

Defining Support-Based Classifications from Aggregated Relational Records

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Abstract

A support attribute can be explicitly defined while remaining unresolved by an aggregated record. This paper examines that distinction in finite models of proposals supported by persons partitioned into groups. It separates attribute representation, definition relative to a language, evidential assignment, and institutional use. After reconstructing the elementary symmetry test for definability, it gives an explicit formula for a fixed threshold of distinct supporters. An exact group-incidence record preserves which groups contain support while omitting individual multiplicity. Under known finite capacities and otherwise unconstrained support relations, the paper proves that every person count between the number of occupied groups and their combined capacity is attainable. This yields sharp threshold judgments and an exact condition for recovery across the whole record format. A fictional dossier distinguishes person counts, group diversity, eligibility, and decisions under uncertainty. Counterexamples address overlapping memberships, shared endorsement constraints, incomplete negative records, and category revision. Verified antecedents in model theory, incomplete databases, numerical aggregation, and conceptual revision locate the inquiry without a claim of priority. The institutional discussion states a conditional premise for answerability and keeps authority distinct from definability. Open questions concern targeted record refinement, joint uncertainty, support meaning, and evaluation in actual practices.

Keywords: relational classification; definability; aggregated records; support thresholds; evidential uncertainty; institutional categories

Research Purpose and Scope

This note states the purpose of circulation, the status of the argument, and the role of research assistance. The paper presents current insights for criticism and wider discussion in public and scholarly settings. It makes no claim to conceptual priority. Similar concepts and arguments may already exist in other disciplines, languages, and intellectual traditions; further comparison may alter the terminology, scope, or contribution.

Definitions organize a bounded inquiry. Conditional formal results carry their stated assumptions, and illustrative cases support conceptual analysis. Empirical validation, wider applicability, and the adequacy of proposed normative premises remain open where the main text identifies those obligations. Relational analysis supplies a method of investigation while stronger ontological questions remain suspended.

AI assistance contributed to source discovery and pre-use verification, argumentative development, formal checking, organization, and drafting. The accompanying project records distinguish retained research insights, newly proposed analysis, and verified literature. The manuscript is prepared for the author's review; circulation and submission decisions remain with the author. Critical responses concerning antecedents, assumptions, counterexamples, and alternative formulations are welcome.

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1 Introduction

This paper examines the definition and evidential application of support-based classifications when records of individual relations are aggregated into groups. The discussion first separates the commitments involved in an attribute, then develops a finite proposal-support model, derives sharp bounds for threshold judgments, and considers their institutional interpretation. The method combines explicit definitions, conditional mathematical arguments, a fictional dossier, and comparison with verified antecedents in logic, databases, and philosophy. The resulting account concerns a bounded representational problem; broader questions about the constitution of social reality remain suspended.

A proposal may be described as supported by several people, supported across several groups, or eligible for consideration. These descriptions can occur together while answering different questions. The first counts distinct supporting persons under an identity convention. The second counts occupied groups under a grouping convention. The third connects some criterion to an institutional status through a rule. Even an exact count leaves open the appropriateness of the rule, the interpretation of support, and the evidence that the recorded relations hold. Keeping these dependencies visible is the central purpose of the paper.

Consider a register that replaces individual endorsements with one mark for each group containing a supporter. The register retains a useful fact: a marked group contains at least one represented supporter. A group containing one supporter and the same group containing three supporters receive the same mark. If the institution later applies a threshold requiring three distinct persons, the retained marks may leave the threshold unresolved. A fully explicit mathematical definition of the threshold does not restore the missing distinctions. Definability and answerability therefore require separate analyses.

The distinction is especially relevant to relational formulations of attributes. A property can be represented by a subset of a carrier or by a predicate over a relational structure. This representation makes membership precise once the subset or predicate interpretation is supplied. A further derivation is needed to show that the property follows from the declared base language. A further evidential argument is needed to connect a represented membership to a person or proposal. A further institutional argument is needed when that classification controls participation, resources, or recognition. Each step can be defensible, but its defense has a different object.

The formal contribution is deliberately specific. A finite population is partitioned into groups of known sizes. For each proposal, an exact aggregate record states which groups contain at least one supporter. With otherwise unconstrained support relations, the possible number of supporters is every integer between the number of occupied groups and their combined capacity. The paper proves both bounds and the attainability of every intermediate count. It then characterizes the thresholds that can be affirmed, refuted, or left open from that record. A second result identifies when the entire aggregate record format supports a fixed threshold for every possible support relation.

The interpreted case also exposes losses that the interval result leaves untouched. Person counts can be recovered while supporter identity remains uncertain. Several proposals can share possible supporters, so separate row bounds can conceal joint constraints. Overlapping group membership changes the counting problem because a single person can witness several group marks. The discussion treats these cases explicitly and identifies the assumptions required to extend the partition model to a wider social setting.

The relevant antecedents already contain substantial resources for this analysis. Elementary model theory relates definability to preservation under structural symmetries. Incomplete-information research studies answers common to compatible complete databases and the special problems of numerical aggregation. Philosophical work on conceptual revision distinguishes inquiry into existing concepts from inquiry into the purposes concepts should serve. The present paper uses these resources to organize a support-classification problem and supplies its own limited derivations. Conceptual and mathematical priority remain outside its claims.

The wider research programme treats attributes alongside other analytical categories of social facts. Here the attribute question receives a local interpretation: a classification of proposals based on repre-

sented support relations. The paper retains overlapping predicates, partial evidential assignments, changing criteria, and the difference between a category's formation and warrant for applying it. The aim is to make the category discussion more accountable to a worked example without making that example an exhaustive account of social attributes.

The manuscript is organized around the dependencies of the argument. The antecedent discussion establishes the intellectual comparisons. The next three sections specify attribute commitments, formal languages, and aggregation. The central derivation and dossier establish the threshold results. Subsequent sections examine overlapping categories, joint constraints, missingness, revision, and institutional authority. Objections and open questions identify the conditions under which the proposed representation should be revised or abandoned.

2 Antecedents in Definability, Incomplete Information, and Conceptual Revision

This section locates the inquiry among several existing approaches while specifying the limits of each comparison. The discussion moves from logical definability to incomplete-data semantics, numerical aggregation, and philosophical concept formation. Each antecedent supplies a distinct resource. Their conjunction motivates an audit of support classifications, while the application and its normative premises remain the responsibility of the present argument.

In elementary model theory, the interpretation of a first-order formula is preserved by an isomorphism when its variable assignment is transported appropriately. An automorphism therefore preserves a parameter-free definable set. Mayr's course notes present the inductive preservation argument and its application to definable sets. With parameters, the corresponding comparison uses automorphisms fixing those parameters. This result supplies a useful obstruction: a proposed attribute that separates structurally interchangeable objects requires additional represented distinctions or named parameters. The proof used below reconstructs the elementary argument within the declared language.[4]

The logical comparison concerns a specified structure and syntax. An attribute can be definable in an enriched language while remaining absent from a smaller base language. The difference is particularly important when a model is introduced with a small collection of primitive operations and later acquires meaningful social predicates. Naming a new predicate determines an interpretation only after an extension has been chosen. Showing that the predicate is an abbreviation for a formula in the original language requires an actual definition. The support model will state its enrichments openly.

