

Human History in Complexes

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Abstract

Human history is encountered through incomplete sources, contested identities, heterogeneous durations, changing relations, and transformations in the conditions of further action. This expanded article develops a theoretical and computational approach to these problems through finite cell complexes and their historically situated interpretations, incorporating earlier event-coupling-primitive and CW-spacetime formalisms. Its starting point is the open-source History Complex within *The Science Infrastructures* (<https://github.com/huangwanhong-maker/The-Science-Infrastructures>). In that implementation, occasions define vertices, repeated appearances of beings define worldline edges, and ties sustained across meetings define attached two-cells; a refined carrier resolves participant-specific presences and signed relations. The paper reconstructs these definitions, derives elementary topological and gauge-theoretic properties, and distinguishes documentary assertions, formal invariants, probabilistic reconstructions, and claims about historical processes. It then places the implementation in dialogue with earlier investigations of generative relational history, evolving constraints, heterogeneous value circulation, future-creation topology, knowledge translation, historical warrant, and epistemic justice. These connections motivate an exploratory extension in which histories are studied at multiple resolutions, interpreters are themselves historically situated, and permissible continuations are separated from observed trajectories without presupposing a universal privileged time. Original vector figures illustrate the record-to-complex pipeline, temporal lenses, signed triangles, multi-resolution histories, a continuation landscape, and the distinction between semantic translation and topological attachment. A synthetic example is worked through algebraically; a repository case is discussed with explicit evidential restrictions. The resulting programme treats topology as a disciplined means of questioning historical records, while distinguishing evidence, generated structure, the grammar of admissible transformations, and realized history. Six theoretical appendices develop the topological, temporal, gauge-theoretic, grammatical and epistemic foundations.

Keywords: history; generative relational theory; CW complex; algebraic topology; spin foam; historical constraints; historiography; provenance; source criticism; temporal epistemology; relational generativity

Open-source implementation: The Science Infrastructures (GitHub; History Complex and history domain).

Manuscript version: Exploratory expanded draft v0.3. The historical canon remains subject to independent verification.

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Notices

Scope and standing. This is an exploratory theoretical and methodological manuscript. Mathematical statements concern explicitly defined finite objects. Illustrative interpretations and proposals for historical modeling are not empirically validated findings. The source-assertion canons in the software may contain mistakes, omissions, coding disagreements and unequal archival coverage.

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Language-model assistance. Development of this exploratory manuscript involved OpenAI ChatGPT for research organization, drafting, bibliographic selection and mathematical exposition. The research directions were supplied by the human author and source texts. The human author remains responsible for source verification, mathematical validity, substantive judgments and any released version. No AI-generated historical narrative is treated as independent historical evidence.

Repository and citation status. Software design and numerical descriptions refer to documentation examined on 24 October 2026. Version-dependent claims should be rechecked against a fixed commit when preparing a public edition. Earlier works of the author are cited as related exploratory proposals, not as independently corroborated empirical evidence.

1 Introduction

History is commonly read through persons, institutions, events, periods, ideas, works, material conditions and their connections. These are indispensable investigative objects, yet their relationships resist containment in a single catalogue. Two intellectual communities may share a work without sharing membership; an alliance may survive particular meetings and alter its significance across them; a correspondence may become an institutional practice; and documents can preserve some interactions while leaving others inaccessible. The research problem concerns the forms in which such partially recorded processes can be represented, reconstructed and critically compared.

A chronology orders assertions. A network represents relations. A geographic map places selected objects. Each illuminates an aspect of historical inquiry; each can also suppress distinctions that matter for interpretation. A timeless network may conflate a relation's attestation with its continued existence. A list of dated events may conceal the loops through which people, texts and organizations re-encounter one another. A spatial or numerical reconstruction can suggest evidentiary certainty that its underlying documents do not warrant. These tensions motivate an exploration of *cellular complexes* as a common but revisable representational apparatus.

The title *Human History in Complexes* identifies a research programme rather than a claim that historical reality literally consists of CW cells. A complex is constructed *relative to* a specified corpus, coding convention, scale and investigative question. The programme asks what such a construction makes visible, which distinctions it inevitably loses, and what interpretive obligations remain once algebraic and computational invariants have been obtained. Our immediate technical point of departure is the open-source History Complex in *The Science Infrastructures* (The Science Infrastructures contributors, 2026e,b,a). Its declared scope spans histories of science, art, music, literature, philosophy and relations among states.

The paper makes four contributions. First, it presents a transparent mapping from documentary assertions to a CW 2-complex and clarifies why the attaching maps of its faces must have historical meaning. Second, it separates an unsigned carrier complex from a refined complex that can accommodate signed relationships and gauge-invariant cycle observables. Third, it gives a small exact example that exposes both a topological invariant and the need to distinguish changing relational signs from topology itself. Fourth, it proposes epistemic and validation criteria through which model-relative descriptions might be translated into defensible historical interpretations.

Our use of history is inclusive: artistic, scientific, intellectual, diplomatic, material and institutional processes may enter the same representational framework. Their equal availability for mathematical description does not imply that they obey a common historical law, that mathematical abstraction exhausts their meanings, or that the same source practices are appropriate to each. We distinguish results established by elementary mathematics, observations reported by the present software, and propositions that require subsequent research.

2 Historiographical and Mathematical Antecedents

The problem of historical duration has a substantial historiographical lineage. Braudel (1958) distinguished event-centered narratives from longer structural durations, while Abbott (2001) examined assumptions about time, turning points and events within social-scientific methodology. Sewell (2005) treated the interaction of situated events with enduring and transformable social structures. Their work warns against inferring historical temporality directly from the clock coordinate used to store records. A date can be an evidentiary convenience; a sequence, a partial ordering or an interval may instead be the primary object of analysis.

Prosopography and network analysis provide another lineage. Structured historical databases organize source-dependent statements about people and relations, with the act of identification remaining part of research rather than an automatic fact (Bradley and Short, 2005). Affiliation structures can pre-

serve the duality of persons and groups (Breiger, 1974). Signed networks add relations of agreement, opposition and structural balance (Cartwright and Harary, 1956; Zaslavsky, 1982); these categories are coding decisions that must be justified domain by domain.

Algebraic topology contributes a different mathematical vocabulary. A graph contains vertices and edges. A CW 2-complex additionally attaches discs by specified closed boundary paths, permitting the computation of first and second homology, fundamental groups and related invariants. These quantities can register cycles unfilled by recorded relations and closed combinations of faces. Filtered complexes and persistent homology investigate which features depend upon a chosen filtration (Edelsbrunner and Harer, 2010). Relation-based simplicial constructions such as Dowker complexes are an alternative to CW carriers and need not return the same answers (Dowker, 1952).

Finally, lattice gauge theory supplies a language for frame transformations and cycle products, and spin-foam formulations organize representation labels on two-dimensional carriers (Kogut and Susskind, 1975; Baez, 1998). Employing this mathematics for historical relations is an *analogy equipped with a precise probabilistic model*. It is neither evidence that human societies are quantum systems nor an inference of historical physical laws. The distinction is essential: equations may be mathematically sound yet historically uninformative if the measurements, sign conventions and validation procedures do not support their interpretation.

The distinctive opportunity lies in the interaction of these antecedents. A history complex can pre-serve repeated encounters and named documentary evidence while enabling exact questions about paths, surfaces and reconstructive uncertainty. Its potential advance over a narrative database or a succession of graphs must be demonstrated through interpretability and source-grounded predictive or explanatory comparisons, rather than inferred from the availability of higher-dimensional mathematics.

3 The Earlier Event–Coupling–Primitive Programme

This section reconstructs the relevant mathematical commitments of the author’s earlier social-spacetime work, particularly *Social Spacetime Topology and Generative Grammars in CW Complexes* (Huang, 2026: Social CW). Its role is to make explicit the relation between that general proposal and the narrower computational History Complex. The two constructions use similar cellular machinery while assigning different representational roles to individual cells. A responsible synthesis therefore requires a translation of models, together with an account of which information and operations the translation preserves.

3.1 Events, couplings and question-relative primitives

The earlier proposal fixes a research question Q and a resolution ρ , and uses a typed two-dimensional CW complex

$$S_{Q,\rho} = (K_{Q,\rho}^{(2)}, \tau, \lambda), \quad K^{(0)} \subseteq K^{(1)} \subseteq K^{(2)}, \quad (1)$$

where 0-cells represent events, 1-cells typed couplings or interfaces, and 2-cells primitives relative to (Q, ρ) . The functions τ and λ summarize, respectively, type assignments and suitable auxiliary labels; neither notation requires a unique internal data schema. For example, a legal act or institutional practice may be an unresolved primitive at one resolution and an internally differentiated subcomplex at another. A primitive is therefore an *analytical boundary*, not an allegedly indivisible thing. A face need not be a human subject, a social actor, or a material body.

Persistent relational beings are instead historically individuated through structured regions, paths, or subcomplexes. This prevents a common category error: regarding a vertex as a person and every edge as a personal relationship. The identity of a scientific institution may involve a succession of events, recurrent coupling interfaces, procedures, records, external relations and periods of reconstitution. Its persistence criterion depends on the investigation and requires evidence; the cellular representation does not confer identity merely because several appearances share a name.

The same paper makes an especially important distinction between the capacity to *encode* information and the capacity to retain organization as *native topology*. Arbitrarily elaborate labels on a two-complex could encode higher cells and whole alternative descriptions. But an algorithm operating only on the carrier's boundary maps need not see those higher incidence relations. The purpose of extending a two-complex to a general CW complex is accordingly structural transparency, not a claim that two-dimensional data are intrinsically incapable of describing higher-dimensional organization.

3.2 A CW extension and the meaning of dimension

For an inquiry that requires higher incidence, the proposed extension has skeletons

$$X_{Q,\rho}^{(0)} \subseteq X_{Q,\rho}^{(1)} \subseteq X_{Q,\rho}^{(2)} \subseteq \cdots \subseteq X_{Q,\rho}, \quad X_{Q,\rho}^{(2)} = K_{Q,\rho}^{(2)}. \quad (2)$$

An n -cell is attached along a map $\varphi_\alpha : S^{n-1} \rightarrow X^{(n-1)}$. The attaching map must be included in the chosen construction; the mere co-occurrence of $n + 1$ social elements does not produce an n -cell. A higher cell can express a hypothesized closure, composite incidence, or organizational unit whose boundary contains relevant lower-dimensional components. It is not an indicator of status, size, historical importance, or moral value. A small contract procedure and a large transnational institution can each admit decompositions involving several cellular dimensions. Their sociological content comes from the type system, evidential interpretation and attachments, not dimension alone.

The distinction between *candidate* and *warranted* attachments also matters. Finding three pairwise cooperations does not prove that the participants formed an integrated organization. A triangular 1-cycle may remain unfilled because a documented collective procedure or closure is missing. Filling it is a modeling assertion that demands evidence and has an algebraic consequence: it can kill the corresponding homology class. Section 13 studies this consequence for a small historical example.

3.3 Structure, grammar and realization

A central commitment of the previous manuscript is that the generated carrier and its generative mechanism should be modeled separately. Its grammar is a triple

$$\Gamma_{Q,\rho} = (\mathbf{T}_{Q,\rho}, \mathbf{R}, \mathbf{A}), \quad (3)$$

consisting of a type system, a family of cellular rewrite rules and rule-specific admissibility conditions. A formal rewrite may be represented by a span of typed patterns $L_r \leftarrow B_r \rightarrow R_r$ that exposes the interface preserved by replacement. Matching the left-hand pattern is only one component of admissibility: an institutional change may require a decision, authority, resources, uptake or normative authorization. The grammar describes a space of admissible transformations; a realized derivation records which transformations were represented as occurring. A static complex therefore does not reveal its grammar, and a grammar does not select one unique realized history.

This point directly strengthens the present work. The History Complex now constructed in software is principally a *representation of a historical record* and a *calculable field configuration* on that representation. A theory of historical generativity requires further objects and evidence: types and interfaces, constraints on rewrites, realizations, records of failed attempts, and the historical evolution of the grammar itself. These are conceptual extensions, not functions whose implementation is certified merely by the existence of the current history domain.

3.4 Three analytical levels and claims of reality

The earlier CW paper maintains an ontological restraint that must also govern the present inquiry. First, ultimate ontology is left open: cellular models do not prove that human history literally consists of cells. Second, the relevant target is historically constituted *relational reality*, including practices, institutions,

Table 1: The principal earlier formal commitments and their implications for this manuscript. “Earlier” denotes the conceptual proposal in Huang (2026: Social CW); “current” denotes the public History Complex implementation.

Object	Earlier social CW programme	Historical-computational interface
Vertex	Event selected at (Q, ρ)	Occasion, a documented aggregate of events
Edge	Typed coupling/interface	Continuing appearance of a being, or refined segment
Face	Question-relative primitive	Sustained tie between selected beings
Relational being	Historically distributed substructure	Named being tracked through records and presences
Higher cell	Explicit organizational incidence	Proposed enrichment; not a present requirement
Generative grammar	Typed rewriting with admissibility	Research extension beyond static construction
Observation	Historically situated manifestation	Coded records with source and uncertainty constraints

actual capacities and constraints. Third, *manifestation and observation* concern what documents, witnesses, classifiers, instruments and subsequent interpretations can make accessible. These levels permit a theorem about a finite chain complex to be fully valid while the associated interpretation of a political coalition remains uncertain. A recorded designation can have real institutional effects while its accuracy remains contested. Formal labels and historically operative meanings should therefore be analyzed together but never identified without an argument.

3.5 Independent but compatible research objects

It would be tempting to declare the current History Complex a direct specialization of Equation (1). That declaration would be too strong. Its occasion cells collect events at a chosen granularity, and its edges are primarily life segments rather than generic interfaces between selected primitive faces. Its tie faces may be processes rather than primitives. The two are compatible *as modeling languages*, subject to a specified reconstruction or interpretation map, but their cellular dimensions are not interchangeable by fiat. The next section establishes a precise distinction between semantic comparison and mathematical maps of carriers.

4 Philosophical Commitments and Representational Levels

The purpose of this section is to identify the philosophical propositions being investigated, the status of the formal carrier, and the distinctions needed for a historically responsible interpretation. A relational approach begins with the observation that historical identities, capacities, categories and institutions are often constituted and sustained through relationships. Such a starting point leaves open both the ultimate ontology of relations and the empirical question of what a particular body of evidence warrants. A cell complex makes certain relational configurations analyzable. It does not resolve these questions by mathematical definition.

4.1 Three levels of assertion

The methodological discipline can be formulated at three levels. The first level suspends commitment to a comprehensive ontology of historical reality. One may investigate historically operative relations while remaining agnostic on whether a chosen mathematical object exhausts the reality of any subject,

event, institution or social order. The second level concerns historically constituted relations, beings and capacities. Here a research account proposes mechanisms that explain the production, persistence and transformation of identifiable possibilities for action and experience. The third concerns manifestation and evidence: which source assertions, codings and interventions allow an investigator to warrant, revise or reject a particular interpretation. The three levels have different validity conditions. A theorem about a chain complex belongs initially to the third level's formal apparatus, even when it is motivated by second-level hypotheses.

The distinction is important when a source names a person, city, empire, religion, language, school or tradition. Such names may be the products of shifting boundary practices. A person may be treated as a continuous historical unit for some research purposes while their statuses and relations change. An institution may persist administratively while its membership, norms and jurisdiction are transformed. Neither example licenses a direct identity between a persistent database key and a fixed ontological essence. The codebook's "being" is a carrier of encoded continuity, an operationally chosen object whose identity must remain contestable (The Science Infrastructures contributors, 2026b).

4.2 The role of emergence and historical constraint

A distinctive proposal in earlier generative relational history is that historically generated constraints may be studied as macroscopic configurations of the relational field, whose changes can affect the repertoire of actions and meanings subsequently available (Huang, 2026a). This supplies a problem for the present paper: can a representation organized by occasions, lives and relational surfaces help identify how constraints were reproduced or transformed? It also supplies a limit. A relational record may exhibit an increase in connectivity or the birth of a homology class without thereby proving an increase in human freedom, generativity, or injustice. Mechanism, interpretation and evaluation must be separately articulated.

A helpful minimal distinction is among (i) realized records, (ii) models of the processes generating those records, and (iii) spaces of admissible historical configurations conditioned by some chosen rules. We may write these as R , G and $\text{Adm}(G, R)$, respectively, without asserting that R is the past itself, that G has been uniquely identified or that admissibility is intrinsic to reality. Several distinct generative mechanisms may induce observationally equivalent records. A model of history should therefore describe its assumptions about what is encoded, what remains latent, and what evidence could discriminate among competing mechanisms.

4.3 Events, couplings and primitives across two semantics

Some antecedent relational two-complexes provisionally interpret vertices as events, edges as couplings and faces as historical or relational primitives. The repository History Complex uses a more specific carrier: occasion vertices, life-segment edges and tie lenses. These conventions can coexist as distinct *levels of representation*; they should not be equated term by term. In the general semantics a coupling can have many mechanisms and durations. In the History Complex an edge is an interval of recorded continuity, and the relationship between two beings occupies an attached surface. The interpretive decision is justified by the need to supply each face with a closed attaching word, whose path composition can be tested.

The distinction also guards against an overly restrictive notion of a person. Vertices do not represent persons in either reading; in the specific carrier they encode occasions, while participant-presences (b, o) appear in its refinement. What resembles a historical entity emerges analytically from an extended path and a body of documentary attribution. This arrangement can support a relational account of entities without requiring a philosophical claim that an individual is literally a mathematical edge. A model should say how its chosen carrier bears on each explanatory question and where it ceases to be informative.

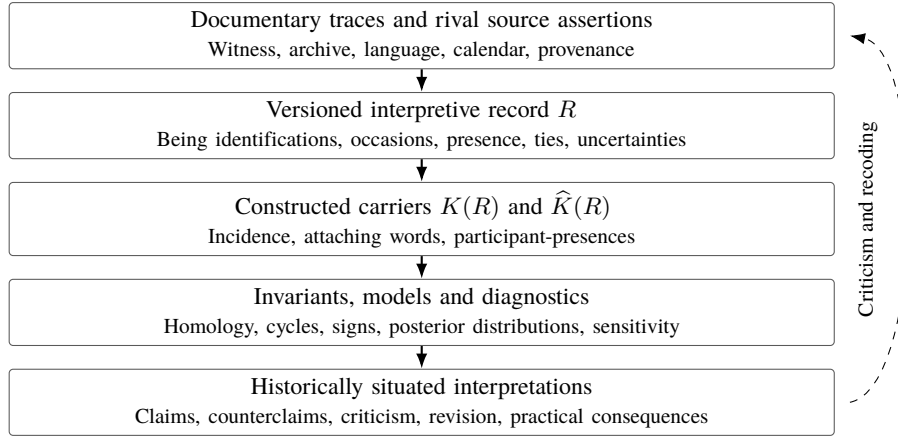


Figure 1: Five distinct research layers. The downward operations formalize sources; the return paths expose every interpretive step to criticism. A mathematical result at the third layer does not, on its own, establish a historical interpretation at the fifth. Original schematic, not empirical data.

4.4 Structural and generative dimensions

Relational complexity, reproduction and generative capacity are analytically distinct. A dense institutional network can circulate information efficiently while excluding challengers. A sparse configuration may contain an opening that enables a new practice, but a topological opening is neither a sufficient nor a necessary condition for political possibility. The earlier framework’s distinction between relational abundance and generative closure therefore suggests examining changes in both recorded structure and the mechanisms determining admissible further configurations (Huang, 2026a,f).

This separation invites a more precise research design. A descriptive quantity such as the Betti number b_1 can be measured on a selected encoded complex. An explanatory construct such as constraints on participation must be operationalized through historically meaningful data: rules, enforcement, resources, competencies, language and permissions. A normative assessment adds yet another question about legitimacy and harm. Combining them in one numerical score would risk obscuring the very mechanisms this inquiry seeks to investigate.

5 Historical Records, Identity and Temporal Resolution

Definition 5.1 (Source-indexed record). At a chosen resolution ρ and for a specified corpus D , a coded historical record is

$$R_{D,\rho} = (B, O, \preceq, P, T, \text{src}, \text{conf}, \text{prov}). \quad (4)$$

Here B denotes record-identified persistent beings (persons, collectives, institutions, states or works); O denotes occasions; $P \subseteq B \times O$ is a presence relation; T is a finite set of typed ties with attested endpoints and, when justified, signed attestations; $\preceq$ is an explicitly chosen order or partial order on occasions; and the final three maps retain evidence, confidence and provenance. An individual assertion may be contested without the dispute being silently collapsed.

