside party are not necessarily cleared by the same multilateral operation.

The counterfactual also exposes an identification limitation. A higher observed reduction after an authorization reform may reflect expansion of the admissible set, changes in participation, or a new distribution of credit risk. Conversely, unchanged observed clearing does not establish that the rule had

no effect: eligible claims, matching cycles, available liquidity or participant consent might be absent at the observed configuration. The model’s monotonicity result applies to *candidate maxima* under fixed amounts and genuinely nested sets. It cannot be invoked as a causal theorem about real economies without confirming those restrictions.

## F.5 Trust, Consent, and the Dual Formation of Credit

Trust should not be represented as a single exogenously fixed probability of compliance. In social and institutional settings, parties may form beliefs through repeated interactions, audit records, restitution, communicative interpretation and contestation of past decisions. Access to a ledger does not guarantee agreement about what the data mean, and a successful accounting reduction may coexist with a dispute about the underlying claim. Accordingly, any proposed trust-dependent clearing mechanism should specify evidence sources, responsible interpreters, revision procedures and the status of unresolved objections. A high-level relation such as  $\Phi_i(z, h) = 1$  must not be treated as an automatically measurable fact.

Trust can enable at least two different mechanisms. It may support performance without per-interaction payments, for example when participants accept reciprocal contributions and shared maintenance rules. It may also make longer credit commitments possible, increasing the stock of recognized monetary obligations even when immediate cash transfers decline. Hence the empirical association between trust and monetary dependence is not generally monotonic. A rule that permits additional bilateral credit may create a new cycle; a rule that supports voluntary multilateral netting may reduce an old one. Their co-occurrence illustrates the possibility of monetary regeneration during monetary dissolution.

Consent requires its own typing. Agreement to share an environmental dataset need not imply consent to monetize its use, to pledge future value as security, or to extinguish an existing receivable. Consent may be revocable or conditional, and the timing of effective authorization may depend on an event ordering rather than a universal clock. When a clearing proposal depends on collective consent, the model must specify what counts as a valid participant, quorum, representation, and remedy for exclusion. For machine-operated agents, the authority to give such consent must be traced to an accountable institutional position rather than inferred from automated execution.

## F.6 Institutional Boundary Shifts and Apparent Demonetization

The disappearance of money at a customer interface can arise through changes in who bears or records monetary obligations rather than the disappearance of claims themselves. Suppose a privately owned AI service previously charges the user per request. It subsequently supplies requests free of charge, while an upstream provider extends credit to the operator, the operator monetizes another service, or a public institution finances the same compute and electricity. Observed user fees fall. The total ledger may nevertheless become larger or change in composition. A claim of “dissolution” must therefore name a ledger boundary and the included forms of financing.

For a fixed represented history  $h$ , let  $\pi_U(x_h)$  select the monetary edges whose debtor and creditor positions are both inside the declared observation boundary  $U$ , and let  $\kappa_U(x_h)$  select the edges crossing that boundary. Write

$$M_U^{\text{internal}} = \mathbf{1}^T \pi_U(x_h), \quad M_U^{\text{cross}} = \mathbf{1}^T \kappa_U(x_h). \quad (120)$$

Neither scalar in Equation (120) measures total social dependence on money: both omit claims outside the boundary and may fail to reveal a subsidy, a capital transfer, or obligations recorded in another institutional unit. Their value lies in forcing explicit disclosure of the observation convention. An increase in one can coincide with a decrease in the other; the signs do not establish a unique historical direction.

A fixed-ledger counterfactual is also insufficient for comparing capitalist and commons-based provision in general. A commons arrangement can change the kinds of claims recorded, the value attributed to contributions, and the way expenditures are pooled. One must therefore compare both (i) the counterfactual *within-ledger* clearing opportunity and (ii) the historical process through which the ledger, its

claimants, and the institution itself are formed. The second comparison is not recoverable from  $Bx$  alone. It requires empirical and interpretive work on production, appropriation, access, and decision-making.

## F.7 Transnational Data Commons and Federated Coordination

A global data infrastructure raises an additional distinction between *interoperability* and *common ownership*. National, scientific, local and other nodes may coordinate observations and models while retaining separate responsibilities for custody, privacy, licensing, and quality control. International weather and ocean observation networks provide a bounded analogy: standards, distributed nodes and data-access rules can support collaboration among organizations with different interests. These analogies show that cooperation is technically and institutionally feasible for some observation activities. They do not establish that every dataset should be shared, that global information coordination eliminates political conflict, or that advanced AI will eliminate credit [26, 22, 16].

A federated clearing or reciprocal-provision system would need layers that remain distinguishable. First, a semantic layer would identify data, model outputs, uncertainty and provenance. Second, an authority layer would identify who can authorize use, conversion, transfer and discharge. Third, a verification layer would preserve an auditable record of actions and corrections, including objections and access restrictions. Fourth, an infrastructure layer would account for energy, equipment, maintenance and ecological consequences. Stronger observation and inference may help agents discover compatible cyclic obligations, but observation alone does not generate a license to clear them.

This framework is also sensitive to unequal participation. Powerful nodes may define the shared schema, make their observations mandatory while charging for downstream computation, or exclude less-resourced participants from infrastructure governance. A global commons label therefore cannot substitute for examining decision rights and material dependencies. The companion paper on global knowledge infrastructure should analyze these issues directly. Here they function as explicit conditions or limitations on claims that distributed information will automatically make monetary mediation unnecessary.

## F.8 Ecological Incidence and Nonmonetary Public Obligations

Ecological impacts require a separate analytical record. Consider the same four-role service with electricity supplied by an energy operation. A shift from paid access to free access, or the full cancellation of the four-party cycle, does not determine whether the underlying generation causes emissions, habitat damage, water withdrawal, heat discharge, or material waste. A valid analysis must attend to the spatial reach of harms, the persons and ecosystems affected, the uncertainty of ecological knowledge and the institutions responsible for monitoring and response. No assumption that every ecological effect can be represented by a transferable monetary claim is made.

Represent a governance-sensitive outcome as an ordered descriptive profile

$$\Omega_i(h) = (r_i, a_i, c_i, q_i, e_i), \quad (121)$$

In Equation (121),  $r_i$  is the remaining monetary-claim record,  $a_i$  records access and participation,  $c_i$  records control of productive conditions,  $q_i$  records contestability and remedy, and  $e_i$  records ecological impacts and obligations. The components can be heterogeneous records rather than commensurable numbers. Accordingly,  $\Omega_i$  is not presented as a scalar optimization target or a complete justice index. It is an observation and accountability protocol.