Libkin's treatment of incomplete information interprets an incomplete database through the complete objects compatible with it and examines information shared across those possibilities. In the relational setting, certain-answer reasoning retains answers holding under every admitted completion. The paper develops a broader account through information orderings. This provides the immediate methodological antecedent for distinguishing a represented support relation from the set of relations compatible with an aggregate record. The present finite interval calculation is a conditional specialization, with its assumptions and proof given directly.[3]

An exact Boolean answer and a useful numerical report can differ. If compatible records yield several different counts, a single certain numerical value may be unavailable even though every count lies within a narrow interval. Console, Libkin, and Peterfreund examine arithmetic and aggregation over incomplete numerical data and discuss limitations of conventional certain and possible outputs. Their proposed framework uses probability distributions associated with marked nulls and retains dependencies created by repeated nulls. The present model uses finite sets of possible support incidences and supplies no probability distribution. The comparison concerns the need to represent uncertainty and dependency in numerical outputs.[1]

Haslanger distinguishes conceptual inquiry into existing concepts, descriptive inquiry directed toward phenomena and extensions, and analytical inquiry into the cognitive or practical work concepts should perform. Her discussion also treats the appropriation of familiar terminology for a revised concept as involving semantic and political considerations. This is a pertinent methodological comparison

for an institution that changes the meaning of “supported” while retaining the same label. The present account does not transfer her substantive analyses of race and gender to proposal records; it uses the inspected methodological distinction to keep description, revision, and justification visible.[2]

These antecedents support several different forms of criticism. A logician can ask whether a predicate is definable from the stated structure. A database researcher can ask which answers survive the loss of individual records. A philosopher can ask whether the selected concept serves a defensible purpose and whether its terminology conceals a change of subject. A participant can challenge the accuracy of a recorded endorsement or the consequences attached to it. Agreement at one level can coexist with unresolved disagreement at another.

The paper’s own contribution is an organized application across these levels. The sharp support-count interval follows from a particular incidence representation, a partition, and finite capacities. The account of category authority introduces an explicit normative premise. The fictional examples test distinctions without establishing their frequency in actual institutions. These boundaries keep the literature discussion from becoming a claim that an existing theory already contains the entire proposal, or that combining familiar ideas establishes a new general theory of classification.

3 Attribute Representation and Its Commitments

This section distinguishes the activities involved in describing a proposal through an attribute. It introduces a compact vocabulary for representation, definition, evidential assignment, and institutional use, then examines their possible combinations. The distinctions prepare the formal model by identifying the questions that a successful calculation can answer and the questions that remain outside it.

Definition 3.1 (Declared attribute). Relative to a carrier X , a Boolean attribute is a specified predicate on X , equivalently a subset of X . A value-valued attribute is a specified map to a declared value domain. A partial attribute includes a domain of applicability or definedness that must itself be recorded.

The word “declared” marks the representational commitment. Selecting a subset tells the model which cases count as members. It does not yet explain the selection. An attribute can be supplied by an analyst, fixed by an institutional rule, interpreted from evidence, or proposed as a regularity emerging through practice. These descriptions concern different aspects of its formation and can overlap. An institutionally established category can be investigated empirically; an analyst’s category can later be adopted institutionally; a participant’s description can challenge both.

Definition 3.2 (Definition relative to a language). An attribute is explicitly definable in a specified language and structure when a formula in that language, with every allowed parameter identified, has exactly the attribute’s extension. A definition relative to a class of structures must state the class on which the defining equivalence is intended to hold.

This definition distinguishes a language-relative result from the loose statement that a property can be “expressed mathematically.” Every chosen Boolean subset can be given a new predicate symbol. That maneuver enlarges the language and supplies an interpretation; it does not exhibit a formula using only the previous resources. A formula can also contain a parameter naming a favored proposal. Its definability then depends on that selection, whose meaning and justification remain substantive questions.

Definition 3.3 (Evidential assignment). An evidential assignment connects a declared attribute to a particular case through records, observations, testimony, or other stated grounds. Its status includes the assumptions under which the available material warrants affirmation, refutation, or an unresolved judgment.

An exactly defined threshold can remain evidentially unresolved. Conversely, a person may have well-supported practical knowledge of a proposal’s support without having written a formal definition. Formal articulation and evidence serve related purposes while retaining independent standards. The

model below isolates one source of evidential incompleteness: aggregation of a stipulated exact individual relation. Actual inquiry can also face inaccurate records, disputed meanings, identity errors, and changing intentions.

Definition 3.4 (Institutional classification rule). An institutional classification rule connects specified conditions to a status within a declared practice. A separate action rule determines the consequences of that status, such as scheduling consideration or allocating a resource.

Suppose a committee stipulates that proposals with three supporters are eligible for discussion. The support threshold supplies a descriptive condition under an interpretation of “supporter.” Eligibility depends on the committee’s rule and its standing to operate that rule. Scheduling a meeting depends on an additional action arrangement. These distinctions are useful even when a single document expresses the entire procedure. A document can record several commitments without making them conceptually identical.

The distinction between a true condition and a status also accommodates constitutive practices. An authorized acceptance can establish a status within an institution even when the information used to reach it later proves mistaken. Whether the status remains valid, becomes reviewable, or is rescinded depends on the institution’s rules. The mathematical model supplies a threshold result; it leaves the legal or organizational consequences to separately established procedures. This is an analytic possibility, not a claim about any particular jurisdiction.

Attributes also differ in their treatment of time. A person’s endorsement at submission can be a historical relation even after withdrawal. Current endorsement is another predicate. A proposal’s status at a decision date can depend on a fixed snapshot or on continuing support. A register that omits the relevant time convention can conflate these targets while containing internally accurate entries. A suitable attribute specification therefore records the temporal question together with its carrier and criterion.

The four activities provide a practical reading discipline. A claim that an attribute has been derived should identify the language and the definition. A claim that a proposal has the attribute should identify the evidence. A claim that the attribute establishes eligibility should identify the institutional rule. A claim that eligibility warrants an action should identify the relevant reasons and authority. The formal sections develop the first two activities in detail and return to the latter two after the limits of aggregation are established.

4 Formal Languages and Declared Support Predicates

This section specifies the resources from which support attributes are defined. It begins with the small generative-relational base used in the wider programme, introduces the additional incidence structure required for this case, and proves the elementary preservation principle behind the definability audit. The discussion then gives an explicit supporter-threshold formula and separates its mathematical interpretation from the meaning of an endorsement.

The base language has a sort of configurations, a sort of mechanisms, a composition operation on mechanisms, an action on configurations, and a declared equivalence on configurations. An action-preservation condition allows equivalent configurations to receive compatible updates. These resources organize transformations and retained distinctions. A person domain, a proposal domain, endorsement incidence, a group assignment, and an eligibility rule are additional structures unless an encoding into the base resources is exhibited.

