

Identifying Unmet Demand under Participation Restrictions in Public Libraries

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Abstract

A public library can restrict an activity and subsequently possess few records of participation in it. Interpreting those records requires a distinction between current use, responses to a specified alternative offer, interests formed through experience, and the public reasons for supporting an opportunity. This paper develops an account of that distinction for shared library spaces and group activities. It reconstructs a binary governance gate under explicit assumptions, derives partial-identification bounds for population coverage and recording error, and examines how capacity and coordination alter the relation between demand and attendance. A history-based counterexample distinguishes an unobserved prior disposition from demand indexed to an actual offer that may itself change interest. A proposed design randomizes formative exposure before a common subsequent offer and identifies the resulting prospective response contrast under stated assignment and measurement conditions. A separate branch-level construction accommodates within-branch interference and distinguishes population weighting from equal branch weighting. Further analysis addresses feedback between observed use and continued access, changing outcome categories, and the preservation of records needed for later comparison. Documented public-library policies and existing professional evaluation practices motivate a prospective study; the numerical examples remain constructed. The philosophical discussion examines preference formation, practical agency, competing uses, refusal, and authority over public priorities. The contribution is an explicit connection among an inferential target, the policy conditions that generate its evidence, the assumptions needed to interpret that evidence, and the decisions left open by it. Empirical implementation and a broader originality review remain research obligations.

Keywords: public libraries; unmet demand; participation; policy evaluation; interest formation

Note on Scope and Research Status

This paper develops an inquiry into unmet demand for participation in public libraries. Its focus is the relation between an opportunity, the activity it permits, the records it produces, and the conclusions drawn from those records. The word demand receives explicit offer, population, and history indices. Ontological questions remain suspended. The proposed categories organize an investigation and remain open to conceptual and empirical criticism.

The manuscript develops the question through conceptual, formal, and philosophical analysis. Definitions and assumptions precede the propositions that use them. Each calculation supplies an interpretive account, a derivation, and conditions under which an empirical application would fail. Constructed numerical examples remain explicitly distinct from field observations. The prospective study design records a research proposal whose implementation and findings remain open.

The new formulations, examples, and proposed instruments await the author's review. The accompanying project files preserve the source discussion, the distinction between original repository ideas and agent-added development, and the access level of each verified source. The comprehensive analytical draft is prepared for author review. Empirical implementation, instrument validation, and a wider confrontation with library research remain open stages of the inquiry.

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Status. This comprehensive manuscript is prepared for author review. The formal results are conditional on declared definitions and assumptions. Published policies supply documentary illustrations; the prospective study and its anticipated analyses remain proposals. Actual demand, implementation effects, and institutional recommendations require further evidence and deliberation.

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Statement on the use of language models. The author selected the research question and authorized comprehensive drafting. Language-model assistance supported source discovery and website verification, conceptual reconstruction, mathematical derivation, counterexamples, argumentative criticism, and preparation in L^AT_EX. Agent-added developments and source-access limits are recorded in the accompanying research files. This version awaits the author’s review of its formulations, arguments, and references. Authorship and decisions about circulation remain with the human author.

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1 Introduction

This paper examines the inference from recorded participation to unmet demand in public libraries. Its immediate setting is a shared space or group activity whose use depends on opening hours, permission, room allocation, staff support, and the presence of other people. The analysis develops the inferential target before introducing a model, derives the consequences of alternative observation conditions, and constructs a prospective study that can investigate the resulting distinctions. Its philosophical question concerns the authority of evidence produced within arrangements that also shape the opportunities and interests being evaluated.

Consider a deliberately hypothetical library in which children may read individually in a room but may hold a group discussion only during a designated session. An annual record shows little discussion outside those sessions. An administrator could accurately describe the recorded activity and still make an unwarranted further inference that additional discussion opportunities would attract little interest. The record was generated under a rule that limits the activity. Whether it reveals the response to a different rule depends on how participation would change when that rule changes. The example concerns an inferential structure. It describes neither an observed library case nor the decision practices of the institutions whose published policies are discussed later.

The same hypothetical example also contains a temptation for the critic. Suppose an additional session opens and attracts participants. It may appear that the later attendance proves the existence of an already formed demand that the earlier rule concealed. Some participants could indeed have wanted the opportunity beforehand. Others might become interested after meeting a friend, discovering the activity, gaining confidence, or experiencing a form of discussion they had never considered. Later participation establishes that an activity occurred within the new arrangement. Its relation to an earlier disposition requires a separate argument. The present paper takes both inferential errors seriously: restrictions can conceal responses to alternative offers, and interventions can help form the interests retrospectively attributed to the earlier population.

The term demand therefore needs a stated object. A person may want a quiet place to complete schoolwork while declining a scheduled discussion group. The same person may accept a group invitation when a friend attends, prefer a different language, or become willing when the time changes. Each description concerns a particular offer under particular circumstances. A useful demand claim specifies the activity, the feasible participation conditions, the population, the time, and the relevant history. A context-free quantity called latent demand would compress these distinctions before their importance has been assessed. The paper introduces binary variables only after stating which contrast they are meant to clarify.

Five questions organize the inquiry. The first concerns what people currently say they would like. The second concerns how they would respond to a specified alternative offer. The third concerns the causal effect of an implemented policy package on participation or other declared outcomes. The fourth concerns how interests change through experience. The fifth concerns the public reasons for supporting an opportunity, including the interests of people whose preferred uses may compete. These questions can inform each other while retaining different evidential and normative requirements. An attendance count, a survey answer, and an assessment of fair access belong at different points in that architecture.

Public libraries offer a particularly useful setting for these distinctions because a single place accommodates several forms of participation. Reading, study, digital access, conversation, organized learning, informal assistance, and public meetings can require different spatial and temporal arrangements. An opportunity can be formally available and practically difficult to use. A room can be fully occupied while additional demand remains invisible. A successful invitation to one group can reduce another group's access. These possibilities make a simple equation between use and public value inadequate even when the count itself is accurate. They also make the elimination of rules an inadequate general response: coordination and finite resources require decisions about shared conditions.

The paper's constructive contribution is an explicit connection among a demand target, the policy conditions that generate its evidence, the assumptions that allow a particular inference, and the deci-

sions that remain open. The mathematical results perform several distinct tasks. A governance gate exhibits observational equivalence. Coverage and recording bounds retain uncertainty that a point estimate would conceal. A timing counterexample separates pre-exposure dispositions from actual-offer responses. Capacity examples expose the difference between individual access and aggregate occupancy. An assignment calculation clarifies the population represented by a proposed experiment. Category bridges preserve what can be compared when an evaluation revises its descriptive vocabulary. Each result is conditional; each is accompanied by the empirical work needed to use it.

Existing evaluation practice already offers resources for such work. Library outcome measurement includes participant learning and perceived change, while official complexity guidance treats adaptation and unexpected effects as substantive evaluation problems. These approaches are interlocutors for the present method. The paper's target is the particular passage from an observed record to a stronger demand claim. Its originality must ultimately be judged by whether the distinctions improve an actual investigation beyond what those existing resources already accomplish.

The study developed here remains prospective. It proposes a documentary audit, a population and access assessment, interviews with users and nonusers, separate measurement of allocation and recording, and a staged intervention where feasible. One component assigns a formative exposure and subsequently makes a common offer to the comparison groups. That design can estimate how the exposure changes the response to the common offer. The offer itself may also form interest, so its effects are included in the target. A second component assigns a branch-level package and measures a fixed baseline cohort, allowing interaction within each branch. Neither component automatically recovers an untouched preference that existed before the study.

The argument proceeds from institutional context and target specification to identification, formation, capacity, assignment, feedback and comparison. It then develops instruments and reporting practices before examining agency, competing uses, authority and objection. The final research obligations retain the questions that the present construction leaves open. A well-designed evaluation should make those questions easier to investigate and contest. Its success consists partly in preventing an administrative record from acquiring an evidential authority that its production conditions cannot support.

The extended argument is organized for several connected reading tasks. A reader assessing a particular demand inference can begin with the target definitions, symbol guide and gate interpretation, then follow the relevant recording or selection model. A reader preparing a study can use the prospective protocol, instrument package and constructed study dossier together. A reader assessing a provision decision can connect the response evidence to the chapters on consequences, agency, competing uses, continuity and alternatives. Each route returns to the same requirement: the evidence should support the claim actually made.

The mathematical development deliberately moves between different domains. Finite-population results preserve integer completions and linked-record constraints. Probability models permit mixture constructions and conditional transport. Assignment results identify the effect of a specified design under a fixed population and interference structure. Decision calculations add a declared value mapping and criterion. The manuscript states these domains locally because combining them without a bridge can create a stronger conclusion than any component establishes.

The worked cases serve a similarly specific purpose. They are constructed examples whose dates, counts, roles and consequences are stipulated. They allow the reader to inspect a complete chain of reasoning and to see where another interpretation remains possible. Documentary policy examples and verified research sources have a separate evidential role. The manuscript attributes only inspected source positions and treats actual branch implementation, questionnaire performance and policy effects as empirical obligations.

The contribution is consequently both constructive and revisable. It supplies a detailed study architecture, conditional results and practical records through which an institution could investigate a contested opportunity. It also proposes an assessment of the method itself using shared evidence dossiers and claim-level review. The framework should earn its practical role by improving warranted conclusions, directing useful inquiry and supporting intelligible public decisions at a proportionate cost. Its formal

detail and length provide occasions for scrutiny, while its success remains a question for that scrutiny and subsequent use.

2 Public Library Rules and Evaluation Practices

This section establishes the documentary and professional context for the analysis. It examines a small set of verified public sources, distinguishes their evidential roles, and identifies the additional observations required for an empirical case. Published rules motivate the opportunity variables, while existing evaluation resources constrain the paper’s account of its own contribution. The discussion treats a rule’s text, its implementation, and a subsequent inference from participation as separate objects of inquiry.

2.1 Published Rules and Practical Allocation

A published rule can establish that an institution formally reserves certain decisions, gives priority to some uses, or specifies conditions for access. It provides a starting point for reconstructing an opportunity. The next step is to investigate how staff and participants encounter that rule in practice. Written availability, perceived availability, successful booking, and actual participation are distinct stages. The institutional illustration below is used only for the first stage.

The Seattle Public Library’s facilities and meeting-room policy gives library administrative and sponsored activities priority over outside uses. It permits applications to be denied for reasons including the availability of space and staff and the frequency of use; branch facilities operate within specified opening-hour and use conditions [20]. These provisions document several dimensions along which the opportunity for a group activity can vary. They establish neither the amount of unmet demand nor an erroneous inference from low attendance by that library.

The analytically relevant object is the connection between an application and the conditions under which it can become an event. Suppose a branch records completed room bookings. That record contains successful applications under an allocation procedure. To interpret an empty booking period, an investigator would need to distinguish an unrequested period from a period unavailable for public booking, a period whose availability was poorly communicated, and a period requested through a channel absent from the exported record. These are questions generated by the model. Establishing which occurs requires local documents, staff accounts, interface inspection and suitable observations.

A rule can also support access by coordinating incompatible claims. A room reserved for a scheduled literacy activity may become unavailable to a newly arriving discussion group. The restriction protects an existing opportunity while constraining another. The appropriate analysis records both relations. Calling the first group enabled and the second group blocked supplies a description of allocation; judging whether that allocation is justified requires further reasons. This distinction allows an investigator to study exclusion without treating the existence of an allocation rule as proof of institutional wrongdoing.

Seattle’s Community Use Agreement also articulates equity, accessibility, attention to unmet needs, and staff responsibilities alongside expectations for shared-space behavior [21]. Its stated commitments matter because they make the documentary context more complex than a list of prohibitions. An empirical investigation could examine how these commitments guide interpretation when uses conflict. Their presence in the policy supplies the institution’s declared position; successful implementation remains a different factual question.

2.2 Existing Accounts of Library Outcomes

Professional documents already recognize that public-library activity serves purposes beyond producing event counts. This subsection identifies the limited antecedent claims used in the manuscript and explains the remaining task for the proposed method. The objective is to situate the argument within an existing field of evaluation, while preserving the repository’s own question.

The IFLA–UNESCO Public Library Manifesto connects services with local needs, community participation, creative development and outreach to people unable to visit [7]. These are professional normative commitments. They support the relevance of asking whose opportunities are being assessed, while leaving the empirical identification of particular needs and the resolution of resource conflicts open. A manifesto can articulate an institutional purpose; it cannot substitute for a local account of how different arrangements affect particular people.

The Public Library Association’s account of Project Outcome describes measurement of perceived changes in knowledge, confidence, application and awareness [15]. That example directly establishes that contemporary library evaluation includes more than attendance. The present analysis therefore asks how a particular outcome survey reaches its respondents, what response history its questions concern, and what conclusions its data support about people who never participated. The same questions can improve an existing outcome instrument without requiring its replacement.

An instrument administered after an activity may be well suited to learning what participants found useful. Its sampling rule gives it a narrower relation to the interests of nonparticipants. That limit follows from the population being observed and the target being asserted. A supplementary outreach sample may add evidence about people outside the event. A repeated follow-up can investigate persistence. An application log can record attempted access. A qualitative account can identify an opportunity that the original survey failed to describe. Combining these sources is a design problem, and each source should retain its own evidential role.

The IMLS Public Libraries Survey documentation distinguishes activity fields and reporting methods, including the method used to report library visits and separate program-attendance items [6]. This is useful administrative context. A field has a defined unit and a collection convention. The demand analysis must inspect those definitions before treating the field as a measure of individual participation, unique participants, or unexpressed interest. The manuscript analyzes no national survey data and draws no prevalence estimate from the documentation.

2.3 Complexity and the Evaluation Comparator

The contribution of the present paper depends on a fair description of the methods with which it is compared. This subsection identifies the relevant competence of adaptive evaluation and then states the narrower issue that remains. The comparison concerns the logic of a demand claim, rather than an opposition between evidence and social change.

The official supplementary guidance on handling complexity in policy evaluation explicitly addresses context, history, adaptation, unexpected effects and continuing learning [3]. The existence of such guidance defeats a general assertion that evaluation can investigate only fixed outcomes in already stable systems. It also supplies a constructive response to the concern that an intervention may generate previously unanticipated activity. Evaluation can follow a changing process and revise the questions it asks.

The narrower issue concerns the identity of the target through that revision. An investigator may begin by asking whether residents already wanted an activity, introduce an opportunity, observe new participation, and end by reporting that the earlier demand was uncovered. Adaptive inquiry can produce valuable evidence during that sequence while leaving the retrospective claim unresolved. The proposed method requires the target to be named at each stage and any shift to be justified. A new question can be better than the original one; its improvement should be recorded rather than allowed to obscure which question the resulting evidence answers.

2.4 Documentary Scope and Empirical Obligations

The sources above establish institutional rules, professional aims, evaluation resources and measurement conventions. An empirical case needs a further chain of evidence. This subsection specifies that chain so that the later thought experiments retain their intended status.

A claim that an actual library inferred absent demand from policy-suppressed activity would require a record of the inference itself. Suitable evidence could include a decision report, meeting minutes, an evaluation document, or a carefully interpreted interview. An analyst's suspicion that administrators probably reasoned this way is insufficient. The record must identify the claim, the data used, the relevant restriction, and the alternative explanations considered. A finding of erroneous inference would then depend on showing the mismatch between the evidence and the asserted target.

Even a documented inferential mismatch would leave the actual magnitude of unmet demand open. It could be small, large, or practically irrelevant to the decision. Conversely, a library may use careful reasoning while still facing substantial unmet demand because resources are limited. The proposed method keeps inferential quality, demand prevalence, distribution and policy justification separate. That separation is necessary for criticism that is both fair to institutions and useful to the people whose opportunities they organize.

3 Evidence from Library Nonusers

Research on people outside current library participation already supplies substantial precedents for the present question. This section places a bounded selection of that research beside the paper's identification task. It first describes the units and instruments of two verified studies, then develops original conditional comparisons involving household aggregation, anticipated use, barrier rankings, behavioral explanations, and intervention targets. The comparisons assess the additional reasoning required for a specific unmet-demand claim; they preserve the studies' own stated purposes.

The central distinction is between an observation's quality and the question to which it responds. A careful survey can measure a distribution of stated expectations. A linked administrative analysis can estimate associations with recorded use. These are valuable achievements. A claim about what particular people would do under a changed booking rule introduces a different target, whose relation to those results requires an explicit bridge. The need for that bridge concerns the new claim, rather than an automatic defect in the earlier study.