**Proposition F.2** (Monetary observational equivalence and governance divergence). *Under the fixed-ledger representation of Appendix C, the pair  $(Br, \mathbf{1}^\top r)$  cannot in general identify a unique governance-sensitive outcome profile  $\Omega_i(h)$ .*

*Proof.* Take two otherwise identical ledgers and the same enacted compatible reduction  $z$  (including  $z = 0$ ). Let one institution provide a verifiable review and appeal procedure for affected nonparticipants

and the other provide no such procedure, while leaving the monetary entries unchanged. Then both arrangements have the same residual  $r = x - z$ , hence the same pair  $(Br, \mathbf{1}^T r)$ , but their contestability records  $q_i$  differ. Therefore the pair cannot identify the complete outcome profile.  $\square$

The proposition does not require a judgment about which remedy system is superior in every context. It demonstrates an information limit: monetary aggregates omit dimensions that are relevant to evaluating a production arrangement. It also strengthens the earlier caution against interpreting perfect multilateral clearing as evidence that all parties, including those excluded from the ledger, are “mutually even.” The expression may describe a local nominal-accounting condition while leaving obligations associated with land, ocean ecology, knowledge contribution, maintenance or political representation unaddressed.

## F.9 Contestability, Power, and the Evidence of Legitimate Discharge

An implementable governance arrangement should expose the difference between a computational recommendation and a binding institutional act. A clearing engine can produce a vector  $z$  and a feasibility certificate. A legal or organizational authority must determine whether the affected claims are eligible for offset and whether the operative rules support discharge. A verifiable execution record must then establish what actually changed. An auditor or affected participant may dispute any of those layers. Confusing them would obscure the location of power, especially when an AI interface makes the entire process appear automatic.

Table 25: Evidence needed to distinguish computational, institutional and ecological assessments.

| Layer                 | Record or test                                                       | Unresolved issue if omitted                          |
|-----------------------|----------------------------------------------------------------------|------------------------------------------------------|
| Accounting            | Typed claim ledger, incidence convention, numerical certificate      | Whether a claimed cycle is actually compatible       |
| Authorization         | Rule version, consent or lawful authority, recognized claim holders  | Whether an algebraic reduction may be enacted        |
| Execution             | Discharge record, effective event, correction and reversal log       | Whether obligations actually ceased to bind          |
| Productive conditions | Energy, compute, materials, maintenance and control rights           | Whether payments shifted upstream                    |
| Participation         | Admission, representation, complaints, appeals, remedies             | Whether affected parties can contest the arrangement |
| Ecological incidence  | Observations, uncertainty, monitoring, impact and remediation duties | Whether monetary clearing conceals external burdens  |

A practical design should also distinguish the right to inspect a calculation from the ability to modify the governing procedure, and distinguish both from access to an independent remedy. Transparency without influence may leave concentrated discretion unchanged. Conversely, privacy, security and the protection of third parties may require that some evidence be reviewed through controlled access rather than fully public data release. The precise balance depends on the legitimate institutional setting and is an open governance problem, not a deduction from the mathematics of circulation.

Our use of “legitimate” is therefore procedural and provisional. It identifies questions about recognized authority, representation, review and effects on affected parties; it does not claim that any single constitutional or economic arrangement uniquely realizes justice. Normative evaluation may also require assessing how claims arose in the first place, especially when monetary obligations encode unequal power over labor, infrastructure or natural resources. Dissolution can eliminate a nominal claim while leaving its historical conditions intact.

## F.10 Comparative Research Design and Limits of Institutional Inference

To compare real cases, the investigator would need more than a dataset of account balances. At a minimum, a study should identify claim types and recognition rules; locate productive dependencies and financing beyond the observed interface; document governance authority, participant rights and amendment procedures; specify what counts as successful settlement; and maintain separate measures of ecological impact and access. A comparison of two ostensibly “free” AI services should ask whether their costs

are met through public appropriation, private cross-subsidy, cooperative work, unpaid contributions, or obligations to outside suppliers. Even then, observational differences need not be causal effects of institutional design. Historical selection, scale, technical capabilities, and external regulation can jointly determine both the rules and the outcomes.

A possible quasi-experimental design would compare bounded changes within one production system when an admissibility rule changes but the ledger and accounting convention remain stable. Proposition F.1 then offers a narrow falsifiable prediction for the model’s *candidate optimum* when one institutional feasible set is genuinely nested inside another. Evaluating enacted discharge requires independent execution records; measuring a change in relational dependence requires production data and boundary audits; and evaluating claims about justice requires participation and ecological evidence. The empirical programme should distinguish all four tasks.

These institutional scenarios support a limited but important conclusion: neither advanced AI capability nor the mathematical possibility of distributed clearing determines a unique monetary order. Authorization rules can close or open clearing opportunities; arrangements for infrastructure finance can move monetary obligations across boundaries; and the same gross reduction can coexist with different participation, control and ecological conditions. Trust, governance and global public facilities therefore matter to the feasibility, durability and legitimacy of particular monetary transformations, but their detailed institutional theories properly belong to subsequent inquiries. The present appendix supplies a comparison protocol for those inquiries without treating any scenario as an inevitable endpoint of evolving production relations.

## G Assumptions, Propositions, and Counterexamples

This appendix audits the logical status and range of the paper’s central claims. Its task is neither to reassert the argument in abbreviated form nor to turn a preliminary theoretical investigation into an empirical prediction. Instead, it identifies what follows from a specified representation, what follows only under additional economic or institutional premises, and what remains a normative or empirical question. A useful theory of spacetime-distributed monetary dissolution must be able to state conditions under which its principal inferences fail. This is particularly important because claims about a decline in monetary dependence can become ambiguous when the analysis moves between consumer interfaces, production networks, legal ledgers, and ecological conditions.

The discussion draws its formal vocabulary from Appendices A–F. The elementary results concerning finite network flows are established relative to typed, compatible and divisible monetary claims. The generative account of new needs and productive relations, by contrast, is a program of conditional explanations rather than a theorem of incidence algebra. The philosophical contributions of value-form analysis and generative perspectives on economic relations provide research orientations, but do not independently demonstrate a particular future monetary trajectory [21, 23]. We therefore mark the evidential status of each inference as explicitly as its formal content.