An enriched incidence language has a proposal sort A , a person sort P , and a binary relation symbol R interpreted as a subset of $A \times P$. The intended reading of $R(a, p)$ is that the represented person p supports proposal a under a stated convention. Both sorts are finite in the main model. Equality on P supplies the distinctness criterion used in counting. The interpretation of a label as one actual person is an evidential commitment that the formal language alone cannot validate.

Definition 4.1 (Person-support threshold). For a fixed positive integer k , the attribute T_k holds of a proposal when at least k distinct members of the person domain stand in the support relation to it.

The defining formula is

$$T_k(a) \iff \exists p_1 \cdots \exists p_k \left(\bigwedge_{i=1}^k R(a, p_i) \wedge \bigwedge_{1 \leq i < j \leq k} p_i \neq p_j \right). \quad (4.1)$$

Equation 4.1 abbreviates a finite first-order formula for each fixed k . The index k specifies the number of quantified variables and conjuncts; it is not a freely varying numerical object inside this language. A uniform variable threshold would require a suitable counting formalism or an additional numerical structure. For $k = 1$, the distinctness conjunction is empty and imposes no further condition.

The formula demonstrates a genuine definitional result in the enriched language. It uses incidence and equality already declared there. It also makes two dependencies explicit. Repeated records of one person do not provide distinct witnesses, and a different interpretation of R can change the extension while the formula stays fixed. Counting names, verified persons, authorized representatives, or expressions of interest would be different representational choices unless their agreement is established.

Proposition 4.2 (Preservation with fixed parameters). *Let an attribute be defined by a first-order formula in a structure, using a declared tuple of parameters. Every automorphism fixing those parameters preserves the attribute.*

Proof. An automorphism preserves the interpretation of each primitive relation and function and is bijective on each sort. Induction on terms therefore shows that evaluating a term and then applying the automorphism gives the same result as first transporting its arguments and then evaluating it. Atomic equalities and primitive relational formulas consequently retain their truth values. Boolean combinations retain truth because their constituent formulas do. For an existential formula, a witness can be transported by the automorphism to obtain a witness for the transported assignment. The inverse automorphism supplies the reverse implication. Universal quantification follows by the same bijective correspondence of assignments. Induction on formulas completes the argument. Fixed parameters retain their original interpretations throughout, so the defined extension is preserved. $\square$

This is the elementary preservation result discussed in the antecedent section, restated to expose the parameter condition. Its value here is diagnostic. Consider a base structure with two configurations a_0, a_1 , one identity mechanism, and equality as the retained equivalence. Swapping the two configurations preserves every base resource. A predicate selecting only a_0 therefore has no parameter-free definition in that base. An added support relation can distinguish the configurations when they are interpreted as proposals. Naming a_0 as a parameter can also define the singleton by equality. These are different enrichments with different explanatory contents.

The example also separates definability from unique determination by a programme. With the same base data, one expansion can mark a_0 as favored and another can mark a_1 . Both may satisfy a description that merely permits an arbitrary additional classification. The base has then supplied insufficient information to select the intended expansion. A particular expansion can still contain definable predicates after its additional parameters or relations are named. The failure concerns the derivation of that selection from the smaller declared resources.

Automorphism preservation is a necessary test and should be used at that strength. This paper neither relies on a general converse nor treats the absence of a visible symmetry as proof that a socially meaningful category has been derived. In finite named datasets, an extension can sometimes be specified simply by listing its members. Such a formula can be correct while offering little explanatory insight or transferable criterion. Uniformity across cases, intelligible reasons, and evidential accessibility are further methodological demands.

Finally, the support relation retains an interpretive obligation. A signature can express endorsement, permission to list a name, attendance, or receipt of information depending on the practice. The formula cannot decide among those meanings without a represented convention and evidence of its application. The formal result is therefore best stated carefully: once a support incidence and an identity convention have been supplied, the fixed numerical threshold has an explicit definition. The next section examines the additional question of recovering its value from an aggregated record.

5 Group Aggregation and the Loss of Individual Witnesses

This section defines the aggregation operation and identifies the information it preserves. It introduces disjoint groups of known capacity, an exact group-incidence record, and the compatible detailed support relations. The discussion then distinguishes the targets affected by aggregation, preparing the sharp bounds in the following section.

Let G be a finite group set and let $q : P \rightarrow G$ be a surjective map. Each represented person belongs to exactly one group in this model. For a group g , its fibre $q^{-1}(g)$ is nonempty and has known size m_g . The group assignment can be an administrative partition, a deliberately selected affiliation, or another declared scheme. Actual multiple affiliations are addressed later. The partition assumption performs mathematical work and carries no claim that social membership is naturally exclusive.

For a detailed support relation $R \subseteq A \times P$, the aggregate relation $B \subseteq A \times G$ is defined by $B(a, g)$ exactly when some p with $q(p) = g$ satisfies $R(a, p)$. An occupied group therefore has at least one supporting person. An unoccupied group has none under this exact record convention. This is stronger than a register containing only a selection of positive reports. Treating an absent mark as a negative statement requires completeness of the aggregation relative to the declared relation.

Assumption 5.1 (Exact partition aggregate). The person domain and its equality criterion are fixed. The group fibres form a finite partition with known positive capacities. The aggregate relation records exactly the existence of support in each group. Apart from these records, the main derivation imposes no restriction on which persons support a proposal or on their support for other proposals.

The assumption makes the first derivation transparent. It allows one person to support several proposals and permits every nonempty subset of an occupied group to support a given proposal. These possibilities can be unrealistic in a particular institution. They are retained initially so that the effect of adding further constraints can be stated precisely. A justified restriction changes the set of compatible detailed relations and can tighten the conclusions.

Definition 5.2 (Compatible support completion). For fixed A, P, G, q and aggregate record B , a compatible completion is a detailed relation $R' \subseteq A \times P$ whose existential aggregation is exactly B . The set of these completions is denoted by $\mathcal{C}(B)$.

Under the exact partition assumptions, every aggregate relation has a completion. Choose one member of each occupied group for each proposal and include those incidences. Include none from the unoccupied groups. Choices for distinct proposals can coexist because the initial model has no restriction on multiple endorsements. This construction establishes nonemptiness, which matters for subsequent claims involving all completions. When additional constraints make a record inconsistent, the resulting empty completion set requires a separate diagnosis.

Several different attributes can now be compared. The existence of any support is preserved: a proposal has a supporting person exactly when its aggregate row contains an occupied group. The number of occupied groups is directly available from the row. The number of supporting persons generally varies across completions. The identity of those persons can vary even when the number is fixed. The independence of their judgments and the reasons for their support require still further information.

An existential mark preserves a witness's existence while forgetting its identity and multiplicity. This is a precise description of the information loss. It avoids assuming that a group itself endorsed the proposal. A group may be occupied because one member acted individually, perhaps against the preferences of other members. Moving from "a member supports" to "the group supports" introduces a different predicate whose constitutive and evidential conditions must be stated.