3.1 Verified Empirical and Theoretical Antecedents

The source comparison uses a deliberately compact description so that the subsequent argument remains distinguishable from the sources' own findings. Sin and Kim analyze household-level annual library use using linked 2002 survey information, with household characteristics and state-level service variables. Their study explicitly discusses limitations of its population and household unit [19]. This supplies a concrete precedent for research on nonuse and a useful case for examining the implications of an aggregate outcome unit.

The DCMS-commissioned Ipsos study surveys 4,243 English nonusers aged seventeen and above. Its methods use an address-based panel with weighting; its questionnaire includes anticipated future use and relative barrier choices. The report develops possible interventions for later testing [8, 9]. These observations support descriptive and design work while leaving realized response to a particular new offer as a separate empirical target.

The theoretical comparison also needs a direct antecedent. Michie, van Stralen, and West's COM-B framework relates capability, opportunity, and motivation to behavior and includes feedback from behavior to those components. Opportunity encompasses external conditions, and motivation extends beyond deliberate intention [14]. The present paper's contribution therefore concerns the identification of a specified response from restricted records, rather than the general discovery that behavior depends on opportunity or changes its later conditions.

3.2 Household Use and Individual Opportunity

A household outcome can be appropriate when the research question concerns whether a household has any contact with a library. Its relation to individual opportunity is less direct. This subsection develops

that distinction through constructed cases, without attributing them to the households in either verified study.

Imagine a household containing four people. One regularly borrows materials, one wants a quiet place to study but finds the available hours incompatible with work, one hopes to organize a discussion group, and one prefers other activities. A positive household-use indicator can represent the borrowing member accurately. It leaves the other members' opportunities unresolved. Calling the household a user remains consistent with all four descriptions. An inference that everyone in that household has adequate access would require additional individual information.

The converse also deserves attention. A household with no recent use can contain a person who would welcome a particular service, a person who tried and failed to gain access, and a person whose current alternatives work well. The aggregate negative indicator preserves their shared absence of recorded use while concealing differences in its production. The identification question asks which of those differences matter for the proposed offer and what evidence can distinguish them. A general nonuser classification alone cannot select one explanation.

Household size can alter the meaning of an any-member outcome even when individual opportunities are otherwise comparable. A larger household has more members who might supply the one positive event needed for the aggregate indicator. An association between household size and household use can therefore contain a mechanical feature of aggregation alongside social or practical mechanisms. Separating those interpretations requires the appropriate model and data. The existence of a plausible aggregation mechanism supplies a rival explanation, not a finding about a particular coefficient in the source study.

Within-household substitution creates a further possibility. One member can collect materials for another, making direct visits a poor measure of the second person's benefit. Conversely, one member's use can consume the household's available travel time, leaving another person's desired activity unrealized. A policy that changes pickup arrangements could affect both members differently. The household outcome remains useful for some questions, but the unit of intervention and the unit of benefit should be specified before evaluating that policy.

These examples suggest a practical instrument choice. A survey can retain a household-level screening question while asking a limited set of person-specific follow-ups for the focal activity. The follow-ups should identify who considered participating, what offer was understood, and which stage of the participation process was reached. They need not demand a complete inventory of every household member's preferences. The objective is to collect the distinctions necessary for the approved target with a proportionate burden.

3.3 Anticipated Use and a Specified Offer

An anticipated-use question asks a person to form a judgment about future conduct. This judgment can reveal expectations, perceived relevance, and the range of activities the person currently associates with the library. Its relation to a particular potential response depends on the offer and history the person imagines while answering.

Consider two respondents who select the same low anticipated-use category. One imagines the existing opening hours and feels satisfied with other resources. The other would welcome a weekend discussion space but assumes that such use is unavailable. Their answers can coincide even though their responses to a concrete weekend offer differ. The difference arises from the imagined offer underlying the answer. A follow-up description can make that offer more explicit, but it also supplies new information and may change the response being elicited.

Two respondents who select a high anticipated-use category can also differ. One has a definite plan tied to a scheduled event. The other expresses a general aspiration that depends on uncertain work hours, companionship, or future health. Both can answer sincerely. A forecast of realized uptake requires a model of those intervening conditions, including how much of them the policy can alter. Treating sincerity as equivalent to predictive accuracy would place an unreasonable burden on respondents' ability to foresee their circumstances.

Time horizon contributes another distinction. A broad expectation over several years permits many offers, personal transitions, and changes in alternatives. A response to one event next month concerns a much more specific opportunity. The broad question can be useful for strategic planning, while the event-specific question can guide implementation. A comparison between their answers should treat the horizon change as part of the target change, rather than interpreting every discrepancy as a measurement failure.

Validation can investigate the relationship between an anticipated-use measure and later participation under documented offers. The analysis would record the offer actually received, the person's opportunity to participate, and the interval between elicitation and observation. It could then assess prediction within relevant conditions. Even a strong predictive relationship would remain conditional on those conditions. Its transfer to a new service, a different population, or a formative introduction would require further evidence.

The possibility of formative elicitation deserves a constructive response. A question that helps someone imagine a new activity can be a useful public engagement practice. Its value need not depend on recovering an untouched preference. The study can document that the elicitation was an introduction and evaluate the resulting opportunity on its own terms. This preserves both the practical role of consultation and the inferential distinction between earlier dispositions and later articulated interests.

3.4 Barrier Rankings and Removal Effects

A barrier ranking organizes what respondents consider more or less important in a stated comparison. An intervention effect concerns what changes when some condition is altered. The relation between them is especially delicate when participation requires several conditions at once. This subsection develops that relation through explicit hypothetical cases.

Suppose a person requires both a suitable time and a suitable room. The person may rank timing as the more serious obstacle because it is the first difficulty encountered or the most burdensome to negotiate. Changing the time alone can leave participation unchanged if the room remains unavailable. Changing the room alone can likewise leave participation unchanged. Changing both can permit the activity. A ranking that accurately describes the person's experience therefore need not predict the effect of a single isolated policy change.

The case also shows why a failed single-component pilot can be misinterpreted. If extended hours produce little uptake while room rules remain restrictive, the result can establish low uptake under that particular combination. It leaves open uptake under extended hours with the relevant room access. A broader conclusion requires either evidence that the room condition is irrelevant or a design that varies it. This is the empirical counterpart of keeping the offer's components explicit.

Substitution produces a different pattern. Suppose either a local room or a workable remote option would support a group. A respondent can regard the absence of the local room as salient while still participating through the remote option. Restoring the local room may change modality and quality without increasing the number of participating people. A policy evaluation focused only on new participants would then miss the target if the intervention was intended to improve an existing experience. The appropriate outcome depends on the reason for changing the offer.

Some barriers concern uncertainty about rules. A person may hesitate because the booking process appears difficult, even when an application would be approved. An explanatory intervention can change perceived feasibility without changing the formal rule. The resulting uptake would support a role for information under that offer. It would leave open whether the earlier uncertainty arose from poor communication, ambiguous practice, a justified memory of prior rejection, or another source. Interview and implementation evidence can discriminate among those mechanisms.

Other barriers emerge only after an initial restriction is removed. People who have never reached the booking stage may be unable to report the scheduling conflicts they would encounter there. A ranking collected under the earlier regime can therefore omit later bottlenecks. This does not make the earlier testimony invalid. It locates that testimony within the person's experienced opportunity structure. A staged study can update the barrier account as participants move through additional stages.

The practical implication is to connect barrier questions with concrete episodes and policy components. A respondent can be asked which stage prevented a particular attempt, whether another condition would still have prevented it, and what changed after a successful attempt. These answers remain reports requiring interpretation. Their episode-specific structure makes the inferential bridge more visible than an isolated ranking of abstract labels.

3.5 Behavioral Explanation and Statistical Identification

A framework that explains behavior through several interacting conditions provides a vocabulary for intervention design. An identification argument asks whether the available observations distinguish the target values within a specified model. This subsection develops the relation between those tasks while preserving the competence of the verified behavioral antecedent.

Suppose an explanatory account says that participation requires adequate skills, practical opportunity, and sufficient motivation. That account directs attention toward several possible reasons for nonparticipation. It can guide interviews, implementation analysis, and candidate changes. Yet a record containing only completed participation may leave several combinations of those conditions compatible with the same observation. The explanatory structure identifies possible mechanisms; additional observation or restriction is needed to choose among them.

Conversely, an identified intervention effect can leave the explanatory mechanism unresolved. A randomized invitation can change uptake under a common offer while operating through information, confidence, social connection, familiarity with staff, or several processes together. The assignment contrast can be credible even when the mechanism account remains incomplete. A mechanism study needs its own measurements and assumptions, particularly when the intervening states affect one another. The result should preserve the difference between learning that a package has an effect and learning why.

The paper's offer-indexed demand variable belongs to neither an abstract motivation category nor a direct measure of all those mechanisms. It is a potential response to a stated offer after a stated history. This definition can incorporate the consequences of capability, opportunity, motivation, and their interactions without claiming to decompose them. Renaming that response "motivation" would collapse the distinction between a condition contributing to behavior and the behavior generated under a complete set of conditions.

The elementary gate representation is correspondingly limited. When the paper writes a fixed response multiplied by a permission gate, it stipulates that opening the gate preserves the relevant response during measurement. A richer behavioral account can challenge that stipulation by identifying learning or other feedback during the opening. The proper response is to revise the observation model and target, as the history-indexed construction does. The framework's richness is a reason for a more careful identification argument.

This relationship also clarifies the local contribution. The paper offers explicit examples of observational equivalence, finite bounds, capacity conflicts, and policy-specific estimands for a public-library problem. It does not claim priority for the general interaction of personal and external conditions. Its originality depends on whether the integrated analysis resolves a consequential inference that the available methods leave implicit or mishandle in a concrete application. That condition remains a substantive standard for later review.

3.6 Descriptive Segments and Public Allocation Objectives

Research can group nonusers by reported characteristics or likelihood of engagement. Such descriptions may help a library choose communication methods and anticipate implementation. Their use in allocating public resources introduces an evaluative objective that should be made explicit. This subsection examines that transition through alternative decision purposes.

One objective is to generate the largest near-term increase in attendance for a given expenditure. A group already interested in familiar services may offer a promising intervention under that objective. A different objective is to extend a fair opportunity to people whose needs are severely constrained.

A group facing multiple barriers may then deserve attention even when immediate uptake is harder to achieve. The conflict concerns the public purpose of the service, not an arithmetic error in the expected attendance calculation.

A third objective can concern the quality or continuity of existing participation. Resources devoted to a small group may preserve an activity that its members cannot reasonably replace elsewhere. A fourth can concern the discovery of currently unarticulated possibilities. Exploratory provision may produce uncertain outcomes while widening the range of uses a public can consider. Each objective calls for different evidence and a different account of acceptable cost and uncertainty. One descriptive segment cannot determine their relative priority.

The evaluator should therefore distinguish a prediction from a recommendation. A prediction states what is expected under a specified intervention and population model. A recommendation combines that prediction with judgments about value, distribution, feasibility, and authority. The two can appear in the same report, provided their connection is visible. Hiding the objective inside a term such as “promising users” would obscure why some people receive attention and others wait.

Descriptive groups should also remain revisable. A person classified through current nonuse can acquire a different history after an accessible introduction or a successful collaboration. The resulting change can affect the relevance of the original group description. An administrative category that remains fixed despite such changes can distort future allocation. Preserving the dates and purposes of classifications helps prevent a provisional research description from becoming an enduring judgment about a person’s civic relevance.

3.7 A Fair Test of the Paper’s Added Value

The comparison concludes by specifying what would establish a useful contribution beyond familiar calls to consult nonusers. The standard requires a concrete inference, a clearly stated target, and an analysis that changes what can responsibly be concluded or done.

An empirical application should identify an actual reasoning step in a policy process. For example, a report may use a participation count to support a claim about response under a different offer. The researcher would reconstruct the original observation regime, identify the implied bridge, and assess whether the available evidence supports it. A successful critique would show exactly where two compatible accounts lead to different target values. A successful constructive contribution would propose feasible evidence capable of narrowing that difference.

The same standard allows an application to vindicate existing practice. A library may already distinguish requests, refusals, scheduling constraints, and formed interests. Its study may have a well-specified population and an appropriately limited conclusion. Applying the present framework could then confirm the inference’s scope while identifying only minor reporting improvements. That result remains valuable as an audit, though it supplies weaker evidence for a claim of methodological novelty.

The paper should also allow abandonment of an unnecessary formalization. If the decisive practical issue is an unambiguous missing field that staff can add immediately, a lengthy latent-variable model may add little. If the issue concerns disputed public purposes, an increasingly precise demand estimate may leave the central disagreement unchanged. The formal work earns its place when it exposes a hidden assumption, distinguishes rival explanations, or supports a better designed study. The verified antecedents strengthen this requirement by showing how much evaluative competence is already available.

4 Participation and Demand Targets

This section fixes the objects that the later models represent. It begins with an activity and an offer, adds population and history indices, and distinguishes several meanings of demand. The definitions are working analytical choices. Their purpose is to make an empirical question precise enough that a reader can identify what would count as evidence for it and which changes would produce a different question.

4.1 Activities, Offers, and Histories

An activity description identifies what participation would involve. A complete offer additionally states the circumstances under which a person could take part. This subsection separates those objects because two offers for the same named activity can support different responses.

Let a denote an offer. In the present setting, its specification includes the activity, location, time, duration, information provided, expected interaction, relevant costs, accessibility arrangements, and allocation procedure. An offer of a discussion session at four in the afternoon differs from an otherwise similar offer at seven in the evening. An offer with a known facilitator differs from a room reservation that requires participants to organize themselves. The analyst should include the dimensions that plausibly matter to the contrast under investigation. An infinitely detailed description would prevent comparison, while an excessively coarse description would conceal consequential differences.

Let h denote the relevant history before the offer. It may include previous opportunities, prior attendance, experiences of acceptance or refusal, and the information through which a person interprets the offer. The notation permits the same current arrangement to produce different responses after different histories. It leaves open which parts of history matter empirically. A researcher should begin with a justified set of candidate historical conditions and revise it when evidence shows that a distinction has been omitted.

Definition 4.1 (Offer-indexed participation). For person i , $Y_i(a, h) \in \{0, 1\}$ denotes a stipulated potential response to the complete offer a following history h . The value one means participation according to a declared event rule and observation window. The response includes any effects of the offer before that participation is measured.

The final sentence is consequential. An invitation can provide information, reduce anxiety, introduce a peer, or make an activity intelligible. If those changes occur before the measured response, they belong to the actual-offer response. A later claim about what the person wanted before the invitation concerns a different object. Section 11 develops that distinction through a counterexample and a constructive study design.

4.2 Population and Observation Windows

A demand proportion requires a denominator. This subsection declares that denominator before introducing an average and explains the choices that a library study must make. The individual indexed by a model should correspond to a recoverable unit in the proposed evidence design.

Fix a finite population I with $N > 0$ members. It might be a baseline cohort of residents, a set of households, or a group of people eligible for a particular activity. These choices answer different questions. Registered borrowers form an administratively convenient population, but a claim about all nearby residents requires evidence concerning people outside that list. Visitors observed during opening hours form a population already conditioned on reaching the building at those times. Changing opening hours can change that population as well as its participation.

The observation window also matters. A person who attends once during a month has a binary monthly participation value of one; a count of four attendances describes frequency. Both may be useful, but they should never be substituted without explanation. A service can gain repeated use among existing users while reaching few additional people. Alternatively, it can reach many new people who attend only once. A single total attendance number combines these patterns. The appropriate unit depends on whether the question concerns access, intensity, persistence, capacity or some other outcome.

For group activity, an individual response may also depend on who else receives an offer. Initially, that peer arrangement is included in a . Later sections make it explicit through assignment vectors and capacity rules. This ordering avoids assuming that an individual response exists independently of a group whose presence constitutes part of the activity itself.

4.3 Demand for a Specified Alternative Offer

The paper’s primary behavioral target is a response to a declared alternative offer. This subsection defines it and distinguishes its observational status from its conceptual meaning. The definition concerns what an investigation seeks to learn; it supplies no automatic procedure for learning it.