### G.1 A Taxonomy of Claim Types and Inferential Status

Let  $\mathfrak{H}$  denote a represented relational history as in Definition A.1,  $h$  a selected configuration with a declared observation boundary, and  $\iota$  the collection of institutional recognition and admissibility rules used for that configuration. Not every statement containing the words “possible,” “emergence,” or “continuation” has the same logical status. For purposes of this audit we distinguish four categories: (M) *mathematical consequences* of explicit assumptions; (E) *conditional economic hypotheses* about production, exchange, and behavior; (I) *institutional propositions* depending on authority, recognized claims and enforceable procedures; and (N) *normative assessments* requiring independent criteria of recognition, participation, justice, or ecological responsibility. Mixed claims must identify which step belongs to each category.

Table 26: Inferential status of representative monetary-dissolution claims. The indicated tests concern the stated layer; they do not substitute for an empirical research design.

| Claim                                                       | Status | Principal assumptions or necessary evidence                                                                |
|-------------------------------------------------------------|--------|------------------------------------------------------------------------------------------------------------|
| Cyclic reductions preserve nominal net positions            | M      | Same typed incidence matrix; compatible claims; $Bz = 0$ and $0 \leq z \leq x$ .                           |
| Eligible reductions extinguish legal obligations            | I      | Enforceable authority, consent or other valid authorization, and effective discharge event.                |
| Longer circuits contribute less to expected clearing        | E/M    | Verified conditional-survival bounds, cycle counts, capacities, and an explicit probability model.         |
| A free Human–AI interface indicates systemic demonetization | E      | Upstream, cross-boundary and public-financing claims must be observed; free access alone is insufficient.  |
| Productive AI autonomy reduces monetary dependence          | E/I    | Boundary-relative material reproduction, resource access, ownership and rules of allocation.               |
| Greater gross clearing improves generative justice          | N      | Independent assessment of participation, productive control, recognition, remedy and ecological incidence. |

This taxonomy prevents an existence proof for an accounting operation from being read as a sociological explanation of why it would be adopted. It also distinguishes the expressive role of a mathematical model from its evidential role: a two-complex may faithfully encode a stipulated set of event couplings without proving that an observed payment is caused by a specific historical process. The thesis that money can dissolve in some relations while being generated in others is a coherent modeling possibility. Determining its frequency, magnitude, and conditions requires a separate observation programme.

## G.2 The Fixed-Ledger Assumptions and Their Dependencies

For a selected compatible monetary type and accounting configuration, write  $G = (P, E)$  for a finite directed multigraph of recognized claim-bearing positions. An edge  $e$  is oriented from debtor to creditor;  $x \in \mathbb{R}_{\geq 0}^E$  records its monetary amount, and  $B \in \mathbb{R}^{P \times E}$  is the creditor-positive incidence matrix. The interpretation requires *typed compatibility*: edges in a proposed clearing operation must be denominated or converted under an agreed legal and computational rule; counterparty, maturity, seniority, restrictions on offset, and relevant collective permissions cannot be silently discarded. In an elementary divisible setting, the algebraically feasible reduction set is

$$\mathcal{Z}(x, B) = \{z \in \mathbb{R}^E : 0 \leq z \leq x, Bz = 0\}. \quad (122)$$

The finite graph and bounded claims make the feasible set in Equation (122) a compact polytope, so an unrestricted linear objective attains an optimum. The inclusion of a mathematical  $z$  in  $\mathcal{Z}$  does *not* mean that a creditor has relinquished a claim. The institutionally admissible subset  $\mathcal{A}_i(h) \subseteq \mathcal{Z}(x, B)$  and an enacted discharge record are separately required, as in Appendix C.

The identity  $B(x - z) = Bx$  is an exact accounting statement. It presumes a fixed incidence matrix and one compatible accounting frame; it does not entail preservation of welfare, purchasing power, exchange rates, claim seniority, risk, or productive capacity. The derived lower bound on gross residual claims,  $\mathbf{1}^\top(x - z) \geq \frac{1}{2}\|Bx\|_1$ , similarly rests on nonnegative residuals and the debtor–creditor orientation. If claims are denominated in different currencies, adjusted for maturity, or converted by a contested exchange rule, the componentwise algebra remains meaningful only after the relevant transformation is declared. In particular, an observed number labeled “gross claims reduced” cannot be compared across incompatible ledgers without reporting units, treatment of collateral, and valuation conventions.

We can state a useful sufficient condition for any cross-history comparison. Let  $h \rightsquigarrow h'$  be a declared continuation or revision with a documented position map  $\psi_P$ , a claim correspondence  $\psi_E$ , and a denomination policy  $\chi$ . Only if those mappings preserve the intended account types, claim identities, and recognition conventions may quantities such as  $\mathbf{1}^\top x_h$  and  $\mathbf{1}^\top x_{h'}$  be read as changes in the *same* observable. In other cases, comparing the totals is a comparison between different measurements. The formalism can represent such changes; it does not resolve them automatically.

### G.3 Cycles, Boundaries, and the Failure of Reverse Implications

Appendix B showed that cycles of the event two-complex and circulations in a monetary-obligation graph inhabit different chain spaces, with distinct interpretation rules. Even within the monetary graph, several seemingly plausible reverse implications fail. The existence of a directed cycle does not imply institutional eligibility. The absence of an observed payment does not imply the absence of an upstream claim. And the acyclicity of a residual graph obtained by one particular reduction does not imply maximal gross cancellation. The counterexample in Appendix D shows an acyclic residual after a short two-edge cycle is canceled, although another admissible four-edge cycle would have produced a larger reduction.

A further issue concerns the boundary of an observed network. Let  $U \subsetneq P$  select positions, and let  $\text{res}_U^{\text{int}}$  retain only edges whose two endpoints belong to  $U$ . For a whole-network circulation  $z$  with  $Bz = 0$ , one might conjecture that the internally retained flow is also a circulation. The conjecture is false: external paths can close a cycle even though they are excluded from the observation boundary.