The current software employs an ordered list of occasions, resolving dates to intervals and sorting them using a deterministic convention (The Science Infrastructures contributors, 2026e,b). Such a total order is necessary for its present worldline construction. It should be understood as a *coding order*, not an independently established universal temporality. Overlapping occasions, duration-bearing events and incompatible calendar conventions introduce partial or ambiguous precedence. One extension would construct complexes for each admissible linear extension of $\preceq$ and report which invariants remain stable under the resulting ensemble.

The term *being* designates a durable index in the coded record. It does not ontologically presuppose a fixed individual substance. Identifications of a person across names, of a political institution through reform, or of a work through translation are themselves historical claims. The repository uses persistent identifiers such as Wikidata items to check some identities; checking identifiers can assist disambiguation but cannot independently validate the social ties they connect (The Science Infrastructures contributors, 2026b,c). At other resolutions, a persistent being may be modeled as a family of subcomplexes with identifiable continuity conditions rather than a primitive node.

The record is composed of assertions. The distinction between *attested sign*, *unattested sign inferred by the model*, and *historical fact claimed by an interpreter* must survive every computational interface. Consider two documentary traditions offering opposed interpretations of a treaty. A model may preserve parallel observations and conditional reconstructions, with neither observation silently promoted to a ground-truth label. This approach draws on factoid-oriented prosopography (Bradley and Short, 2005).

An occasion has a type, an interval or date, a collection of presences and an evidence trail. A tie connects two coded beings and is attested at one or more joint occasions. A relation involving multiple participants may be represented by an explicit collective object or by higher-arity data retained outside the binary carrier. Binary reduction is useful for constructing faces, but it can obscure asymmetric participation, mediation, exclusion or nested institutions. The reduction therefore belongs to the model's documented limitations.

We use a crucial distinction throughout: the *realization history* is a set of represented and interpreted occurrences supported to varying degrees by documentary evidence, whereas the *continuation space* names admissible modifications of the representational state under specified rules. These categories concern epistemic and modal status. Neither requires a metaphysically privileged “present” or an unexamined linear arrow of time.

6 Constructing the History Complex

For the remainder of the initial construction, assume a finite total order $<$ compatible with the record's adopted chronology. For each $b \in B$ write its ordered occasions as $O_b = (o_{b,0}, \dots, o_{b,m_b})$. A tie $t \in T$ with endpoints $a(t)$ and $b(t)$ has meetings $M_t = (m_{t,0}, \dots, m_{t,k_t})$ within its coded life. At least one joint attestation anchors the life of a tie, and only meetings occurring in that life determine its faces. Ties of a single meeting produce no lenses.

Definition 6.1 (Coarse historical 2-complex). The CW 2-complex $K(R)$ consists of (i) one vertex v_o for each occasion o ; (ii) an oriented edge $e_{b,i}$ from $o_{b,i-1}$ to $o_{b,i}$ for every successive appearance of b ; and (iii) a 2-cell $f_{t,i}$ for successive meetings $m_{t,i-1}, m_{t,i}$ of each tie t . Its attaching word follows $a(t)$ between those meetings and returns along $b(t)$ in reverse.

At the cellular-chain level the face boundary is

$$\partial_2 f_{t,i} = [a(t); m_{t,i-1}, m_{t,i}] - [b(t); m_{t,i-1}, m_{t,i}], \quad (5)$$

where each bracket represents the oriented sum of worldline edges along that interval. Both brackets share their endpoints, so $\partial_1 \partial_2 f_{t,i} = 0$. This is the minimal condition that turns the encoded relation into a valid cellular face. A list assigning one “fact” to each face without an attaching map would define an incidence structure, but it would not suffice for the topological quantities used here.

Proposition 6.2 (Cell counts and Euler characteristic). *With the conventions above, the number of vertices, edges and faces and the Euler characteristic satisfy*

$$V = |O|, \quad E = \sum_{b \in B} n_b, \quad F = \sum_{t \in T} k_t, \quad \chi(K) = |O| - \sum_b n_b + \sum_t k_t. \quad (6)$$

Proof. The vertices and cells are indexed exactly by the respective record constructions. The Euler characteristic of a finite CW complex is the alternating sum of its cell numbers. $\square$

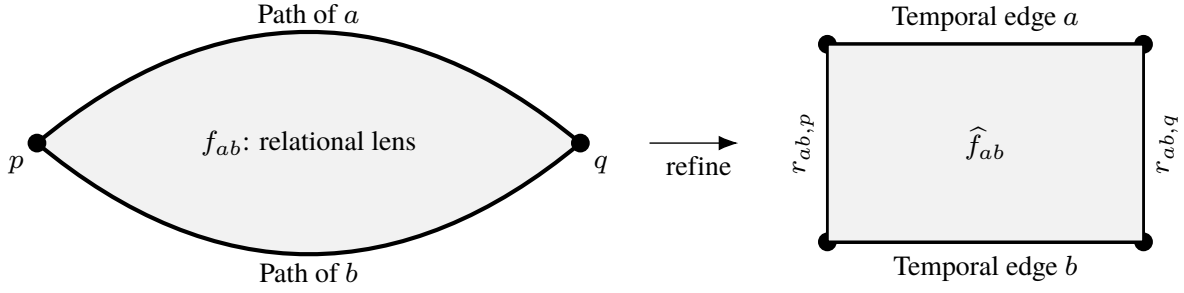


Figure 2: An original schematic of a historical tie as a lens between two worldline paths, and its refinement with participant-presences and rungs. The depicted shape is a visualization of an attaching map, not a literal metric or spatial distance.

With integer or field coefficients, the cellular boundary operators $\partial_2 : C_2(K) \rightarrow C_1(K)$ and $\partial_1 : C_1(K) \rightarrow C_0(K)$ determine

$$H_1(K) = \ker \partial_1 / \text{im } \partial_2, \quad H_2(K) = \ker \partial_2. \quad (7)$$

275 An H_1 class is a cycle in the encoded encounter structure not filled by its encoded faces. It may suggest a place for source investigation, but does not establish that a real historical relationship was missing. A nontrivial H_2 class represents a closed combination of relational surfaces. The distinction between topological absence *in the record* and historical absence *in the world* is categorical, not merely a small statistical caveat.

280 6.1 The refined complex and local relations

A refined complex $\hat{K}(R)$ opens an occasion into its participant-specific presences (b, o) . The vertices are presences. Worldline segments form longitudinal edges. A tie creates a transverse rung at each joint meeting. Consecutive rungs of a tie are joined by quadrilateral lenses, while triplets of pairwise ties at an occasion form local triangular faces. The boundary of a lens is

$$\partial f_{t,i} = a[m_{i-1}, m_i] + r_{t,m_i} - b[m_{i-1}, m_i] - r_{t,m_{i-1}}. \quad (8)$$

285 Under the specified collapse, transverse rungs of an occasion are identified and the coarse encounter complex is recovered. This refinement is conceptually important: the coarse representation summarizes a meeting as one vertex, while the expanded representation preserves which pairs stand in which relations at the meeting. A signed relationship cannot in general be represented by a gauge connection on the coarse carrier with the desired face holonomies when local triangular relations change frustration (The Science
290 Infrastructures contributors, 2026e).

6.2 Joining histories and choosing resolution

The repository can join multiple disciplinary canons along independently identified beings, occasions and ties. Such a join is not generally identical to taking the union of complexes already built separately: shared worldlines can be subdivided by newly introduced occasions, and previously distinct tie records may be
295 merged (The Science Infrastructures contributors, 2026e). An integrated complex therefore depends on both entity resolution and the gluing policy. Interdisciplinary structure becomes measurable only once the assumptions underlying the join are explicit.

A family $\{K(R_{D,\rho})\}_\rho$ over several levels of resolution may be more informative than one supposed canonical history. Maps among these complexes should record whether refinement merely subdivides
300 cells, adds evidence, changes identity decisions, or alters the inferential model. A change in Betti numbers caused by an added source is different from a change caused only by a subdivision. In particular,

invariants preserved under subdivision should not be confused with invariants stable under empirical recoding.

7 Alternative Complexes and the Choice of Historical Carrier

305 The construction $K(R)$ is one defensible response to a specific representational question. Historical encounters admit several other cellular or relational realizations, each preserving different information. Comparing these alternatives helps distinguish the results dependent on the chosen carrier from findings resilient across representations. The repository itself offers Dowker complexes of co-presence, the cellular construction with filled shells, and time-filtered complexes (The Science Infrastructures contributors, 2026e,a).
310 2026e,a). A rigorous programme ought to study such variation as substantive modeling uncertainty rather than a nuisance to be hidden.

7.1 The incidence relation and Dowker constructions

Let $P \subseteq B \times O$ record which beings appear at which occasions. Two Dowker constructions arise. The first is a complex on B whose simplices consist of finite sets of beings that share an occasion. The second
315 is a complex on O whose simplices consist of finite sets of occasions sharing a being. These objects express overlapping information about co-presence. They can be compared using Dowker's theorem on the homology of relations (Dowker, 1952). Because a shared occasion produces a high-dimensional simplex even when no substantive tie is attested, co-presence and relation should not be confused. The Dowker object answers a question about joint appearance; the History Complex's lenses ask about a
320 persisting pairwise relationship, with its own source and coding conditions.

7.2 Hypergraphs, nerves and simplicial closures

A history recorded as a sequence of meetings of varying membership naturally defines a temporal hypergraph: each occasion carries a finite participant set. Taking its downward simplicial closure adds every possible subset of each meeting as a simplex. This can be useful for studying collective presence, but
325 has an interpretive cost. The presence of three participants at the same gathering does not necessarily mean that all three pairs had independently documented ties. A simplicial triangle introduced by closure may fill a cycle merely because a meeting was entered, while a tie-based cellular face demands an attested relation sustained through meetings. The mathematical structures are both legitimate; the source obligations differ.

330 A nerve construction can instead be formed from a cover by institutions, archival collections, places, social milieus or periods. Its simplices express nonempty intersections of chosen coverings. Nerve-based topology can reflect shared fields of practice, but its meaning depends strongly on the nature and scale of the cover. The category of an "intersecting historical field" should therefore be declared before its homology is interpreted as a historical property.

335 7.3 Directed paths and partially ordered histories

A chronology of records is not generally equivalent to an objective causal order. Some documentary sequences are compatible with a partial ordering, with uncertain and overlapping dates. Directed path complexes and directed flag constructions provide potential alternatives when direction or admissible path composition is central to the question. Their use can also prevent temporal direction from being
340 erased by ordinary undirected homology. Yet directedness does not imply causality, just as a dated citation does not prove direct intellectual transmission. An investigator must distinguish the order imposed for an algorithm from evidence of influence or responsibility.

Table 2: Comparison of candidate historical carriers. The interpretive differences are part of the research design.

Carrier	Primitive encoded object	Primary interpretive risk
Temporal hypergraph	Joint attendance	Co-presence mistaken for interaction
Dowker complex	Intersections of presence relation	Simplices treated as documented ties
History CW complex	Occasion paths and relational lenses	Missing faces treated as historical absence
Refined CW complex	Participant-presences and signed links	Binary signs treated as complete social meaning
Relative complex	A chosen subhistory and boundary	Chosen boundary treated as natural
Directed paths	Ordered or admissible traversals	Documentary order treated as causation

7.4 Relative complexes and boundary conditions

For a historical subfield $L \subseteq K$, relative homology $H_n(K, L)$ can describe cycles and boundaries modulo a chosen, explicitly named context. This is attractive when studying historical changes against a recognized boundary of institutions, languages, jurisdictions or records. The mathematical boundary is a declared comparison object, not an automatically objective historical perimeter. A complementary approach is to attach higher-dimensional cells to close existing two-cycles. Such a completion changes H_2 and answers a different question from the original 2-complex. In particular, filling closed shells should be recorded as a modeling choice, not mistaken for additional attested historical relationships.

7.5 A comparison criterion

The preferred carrier depends on the task. For documentary transparency, a representation that returns every algebraic object to source assertions may be especially useful. For an encounter's internal structure, a refined or hypergraphic model may preserve more information. For long-run cross-disciplinary structure, a cellular carrier with an explicit method of joining records may facilitate computation. The defensible criterion is therefore task-sensitive fidelity: which objects, morphisms, uncertainty properties and invariants does the historical question require? A comparison should report what changes when the carrier changes, including changes in the questions the formalization makes answerable.

8 Translations Between Historical Cellular Representations

This section treats the coexistence of several legitimate cell conventions as a mathematical and methodological problem. The initial History Complex is useful precisely because its choice of cells makes continuity between documented occasions computable. The event-coupling-primitive model is useful because it exposes interface structure and an explicit grammar. Translation between them should preserve the history questions to which an analysis commits, while recording which structures cannot be preserved without changing the carrier.

8.1 The distinction between semantic maps and chain maps

Let $H(R)$ be the repository construction from a record R , and let $S_{Q,\rho}(R)$ be a social-complex reconstruction under question Q and resolution ρ . A *semantic correspondence* Θ may associate a life segment in $H(R)$ with a persistent substructure in $S_{Q,\rho}(R)$. It can even link a tie face to several typed couplings and their generative history. In general, however, such a correspondence is a relation between interpreted structures, not a continuous map of topological spaces.

A cellular map $f : X \rightarrow Y$ that respects skeleta induces homomorphisms $f_\# : C_n(X) \rightarrow C_n(Y)$

satisfying

$$\partial_n^Y f_{\#,n} = f_{\#,n-1} \partial_n^X. \quad (9)$$

If this identity is absent, a semantic transformation cannot automatically transport homology classes. In particular, a map that sends a repository edge to a face in the general model is dimension-shifting. Without further constructions, it is *not* a degree-zero cellular chain map. One may instead use a common refinement, a bicomplex, a correspondence, a diagram of spans, or a derived construction that records the change of degree. Each choice carries technical obligations of its own.

Definition 8.1 (Question-relative comparison package). A comparison package for two historical representations is a tuple

$$\mathcal{P} = (Q, \rho, X, Y, D_X, D_Y, \Theta, \mathcal{I}), \quad (10)$$

where X, Y are carriers, D_X, D_Y their typed interpretations and evidence maps, Θ a specified partial translation of represented historical claims, and $\mathcal{I}$ the collection of observables or properties the comparison intends to preserve. The package may include a cellular map, but is not assumed to do so.

8.2 A counterexample to dimension-free identification

Consider three occasions o_0, o_1, o_2 connected by three life-continuity edges and suppose a tied historical process is modeled by an attached two-cell. The resulting cell may fill a cycle. In another model, the same three occasions may remain three event vertices, while the historically significant primitive is an institution represented by a face incident through a different set of coupling edges. Both descriptions can be defensible for their own questions. They need not have isomorphic C_1, C_2 , or boundary matrices, nor the same Betti numbers. Their disagreement can arise from *different questions* and *different cells*, prior to any dispute about the underlying documentary record.

This example motivates a three-part discipline. First, formulate the problem in ordinary historiographical terms. Second, specify what the cells and attachment maps mean for that question. Third, identify the invariant or relation being compared and the maps warranting that comparison. If an alleged historical finding disappears when reasonable representations are changed, it may be a fragile coding-dependent feature rather than a demonstrated property of historical organization.

8.3 When a common refinement can help

Suppose Z resolves the relevant events, identity claims, tie intervals, coupling interfaces and primitive distinctions used by both X and Y . A span

$$X \xleftarrow{p} Z \xrightarrow{q} Y \quad (11)$$

can provide a comparison when p and q have documented semantic meanings. Neither arrow is automatically a homotopy equivalence. If they are cellular and are shown to induce isomorphisms on designated homology groups, the corresponding invariants may be transported; if not, the changes themselves are analytical results about the representation. For instance, an additional institutional 2-cell can change H_1 by imposing a previously absent relation among edge cycles. This is precisely why a higher-order interpretation cannot be treated as a costless relabeling.

When no suitable common carrier is available, a provenance-preserving crosswalk may be more honest than a forced mathematical functor. Such a crosswalk records source assertions, alternative identifications, unresolved semantic differences and researcher decisions. It permits future refinement without claiming lossless translation. The inability to translate one construction fully is a research finding when it is documented and delimited.

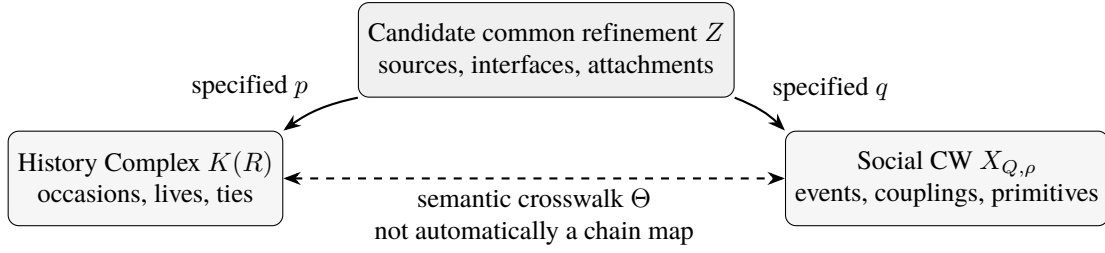


Figure 3: A comparison through a hypothesized common refinement. Arrows express proposed, typed translations, not automatic homeomorphisms or homology isomorphisms.

8.4 Structural, generative and historical equivalence

The previous CW manuscript distinguishes several relations that are sometimes conflated. *Informational equivalence* asks whether a decoder can recover specified data. *Topological or structural equivalence* concerns the cell structure, incidence, homology or chosen topological equivalence. *Generative equivalence* concerns the reachable transitions admitted by grammars. *Historical equivalence* additionally examines realized derivations and provenance. A pair of structures can share Betti numbers while differing under every stronger criterion. Even isomorphic endpoints need not share their generative histories.

To state the distinction, let $\text{Reach}_\Gamma(X)$ be the class of reachable modeled configurations and $\text{Der}_\Gamma(X, Y)$ the realizable derivations from X to Y . An equivalence of endpoint complexes need not induce

$$\text{Der}_{\Gamma_1}(X_1, Y_1) \cong \text{Der}_{\Gamma_2}(X_2, Y_2). \quad (12)$$

Indeed, the same endpoint may be reached by one authorized revision in one institution and by numerous informal practices in another. Their endpoint structures may be observationally similar although their authority, reversibility and legitimacy differ. Equivalence claims should therefore state their *axis* and their evidential warrants.

8.5 A preservation ladder

For comparative work we propose a preservation ladder, ordered by increasingly demanding evidence rather than by any universal hierarchy of scientific merit: equality of selected summary statistics; preservation of a selected homology group; compatible incidence and typing; preservation of admissible rewrite transitions; preservation of realized genealogies; and preservation of documentary warrant and interpretive alternatives. Movement upward generally requires data unavailable to a summary topology alone. A practical system may legitimately stop at a lower level if the research question warrants it. It should disclose that choice.

9 Signed Relations, Gauge Transformations and Inference

For some historically documented relationships, a coding scheme can assign a sign $u_{t,o} \in \{+1, -1\}$ at an occasion: for example, a convention distinguishing cooperation from antagonism in a particular relation type. Such signs are highly lossy. They should be introduced only when the codebook defines the meaning of both values, the conditions for an ambiguous or missing value, and the reliability of multiple coders. They do not measure moral value, legitimacy or causal responsibility. The current History Complex offers a preliminary typed codebook and records which signs are documented (The Science Infrastructures contributors, 2026b).

The expanded complex can carry a $\mathbb{Z}_2 = \{\pm 1\}$ link variable g_e on each oriented edge, with $g_{\bar{e}} = g_e^{-1} = g_e$. For each vertex v a change of frame $\phi_v \in \mathbb{Z}_2$ acts on a link $e : v \rightarrow w$ by

$$g_e \longmapsto \phi_v g_e \phi_w. \quad (13)$$

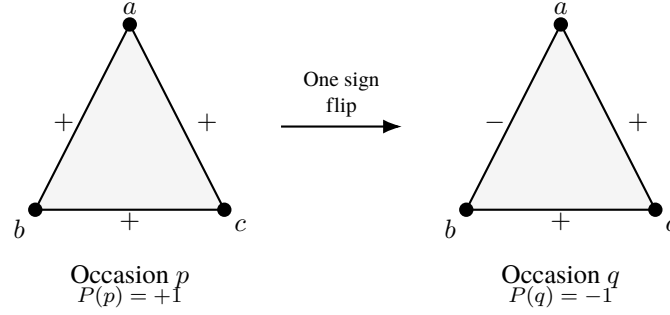


Figure 4: A synthetic signed triangle at two joint occasions. A single coded tie flips, changing the triangle product while leaving the underlying coarse topological carrier unchanged. This is an illustrative algebraic example.