Definition 4.2 (Demand relative to an offer). For a specified alternative a^* and prior history h , define

$$D_i(h; a^*) = Y_i(a^*, h), \quad \theta(h; a^*) = \frac{1}{N} \sum_{i \in I} D_i(h; a^*). \quad (1)$$

The first quantity is individual offer-indexed demand, and the second is its finite-population proportion.

Equation 1 is a working behavioral definition. It places the alternative offer inside the target, making the dependence on practical conditions explicit. The target can remain scientifically useful even when current data only bound it. Its interpretation becomes unstable when the offer cannot be described consistently across people or when the activity’s meaning depends on an unspecified future negotiation. In such cases the investigation should first clarify the proposed practice, or retain several possible offers and report how the conclusions vary among them.

The word unmet adds a comparison with a current arrangement. A person can have $D_i(h; a^*) = 1$ while already participating in a satisfactory equivalent opportunity elsewhere. Another person may have access to an alternative that requires substantially more travel or carries an unwanted condition. Deciding whether those opportunities count as adequate substitutes requires a declared comparison standard. The mere existence of some route to activity is weaker than the claim that a person’s relevant demand is adequately served.

A narrow descriptive definition can identify people who would participate in a^* while being unable to participate through a specified current route. A broader policy definition might additionally require that the alternative be suitable, that its absence matters, and that supporting it falls within the library’s role. The manuscript uses the narrow target for conditional mathematical results and examines the additional evaluative premises later. This prevents a binary variable from silently deciding what the institution owes a person.

4.4 Expressions, Dispositions, and Recorded Events

The same person can supply several kinds of evidence about participation. This subsection distinguishes an expressed response, a modeled prior disposition, and an event record. Their relations require explicit bridges.

Let $S_i(q, h)$ be an answer to question q after history h . A positive answer can express interest, a request, politeness, a conditional intention, or a judgment about what the library should provide. A well-designed question clarifies which of these is being sought. The answer deserves to be preserved in its own terms. Equating it with $D_i(h; a^*)$ additionally assumes that the question specifies the relevant offer and that the response predicts behavior in the required way. Those are empirical claims.

Let W_i^- denote a hypothetical disposition immediately before a formative exposure. This latent state is distinct from the actual-offer response in Equation 1. If the offer forms interest before participation, the person can have a low pre-exposure disposition and a positive actual-offer response. A model that introduces W_i^- must explain its interpretation and the evidence that could distinguish its possible values. Otherwise the symbol remains an unvalidated latent modeling device.

Finally, let R_i denote a recorded event. Its relation to actual participation depends on the recording rule. A booking may be canceled; an unbooked informal activity may occur; a single person may generate several entries. The record is a trace produced by a particular procedure. Its use as evidence is proposition-specific. A confirmed attendance can establish that participation occurred within a declared window while leaving prior interest, counterfactual participation and public value unresolved.

4.5 Permission, Practical Access, and Choice

The final distinction prepares the governance gate. Permission removes one constraint; practical access concerns the remaining conditions required to act; choice concerns the response within those conditions. A coherent model must state where it places each factor.

Write G_i for permission and K_i for a simplified practical-access indicator. A person may be permitted to join an activity while lacking transport, time, required equipment, confidence about the rule, or a necessary companion. If these conditions enter K_i , the analyst needs evidence for their classification. A failure to attend cannot by itself reveal which condition failed. Multiple barriers may coexist, and removing one may leave behavior unchanged because another remains decisive.

The choice to decline an accessible opportunity also deserves analytical space. An evaluation concerned with expanding participation should preserve the possibility that a person has considered the offer and prefers another use of time. A record of refusal can be informative without becoming a permanent classification of the person. A later change in circumstances or interest may produce a different response. The target's history and offer indices allow that change to be described without treating either response as an error in the person's identity.

5 Symbol Domains and Model Boundaries

The manuscript uses several small models for different questions. This section supplies a reading guide to their symbols and boundaries. It identifies the response variables that must remain distinct, explains the change between finite-population and probability formulations, and states how local notation is introduced. The guide organizes the mathematics; each later result still carries its own complete definitions and assumptions.

The models form a connected argument without constituting one universal equation for library participation. A gate model isolates an inference under restriction. A recording model studies a channel. An assignment model compares policies. A value model adds evaluative premises. Transferring a quantity from one model to another requires a bridge that preserves its target and domain. Similar notation or a shared numerical range supplies insufficient justification for that transfer.

5.1 Response, Disposition, Request, and Record

Table 1 identifies the most consequential distinctions. The actual-offer response $D_i(h; a)$ is defined through participation under a specified offer and history. It includes effects of that offer before participation is observed. The latent pre-exposure state W_i^- belongs to a separate modeling question and needs its own interpretation. The request variable Q_i in the capacity model refers to a specified willingness to request before allocation, while an actual submitted application is a recorded action under its own conditions.

The table also makes a local reuse of S explicit. In the question-response notation, $S_i(q, h)$ is an answer. In the participant-selection model, $S(a)$ is participation under offer a . The latter model introduces its domains before deriving the bounds. An answer from the former model cannot be substituted for the latter participation indicator merely because the letter agrees.

The distinction between Y and R is similarly target specific. An administrative booking can be the actual event for a question about submitted bookings and a fallible proxy for a question about attendance. Its epistemic role follows the proposition under study. Calling a record administrative does not make it intrinsically weak; calling it direct does not establish that it measures every target of interest.

5.2 Finite Populations and Probability Laws

Several results begin with a fixed list of N people or records. Binary totals in those models are integers, and exact attainable proportions lie on the corresponding lattice. Other results concern probabilities or

Symbol or family	Local analytical role	Boundary requiring attention
$D_i(h; a)$	Response under an actual specified alternative offer	Offer formation, history, peers and allocation belong to the target
W_i^-	Modeled disposition before exposure	Latent interpretation requires a bridge; opening response need not identify it
Q_i	Reference request willingness before admission	A reported intention, actual submission and capacity-rationed attendance remain separate
Y_i, R_i	Actual event and its recorded trace	Recording accuracy, unit and coverage require evidence
$S_i(q, h)$	Answer to a specified question	The answer's meaning and behavioral transfer need validation
$S(a), Z(a), R^q(a)$	Participation, participant outcome and questionnaire completion in the selected-outcome model	Local notation; experience may be inapplicable outside participation
Ω	Sequential-policy importance weight	A probability ratio, distinct from the latent state W^-

Table 1: Core symbol domains. The table is a navigation aid; the linked models define their own populations, histories and assumptions.

expectations under a population law. Mixtures can fill an interval in the probability model even when a fixed finite list can attain only selected points.

The difference matters when a result is described as sharp. Sharp endpoints mean that the lower and upper values can occur under the stated model. An exact finite set additionally identifies which interior values are possible. The linked-event example has gaps larger than a single-record increment because event membership couples several records. A continuous interval can summarize such a set while preserving a warning that it is a convex summary.

The probability law can be identified even when one realized sample leaves many finite completions. The sampling chapter derives this distinction directly. Repeated-sampling distributions, exact population quantities and one observed dataset should therefore be named separately. A confidence statement about an estimator is a further object with its own design assumptions.

Counts also need local names. The classification model uses a false-negative cap M . The sampling chapter writes the population positive count as N_+ to distinguish it from that cap. The audited total p_A counts every actual positive in the audited set, including positives missed by the original classifier. Reading it as only the correctly labeled positive cell would change the residual-budget calculation.

5.3 Offers, Policies, and Histories

The symbol a usually denotes an offer. A policy can specify how offers are chosen over histories, and an assignment indicator identifies which policy a branch receives. These objects differ in level. A comparison of two offers under a fixed history is narrower than a comparison of two adaptive policies that generate different histories.

The sequential model uses Ω for a product of assignment-probability ratios. This weight changes how observed histories contribute to a policy-value calculation under specified common-process and support assumptions. It is an algebraic weight, not a psychological state. The model permits formative transitions within the shared history process and leaves pre-exposure latent interpretation separate.

The decision chapter uses r_L and r_U for losses at the lower and upper response endpoints. These symbols avoid reusing the offer symbol a inside the regret derivation. Its policy lottery chooses between complete alternatives. A branch assignment used for research and a lottery for individual places have different targets and consequences, even though each can involve a probability between zero and one.

An offer can be individually interpretable without being jointly deliverable to everyone at once. A hypothetical guaranteed place gives a reference response. A system with finite capacity needs an alloca-

tion rule and a model of joint participation. The capacity and transport chapters retain this distinction so that sums of individual reference responses are not silently reported as achievable attendance totals.

5.4 Bridges between the Models

A useful reading practice is to identify the bridge whenever the argument changes models. Moving from records to actual events requires a measurement relation. Moving from actual events under assignment to a causal contrast requires a design and consistency conditions. Moving from a source population to a target population requires comparability and support. Moving from participation to a provision decision requires evaluative premises and legitimate authority.

Some bridges are conditional mathematical assumptions. Others require field evidence, interpretation or public judgment. The manuscript separates these roles in each construction. A proof establishes what follows if its assumptions hold; it supplies no automatic evidence that a particular institution satisfies them. A documentary illustration establishes the inspected rule or instrument content within its access limits; it supplies no invented study result.

The guide is intended to make the extensive argument easier to inspect without flattening its distinctions. Readers can follow the target relevant to their question through the corresponding model and return to the full derivation where necessary. The recurring requirement is continuity of meaning: the quantity used in a conclusion should remain the quantity defined, observed, bounded or valued in the steps that support it.

6 The Governance Gate

This section reconstructs the simplest formal example that motivates the paper. It states the domains and assumptions, proves the resulting ambiguity, and then examines the interpretation of an enabling intervention. The example is deliberately small. Its usefulness comes from exposing exactly which additional assumptions would make an inference from activity to demand valid.

6.1 The Binary Construction

The construction represents a single specified activity within a single observation window. It temporarily holds the relevant history and alternative offer fixed. This simplification allows the inferential role of a restrictive rule to be examined separately from recording error, capacity and formation.

Let $D, G, Y \in \{0, 1\}$. The variable D is the response that would occur under the fully specified enabled offer, G indicates whether the activity is enabled, and Y is actual participation. Adopt the following explicit assumption.

Assumption 6.1 (Fixed-response gate). The alternative offer is sufficiently specified that D is well defined. Changing G affects participation only through permission; every other required condition is held fixed and sufficient. Participation is measured accurately within the declared window, and

$$Y = GD. \tag{2}$$

Equation 2 is a deterministic conjunction. It requires no statistical independence between G and D . Permission could be assigned preferentially to people believed likely to attend, and the equation could still hold for each person. The restrictive assumption concerns the substantive completeness of the bridge from the alternative offer to the actual event. Under permission, the equation makes participation equal to the declared response. Under restriction, it forces participation to zero.

Proposition 6.2 (Ambiguity under a closed gate). *Within the fixed-response gate model, the observation $(G, Y) = (0, 0)$ is compatible with both $D = 0$ and $D = 1$. The value of D is therefore unidentified by that observation alone.*

Proof. Substituting $G = 0$ into Equation 2 gives $Y = 0 \cdot D = 0$ for each value of D . The two admissible assignments have equal observed values and different target values. Any function of the observed pair must return the same answer for both assignments, so it cannot recover both target values correctly. $\square$

The proof establishes a limit of the specified observation, not a limit of every conceivable inquiry. A request made before restriction, a suitable interview, an independently observed alternative opportunity, or a justified intervention could supply additional evidence. The result also leaves the prevalence of the two possibilities open. An institution could happen to face little demand even though the inference from zero use was insufficient. The logical defect lies in treating one compatible explanation as the conclusion without a reason that excludes the other.

6.2 Population Bounds from Permission Records

The individual example extends to a population when permission is known for each person. This subsection derives the strongest conclusion available under the same gate assumptions and unrestricted responses among the blocked units. The derivation retains the finite nature of the population.

Let I be a finite nonempty population containing $N \geq 1$ people. Suppose $Y_i = G_i D_i$ and all G_i and Y_i are observed. Define

$$g = \frac{1}{N} \sum_i G_i, \quad y = \frac{1}{N} \sum_i Y_i, \quad u = \frac{1}{N} \sum_i (1 - G_i) D_i.$$

The quantity u is the blocked response proportion under the stipulated alternative offer. Partitioning the demand total by permission gives

$$\theta = \frac{1}{N} \sum_i D_i = \frac{1}{N} \sum_i G_i D_i + \frac{1}{N} \sum_i (1 - G_i) D_i = y + u. \quad (3)$$

Proposition 6.3 (Sharp gate bounds). *Under the finite-population gate model,*

$$0 \leq u \leq 1 - g, \quad y \leq \theta \leq y + 1 - g. \quad (4)$$

Every value obtained by adding an integer multiple of $1/N$ within these limits is attainable. The displayed intervals are the convex hulls of the finite attainable sets.

Proof. There are $N(1 - g)$ blocked people, each with an unrestricted binary D_i . Their sum can equal any integer from zero through $N(1 - g)$. Assigning one to exactly that many blocked units constructs each value. Equation 3 then gives the corresponding value of θ . Observations on permitted people remain unchanged throughout. $\square$

The finite qualification matters. With ten people and three unresolved responses, the exact possible totals are integers. Reporting a continuous interval is useful shorthand for bounds, but it should not be mistaken for the exact finite set. In a population-probability formulation, mixtures can fill the interval. Neither formulation supplies a sampling confidence level. Identification concerns compatibility with an observation law or a complete finite record; inference from a sample adds another source of uncertainty.

6.3 Additional Practical Gates

Permission can fail to be the final condition for participation. This subsection adds one practical gate and examines how the interpretation changes. The objective is to preserve the simple model's conceptual clarity while withdrawing an unsupported bridge.

Let $K_i \in \{0, 1\}$ represent the joint sufficiency of additional practical conditions. A stronger conjunction model is

$$Y_i = G_i K_i D_i. \quad (5)$$

If $G_i = 1$ and $K_i = 0$, a zero event is compatible with either demand value. Permission records alone therefore cease to identify D_i among every permitted person. The coverage set in the previous proof must be reduced to those for whom the complete enabling bridge is justified. An analyst who retains the larger set silently treats a logistical or social barrier as an expression of preference.

The conjunction is still a model. Some practical conditions affect the attractiveness of the offer itself. A difficult journey can make the activity less desirable, and a trusted companion can change what participation means. Whether to place such a condition in K_i or inside the offer-indexed response depends on the research question. Both choices can be coherent if their meanings remain explicit. Trouble arises when the analyst changes that placement during the argument and then compares quantities that were defined under different decompositions.

Multiple barriers also produce interaction. Suppose transport and an available time slot are both necessary. Providing transport while keeping the incompatible schedule may leave attendance unchanged. This unchanged outcome does not establish that transport was irrelevant to the person's feasible route. It establishes that the implemented change did not complete that route under the remaining conditions. A study of single modifications should therefore distinguish their marginal effects within the tested arrangement from the importance of each condition in a jointly enabled alternative.

6.4 The Interpretation of Opening

An enabling intervention is a possible source of information, but its target depends on its full effects. This subsection identifies the exact inference available from the binary model and the additional burden created by real opportunities.

Under Equation 2, changing G to one produces $Y = D$. The stipulated alternative-offer response is thereby observed. A claim that the same observation recovers a pre-existing psychological disposition needs the further assumption that the opening leaves that disposition unchanged before measurement. The original equation contains no independent measure of such a disposition. Its D is defined by the specified response bridge. Keeping that interpretation explicit prevents a useful elementary result from becoming an unsupported psychological conclusion.

A public announcement may teach residents that a kind of activity is possible. A facilitator may help them understand it, and the first session may create relationships that make later sessions attractive. Those effects can be part of a successful policy. Their presence changes the retrospective question. Evidence of success under the new offer can support retaining the opportunity without establishing that the same interest had already existed in a fully formed state. Section 11 develops this point without discarding the positive role of interventions in learning about alternatives.

The gate model remains worth retaining because it isolates a valid elementary objection. Its proper use is diagnostic: examine whether a record was produced under conditions that make competing demand states observationally equivalent. The subsequent investigation determines whether additional sources, different offers or stronger assumptions can resolve the ambiguity. The equation begins that investigation by exposing its dependency; it never completes the institutional or philosophical judgment on its own.

7 Permission, Enforcement, and the Interpretation of the Gate

The product model depends on what permission and participation mean in the application. This section examines that dependence directly. It distinguishes a formal rule, an effectively controlled route and an outcome defined through administrative approval. It then considers observational tests, model revision and the limits of attributing nonparticipation to a single factor. The objective is to retain the useful conditional derivation while making its empirical and conceptual burdens explicit.