**Proposition G.1** (Noncommutation of internal restriction and circulation). *There exist a finite directed claim graph  $G$ , a nonnegative vector  $z$  satisfying  $Bz = 0$ , and a proper set of positions  $U$  such that the internal-edge restriction  $z_U = \text{res}_U^{\text{int}} z$  satisfies  $B_U z_U \neq 0$ , where  $B_U$  is the incidence matrix of the internal induced graph.*

*Proof.* Take three positions  $A, B, C$  and unit edges  $A \rightarrow B$ ,  $B \rightarrow C$  and  $C \rightarrow A$ , and set  $z_e = 1$  on each edge. The whole-network flow is balanced at every position, so  $Bz = 0$ . For  $U = \{A, B\}$  the only internal edge is  $A \rightarrow B$ . It produces net incidence  $-1$  at  $A$  and  $+1$  at  $B$  in  $B_U z_U$ , and therefore  $B_U z_U \neq 0$ .  $\square$

This elementary construction explains why tracing only bilateral transactions within a jurisdiction or platform may fail to reveal the larger closed route through which reciprocal claims could be canceled. Conversely, a locally balanced ledger may conceal boundary-crossing environmental or financing obligations that are absent from its monetary nodes altogether. The correct inferential practice is to specify the boundary and its cuts; expand the graph when necessary; and avoid equating an internal net-zero observation with comprehensive settlement. The proposition is mathematical. Whether any of its edges is legally set off is a further institutional matter.

Homological reasoning requires an analogous caution. Adding a two-cell can render an existing one-cycle homologous to zero while leaving the directed one-skeleton unchanged. Consequently, a claim such as “the disappearance of a first-homology class means a debt has been extinguished” is invalid without a separately specified, justified map from topological transformations to recognized discharge procedures. In the event-complex interpretation, vertices denote events, edges couplings, and faces model-declared primitives; none of these is automatically a legal debtor or a creditor.

### G.4 Path Fragility, Capacity, and Approximation Counterexamples

The intuition that longer paths are more fragile depends on at least three analytically distinct factors: conditional survival of an individual proposed circulation, the growth rate of the number of candidate circulations, and their monetary capacities. Under sequential conditional success bounds not exceeding  $q < 1$  for each additional dependency, a length- $L$  circuit has modeled survival probability no greater than  $q^L$ . When the number of length- $L$  circuits is bounded by  $n\Delta^L$  and each has capacity at most  $\bar{c}$ , a geometric bound results if  $\Delta q < 1$ . The proof and sharper weighted-transition alternatives appear in Appendix D. These are sufficient *model conditions*, not generally necessary conditions, and they do not follow from path length by itself.

**Proposition G.2** (No class-uniform monetary tail bound without capacity control). *Fix an integer cutoff  $K \geq 1$ . Over the class of finite compatible directed-claim networks with nonnegative but unbounded edge capacities, there is no finite uniform upper bound on the gross clearing objective lost by excluding all cycles longer than  $K$ .*

*Proof.* Choose a directed simple cycle of length  $L = K + 1$ , with every edge having capacity  $M > 0$ . It is the graph’s only directed simple cycle. The unrestricted gross reduction is  $LM$  by canceling  $M$  on each edge. The cutoff model, which admits only cycles of length at most  $K$ , has gross reduction zero. Its omission gap is  $LM$ , which exceeds any proposed finite uniform bound when  $M$  is chosen sufficiently large.  $\square$

The construction is deterministic; a stochastic guarantee must additionally justify its capacity and survival assumptions. For instance, if all edges on a long cycle depend on a single common approval event that succeeds with probability  $p$ , then the circuit survival probability is  $p$ , not  $p^L$ . Marginal reliability alone cannot justify multiplying edge probabilities. Under a common shock, a path can be long and nevertheless survive or fail as a single correlated unit. Conversely, even a short circuit may be impossible if one edge involves a prohibited offset or nontransferable obligation.

A second counterexample concerns optimization. Let  $A \rightarrow B$ ,  $B \rightarrow A$ ,  $B \rightarrow C$ ,  $C \rightarrow D$ , and  $D \rightarrow A$  each have unit capacity. Canceling the two-edge circuit yields a reduction of 2 and leaves an acyclic residual. Canceling the four-edge circuit instead yields a reduction of 4. Hence the theorem that *an unrestricted optimum has an acyclic positive residual* does not entail its converse. Any empirical or computational demonstration of near-optimality must use a valid upper-bound certificate, an exact solver, or an appropriate conditional tail bound, not merely visual inspection of residual cycles. Proposition C.6 supplies an exact primal–dual gap identity for the unrestricted finite linear programme; its use for restricted clearing must be qualified according to the declared feasible set.

## G.5 Productive Autonomy and the Inference from Free Access

The emergence of highly capable AI systems motivates the hypothesis that some human–human and human–AI exchanges may cease to require monetary transactions. The logically weaker premise is that a particular AI service can meet a task or provide an output. The stronger premises needed for a systemic inference concern reproduction of its conditions of production: energy conversion and access; reliable computing and replacement hardware; observation and data acquisition; materials and maintenance; ecological resources; and valid institutional rights. Moreover, collective AI-network autonomy is not the same as the autonomy of any particular operational model. Both are relative to a declared boundary and target, as Definition A.3 emphasizes.

Consider two possible configurations with indistinguishable zero-price user interfaces. In the first, a publicly provided compute node supplies an AI service without a usage charge and funds electricity and equipment through a separate budget. In the second, an automated private intermediary offers the same service for free while accumulating payables to energy and equipment suppliers. The observations at the user interface are identical; upstream claims differ. We therefore cannot infer the monetary status of the whole production network from the absence of user payments. Appendix E provides quantitative illustrations of this nonidentification.

The ordering  $H-H \prec H-AI \prec AI-AI$  can be investigated as a *conditional dependency hypothesis*, with  $\prec$  explicitly interpreted as an explanatory dependence or model-specific transition order. It is not an ordering theorem and not a necessary chronological sequence. For example, a closed internal AI system governed through shared allocations might eliminate its own bilateral internal invoices while people continue to pay human professionals for access, skill, trust, or specific relationships. Conversely, a free user interface may coexist with intense AI–AI credit and finance. Institutional and technological changes can produce these states without contradicting the mathematical clearing results.

The phrase “need for others” likewise identifies a source of possible relational and productive demand, not a sufficient cause of monetary exchange. Strong dependence may coexist with reciprocal gifts, public provision, common stewardship, and other nonmonetary coordination. Weak direct dependence may coexist with monetary access fees imposed by ownership or exclusion rules. New capabilities can remove one need while creating other needs for evaluation, repair, ecological supervision, recognition, specialized knowledge, or participation. These alternative pathways are integral to the generative account, not exceptions to be ignored.