For a fixed proposal a , let $H_a = \{g \in G : B(a, g)\}$ be its occupied groups. A group-diversity threshold can ask whether $|H_a|$ reaches a specified level. The person threshold in Equation 4.1 asks whether enough distinct persons support the proposal. Either criterion may serve an institutional purpose, but their agreement requires a special case, such as every group having capacity one. In the general model, substituting one count for the other changes the target.

The word “aggregation” can also conceal a change in carrier. The detailed relation concerns person labels; the aggregate relation concerns group labels. Distinctness in the latter domain differs from distinctness in the former. Two groups can be distinct while the persons actually relevant to a broader, overlapping membership scheme coincide. This section’s partition convention prevents that complication within the main result. Its later removal will show why the convention must remain visible.

Aggregation can be appropriate for disclosure, storage, or a limited reporting purpose. The present analysis supplies no general preference for retaining every individual record. It identifies the queries for which a chosen representation is adequate and the uncertainty that accompanies its limits. A responsible report can state that a threshold remains unresolved without demanding indiscriminate collection or publication of personal information.

6 Sharp Bounds for Support-Threshold Judgments

This section derives the information about person counts retained by an exact partition aggregate. It proves the attainable count interval, converts that interval into a threshold judgment, and distinguishes recovery at one record from adequacy of a record format across all inputs. The final argument identifies the information required by a family of threshold queries. Each result uses the assumptions stated in the preceding section.

For a fixed proposal a and a completion R' , write $N_a(R')$ for the number of distinct $p \in P$ satisfying $R'(a, p)$. Define the two bounds

$$L_a = |H_a|, \quad U_a = \sum_{g \in H_a} m_g. \quad (6.1)$$

Equation 6.1 assigns one mandatory witness to every occupied group for the lower bound and allows every person in those groups for the upper bound. Empty occupied-group sets give both bounds equal to zero. The sum counts disjoint fibres, which is why the partition assumption matters.

Proposition 6.1 (Attainable person counts). *Under the exact partition aggregate assumption, the counts compatible with a proposal’s aggregate row are precisely the integers from L_a through U_a .*

The result can be written as

$$\{N_a(R') : R' \in \mathcal{C}(B)\} = \{L_a, L_a + 1, \dots, U_a\}. \quad (6.2)$$

Equation 6.2 is an exact possibility statement. It assigns no likelihood to the counts and makes no assertion that all compatible completions describe the actual situation.

Proof. Every occupied group contains at least one supporter in a compatible completion. Because different fibres are disjoint, these mandatory supporters are distinct. Hence the count is at least L_a . Every supporter belongs to an occupied group, since a supporter in an unoccupied group would contradict the exact aggregate. Each occupied group contributes at most its capacity. Summing these capacities gives the upper bound U_a .

To establish attainment, first handle $H_a = \emptyset$. Exactness then forces the empty support set for this proposal, giving count zero. Otherwise enumerate the occupied groups as $g_1, \dots, g_h$, where $h = L_a$. Choose one person from each group. This produces the lower count and leaves $m_{g_i} - 1$ unused places in group i .

Let n be any integer with $L_a \leq n \leq U_a$. The number of additional supporters required is $r = n - L_a$. It is nonnegative and is at most the sum of all unused places. Starting with the first occupied group, choose as many additional distinct members as possible up to its unused capacity and the remaining requirement. Subtract that number from the requirement and proceed to the next group. If a positive requirement remained after the last group, the initial requirement would have exceeded the total unused capacity, contrary to its bound. The construction therefore adds exactly r persons.

The resulting support set has size n , intersects every occupied group, and contains no member of an unoccupied group. Its aggregate row is exactly the observed row. For the other proposals, choose one supporter in each of their occupied groups. These choices coexist with the constructed row because support for different proposals is unconstrained. They complete a relation in $\mathcal{C}(B)$ with the required count. Thus every integer in the interval is attainable, and the bounds exclude every integer outside it. $\square$

Definition 6.2 (Threshold status relative to a record). A threshold is affirmed by a nonempty completion set when it holds in every completion, refuted when it fails in every completion, and unresolved when some completions satisfy it and others fail it.

Proposition 6.3 (Sharp threshold classification). *For a fixed positive threshold k , the exact aggregate affirms $T_k(a)$ when $L_a \geq k$, refutes it when $U_a < k$, and leaves it unresolved when $L_a < k \leq U_a$.*

Proof. If the lower bound reaches the threshold, every compatible count reaches it. If the upper bound falls below the threshold, every compatible count falls below it. In the remaining case, attainment supplies a completion with count $L_a < k$ and another with count $U_a \geq k$. Hence both truth values occur. Conversely, whenever the threshold is affirmed, the attainable lower count must satisfy it; whenever it is refuted, the attainable upper count must fail it. These observations establish the stated conditions in both directions. $\square$

The result permits an exact answer even when an exact count remains unavailable. For example, a count interval from three through six affirms a threshold of three. A count interval from one through three refutes a threshold of four. The retained answer depends on the target query. Describing an aggregate record simply as “sufficient” leaves that target unspecified.

The distinction between a local record and an entire record format is equally important. Fix the person partition and a threshold k . Consider all detailed support relations for a chosen proposal and the operation that replaces its supporting persons with its occupied groups. Some rows may determine the threshold while others remain unresolved. Adequacy for the format requires a correct answer for every row the format can produce.

Proposition 6.4 (Global threshold recovery under a fixed partition). *For a fixed proposal, person partition, and positive threshold k , the threshold factors through the occupied-group record for every detailed support set exactly when no group subset $H \subseteq G$ satisfies $|H| < k \leq \sum_{g \in H} m_g$.*

Proof. Suppose such a group subset exists. The attainment construction supplies two detailed support sets with exactly that occupied-group record: one has $|H|$ persons and the other has the combined capacity. The first fails the threshold and the second satisfies it. A function of the common aggregate row would have to return the same value for both and therefore cannot recover the threshold correctly.

Conversely, suppose every group subset avoids the stated inequality. For any observed row, either its number of occupied groups reaches k or its combined capacity is below k . The sharp threshold proposition then gives a unique Boolean answer for that row. Assign that answer to the row. The assignment recovers the threshold on every detailed support set because each set belongs to the completion set of its own aggregate. This constructs the required factorization. $\square$

The statement is a specialized form of a familiar fibre-constancy criterion: a target can be recovered from a record when equal records always give equal target values. Its content here is the explicit condition on the group capacities and the chosen threshold. For $k = 1$, the existence of an occupied group always settles the answer. For a threshold larger than the total person population, every answer is false. Intermediate thresholds can fail even under an otherwise accurate aggregate format.