A rule can state what is permitted without determining everything people do. People may misunderstand it, act outside it or receive an exception. Conversely, an activity can be formally permitted while remaining difficult to arrange. The gate model can represent particular versions of these processes, but

the choice of representation requires evidence. A normative statement should not acquire the force of a behavioral law merely because both can be written with a binary indicator.

7.1 Formal Permission and Actual Activity

Suppose G records whether the written rule permits an activity, while Y records whether the activity actually occurs. The gate equation implies $Y \leq G$. A case with $G = 0$ and $Y = 1$ contradicts that implication. Such a case can arise conceptually when people conduct an unauthorized activity. The model therefore needs an explicit restriction concerning compliance, enforcement or the route included in the outcome.

This observation leaves the written rule's normative content intact. A prohibition can remain a prohibition when it is violated. Coding every observed activity as permitted would erase that distinction. The investigator should preserve the rule status and examine whether the outcome occurred through an exception, a different route, a recording mistake or behavior outside the rule.

An exception requires its own timing and authority record. A staff member may authorize an activity under a provision that the initial code overlooks. In that case the relevant permission indicator may have been misclassified. Alternatively, staff may tolerate an activity without formally approving it. The study should distinguish these arrangements where they matter to the target. A single label of allowed can conceal different expectations and routes for future participants.

The actual activity also needs definition. Informal conversation in an open area can occur despite a rule restricting reserved discussion events. If Y concerns any conversation, the reservation rule may fail to be a necessary condition. If Y concerns a formally organized event using a controlled room, the rule can have a different relation to occurrence. The model should identify the route and activity before interpreting a counterexample.

7.2 Approved Events and a Definitional Gate

A different outcome can make the restriction hold by construction. Suppose Y means that an event appears in the approved-meeting register, and let G denote the same event-specific approval predicate and time window used to define that register. This local definition can differ from the written-rule permission indicator considered above, particularly where exceptions are authorized. Approval is then part of the outcome's definition. An unapproved activity can occur physically while remaining outside Y . Under this aligned coding, $Y \leq G$ holds definitionally. The full product $Y = GD$ still requires the fixed-response bridge; the approval classification alone does not establish it.

This construction can be useful for a question about approved access. It identifies the administrative route through which the institution recognizes and records an event. The target must retain that scope. A zero approved-event count does not establish zero activity of every related kind, and a change in approval categories can alter the count without an equivalent change in actual practice.

The definitional relation also changes the evidential role of an observed pattern. Finding that every registered approved event had permission supplies little independent support for the claim that the rule prevented unapproved activity. The register was designed to contain approved events. Evidence about broader behavior would require an observation process capable of recording activity outside that category.

The study should therefore ask whether the gate is an empirical restriction, an administrative classification or a combination. An empirical restriction can be challenged by observed violations within the defined route. A classification can be assessed for its consistency and suitability for the target. Combining them without explanation risks presenting a consequence of the database definition as evidence of a causal mechanism.

The distinction is especially important for historical comparison. An institution may begin recognizing an activity that previously occurred informally. The approved-event count rises, while the broader practice changes in a more complex way. Recognition can have real consequences for resources, legit-

imacy and continuity. Those consequences deserve investigation on their own terms rather than being compressed into an unsupported assertion that activity began at the moment of registration.

7.3 Effective Control of a Specified Route

The gate can also describe a substantively controlled route. A room may require an allocation that only the institution can provide, or a scheduled activity may require staff support that is unavailable without approval. In such cases a permission decision can be closely connected to the practical possibility of participation through that route.

The connection still needs a description of implementation. The institution may lock the room, withhold a booking confirmation, decline staffing or refuse entry. These are different actions with different records and possible exceptions. A written prohibition alone does not establish which action occurred. The study should identify the mechanism by which the specified route was made unavailable.

A controlled route is narrower than the entire activity. A group unable to reserve the room may meet elsewhere, change its format or abandon the proposal. Those responses can matter to the public decision. The gate result concerning the room route remains valid within its scope, while a claim about overall opportunity requires the alternative-provision analysis.

Effective control can also vary over time. Staff may apply a rule consistently in one period and interpret it differently later. Applicants may learn how to obtain an exception or which information a request requires. A single permanent permission indicator can miss these changes. The history-indexed target and process record should preserve the relevant sequence rather than assume that a rule's text fully specifies its implementation.

The evaluator can use several kinds of evidence to assess the route: policy versions, application responses, booking records, observations of access and participant accounts of attempts. Each source has a distinct role. A written rule establishes stated conditions; a response letter can establish a particular decision; an observed denial can establish an event in implementation. Their agreement can strengthen the route description when their provenance and scope are clear.

7.4 The Information Content of the Product Model

The simple product places strong restrictions on some observations and leaves other quantities free. This subsection explains that asymmetry so that the model's apparent precision does not exceed its actual information content.

When the fixed-response bridge holds and $G = 1$, the observed Y identifies D for that specified alternative offer. When $G = 0$, the equation restricts Y to zero while allowing either value of D . The model is therefore informative about the relation between the gate and the observed event, yet incomplete about response behind a closed gate. Those are compatible properties.

The same observed closed-gate record can be fitted by many assignments of blocked responses. This flexibility is the reason for the identified set. It should not be treated as evidence that any particular assignment is likely. The lower and upper constructions establish logical attainability under the retained assumptions; they provide no probability distribution over the compatible worlds.

An analyst can always introduce a latent variable whose unobserved values fill the missing part of an equation. The substantive question is whether that variable has an interpretable target. Defining D through a complete alternative offer supplies such a target when the offer is coherent. Calling the remaining free values an underlying desire without an operational or theoretical bridge would add a stronger interpretation than the algebra provides.

The product also leaves the permission assignment process unrestricted in the basic finite model. Permission can depend on prior applications, staff judgments or expected use. This fact explains why the deterministic equality requires no statistical independence between G and D . It also explains why a comparison between permitted and blocked people cannot automatically identify a causal permission effect. Assignment selection and the individual bridge are separate issues.

The model's value lies in this explicit separation. It can show exactly which responses the current route reveals and which remain unresolved. It becomes less useful if the analyst treats its latent completion as an established account of motivation or uses the equation to bypass the assignment and offer-comparability questions.

7.5 Factor Placement and the Meaning of a Barrier

A practical condition can be represented as a gate or included inside the offer to which a response is defined. This subsection examines how that choice affects the interpretation of a claim about barriers. The choice should follow the question and remain stable through the derivation.

Consider travel. A model can define D as response under an offer with transport already provided, then use a practical condition to represent the availability of that transport. Another model can define response to an offer that includes the person's actual journey and cost. The two targets answer different questions. A person can have a positive response to the supported offer and a negative response to the actual-cost offer without inconsistency.

The same issue arises with companions, facilitation and confidence in the invitation. A trusted companion can make an otherwise impossible journey feasible, or change the experience that the person values. Facilitation can supply necessary coordination or transform the activity's content. The boundary between a condition of action and a feature of the desired activity is therefore partly a modeling choice that needs an explicit purpose.

The study should avoid moving a factor merely to protect a favored conclusion. If transport cost was initially part of the specified offer, a low response under that offer cannot be reinterpreted as high demand behind a separate transport gate without defining and investigating the revised alternative. The revision may be useful, but it changes the question. Both versions and the reason for the change should remain in the record.

A barrier claim can also have several meanings. It can state that a condition makes the specified route infeasible, that removing it increases participation on average, or that it is part of a jointly sufficient arrangement. The component analysis shows why these statements need not coincide. Removing one necessary condition's absence can leave attendance unchanged when another condition still fails. An average effect can also conceal heterogeneous response profiles.

These distinctions improve the language of findings. A report can state that the current schedule conflicts with a documented requirement, that an assigned scheduling package changed attendance, or that a joint offer appears necessary under a specified model. Each statement has a different evidence base. The broad word barrier should be connected to the particular claim being made.

7.6 Testing the Gate and Revising the Model

A model should identify the observations that challenge its application. For the simple gate, an accurate event with $G = 0$ is a direct challenge when permission and occurrence are independently defined. A permitted nonparticipant is not a challenge by itself, because the model allows $D = 0$. The difference follows from the stated equation.

A challenge can prompt several responses. The permission record may be wrong, the event may belong to a different route, the outcome may be misclassified or the necessary-condition assumption may fail. The investigator should assess these possibilities through independent evidence. Recoding the variables solely to make the equality true would remove the model's empirical content.

Additional practical gates can repair an incomplete bridge when evidence identifies omitted conditions. The repair should state those conditions and how they will be observed or constrained. Introducing an unrestricted residual gate for every unexplained nonparticipant can make the model fit almost anything while weakening its capacity to answer the demand question. The value of the repair depends on whether it adds interpretable and investigable structure.

The same standard applies to history. Evidence that earlier rejection changed later expectations can justify a history-sensitive offer response. It should lead to a concrete account of the relevant sequence

and a revised target. Invoking history as an unspecified explanation for every result would supply little more than a reminder that circumstances matter.

The conditional bounds survive these discussions at their appropriate level. A proof can remain mathematically valid even when a particular library fails its assumptions. The application should then use a revised model or a narrower claim. Preserving the distinction between proof and empirical fit allows the research to correct its formalism without discarding the useful reasoning that motivated it.

7.7 Model Adequacy and Institutional Interpretation

The final assessment concerns whether the gate improves the institutional argument. Its simplicity can help identify a missing inference, but it can also conceal the differences among permission, implementation, recording and choice if introduced too quickly.

A useful application begins with the actual rule and route. It identifies the activity, population, time window and alternative offer. It states whether the gate is enforced, definitional or supported by observed compliance. It then derives the consequence that the evidence warrants and identifies the remaining research task. This sequence makes the mathematical construction reviewable by people who understand the practice.

The institution may discover that its existing evaluation already preserves these distinctions. In that case the gate provides a compact reconstruction rather than a criticism of current reasoning. It may discover a specific report that treats a closed route's zero count as proof of low response to an alternative. The model then identifies the unsupported step, while the subsequent inquiry determines whether the alternative would attract participation.

The philosophical restraint is equally important. The model need not settle what interests ultimately are or whether social facts belong to a particular ontology. It uses a defined response target and a specified institutional process to examine an inference. The wider questions about meaning, agency and public responsibility remain available for discussion through their own arguments.

The gate is therefore best retained as a carefully limited formal instrument. Its assumptions are demanding enough to require investigation, its consequences are simple enough to derive completely, and its counterconditions are concrete enough to challenge. Those features make it useful for a comprehensive account of demand precisely when the paper resists treating the product as a complete theory of participation.

8 Population Coverage and Recording Error

This section develops two sources of uncertainty that survive even when the activity and alternative offer are clearly specified. The first concerns which people are represented in the evidence. The second concerns which actual events enter the record. The derivations keep these mechanisms separate, identify the strongest bounds available under stated assumptions, and explain why a precise observed proportion can still leave the policy target uncertain.

8.1 Known Responses and Unobserved People

Suppose the relevant binary response is accurately known for some members of a fixed population. This subsection derives the basic coverage bound without assuming that those observed people are representative. The calculation is an elementary application of established partial-identification reasoning. Manski and Molinari examine the unresolved distributional component created by nonresponse and the information consequences of questionnaire design [13]. The local construction makes the finite-population interpretation explicit.

Let $V \subseteq I$ be the people whose D_i values are known, and define $q = |V|/N$ and $s = N^{-1} \sum_{i \in V} D_i$. The quantity s uses the full population denominator. By contrast, the proportion among known respondents is s/q when $q > 0$. The two quantities answer different questions and should receive different labels in a report.

Proposition 8.1 (Coverage identification region). *With accurate binary responses on V and unrestricted responses outside V ,*

$$\theta \in [s, s + 1 - q]. \quad (6)$$

For a finite population, the attainable values are the lattice values in this interval generated by the unobserved binary responses. Both endpoints are attained.

Proof. Write

$$\theta = s + \frac{1}{N} \sum_{i \notin V} D_i.$$

There are $N - |V|$ terms in the remaining sum, each between zero and one. Assigning all zero attains the lower endpoint, and assigning all one attains the upper endpoint. Assigning one to any intermediate number of unobserved people constructs each intermediate lattice value. $\square$

The width $1 - q$ measures unresolved coverage under this model. Increasing the number of questions answered by the same observed participants may improve understanding of their experiences while leaving this width unchanged. An additional independent response from someone previously outside V reduces the unresolved component, provided it measures the same target accurately. The relevant gain depends on reaching new units and preserving the target, rather than simply adding more entries to the dataset.

8.2 A Constructed Coverage Example

A finite example clarifies the distinction between a respondent proportion and a population conclusion. The numbers in this subsection are constructed solely to illustrate the derivation. They describe no library survey.

Suppose a declared population contains 100 people. A suitable instrument accurately identifies the target response for 40 of them, of whom 24 would participate in the specified offer. Then $q = 0.4$, $s = 0.24$, and the respondent proportion is 0.6. Equation 6 gives $\theta \in [0.24, 0.84]$, with steps of 0.01. Reporting that sixty percent of the whole population would participate introduces a representativeness assumption that the count itself never supplied.

Suppose the investigator reaches another 30 people and obtains 9 positive responses under the same valid instrument. The accumulated known positive count becomes 33, coverage becomes 70 people, and the bound becomes $[0.33, 0.63]$. The additional responses reduce the unresolved region even though the newly observed group has a lower positive proportion. A revision toward a lower estimate need not represent failure of the inquiry. It can be the consequence of reaching people omitted by an earlier sampling route.

Now change the interpretation of the instrument. Suppose it asks whether the activity sounds interesting, while the target is actual participation in a specified evening session. The same arithmetic provides bounds on that stated interest response, assuming accurate recording of answers. Transferring those bounds to behavioral participation requires a separate calibration or an explicitly conditional bridge. The numerical precision of the arithmetic cannot repair a mismatch between the question asked and the target reported.

8.3 Recording Sensitivity

Coverage concerns people whose responses remain unknown. Recording sensitivity concerns events that occur but fail to enter an administrative trace. This subsection derives a separate population-law bound and identifies the conditions that make it useful.

Let Y indicate actual participation and R indicate a recorded participation. Assume recorded positives are actual events, so $R \leq Y$. Write $p = \Pr(Y = 1)$ and $r = \Pr(R = 1)$. Suppose an independent assessment warrants a lower bound $\lambda > 0$ on recording sensitivity:

$$\Pr(R = 1 \mid Y = 1) \geq \lambda \quad \text{whenever } p > 0.$$

This assumption concerns the chance that an actual event is recorded. It cannot be established by inspecting recorded events alone.

Proposition 8.2 (Participation bounds under recording loss). *Under the stated absence of false positives and the positive sensitivity bound,*

$$r \leq p \leq \min\{1, r/\lambda\}. \quad (7)$$

Every value in the displayed interval is compatible with a suitable recording law satisfying the assumptions.

Proof. Since $R \leq Y$, $r \leq p$. If $p > 0$, total probability gives $r = \Pr(R = 1 \mid Y = 1)p \geq \lambda p$, because the term conditional on $Y = 0$ vanishes. Rearrangement gives the upper bound. Conversely, for any admissible $p > 0$, choose sensitivity r/p , which lies between λ and one, and allow no false positives. This constructs the required law. If $p = 0$, then $r = 0$, and the law is immediate. $\square$

When $r = 0$ is a known population probability and $\lambda > 0$, the bound implies $p = 0$. A sample containing zero recorded events is a different object. Sampling variation may produce that sample even when $r > 0$. Thus a report must distinguish an identification calculation using an observation law from an uncertainty statement based on finitely many observations. Adding a confidence interval requires a sampling design and its assumptions. The present proof alone supplies neither.

If recording sensitivity could be zero, an empty record is compatible with arbitrarily frequent activity. Informal discussion outside a formal booking system illustrates the kind of mechanism that would need investigation. The example supplies a possibility, not evidence that any particular library system misses such activity. A direct observation sample can assess the relationship between actual events and administrative entries if its own coverage and observer effects are examined.

8.4 Combining Observation Channels

Several channels can produce stronger evidence than one, but their combination requires attention to dependence and target alignment. This subsection explains the operational meaning of corroboration without imposing an unwarranted independence model.