The product $h_f(g) = \prod_{e \in \partial f} g_e$ around a face is invariant under this transformation. Because a being's temporal edges form a forest, one can choose a temporal gauge with temporal links $+1$ and retain one residual switching transformation per worldline (The Science Infrastructures contributors, 2026e; Zaslavsky, 1982).

For a tie t coded at two joint occasions p, q , the plaquette or lens holonomy reduces to

$$h_{t;pq} = u_{t,p} u_{t,q}. \quad (14)$$

It equals -1 when the coded sign flips. For a triangle of beings a, b, c at occasion o , the local product is

$$P_{abc}(o) = u_{ab,o} u_{bc,o} u_{ac,o}. \quad (15)$$

It remains invariant when the signs incident to one being are jointly switched. Structural-balance terminology calls a negative product *frustrated* (Cartwright and Harary, 1956; Zaslavsky, 1982). In this paper, frustration is an observable of the specified sign convention, not a judgment that its participants were historically irrational or socially defective.

Proposition 9.1 (A discrete compatibility identity). *Suppose three pairwise ties among a, b, c are encoded at two joint occasions p, q . Then*

$$P_{abc}(q) = P_{abc}(p) h_{ab;pq} h_{bc;pq} h_{ac;pq}. \quad (16)$$

Proof. Substitute Equation (14) into the right side. Every sign at p appears once in $P(p)$ and once in the three lens products and hence squares to $+1$; the remaining three factors multiply to $P(q)$. $\square$

Equation (16) is a discrete compatibility identity, sometimes described in the language of a Bianchi identity. It does not imply that this relation among historical assertions is a physical field equation. It is useful primarily because it reveals which signs and local triangles must be represented for the model to be internally consistent.

9.1 A conditioned ensemble, not a reconstruction of certainty

The current implementation employs a simple Gibbs family in which lens terms favour persistence and triangle terms favour balance. For a collection $F(\hat{K})$ of faces, define a statistical weight

$$Z(\beta) = \sum_{g \in \mathbb{Z}_2^{E(\hat{K})}} \exp\left(\sum_{f \in F(\hat{K})} \beta_f h_f(g)\right). \quad (17)$$

Conditioning on source-attested signs produces a posterior over signs left unattested by the record. A fitted parameter β_c for a tie type represents an assumed propensity for persistence, while a balance coefficient β_Δ describes the fit of triangle products to the adopted coding. These are model parameters, and their estimation is vulnerable to selection bias and dependence among ties.

Using $e^{\beta h} = \cosh \beta (1 + h \tanh \beta)$, summation over each independent link sign eliminates face selections with odd incidence on any edge. In the unconstrained edge formulation this yields

$$Z = 2^{|E|} \left(\prod_f \cosh \beta_f \right) \sum_{z \in \ker(\partial_2 \bmod 2)} \prod_{f: z_f=1} \tanh \beta_f. \quad (18)$$

Thus the combinatorial expansion is indexed by mod-two 2-cycles, conventionally described as spin foams (Baez, 1998; The Science Infrastructures contributors, 2026e). Equation (18) is an exact algebraic identity for the specified finite gauge partition function. Its existence alone establishes neither empirical fit nor a quantum character of historical relations. The case of negative coefficients requires care because expansion weights then need not be nonnegative probabilities.

For the narrow purpose of interpreting a model of historical ties, the most useful quantities may be simpler than the full foam expansion: posterior sign probabilities, trajectory-conditioned persistence, uncertainty bands for a network cut, and the sensitivity of results to the coding of a disputed event. In contrast, importing non-Abelian representation labels or continuum quantum-gravity dynamics would require new measurements and an independently justified semantics. The repository currently describes higher-group dynamics as exploratory rather than fitted (The Science Infrastructures contributors, 2026d).

10 Formal Models, Interpretation and Physical Analogies

Mathematical analogies can provide powerful organizing devices while also inviting false equivalences. In this paper, topology, gauge theory and spin foams perform different tasks. Cellular topology supplies a representation-independent calculus once a specific cell attachment is fixed. A gauge model supplies a language for transformations of local frames and invariants of signed relational coding. Spin-foam expansion supplies a combinatorial rewriting of a chosen statistical partition function. None of these operations directly identifies causes, moral values, physical forces, or laws of history.

10.1 Gauge symmetry as a coding invariance

The $\mathbb{Z}_2$ transformation described in Equation (13) corresponds, in temporal gauge, to a systematic reversal of the signs of all ties incident to one being. The invariance of loop and triangle products can be interpreted as robustness under this specific switching operation. It is meaningful to ask whether such a switching operation preserves the historical content of the chosen codebook. If $+1$ and -1 denote cooperation and conflict as independently meaningful categories, reversing all ties around an actor may alter empirical meaning, even though a gauge partition function remains invariant. The physical interpretation of gauge symmetry therefore cannot simply be inherited by historical semantics. One must distinguish *formal invariance of observables* from *substantive equivalence of coded descriptions*.

This is a productive criticism rather than a rejection. Gauge-invariant quantities may show which combinations of signs have a mathematical interpretation independent of arbitrary representational orientations. Where the signs have fixed archival meanings, the gauge model remains an explicitly chosen generative statistical family, and the symmetry is a modeling assumption to investigate. The possibility of switching-invariant signed-network analysis supplies one comparison, not a guarantee of historical semantic invariance (Zaslavsky, 1982; Cartwright and Harary, 1956).

10.2 The spin-foam expansion and its standing

Equation (18) exhibits a mod-two cycle expansion of the unconstrained partition function on the refined complex. In physical theories, a spin foam is usually endowed with representation labels, amplitudes and an interpretation derived from a physical action or related dynamical construction (Baez, 1998). Here the term names an algebraic configuration of selected cells in a finite coded model. Its presence does not imply quantum probabilities, general relativity, or a physical spacetime. The historical source record

also does not measure the geometric quantities from which a non-Abelian representation or amplitude could be identified. Extensions to other groups require new observables and an explicit account of how such observations constrain representation labels.

The model is nevertheless useful because it makes one form of dependence across relations exact. Selecting face terms in the character expansion under an edge parity constraint shows how a local relational description participates in a globally admissible set of cycles. The corresponding notion of collective constraint may inspire historical research questions about repeated alliances, institutional commitments or the continuity of cooperation, while each substantive interpretation must be checked with appropriate evidence.

10.3 Equivalence is relative to a problem

The earlier work on the equivalence of mathematical representations of consciousness foregrounds an issue that also applies to history: descriptive equivalence may be local to an observation, invariant or question, even where the underlying formulations differ (Huang, 2026h). For two historical representations K and K' , equality of Betti numbers is weaker than homotopy equivalence; homotopy equivalence is weaker than agreement about documentary provenance, chronology or causal dependencies. Conversely, two formalizations can differ topologically while supporting the same well-evidenced substantive explanation. The levels of agreement should be declared before comparing models.

10.4 An interpretive safety condition

A responsible reading of an invariant asks four questions: what is being counted or compared mathematically; which source operations generated the object; which alternative coding choices could change it; and what historical inference would require additional evidence? A change in H_1 may suggest an altered documentary circulation structure, but it does not independently show the destruction of a human community. An increase in estimated network balance may follow from a diplomatic treaty, from a recoding convention, or from the exclusion of opposing voices. The methodological value of the model lies partly in making these distinct possibilities visible and testable.

11 An Exact Minimal Example: Three Lives and Two Occasions

Consider a synthetic, explicitly nonhistorical record with two occasions $p \prec q$ and three beings a, b, c , each present at both. Each unordered pair has one continuing tie attested at both occasions. The coarse complex K contains two vertices, three parallel worldline edges, and three lenses attached between each pair of edges. Its cell counts are $V = 2$, $E = 3$, $F = 3$ and $\chi = 2$. The boundaries of the faces may be written

$$\partial f_{ab} = e_a - e_b, \quad \partial f_{bc} = e_b - e_c, \quad \partial f_{ac} = e_a - e_c. \quad (19)$$

Since the three edges connect the same two vertices, $\text{rank } \partial_1 = 1$. The image of ∂_2 has rank two, so

$$b_0 = 1, \quad b_1 = (3 - 1) - 2 = 0, \quad b_2 = 3 - 2 = 1. \quad (20)$$

A generator of $H_2(K; \mathbb{Z})$ is $f_{ab} + f_{bc} - f_{ac}$ with orientations as in Equation (19). Its boundary cancels exactly. At this scale the three lenses form a closed surface.

Now let all three tie signs be positive at p . At q , suppose the signs of ab and bc remain positive but the sign of ac becomes negative. Then $P_{abc}(p) = +1$ while $P_{abc}(q) = -1$. The holonomies are $h_{ab} = h_{bc} = +1$ and $h_{ac} = -1$, and Equation (16) is satisfied. The change of frustration is information about the coded field, although the coarse complex still has precisely the same Betti numbers. This demonstrates why topology and signed-relational dynamics should be kept distinct.

The refined complex $\hat{K}$ has six presence vertices, three longitudinal edges, six rungs and five 2-cells (three longitudinal lenses and one triangle at each occasion). Its Euler characteristic is likewise

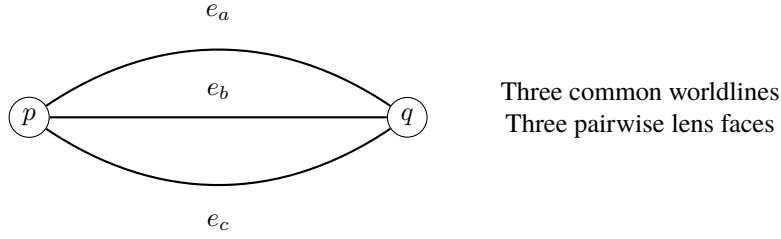


Figure 5: One-dimensional skeleton of the synthetic record. The three 2-cells attached along the pairs of paths are described algebraically in Equation (19). Their filling is not fully shown in this schematic.

550 $6 - 9 + 5 = 2$. The transverse triangles allow the $+1$ and -1 products at the two occasions to be represented without demanding an inconsistent gauge connection on the coarse carrier. The example is small enough to verify exactly and suggests a first unit test for the computational implementation: independent calculations of its chain matrices, Betti numbers and the compatibility identity must agree.

555 A richer empirical example should then vary documentary completeness while keeping the synthetic ground truth known. Removing one attestation leaves topology unchanged if the encoded tie's life remains established, but enlarges uncertainty in the associated sign. Removing the tie itself changes the set of faces and may change homology. This difference gives a practical test of whether the application distinguishes missing relational knowledge from missing relational structure.

12 Scale, Persistence and the Geometry of Historical Resolution

560 A historical complex has no privileged grain. The same documentary world can be organized as single-day occasions, prolonged assemblies, schools spanning generations, or transnational intellectual movements. A claim about historical topology therefore depends on the resolution at which an occasion is distinguished, on the criterion for a persistent being, and on the documentary threshold for recording a tie. This section explores how multi-resolution analysis can preserve such dependencies rather than
565 conceal them beneath an apparently unique representation.

12.1 Refinement, extension and revision

Let R_ρ be a record at resolution ρ and let $K_\rho = K(R_\rho)$. At least three kinds of transformation can occur. A *cellular refinement* replaces cells with smaller cells while preserving the underlying represented geometric object; subdivision ought to preserve homology. An *evidential extension* adds assertions,
570 beings or occasions that change the represented history, and it may change the homotopy type. A *revision* changes the identity, dating or classification of assertions already present; its map need not be injective or homology preserving. Writing all three as a single “increase of resolution” would incorrectly suggest that every change is a harmless subdivision.

A refinement analysis consequently needs an explicit morphism $r_{\rho\rho'} : K_\rho \rightarrow K_{\rho'}$ or a comparison span through a common object. If the construction depends on a changed identity decision, no canonical direct map may exist. One can instead construct a diagram $K_\rho \leftarrow Q \rightarrow K_{\rho'}$ that records which cells have traceable correspondences. The problem of finding Q is itself a historical matching problem requiring source and interpretive warrants. Where the map exists, induced maps $H_k(r_{\rho\rho'})$ permit the question of which classes persist under the stipulated change. Where it does not, a bare comparison of Betti numbers
580 is often insufficient.

12.2 A filtration and its limits

Suppose that each occasion o has a chosen filtration value $\tau(o)$, perhaps the lower bound of a dated interval. A nested sequence of subcomplexes $K_{\tau_1} \subseteq K_{\tau_2} \subseteq \dots$ yields persistent homology (Edelsbrun-

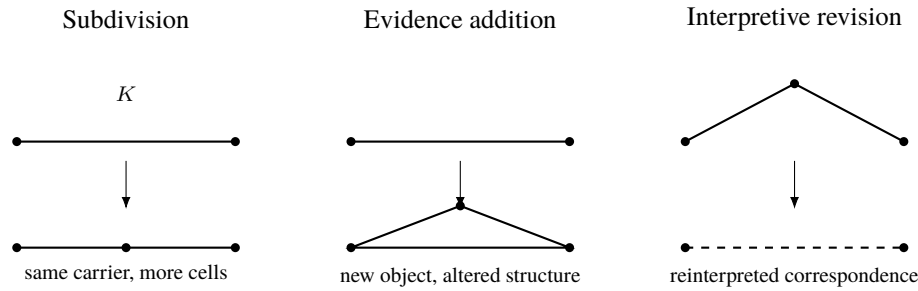


Figure 6: A schematic distinction among subdivision, evidential extension and interpretive revision. Only the first transformation is required to preserve the represented topological object. These are modeled operations, not empirical chronologies.

ner and Harer, 2010). The filtration rule must specify how a face enters when its boundary involves occasions that have already appeared, and whether intervals are ordered by their earliest, latest or conventionally resolved date. For uncertain dates, one may construct a family of admissible filtrations, or use interval-parameterized constructions instead of a single total chronological order.

A persistent class records the lifespan of a particular homological feature under that filtration, not the duration of a real person's influence. A long bar may arise because the corresponding coded path is resistant to later face additions; it may also reflect sparsity of surviving sources. The interpretation should examine how the bar responds to archives newly included, ties recoded, and changes in the definition of an occasion. Persistence across thresholds of source confidence is an additional, methodologically different question.

12.3 Historical duration and slow variables

Long-duration historiography identifies processes unfolding at different characteristic rates, including material conditions, institutional reproduction, intellectual formation and episodic events (Braudel, 1958; Abbott, 2001). Earlier generative relational history proposes that slow macroscopic constraints and fast actions may be coupled rather than ontologically separated (Huang, 2026a). The computational problem is how to define observables capable of showing when a slow variable changes and what evidence would distinguish it from an aggregate of fast fluctuations. A time slice of relationships can be compared with a moving window, but neither representation by itself proves a mechanism of phase transition.

One possible approach is to define a sequence of measured relational descriptors $z_\rho(t)$ and a separately justified constraint estimate $q_\rho(t)$, then examine cross-scale response functions. For example, a locally estimated relation between change in a source-grounded institutional restriction and the frequency of certain coded encounters would be a testable hypothesis. The relation between two quantities must be designed so that it is not merely an artifact of the shared codebook. Apparent sudden changes should be examined against archival visibility, selection practices and changes in calendar conventions.

12.4 An interval-aware version of the model

If an occasion is dated by an interval $I_o = [\ell_o, u_o]$, an admissible chronological ordering $\prec$ should be compatible with all secure precedence relations, including $u_o < \ell_p \Rightarrow o \prec p$. The set $\Omega(R)$ of admissible total extensions can be studied as an uncertainty object. The collection $\{K(R; \prec) : \prec \in \Omega(R)\}$ is then a family of complexes over order uncertainty, rather than a unique complex generated by an arbitrary file order. Some homology classes may survive every admissible order while others depend on date resolution. The distinction can be reported without requiring a probability distribution over orderings; when probabilities are introduced, their assumptions and provenance should be documented.

The construction invites a qualified invariant: a property is *order-robust* over $\Omega(R)$ if it holds for every allowed order. It is *resolution-robust* over a stipulated family of codings if it holds under every supported

encoding choice. These are properties of a research protocol and its evidence, not transcendent features of all historical reality.

13 Higher-Order Social Constitution and Cellular Attachment

The purpose of this section is to distinguish historical association from compositional constitution. Network analysis commonly represents ties among actors, sources or events. A higher-dimensional cell model can ask whether several such ties participated in the historical formation of an organized collective, rule, practice or fact. Its additional strength depends on evidentially justified attaching maps and on the question being investigated, rather than the automatic filling of every small cycle.

13.1 Closure as a historical claim

Suppose a record attests that three institutions a, b, c exchanged letters in pairwise channels. The 1-skeleton may contain a triangular cycle $z = e_{ab} + e_{bc} + e_{ca}$, with orientations fixed consistently. There are at least two possible reconstructions. One leaves the cycle unfilled because no joint body or common procedure is established. The other attaches a 2-cell f satisfying $\partial_2 f = z$ after corroborating a constituted joint agreement. In the first reconstruction $[z]$ may define a nontrivial H_1 class; in the second it becomes a boundary. This difference can be checked exactly over a stated coefficient ring.

The attachment itself is not evidence for an agreement. It is the formal consequence of a *coding decision* that claims an agreement on separate evidential grounds. In humanities research, the interpretive distinction among conversation, coordination, legal constitution and shared practice is frequently more important than the count of triangles. One must therefore record the source assertions on which the decision rests and distinguish nominal constitution from social effectiveness.

Proposition 13.1 (A filled cellular cycle becomes homologically trivial). *Let X be a CW complex, let $z \in Z_1(X; \mathbb{F})$ be a cellular cycle and let $Y = X \cup_{\varphi} D^2$ attach a two-cell f whose cellular boundary is z . Then the image of $[z]$ in $H_1(Y; \mathbb{F})$ vanishes.*

Proof. In $C_2(Y; \mathbb{F})$ one has $\partial_2 f = z$, hence $z \in B_1(Y; \mathbb{F})$. Therefore its image is the zero homology class. The statement requires the specified attaching boundary; it makes no claim about the historical truth of the proposed filling. $\square$

13.2 Why larger history requires more than triangles

A complex history of scientific innovation can involve the creation of a discipline that coordinates laboratories, journals, standards, instruments and pedagogical curricula. Pairwise ties among individual researchers provide valuable information but may leave the disciplinary architecture implicit. A higher-cell representation can preserve specific incidence among constituted working groups, shared procedures and larger coordinated structures. Such a structure can be represented by a 3-cell attached to a suitable 2-skeleton only if a meaningful spherical attaching diagram has been specified. Arbitrary collections of three-dimensional labels on documents do not establish the necessary boundary.

Conversely, a genuinely high-order historical structure may be most transparently modeled as a *hyperedge*, a typed relation, an institution-specific object, a sheaf of local information or a category of processes. CW cells are tools rather than an obligatory ontology. A methodological advantage arises when the question concerns filling, gluing, relative topology, higher incidence or homological obstructions. For the study of legal jurisdictional overlap, a typed hypergraph may prove clearer; for coupled local norms and incompatible records, a presheaf-like account may better express restrictions and failed global compatibility. The appropriate carrier should be chosen after the research question is framed.

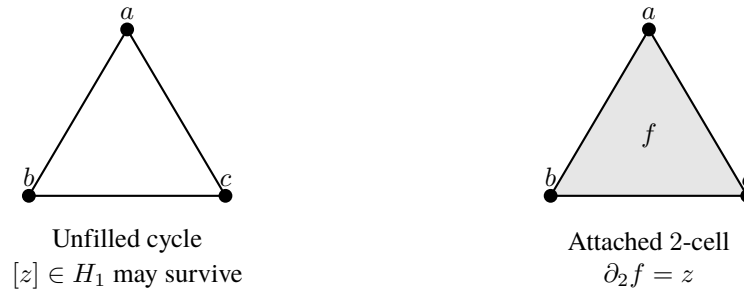


Figure 7: The same observed triangular incidence can be interpreted as an unfilled 1-cycle (left) or as the boundary of a warranted constitutive 2-cell (right). This is a synthetic modeling comparison, not a reconstruction of any historical coalition.

13.3 Institution formation, persistence and dissolution

660 An institution can be analytically decomposed into constitutive acts, recurrent performances and inter-
 faces with an environment. Formation may add cells, strengthen type constraints or make a previously
 inadmissible rewrite admissible. Persistence may involve continued enactment even when foundational
 actors depart. Dissolution can remove an operative coupling, disable admissible actions, transform au-
 thority or gradually empty a constituted body of effective capacity. These processes are historically
 665 different and need not yield different untyped Betti numbers.