Proposition 6.5 (Information for the complete threshold family). *For a fixed proposal and finite person domain, a summary of detailed support sets determines every positive person-count threshold exactly when it determines the exact person count. Consequently, count is a coarsest summary for this family of queries, up to a change of labels.*

Proof. Knowing the count determines every threshold by comparison. For the converse, suppose two detailed support sets share a summary and have different counts, with $n < m$. The threshold $n + 1$ is within the possible positive range because $m \leq |P|$. It fails for the first set and holds for the second. A shared summary therefore cannot determine every threshold. Every fibre of a sufficient summary must have a constant count, which permits a count function on its summary values. Count itself supplies all threshold answers, establishing the stated coarseness. $\square$

This last result concerns the exact count of represented persons for one proposal. It supplies neither supporter identity nor an account of how support was formed. It also leaves open the design of a privacy-preserving or otherwise justified way to verify a count. A record can be economical for a declared query family while remaining inadequate for attribution, independence, consent, or historical revision.

7 A Proposal-Support Dossier and Its Interpretations

This section applies the derivation to a small fictional register. The dossier fixes capacities and recorded group incidences, computes the compatible person counts, and compares two possible institutional criteria. Its purpose is to make the difference between a defined attribute, an evidential result, and a decision rule concrete. The numbers are constructed witnesses and carry no empirical claim.

Take three disjoint groups with capacities two, three, and one, respectively. Denote them by g_1 , g_2 , g_3 . The person domain therefore contains six represented persons. Four proposals have the aggregate rows shown in Table 1. The person threshold is fixed at three. For comparison, the table also records the group-diversity threshold of at least two occupied groups.

For proposal a , the two occupied groups require two distinct supporters. Their combined capacity is three. A completion containing one member of g_1 and the sole member of g_3 gives count two; adding the other member of g_1 gives count three. Both completions produce the same aggregate row. The three-person threshold is therefore unresolved even though the record is exact under its own convention.

Proposal b has support in one group of capacity three. The aggregate is compatible with one, two, or three supporting members. It refutes the two-group criterion and leaves the three-person criterion unresolved. The difference is substantive. An institution seeking a minimum number of distinct participants may regard three supporters from one group differently from an institution seeking participation across groups. The arithmetic can represent either purpose after it has been selected; it cannot establish which purpose is appropriate.

Proposal c has at least one supporter in each of three disjoint groups. Its lower count is three, so the three-person threshold is affirmed. The exact number of supporters remains unresolved between three and six. This row demonstrates why demanding an exact count before every threshold judgment can be unnecessary. The relevant evidential task is to settle the selected proposition, with the assumptions that make the lower bound reliable.

Proposal d has an empty row and therefore count zero under exact aggregation. This conclusion changes if the register records only endorsements received through a particular channel or omits unprocessed reports. In that alternative convention, an empty row reports absence of recorded support, leaving the existence of support in the declared relation open. The difference is an evidential issue about the record's completeness. The report should state its completeness convention explicitly.

Now suppose a fictional committee uses the three-person threshold to determine eligibility for a discussion session. The evidence permits an affirmed threshold for c , a refuted threshold for d , and unresolved thresholds for a and b . The institution still needs a policy for unresolved cases. It might request limited additional evidence, provisionally include them, defer the decision with an appeal route, or apply another publicly justified arrangement. These actions distribute burdens differently. The mathematical result reports uncertainty; selecting an action under uncertainty is a further decision.

If the committee silently treats unresolved cases as failures, its decision procedure becomes stricter than the declared three-person condition. In the completion where proposal a has three supporters, that

Table 1: Fictional aggregate records under a three-person threshold and a two-group threshold

Proposal	Occupied groups	Possible person count	Three-person status	Two-group status
<i>a</i>	g_1, g_3	2, 3	Unresolved	Affirmed
<i>b</i>	g_2	1, 2, 3	Unresolved	Refuted
<i>c</i>	g_1, g_2, g_3	3, 4, 5, 6	Affirmed	Affirmed
<i>d</i>	None	0	Refuted	Refuted

procedure excludes a threshold-satisfying proposal because its retained record lacks the needed information. This may be a consciously adopted burden-of-proof rule, but it should be identified and defended as such. Calling the procedure a direct evaluation of the threshold would conceal its additional evidential requirement.

An opposite policy can also require justification. Treating every unresolved case as satisfying the threshold includes completions where the actual count is below three. A presumption of inclusion may be defensible for low-cost deliberation, while another context may support a different presumption. The dossier establishes the relation between the policy and the compatible counts. Its normative assessment depends on the stakes, the affected participants, and the alternatives available.

A targeted refinement can resolve proposal *a* without disclosing the complete support relation. An authorized verifier could attest whether both members of g_1 support it, provided the attestation's identity and endorsement checks are adequate. For the fixed threshold, that additional distinction is enough. Publishing the names, reasons, or unrelated endorsements of every supporter would supply more information than this particular query requires. Verification and disclosure are separate design questions.

The dossier also shows why a resulting eligibility label should retain its genealogy. If the institution later switches to the two-group criterion, *a* becomes affirmable from the existing aggregate, while *b* remains below the new threshold even in a completion with three supporters. The changed classification can arise from a revised target with unchanged support. A longitudinal account should identify the criterion used at each decision and preserve the earlier evidence and reasons where appropriate.

8 Overlapping Attributes and Constraints among Support Records

This section examines three extensions that challenge a simple row-by-row partition analysis. The discussion distinguishes overlapping substantive predicates from their joint coding profiles, multiple group membership from a partition, and shared constraints across proposals from independent completion choices. The examples identify the premises that must change before the central interval result can be applied to these cases.

Several attributes can hold of one proposal simultaneously. A proposal can have at least three supporters, involve several neighborhoods, address an urgent need, and remain costly to implement. A family of Boolean predicates represents these dimensions without requiring exclusivity. Their joint membership profile creates a partition into combinations of truth values, but this derived partition does not turn the original substantive categories into mutually exclusive alternatives.

Definition 8.1 (Joint attribute profile). For declared Boolean attributes $A_1, \dots, A_r$ on a common carrier, the joint profile of a case is the tuple of its membership values. Two cases share a profile when they agree on every declared attribute. The profile classes partition the carrier, while the individual attribute extensions may overlap.

There are at most 2^r Boolean profiles, and some combinations can be impossible because the attributes imply or exclude one another. With person thresholds, satisfying a higher threshold entails satisfying each lower one. A profile that affirms a threshold of four while refuting a threshold of three is inconsistent with the common count interpretation. Coding every attribute independently and then combining the labels requires attention to such dependencies.

Evidential uncertainty introduces another distinction. An unresolved label reports variation across compatible completions. It need not assign a third truth value to the underlying attribute in an actual complete situation. Two unresolved attributes can be tightly connected: exactly one may hold in every completion, or one may imply the other. A tuple of marginal uncertainty labels can omit these relations. Preserving the joint completion set, or a justified representation of its constraints, gives a more faithful account when those dependencies matter.

Group membership can overlap in a different sense. Suppose one person belongs to two groups and supports a proposal. Both groups become occupied under existential aggregation, yet there is only one supporting person. Counting occupied groups as mandatory distinct witnesses now overstates the lower count. The partition proof fails at the point where witnesses drawn from different groups were assumed distinct.