An application log, a staff notebook, a participant account and an attendance record may concern the same event. Agreement among them can improve confidence about its occurrence. Their agreement is weaker evidence of independent confirmation if every later entry copied the first application. The study should preserve the provenance of each record and distinguish independent observation from repeated representation. The useful question is what new constraint a source places on the possible histories, rather than how many files repeat the claim.

Different channels may also measure different stages. A request concerns an attempt to obtain access. A confirmed booking concerns authorization. An attendance record concerns an occurrence within a measurement procedure. A follow-up response concerns a later interpretation. Joining these records can reveal where an intended activity ceased to progress. Collapsing them into one participation variable would discard the mechanism that the inquiry seeks to understand. A minimal linked event record should therefore preserve stage, time, unit, source and uncertainty before producing any summary measure.

An apparent discrepancy can be informative. A participant may report an unsuccessful attempt that the booking database lacks because requests were accepted only through another route. Alternatively, the participant may remember a different branch or date. The investigation should retain these possibilities and seek discriminating evidence. The absence of an administrative entry can support a claim about that recording channel while leaving the underlying attempted activity unresolved.

8.5 Denominator Revision and Remaining Uncertainty

The final coverage issue concerns a changing or incompletely enumerated population. This subsection states why a study needs both a sampling frame and a record of who falls outside it.

A public library serves people whose practical connections extend beyond a single administrative boundary. Residents may use another branch, commuters may visit near work, and people temporarily staying in an area may lack an entry in the chosen frame. A study can legitimately choose a bounded population for tractability. Its report should name that population and separate direct findings from proposed transport to a wider public. The mathematical bounds above depend on a known N ; uncertainty about the denominator creates an additional layer that should be represented separately.

Policy can also change who appears in a visitor sample. If a new program brings in previously absent people, the after-policy visitor proportion combines changes in behavior with changes in sample composition. A fixed baseline cohort answers a question about the original population. A repeated cross-section answers a question about each period's observed population. Both can be valuable. The investigator should resist treating their numerical difference as an individual change without a linking design.

Coverage, recording, sampling and target definition therefore produce different uncertainty statements. Improving one can leave the others open. A careful report makes each uncertainty visible and describes which proposed evidence would reduce it. That practice permits an evaluation to become more informative without promising a point estimate before its assumptions have been justified.

9 Classification Error and Validation

An observation can be present and still represent the wrong event. A booking can be counted as attendance, a participant can be entered twice, or a shared activity can be omitted because its description falls outside a reporting category. This section separates these errors from incomplete population coverage. It first derives finite bounds under explicit error budgets, then examines probability models for an imperfect binary record. The final discussion develops the validation work needed to make either approach credible in a library study.

The mathematical target throughout this section is a specified binary attribute of a fixed population. For example, it can be whether a person participated in a named activity under a documented offer during a stated period. This target is already narrower than a general interest in libraries. Correcting its records improves knowledge of that attribute. A subsequent claim about participation under a different offer requires an additional argument. This separation prevents a technically sophisticated correction from concealing a change in the question being answered.

9.1 Finite Error Budgets in a Partly Observed Population

Finite counting supplies a transparent starting point because every uncertain quantity has an identifiable location in the records. Let the target population contain $N > 0$ people. A covered subset V contains n people, and each covered person has a binary recorded label R_i . Exactly k of those labels are positive. The actual target attribute is Y_i . A false positive has $R_i = 1, Y_i = 0$; a false negative has $R_i = 0, Y_i = 1$. The definitions concern the stated event, rather than an observer's overall judgment of the person's interest or conduct.

Assumption 9.1 (Finite classification budgets). The population and covered subset are fixed, with $0 \leq k \leq n \leq N$. Among covered records, the number of false positives is at most the nonnegative integer F , and the number of false negatives is at most the nonnegative integer M . Every assignment of true labels satisfying these separate caps is admissible. The labels of uncovered people are unrestricted.

The assumption can describe a sensitivity analysis even when the evaluator cannot defend one preferred pair of caps. For example, the report can display conclusions under zero, two, and five possible false positives. Each column then answers a conditional question. A column acquires an empirical interpretation only through evidence supporting its restriction. The use of integer caps makes this status particularly visible: a statement that at most two events were miscoded is a substantive statement about those events, not a property supplied by algebra.

Proposition 9.2 (Sharp bounds under finite error budgets). *Under the finite classification budgets, the possible population totals $\sum_i Y_i$ are every integer from*

$$\max\{0, k - F\} \text{ through } \min\{n, k + M\} + N - n. \quad (8)$$

The corresponding proportion has these endpoints divided by N , with attainable values separated by $1/N$.

Proof. Let f_0 be the actual number of false positives and m_0 the actual number of false negatives in V . The number of true positive attributes in V is $k - f_0 + m_0$: begin with positive records, remove their errors, and add the positive attributes hidden among negative records. The feasible values obey $0 \leq f_0 \leq \min\{F, k\}$ and $0 \leq m_0 \leq \min\{M, n - k\}$. The smallest covered total uses the largest allowed f_0 and sets $m_0 = 0$. The largest covered total sets $f_0 = 0$ and uses the largest allowed m_0 . The $N - n$ uncovered people contribute any integer between zero and $N - n$. Adding these possibilities gives the displayed endpoints. Consecutive changes in f_0 , m_0 , and the uncovered positive count attain every intermediate integer. The unrestricted assignment clause ensures that each such construction belongs to the model. $\square$

The result separates three operations that a single correction factor often merges. False positives reduce the lower endpoint because some observed positive events may disappear on review. False negatives increase the upper endpoint within the covered population. Missing people contribute a further interval beyond the covered population. Improving attendance coding can therefore leave a wide population range when the coverage frame remains small. Conversely, a census of poorly classified records can remain uninformative despite complete enumeration.

Consider a constructed population of one hundred people with sixty covered records and twenty-four positive labels. Suppose the analysis allows at most three false positives and six false negatives. The lowest possible actual positive total is twenty-one. The largest covered total is thirty, and the forty uncovered people can increase the population total to seventy. The proportion therefore lies on the finite lattice from 0.21 to 0.70. These numbers are a calculation example. They are unrelated to a measured library population or a documented error rate.

The interval's width reveals the practical priority of further work. Perfect classification of the sixty existing records could at most remove the uncertainty contributed by their possible errors. The uncovered forty people would still permit a forty-point range. That observation supports an investigation of coverage when the target is population prevalence. It leaves open other reasons for reviewing the existing records, including fairness to a person whose attendance was disputed or the discovery that the entire event definition was being applied inconsistently. Statistical interval reduction is one reason for an audit among several.

9.2 Audited Records and Residual Restrictions

An audit can replace uncertainty about particular labels with verified information. The mathematical update depends on what the audit verifies and on whether the original error restriction concerns all covered records or only the records left unaudited. This subsection derives the update for an error cap that applies to the original covered set.

Let $A \subseteq V$ be an audited subset. For each person in A , suppose a credible review determines the actual Y_i under the same event definition. Write $p_A = \sum_{i \in A} Y_i$ for the audited actual positive total. This includes audited people whose original labels were negative. Let f_A and m_A be the false-positive and false-negative counts discovered in the audit. The unaudited set $U = V \setminus A$ has size n_U and k_U positive labels. The original caps leave residual budgets $F' = F - f_A$ and $M' = M - m_A$.

Proposition 9.3 (Audited error-budget bounds). *If F' and M' are nonnegative, the sharp population-total bounds are*

$$\begin{aligned} L_A &= p_A + k_U - \min\{F', k_U\}, \\ U_A &= p_A + k_U + \min\{M', n_U - k_U\} + N - n. \end{aligned} \quad (9)$$

A negative residual budget makes the original model incompatible with the verified audit.

Proof. The audited set contributes the fixed total p_A . The remaining covered records obey the same counting argument as before, with observed-positive count k_U and residual caps F', M' . The uncovered population again contributes any total from zero to $N - n$. Adding these components proves the bounds and their lattice sharpness. If the audit has already found more errors of a kind than the original cap permits, every completion violates that cap. A compatible population cannot then be constructed. $\square$

The incompatibility clause is a useful protection against cosmetic model repair. Suppose the initial analysis assumes at most two false-positive records and an audit verifies three. Replacing the residual budget by zero would make the contradiction disappear from the formula while preserving the language of the original assumption. The correct response records that the assumed cap failed. The evaluator can then adopt a revised cap, explain its justification, and calculate a new range. The revised analysis has a traceable evidential basis.

A constructed update illustrates the information retained by this procedure. Return to the population of one hundred, with sixty covered records, twenty-four positive labels, and caps of three false positives and six false negatives. Audit twenty records containing eight originally positive labels. Suppose the review finds two false positives and three false negatives. The audited actual positive total is nine. Forty covered records remain, with sixteen positive labels. The residual caps are one false positive and three false negatives. The lower population total is then $9 + 16 - 1 = 24$, and the upper total is $9 + 16 + 3 + 40 = 68$. The interval narrows from $[0.21, 0.70]$ to $[0.24, 0.68]$.

The audit has several kinds of value in this example. It determines the status of twenty people, rules out some extreme completions, and tests the plausibility of the proposed error restriction. Its modest effect on the overall interval also shows why a report should retain a decomposition by uncertainty source. An evaluator who reports only the remaining wide range might underestimate the audit's contribution. An evaluator who reports only the number of reviewed records might overstate its resolution of population coverage. The decomposition makes both assessments more precise.

Some studies can defend only a cap on the total number of errors, combining both directions. Let that cap be E . After the audit, the remaining budget is $E' = E - f_A - m_A$. The same endpoint calculations use E' for each direction separately: the lower extreme spends the available budget on false positives, while the upper extreme spends it on false negatives. These are different possible worlds. They cannot be combined into one world containing both maximum error counts when their sum exceeds E' . The endpoint result remains valid because each endpoint uses only one direction of correction. A claim about the joint pattern of mistakes must retain the shared constraint.

9.3 The Meaning of an Audit Standard

The phrase “actual positive” requires a practical interpretation before the counting argument can guide data collection. An audit standard specifies what evidence resolves the target label and how conflicting evidence is handled. For attendance at a scheduled session, a corroborated event log may sometimes resolve a coding mistake. For informal discussion, the boundaries of the event may remain disputed even after every available record has been read. The second problem concerns the classification rule itself.

A review can therefore produce three outcomes: a verified positive under the stated rule, a verified negative, or an unresolved case. Assigning unresolved cases to the negative category merely to complete a spreadsheet changes the measurement model. A defensible implementation keeps them unresolved and either treats their labels as unrestricted or supplies an explicit compatibility set. The finite bound then contains the remaining possibilities. This treatment preserves useful uncertainty without attributing hidden interest to the person.

Disagreement among reviewers also has several interpretations. One reviewer may have overlooked a record. Two reviewers may apply different event definitions. A participant may describe the activity through categories that the administrative scheme lacks. These situations call for different repairs: additional evidence, clarification of the existing rule, or reconsideration of the categories. A single agreement

percentage cannot select among them. It summarizes a pattern of judgments whose substantive meaning still requires investigation.

The audit protocol should retain a short reason for every change of label. Examples include duplicate registration, cancellation before participation, attendance entered under a different session, or activity excluded by the current coding definition. Such reasons allow the analyst to distinguish a clerical mechanism from a disagreement about the target. The reasons can also expose an institutional process that repeatedly produces the same mistake. A corrected total alone would conceal that process and make future errors harder to prevent.

Verification can itself create information. Contacting a participant to ask whether an informal meeting occurred may prompt a more articulated account of the event. This can improve the record while changing the form of the available testimony. The audit should preserve the earlier description and the later clarification, with their dates and elicitation conditions. Agreement between them can support a stable coding decision; a change in description can be meaningful evidence about articulation. Both remain different from direct observation of a pre-exposure disposition.

9.4 Probability Models for Imperfect Records

Finite error caps describe an actual collection of records. A probability channel describes how labels relate across a population law. Both can be useful, but their parameters have different evidential requirements. This subsection derives the binary channel identity directly and uses it to show which features of an imperfect record preserve information.

Let $p = \Pr(Y = 1)$ be the true positive proportion in the target population law, and let $r = \Pr(R = 1)$ be the positive-record probability. Define sensitivity $s = \Pr(R = 1 \mid Y = 1)$ and false-positive probability $f = \Pr(R = 1 \mid Y = 0)$. When p is zero or one, the conditional probability for the absent group has no observed cases; it can be treated as a stipulated parameter of the channel. Partitioning by Y gives

$$r = sp + f(1 - p) = f + (s - f)p. \quad (10)$$

This identity uses the law of total probability. It requires a common target definition and a correctly specified channel; it requires no claim that recording errors are morally or institutionally neutral.

Adjustment for imperfect binary measurement has a long methodological history. Rogan and Gladen's original abstract identifies the problem of estimating prevalence from an imperfect screening result [17]. The algebra here is an explicit elementary derivation for the present observation problem. The analysis does not transfer an uninspected uncertainty procedure from that source.

If $s \neq f$, equation (10) has the unique candidate solution

$$p = \frac{r - f}{s - f}. \quad (11)$$

The candidate must lie in $[0, 1]$. A value outside that interval reveals an incompatibility among the observed population rate and the stipulated channel parameters. It supplies a reason to review the model or the measurements. Automatically truncating the candidate to zero or one can be a chosen estimation rule, but it changes the exact identification claim and requires a separate explanation.

The denominator has a direct interpretation. If positive and negative attributes produce positive records with the same probability, the record does not distinguish the two groups. Formally, when $s = f$, the observed rate equals that common probability for every p . If $r = s$, every prevalence is compatible. If $r \neq s$, none is compatible. A high rate of apparently correct labels in a population dominated by one category can coexist with weak discrimination between categories. Informational usefulness depends on the separation of the channel responses.

A known inverted channel is instructive. Suppose $s = 0.2$ and $f = 0.8$. A positive record is more likely for an actually negative attribute than for an actually positive one. If these channel probabilities are credible and $r = 0.5$, the underlying proportion is 0.5. The misleading label can be reinterpreted because its relation to the target is known. The case is a mathematical counterexample to equating

ordinary coding accuracy with identification. It supplies no reason to prefer confusing administrative labels, whose practical costs remain substantial.

The channel can also be weakly informative. Suppose s and f are close. Then a small change in r , or a small error in either channel parameter, can produce a large change in the inverted value. This follows from division by the small separation $s - f$. Exact identification under known parameters can therefore coexist with poor practical precision when those parameters are estimated. A report that advertises a unique answer should disclose whether that uniqueness rests on highly uncertain calibration.

9.5 Uncertain Channels and Compatible Prevalence

Validation usually supports a range of plausible channel parameters. This subsection derives the compatible prevalence set when sensitivity and the false-positive rate vary independently within declared intervals. The independent admissibility of their combinations is a substantive condition of the model, distinct from independence of the underlying events.

Assume $s \in [s_L, s_U]$ and $f \in [f_L, f_U]$, with the full rectangle of parameter pairs allowed. At fixed p , the smallest predicted positive-record rate is obtained at (s_L, f_L) because the coefficients p and $1 - p$ are nonnegative. The largest is obtained at (s_U, f_U) . Every intermediate rate is achieved by moving continuously along the line segment between those corners. Consequently the sharp compatible set is

$$\mathcal{P}(r) = \{p \in [0, 1] : f_L + (s_L - f_L)p \leq r \leq f_U + (s_U - f_U)p\}. \quad (12)$$

The construction proves sufficiency as well as necessity: every retained value corresponds to an admissible channel, rather than merely passing a loose numerical test.

When $s_L > f_U$, every admissible channel distinguishes the target in the same direction. Both relevant slopes are positive, and solving the inequalities gives

$$\begin{aligned} L &= \max \left\{ 0, \frac{r - f_U}{s_U - f_U} \right\}, \\ U &= \min \left\{ 1, \frac{r - f_L}{s_L - f_L} \right\}. \end{aligned} \quad (13)$$

The set is empty when $L > U$. Outside the uniformly separated case, the general inequalities in (12) remain the safer representation; they preserve cases with zero or reversed slopes.

For a constructed example, let $r = 0.30$, let sensitivity range from 0.75 to 0.90, and let the false-positive probability range from 0.05 to 0.10. The lower compatible prevalence is $(0.30 - 0.10)/(0.90 - 0.10) = 0.25$. The upper is $(0.30 - 0.05)/(0.75 - 0.05) = 5/14$, approximately 0.357. The observed record rate of 0.30 lies inside this range, but that coincidence gives it no special identification privilege. Its relation to the true proportion depends on the admissible balance of the two error directions.