For an illustrative institution, let $I_h \subseteq X_h$ be its relationally instantiated subcomplex under a docu-
 mented historical stage h . A persistence rule Π may compare supported organizational predicates along
 a history; it should not be reduced to literal preservation of every cell. Different officeholders may main-
 tain a legal procedure while changes of purpose, authority and membership produce a different institution.
 670 A useful model therefore combines local continuity witnesses, constraints, types, authority relations and
 provenance rather than only topological inclusion.

13.4 Shared boundaries and contested constitution

Two organizations may share participants, resources, legal norms and archival institutions without being
 one organization. Formally, the intersection $I_1 \cap I_2$ can be nonempty without a justified higher cell
 675 expressing their merger. Conversely, a composite institution can be constituted despite weak personal
 ties when law, infrastructure and recurrent procedures supply alternative interfaces. A historical recon-
 struction should make the criterion of composite identity explicit, and should keep contested constitution
 visible as alternative candidate complexes or weighted claims. In particular, a historian should not equate
 the absence of a documented attachment with proof that no historical relation existed.

680 13.5 Dimension, scale and the research question

The n in an n -cell counts topological dimension within a chosen CW representation; it is not a tem-
 poral index or level of causal power. A 4-cell does not describe a historically later or more advanced
 society than a 2-cell. Likewise, an intercontinental economic system can be modeled with relatively low-
 dimensional cells, while a local constitutional configuration can have higher incidence dimension. The
 685 earlier CW programme emphasizes precisely this independence of cellular dimension, generative stage
 and institutional scale. The present proposal retains it in every comparison of world history, diplomatic
 history, knowledge history and the arts.

14 Historical Interpretation Across Complexes

14.1 Multiple durations, registers and disciplinary boundaries

690 A complex acquires historical significance through the questions to which its cells and mappings are answerable. At one scale, an exchange of letters may be an occasion. At another, a sustained correspondence may be a relatively persistent tie. A collection of works may constitute an intellectual movement only after interpretive criteria for recognition and participation have been stated. This plurality resonates with historiographical approaches to heterogeneous durations (Braudel, 1958; Abbott, 2001; Sewell, 2005). It discourages equating the software's vertical plotting axis with an ontologically fundamental time dimension.

The repository's coloured tie categories distinguish six subject areas. At a coarse resolution, a face may have the theme "music" while touching vertices and worldlines that also participate in political or philosophical histories. A subcomplex assembled from one theme is not an isolated historical substance. 700 Its boundary and intersections matter: a work dedicated to a sovereign may circulate aesthetic, social and political significances that no single colour captures. A practical extension would allow a tie to have multiple typed, source-linked interpretive annotations rather than require one exclusive disciplinary label.

Persistent beings, understood as coded identities, may support biographies and institutional histories. 705 Yet persons, institutions and works are not mathematically or ethically interchangeable. A treaty text has documentary continuities different from those of a human life; an institutional entity can reproduce itself through changing membership; and states have contested continuity criteria. A generic path construction provides a shared syntax while leaving these domain-specific semantics open to explicit review.

14.2 Loops, closures and historically situated silence

710 The homology of a complex can locate closed encounters that are not filled by attested relational lenses. This is a useful research heuristic. A loop might identify a set of persons known to have attended overlapping occasions without any documented continuing tie between some pairs. It could prompt investigation of archives, correspondence, exclusion mechanisms or the decisions by which the corpus was assembled. But the logical direction is one-way: a gap in coding motivates inquiry; it does not prove a previously hidden connection or its absence. 715 The absence may result from inaccessible archives, unequal preservation, disciplinary classification, or a coding decision.

Likewise, a closed two-dimensional shell is a combination of encoded continuing relations whose oriented boundaries cancel. It may help describe a relatively interdependent configuration, yet its mathematical closure does not by itself certify social cohesion or historical stability. Historical meanings 720 require looking at the types, content and consequences of the encoded actions. An invariant can remain stable while the field values, participants' interpretations or material effects change substantially, as the example in Section 11 shows.

14.3 Epistemic asymmetry, exclusion and justice

725 Documents are not random samples of lived relations. Their production, survival, cataloguing, digitization and scholarly interpretation all involve asymmetries of institutional authority and resources. Famous political figures and canonical artists are often more fully recorded than those who performed caregiving, translation, domestic labour or informal knowledge exchange. The resulting relational geometry can concentrate around the most documented people, making their apparent centrality a joint effect of historical practice and archival selection.

730 A defensible methodology should retain source provenance, record disputed identifications, disclose uncertainty and state whose participation is likely underdocumented. It should also address representational and epistemic biases in the selection and interpretation of the corpus. Ethical and justice-oriented

evaluation asks who becomes visible or remains absent when a historical complex is constructed, whose testimony receives a confidence score, and who is positioned to contest the model's classifications. Such questions are substantive methodological demands, not optional commentary added after the mathematics.

14.4 Possible continuations and historically generative structures

A record-derived complex represents one epistemically constrained organization of historical assertions. Generativity concerns which transformations of relational structure and its interpretation are admissible, how they are constrained, and how those constraints can themselves change. To investigate this formally, define a family of states

$$\mathfrak{S} = \{(R, K(R), \widehat{K}(R), \Theta, \Pi) : \text{the stated consistency requirements hold}\}, \quad (21)$$

where Θ collects explicit model parameters and Π records evidentiary provenance. Let Γ be a typed set of transformations among states, including addition of an assertion, revision of an identity resolution, splitting an occasion, ending a tie and admissible structural rewriting. Their domain and codomain conditions form part of the definition. A transition relation

$$S \xrightarrow{\gamma} S', \quad \gamma \in \Gamma, \quad (22)$$

does not assert that S' occurred, is desirable, or is an inevitable successor. It specifies a representational possibility under a chosen rule and is distinct from the realized, source-supported history. A full model might attach values or fields to such possible transformations and study how conditions for further transformations evolve. The current software does not validate this wider generative programme; it provides a structured point of departure for it.

To avoid a predetermined chronological metaphysics, any use of a total order, causal order or arrow associated with the rewrite rules must be introduced as an assumption and investigated. For example, causal explanation normally demands evidence stronger than the temporal succession of two occasions. Rewriting a record after new evidence appears is also a change in knowledge, not necessarily a change in the represented historical world. Generative epistemology must distinguish the two.

15 Historical Constraint and Generative Transformation

The preceding sections construct and analyze records of realized encounters. The purpose of this section is different: it considers how a topological representation might participate in a theory of changes to the conditions under which further relational configurations become possible. The proposal is informed by earlier work on historical constraint evolution and criticality (Huang, 2026a,f). It is presented as a modeling agenda, not a deduction of historical freedom from homology.

15.1 A schematic state with mutable admissibility

Let X denote a set of descriptions of historical configurations, and let q denote an organized collection of material, institutional, epistemic and relational constraints, whose semantics are supplied by a domain study. A family of set-valued admissibility maps can be expressed as

$$\text{Adm} : Q \times X \longrightarrow \mathcal{P}(X), \quad (q, x) \longmapsto \text{Adm}(q, x). \quad (23)$$

A continuation relation $x \rightsquigarrow_q x'$ means $x' \in \text{Adm}(q, x)$. We may separately posit a mechanism Upd through which a realized interaction or collective activity changes q . Neither function is known in advance for human history. Equation (23) is a scaffold for formulating questions about the constitution of possibilities, and it should not be mistaken for an already established historical law.

A model might represent x by some enriched history complex carrying source-grounded labels. But the state of the model cannot generally be reduced to its Betti numbers or even to its entire combinatorial type. Institutions may differ in enforcement practices and ethical meanings while sharing an identical cell structure. The constraint variable must therefore remain typed by the inquiry; examples include available methods of scholarly correspondence, legal rights of association, physical transportation capacity, access to archives, or opportunities to challenge an official narrative.

15.2 Generation of conditions and the problem of the stage

A central question for a generative history is how the stage on which action occurs is itself constituted. If one assumes a fixed configuration space and immutable update rule, the model can explore possibilities already defined by the rule but cannot describe the historical generation of its own categories. An expanded representation may instead allow changes in the set of objects, edge types, allowed attachments and interpretive vocabularies. Such a representation must still specify a meta-language in which these changes are described; recognizing this dependence prevents the formalism from presenting itself as an absolute view outside history.

The issue can be stated through an evolving family of descriptions $\{X_\alpha\}$ and typed transformations $F_{\alpha\beta} : X_\alpha \rightrightarrows X_\beta$ that need not be endomorphisms of one fixed state space. Such transformations may create a new kind of institution, a new method for observing claims, or a new category through which individuals can be recognized. This is a more ambitious formal project than changing the values of links in one existing complex. It requires an account of category generation, comparability across representations, and the documentary evidence for a historical transformation of rules.

15.3 Continuity, criticality and bifurcation

The vocabulary of dynamical systems can describe candidate mechanisms. A system may spend a long interval near a metastable regime, accumulate perturbations that have little visible effect, and then exhibit a qualitative reorganization near a stability threshold. Yet one should not infer a mathematical bifurcation from a historical episode merely because the event was abrupt or momentous. A claimed bifurcation must specify control parameters, observable state variables, rival mechanisms and evidence of a qualitative alteration in the admissible dynamics.

The earlier account of historical generativity distinguishes the trigger of a transition from the slower preparation that renders such a trigger effective (Huang, 2026a). Within the present framework, this suggests coupling record-visible interactions to independent descriptors of institutional constraint. An alteration of a two-complex, for example the addition of a new channel of correspondence, might be a sign of expanded relational access. Whether that alteration changes political possibility depends on the additional mechanisms of enforcement, legitimacy, resources and collective interpretation. The model should preserve this causal incompleteness instead of concealing it with an appealing topological picture.

15.4 Historical objectives and what remains unoptimized

Human actions can be organized around competing structural, generative and normative goals. An institutional goal may optimize participation under a known set of rules; a generative goal may seek to create a new practice; a normative commitment may refuse to treat certain persons, rights or values as instrumental to a fixed objective. A model can represent the first kind through a functional evaluated on known states, but a fixed numerical objective may inadequately describe the latter two. Historical generativity raises the possibility that the criteria of evaluation themselves emerge, become contested and change.

For this reason the paper does not postulate one global score for the value of human history. It distinguishes measurable structural properties, locally articulated judgments, and critical questions about who can participate in revising those judgments. This aligns the mathematical programme with a broader commitment to epistemic humility: the formalism is a tool through which agents can investigate, nego-

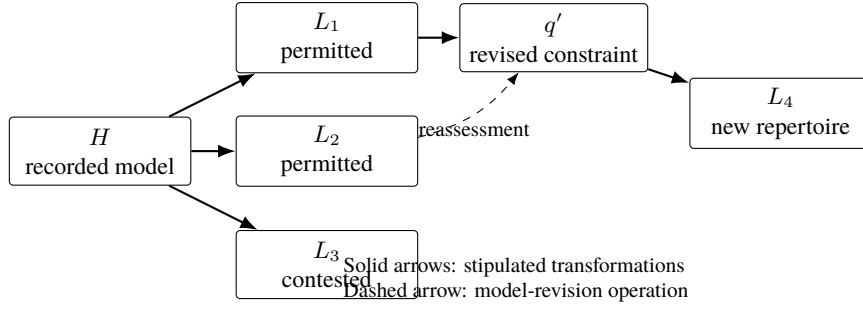


Figure 8: A representation-relative continuation landscape: recorded configuration, admissible alternatives and transformations of the constraint description. The diagram distinguishes modal structure from a realized trajectory and assumes no universally privileged time coordinate.

815 tiate and revise representations of their histories, rather than a device for authoritatively ranking their futures.

16 Cellular Generative Grammars and Historical Derivations

820 This section develops the grammar mechanism inherited from the earlier CW work and connects it to a theory of historical transformations. It introduces a distinction between what a structure is, what changes it can admit, and which changes a record supports as realized. The formal development is exploratory: local rewrite rules are useful descriptions of an analytical possibility space, not a demonstrated universal syntax of human societies.

16.1 Typed rewrite interfaces

825 Let $\mathcal{X}_{Q,\rho}$ be a class of finite typed CW representations. A rewrite rule is represented schematically by the span

$$r : L_r \xleftarrow{\ell_r} B_r \xrightarrow{r_r} R_r, \quad (24)$$

830 where L_r is the recognized pattern, B_r the preserved interface, and R_r the replacement pattern. To define a valid rewrite, the application must specify how L_r matches a complex X , how cells outside the match remain attached, how dangling incidence is handled, and what historical conditions A_r allow the transformation. A formal left-to-right replacement that breaks the boundary of an existing higher cell is generally inadmissible unless an explicit rule for its attachments is also supplied. A legal amendment may preserve institutional continuity while replacing authority interfaces; a scientific innovation may add instrument and evidential cells while retaining a previous community.

For a match m and context c , let

$$X \xrightarrow[(r,m,c)]{\Gamma} X', \quad A_r(X, m, c, h) = 1 \quad (25)$$

835 denote a well-defined, admissible step. The expression makes history h an argument because the same apparent structure may permit different actions after a legal precedent, breach of trust, war, recognition, new technique or loss of material capacity. The admissibility of a step is an empirical and interpretive modeling claim. A mathematically legal replacement in a typed complex need not be socially or legally permitted.

16.2 Admissible transformations and realized trajectories

840 The successor set

$$\text{Succ}_\Gamma(X; c, h) = \{X' : \exists(r, m) X \xrightarrow[(r,m,c)]{\Gamma} X'\} \quad (26)$$

contains modeled possibilities. An evidenced derivation consists of a sequence or partially ordered diagram of particular realized steps with documentary support. The set Succ_Γ and the realized derivation belong to different categories of assertion. The former depends on a generative model and its assumptions; the latter depends on what historical events and transitions are evidenced. Possible transformations need not be merely unrealized future events. They can include alternative classifications, counterfactual constructions, or branches whose ordering with the observed derivation is incompletely specified.

To represent revision of the rules themselves, use a richer state

$$Z_h = (X_h, \Gamma_h, D_h, \mathcal{E}_h), \quad Z_h \rightsquigarrow Z_{h'}, \quad (27)$$

where D_h includes fields and operative capacities, and $\mathcal{E}_h$ represents the available evidence and interpretive configuration. A change in X may occur without change in Γ , and vice versa. Updating Γ can change the admissible space even when an observed institution looks unchanged. Changes in an observer's evidence can alter a reconstruction without entailing a historical transformation of the institution. These distinctions prevent the method from turning every database revision into an event in the historical world.

16.3 Critical transitions and grammar revision

A grammar can encode alternative rules that are ordinarily prohibited by a background constraint. In a political crisis, a change in authority, social coordination, resources or collective expectations might modify that constraint. The phrase *critical transition* should then receive a testable operational meaning: a measure of available states, admissibility or sensitivity changes sharply relative to the specified model and uncertainty. A computed increase in the number of rewrite matches is a property of the model; interpreting it as an expansion of political freedom requires independent normative and institutional analysis.

The earlier *Generative Relational History* paper (Huang, 2026a) proposes that constraints may be regarded as a macroscopic phase of the same relational field as the activities they condition. The present paper translates that claim into a modest computational interface. We can estimate a slow structural descriptor $q_h = \mathcal{P}(X_h, D_h, \Gamma_h)$ and investigate whether certain transitions in q_h precede changes in admissibility. This is an inference programme, not a proof of a physical phase transition. The projection $\mathcal{P}$, the timescale, the comparison cases and the historically meaningful criterion of constraint change must be defined.

16.4 Why a generative grammar is not an archive

An archive contains traces of documentary production and preservation. It may support reconstruction of recognized actors, acts, constraints and narratives. A grammar is an inferred or stipulated model of admissible transformation. Even complete preservation of an archive would not uniquely identify the rules that generated its contents: multiple grammars can produce the same surviving histories. Equally, a plausible grammar cannot substitute for evidence that a particular historical episode occurred. The distinction is related to underdetermination in inverse problems. A responsible inference procedure represents a set or posterior family of candidate grammars, evaluates them out of sample where possible, and records interpretive disagreement rather than presenting one fitted rule as the hidden law of history.

16.5 Path-dependence and noncommuting interventions

A striking feature of historical generation is that the order of operations can matter. Suppose r is an institutional recognition procedure and s is the formation of a shared technical standard. Both may be admissible on a starting complex but the intermediate structure after r may enable a different application of s than the intermediate structure after s enables of r . Formally,

$$R_s(R_r(X)) \not\cong R_r(R_s(X)) \quad (28)$$

can occur when both compositions are defined. The symbol $\cong$ must be interpreted by a stated equivalence, perhaps typed cellular isomorphism. Historical path-dependence is richer than this algebraic noncommutation: one also needs evidence of distinct processes, resources, expectations and uptake. Conversely, commuting structural outcomes do not guarantee equivalent historical experience or justice.

16.6 The limits of predictability

The grammatical approach can clarify conditional possibility without claiming to forecast history. The evolution of the rule set may depend on unmodeled institutions, new language, changing technologies, environmental conditions and the interventions of interpreters themselves. Historical creativity can modify the representational vocabulary through which alternatives are described. Any finite grammar is therefore an approximation relative to a research question. Its revision should be treated as part of inquiry rather than an embarrassing exception to be concealed.

17 Historical Constitution, Agency and the Evolution of Constraints

The central philosophical task of this section is to connect cellular representations to the author's earlier theory of historical generativity without importing an unquestioned physical ontology. Huang (2026a) argues that historically operating constraints and the activity they condition should be studied as mutually involved aspects of the same relational field. This proposal points toward a dynamics of changing admissibility rather than a linear story in which an otherwise fixed world simply accumulates events. A cellular model can help formulate the question when it keeps the relevant levels separate.

17.1 Constraint as enacted and reproduced

A law, scientific standard or institutional convention may constrain practice partly because participants recognize it, material systems reproduce it, organizations enforce it and alternative pathways become expensive or unintelligible. The constraint can persist across individual turnover because its conditions of reproduction are distributed. The analysis therefore seeks both the operative relation and the historical mechanisms maintaining it. A legal text may articulate a rule while effective compliance depends upon distinct enactments, expectations and authority relations. A formal model should record both the textual manifestation and the structures whose coupled operation sustains the constraint.

Let C_h denote a modeled constraint descriptor reconstructed from an evidential history h . The interpretation

$$C_h = \mathcal{P}_{Q,\rho}(X_h, D_h, \Gamma_h; \alpha_h) \quad (29)$$

uses a projection $\mathcal{P}$ and historically situated interpretive parameters α_h . This notation does not assert that a social constraint is a measurable scalar or that $\mathcal{P}$ is unique. A descriptor may be a partially ordered family of institutional capacities, authorization conditions, norms and resource dependencies. The question is whether its inferred variation explains or predicts changes in admissible activity more responsibly than a simpler alternative.

17.2 Slow variables and actor horizons

Historical inquiry frequently faces mismatched durations. Political appointments may change quickly while educational institutions, linguistic categories and property regimes change slowly. A selected actor can observe some changes and lack access to others. The earlier historical-generativity paper relates this to scale separation and the difference between participant and historical observer. In a model, slow and fast variables should be estimated only under a specified chronology or ordering convention; the category "slow" is always relative to a process, observation frequency and chosen scale. An archivist's

retrospective reconstruction may reveal a long transformation that participants experienced as a collection of locally disconnected developments. Such retrospective availability should not be mistaken for a privileged ontological standpoint outside history.

A hypothetical multiscale model could separate local interactions x_h , institutional descriptors c_h and the evolution of the rules governing each:

$$x_{h'} \in F_h(x_h, c_h), \quad c_{h'} \in U_h(c_h, x_h, \mathcal{H}_h), \quad (F_{h'}, U_{h'}) \in \mathcal{V}_h(F_h, U_h, \mathcal{H}_h). \quad (30)$$

The nested set-valued notation expresses incomplete specification; it is not a recovered law. The index h' is an ordered comparison stage within a chosen model, not a universal cosmic present. One can model loops of historical revision without positing an infinite hierarchy of ultimate metarules. At any finite horizon, some components of $\mathcal{V}$ remain external modeling assumptions that can themselves be questioned.

17.3 Goals, normative remainder and the possibility of change

The earlier GRH account distinguishes structural goals, generative goals and commitments that resist complete absorption into goal optimization. In this manuscript the distinction is useful for separating interventions that optimize activity within an existing grammar from interventions that modify admissibility and from normative commitments that discipline such modification. A constitution that permits amendment may provide a procedure for changing legal rules while reserving rights and constraints on legitimate amendment. Formal modeling should therefore avoid treating every value as an objective function to be maximized. Non-optimization can itself express a historically constitutive commitment.