A suitable extension replaces the function q with an explicit person-to-group membership relation. For a chosen support set, its aggregate is the union of the group memberships of its supporters. Recovering the smallest possible person count then requires finding a support set whose combined memberships match the observed occupied groups. A single person may cover several occupied groups. Conversely, a person belonging to an unoccupied group cannot be included in an exact completion, because their support would create a mark in that group as well.

This extension can introduce inconsistency. If the only potential supporter belonging to an occupied group also belongs to a group recorded as unoccupied, the two marks may be incompatible with the membership relation. Such a case should be reported as a conflict among record assumptions. Applying universal quantification to an empty completion set would otherwise make both an attribute and its negation vacuously universal. That formal fact provides a reason to test consistency before assigning ordinary evidential status.

Joint constraints across proposals create a third problem even when groups remain disjoint. Consider one group containing two persons and two proposals, a and b . Both proposals have that group marked as occupied. Under the unconstrained model, each proposal separately has a possible count of one or two. Add the rule that each person can support at most one of these proposals. Every compatible completion must allocate at least one person to each proposal, and only two persons are available. Each proposal therefore has exactly one supporter.

Claim 8.2 (Constraint-sensitive threshold status). In the two-person, two-proposal example, the shared one-endorsement constraint refutes a threshold of two for each proposal, although the unconstrained row intervals leave that threshold unresolved.

Proof. Each occupied row requires at least one supporter. The constraint makes the two support sets disjoint. Their combined size is therefore at most two and at least two. If either proposal had two supporters, the other would have none, contradicting its occupied row. Both counts must be one. Either assignment of the two distinct persons to the proposals supplies a completion, so the constrained record is consistent. $\square$

The original row intervals remain valid outer bounds after this restriction, because every constrained completion was previously permitted. They cease to be sharp. Treating the upper endpoints for different rows as jointly attainable would introduce a further error. The main attainment proof extended a chosen row by making independent choices for other proposals; the one-endorsement constraint removes that freedom. The failed proof step identifies exactly where the additional information changes the result.

The status of the constraint itself matters. A normative rule permitting one endorsement per person need not establish that the actual record complies with it. If the task is to audit possible multiple endorsements, restricting completions to rule-compliant histories would assume the target of the audit. The constraint should enter as an established fact only when its enforcement or independent evidence supports that interpretation. Otherwise compliant and noncompliant possibilities may need separate representation.

These extensions show a common methodological pattern. The representation should preserve the distinction relevant to the query, the dependencies among the retained records, and the status of assump-

tions that eliminate possibilities. Overlapping predicates, overlapping memberships, and shared endorsement constraints are different structures. A single informal appeal to “complexity” would obscure the precise correction each one requires.

9 Missing Evidence, Temporal Scope, and Category Revision

This section examines changes in evidential and conceptual state. It distinguishes missingness from falsehood and inapplicability, states the effect of genuine information refinement on compatible counts, and contrasts that effect with changes in the target criterion or underlying support relation. The aim is to preserve a traceable interpretation when classifications change over time.

An unmarked cell can have several meanings. Under the exact aggregate model, it records that no person in the group supports the proposal. Under a partial reporting convention, it can indicate that the group’s support has not been checked. In another system, the group may be outside the proposal’s eligible constituency, making the question inapplicable. An entry can also be disputed because two sources conflict. Treating these states as a single zero can make an otherwise correct computation answer a different question from the one participants intended.

A practical record should therefore identify the evidence convention before evaluating a predicate. A positive report, a verified negative report, an unchecked cell, and an inapplicable cell may need separate statuses. Inapplicability itself requires a declared rule; an excluded constituency can be politically consequential. A disputed cell should retain the competing assertions and their provenance when those differences matter. The formal representation can encode these statuses, but the reasons for using them belong in the interpretive account.

The compatible-completion approach can accommodate partial evidence. Define a set of detailed support relations satisfying the information actually established. A positive group report requires a witness. A verified negative report excludes one. An unchecked group places no such restriction. Additional reliable information can narrow the set. The earlier interval formula must be reconsidered because its upper bound relied on every unoccupied group being genuinely empty of support.

Proposition 9.1 (Refinement under a fixed target). *Let two nonempty completion sets describe the same person domain, support meaning, and count target, with the later set contained in the earlier set. The minimum possible count can only increase or stay fixed, and the maximum can only decrease or stay fixed. A threshold affirmed or refuted on the earlier set retains that status on the later set.*

Proof. Every count attained in the later set was already attained in the earlier set. Its minimum is therefore at least the earlier minimum, and its maximum is at most the earlier maximum. If every earlier completion satisfied a threshold, every member of the smaller set satisfies it. The same argument applies when every earlier completion failed it. Nonemptiness ensures that the minima, maxima, and ordinary evidential statuses remain meaningful. $\square$

The proposition is a statement about additional information under a fixed interpretation. An apparent reversal can reveal that another kind of change has occurred. Correcting an erroneous negative report may require replacing the earlier completion set with a different, nonnested set. A supporter can withdraw, changing the underlying time-indexed relation. An institution can change the threshold, changing the target. Each change deserves a distinct record because its implications for previous decisions differ.

Suppose proposal a in the dossier receives a new third endorsement. A later threshold affirmation may reflect a change in support. Suppose the third endorsement had already existed but was omitted from the retained aggregate detail. A later affirmation may reflect improved evidence about the same historical situation. Suppose the institution changes its rule from three persons to two groups. The existing aggregate can then support a different classification without either new endorsement or improved person-level evidence. The visible label alone cannot distinguish these trajectories.

Temporal indexing helps make the question explicit. A predicate can concern endorsement at submission, endorsement at the decision date, or endorsement throughout a specified interval. A continuing-

support rule must address withdrawal, replacement, and the relevance of past decisions. A historical-support rule can preserve a past fact while acknowledging a person's current disagreement. The model does not select one temporal interpretation for all purposes.

Category revision also changes the meaning of comparability. If one year's eligible proposals were identified by individual count and another year's by group diversity, comparing the two eligibility totals directly combines changes in proposals with changes in criteria. The earlier criterion can be replayed only if the retained records support it. The present recovery theorem therefore has a prospective implication: a record adequate for today's rule may prevent a future comparison under another rule.

This possibility provides a reason to document expected future queries, while leaving record expansion proportionate. Retaining every conceivable distinction is neither feasible nor automatically justified. An institution can state the purposes its records support, the queries they leave unresolved, and the procedures for revising those purposes. A discontinuity in a reporting series can be intellectually responsible when its causes and consequences are explained.

Finally, classification can influence the practice it describes. A threshold can encourage additional endorsements, strategic group assignments, or efforts to clarify what support means. These are hypotheses about uptake and response. They require evidence of actual behavior and cannot be derived merely from writing the threshold formula. If such effects are modeled, the response rule and its empirical basis should be added explicitly. The static counting result remains valid within its stated snapshot while its role in a changing institution becomes a separate inquiry.

10 Institutional Interpretation and Answerability for Classification

This section examines the transition from a support attribute to an institutional decision. It separates descriptive findings, constituted statuses, and action rules, then states a conditional normative premise for answerability. The discussion uses the proposal dossier to identify concrete responsibilities without making them consequences of the counting theorem.