The interval can conceal restrictions known from validation. For example, a coding rule that increases sensitivity may also increase false positives. The feasible channel pairs might lie along a curve or in a smaller region, excluding the rectangle's extreme corners. Applying the rectangle remains a conservative enlargement when it contains the actual feasible set. Calling its bounds sharp for the narrower validation model would be inaccurate. The evaluator should preserve joint restrictions when they affect the conclusion and when the data can support them credibly.

9.6 Transporting Validation across Library Conditions

The final step from a channel calculation to a library conclusion concerns transport. A validation exercise observes a relation between labels and an audit standard under particular conditions. Applying that relation to a different setting requires evidence that the relevant recording process remains comparable. This subsection identifies the conditions most likely to matter for the proposed study.

A staff-supervised scheduled session and an informal unsupervised meeting can differ in the probability of receiving a record. A registration system may detect the first efficiently while giving the second

no route into its fields. Calibration from the scheduled session then addresses the wrong recording mechanism for the informal activity. The problem persists even when both events are beneficial library uses. Their value supplies no mathematical equivalence between their observation channels.

Language and accessibility can also affect the relation between participation and its record. A form completed through an interpreter, an oral account entered later by staff, and a self-completed digital response may produce different error patterns. These possibilities are research hypotheses until investigated. Their practical implication is to preserve collection mode and relevant process information, then examine whether one pooled channel is defensible. Automatically dividing people into fixed demographic error classes would risk replacing a process explanation with a label.

Time matters because implementation changes the channel. Staff may become more familiar with a new category, participants may learn to register informal events, or software may begin to reject incomplete entries. A rising record count can then contain a measurement change alongside any activity change. Repeating a small, well-defined validation exercise after a process change can be more informative than adding the same number of unvalidated records. The design should state which continuity claims each repeat is intended to assess.

Channel correction also leaves the observation occasion intact. If a person participates after an inviting introduction and an explanation of the activity, the corrected record measures that participation. It cannot be reinterpreted as the response to an earlier offer with less information without a history bridge. Even perfect classification of the later event preserves the distinction between revealing an existing disposition and forming a response through the event. Better measurement sharpens the declared target; target revision remains a separate inferential act.

The practical deliverable is therefore a validation record containing the event definition, covered population, collection mode, audit evidence, unresolved classifications, error restrictions, and transport arguments. The numerical bounds summarize the consequences of that record. Their credibility comes from the visible connection between restrictions and evidence, together with a willingness to revise restrictions when the audit contradicts them.

10 Sampling and Identification

A sample can be well designed while a claim about demand remains weakly identified, and a target can be identified by a sampling law while one small sample leaves substantial uncertainty. This section makes those statements precise through finite counting. It then examines the practical transitions from a population frame to invitation, contact, consent, response, and measurement. The objective is to locate the assumptions behind a population claim without treating every absence of an observation as the same problem.

The distinction matters especially when the recorded count is zero. Zero events in an administrative file, zero positive responses among volunteers, and zero positives in a probability sample have different evidential meanings. Each can be useful when its production process is known. Their common numerical value supplies no common observation model. The derivation below begins with an unusually transparent design and then identifies the additional obligations created by a real library study.

10.1 A Fixed Population and a Randomly Selected Subset

The first model holds the target population and its binary labels fixed. Uncertainty arises from selecting which people to observe. Let $N > 0$ be the population size, and let exactly N_+ people have the specified positive attribute $Y_i = 1$. Select a subset of n people uniformly from all subsets of that size, where $0 \leq n \leq N$. Every selected person is measured accurately under the same target definition. The observed positive count is K .

These conditions describe simple random sampling without replacement. A person appears at most once, every subset of the required size has the same selection probability, and measurement succeeds for

everyone selected. The model is intentionally separate from a convenience survey at a library door. It provides a clear reference against which the latter's actual observation process can be assessed.

Proposition 10.1 (Finite sampling law). *Under the stated design,*

$$\Pr_{N_+}(K = k) = \frac{\binom{N_+}{k} \binom{N-N_+}{n-k}}{\binom{N}{n}}, \quad \mathbb{E}_{N_+}[K] = \frac{nN_+}{N}. \quad (14)$$

A binomial coefficient is zero when its requested selection is impossible. For $n > 0$, distinct values of N_+ yield distinct sampling laws.

Proof. There are $\binom{N}{n}$ equally likely sampled subsets. A subset with exactly k positives selects k of the N_+ positive people and $n - k$ of the $N - N_+$ negative people. Multiplying the two selection counts gives the numerator. For the expectation, each fixed person has inclusion probability n/N by symmetry. Express K as the sum of the inclusion indicators of the N_+ positive people and use linearity of expectation. The result is nN_+/N . When $n > 0$, changing N_+ changes this expectation, so the entire probability distribution cannot remain the same. $\square$

The final statement is identification from the full sampling law. If an analyst knew the entire distribution of possible sampled counts under this design, its expectation would determine N_+ . That statement concerns a model's mapping from the target quantity to an observable distribution. It does not assert that one sampled count determines N_+ . A single count can be consistent with many distributions, even when those distributions differ.

This distinction can be obscured by ordinary language. Saying that a sample "identifies demand" may mean that its design permits a population estimate, that a mathematical model has a unique parameter for each observation law, or that a particular dataset establishes a narrow interval. These meanings should be separated in a research report. The first concerns a defensible design, the second a theoretical property, and the third the information actually available after the study.

10.2 The Completion Range of a Realized Sample

The same finite model supports a different question once one subset has been observed. Suppose the selected n people include exactly k positives. The remaining $N - n$ people have unobserved fixed labels. The possible population totals are then every integer from k to $k + N - n$.

Proposition 10.2 (Realized finite completions). *For an accurately measured sample with k positive labels, the set of population totals compatible with the realized labels is*

$$\{k, k + 1, \dots, k + N - n\}. \quad (15)$$

Every such completion assigns positive probability to the observed sampled subset under the uniform design.

Proof. The observed labels contribute the fixed total k . Assigning any number of positives from zero through $N - n$ to the unselected people gives every displayed total. Every assignment leaves the observed labels unchanged. The uniform sampling design assigns the selected subset probability $1/\binom{N}{n}$ independently of those labels, so each completion can produce that subset with positive probability. $\square$

The completion set is a logical compatibility statement. Sampling inference can give different weights or confidence assessments to its members because some totals make the observed result less likely. Those inferential assessments use the sampling probabilities. They are stronger than bare compatibility because they introduce a probabilistic criterion for assessing the data. A report should identify that criterion explicitly and state its coverage or calibration justification.

Consider a constructed population of ten people, a sample of two, and an observed count of one positive. The population may contain any total from one to nine positives. A total of one gives nine

positive-containing samples among the forty-five possible two-person subsets, while a middle prevalence produces a different distribution of counts. The exact frequency calculations come from equation (14). The observation of one positive preserves many logical completions even though their probabilities differ.

The limiting cases help clarify the model. If $n = N$, every person is observed and the completion set contains only $N_+ = k$. If $n = 0$, the data leave every total from zero through N possible, and the sampling distribution is the same degenerate zero count for all N_+ . The full-law identification result therefore requires $n > 0$. These cases also show that sampling design and coverage quantity enter different parts of the argument.

10.3 Zero Counts and the Probability of Missing Positives

A zero sample deserves an explicit calculation because it often invites an overconfident interpretation. Setting $k = 0$ in the sampling law gives

$$\Pr_{N_+}(K = 0) = \frac{\binom{N-N_+}{n}}{\binom{N}{n}}. \quad (16)$$

The numerator counts subsets composed entirely of negative people. The probability is positive whenever the population contains enough negatives to fill the sample. A zero count therefore remains possible under positive population totals, subject to that finite constraint.

For a constructed population of twenty people, suppose four possess the specified positive attribute and five are sampled. A zero-positive sample has probability $\binom{16}{5}/\binom{20}{5}$, which is 4368/15504, about 0.282. Thus almost three tenths of the possible samples under this constructed design contain zero positives despite a population proportion of one fifth. The example supplies a concrete warning against equating a sample zero with a population zero. Its numerical value depends on the sample size, population size, and fixed positive count.

Increasing the sample size changes this probability under the stated design. Once more than sixteen of those twenty people are sampled, a zero count is impossible because the sample cannot be filled by negative people alone. Before that point, the probability can become small while remaining positive. A study can use such probabilities to assess which population totals are consistent with a chosen error-control procedure. The procedure must be specified; the word “unlikely” alone supplies no agreed threshold or repeated-sampling guarantee.

The distinction also applies to recording models. If a population recording law has a strictly positive sensitivity bound and exactly zero positive record probability, some earlier results can force the true positive probability to zero. An observed sample containing zero records does not establish that its population recording probability is exactly zero. The sampling calculation is the missing step. Combining these two levels without that step would turn a conditional population result into an unsupported conclusion from finite data.

The library study should therefore state whether a zero refers to scheduled sessions, recorded requests, invited people, responding people, or a probability sample of the declared population. It should also state whether zero was measured directly or produced by a reporting rule. A field that defaults to zero when staff leave it blank combines missingness with a negative count. The sampling law for true zeros cannot repair that coding choice until the underlying missing values are distinguished.

10.4 Invitation, Contact, and Completed Measurement

Real data collection usually has several stages between population membership and a usable observation. This subsection treats those stages separately because a probability invitation design can be followed by selective contact or response. The initial randomization remains valuable, but the completed observations need the model that describes their actual production.

Suppose addresses are sampled randomly from an eligible frame. Some invitations fail to arrive, some reach people outside the intended unit, some recipients decline, and some completed forms omit

the question about the relevant activity. The final set of complete answers is generally a subset of the initially selected sample. Its inclusion probabilities depend on those later stages. Treating it as if it were the original simple random sample requires a justification about the relation between response and the target attribute.

The appropriate record retains a denominator for each transition. The number invited describes the design's initial reach. The number contacted describes successful delivery. The number eligible describes the target definition applied to those screened. The number consenting describes participation in the study, and the number with a usable target measure describes the final analytical coverage. Each denominator answers a different question. A high completion rate among eligible respondents can coexist with limited contact coverage of the original frame.

This decomposition also supports practical repair. If failed delivery concentrates in a known address type, improving contact methods may expand coverage. If a long instrument produces late item omissions, revising its sequence may reduce burden. If people decline because they distrust the study sponsor, shortening the instrument may leave the principal problem untouched. The data collection log should permit these rival mechanisms to be examined without attributing an unobserved motive to every absent respondent.

Participation in the study can itself depend on an interest in libraries. A volunteer who accepts an interview about public space may differ from a person who never reads the invitation. That possibility warrants a selection analysis; it does not establish the direction or magnitude of bias. Some people with strong unmet interests may respond eagerly, while others may have withdrawn after repeated disappointment. Some satisfied nonusers may be willing to discuss their alternatives. The observation process must be investigated rather than inferred from a single stereotype of the nonrespondent.

10.5 Weighting and the Variables Available for Adjustment

Weighting can align an observed sample with known population characteristics. Its interpretation depends on which inclusion probabilities or response relations the weights represent. This subsection develops that distinction without prescribing a particular weighting estimator for data that have yet to be collected.

Design weights can account for known unequal sampling probabilities. A person selected with a smaller probability represents more population units under the corresponding design calculation. Calibration can further align weighted totals with external population information. These operations can improve a study's relationship to a declared frame. They remain dependent on accurate frame information, correct implementation, and a clear target population.

An adjustment for age and residential area cannot automatically resolve a selection difference related to an unmeasured history of exclusion. Within each adjusted group, respondents and nonrespondents can still differ in the target response. The claim that the adjustment suffices is therefore a substantive conditional-independence or representativeness claim. It should be stated in relation to the actual target and measured variables. Adding more variables can help when they describe relevant selection mechanisms; it can also create unstable weights or sparse groups when the data offer little support.

The target can remain wrong even after a weighting procedure succeeds on its own terms. A representative distribution of answers to a future-use question is a valuable descriptive result. It becomes a representative distribution of behavior under a specified alternative offer only through a validated relationship between the answer and that behavior. Weighting addresses which people the answer represents. Measurement validation addresses what the answer means. The two tasks can support one another, while retaining separate assumptions.

A useful report presents an unweighted description, the role of each adjustment, the population totals used for calibration, and the remaining coverage limitations. Sensitivity analyses can then ask how conclusions change under plausible differences within adjustment groups. The report should avoid treating an adjusted point estimate as a comprehensive summary of every uncertainty layer. Sampling variability, selection assumptions, measurement error, and target interpretation remain distinguishable even when one final estimate is needed for a decision.

10.6 Population Frames and People Outside Administrative Lists

The frame is the practical list or mechanism through which a person can enter the sampling design. A library membership database, a residential address list, and recruitment through community organizations each define different routes into observation. This subsection examines their relation to the population named by an unmet-demand claim.

A membership database provides useful information about registered users. It may also include inactive members whose recent participation is zero. People who never registered have a different route into the study, if any. When the research question concerns the surrounding public, those people remain relevant even if the existing administrative system has no record for them. Their absence is a frame limitation, separate from nonresponse among people who were actually invited.

A residential frame can reach beyond registered users, but the target may include people whose relationship to the locality is more complicated than one stable residence. Workers, students, temporary residents, people with unstable accommodation, and users of services across administrative boundaries may have a legitimate interest in the facility. The study needs an explicit inclusion rule connected to its policy question. Listing every possible relationship without choosing a target would make the denominator indefinite; choosing a narrow target without disclosing its exclusions would overstate the resulting population claim.

Recruitment through organizations can improve contact with groups poorly represented in a conventional frame. It also depends on organizational membership, trust, and the organization's selection of potential participants. A combined design should retain the recruitment route and address duplicate inclusion. Counting a person once in each route can inflate apparent reach. Deduplicating without preserving route information can hide which methods made participation possible. A suitable linkage record supports both accurate counting and assessment of coverage.

The philosophical question of who belongs to the relevant public cannot be settled by selecting the easiest available list. The empirical study can make consequences of alternative boundaries visible: which people become eligible, what outcomes can be measured, and which resource claims are included. The authority to choose a policy denominator then requires public justification. A clearly defined narrow study can inform that deliberation without claiming to have represented every affected person.

10.7 Sampling a Response That the Study Helps Produce

The preceding calculations treat the binary labels as fixed during measurement. Some library questions concern a response that arises through the invitation or instrument itself. This subsection connects sampling discipline to the earlier distinction between pre-exposure disposition and actual-offer response.

If the invitation describes a previously unfamiliar activity, it may change what the recipient can imagine doing at the library. The sampled response then belongs to a history containing that description. A probability sample can still support inference about responses under this specified elicitation regime. The design's rigor is preserved when the target is stated honestly. The difficulty arises when that result is relabeled as a measure of an unchanged disposition that existed before the invitation.

Randomizing the content of an invitation can investigate its formative role. For example, one group can receive a minimal description and another a more concrete illustration, followed by a common observation opportunity. The resulting contrast concerns the two exposure histories under that common offer. It also requires the assignment and interference assumptions already specified for the exposure design. The common offer can itself form interest, so the contrast retains that final intervention in its target.

Sampling and assignment solve different parts of this design. Sampling concerns how the study population relates to a wider public. Assignment concerns comparability of exposure conditions within the study population. Measurement concerns the observed outcome after those exposures. A design can be strong on one part and weak on another. An internally credible exposure effect among consenting participants, for example, can leave its transport to people who declined the study unresolved.

The appropriate conclusion names all three boundaries. It states the people represented, the histories compared, and the response measured. This discipline gives a positive role to research on formed

interests. A library can learn whether an introduction creates valued opportunities without claiming to recover a preference distribution that existed before any introduction was made. The retrospective question remains available as a distinct research problem with its own interpretation and evidence gates.

11 Interest Formation through Experience

This section examines the temporal relation between an opportunity and the interest expressed through participation. It distinguishes a prior latent disposition from the response to an actual offer, constructs an exact counterexample to their conflation, and develops a prospective comparison that remains useful when experience changes interest. The argument also derives what marginal response rates can reveal about individual changes.