A modeled actor's intention is not observable merely through graph incidence. Goals may be inferred from texts, testimony, practice, and contextual interpretation, each with uncertainty. In particular, the persistence of a tie cannot prove shared goals; a hostile relationship may reproduce itself while constraining both participants. An analytical framework should accommodate contested intention, strategic action and the influence of unanticipated consequences on later conditions.

17.4 Criticality, perturbations and alternative explanations

A small perturbation can accompany a large change when an institution is near a stability boundary. This is a plausible mechanism analogized from dynamical systems, not proof that a particular revolution or cultural movement was a phase transition. An empirical test would specify an order parameter, estimate sensitivity under data uncertainty, distinguish external shocks from endogenous accumulation, and compare against explanations based on discrete reforms, shifting coalitions and archival selection. One could examine whether structural sensitivity increases before an independently dated institutional change, but the evidence must support both the ordering and the counterfactual interpretation.

The virtue of cellular modeling is its capacity to distinguish a rule-level transition, a changed attachment and an altered observable. Its risk is overidentifying these distinct changes with one scalar such as Betti number. A history can transform in power, governance, meaning or permissible speech while leaving its untyped topology unchanged. A history can also undergo a large Betti-number change because new archival collections were added, even when there was no correspondingly abrupt institutional change. Every proposed criticality measure therefore requires a sensitivity analysis for representational coverage.

17.5 Agency and situated historical explanation

Persons and collectivities can participate in the rewriting of their conditions without standing outside those conditions. The GR account of agency emphasizes historically constituted capacities and constraints. A cellular model should consequently avoid encoding actors as external omniscient editors of the complex. A negotiated agreement, protest, invention or scientific discovery appears as a situated

transformation undertaken through material and interpretive interfaces. Its effects can be analyzed at multiple resolutions, including intended changes, unintended couplings, changes of grammar and transformations in the conditions of inquiry itself.

The theory thereby invites an empirical middle path. It neither eliminates intentional agency in favor of topology nor treats social structures as merely static backgrounds. It asks which historically supported mechanisms allow particular actors to reproduce or revise the rules through which later agency is constituted. The answer will differ across research questions, institutions and societies; the framework defines questions of comparability rather than a single mechanism of history.

18 Spin-Foam-Inspired Histories and the Limits of Physical Translation

This section situates the History Complex within the author's broader spin-foam proposals for social science. The objective is to reconstruct a spectrum of possible mathematical uses of the term *spin foam*, rather than allow a familiar physical expression to conceal distinct formal commitments. The public repository presently implements a specific finite $\mathbb{Z}_2$ lattice gauge model. Earlier writings and the repository's design note describe larger possibilities involving group-valued connections, amplitudes, histories and eventual coarse-graining. Those possibilities are intelligible research hypotheses but are not equivalent to an implemented model of quantum gravity or a demonstrated physical theory of society.

18.1 Three levels of spin-foam language

At the most permissive level, a *spin-foam-inspired carrier* is an oriented two-complex with face and edge labels, local compatibility conditions and perhaps weights. Such a structure can model processes, institutions, value types or dynamical constraints without selecting a physical gauge group. At a second, mathematically stronger level, a group or representation theory is specified: oriented edges receive holonomies in a group G , faces receive loop observables or representations, and local weights are constrained by chosen invariances. At a third level, a statistical or quantum model supplies an action or amplitude and a sum or integral over appropriate histories, together with a justified interpretation of its outputs. Moving between levels requires explicit additional assumptions.

The present software is situated at the second level in a bounded sense: it implements a $\mathbb{Z}_2$ connection and weights on the historically constructed complex. It does not follow that cultural relations are quantum states, that the fitted parameters have the units or universality of physical couplings, or that an amplitude-based sum over geometries has been computed. The idea note expressly leaves more elaborate groups, a generating field theory, and a condensate model for future investigation (The Science Infrastructures contributors, 2026d).

18.2 Oriented connections and local frame changes

For an oriented 1-cell e , a G -connection assigns $g_e \in G$ with $g_{\bar{e}} = g_e^{-1}$. An ordered face boundary $\partial f = e_1^{\epsilon_1} \cdots e_k^{\epsilon_k}$ gives the holonomy

$$H_f = \prod_{i=1}^k g_{e_i}^{\epsilon_i}, \quad (31)$$

where ordering is essential for a non-Abelian group. Under a vertex frame transformation $k_v \in G$, g_e transforms according to its endpoints and H_f changes by conjugation at the base point. A class function $w_f(H_f)$, such as a suitable character-based term, is then gauge invariant. The finite-group case avoids analytic questions of Haar-measure normalization; continuous groups require additional analytic structure.

For $G = \mathbb{Z}_2 = \{\pm 1\}$, inversion has no effect and the face product reduces to the product of edge signs. A frustrated face has holonomy -1 in the chosen sign convention. This expresses a coding inconsistency around that loop *relative to the representation*; it is not an automatically verified contradiction in motives,

a proof of collective injustice, or evidence that three people experienced psychological tension. Gauge invariance expresses redundancy of local frame assignments, not empirical invariance under rewriting historical evidence.

18.3 An example of a local conditional model

1010 A finite signed-history ensemble could have weight

$$P(g \mid R, \theta) = Z(R, \theta)^{-1} \exp \left[\sum_{f \in K_2(R)} \beta_f \chi(H_f) + \sum_{e \in K_1(R)} \eta_e \psi(g_e, R) \right], \quad (32)$$

provided Z is finite and positive. Here χ is a chosen class function, ψ is a stipulated data-likelihood or coding feature, and θ collects parameters. The second term may break local gauge invariance unless its transformation properties are designed accordingly; it should not be described as gauge invariant merely because the first term is. The exact implemented model and sign conventions must be verified from the repository before Equation (32) is used to replicate its numerical outputs. Equation (32) is a *general illustrative family*, not a reproduction of a particular executable likelihood.

More generally, a formal amplitude model might assign local factors to faces, edges and vertices,

$$\mathcal{A}(K, \xi) = \prod_{f \in K_2} A_f(\xi_f) \prod_{e \in K_1} A_e(\xi_e, \xi_{\partial e}) \prod_{v \in K_0} A_v(\xi_{\text{star}(v)}), \quad (33)$$

with an appropriate interpretation of the state labels ξ . The construction requires consistency of domains, attachment and, where appropriate, orientation. A sum $\sum_{K, \xi} \mathcal{A}(K, \xi)$ has no established social meaning until a measure, model class, convergence strategy, boundary condition and epistemic target are supplied. Treating the sum as a population of actual alternative histories would be an additional interpretive assumption.

18.4 Historical semantics of faces and vertices

The author's prior CW manuscript and the History Complex assign different meanings to faces. In the former, faces are selected primitives, and the coupling weights may describe compatibility among them. In the latter, faces are continuing ties between named beings. The local gauge model can be placed on either carrier, but the empirical meaning of the same face holonomy changes when the attachments change. Consequently, a physical derivation does not rescue a poor historical coding scheme: it operates faithfully on the supplied complexes, including their assumptions and omissions.

1030 This difference also clarifies the proposal for *social spin foams* in the author's wider research programme. One paper on power and governance, reported in the repository's design note, uses local non-negative coupling weights and ensemble reconstruction without committing to a physical group or action. A separate street-dynamics specification associates event vertices, continuity interfaces and persistent histories with a neighborhood's heterogeneous processes. These are related projects with different variables and target questions. The present paper takes them as comparative designs; it does not silently pool their conceptual primitives into one supposedly established ontology.

18.5 Field, power and normative evaluation

A gauge field or amplitude can represent an interaction pattern under assumptions. It does not prescribe legitimate power, justice or institutional authority. The earlier power programme distinguishes the capability to reshape a historical landscape from the success of a particular action. In the current setting, one can compare how an intervention changes the distribution of reachable typed structures, while separately evaluating rights, participation, exclusion, harm and the legitimacy of imposed categories. Such evaluation requires explicit normative principles. A sign assigned to a tie is particularly poor as a scalar proxy

Table 3: Levels of mathematical commitment when interpreting “social spin foam.” The table is a conceptual taxonomy, not an implementation status claim for every earlier proposal.

Level	Mathematical commitment	Historical inference requiring separate warrant
Cellular carrier	Typed events, interfaces, faces, attachments	That these are adequate social primitives and relations
Finite gauge theory	Group connections, face holonomies, model weights	That observed labels justify the connection semantics
Amplitude / state sum	Boundary data, amplitudes, measure and summation	That amplitudes represent a historical or explanatory mechanism
Generating field theory	Dynamics that generate complexes	That a generating action is empirically motivated
Effective continuum	Limits, scaling, order parameters	That coarse-grained quantities track historical constraints

for justice: an antagonistic relation may involve principled resistance to domination, while a positively coded tie may involve exploitation or coercive dependence.

18.6 A non-teleological research direction

The author’s historical-generativity work rejects the introduction of an extremized social action merely because variational principles work in physics. That restraint is retained here. We can study weighted ensembles, transformations and continuum candidates without assuming that human history minimizes energy, optimizes utility, maximizes generativity or realizes a predetermined collective end. Any such principle would demand its own derivation and testing. In the absence of such evidence, the spin-foam analogy is a tool for structurally precise hypotheses rather than a law of human destiny.

19 Temporal Ordering, Historical Continuation and Modal Structure

A historical model routinely uses dates, sequences and expressions such as “before” and “after”. The explanatory status of those terms cannot be taken for granted. Documentary calendars are measurement conventions, partial causal orders are empirically contestable, and the temporal parameter of a computation need not identify an absolute temporal structure of the processes it represents. The earlier continuation-complex inquiry makes the distinction between represented histories and admissible transformations an explicit methodological requirement (Huang, 2026c).

19.1 Chronology as a representation

The repository converts several historical date conventions into Julian day numbers and uses deterministic tie-breakers for overlapping intervals (The Science Infrastructures contributors, 2026b). These conventions make a program reproducible. They do not warrant an interpretation that the file’s total order is a unique order of causation or social meaning. A prolonged congress may overlap many letters and audiences, and the order of its encoding may impose a sequence where the underlying historical relation is nested, simultaneous, disputed or only partially ordered. In some settings, the relevant temporal structure is a precedence relation in a category of events; in others, a chronology constructed from heterogeneous testimony is the research object.

Let $(O, \preceq)$ be a locally justified partially ordered set of occasions. A topological construction that requires total order should explicitly state the chosen linear extension ℓ and the stability of its findings under other allowed extensions. In cases of cyclic social influence, ordering of records and cyclic causal

feedback should also be distinguished: a feedback process can unfold through numerous occasions without giving the same occasion a cyclic temporal precedence relation.

19.2 History-relative continuation

Suppose a finite represented history H is a subcomplex of a candidate larger relational structure L under an inclusion or another declared embedding $i : H \hookrightarrow L$. We may write a relative pair (L, H) , and investigate $H_n(L, H)$ or a diagram of such pairs, while allowing the structural and interpretive status of H itself to be revised. This is an exploratory mathematical picture, not a claim that the past literally exists as a frozen geometric subset. It becomes informative when the admissibility conditions on L are stated and when competing continuations can be interpreted through recognizable types of evidence and action.

An alternative is a *span of histories* $H \xleftarrow{a} Q \xrightarrow{b} L$, useful where a representational change alters the category of objects. In this formulation a continuation is not merely the attachment of cells to an immutable past: it may include translation, consolidation, reinterpretation and the generation of new forms of relation. When past documents are reinterpreted, their earlier material existence and their changing social meaning can be modeled separately rather than collapsed into one claim about retroactively changed events.

19.3 Modal availability and realized trajectories

A possible continuation is not simply an event waiting to occur. A model's set of admissible successors depends on assumptions, institutions, agents and epistemic reach. The realized trajectory, to the extent it is reconstructible, is only one element or compatible trace of those possibilities. Conversely, an alternative permitted by a symbolic rule may have been materially impossible, politically unimaginable, or ethically prohibited. Thus the model should use typed modalities: formally admissible, materially feasible, institutionally permitted, socially intelligible, and evidentially supported. Each type places a different burden on historical argument.

This distinction is important for counterfactual history. A well-formed topological attachment does not establish a meaningful historical counterfactual. Credible counterfactual analysis requires explicit preservation of relevant background conditions, a reasoned intervention, and an account of which causal relations are being altered. A continuation complex can help organize these assumptions visually, but its mathematical freedom should not be mistaken for evidence about the set of alternatives actors faced.

19.4 Historical self-reference

Historians and archival institutions participate in history through their acts of collecting, translating, indexing, publishing and contesting evidence. The record is therefore an intervention in the social world that may change further possibilities of knowledge. A later interpretation can affect what is taught, what receives support and what is preserved; it does not thereby establish the truth of its historical claims. This creates a coupled inquiry into the history being reconstructed and the history of its reconstruction. Making that coupling visible is a conceptual strength of the proposed research infrastructure.

20 Value Circulation, Power and Historiographical Justice

A cell complex can support a formal question about circulation, but a circulation observable is not yet a theory of value. The earlier study of heterogeneous value circulation investigates how incompatible kinds of value might be traced through relational two-complexes without reducing them to one commensurable magnitude (Huang, 2026b). In a historical application, scientific knowledge, an aesthetic experience, diplomatic recognition, institutional authority and material wealth may all be consequential; they have distinct modes of production, recognition, transfer and loss. A single gauge sign cannot express their

differences, and a closed homology cycle cannot tell us whether something worth preserving circulated within it.

20.1 Typed value fields and interpretive dependence

A tentative extension attaches a vector of typed observations or interpretations $v_f \in V_{c(f)}$ to certain faces, with the value space V_c dependent on the relation type c . The objects V_c need not be normed spaces. For some themes they may be posets of recognized statuses; for others they may be categories of textual practices or empirically measured resource flows. To relate one historical domain to another requires a specified transformation $T_{cd} : V_c \rightrightarrows V_d$, including its domain of legitimacy and its possible losses. Such transformations are historical practices subject to criticism, not universal exchange rates.

An example concerns a cultural work that circulates between courts, schools and public institutions. The work's reuse may produce political value for a patron, educational value for a school and recognition for an artist; these effects need not be exchangeable. A typed relational record would preserve the different interpretive acts and their source evidence. If a formal circulation operator is defined on its faces, its outputs must remain interpretable as quantities or structures warranted by the coding, rather than metaphorical fluids whose numerical existence is presumed.

20.2 Power as capacity to alter the conditions of relation

The earlier work on power describes asymmetry through historically constituted repertoires and the reproduction of differential capacities (Huang, 2026e). This motivates a distinction between an actor's position in a recorded network and the actor's power to change the network's rules of generation. Degree, betweenness and participation in many cycles can be informative descriptive statistics, but power over admission, classification, publication or coercion may be exercised through fewer visible contacts. The relevant mechanisms may lie in funding, exclusion rights, violence, legal authority, control of archives, and the capacity to invalidate another group's testimony.

Formally, if a model includes a family of admissibility correspondences $\text{Adm}(q, x)$, one candidate power question is how an intervention a changes the correspondence. For an appropriately defined family of measures or orders on reachable possibilities, one might study $\Delta_a \text{Adm}$, its distribution across differently situated subjects, and which institutional procedures enable contestation. This is a proposed research strategy. It neither presumes that maximizing the number of reachable states is just nor that every expansion of institutional capacity benefits those affected by it.

20.3 Archival power and future knowability

Historical power also shapes the survival and discoverability of evidence. Institutions choose which documents to preserve, which languages to translate, whose authorship to attribute and which classification schemes to maintain. A graph drawn from digitized documents may consequently reproduce archival inequality. An inquiry into historiographical justice therefore examines both the world encoded by a corpus and the conditions under which a corpus becomes possible. The earlier generative-epistemic and historical-truth inquiries foreground the difference between authenticating a record and warranting its substantive claim, as well as the future conditions of independent re-interpretation (Huang, 2026g,d).

Provenance-aware software can support such criticism by keeping competing assertions, sources and version histories available. It cannot undo past archival destruction; nor should it claim to infer missing voices from structural gaps alone. The ethical response to a topological silence includes more than searching for a plausible hidden edge: it may involve preserving uncertainties, documenting missing sources, consulting affected communities, and recognizing when no defensible reconstruction is available.

20.4 Normative evaluation without a universal evaluator

The paper treats descriptive complexity, historical success and ethical legitimacy as different categories. A highly coordinated oppressive system may be topologically robust. A conflictual triangle may embody justified resistance to injustice. A value circuit may reproduce asymmetrical extraction as readily as reciprocity. A useful normative research programme asks whose capacities, rights and opportunities for mutual recognition are affected, how those affected can challenge the interpretation, and what material conditions make such challenge possible. No homology group establishes answers to these questions. Formal inquiry can make the structure of some problems clearer while leaving historical judgment publicly contestable.

21 Epistemic Translation, Knowledge Pressing and Interpretive Plurality

The complex is one representation of historical material. Historiography also entails translation among the concepts, languages, institutions and practices through which history becomes intelligible. Earlier work on knowledge pressing proposes studying not merely the translated outputs of heterogeneous systems but the generative organizations that produce them (Huang, 2026d). Applied to history, this means asking how a reconstructed practice generated its categories and interpretations, and how those categories might be related to the historian's own without assuming that either system is transparent or universally complete.

21.1 Documents, interpreters and the production of warrants

A source assertion is not itself a historical fact, even when it is accurately transcribed. Its evidential use depends on authentication, genre, authorship, language, circumstances of production, corroboration and the research question. For example, a ceremonial declaration may be reliable evidence that a certain formula was publicly pronounced, while remaining weak evidence about private intentions or subsequent compliance. The source should retain these distinctions when coded as an occasion or tie. A model that collapses them into one "confidence" number risks conflating authenticity, interpretation, referential accuracy and causal relevance.

The repository's factoid record model is an important starting point: conflicting assertions survive in the record, rather than being silently reconciled by the construction (The Science Infrastructures contributors, 2026b). A further development would maintain separate source-specific interpretations $I_j(s)$ and warranted relationships among them, perhaps including translation provenance, philological alternatives, disagreements and unresolved reference. The historical object generated downstream would then be conditional on interpretive hypotheses rather than presenting one harmonized corpus as reality.

21.2 Translation between generative systems

Suppose two historical communities use symbolic practices G_A and G_B to produce descriptions, roles or norms. A translation is not necessarily a function taking individual tokens of one system into tokens of the other. It may require a pair of interpretive interfaces Φ_A and Φ_B and a partially defined relation among generative operations, preserving what the study regards as consequential: inferential roles, normative permissions, consequences of action, or historically situated use. Several descriptions can map into one translated label, with loss of the distinctions that originally made them meaningful.

This is directly relevant when combining canons of scientific, artistic and diplomatic histories. The same figure may occupy different categories in different disciplinary accounts, and the act of integration can change the apparent prominence of that figure. Cross-disciplinary gluing is therefore a translation problem as well as a database identity problem. The conservative policy is to retain the original disciplinary record, state the proposed identity and type correspondences, and report which algebraic outputs depend on them.

21.3 The archive as a participant in relational history

Every act of description can alter the conditions for later description. When a text is indexed or digitized, previously disconnected records may become searchable together. When a scholarly article standardizes a category, other researchers may adopt it; the category can then influence the production of additional sources. These reflexive effects are historically real, even though the formal complex representing an earlier episode remains a retrospective construction. A generative epistemic infrastructure should therefore distinguish an archival object's provenance from the provenance of its annotations and subsequent uses, and preserve revision trails that allow later interpreters to test earlier inferences.

21.4 Plurality without arbitrariness

Interpretive plurality should not collapse into unrestricted relativism. Evidence constrains claims; established documentary contradictions cannot be eliminated through preference. Nevertheless, source criticism may support multiple defensible codings when categories are historically contested. Their coexistence is an opportunity for structured comparison. One can construct a model space $\mathcal{M}(R)$ of permissible coding choices and evaluate a claim φ across it. When φ holds under all source-supported alternatives it has a different epistemic status from one whose truth depends on a disputed attribution. That is a useful contribution of computation to historical inquiry, provided the family of alternatives was itself constructed responsibly.

22 Historical Reconstruction, Interpretive Translation and Epistemic Justice

This section develops the epistemic side of the complex model. Histories are not presented to inquiry as fully specified sets of events with unproblematic identities. They are investigated through documents, artifacts, inscriptions, collections, testimony, oral traditions, measurement systems and inherited categories. These traces are generated and preserved through social relations that can enable or obstruct later knowledge. A computational history should therefore model the formation of the record and the changing interpretive apparatus as part of its research object.