A formal definition can make a classification reproducible while leaving its purpose contested. A three-person threshold might aim to establish that a proposal concerns more than an isolated preference. A two-group threshold might aim to encourage a wider distribution of participation. Either aim can be questioned. Some valuable proposals originate with a single affected person, and some apparent diversity can be produced by closely coordinated organizations. A numerical criterion is a selective representation of a purpose, with cases that expose the distance between the representation and its rationale.

This distance also concerns the unit counted. Distinct represented persons can have dependent reasons, common information, or coercive relations. Distinct groups can share members, leadership, funding, or administrative origin. The person-count theorem establishes a property of incidence under identity and partition assumptions. It gives no measure of deliberative independence, seriousness, or merit. A claim about those qualities requires additional definitions and evidence.

The interpretation of support can itself be contested. A participant may agree that a proposal deserves discussion while opposing its adoption. Another may permit public association with an initiative while withholding agreement with a particular formulation. A record that compresses these attitudes into a single endorsement predicate can be appropriate for a narrowly explained task. Its reuse for a different task can misrepresent the original act. Recording the question presented to supporters and the scope of their response helps preserve the intended relation.

Assumption 10.1 (Standing in consequential classification). When an institution uses a classification to impose material burdens or distribute access to a shared opportunity, affected participants have a defeasible standing to receive an intelligible account of the criterion, relevant evidential limits, and available routes of correction or challenge. The form and extent of this standing depend on the context and require independent normative defense.

The premise is proposed explicitly because the mathematical results alone leave institutional duties open. Its justification can appeal to the participants' stake in the decision, the institution's exercise of

power, and the possibility of avoidable misclassification. Competing considerations include privacy, the costs of review, confidentiality, and the burdens placed on other participants. The paper treats the premise as a basis for conditional argument, inviting criticism of its formulation and scope.

Claim 10.2 (Answerability for an unresolved threshold). Under the standing premise, an institution relying on an aggregate record that leaves its declared threshold unresolved owes an account of the additional decision convention it applies to that uncertainty when the resulting classification materially affects participants.

The reason proceeds in steps. The aggregate record permits both threshold-satisfying and threshold-failing completions. A determinate decision therefore requires something beyond evaluation of the threshold from that record. The additional element may be a burden of proof, a presumption, supplementary evidence, or discretionary judgment. When the decision materially affects a participant, the standing premise gives that participant a defeasible claim to understand and challenge the relevant additional element. The conclusion concerns answerability; it does not by itself select the decision that must follow.

This account preserves the distinction between error and disagreement. A computation can mistakenly count one person twice, yielding an error relative to the declared identity convention. A participant can disagree with the convention's use of household or organizational categories. A threshold can be correctly applied while imposing an unjustified burden. Each problem calls for a different response: correcting a record, revising a representation, or reconsidering the rule and its consequences.

Institutional authority can also be distributed. One office may maintain person identifiers, another may verify endorsement, a committee may select the criterion, and an appeal body may review decisions. Such distribution can improve scrutiny or fragment responsibility. A useful account identifies which actor warrants each transition rather than placing the entire classification under one undifferentiated label of "the data." The formal chain helps reveal these transitions even when the actual institutional arrangement is more complex.

Privacy does not require accepting uninterpretable decisions, and answerability does not require publishing every personal relation. A report can disclose the criterion, the verified count or threshold result, the relevant completeness assumptions, and a protected correction channel. Some settings may justify independent verification without public supporter lists. The precise mechanism requires further design and legal or ethical assessment; the conceptual point is that evidential sufficiency and unrestricted disclosure are different objectives.

The philosophical position is therefore modest. Classifications can be constructed for purposes while remaining answerable to evidence and criticism. The fact that a criterion is selected does not make every application arbitrary. Once a criterion and interpretation are fixed, the derivation identifies genuine constraints on what follows from a record. Conversely, satisfying those constraints leaves the purposes and institutional consequences open to further argument. Objectivity in this setting can involve publicly stated assumptions, reproducible reasoning, and correction of error without requiring an account of categories detached from all practical interests.

11 Objections and Limits of the Support Representation

This section tests the proposal against objections concerning mathematical contribution, social interpretation, evidence, and institutional scope. The discussion identifies which objections call for a narrower conclusion and which require changing the representation itself. The aim is to make the paper's revision conditions sufficiently clear for independent criticism.

The first objection concerns the familiarity of the mathematics. The interval proof distributes a number of persons across finite disjoint groups; the recovery criterion tests constancy on record fibres. These are elementary constructions. Their role is to resolve a concrete conceptual question with explicit assumptions. The contribution should be assessed through the clarity of the support-classification appli-

cation, the separation of commitments, and the usefulness of its counterconditions. If those distinctions add little to existing practice, the paper's practical importance should be revised accordingly.

A second objection is that the chosen formal language makes the distinction between supplied and definable attributes too easy. Once support incidence is primitive, a threshold is straightforwardly definable; before it is primitive, the base model lacks the relevant content. That objection identifies part of the intended lesson. A sparse general framework can organize a range of phenomena while requiring substantive enrichments for particular inquiries. The paper makes those enrichments visible and then examines a further, less immediate problem: a definable attribute can lose answerability under aggregation of its primitive witnesses.

A third objection concerns the relationship between counting and support. Endorsement can be graded, conditional, ambivalent, or withdrawn. A binary relation can erase these differences. The appropriate response depends on the task. A clearly specified procedural question may permit a binary predicate, such as whether a person has authorized a particular statement at a specified time. A broader inquiry into commitment may need several predicates, graded descriptions, or qualitative evidence. The current result should be abandoned as a sufficient account when the binary interpretation fails the intended question.

The fourth objection concerns person identity. A mathematically distinct label can represent the same person as another label, and one shared account can represent several persons. The theorem assumes an accurate person domain. It does not solve identity verification or duplicate detection. If the identity relation remains uncertain, compatible completions should vary over both incidence and admissible identity assignments. The resulting bounds may change substantially. Reusing the current count while treating identity uncertainty as a minor implementation detail would conceal a premise of the result.

A fifth objection targets the partition. Social affiliations frequently overlap, and assigning each person to one group can suppress the relations that matter. The overlapping-membership example establishes that the partition formula cannot simply be reused. A declared single-affiliation convention can still serve a particular report, provided its selection rule and limitations are acknowledged. When the inquiry concerns actual plural affiliations, an explicit membership relation and a different counting analysis are required.

A sixth objection concerns the completion set. The unconstrained model allows support relations that may be practically implausible, and its interval can therefore appear too broad. Additional evidence can narrow the possibilities. A probability model can also distinguish more and less likely completions if its interpretation and calibration are defended. The present account leaves probabilities unassigned because the aggregate relation and capacities alone supply none. Equally weighting all detailed completions would introduce a new assumption whose implications could depend on the chosen representation.