11.1 Pre-exposure States and Actual-Offer Responses

The first task is to distinguish two targets that ordinary language can both call prior demand. This subsection fixes the timing and states the empirical burden of a latent disposition. A response to an offer includes what the offer does before response; a pre-exposure state concerns the person immediately before those effects.

Let $W_t^- \in \{0, 1\}$ be a proposed latent disposition before the opportunity at time t , and W_t^+ its value after any immediate effects of that opportunity. Let G_t indicate opening, and suppose the recorded participation is

$$Y_t = G_t W_t^+. \quad (17)$$

This is a constructed dynamic model. The symbols W_t^- and W_t^+ have no automatic psychological interpretation. A substantive application needs a reason to treat the proposed state as meaningful and evidence linking it to observations. A binary model may clarify a distinction even when a continuous or qualitative account is needed for actual experience.

The demand definition in Equation 1 concerns the response to a complete actual offer. If opening changes W^- into W^+ before participation, those effects belong inside that demand target. It would be an equivocation to define demand this way and then claim that its value equals W^- merely because the offer began after W^- existed. A hypothetical offer that opens access while holding the disposition fixed would define a different, controlled response. The coherence and feasibility of that control require their own argument.

This distinction repairs an attractive but unstable way of criticizing policy-produced evidence. The claim that opening forms interest can challenge an inference about a prior disposition. It need not undermine an inference about how people respond to that very opening. The two targets can differ in identifiability while sharing the same observed event. A precise evaluation should preserve the useful prospective target and state what remains unknown about the historical one.

11.2 A Counterexample with Formation before Recording

The following two worlds demonstrate the timing issue. Their purpose is to establish a conditional identification limit through an explicit construction. They are possible models of an observation interface, not descriptions of library users.

Example 11.1 (Different initial dispositions with equal opening responses). In world A, set $W_t^- = W_t^+ = 1$ at every date. In world B, set $W_0^- = 0$ and use the update

$$W_t^+ = \max\{W_t^-, G_t\}, \quad W_{t+1}^- = W_t^+. \quad (18)$$

Both worlds record participation according to Equation 17. Thus world B's first opening forms the positive disposition before the participation measure.

Proposition 11.2 (Initial-state ambiguity under the opening interface). *For every fixed opening sequence, the observable sequences (G_t, Y_t) are equal across the two worlds and satisfy $Y_t = G_t$. The same equality holds in law for any randomized adaptive opening policy whose decisions depend only on past opening and participation records. The initial latent state W_0^- differs across the worlds. The actual-offer response to opening at the initial history equals one in both worlds.*

Proof. In world A, Equation 17 immediately gives $Y_t = G_t$. In world B, a closed date has $G_t = 0$, so its recorded response is zero. At the first open date, the maximum in Equation 18 equals one, so the response is one. Thereafter the state remains one, and again $Y_t = G_t$. This proves equality for fixed sequences.

For an adaptive policy, use the same random seed in both worlds. Equal observable histories give the same next policy decision, and the preceding calculation gives the same next response. Induction establishes equality of every finite observable history, hence equality of its law. Nevertheless, W_0^- is one in A and zero in B by construction. Finally, an actual initial opening produces response one in each world, so demand indexed to that actual opening agrees across them. $\square$

The last sentence of the proposition is as important as the ambiguity result. The counterexample identifies which claim fails. It prevents a statement about an unobserved prior disposition from being reported as a failure to identify every relevant demand target. An institution deciding whether to continue an opportunity may care about its actual response and consequences, even while the history of interest formation remains uncertain.

Several additions could distinguish the worlds. A pre-opening observation that responds differently to W_0^- would supply a new channel. A warranted lower bound on the time required for formation could permit an earlier measure. A theory excluding the update in Equation 18 would narrow the model class. Each addition needs justification. A baseline questionnaire is not automatically a neutral channel: describing an unfamiliar activity can itself create an interest or alter how a person understands the available possibilities.

11.3 Formative Exposure Followed by a Common Offer

A useful study can investigate formation prospectively without claiming to observe an untouched prior preference. This subsection defines such a target and derives its identification under simple assignment assumptions. Later sections address interference and branch-level implementation.

Let $E_i \in \{0, 1\}$ be assignment to a formative exposure, such as a clearly specified introductory experience. After the exposure period, both arms receive the same subsequent offer c . Define

$$V_i(e; c) = Y_i(c, h_i(e)), \quad \tau_c = \mathbb{E}[V_i(1; c) - V_i(0; c)]. \quad (19)$$

The history $h_i(e)$ includes the assigned exposure and the consequences included in its specification. The common offer c includes its own information, timing and possible formative effects before the response is recorded. The contrast asks how the earlier exposure changes the response to this common later opportunity.

Assumption 11.3 (Simple exposure comparison). Assignment E_i is independent of both potential responses in Equation 19; each arm has positive assignment probability. The declared offer and exposure versions are implemented consistently, observation is complete and accurate, and a person's response is unaffected by other people's exposure assignments in this simple comparison. The later branch design replaces this last restriction with a declared branch-level interference boundary.

Proposition 11.4 (Identification of a common-offer contrast). *Under the simple exposure assumptions,*

$$\mathbb{E}[V_i^{\text{obs}} \mid E_i = 1] - \mathbb{E}[V_i^{\text{obs}} \mid E_i = 0] = \tau_c. \quad (20)$$

Proof. Consistency gives $V_i^{\text{obs}} = V_i(e; c)$ in arm e . Independence then yields $\mathbb{E}[V_i(e; c) \mid E_i = e] = \mathbb{E}[V_i(e; c)]$. Applying this equality to both arms and subtracting proves Equation 20. $\square$

The result identifies an exposure effect on the specified later response. It leaves the earlier latent W_i^- unresolved unless a separate bridge is supplied. It also leaves open whether the observed effect reflects information, skill, trust, familiarity, peer relations or another component of the exposure. A more detailed mechanism study requires additional measures or interventions and their assumptions. The virtue of the design is that its primary contrast remains meaningful without pretending that the common offer merely reads a pre-existing value.

For a library application, the comparison arm should be described positively and fully. It might retain ordinary service with the same final invitation, while the exposure arm receives an earlier introductory session. The package may differ in staff contact, social attention and time as well as activity experience. Those differences belong to the implemented treatment. A later claim about a single component would require an additional comparison that separates it. Calling the treatment familiarization does not make every other effect disappear.

11.4 Marginal Rates and Individual Changes

Even a well-identified average contrast leaves the pairing of individual potential outcomes unresolved. This subsection derives the resulting bounds for the fraction whose participation would increase. The calculation prevents a positive average effect from becoming an unsupported claim that the same change benefits every person.

Suppress c and write $p_0 = \Pr(V(0) = 1)$ and $p_1 = \Pr(V(1) = 1)$. Let $\gamma = \Pr(V(0) = 0, V(1) = 1)$ be the proportion of positive individual changes and $\ell = \Pr(V(0) = 1, V(1) = 0)$ the proportion of negative changes. These are joint potential-response quantities. A parallel-arm experiment ordinarily observes one response per person under its assigned history.

Proposition 11.5 (Bounds on positive individual changes). *Given only the two binary marginals,*

$$\max\{0, p_1 - p_0\} \leq \gamma \leq \min\{1 - p_0, p_1\}, \quad p_1 - p_0 = \gamma - \ell. \quad (21)$$

The bounds are sharp over unrestricted joint distributions with those marginals.

Proof. Let $q_{ab} = \Pr(V(0) = a, V(1) = b)$. Set $q_{01} = \gamma$. The marginals and normalization require

$$q_{11} = p_1 - \gamma, \quad q_{10} = p_0 - p_1 + \gamma, \quad q_{00} = 1 - p_0 - \gamma.$$

Nonnegativity of these four cells gives the displayed lower and upper bounds. Conversely, any γ in those bounds makes all four cells nonnegative and summing to one, constructing a compatible distribution. Since $\ell = q_{10}$, subtraction gives the final identity. $\square$

Suppose, in a constructed population-law example, $p_0 = 0.3$ and $p_1 = 0.5$. The average increase is 0.2, while the positive-change fraction can range from 0.2 to 0.5. At the lower endpoint there are no negative individual changes; at the upper endpoint their fraction is 0.3. Both patterns yield the same marginal rates. An assumption that exposure never reduces anyone's response would identify $\gamma = p_1 - p_0$, but that monotonicity assumption needs substantive support. A group experience can discourage some people while attracting others.

11.5 Historical Interpretation and Practical Relevance

The section's distinctions have an interpretive consequence. A policy can reveal behavior, produce experience, and change interest in one continuous process. This subsection explains how to preserve those relations without forcing a choice between discovery and formation as mutually exclusive descriptions of an entire program.

Different people can follow different histories. One participant may have requested the activity for years; another may join after a friend attends; a third may discover that the activity differs from the initial description and decline further involvement. A population average compresses those trajectories.

Documenting their diversity can explain why a policy works for some people and what revisions would make it more suitable for others. The histories can matter normatively even when they are unnecessary for identifying the average response contrast.

The retrospective question also needs a purpose. If a library is deciding whether to continue a beneficial activity, a claim that demand existed before the pilot may be unnecessary. If the inquiry concerns whether earlier requests were ignored, historical evidence becomes central. If the question concerns whether the institution manipulated interests to secure support, the process of formation and the available alternatives require investigation. Naming the practical question prevents a technically demanding historical target from being pursued simply because the word demand appears to require it.

The constructive result is therefore a more differentiated research agenda. Actual-offer response, exposure effects, individual change patterns and prior dispositions can each become legitimate objects of inquiry. Their meanings, identification conditions and public significance differ. Keeping them distinct preserves what an intervention teaches while retaining the uncertainties that its own formative role creates.

12 Time, Delay, and Response Windows

The timing of an offer can determine whether its conditions are usable. This section examines deadlines, administrative delay, planning requirements and the window within which a response is counted. It develops one constructed application sequence and distinguishes the evidence supplied by dates from a causal claim about earlier permission. The discussion then considers repeated offers and the interpretation of persistence, postponement and withdrawal.

A policy can grant permission in its final record while leaving too little time for a person to act. A person can also postpone a response deliberately, wait for other members or change priorities before a deadline. These sequences can produce the same terminal attendance value. Preserving the temporal route helps explain what the value represents and which alternative offer the demand claim concerns.

12.1 The Timing Components of a Complete Offer

An offer includes more than the date of the activity. It can contain a notice date, an application deadline, a promised reply period, an allocation date, a confirmation deadline and an activity window. Each component can matter to participation. The study should retain the timing dimensions relevant to the proposed contrast.

A person may need time to arrange care, travel, work or companionship. A group may need to circulate materials, confirm essential roles or find a substitute. These requirements can vary among people and activities. A single rule that provides the same number of days to everyone can therefore create different practical conditions. The evaluator should investigate the relevant planning process rather than assume that a formally uniform deadline produces a uniform opportunity.

The temporal description should distinguish promised timing from actual delivery. An institution can intend to reply within a week and sometimes reply later. A participant can receive a message later than the system records sending it. A booking can be approved while details needed to use it remain unresolved. The relevant target may concern the promised policy, the implemented policy or a specified distribution of implementation delays. These are different offers.

The study need not record every timestamp with equal precision. It should identify which transitions bear on the question and preserve uncertainty where the exact date is unknown. A remembered sequence may establish that clarification arrived after a deadline while leaving the number of days uncertain. That can be sufficient for one process conclusion and insufficient for a detailed duration model. Precision should follow the evidence and target.

12.2 A Constructed Sequence of Applications and Replies

Consider a fictional meeting scheduled for day fourteen, with formal requests closing on day seven. The illustration stipulates adequate capacity and accurate process dates. Four people encounter the opportunity through different sequences. The case separates what can be established from those sequences from what remains a counterfactual response question.

Person A applies on day two. A documented care arrangement must be confirmed by day seven for attendance to be practicable. Permission arrives on day ten, and A is absent from the meeting. The record establishes an eventual grant and a reply after the specified planning deadline. Earlier permission would remove that documented timing obstacle under the stipulated arrangement. It would still leave willingness and any other conditions relevant to actual attendance to be assessed.

Person B applies on day six and receives permission that day. B attends on day fourteen. This sequence establishes actual participation under the delivered offer history. It does not establish that B held a particular latent interest before the invitation or that every earlier approval would have produced the same response. The observed event has a clear scope without those additional claims.

Person C inquires on day two and is told to await clarification before submitting a formal request. Clarification arrives on day eight, after the request deadline. C never appears in the formal application count and does not attend. The retained inquiry record establishes contact and the timing of the route. A report limited to applications would omit this part of the process. The inquiry's substantive meaning should be preserved from its content rather than inferred entirely from its date.

Person D receives an intelligible and timely offer, considers it and reports a decision to decline. The route remains open through the deadline. The record supplies evidence of a reported considered refusal under that offer. It should remain possible to investigate whether the description was understood and whether an unreported constraint mattered. The study can acknowledge the reported choice without turning D into a permanent nonuser category.

The formal record contains two applications, two grants of permission and one attendee. Those totals are compatible with the four distinct sequences. A terminal permission indicator on day fourteen codes A and B alike. It omits the difference in usable planning time. A zero-application indicator codes C and D alike. It omits the difference between a delayed inquiry route and a reported decision to decline. The sequence record recovers those distinctions while leaving the stronger causal questions open.

12.3 Eventual Permission and a Timely Opportunity

The binary gate can be defined at a single time or over a process. This subsection examines the choice needed for a delay-sensitive demand question. The aim is to prevent the gate from absorbing a temporal difference without making that difference visible.

If permission is coded only at the activity date, A in the constructed case has permission. The practical-access indicator must then include the failed planning condition. If permission is defined as a grant by the date required for the specified offer, A fails that timely-permission condition. Either formalization can be coherent if its definitions are explicit. The substantive process should remain the same across the notation.

The model becomes misleading when the analyst moves between those definitions unnoticed. A claim that everyone was permitted may refer to eventual grants, while a conclusion that permission posed no obstacle may require timely grants. The implication fails because the timing condition changed. The remedy is to state the relevant grant window and retain the dates that support its classification.

Timely opportunity also includes information. A room can be technically available while applicants lack a clear statement of its conditions before they must decide. A late clarification can change the practical offer even when the permission rule itself remains constant. The study should identify whether uncertainty concerns the existence of a route, the activity allowed, a cost or the expected response time. These distinctions guide different improvements.

A process audit can establish that a requirement and an institutional reply were temporally incompatible. It can identify where an application could not proceed under the stated sequence. Such a finding

is valuable even when counterfactual attendance remains unidentified. It can support reviewing a reply procedure or clarifying information. The public decision need not demand proof of an exact attendance gain before recognizing a documented process failure within its stated responsibilities.

12.4 Causal Interpretation of Administrative Delay

Observed delay is often associated with features of the request or applicant. This subsection explains why a comparison of late and early replies requires care. The source of the delay matters to the causal question.

Complex requests may take longer to assess and may also concern activities that are harder to organize. Staff may prioritize urgent cases, familiar applicants or requests with complete information. Applicants who can supply missing details rapidly may receive quicker replies. A difference in attendance between early and late replies can therefore reflect several processes besides the effect of elapsed time.

A process record can help identify these alternatives. It can retain when information was requested, when it arrived, why a decision remained pending and what rule governed prioritization. Such records improve the description of the sequence. They do not automatically create exchangeability between early and late cases. A causal estimate of reply timing would require a defensible design or explicit assumptions about the relevant selection process.

A prospective service change might create a clearer comparison. For example, an institution could implement a specified reply protocol through an authorized assignment design while preserving necessary review requirements. The evaluated treatment would be that protocol, including any changes in staffing or communication needed to deliver it. A finding about the protocol should not be translated into the isolated effect of one fewer day unless the design supports that narrower component claim.

The study should also examine the meaning of a faster reply. An immediate but ambiguous message can leave the practical uncertainty unchanged. A slower complete answer may provide a usable plan. The outcome should follow the information needed for action, rather than treating elapsed days as a complete measure of service quality. The timing variable acquires its interpretation through the task and decision process.

12.5 Waiting, Coordination, and Participant Choice

Participants can wait for reasons that the institution cannot observe directly. This subsection distinguishes deliberate postponement, conditional commitment and waiting imposed by an unresolved route. The differences matter to the interpretation of response time.

A person may prefer to decide after learning who else will attend. A group may wait for an essential member's schedule. Someone may be interested in several activities and choose among them near the deadline. These sequences can be part of practical agency. A short delay between invitation and request should not be automatically interpreted as weak interest.