22.1 The record as a historically produced structure

Let H denote a hypothesized historical process and D the available documentary corpus. The record may be described by an observation construction

$$D \sim \mathcal{O}(H, M, \kappa), \quad (34)$$

where M represents archival mediation and κ an interpretive and coding protocol. Equation (34) is schematic. Its purpose is to resist the identification of missing documents with missing historical relations. The same process may leave different traces according to literacy, censorship, media survival, institutional recordkeeping, accessibility and the practical interests of later collectors. The observation channel can be selective, dependent on the investigated events, and historically mutable. The convenient statistical assumption that observations are missing at random is frequently indefensible for historical archives.

Historical records can also become historically operative in their own right. Publishing a political manifesto, verifying a record, translating an archive or generating a public narrative may change expectations and institutions. The distinction between source and event is therefore contextual: a document can be evidence of an earlier event and itself an event in a later relational history. Maintaining separate identifiers and provenance links permits the model to represent both roles.

22.2 Record authentication and historical truth

Cryptographic authentication can support integrity, custody and attribution under specified technical assumptions. It cannot establish that the substantive claims of a signed document are true. In particular, a valid signature may verify the relation of a text to a key while leaving motive, context, completeness and factual assertions contested. The earlier GR epistemological work develops this difference between authenticated records and warranted historical knowledge. The present model implements the distinction as an evidential chain: identify the claim, its documentary basis, the verification operation, the interpretation, the coded cell or relation, and the downstream computed consequence.

A cell without documentary justification is not rescued by inclusion in an elegant foam. A mathematically sound homology computation certifies the invariant of the input chain complex. It does not validate the source assertions or the selected semantic interpretation of its cells. Ideally each inferred cell carries a traceable dependence on specific assertions; any derived invariant is accompanied by a record of which contested assertions can change it.

22.3 Incompatible historical ontologies and partial translations

One community may understand a political practice through kinship, another through property, another through legal status, and another through ritual relations. A universal vocabulary imposed at ingestion can erase precisely the differences that make the archive significant. The author's work on *knowledge pressing* suggests a deeper task: infer the generative representations through which a community produces and interprets meaningful structures, rather than simply translate its observed statements into an existing vocabulary. Applied to historical research, this calls for the representation of local categories, rules of classification, permissible claims and the episodes of interpretation through which another system becomes intelligible.

A partial interpretive translation

$$T_{AB} : \text{Claims}_A \rightarrow \text{Claims}_B \quad (35)$$

may fail to be defined on concepts that have no defensible counterpart. Even where defined, it may preserve only limited properties, such as event identity, a source reference or one institutional relation. An attempt to fill every gap by nearest-neighbor semantic similarity can generate false consensus or erase historically significant forms of otherness. Recording untranslated residues and incompatible categories is a productive outcome when it improves subsequent inquiry.

22.4 Epistemic asymmetry and future knowability

Power affects not only what social relations occur but also which traces survive, who receives interpretive authority and which forms of explanation become available. An imperial archive, a court registry, a canon of art or an academic citation network may concentrate preservation and public visibility. A computational history drawn primarily from those materials can reproduce their asymmetries even when every algorithmic step is internally correct. This is a structural concern: the conditions of knowledge production can shape which later histories can be reconstructed at all.

A useful audit should ask which persons, places, languages and types of institution are systematically absent; which categories come from powerful documentary producers; which ties are easier to attest because they passed through formal bureaucracies; and which populations are assigned undifferentiated collective identities. Such audits require historically appropriate baselines, not a naive demand that every time or region contribute equal numbers of records. The normative question is whether the infrastructure preserves contestability, correction and the potential for future interpreters to recover what was suppressed or overlooked.

22.5 Observation uncertainty as a family of complexes

A categorical assertion that a tie existed may be contested. Rather than force a false dichotomy, a reconstruction may associate source-supported hypotheses with families of candidate complexes $\{K_j\}$. Their invariants can be compared,

$$\{(\beta_0(K_j), \beta_1(K_j), \beta_2(K_j)) : j \in J\}, \quad (36)$$

where J indexes defensible coding hypotheses or draws under a documented statistical model. Probability weights, if used, require a justified likelihood and dependence assumptions. An interval of observed Betti numbers over all defensible reconstructions may be more honest than a point estimate with spurious decimal precision. Disagreement about ontological categories can be preserved as alternative typed models rather than resolved by averaging incommensurable codes.

22.6 Interpretation as intervention

Historians, software developers and institutions can change the conditions under which history is known. Correcting one person's identity, translating an archive, digitizing a corpus or exposing a provenance trail alters the reconstructed complex and may enable new historical questions. Such interventions should be logged as changes of *epistemic access*, unless separate evidence supports an effect on historically operative relations. If publication of the new account later changes public practices, that effect becomes a new event with its own history. This reflexive loop is a reason to design a versioned, provenance-aware infrastructure rather than to imagine a final timeless database of history.

23 Computational Illustration: The Initial History Canons

The repository's draft History Complex offers six initial canons that include Athens, Renaissance Florence and Rome, the Scientific Revolution, Europe in 1785–1828, European alliances and Vienna. The application provides a three-dimensional foam view, a loom of individual lifelines and ties, network cuts at selected moments, topological readings and a provenance-oriented record view. These functions illustrate how a user might move between the documentary basis and the model's algebraic output (The Science Infrastructures contributors, 2026a).

The development decision for the history domain describes the first European canon as containing 33 coded beings, 92 occasions and 57 ties. It also states explicitly that the canon was drafted by a language model and had not been checked against all underlying sources (The Science Infrastructures contributors, 2026c). Those numbers document the *scope of an initial software fixture*, not the size of the historical world it purports to represent. The implementation's theory note reports 66 mod-two foam degrees of freedom for that selected European canon and illustrative reconstruction results for certain diplomatic relations (The Science Infrastructures contributors, 2026e). In this manuscript they remain *repository-reported* quantities. We have not independently run the engine against the same commit, audited the documentary assertions or reproduced the inference chains.

The appropriate reading of a case interface proceeds in stages. A historical occasion selected in the visual foam should expose its documentary assertions and participants. The viewer can follow a persistent being's coded appearances, then inspect the lenses generated by relationships that have more than one relevant meeting. A topological loop should open into the exact sequence of people and occasions defining it, thereby returning a formal invariant to inspectable archival questions. A posterior sign inferred for an unattested meeting should display its uncertainty, the original attested signs and the source of the priors. These are important advantages of transparent computational histories over pictures whose semantic provenance has been flattened into visual style.

The programme also reveals its present limits. The six canons are heterogeneous in their source coverage; they do not establish a representative cross-section of human history. The coded sign categories

are a coarse mixture of relation types and may change meaning across artistic, interpersonal and diplomatic material. The gauge parameters fitted to sparse, purposively sampled ties cannot be interpreted as universal historical constants. In such circumstances, the application's greatest immediate research contribution may be a transparent procedure for asking where apparent historical regularities come from, rather than a catalogue of new historical laws.

We therefore describe the computational demonstration as a *proof of representational feasibility*: structured source assertions can be transformed into well-defined complexes, inspected by algebraic invariants and linked to conditional relational models. Valid historical inference requires further validation, comparison and criticism. That distinction should remain visible in abstracts, presentations, software interfaces and public discussion of the findings.

24 The History Complex as a Research Infrastructure

The practical contribution of the repository is not exhausted by its initial canons. Its architecture separates a domain library of historical records and transformations from a general mathematical foundation and the applications through which readers explore results (The Science Infrastructures contributors, 2026c,a). That separation has methodological significance. An analysis of human history should not depend on a particular graphical interface; likewise, an archival coding rule should be testable without requiring an interactive visualization to infer its semantics.

24.1 Executable separation of concerns

The history domain reads source-linked JSON-lines records, constructs the complex, and exposes mathematical and relational readings. The general mathematical foundation computes exact topological invariants, while a separate application renders the foam, the loom, time cuts, loops, provenance and gauge-model outcomes. Such division makes it possible to test deterministic components separately from subjective judgments. Reusability is strongest when a library declares precisely which types and invariants it exports, and an application declares which additional interpretations it introduces.

A research implementation needs an explicit provenance chain:

$$\text{source} \xrightarrow{\text{transcribe}} \text{assertion} \xrightarrow{\text{code}} R \xrightarrow{\text{construct}} K(R) \xrightarrow{\text{compute}} \text{invariant} \xrightarrow{\text{interpret}} \text{claim}. \quad (37)$$

Each arrow should be associated with a method, operator or accountable act, together with its version, uncertainty and responsible parties. The reverse investigative path is equally important: a displayed figure should allow a reader to inspect the classes and chains producing it, the coded relations producing those chains, and the source assertions supporting the coding. Transparency alone does not establish accuracy, but without this backward route many model claims become effectively impossible to evaluate.

24.2 Visualization as an analytical interface

The repository's three-dimensional foam shows repeated appearances and tie films through a drawn chronology; the loom emphasizes individual continuity; the moment view displays active relations; and the provenance view returns to sources (The Science Infrastructures contributors, 2026a). Their different encodings can support different questions. A 3D view makes thick historical interaction visually intuitive but can obscure occluded edges and promote spurious geometric interpretations. A loom may clarify sequential relations while compressing the importance of events that are not directly tied to named participants. A moment network facilitates local comparison but risks treating unobserved sign values as verified contemporaneous states.

A responsible visualization should encode what is documentary, what is derived by deterministic rules, and what is inferred probabilistically through visibly different styles, not merely through colour changes. Screen readers, captions, static printable diagrams and tabular exports should be supported for

substantive scientific accessibility. A viewer should be free to select the modality best suited to the task; a 3D animation is an optional exploration rather than a required form of evidence.

24.3 Reproducibility and the meaning of exactness

Exact Smith normal form or integer homology computation can establish exact algebraic results for a given complex. This precision should be protected as a mathematical achievement. At the same time, it does not make the source record exact. A build can be perfectly reproducible and generate a misleading historical interpretation from incorrectly dated events or a biased set of subjects. Reproducibility requirements must therefore span both the computational and documentary portions of the workflow: retained raw sources, stable citations, coding records, codebook versions, explicit joins, test fixtures, executable source, and structured uncertainty.

24.4 Proposed development interfaces

Several interfaces merit separate work. The first is a versioned uncertain-record model in which rival date, identity and tie choices can coexist. The second is a provenance-aware morphism system relating revised and expanded complexes. The third is a disciplined inference interface separating observed signs, prior assumptions, conditioned reconstructions and posterior diagnostics. The fourth is a comparison service capable of testing representation stability across different carriers. The fifth is a historian-oriented review workflow that supports correction without overwriting the historical record of a previous research decision. Each development should be evaluated against a specific scholarly activity rather than pursued merely because it can be built.

25 Research Obligations and Validation Programme

The framework's mathematical coherence can be checked independently of the historical claims. A multi-stage evaluation should address at least the following forms of validity.

Construction validity. Every 2-cell must have a closed, explicitly reconstructible attaching path; the implementation must satisfy $\partial_1 \partial_2 = 0$ exactly. Synthetic fixtures should include isolated beings, repeated meetings, simultaneous occasions, long-lived ties, tie termination, identity collisions and disputed dates. Computed Euler characteristics and homology groups should match independently constructed boundary matrices. The two-occasion example in Section 11 supplies a minimal transparent check.

Chronological and representational sensitivity. Records with interval dates should be recoded under several justified precedence choices. Compare homology and inferred sign distributions under source-supported alternative orderings. Test mappings between coarser and finer resolutions, separating cell subdivision from genuine addition or removal of information. Report a stability interval or distribution over plausible complexes where the order cannot be justified uniquely.

Documentary and coding reliability. Source assertions should be checked against independent scholarly or primary references, with disagreements preserved. A second coder should classify a stratified sample of tie signs and occasions, and inter-coder agreement should be reported rather than inferred. The same corpus should be coded using alternative defensible sign definitions. Discrepancies between ontology, codebook and practice must be analyzed before publishing social interpretations (Krippendorff, 2018; Bradley and Short, 2005).

Table 4: Claims, mathematical objects and minimum evidence required for interpretation.

Reported quantity	What it establishes	Additional requirements for a historical claim
χ, H_1, H_2	Invariant of the chosen coded complex	Verified coding, appropriate resolution, robustness to archive coverage
Flip or triangle product	Invariant of a signed configuration	Defined sign semantics, reliable attestations, coding agreement
Posterior sign mean	Conditional model expectation	Calibration, sensitivity to priors and sampling, validation against withheld evidence
Power divergence	Difference between selected model distributions	A defensible intervention, stable baseline and separate causal argument
Interdisciplinary face inter-sections	Contact within a coded theme assignment	Justified disciplinary categories and analysis of excluded relations

Model fit and baselines. Hold out genuinely attested relationship signs, fit the model on remaining evidence, and assess calibration, log loss and error by tie type, source and period. Compare against persistence-only Markov predictors, ordinary signed dynamic networks, simple independent-sign baselines, and models using no topological features. The comparison should test whether the 2-complex structure contributes measurable value rather than merely re-encoding observed degrees and dates. Posterior uncertainty must not be presented as a verified probability of historical truth unless an appropriate calibration study supports that interpretation.

Epistemic and ethical evaluation. Document who enters the canon and who is likely absent; examine whether source density predicts apparent network centrality or model-assigned “power.” Include alternative corpus-selection procedures and underrepresented institutional or social positions when the record permits. Interpretation panels should distinguish an observed relationship, a reconstruction conditional on a model, and a narrative judgment. Correction procedures must allow a contested assertion to be traced through all derived results.

Three open theoretical questions merit particular emphasis. First, which equivalences among documentary recodings preserve historical interpretation, rather than only homology? Second, can changing patterns of relational constraints be inferred with identifiable uncertainty from records selected for preservation and canonization? Third, how can transformations of historical structures and changes in historical knowledge be represented together while remaining conceptually distinguishable? These questions delimit the transition from an exploratory computational formalism to a richer programme of historical inquiry.

26 Comparative Historical Research and Validation Design

A methodology for historical complexes should be judged by its capacity to support responsible inquiry, comparison and correction. At present the most important unresolved questions are not how many figures can be produced or how elaborate a field theory can be overlaid, but how to demonstrate that the encoding contributes explanatory power beyond more familiar historical methods. The following research designs distinguish formal correctness, documentary reliability, descriptive relevance and explanatory validity.

26.1 A prospective comparison of coders

Select a small, bounded corpus where relevant primary and secondary sources can be obtained and contested claims can be inspected. Pre-register a coding protocol for occasions, identities, presence, relation types, signs and endings. Ask at least two independent coders to construct the records, and retain their differences. Agreement should be computed separately for the type of occasion, the presence relation, the existence of a tie, its colour, its sign and its date. These are different measurement tasks. A single

aggregate inter-coder agreement statistic may mask disagreement on the very relationships that generate topological classes. Where a comparison statistic such as Krippendorff's α is suitable, the relevant distance function and missing-value policy must be specified (Krippendorff, 2018).

26.2 Competing historical explanations

Choose a documented historical episode for which historians have proposed rival interpretations about the circulation of ideas, the coordination of institutions or the stability of relationships. Fit the representation under each defensible coding hypothesis, and examine which invariants and conditioned sign probabilities are genuinely discriminating. A model is informative when it makes a consequence of a proposed interpretation inspectable and potentially falsifiable. A result that is the same under every hypothesis may still aid visualization but cannot discriminate between their causal claims.

26.3 Placebo and perturbation tests

Perturb the record in controlled ways. Shuffle dates only where documentary constraints permit it, remove classes of sources, change tie persistence assumptions, and vary identity resolution and record density. Compare observed invariants with results from synthetic baselines that preserve selected marginals. A significant discrepancy from a naive random model is not by itself evidence of a historical mechanism; the null model must correspond to an interpretable rival account. Where a topological feature disappears after removing one disputed document, its dependence on that document should be communicated rather than suppressed.

26.4 Prediction and reconstruction as different tests

For the signed field, withhold a sample of attested signs and ask the fitted model to predict them from other attestations and correlations, using a procedure designed to prevent leakage between observations of the same tie. Report proper scoring rules, uncertainty calibration, error by relation type and the strength of simple baselines such as the last-attested sign. The value of the gauge model is not established unless its conditional predictions or structural insights outperform justified alternatives. Prediction of a withheld coded sign is also different from reconstructing an undocumented historical intention. The latter requires evidence not supplied by a binary field alone.

26.5 Triangulation with qualitative history

A useful evaluation includes interpretive reading by domain historians. Does the model reveal an overlooked interaction worth checking in an archive? Does it obscure a conceptually crucial difference among actors? Can a visualization facilitate a more defensible comparative argument than a table, timeline or narrative? These questions can be studied through transparent user tasks and recorded criticism. The objective is a historically warranted improvement in understanding rather than the maximization of algebraic complexity.

Table 5: A validation ladder separating kinds of warrant. Passing a lower rung does not imply passing the next.

Level	Main test	Inference permitted
Formal consistency	Boundary composition, integer homology, synthetic invariants	The chosen finite objects are computed correctly
Record reliability	Independent coding, source checks, provenance audit	Assertions are represented reproducibly
Measurement validity	Construct and cross-domain tests	The codebook operationalizes a stated concept
Model calibration	Held-out prediction, ablations, baselines	A conditional model has quantified performance
Historical explanation	Rival hypotheses, critical source reading	A mechanism is more adequately supported
Ethical legitimacy	Contestability, rights and situated review	A specified application has justified safeguards

27 An Illustrative Comparative Research Design

This section demonstrates how the expanded framework might inform social-scientific historical investigation. It proposes a comparative protocol for studying the formation of a hypothetical transregional scholarly network. The example is deliberately synthetic: names, occasions, relations and numerical results are constructed to expose methodological choices, not to make new claims about an actual historical society.

27.1 Three observational layers

Assume three archives: a correspondence collection, the records of a scholarly society and an oral-history collection. The first supplies dated exchanges and authorship claims, the second gives formal membership and rule changes, and the third supplies recollections of informal collaboration. They differ in the units they observe, the degree of temporal precision and their selection mechanisms. A coarse History Complex can use documented occasions as vertices, identified correspondents as continuing appearances and attested collaborations as tie faces. A general social CW model can instead represent discrete acts of communication as event vertices, typed channels as couplings and selected research projects or institutional practices as faces, with possible higher attachments for collectively constituted scholarly bodies.

The two descriptions should be assembled independently from the source records and connected by documented semantic correspondences. If an inferred scientific movement appears as a single node in a network summary, the analyst should not silently infer that it was a stable, unified actor. The institution's organizational claim must be assessed through its own documents, continuity and contextual evidence. The comparison asks whether topology, grammar and historiography produce complementary insights rather than merely matching decorative pictures.

27.2 Competing hypotheses

One hypothesis is that the network's apparent expansion reflects increased collaboration. Another is that correspondence was selectively digitized from later years. A third suggests a change in what the society counted as a collaboration, while a fourth attributes the shift to an operative rule change. These hypotheses can produce overlapping growth in an observed graph. A useful study would compare multiple observables: the coverage of archival series, verified identities, newly attested couplings, independently recorded constitutive decisions, rule changes and sensitivity of topological features to alternate codings.

A candidate empirical model should be tested against baselines that do not require a new theory. Temporal graph models can capture changing edges; two-mode networks can express persons and societies;

event-history analysis can estimate transitions; process tracing can investigate mechanisms; hypergraphs can encode co-participation; and conventional documentary historiography can interrogate the sources. The cellular construction contributes if explicit attachment, higher-order closure, structural genealogy or topological obstructions explain otherwise obscured distinctions. A negative comparison, in which ordinary network analysis does as well, would be informative.

27.3 Predesignated measurements

One might report Betti numbers and representative cycles at each documented construction stage, sensitivity to source exclusion, agreement among independent coders, and the consequences of changing temporal granularity. However, topology should be interpreted only alongside historical diagnostics: which source assertions support each cell, which archival gaps alter the result, which conclusions survive reclassification, and whether the relevant institutional change was independently established. A formal rule model should also be evaluated by held-out or prospectively selected episodes rather than only the histories used to invent its grammar.

An appropriate design could preregister contrasts in the form

$$\Delta_Q = \mathcal{L}(M_{\text{baseline}}; D_{\text{test}}) - \mathcal{L}(M_{\text{complex}}; D_{\text{test}}), \quad (38)$$

where $\mathcal{L}$ is a task-appropriate predictive or reconstruction loss. Positive Δ_Q would indicate better measured performance for a stated task and dataset, not proof of a superior ontology. For qualitative inquiry, a transparent case-comparison protocol can replace a numerical loss; it should state beforehand which historical distinctions each formalism is expected to preserve.