## G.6 Institutional Admissibility, Power, and Ecological Incidence

A gross clearing optimum ranks candidates by a specified accounting objective. Its numerical value contains no information about the validity of consent, the distribution of decision rights, or the fairness of procedures. The fixed-ledger scenarios of Appendix F give differing candidate reductions under different eligibility rules, but neither larger cancellation nor greater restriction is an intrinsic ethical ranking. A restriction may protect a nonwaivable interest; an expansion of eligibility may improve coordination or permit appropriation. Assessment requires reasons, evidence, and institutions suited to the affected parties.

**Proposition G.3** (Normative and ecological non-identification from monetary clearing). *Suppose the observational record is limited to a recognized monetary ledger  $(B, x)$  and its enacted reduction  $z$ . If the model places no functional constraint linking those records to the ecological-incidence record  $\mathbf{e}$  or participatory-governance profile  $\mathcal{J}$ , then  $(B, x, z)$  does not identify  $\mathbf{e}$  or  $\mathcal{J}$ .*

*Proof.* Take two represented configurations with the same positions, claims, and enacted reduction, and assign them distinct allowed environmental-impact records or participation procedures. The premise contains no relation forbidding this difference. The observations  $(B, x, z)$  are identical in both configurations, but  $\mathbf{e}$  or  $\mathcal{J}$  differs, so neither is identified by those monetary data alone.  $\square$

The proposition does not deny that detailed environmental measurements, material-flow accounts, legal evidence, or field observations can constrain and help explain these outcomes. It states the information limit of a *monetary-only* description. A claim that a three-party netting cycle has discharged all monetary receivables is compatible with an unresolved habitat injury, a nonconsenting community, or a contested allocation of the infrastructure used to produce the value. Similarly, the absence of monetary debt to an ecosystem does not establish the absence of an ecological obligation.

Trust adds another identification problem. A new verified assurance mechanism may reduce the need for cash-on-delivery; the same mechanism can also expand credit by supporting deferred payment. Thus the direction of monetary change cannot be inferred from “greater trust” as a single scalar input. To study generative trust, a future investigation must reconstruct the relational history of commitments, evidence, correction, dissent, enforcement, and restitution. Normative justification cannot be reduced to computational admissibility, and operational automation does not grant an AI system independent legal authority to forgive someone else’s debts.

## G.7 Falsification Tests and Reporting Requirements

A useful research programme should announce observable differences between competing mechanisms. The following tests are proposed as methodological requirements, not as claims that the relevant datasets currently exist in adequate quality. They build on the empirical research continuation in Section 14 and the boundary analysis of the preceding appendices.

Reporting should preserve the underlying observation convention. For each proposed result, a reproducible ledger study should disclose (i) the unit and claim type, (ii) the selected positions and omitted boundary crossings, (iii) whether events or positions are primitive in the chosen representation, (iv) the set of candidate cycles and the admissibility rule, (v) whether reported clearing is potential, authorized, or enacted, (vi) the optimization criterion and bound on approximation error, and (vii) the independent nonmonetary outcomes relevant to the inference. The same reporting structure can be used across capitalist, public, cooperative, and commons-based cases without defining any one arrangement as the default counterfactual.

Empirical falsification of a historical-ordering hypothesis also requires explicit units of comparison. A decrease in payments per user interaction, a reduction in gross recognized claims, a decrease in monetary financing of energy, and a decline in the ability of an intermediary to exclude users are not interchangeable variables. If a study switches among them without disclosure, an apparent monotonic trend may be a measurement artifact. Since economic networks change their membership and accounting

Table 27: Proposed tests for distinguishing monetary-dissolution mechanisms. “Evidence” refers to the data required to attempt the test, not to evidence already obtained.

| Proposed mechanism                             | Required evidence                                                                                      | Potential disconfirming observation                                                                    |
|------------------------------------------------|--------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------|
| Net cancellation through multilateral circuits | Typed claims, authorization records, discharge events, and boundary changes                            | No eligible balancing circuit or no effective discharge despite proposed arithmetic reduction          |
| Reduction in Human–AI monetary dependence      | User payments plus upstream energy, equipment, credit, taxes, and subsidies                            | User-facing fees vanish while total measured dependence merely shifts to other nodes                   |
| Length-related computational decay             | Cycle counts, capacities and justified conditional-survival estimates                                  | Common shocks, branching growth or high-capacity long circuits defeat stated tail bound                |
| Productive autonomy reduces external finance   | Reproduction targets, material dependencies, access rights and financing across several configurations | Autonomous task execution coexists with persistent paid inputs or exclusion rights                     |
| Institutional change permits coordination      | Before/after eligibility rules, independent consent evidence and executed records                      | Claimed permission has no legal effect or altered positions invalidate fixed-ledger comparison         |
| Monetary clearing improves broader outcomes    | Participation, remedies and ecological monitoring alongside money ledgers                              | Monetary reductions coincide with exclusion, uncompensated harm or deteriorating ecological conditions |

boundaries, a comparison should also state whether its unit is a fixed cohort, a fixed productive service, an institutional jurisdiction, or a mapped series of relational configurations.

## G.8 Economic Identification and Institutional Counterfactual Audit

The paper’s economic claims require a separate falsification protocol beyond algebraic validity. Three common category errors should be treated as independently testable threats. First, a zero user price may hide subsidies or upstream monetary claims. Second, a transfer between AI processes may be a nonmonetary internal allocation rather than credit between recognized claim-bearing positions. Third, a recorded reduction in balances may be a proposed netting instruction, a write-off, a cash payment, or a legally effective discharge. These mechanisms can produce similar dashboards but carry different implications for the paper’s central hypothesis. The evidential unit must be a typed and institutionally situated claim or productive dependency, not a bare digital transaction.

A pre-registered scenario comparison should specify: (i) the physical service and service-quality measure; (ii) the included and excluded production roles; (iii) whether each role can bear a claim under the governing arrangement; (iv) a comparable observation or history relation; (v) a complete mapping from obligations created to obligations transferred, paid, or discharged; (vi) energy, data, materials and maintenance required for continuing production; and (vii) remedies and ecological externalities recorded independently of money. Reporting only a gross-clearing optimum does not identify any change in these other dimensions. Where monetary aggregates cannot be combined across denominations or jurisdictions, the study should report typed components rather than apply an unverified exchange-rate normalization.

Two deliberately conflicting hypothetical observations are informative. In case A, AI–AI operations within a publicly operated laboratory use internal nontransferable allotments from the outset, while human access requires a paid membership. Under the ledger definitions of this paper, AI–AI monetary mediation can be absent while H–AI monetary mediation persists. In case B, a privately financed platform offers free human access while independent energy and computing companies maintain payable invoices and credit. The apparent ordering is reversed at the visible interface. Both cases are consistent with the same technological abilities. Thus an economic study can examine conditional tendencies across specified institutions, but cannot confirm a universal H–H/H–AI/AI–AI chronology by selecting one illustrative pathway.