A seventh objection concerns the feasibility of institutional explanation. Participants may lack the time or technical training to evaluate a formal account. The standing premise therefore calls for an intelligible explanation appropriate to the decision, not a demand that every participant inspect notation. A statement such as "the record establishes two to three supporters, and the rule requires three" can communicate the relevant uncertainty directly. The derivation supports that explanation and permits technical scrutiny when needed.

An eighth objection is that the normative discussion remains conditional. The paper does not provide a complete political theory of participation or a universal justification for public thresholds. That scope is intentional: the central formal result addresses recovery from an aggregate record, while the standing premise identifies one route from consequential classification to answerability. A fuller normative argument should examine institutions, burdens, and competing rights in a particular setting. The institutional conclusions remain conditional on that further argument; the count theorem retains its mathematical assumptions.

Finally, a dynamic objection concerns feedback. Announcing a threshold can change endorsement behavior, the formation of groups, or the meaning of the recorded act. A static snapshot omits those processes. It can still support a carefully time-indexed evidential judgment, but claims about policy effectiveness require a dynamic and empirical account. The present paper contributes a representation

audit that such an account can use; it leaves the effects of thresholds on participation as an open research question.

12 Open Questions and a Research Design for Classification Audits

This section identifies research questions generated by the formal and philosophical analysis. It separates extensions of the mathematics from investigations of record practices and institutional interpretation, then proposes a bounded evaluation design. The questions concern the adequacy and consequences of the representation in particular classification practices.

Open Problem 12.1 (Witness recovery under overlapping memberships). Characterize useful and computationally tractable bounds on person counts when individuals belong to several groups and the aggregate records exact existential group incidence, with membership and identity uncertainty represented explicitly.

The partition result provides a baseline for this problem. An extension should identify the information needed to distinguish one person occupying several groups from several distinct persons. It should also detect inconsistent aggregate patterns and state the effect of incomplete group coverage. A successful formal result would need an interpreted reporting task to explain why its additional complexity is warranted.

Open Problem 12.2 (Targeted refinement of an unresolved threshold). Given a compatible support-completion set and a fixed threshold, identify limited additional queries that resolve the threshold under declared constraints on burden, privacy, reliability, and cost.

The question concerns both information and authority. A query that mathematically separates the remaining truth values may demand evidence that an institution has insufficient standing to request. Another query may preserve privacy but rely on a verifier whose reliability needs assessment. The research task is to compare admissible refinements under explicit conditions, keeping the normative constraints separate from the count calculation.

Open Problem 12.3 (Communication of joint evidential uncertainty). Develop and evaluate ways to communicate unresolved attribute profiles while preserving relevant dependencies among them, including implications between thresholds and shared constraints across proposals.

A collection of independent uncertainty labels can be easier to read while concealing impossible combinations. A full completion set can be accurate while overwhelming its audience. Intermediate explanations might state a small number of attainable profiles or the particular dependency relevant to a decision. Their usefulness should be evaluated with the people who interpret and are affected by the reports. Comprehension, burden, and the possibility of challenging a premise are separate outcomes.

A prospective audit study could begin with one consenting institution whose existing process already records proposal support. The first phase would document the actual meanings of support, person identity, group membership, absence, and eligibility. Researchers would distinguish written rules from observed practice and record disagreements about interpretation. The study should avoid assuming that the fictional partition model fits the institution merely because its database contains a group field.

The second phase would select a fixed historical or synthetic corpus with a justified reference interpretation. Where personal records are involved, access and disclosure would follow the applicable agreements. Researchers would construct the aggregate view used by the institution and compare its declared target queries with those recoverable from that view. The analysis would classify exact answers, valid bounds, unresolved targets, and inconsistent assumptions. A disagreement about the reference interpretation would remain visible alongside any agreed reference labels.

The third phase would compare explanations of the same cases. One explanation could provide the institution's usual classification, another the relevant count range and decision convention, and another

a limited account of the critical assumption. The design would assess understanding of the threshold, recognition of uncertainty, ability to identify a correction route, and time or cognitive burden. A participant's preference for one explanation would be relevant evidence while remaining distinct from proof of institutional legitimacy.

Several rival explanations should be retained. Clearer prose, additional attention, and the specific completion analysis provide competing explanations of an apparent improvement. Participants may already understand the uncertainty through informal practices. A more detailed report may increase anxiety or deter participation. A threshold may be so rarely unresolved that an elaborate reporting procedure brings little benefit. These possibilities support a proportionate evaluation of the proposed audit under its actual conditions of use.

The study's claims should remain tied to its design. A retrospective record comparison can establish discrepancies under stated interpretations. An explanation study can provide evidence about comprehension in its sample. Establishing effects on participation, fairness, or the formation of support requires further observation and appropriate causal reasoning. Wider normative conclusions require argument about the institution's purposes and the standing of affected participants.

The philosophical questions remain equally active. Further work can investigate how participants negotiate the meaning of support, how category labels acquire authority through repeated use, and how institutions revise a criterion without concealing earlier decisions. The present paper contributes a precise location for these questions: the transitions between a declared predicate, a retained record, an evidential assignment, and a consequential institutional classification.

13 Conclusion

This section states the completed argument and its remaining boundaries. The paper has examined support-based attributes through a sequence of distinct commitments: representing a classification, defining it in a language, establishing its value from evidence, and connecting it to an institutional status or action. The finite aggregation model makes the independence of those commitments concrete and supplies explicit conditions for a limited family of answers.

A fixed threshold of distinct supporters has a first-order definition once support incidence and person equality are declared. That definition does not supply the relation's interpretation or establish its value from every record format. Aggregating individual support into occupied groups preserves support existence while potentially losing the multiplicity needed by the threshold. Under a finite partition with known capacities and exact existential records, the possible person counts form a sharp integer interval. The lower and upper endpoints then characterize affirmation, refutation, and unresolved threshold status.

The derivation also distinguishes recovery at a particular record from adequacy of the aggregate format for all inputs. Some rows settle a threshold even while the count remains uncertain. A format supports the threshold globally only when every occupied-group subset avoids straddling it. Exact count is a coarsest summary for the complete family of person-count thresholds, while other questions can require information about identity, dependence, reasons, or time.

The counterexamples identify the limits of that result. Overlapping group membership can make one person witness several marks. Shared endorsement constraints can make individually possible counts jointly impossible and can sharpen a row's result. Missing negative evidence can invalidate the upper bound. Changes in support, improvements in evidence, corrections of earlier records, and revisions of the criterion can all alter a classification through different processes. A durable account should preserve those differences.

The institutional discussion introduces a further conditional argument. When consequential classifications affect participants who have standing to seek an account, an institution should explain the decision convention used where its retained evidence leaves the declared threshold unresolved. The mathematical analysis identifies the additional convention; the normative premise supplies the reason for answerability. The paper leaves the full justification and implementation of that premise open to contextual criticism.

The resulting proposal is a bounded method for clarifying an attribute's dependencies. It presents current insights for discussion and acknowledges antecedents in logical definability, incomplete-information research, and philosophical concept formation. Further work should test the representation against actual practices, refine its treatment of uncertain and overlapping relations, and examine the consequences of different ways to report and govern classifications.

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