27.4 A falsification-friendly outcome matrix

The study can produce several outcomes. The complex model may reveal robust higher-order closures corroborated by independent documents. It may produce unstable topology dominated by a few disputed sources. A proposed grammar may fit one archive but fail on another. Two representations may disagree because they encode genuinely different research questions, or because their source processing contains a reproducible error. The framework is productive when these alternatives are made inspectable. A formally ambitious model without such discriminating tests risks attaching mathematics to a historical narrative without improving its evidential standing.

27.5 Reproducible historical software

The existing infrastructure's separation of foundations, domain rules, applications and documentation is well suited to this programme (The Science Infrastructures contributors, 2026a,b). A larger implementation would retain immutable source assertions, canonical record identifiers, alternative coding decisions, source confidence annotations, the exact complex-building algorithm, chain matrices, rule definitions, and provenance for derived observables. Analytical results should be reproducible at a pinned commit with a declared runtime and a license-consistent dataset. Visualizations are valuable for exploration, but the printed paper should explain the mathematical objects independently of an interactive browser.

28 Research Problems and Theoretical Boundaries

The proposed framework invites several lines of investigation whose mathematical and historical requirements should remain distinct. Their importance lies in the obligations they create for subsequent work, rather than in a claim that the present construction has already resolved them.

28.1 Historically meaningful morphology

One question concerns whether relational complexes can recover a notion of morphogenesis relevant to social history. A new institution may reorganize attendance, relation types, authority and memory simultaneously. A model should distinguish the creation of a new documented tie, a restructuring of the conditions governing ties, and a change in the archival observability of ties. These operations need not have identical topological signatures. A theory of historical morphology would connect them to specific, testable mechanisms of institution formation and transformation, possibly comparing CW rewritings with category-theoretic diagrams or graph-grammar models.

28.2 Cross-domain semantics and heterogeneous relations

The six discipline colours are a convenient initial typology, while many historically important phenomena cross their boundaries. A scientific text can also be an instrument of diplomacy and an object of aesthetic judgment. A model may support overlapping colour classes, typed higher-order relations, or a category of interpretive translations, but it must preserve the difference between a shared historical object and a shared interpretation. Extending the formal carrier is straightforward compared with justifying the semantics of its added objects.

28.3 Inference with missingness mechanisms

Historical missingness is strongly nonrandom. Works are destroyed, communications censored, some persons excluded from official records, and digitized archives assembled according to contemporary priorities. A statistical model of unattested ties must eventually include models of why evidence might be missing and what can remain fundamentally unidentifiable. In particular, the absence of an edge from the present complex may be due to a real absence of interaction, insufficient preservation or the omission of a source. Without observations capable of distinguishing these cases, posterior probabilities should be sensitive to assumed missingness regimes. Source uncertainty and model uncertainty should be reported separately.

28.4 Evolving representational grammars

A historically situated ontology may allow the repertoire of permissible categories to change. A language, institution or artistic movement can generate distinctions unknown to earlier codebooks. A promising but difficult task is to model changes in the grammar by which relations are represented, rather than merely adding more items to an unchanged vocabulary. Such a programme would connect the History Complex to the earlier work on knowledge pressing and formal representations of generativity (Huang, 2026d,c). Its strongest form requires an account of why one translation or categorical extension is warranted by the historical material.

28.5 Identity, subject formation and contested collectivities

Persistent paths provide a useful structural encoding but may conceal the historically mediated production of the very entities they are used to track. An individual subject can change commitments, practices and recognized status; a collective identity may be reconstituted through governance, dispute and exclusion. An enriched model might keep identity hypotheses and their supporting documentary relations as first-class objects. This could help distinguish the continuity of a name, the continuity of a legal person, continuity of practices, and continuity of a group recognized by others. Such plural identities require explicit domain semantics and should not be resolved through record linkage alone.

28.6 Counterfactual and normative caution

Finally, the mathematical flexibility of continuing a complex must be constrained by source-grounded causal and ethical reasoning. A coherent model can generate continuations that no historical actor could have imagined or enacted. Conversely, a model's failure to represent an alternative does not imply that it was historically unavailable. A generative research infrastructure should make this epistemic asymmetry explicit and enable communities to challenge the categories and omissions on which its claims depend. This is especially important when the application bears on contested political histories, collective trauma, or the interpretation of marginalized groups' experiences.

29 Conclusion

The proposal investigated here is to study human history through source-indexed cellular complexes while retaining the epistemic distinctions that historical research requires. A coarse CW complex organizes occasions, the coded persistence of beings and repeated relationships into vertices, edges and faces with meaningful attaching maps. Its homology characterizes particular patterns of connectedness, unfilled cycles and closed relational surfaces. A refined complex preserves local ties and supports signed gauge observables and conditional reconstructions. None of these formal results, taken in isolation, establishes an event's meaning, a causal explanation or the completeness of the documentary record.

The initial computational implementation demonstrates that a cross-disciplinary historical corpus can be represented by this machinery, and that its assertions can in principle be returned to view together with computed invariants. The paper's exact synthetic example exhibits the decisive distinction between topology and changing relational signs. Its historiographical analysis identifies a further distinction between coded realizations and modal spaces of permissible transformations. These distinctions create room for inquiry into historical generativity without assuming that a chronology is an ontologically primary timeline or that a probabilistic reconstruction is a recovered fact.

The next substantive advances depend on documentary verification, semantic accountability, reproducible model comparisons and critical evaluation of exclusion and epistemic asymmetry. If these obligations can be met, cellular complexes may become a productive language in which historians and mathematical researchers compare heterogeneous histories, formulate precise questions about their connections and revise their representations in response to new evidence.

A Additional Formal Results and Checks

This appendix collects elementary calculations that permit independent examination of the carrier without appealing to historical cases. Their claims are conditional on the finite record and attaching maps defined in Section 6.

A.1 Cellular consistency

For a worldline segment $e_{b,i} : p \rightarrow q$, $\partial_1 e_{b,i} = v_q - v_p$. For each tie face whose boundary runs along two paths with common endpoints,

$$\partial_1 \partial_2 f_{t,i} = (v_q - v_p) - (v_q - v_p) = 0. \quad (39)$$

Thus the cellular chain groups and boundary maps form a chain complex. This identity is insensitive to the historical truth of an assertion: it is a property of the declared attachment map. Record validity and source warrants are separate checks.

A.2 Homological rank calculation

Over a field $\mathbb{F}$, for a finite connected CW 2-complex one has $b_0 = 1$, $b_1 = E - V + 1 - \text{rank } \partial_2$, and $b_2 = F - \text{rank } \partial_2$. Consequently $\chi = V - E + F = 1 - b_1 + b_2$. Torsion information in integer homology requires an integer computation such as Smith normal form. These formulas are useful for regression tests but do not identify a historian's substantive categories with homological generators.

A.3 A miniature shell and its sign field

Take two occasions $p < q$, and three record-identified beings a, b, c , each present at both occasions. Each being supplies one edge from p to q . For each pair ab, bc and ac , suppose one tie is valid across both meetings; it supplies one face. The coarse complex therefore has $(V, E, F) = (2, 3, 3)$, $\chi = 2$ and $H_1 = 0$, $H_2 \cong \mathbb{Z}$ under the displayed attaching maps. Let the three signs at p be $+1, +1, +1$ and at q be $-1, +1, +1$. Then $P(p) = +1$, $P(q) = -1$, and exactly one lens flip occurs, satisfying Equation (16). This changes the field interpretation but leaves K unchanged. The example proves no empirical claim about the meaning of a frustrated triangle.

A.4 A note on character expansion and gauge fixing

Equation (18) assumes a sum over all link variables on the specified refined carrier. It follows by writing $e^{\beta_f h_f} = \cosh \beta_f (1 + h_f \tanh \beta_f)$ and expanding the product. For a selected set S of faces, the link sum of $\prod_{f \in S} h_f$ vanishes if any edge occurs with odd parity. The surviving subsets are precisely the mod-two 2-cycles. The factor $2^{|E|}$ counts independent summed link variables before fixing gauge. In a gauge-fixed or source-conditioned ensemble, normalization and surviving constraints must be derived again from the actually summed degrees of freedom; the unconditional formula cannot simply be used unchanged.

A.5 Closed-shell compatibility

For three pairwise ties between the same two joint occasions, the oriented face combination $f_{ab} + f_{bc} - f_{ac}$ has zero cellular boundary. The identity $P(q) = P(p) \prod_{t \in \{ab, bc, ac\}} h_t$ is true algebraically for the signed record. The requirement to place local triangular faces in the refined complex arises when the recorded sign change would be incompatible with treating only the coarse worldline edges as gauge variables. This is a representational-consistency result, not a physical Bianchi law applied to societies.

B Outline of a Reproducible Historical Coding Protocol

The protocol below is a proposed research extension; it does not certify that the existing canons already satisfy all of these conditions. Its goal is to make future applications auditable, including the revisions and disagreements through which a historical interpretation develops.

Corpus declaration. Record the research question, geographic and chronological scope, source corpus, acquisition route, permissions, excluded source classes and relevant linguistic competence. Retain stable identifiers and copies or archive links where legally and ethically appropriate.

Source-critical layer. Record documentary genre, authorship claims, material authenticity, provenance, dating uncertainty, translation, conflicting readings and interpretive limitations. Authentication of a document and accuracy of its substantive account are distinct predicates.

Assertion layer. Preserve every source-grounded assertion with its reference, textual support, confidence semantics, author or model drafter, review status and version history. Allow disagreements to coexist without silently overwriting earlier entries.

Identity and occasion layer. Define rules for personal, corporate, geographic and textual identity. Declare the treatment of overlapping intervals, uncertain dates, long-duration occasions and split or merged appearances. Record reasons for joins across historical corpora.

Relation layer. Declare relation types, boundaries, termination criteria and the evidential conditions for any sign. Reserve explicit missing and contested values. Report agreement across coders and illustrate hard coding decisions with anonymized or public-source examples.

Construction layer. Fix the algorithm that converts assertions into K and $\hat{K}$, together with codebook version, dependency versions, execution environment and tests. Preserve a map from every cell back to the assertions and operators that generated it.

Interpretive layer. Separate exact invariants, conditional model estimates and substantive historical claims in the paper and interface. Provide rival interpretations, perturbation tests, uncertainty and the mechanism by which qualified investigators can contest and amend the record.

Ethical layer. For histories involving vulnerable living persons or sensitive groups, perform a separate assessment of privacy, attribution, consequences of publication and appropriate community consultation. Open-source publication of a model is not, by itself, permission to disseminate all data to which the model could be applied.

C Foundations of CW Complexes and Cellular Homology

This appendix develops the mathematical apparatus assumed throughout the main text. It distinguishes a CW topology from a diagram of historical associations, describes the relevant chain algebra, and records elementary results needed for the interpretation of attachments. Its definitions follow standard algebraic topology; their proposed social semantics are additional hypotheses, developed separately in Sections 3–8.

C.1 Cells, characteristic maps and skeletons

A finite CW complex X is built inductively from disjoint open cells. Its n -skeleton $X^{(n)}$ is obtained by attaching finitely many n -disks to $X^{(n-1)}$ along continuous maps from their boundary spheres. An attaching map $\varphi_\alpha : S^{n-1} \rightarrow X^{(n-1)}$ and the corresponding characteristic map $\Phi_\alpha : D^n \rightarrow X$ specify how a cell meets the existing structure. The attaching map can identify multiple boundary arcs or have nontrivial degree. Merely supplying counts of vertices, edges and faces does not fully define X . This is especially important for CW complexes with loop edges, nontrivial attaching words or repeated vertices in a face boundary.

A regular CW complex has attaching behavior restrictive enough that each closed cell is homeomorphic to a closed ball under its characteristic map; many useful CW complexes are not regular. For example, the canonical CW structure on a sphere S^2 may consist of one 0-cell and one 2-cell whose boundary collapses to the point. A naive simplicial description with distinct face vertices cannot express this without subdivision. In a historical model, a repeated occasion or recurrent process may similarly require a non-simplicial attaching specification if the representation is to remain faithful to a proposed carrier.

C.2 Cellular chains and boundary maps

For an oriented finite CW complex and coefficient ring R , the chain module $C_n(X; R)$ is freely generated by the n -cells. Its boundary maps

$$\cdots \xrightarrow{\partial_3} C_2(X; R) \xrightarrow{\partial_2} C_1(X; R) \xrightarrow{\partial_1} C_0(X; R) \rightarrow 0, \quad \partial_n \partial_{n+1} = 0 \quad (40)$$

form a chain complex. For regular cell complexes with suitable orientations, boundary coefficients can be read from signed incidence. For a general CW complex, the coefficients are cellular incidence degrees; they require the characteristic maps, not just adjacency. Matrix multiplication gives a computational check for $\partial_{n-1} \partial_n = 0$. Failing this check indicates an invalid chain specification or inconsistent orientation rather than an unusual historical phenomenon.

Cycles are $Z_n = \ker \partial_n$; boundaries are $B_n = \text{im } \partial_{n+1}$. Homology is the quotient $H_n(X; R) = Z_n / B_n$. Over a field $\mathbb{F}$, the Betti number $\beta_n = \dim_{\mathbb{F}} H_n$ satisfies

$$\beta_n = \dim C_n - \text{rank } \partial_n - \text{rank } \partial_{n+1}. \quad (41)$$

This identity supplies an exact consistency check for computations when boundary matrices are available. Over $\mathbb{Z}$, torsion is also possible; field-valued Betti numbers do not detect the full integral homology structure. A social interpretation should specify the coefficient ring and avoid treating different coefficient choices as mutually interchangeable.

C.3 Euler characteristic and dimension bounds

For a finite complex X , the Euler characteristic is

$$\chi(X) = \sum_{n \geq 0} (-1)^n c_n = \sum_{n \geq 0} (-1)^n \beta_n(X; \mathbb{F}), \quad (42)$$

where c_n is the number of n -cells. For a two-complex, $H_n = 0$ for $n > 2$. The characteristic is invariant under homotopy equivalence but not under arbitrary source-data additions or changes in the modeling convention. A growth in β_2 can signal a closed surface-like cycle in the representation, not necessarily the closure of a diplomatic process or the formation of a self-contained community.

C.4 Relative homology and historical boundaries

When a subcomplex $A \subseteq X$ represents a declared boundary, source subset, or comparison region, the relative chain complex $C_\bullet(X, A) = C_\bullet(X) / C_\bullet(A)$ leads to relative homology $H_n(X, A)$. The long exact sequence

$$\cdots \rightarrow H_n(A) \rightarrow H_n(X) \rightarrow H_n(X, A) \xrightarrow{\delta} H_{n-1}(A) \rightarrow \cdots \quad (43)$$

relates the boundary structure to the larger complex. In a proposed historical analysis, this can be used to compare a network before and after the addition of a documented institutional relation, provided A and X are genuinely linked by a subcomplex inclusion. If the new source data force recutting of life segments or reidentification of records, the original carrier may not sit inside the reconstructed one as a subcomplex. Then a relative comparison requires a new explicit map or common refinement.

C.5 A bounded interpretation of topology

Homology records certain equivalence classes of closed chains under filling; it ignores much of a complex's geometry, typing, probability and history of construction. Two societies with identical Betti numbers can differ in every relevant institutional and ethical respect. Homotopy groups, local systems, sheaf cohomology or typed chain invariants can capture additional distinctions, but their use depends on a justified model. Algebraic topology is therefore a toolbox for inspectable questions about a reconstruction, not a scale of cultural maturity.

D Event Structures, Partial Orders and Relational Time

This appendix clarifies the temporal assumptions required by historically evolving cell complexes. An input dataset commonly supplies calendar dates or documentary date ranges. Such coordinates are indispensable for many research questions but need not be promoted into an ontology of universal time. The author's earlier value-conversion formalism investigates partial orders, event structures, antichains, order ideals and order-sensitive operations; those distinctions are adopted here as candidate interfaces for future extensions.

D.1 Order, chronology and observability

A partially ordered set $(E, \preceq)$ represents a relation that is reflexive, antisymmetric and transitive. Depending on the research question, $e \preceq e'$ may denote evidenced causal precedence, dependency of an institutional step, or a specified generative prerequisite. It should not automatically denote chronological priority. By contrast, a total ordering of recorded occasions can be imposed to compute a repository construction even when historical time information consists of broad intervals or uncertain dates. When ties are built between adjacent documented appearances in that order, changing uncertain date resolutions may change which edges exist; this is a legitimate sensitivity question.

An antichain is a set of pairwise incomparable events. An order ideal $I \subseteq E$ is downward closed: $e' \in I$ and $e \preceq e'$ entail $e \in I$. Order ideals can represent finite, prerequisite-closed domains of a derivation without requiring every event to be assigned a universal time. A linear extension is a total order compatible with $\preceq$; different linear extensions can help implement computations, but an algorithm must not infer an ordering difference between incomparable events merely from the chosen extension.

D.2 Prerequisites and contradictory testimonies

A modeled legal procedure may require drafting and authorized approval before promulgation. This generates a prerequisite relation among procedural events. A historian can separately encode contested witness dates and documentary chronology. Their disagreement is informative: perhaps a document was backdated, an act had retroactive legal effect, or the sources record distinct kinds of occurrence. Collapsing temporal evidence, procedural prerequisite and causal explanation into a single edge type can conceal the very uncertainty the historical inquiry seeks to investigate.

An event structure may also include conflicts or incompatibilities beyond order. Two rival codings of one document could be represented as incompatible observations even when their source assertions refer to the same dated occasion. This permits the study of alternative historical reconstructions without representing them as two co-occurring realities. The formal language used for conflict should distinguish logical incompatibility in the coded claims from historically operative social conflict.

D.3 History-indexed grammars

A derivation can be indexed by a finite ordered diagram H rather than one scalar time axis. Let X_h and Γ_h be the carrier and grammar attached to $h \in H$. For an allowed relation $h \prec h'$, a specified rewrite induces $X_h \rightsquigarrow X_{h'}$ under the relevant admissibility predicate. The index set might be a chain, a partially ordered event structure, or a category in which arrows record different kinds of admissible comparison. Nothing about the notation determines an ontological arrow of time; the orientation and semantics come from the model.

One should distinguish a partial order over recorded events, a precedence order over rule applications, and a modal ordering among admissible transformations. They may be related but are not interchangeable. A model can describe several possible continuations of a represented structure even when their eventual realization is not defined. Likewise, an alternative counterfactual branch may remain a disciplined thought experiment rather than an unrealized event awaiting a privileged future instant.

D.4 Path dependence without a universal path metric

Suppose transformations r, s are both available in a given configuration. If only one order is admissible, or if $r \circ s$ and $s \circ r$ produce different typed outcomes, then order matters in the modeled history. This formal property does not require a continuously varying clock. Nor does it suffice to prove an empirical causal claim: independent historical evidence must establish that the operative processes had the relevant dependency. A theory of historical pathways can therefore combine partial-order constraints with the richer grammar, provenance and interpretive layers described in the main text.

E Group Connections, Gauge Invariance and Spin-Foam Weights

This appendix supplies the limited group-theoretic foundations required for the paper's gauge discussion. It keeps the mathematical definitions separate from any inference that social ties possess a literal gauge-field ontology. The physically inspired machinery appears in the author's earlier value-conversion and social-spacetime work as a candidate language for relational organization. Its empirical justification is a distinct research obligation.

E.1 Discrete connection on a two-complex

Let K be a finite oriented two-complex and G a finite group. Choose one orientation for each geometrical edge and associate $g_e \in G$. The reverse edge receives g_e^{-1} . For a face f with an ordered attaching word, its holonomy is Equation (31). If f is not a simple embedded polygon, the attaching word, repeated edges and orientation exponents must still be represented; an unordered set of face edges is insufficient when G is non-Abelian.

A vertex-gauge transformation $k : K_0 \rightarrow G$ acts on an edge oriented $s(e) \rightarrow t(e)$ by

$$g_e \mapsto k_{s(e)} g_e k_{t(e)}^{-1}. \quad (44)$$

The face holonomy then transforms by conjugation at the selected base vertex: $H_f \mapsto k_{v_f} H_f k_{v_f}^{-1}$. Therefore a class function $W : G \rightarrow \mathbb{C}$ satisfying $W(aga^{-1}) = W(g)$ is suitable for a gauge-invariant face weight. In the Abelian case the holonomy itself is invariant.

E.2 The $\mathbb{Z}_2$ case and cohomology

For $G = \mathbb{Z}_2 \cong \{\pm 1\}$, a connection is equivalently a 1-cochain with coefficients in $\mathbb{F}_2$ after the identification $g_e = (-1)^{a_e}$. Its plaquette sign is

$$H_f = (-1)^{(\delta a)(f)}, \quad (45)$$

where $\delta : C^1(K; \mathbb{F}_2) \rightarrow C^2(K; \mathbb{F}_2)$ is the cellular coboundary. A gauge transformation changes a by a 0-coboundary: $a \mapsto a + \delta\lambda$. Then $\delta a \mapsto \delta a + \delta^2\lambda = \delta a$. Hence face plaquettes are unchanged. The proof is a direct consequence of $\partial^2 = 0$ and does not depend on the social meaning of the edge signs.