Even when comparison data are collected, causal identification remains a separate task. If institutional rules, service volume, input prices or claim-reporting standards move concurrently, observed demonetization can be attributable to multiple mechanisms. An adequate analysis should disclose these changes and, where possible, conduct sensitivity checks using stable cohorts, constant-price service measures, upstream counterfactuals, alternative observation boundaries and negative-control claim classes. Lack of such evidence should be explicitly coded as an unresolved identification problem rather than a

positive observation of monetary dissolution. This register complements the mathematical claim-status distinctions earlier in this appendix.

## G.9 Limitations Register and Priorities for Integrated Revision

Several limitations now deserve explicit placement in the manuscript’s subsequent verification pass. *Formal limitation*: the finite typed-flow model gives exact conditional results, but multi-currency claims, nondivisible contracts, priority and insolvency rules, and strategic bargaining require distinct models. *Topological limitation*: the event two-complex is a chosen representation; its homology does not itself identify economic transactions, motivations, or legal rights. *Statistical limitation*: truncation certificates depend on calibrated or defensibly bounded probabilities, capacities and path counts; strong correlations and unobserved edges can defeat the proposed conditions. *Economic limitation*: the human–human, human–AI and AI–AI ordering remains a comparative hypothesis, and autonomous production is always defined relative to a productive target and observation boundary. *Institutional limitation*: candidate clearing, lawful authorization and effective discharge must be recorded as distinct states. *Ethical limitation*: gross monetary reduction does not determine whose value circulates, who can contest a claim, or how ecological harms are addressed.

The final integrated revision should audit every occurrence of the words “necessary,” “sufficient,” “equivalent,” “optimal,” “autonomous,” and “dissolution” against these distinctions. The theorem and proof registry should distinguish mathematical statements that are exact for specified finite models from claims of relevance to observed economies. Numerical examples should be reproducible, and comparisons should identify their claim boundaries, counterfactual rules and units. The literature review should verify both the accuracy of foundational citations and the extent of the prior author’s contribution; theoretical inspiration is neither a substitute for proof nor a claim of direct formal derivation.

The present appendix therefore supports a bounded conclusion. Distributed monetary dissolution is mathematically representable through particular changes in compatible obligations and their authorized settlement, and it is economically imaginable through broader transformations in production and relational coordination. The frameworks do not establish a universal path toward the elimination of money. Persistent needs, new forms of exchange, power, ecological dependence, and historically changing institutions leave the problem open. This combination of formal specificity and theoretical openness is the methodological basis for the companion studies on global knowledge infrastructure, ecological intelligence, autonomous production, and generative trust.

## H Intellectual Genealogy and Connections to Previous Research

The purpose of this appendix is to identify the distinct intellectual sources, exploratory predecessors, and original proposals that meet in the present investigation. Its function is methodological as well as bibliographical. A theory of monetary transformation draws on concepts developed for different explanatory tasks: the social form of value, the institutions that recognize claims, the conditions of monetary exchange, the generation and circulation of heterogeneous value, and mathematical representations of historically constituted relations. Similar vocabulary across these tasks does not establish that their underlying objects or conclusions are interchangeable. The present account therefore treats an intellectual genealogy as a set of *qualified conceptual connections*, not as a single sequence of intellectual inheritance or a proof of the paper’s hypotheses.

Three attribution categories will be kept separate throughout. An *intellectual inspiration* can motivate the questions or mode of attention pursued here without supplying the formal premises of the model. A *theoretical resource* can provide an established distinction or result used under identifiable assumptions. An *exploratory predecessor* can contain an earlier formulation of a question, proposal, or candidate representation, while remaining subject to independent evaluation. A fourth category, a *contribution of the present paper*, concerns the particular synthetic argument, typed models, conditional mathematical consequences, and deliberately unresolved hypotheses developed here. These categories are not state-

ments about the relative importance of researchers. They protect the correspondence between what a source establishes and what the present work claims.

## H.1 Value Forms, Production Relations, and Historical Specificity

An essential political-economic question concerns the relation between the social organization of production and the forms through which its products are recognized and exchanged. Marx's analysis of the commodity, value-form, money, and capital provides a systematic historical account of this question within commodity-producing and capitalist relations [21]. It informs the present paper's distinction between growing productive capability and changes in the social conditions of access, ownership, control, and exchange. If automated systems produce a service at low marginal labour cost while ownership of energy infrastructure, materials, computing equipment, and distribution remains concentrated, the organization of monetary access may persist. Conversely, a change in the institutions governing production may alter monetary claims even without an equivalent increase in autonomous productive capability.

This conceptual connection has limits. Marxian value categories arise within a historically specified theory of social labour and capital. They cannot be transferred without argument into a general measure of every ecological, expressive, affective, or AI-generated contribution. Nor is a reduction in a claimholder's gross nominal obligations equivalent to a change in the capitalist form of production. The present paper therefore does not identify the disappearance of payment tokens with the overcoming of alienation, or treat a computational model of monetary clearing as a derivation of Marx's theory of value. Rather, the Marxian perspective motivates scrutiny of *which production relations* support monetary mediation and *whose control* over productive conditions changes when a payment interface is removed.

Related historical and sociological inquiries make the institutional specificity of monetary relations equally important. Polanyi's treatment of markets and social institutions illuminates the public and political organization of apparently self-regulating exchange [24]. Simmel examines the forms of social association and abstraction through which money mediates relations among persons [25]. Neither approach yields a universal law predicting a sequence of human-human, human-AI, and AI-AI monetary dissolution. Their contribution here is to widen the analysis beyond technical exchange efficiency: fees for admission, membership and relationship access are institutionally constituted claims associated with encounters and associations, not mechanical consequences of a pre-existing subjective need.

## H.2 Money, Credit, Memory, and the Institutional Recognition of Claims

A second body of scholarship addresses what monetary instruments do and why they become acceptable. Search-theoretic work models monetary acceptance when trade faces decentralized matching or exchange frictions [19]. The inquiry into money and record-keeping associated with the formulation that "money is memory" examines circumstances in which reliable records can replace some functions of a monetary medium [20]. These are important resources for the present hypothesis that improved computational information, matching, recognition, and coordination may reduce particular kinds of monetary mediation. The models' assumptions, however, are substantive: neither greater database capacity nor more accurate prediction is in itself a valid contractual cancellation, a transfer of ownership, or an enforceable settlement.

Institutional and credit-oriented approaches investigate monetary units, authoritative acceptance, obligation, and social structures beyond a pure matching mechanism. Ingham's analysis of money as a social relation and Goodhart's contrast between conceptual approaches to money sharpen the distinction between transferable instruments and the legal or political arrangements supporting a monetary system [15, 5]. These views offer important correctives to technological determinism: a perfectly informed pair of AI systems can still face differently recognized claims, exclusions, priority rules, or incompatible forms of account. Conversely, compatible claims in a permitted clearing system may be netted even when the participating organizations have limited insight into each other's comprehensive productive histories.

The present paper distinguishes money, currency, credit, internal accounting units, monetary claims, and enacted discharge. It uses a directed typed obligation ledger for one bounded mathematical question: which nonnegative compatible claims can be reduced while preserving nominal net positions? That analysis belongs to a chosen accounting and institutional layer. It does not purport to provide an exhaustive explanation of the historical origin of sovereign money, all credit creation, taxation, financial intermediation, or the political constitution of units of account. In particular, whether an automated productive role corresponds to an independently recognized debtor or creditor is a question about institutional positions, not an inference from the technical identity of a software agent.

### H.3 Generative Perspectives, Heterogeneous Value, and Commons

The generative orientation of this research asks how needs, productive capacities, social relations, institutions, and possibilities for further creation arise through historically situated interaction. One intellectual inspiration for looking at economics from this perspective is the theory of Generative Justice, which attends to the creation and bottom-up circulation of labour, ecological, and expressive forms of value and to the conditions under which those who generate value participate in its circulation [3]. This acknowledgment concerns a perspective that helped motivate the questions asked here. It does not attribute the present monetary-dissolution hypotheses, the AI production-network typology, or the two-complex and clearing results to that source.

This distinction also helps prevent a normative shortcut. Nonmonetary exchange is neither a necessary nor a sufficient condition for the ethical aims considered in generative political economy. A publicly funded service may expand access while transferring costs to populations absent from a chosen ledger; a privately operated service may charge fees while ensuring substantial consent and remedy; a nominally commons-based system may be subject to exclusion, extraction, or unacknowledged maintenance burdens. Commons scholarship, particularly the study of plural governance arrangements and their concrete rules, informs the requirement to evaluate participation, monitoring, sanctions, dispute resolution, and collective decision-making as distinct institutional features [23]. The present paper does not infer a unique governance ideal from the mathematical amount of clearing achieved.

The term *heterogeneous value* is also used with care. Monetary denominations can record a locally accepted numerical obligation without representing the entirety of labour, knowledge, ecological maintenance, relational significance, or possibilities for further creation. The author's earlier inquiry into heterogeneous value circulation explored this distinction in a relational and topological setting [11]. The present paper operationalizes a narrow monetary slice through typed, compatible, nonnegative claims and retains independent records of other consequences. Such a slice is useful precisely because its limits can be specified. No claim is made that economic and ecological values become mutually convertible merely through a higher-dimensional representation.

Accordingly, the paper neither identifies Marxian socially necessary labour time with the heterogeneous-value record nor infers a general equivalence scale from a chosen monetary unit. It also does not equate the normative aims of generative justice with the numerical objective of maximizing gross clearing. These are distinct explanations and evaluative standpoints; their productive interaction consists in illuminating different aspects of a specified economic configuration.

### H.4 Relational Histories and the Scope of Topological Representation

The mathematical vocabulary of chain complexes, cellular boundaries, cycles, homology, and changing complexes is drawn from established algebraic and computational topology [6, 1]. The present programme explores one use of that vocabulary for representing events and couplings within historically constituted production relations. Here, an event is not automatically a person or firm: vertices of a relational event complex designate events, edges designate couplings, and two-cells require an expressly specified interpretation. Agents and claim-bearing positions may be reconstructed or recorded through additional structure. A separate directed obligation multigraph records recognized claims and their monetary direction. These are distinct mathematical objects, even where explicit mappings connect them.

A topological first cycle may represent an unfilled relational loop, and a graph's directed cycle may represent a compatible reciprocal liability circuit. The two concepts do not coincide. Filling a triangular one-cycle with a two-cell can eliminate its first homology class without removing any directed edges from the obligation graph; changing legal eligibility can extinguish a claim without altering the topology of the event complex. Appendix B develops these counterexamples. Therefore, neither the existence of a topological loop nor the vanishing of a homology group is, without further assumptions, a monetary-clearing theorem.

The *spacetime-distributed* qualifier is likewise a modeling commitment requiring interpretation. It refers here to how monetary claims, productive events, commitments, and clearing possibilities can be associated with different locations in a relational history and with nontrivial dependency or partial-order structures. It does not assert that all economies possess one universal external clock or that an admissible continuation is simply a predestined future awaiting realization. The study may use a chosen coordinate or filtration to calculate a particular example, but the observational convention and the manner in which that representation arises are part of the model. The author's broader generative relational economics research programme and the earlier heterogeneous-value paper supply exploratory questions and possible representations [9, 11]; neither is presented as empirical certification that one two-complex realizes social reality uniquely.

## H.5 Exploratory Predecessors in the Monetary Research Programme

Several preceding papers developed parts of the present question as independent thought experiments. *Money in AI-Operated Production Networks* explores how monetary mediation might change when AI performs productive functions [12]. *Spatiotemporally Distributed Monetary Dissolution in AI Production Networks* formulates the question of nonlocal and cross-event reduction of monetary dependence [14]. *A Generative Grammar of Value Circuits in AI Production Networks* explores how possible value circuits might be generated or classified [7]. *Normative Renormalization in Distributed Monetary Dissolution* asks how multi-scale representations and normative considerations might bear on economic circulation [13]. The current manuscript draws on this programme's sequence of questions while reconsidering the underlying distinctions and its conceptual scope.

Those exploratory works should not be cited as independent empirical confirmations of a future post-monetary economy. Their roles here are more particular: they locate earlier proposals concerning AI production, heterogeneous circuits, distributed disappearance of monetary mediation, and approximation across scales. The present manuscript revises and connects these proposals to a typed account of recognized liabilities, formal clearing bounds, relational histories, productive reproduction, and open institutional contingencies. It does not claim originality for established network-flow identities, ordinary graph-cycle decomposition, or standard algebraic topology. Its distinctive contribution lies instead in the articulated distinctions, conditional synthesis, and set of questions and demonstrations produced by their application to the monetary-dissolution hypothesis.