Not every assignment of face signs is achievable from edge variables on a fixed complex. For example, products of face holonomies over a closed oriented 2-cycle are constrained when each edge appears with canceling incidence. That algebraic constraint can be a valuable consistency check in a numerical implementation. It would be a mistake, however, to interpret a gauge constraint as evidence that particular historical assertions are jointly true.

E.3 Finite partition functions and inference

For a model with gauge-invariant local weights, an illustrative finite partition function is

$$Z_K(\theta) = \sum_{g \in G^{K_1}} \prod_{f \in K_2} W_{f,\theta}(H_f). \quad (46)$$

If the weights are nonnegative and $Z_K > 0$, normalization yields a probability measure. If the weights are complex amplitudes, the same expression is not in general a probability distribution; extra interpretive and analytic structure is required. For continuous compact G , the sum is replaced by a product Haar integral with appropriate normalization, and one may perform a character expansion into representations. Such mathematical machinery is familiar from lattice gauge theory and spin-foam constructions (Baez, 1998; Kogut and Susskind, 1975). An applied historical model must separately explain what the labels and boundary conditions mean and what observations identify its parameters.

E.4 Boundary states and changing complexes

If a historical region is modeled as a two-complex with designated boundary, one can condition a gauge ensemble on boundary edge data or integrate over them according to a declared procedure. In physical spin-foam models, boundary states carry established mathematical interpretations. In social science, a “boundary state” might encode an institutional interface, the accessible record at the edge of a comparison period, or a prescribed counterfactual intervention. These are conceptually different uses of the word *boundary*; they should not be treated as equivalents without an explicit map.

For a family of different complexes K , a sum over carriers requires a set of allowed complexes, a weight or measure on them and conventions for automorphisms and boundaries. A field theory that generates complexes would additionally specify fields, interactions and a perturbative or nonperturbative interpretation. Nothing in the present historical documents identifies such an action uniquely. The research proposal can investigate it, while the present manuscript refrains from claiming a derived social quantum dynamics.

F Cellular Rewriting, Generative Equivalence and Proof Obligations

This appendix specifies a formal route by which the earlier CW grammar can be applied to history. It is deliberately methodological. A complete double-pushout rewrite calculus for arbitrary CW complexes, with all relevant categorical hypotheses, is outside the manuscript’s proved results. The formal definitions below therefore identify constructions and proof obligations rather than claim that an unrestricted rewrite operation has already been established.

F.1 States, interfaces and rewrite candidates

Let $\mathbf{CW}_\tau$ denote a chosen category of finite typed cellular structures and admissible structure-preserving maps. To model local replacement, choose a diagram $L \xleftarrow{\ell} B \xrightarrow{r} R$ within that category. A match $m : L \rightarrow X$ identifies the occurrence to be replaced, while B defines the interface preserved. In a suitable adhesive category of graphs, familiar double-pushout methods can construct a complement and replacement under gluing conditions. CW complexes with attaching maps, possible cell identifications and heterogeneous types require explicit restrictions before the same categorical results are invoked. In particular, all incident cells of a removed cell must be treated consistently or the rewritten object may fail to be a CW complex with the intended attachments.

A safe finite implementation may therefore operate on a restricted combinatorial class such as regular cell complexes with typed incidence data and verified boundary matrices. Every candidate rewrite should check that attachments of existing cells remain valid and that its resulting boundary operators satisfy Equation (40). The restriction is a useful engineering choice; it does not establish that historical structures are intrinsically regular cell complexes.

F.2 Generative languages and realized genealogies

1850 Given an initial state X_0 and a grammar Γ , define the reachable language

$$\mathcal{L}(\Gamma, X_0) = \{Y : \exists n \geq 0, X_0 \Rightarrow_{\Gamma}^n Y\}, \quad (47)$$

where $\Rightarrow^n$ denotes an admissible finite derivation. A particular modeled history is a derivation trace $h = (X_0, r_1, X_1, \dots, r_n, X_n)$ together with evidence, conditions, and the chosen ordering or partial ordering of rewrite events. The trace is a different object from the endpoint X_n . Repeated generation of isomorphic endpoints does not imply that the traces are equivalent.

1855 One can define endpoint equivalence, trace equivalence and grammar equivalence under separately chosen criteria. Grammar equivalence relative to an observation functor F might require equivalence of F -observable reachable structures, but that is weaker than equivalence of raw derivation languages. An empirically useful account must make the observational criterion explicit and examine whether it conflates histories that matter ethically or institutionally.

1860 F.3 Why endpoint topology does not identify history

Take a finite cell complex X with an edge e and consider two derivations. In one, a transformation subdivides e into two consecutive edges and later merges them under a declared inverse operation. In the other, no rule is applied. Both end at structures isomorphic to X . Even the intermediate subdivision is homeomorphic to the original carrier, so Betti numbers at all observed endpoints can agree. Yet the traces have different lengths, rules, and provenance. If the intermediate operation represented a legal proceeding, an educational experience or a scientifically productive failed attempt, eliminating it on grounds of endpoint topology would erase historically significant information. This synthetic example proves the logical insufficiency of endpoint homology for identifying derivational history.

F.4 Grammar updates and self-reference

1870 A rewrite on a cellular state should be distinguished from a rewrite of the grammar describing admissible state changes. One can represent grammar changes through a higher-level map $\Gamma_h \rightsquigarrow \Gamma_{h'}$ triggered by a rule that specifies evidence of authorization or social uptake. A statute's amendment, a standard's revision and an informal change of practice can all affect admissibility, but their evidence and effect conditions differ. Representing them with one abstract operator may aid comparison, provided the operative semantics are supplied by domain-specific types and predicates.

1875 Self-reference generates a further difficulty: some admissibility decisions are expressed and negotiated by actors who are also constituted by the institutional system. An external evaluator who always knows the true rule and true social identity is therefore a poor default. A more careful model allows multiple interpreters to propose differing rule classifications, records the authority under which a resolution was made, and preserves unresolved disputes. The grammar can become an object of historical argument, not a timeless oracle.

F.5 A concise verification checklist

1885 A proposed grammar is formally meaningful when its category of states and morphisms is defined; each rule's left pattern, preserved interface and replacement are well typed; the gluing operation is valid; the resulting object satisfies cell-complex and boundary consistency; the admissibility predicate is explicit; and the derivation log identifies the applied rule and evidence. It becomes an empirical theory only through independently assessed records, informative comparisons, and sensitivity analyses. Finally, its use in governance or historical evaluation requires distinct normative arguments about authority, rights and contestability.

G Historical Evidence, Source Dependence and Identifiability

This appendix develops a basic information model for the provenance-sensitive historical complexes proposed in the main text. The goal is to make the inferential status of a cell visible. A source report, a historian’s coding decision, a constructed cell and a topological invariant occupy different levels of description and should be traceable without being conflated.

G.1 Assertions and support relations

Let S be a set of source items, A a set of atomic historical assertions and C a set of coding claims. Each assertion $a \in A$ is related to one or more sources through a support map $\text{supp}(a) \subseteq S$, which records relevant passages and documentary provenance. A coding claim may assert that two recorded names refer to one individual, that an occasion is shared, that a particular relation persisted during a bounded interval, or that a collective institution was constituted. Such claims have reasoned statuses: supported, contested, provisional, rejected or unassessed, rather than a universal scalar truth value imposed by software.

A reconstruction function

$$\mathcal{C}_{Q,\rho} : (S, A, C, \kappa) \longmapsto X_{Q,\rho} \quad (48)$$

uses an explicit coding protocol κ to produce a finite complex. Its dependence on Q and ρ expresses question-relative selection. A supplementary trace map should associate each generated cell to its supporting coded assertions; it should also record algorithmic derivations, such as the creation of a life segment between adjacent appearances, which may be derived from a rule rather than directly attested. Losing this distinction can lead a reader to mistake an algorithmically interpolated edge for a directly recorded encounter.

G.2 Identifier resolution and joins

Two historical datasets may use the same string for different persons or multiple names for one person. A joint record requires a documented equivalence relation on identities, supported by identifiers, sources and a declared matching rule. Joining records can create new occasions along an established life history, which recuts segments in the resulting complex. Hence constructing a complex after joining source records may not equal taking the set-theoretic union of independently constructed complexes. This feature is explicitly recognized in the repository’s domain theory (The Science Infrastructures contributors, 2026e). The difference is consequential for chain maps, filtrations and incremental computations.

G.3 Sensitivity to one disputed assertion

Suppose a face f is supported only by one contested claim. Adding or removing f can change rank ∂_2 and therefore β_1 . The magnitude of this change is bounded by one for a single new two-cell over a field when the rest of the chain groups are fixed, but the interpretive consequence may be large if the face is presented as evidence of a united coalition. This example motivates computing a cell-level dependency graph for every highlighted invariant. A statement such as “the network contains two holes” should be accompanied by a record of which tie or constitution claims cause those holes, and which alternative codings change the result.

Adding several records may alter identity resolution, occasion ordering and the entire segmentation, so the fixed-chain-group bound need not apply to such a join. Computational incremental updates should explicitly identify whether they are adding cells to an unchanged carrier or rebuilding the carrier after recutting. This distinction is a prerequisite for meaningful topological persistence across data releases.

G.4 Alternative reconstructions and uncertainty

1930 Let Ω denote a set of defensible coding assignments. The family $\{X_\omega : \omega \in \Omega\}$ supports interval-valued or distribution-valued summaries of a topological observable T . One can report

$$\text{Range}_\Omega(T) = \{T(X_\omega) : \omega \in \Omega\}. \quad (49)$$

If a probability measure on Ω is proposed, its source-dependence assumptions should be stated. Two biographies drawing from the same original letter are not independent confirmations merely because they are separate books. Archival gaps, dependence among published narratives and shared coding biases can
1935 cause spurious precision even when the formal ensemble is computed without error.

G.5 Limits of historical identifiability

Multiple historical processes may leave the same set of surviving documents. Multiple generative grammars may produce an identical observed derivation. Multiple complexes may be consistent with an archive when identity and duration are uncertain. A statistical fit cannot make a parameter historically
1940 identifiable unless the observation model and comparisons constrain it. A useful model should distinguish structural nonidentifiability, practical limitations of finite data, and semantic noncomparability between historical categories. The first may call for a proof or counterexample; the second for additional sources; the third for a richer interpretive framework.

An accountable research infrastructure therefore preserves both outcomes and the route by which they were inferred: primary and secondary sources, provenance chains, coding judgments, changes of
1945 ontology, revisions of software, and computed results. Its purpose is not to guarantee one final view of history, but to support criticism, recovery, and responsible reanalysis.

H Source Alignment and Theoretical Genealogy

This appendix records the provenance of the manuscript's theoretical concepts. Its purpose is bibliographic and methodological: to distinguish ideas recovered directly from prior manuscripts and repository
1950 documentation from new synthesis proposed here. A previous paper's presence in this genealogy indicates an intellectual relation, not independent empirical confirmation or priority over broader mathematical literature.

Table 6: Alignment of selected earlier writings, repository objects and the present paper.

Earlier source	Source-supported commitments	New use or bounded extension in this paper
<i>Social Spacetime Topology and Generative Grammars in CW Complexes</i> (Huang, 2026: Social CW)	Vertices as events; edges as typed coupling interfaces; question-relative primitive faces; higher-cell attachments; a separate grammar $\Gamma = (\mathbf{T}, \mathbf{R}, \mathbf{A})$; multi-axis equivalence; ontological suspension.	Explicit comparison with the occasion–life–tie convention and with the public software; chain-map and common-refinement obligations.
<i>A Theory of Historical Generativity under Constraints</i> (Huang, 2026a)	Historically constituted constraints, slow variables, actor horizons, endogenous revision and caution about importing action principles.	Constraint projection and testable distinctions between rule changes, structure changes and evidence revisions.

Earlier source	Source-supported commitments	New use or bounded extension
<i>A Preliminary Formalism for Heterogeneous Value Conversion</i> (Huang, 2026: Value Conversion)	Partial-order event structures, conversion operators, order sensitivity, emergent nomos, value attribution, ontology/evidence distinction and spin-foam resources.	A more general historical grammar of admissible transformation and a source-sensitive interpretation of operator order.
<i>Heterogeneous Value Circulation in Relational Spacetime</i> (Huang, 2026b)	Exploratory value circulation and topological representation, as described in the publication record and manuscript v0.2.	Distinguishing circulation pathways and legitimacy from untyped cycles and signed relations.
<i>The Topological Epistemology of Future Creation</i> (Huang, 2026c)	Continuation-complex questions and historical dependence of generative possibilities, as recorded in the earlier bibliographic entry and research programme.	Explicit separation of modal successor sets from recorded derivations, with no privileged global time.
<i>Knowledge Pressing in Evolving Relational Spacetime</i> (Huang, 2026d)	Translation and inference of generative representations across heterogeneous systems, as described in the earlier bibliographic record.	Partial interpretive translation with explicit non-translatable residues and provenance.
Repository idea note and history theory (The Science Infrastructures contributors, 2026d,e)	Construction from occasions, beings, presences and ties; $\mathbb{Z}_2$ gauge model; relation to previous social spin foams and future field-theory proposals.	Distinguishing implemented claims from open proposals and comparing semantics across cellular conventions.

The available direct textual sources for this revision include the eighty-eight-page draft *Social Spacetime Topology and Generative Grammars in CW Complexes* (Section 18, dated 6 October 2026), the seventy-six-page *A Theory of Historical Generativity under Constraints*, and the ninety-two-page *A Preliminary Formalism for Heterogeneous Value Conversion*. These were inspected directly. Additional author publications appearing in the earlier v0.2 bibliography or the supplied publication listing are used conservatively as programme-level references when their complete manuscripts have not been inspected in this revision. The exact definitions of those additional works should be checked against their authoritative released text before final publication.

H.1 Distinctions to preserve across the programme

A cell's dimension is a modeling choice, while a historical process's organization requires evidence. A generative grammar states modeled admissibility; a realized derivation states a supported trajectory. A signed tie is a coding object; an ethical judgment requires criteria. A mathematical invariant can be reproducible even when the record's completeness is disputed. A self-consistent field theory does not thereby become a physical or social ontology. A source that proves a document was issued does not by itself establish the truth of each assertion in the document. These distinctions are central to the enlarged research programme and remain open to further refinement.

Data, code and reproducibility

The implementation, record codebook, domain theory and design decision are accessible at <https://github.com/huangwanhong-maker/The-Science-Infrastructures>. The manuscript's self-

contained synthetic example is logically reproducible from its stated definitions; the original technical diagrams are supplied as editable TikZ code. This text does not certify an independent run of all software fixtures, nor does it provide a source-verified reconstruction of any historical canon. A reproducibility package for empirical historical claims would need a fixed commit hash, immutable datasets, checked source references, the precise coding protocol, independent coding, code/runtime versions, and uncertainty analyses.

References

- Abbott, A. (2001). *Time Matters: On Theory and Method*. University of Chicago Press.
- Baez, J. C. (1998). Spin foam models. *Classical and Quantum Gravity*, 15(7), 1827–1858. doi:10.1088/0264-9381/15/7/004.
- Bradley, J., and Short, H. (2005). Texts into databases: The evolving field of new-style prosopography. *Literary and Linguistic Computing*, 20(Suppl.), 3–24. doi:10.1093/lc/fqi022.
- Braudel, F. (1958). Histoire et sciences sociales: La longue durée. *Annales*, 13(4), 725–753. doi:10.3406/ahess.1958.2781.
- Breiger, R. L. (1974). The duality of persons and groups. *Social Forces*, 53(2), 181–190. doi:10.1093/sf/53.2.181.
- Cartwright, D., and Harary, F. (1956). Structural balance: A generalization of Heider’s theory. *Psychological Review*, 63(5), 277–293. doi:10.1037/h0046049.
- Dowker, C. H. (1952). Homology groups of relations. *Annals of Mathematics*, 56(1), 84–95.
- Edelsbrunner, H., and Harer, J. L. (2010). *Computational Topology: An Introduction*. American Mathematical Society.
- Kogut, J., and Susskind, L. (1975). Hamiltonian formulation of Wilson’s lattice gauge theories. *Physical Review D*, 11, 395–408. doi:10.1103/PhysRevD.11.395.
- Krippendorff, K. (2018). *Content Analysis: An Introduction to Its Methodology* (4th ed.). SAGE.
- Sewell, W. H., Jr. (2005). *Logics of History: Social Theory and Social Transformation*. University of Chicago Press.
- Zaslavsky, T. (1982). Signed graphs. *Discrete Applied Mathematics*, 4(1), 47–74. doi:10.1016/0166-218X(82)90033-6.
- The Science Infrastructures contributors. (2026a). *History Complex Application and Protocol*. Project repository; accessed 24 October 2026.
- The Science Infrastructures contributors. (2026b). *Codebook of the History Domain*, version 1 (draft). Project repository; accessed 24 October 2026.
- The Science Infrastructures contributors. (2026c). *ADR 0041: The History Domain and the History Complex*. Project repository; 24 October 2026.
- The Science Infrastructures contributors. (2026d). *The History Complex: Histories as Social Spin Foams, from the Record to the Formalism* (idea note). Project repository; accessed 24 October 2026.
- The Science Infrastructures contributors. (2026e). *The History Complex: Theory and Construction* (draft). Project repository; accessed 24 October 2026.

Hatcher, A. (2002). *Algebraic Topology*. Cambridge University Press. <https://pi.math.cornell.edu/~hatcher/AT/AT.pdf>.

Whitehead, J. H. C. (1949). Combinatorial homotopy I. *Bulletin of the American Mathematical Society*, 55, 213–245.

2015 Forman, R. (1998). Morse theory for cell complexes. *Advances in Mathematics*, 134(1), 90–145.

Huang, W. (2026). *Social Spacetime Topology and Generative Grammars in CW Complexes*. Working discussion paper, 88-page draft (Section 18), 6 October 2026. Manuscript inspected for this revision; publication identifier not independently established.

2020 Huang, W. (2026). *A Preliminary Formalism for Heterogeneous Value Conversion in the Generative Relational Framework*. Working discussion paper, 92-page draft, inspected for this revision; publication identifier not independently established.

Huang, W. (2026a). *A Theory of Historical Generativity under Constraints: Towards Generative Relational History*. Knowledge Commons. doi:10.17613/q3ya0-6f476. Exploratory research.

2025 Huang, W. (2026b). *Heterogeneous Value Circulation in Relational Spacetime: A Topological Formalism*. Knowledge Commons. doi:10.17613/kw3jt-5qw23. Exploratory research.

Huang, W. (2026c). *The Topological Epistemology of Future Creation: A Two-Complex Formalism for Spacetime Continuation*. Knowledge Commons. doi:10.17613/j17xx-0x835. Exploratory research.

Huang, W. (2026d). *Knowledge Pressing in Evolving Relational Spacetime: Inference and Translation of Generative Representations*. Knowledge Commons. doi:10.17613/h9x7f-dfp21. Exploratory research.

2030 Huang, W. (2026e). *A Grammar of Power: Repertoires and the Reproduction of Asymmetry*. Knowledge Commons. doi:10.17613/vpjan-0ss17. Exploratory research.

Huang, W. (2026f). *Criticality or Controllability? The Historical Evolution of Generative Landscapes*. Knowledge Commons. doi:10.17613/m64j0-hm816. Exploratory research.

2035 Huang, W. (2026g). *Generative Epistemic Injustice*. Knowledge Commons. doi:10.17613/s2eps-9k130. Exploratory research.

Huang, W. (2026h). *Equivalence Structures in Mathematical Models of Consciousness: A Preliminary Algebraic and Geometric Investigation*. Knowledge Commons. doi:10.17613/4y3wp-x3251. Exploratory research.

2040 Huang, W. (2026i). *Defining Relation in Generative Relational Being: A Position Paper*. Knowledge Commons. doi:10.17613/f9pwy-x0326. Exploratory research.

Huang, W. (2026j). *Interpreted Temporal Fields: Anticipation and the Composition of Generative Assessment*. doi:10.17613/3j9m5-ptn16. Exploratory research.

Huang, W. (2026k). *Braid Groups and Procedural Justice: Path-Dependence in Generative Relational Being*. Knowledge Commons. doi:10.17613/c2qm7-mqf03. Exploratory research.