Two adjacent explorations illuminate parts of the institutional and epistemic boundary. The earlier discussion of credible commitments, contestability, and remedy identifies difficulties that can persist under asymmetric circulation even if gross obligations decline [8]. The public-knowledge governance inquiry considers how provenance, procedural authority, and responsible participation matter for infrastructures that represent and circulate knowledge [10]. They provide relevant research continuity, but neither establishes the operational existence of a global public knowledge network or the legal efficacy of any proposed clearing system. The present paper's introductory discussion of trust and global infrastructure is correspondingly preparatory; the dedicated infrastructure and governance studies must justify their own architectures and institutional assumptions.

## H.6 A Contribution-Boundary Map

Table 28 records where the present paper uses earlier concepts and where its own claims begin. The column of limitations is as important as the proposed extension: it prevents inspiration, formal analogy,

bibliographical continuity, and substantive proof from being treated as the same kind of support.

Table 28: Selected intellectual resources and the limits of the present paper’s use of them. Entries indicate conceptual roles, not scholarly priority or a hierarchy of authorities.

| Resource or research strand                  | Contribution to the present inquiry                                                              | Boundary of the inference                                                        |
|----------------------------------------------|--------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------|
| Value-form and production-relations analyses | Historical specificity of commodification, capital and control over productive conditions        | Monetary netting alone cannot establish a transformation of production relations |
| Money, credit and exchange theories          | Medium, unit, recognized credit and record-keeping as analytically different functions           | Efficient matching or perfect memory is not an authorized discharge rule         |
| Generative economics and commons             | Attention to value generation, circulation, participation and plural institutional forms         | Nonmonetary exchange does not by itself constitute justice or shared control     |
| Relational/topological exploratory programme | Event structures, heterogeneous value circuits and candidate two-complex representations         | Cellular homology does not determine legally admissible clearing                 |
| Earlier AI monetary thought experiments      | Possible transformations, network migration, cross-event dissolution and multi-scale conjectures | Prior hypotheses are not independent empirical validation                        |
| Present manuscript’s typed clearing model    | Conditional feasible reductions, residual bounds, counterexamples and truncation tests           | Exact ledger results do not predict social adoption or ethical outcomes          |

This differentiation clarifies how the mathematical propositions in Appendices A–G are to be understood. Some are proofs of propositions under explicit assumptions of compatible divisible obligations and selected network boundaries. Others are counterexamples that defeat a proposed converse or demonstrate the limits of a measurement. The economics of why particular claims are created, recognized, enforced, or replaced is a distinct investigation. The framework for monetary dissolution is therefore a set of linked, but nonidentical, problems: the formation of relational needs, the constitution of exchange and claims, the algebra of permissible reductions, the reproduction of productive conditions, and the institutional and ethical consequences of actual discharge.

## H.7 Attribution, Verification, and the Limits of Self-Citation

An intellectual genealogy also requires a practice of bibliographic restraint. Public listings establish that a title or abstract was made available, not that every mathematical argument in the underlying manuscript has been independently checked. A stable DOI can identify a public research object without establishing external review or experimental confirmation. Accordingly, the author’s earlier publications are identified as exploratory papers and used only for the scope of their publicly documented ideas. Where an earlier bibliographic entry was verified through a public listing rather than inspection of the complete source, the associated reference record and verification ledger preserve that limitation. Such works are not used as a substitute for proofs or independent data in the present manuscript.

For the established literature, attribution should follow the particular proposition actually borrowed. A discussion of commodity value and the social form of money should cite Marx’s relevant analysis; a claim about record-keeping and monetary functionality should identify the scope of the memory model; a statement about collective institutions should not transform a study of governance diversity into an endorsement of one economic regime. Likewise, the generative perspective should be acknowledged where it informs the research framing, without repeatedly foregrounding an individual source as an authority for claims that the source did not make. References and the source ledger are intended to support these distinctions, and the final bibliographic audit remains necessary before public release.

The present manuscript is a substantial exploratory discussion, not a report of observed AI–AI economies in which monetary relations have disappeared. Its numerical cases are constructed examples. Its proof obligations are bounded by the hypotheses and mathematical definitions given in the relevant propositions. The proposed order of changes in human–human, human–AI and AI–AI exchange is an economic hypothesis susceptible to empirical challenge, not an implication of AI capability alone.

The possibility of new obligations after local clearing, and the continued presence of ecological or participatory consequences, prevent a ledger result from being redescribed as the disappearance of value circulation or dependence on others.

## H.8 Research Continuations and Original Contribution

The paper's principal research move is to reconnect problems often discussed separately: reasons for seeking others, historically contingent monetary intermediation, productive and legal dependence, the partial clearing of compatible obligations, and the possibility that new relations create new claims. The contribution is not a single prediction about a date at which money ceases to exist. It is a set of distinctions, conditional models and counterexamples permitting more precise inquiry into how monetary mediation may be formed, locally reduced, migrated or regenerated across changing networks. A monetary reduction at one interface can be real and valuable without demonstrating the absence of money elsewhere. A clearing operation can reduce gross claims without changing net positions or the productive and ecological relationships from which those claims arose. A rise in computational power can facilitate certain forms of coordination without determining what participants recognize as legitimate.

The wider research programme leaves several complementary tasks for independent investigations. Global observation and knowledge infrastructure must address data interoperability, provenance, asymmetric participation, access rights, and epistemic error. Marine observation and underwater robotics must be evaluated as scientific and ecological undertakings in their own right. Autonomous AI production infrastructure requires bounded assessments of energy, materials, repair, replacement, governance, and environmental burden. Transnational trust and public governance require an account of commitments, institutional credibility, disagreement and effective remedy. Existing international weather and ocean-observation systems illustrate possibilities for cooperative information exchange, while not establishing the feasibility of an unqualified global data commons or any particular monetary transition [26, 22, 16, 4].

These continuations do not have to resolve into one global monetary equilibrium. They instead offer domains for comparative and longitudinal inquiry into local needs, public knowledge, productive autonomy, heterogeneous value, and the ethics of coordination. The provisional thesis of this paper is correspondingly bounded: monetary mediation may change in multiple, uneven and sometimes opposing directions within evolving relational production networks. The durability of such changes and their institutional and ecological legitimacy remain questions for theory, measurement, and public practice rather than conclusions supplied by the formalism alone.

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