Waiting can also be costly. Keeping an evening open may prevent another commitment, and uncertainty about a room can make coordination harder. A person can withdraw because the cost of continued uncertainty becomes too high. The eventual record may show a voluntary withdrawal, while the process includes a constraint produced by the delayed offer. The study should preserve the account of that sequence and distinguish the person's action from the conditions under which it occurred.

Group waiting introduces dependence. One person's confirmation can trigger several others, while an unresolved role can hold the whole request open. The order of replies may therefore reveal coordination without measuring independent individual propensities. A model that treats each application time as an isolated response should justify that restriction. The group-composition analysis provides a way to retain shared conditions where they matter.

The institution can make waiting more intelligible by communicating a credible decision schedule and the status of an application. That may help participants plan even when immediate approval is impossible. Its effectiveness remains an empirical question. A promised schedule that is repeatedly

missed can change expectations in later offers, connecting current delay to the history-sensitive response model.

12.6 Observation Windows and Delayed Participation

The study must choose when to count a response. This choice defines the outcome rather than merely setting an administrative reporting date. A person who participates after the initial window can change the answer to a longer-horizon question while leaving the earlier outcome unchanged.

Suppose a common offer includes a meeting on day fourteen and a second on day twenty-eight. If the target is any participation by day fourteen, attendance at the later meeting lies outside it. If the target is any participation across both meetings, the later event belongs in the outcome. The offers may also differ because the second meeting occurs after publicity, accounts from the first participants or further coordination. A longer window can include a different formative history.

Extending follow-up after seeing a low initial count can be reasonable when the research question requires a longer horizon or implementation delayed a promised opportunity. The earlier result should remain recorded. The report should identify the reason for extension and state whether the target changed. Repeatedly lengthening the window until a favorable count appears would make the stopping rule part of the result and require a corresponding analysis.

Long follow-up also increases the possibility of changing circumstances. People can move, begin different work schedules or find substitutes. A later absence may concern a different practical setting from the original offer. The population-change and history analyses help preserve those differences. A single label of nonparticipation across a long period can conceal several distinct response opportunities.

The design can predefine primary and secondary windows linked to substantive questions. A short window may concern immediate response to a credible invitation. A later window may concern persistence or a developing practice. Each needs its own offer and history description. The later result should supplement the earlier one through an explicit relation, rather than retrospectively replacing it.

12.7 Repeated Invitations and Accumulated Exposure

A repeated invitation can provide another chance to act and another occasion for formation. This subsection examines why repeated offers cannot always be treated as repeated measurements of a fixed prior demand state.

A person may ignore the first notice, understand the second and attend after a friend mentions the third. The final response belongs to that sequence. Repetition may improve awareness, establish credibility or enable coordination. It may also create irritation or pressure. The direction and significance of these effects require evidence; the model should permit them rather than assuming that each invitation simply reveals the same unchanged disposition.

The sequential-policy framework can represent the sequence through history-indexed decisions and responses. A policy that sends another invitation after silence differs from one that stops. Evaluating those policies requires support for their relevant histories and a clear outcome horizon. A probability of eventual uptake under repeated contact is a prospective policy target. It supplies no automatic estimate of interest before the first contact.

Repeated contact also changes the meaning of refusal. A person can decline one date while remaining open to another, or decline the activity and prefer no further invitations. The instrument should preserve that distinction where the service requires it. Treating every refusal as a temporary obstacle can disrespect the expressed decision; treating every date-specific refusal as permanent can close a useful route. The study should record the scope of the answer.

The resource cost of repetition belongs to the policy. Staff time spent following up can be essential to the observed response. A later rollout that omits that work may constitute a different offer sequence. Transport therefore requires a comparison of contact histories as well as activity content. A successful sequence should be described with enough detail that the institution knows what it would need to reproduce.

12.8 Temporal Evidence in a Public Decision

The timing analysis can improve a decision without reducing it to a duration statistic. This subsection identifies how the sequence record should enter the final report and which claims it can support.

The report can distinguish unresolved inquiries, submitted applications, timely decisions, late decisions, allocations, cancellations and actual attendance within each stated window. It can identify documented incompatibilities between process timing and participation requirements. It can also preserve reported reasons for waiting or declining, with the limits of those accounts. These facts provide a more precise basis for reviewing the route than a terminal permission rate alone.

A recommendation to clarify reply expectations can rest on evidence that applicants repeatedly lacked information needed to plan. A recommendation to change a deadline may require showing how the current timing interacts with the proposed activity. A claim about the resulting attendance gain needs a further causal argument. The strength of the recommendation's empirical language should match that distinction.

The constructed case remains useful at this final stage. A and C reveal different timing problems; B supplies an actual participation event; D supplies a reported considered refusal. A single policy response may help A and C while leaving D's decision appropriately unchanged. The institution can learn from the differences without treating participation by every person as the only acceptable outcome.

Time thus belongs inside the opportunity being evaluated. Eventual permission, timely access and actual response are connected but distinct. Preserving their sequence makes the offer-history target more concrete, supports proportionate process improvements and prevents a late administrative success from becoming unsupported evidence that the earlier opportunity was usable.

13 Capacity and Coordinated Participation

This section adds shared resources and interaction to the demand analysis. It first examines admission under a fixed capacity, then studies an activity that requires a minimum number of participants. The examples show how individual access, total occupancy and collective success can diverge. Each example is a conditional construction whose empirical use depends on the actual allocation and coordination mechanisms of the activity concerned.

13.1 Admission under a Fixed Capacity

A room can admit only a bounded number of people during a session. This subsection makes that constraint explicit and derives what an attendance total reveals about the number of candidates. A candidate is someone who would seek admission under the stipulated offer; this remains distinct from a person who actually receives a place.

Let I contain N people, let G_i indicate eligibility, and let Q_i indicate willingness to request admission under declared conditions. This request target differs from the actual-offer participation response D_i , which already includes the evaluated offer's allocation conditions. Define the candidate set $S = \{i : G_i Q_i = 1\}$. Let C be an integer capacity between zero and N . A stipulated priority ordering admits the first C members of S , or all members if there are fewer. Assume every admitted candidate uses the place. For actual attendance indicators Y_i , this gives

$$\sum_i Y_i = \min\{C, |S|\}. \quad (22)$$

The ordering could represent a declared booking sequence in a model. Its empirical interpretation requires evidence about how priority is actually assigned and how exceptions are handled.

One could alternatively define the candidate response through an individually guaranteed-place reference offer. That reference can clarify an individual's willingness while remaining jointly infeasible

for the entire population. The sum of such reference responses is not automatically an achievable attendance total under the capacity-limited policy. The distinction prevents allocation from being counted once inside a participation target and again inside the admission rule.

Proposition 13.1 (Saturation of an attendance total). *Within the fixed-capacity model, an observed total equal to C is compatible with every candidate count $|S|$ from C through N . An observed total below C identifies the candidate count only under the assumption that every candidate receives and uses an available place.*

Proof. For each $|S| \geq C$, Equation 22 gives the same total C . If the observed total is $m < C$, the minimum can equal m only when $|S| = m$. The latter inference uses the displayed allocation rule, including its complete admission and attendance bridge. $\square$

The proof makes a practical point about saturated records. A full session can show that at least enough candidates existed to fill it while providing little information about additional interest. A booking system that stops accepting requests once the room is full may suppress even attempted demand from its records. A waitlist can provide another channel, although joining a waitlist depends on knowing it exists and believing that a place may become available. Each additional channel has its own selection mechanism.

The converse inference also requires care. An incompletely filled room may reflect late cancellations, an inaccessible booking route, insufficient notice, or coordination failure. Those mechanisms violate the simple bridge from candidacy to realized admission. Observed empty places therefore identify low candidacy only after the relevant alternatives have been addressed. Capacity data become informative when linked to applications, allocations, cancellations and attendance, with distinct timestamps for each stage.

13.2 Individual Displacement within Aggregate Stability

Increasing eligibility can leave total occupancy unchanged while changing who participates. This subsection constructs that possibility and explains why aggregate monotonicity is weaker than a statement about each person's access.

Take two people, a and b , capacity $C = 1$, and priority order a before b . Initially only b is eligible and willing, so $S = \{b\}$ and $Y_b = 1$. Expand eligibility to include willing person a . The candidate set becomes $\{a, b\}$, the priority rule admits a , and Y_b becomes zero. Total occupancy remains one. A count-only evaluation would miss both the newly obtained place and the displaced use.

Under a fixed candidate response and fixed priority rule, enlarging S cannot reduce $\min\{C, |S|\}$. That aggregate monotonicity is a direct property of the minimum function. It leaves individual outcomes open, as the example shows. If expansion also changes room configuration, staff workload, schedules or willingness, even aggregate monotonicity requires renewed analysis. A proof about set inclusion under fixed conditions should never be extended to a policy package that changes those conditions.

The normative interpretation is also distinct. Displacement can be justified, unjustified, temporary or avoidable through another arrangement. A new priority may repair earlier exclusion, while a poorly designed reservation system may favor those who can respond quickly to announcements. The mathematics identifies the distributional question. It supplies no automatic answer about fair priority. A study should document the people and uses affected, the rule that allocates places, and the reasons offered for that rule.

13.3 Participation Requiring Other Participants

Some library activities require coordination before they can exist. This subsection models a minimum group size to show why low individual uptake can reflect the offer pattern rather than weak interest. The construction holds every person's willingness fixed and places the uncertainty in who receives an offer.

Suppose n people would each participate if at least k of them were offered the activity, where $1 \leq k \leq n$. Each independently receives an offer with probability p . Let Z_i indicate an offer and $X = \sum_i Z_i$.

The activity occurs when $X \geq k$, and every offered person then participates. Total participation is $X\mathbf{1}\{X \geq k\}$.

Proposition 13.2 (Expected coordinated participation). *Under the independent-offer model,*

$$\mathbb{E}[X\mathbf{1}\{X \geq k\}] = np \Pr\{\text{Bin}(n-1, p) \geq k-1\}. \quad (23)$$

Proof. Rewrite total participation as

$$X\mathbf{1}\{X \geq k\} = \sum_{i=1}^n Z_i \mathbf{1}\left\{\sum_{j \neq i} Z_j \geq k-1\right\}.$$

For each term, independence separates the expectation of Z_i , equal to p , from the probability that the remaining $n-1$ offers reach the threshold. Their sum has the displayed binomial distribution. Summing the n equal terms proves the formula. $\square$

For $k = 1$, the coordination condition is automatic and the expression reduces to np . For $k = n$, the activity requires every offer to be made, so expected participation is np^n . A low offer probability can then yield very little activity even though everyone in the constructed model is willing under the complete group offer. The result illustrates a mechanism through which fragmented invitations produce weak observed uptake.

Real groups usually depend on more than their size. Particular friendships, language practices, skills, relationships of trust and incompatibilities can matter. An offered set of five people may differ greatly from another set of five. The binomial example therefore performs one narrow task: it demonstrates that independent invitations and collective participation need different models. An empirical study should inspect the relevant group structure before treating anonymous group size as sufficient.

13.4 Interference and the Unit of Intervention

Capacity and coordination both make one person's outcome depend on conditions assigned to others. This subsection connects that observation to the choice of experimental unit. It states the relevant methodological antecedent without transferring a general statistical result beyond its assumptions.

Hudgens and Halloran distinguish causal effects in settings where individuals' outcomes depend on others' treatment assignments and show how inference depends on the assignment structure [4]. Their analysis provides an established antecedent for the present interference problem. A public-library study should consequently define whether the intervention concerns an individual invitation, a group composition, a room schedule or an entire branch. Each choice creates a different potential-response object.

Assigning individuals independently can be useful for some invitations, but it can also divide the very group needed for an activity. Assigning a whole branch can retain within-branch interaction inside the treatment effect while creating another burden: people may travel across branches or share information between them. The study must justify the boundary within which interference is treated as part of the package and investigate effects across that boundary. A unit is a substantive feature of the design, not merely a convenient row in a dataset.

13.5 Implications for Demand Interpretation

The section's models identify three distinct reasons that participation can be an incomplete demand measure. A restrictive gate can prevent candidacy from becoming activity. A capacity limit can compress several candidate totals into one occupancy total. A coordination rule can make individually willing people unable to realize a group activity under fragmented offers. Each mechanism suggests different supplementary evidence and different interventions.

An evaluation should therefore record application and allocation stages, measure the opportunity actually offered, and preserve the relational conditions needed for group participation. It should also report

changes in who receives scarce opportunities, since stable total use can conceal substantial redistribution. Those practices allow participation evidence to inform a public decision while retaining the differences among demand, feasibility, realization and fair allocation.

14 Joint Components of a Library Offer

A person may require both a suitable schedule and a workable booking route before a shared activity becomes feasible. Another person may be able to participate when either of two changes occurs. This section distinguishes those response patterns from the average effect of changing each component. It derives sharp bounds from four offer-specific means under an explicit individual monotonicity assumption, then examines the assumption's failure under capacity, interference and formative experience.

The distinction has established methodological antecedents. VanderWeele and Richardson analyze binary potential responses, individual monotonicity, and the relation between response interactions and statistical interaction terms. They also distinguish a counterfactual description from a stronger claim about an underlying mechanism [22]. The derivation here is a local reconstruction for the library question. It does not claim that the general distinction or the resulting statistical reasoning is newly discovered.

14.1 Two Specified Components and Four Complete Offers

Let A and B be two binary components of a complete offer. For illustration, $A = 1$ could provide an evening schedule and $B = 1$ an accessible booking route. Their zero values must also be specified: a different schedule and the existing booking route, for example. A value of zero cannot stand merely for an undefined absence of improvement. Each of the four combinations should describe a meaningful offer whose other relevant conditions are stated.

Let $Y_i(a, b)$ be unit i 's binary response to combination (a, b) after a fixed prior history. The unit can be a person under a justified interference restriction or a group with a binary activity outcome. A branch-wide change that alters peers and allocation requires a corresponding whole-policy response definition. The notation records a comparison among complete interventions; it does not identify a component independently of the setting in which it operates.

For the simple individual interpretation, changing A or B must leave the omitted background sufficiently specified. If an evening schedule also changes staffing, transport or the people likely to attend, those features either belong in the intervention version or require an argument that they are irrelevant to the target. Calling the change a schedule component does not prevent it from having several consequences. The study should define the contrast it can deliver rather than an idealized contrast that has no practical implementation.

The four potential responses share a declared prior history for the individual. They can each include formation that occurs after the corresponding offer is received. Giving the four offers successively to the same person generally produces four different histories, because earlier experiences can change later responses. A sequence of observed answers therefore does not automatically reveal the four potential responses used here. The sequential model provides the appropriate alternative when those changes matter.

14.2 Individual Monotonicity and the Response Profiles

The following assumption narrows the joint response patterns. Its content is stronger than a favorable average effect, and its plausibility depends on the actual offer components.

Assumption 14.1 (Individual component monotonicity). For every unit, changing either component from zero to one weakly increases the binary response while holding the other component fixed:

$$Y_i(1, b) \geq Y_i(0, b), \quad Y_i(a, 1) \geq Y_i(a, 0), \quad a, b \in \{0, 1\}. \quad (24)$$

The response definition and prior history remain fixed across these comparisons.

Table 2: Binary response profiles under individual component monotonicity

Response profile	$Y(00)$	$Y(10)$	$Y(01)$	$Y(11)$	Proportion
Negative under all four offers	0	0	0	0	q_0
Conjunctive response	0	0	0	1	$q_{\wedge}$
A -responsive	0	1	0	1	q_A
B -responsive	0	0	1	1	q_B
Disjunctive response	0	1	1	1	$q_{\vee}$
Positive under all four offers	1	1	1	1	q_1

For example, an easier booking route may appear likely to preserve every response that occurred under the older route. That expectation can fail if the easier route increases competition for places or changes a group’s anticipated composition. A later hour can suit some participants while excluding others. Monotonicity is therefore a conditional model restriction, not a consequence of naming a component an improvement.

Under the assumption, only six binary profiles are possible. Table 2 lists them in the order (00, 10, 01, 11). The profile labels describe responses to these four offers. A person labeled always positive in the table can refuse another activity or another history; a person labeled never positive can respond under an offer outside this comparison.

The enumeration follows directly from the restriction. A positive response at 00 forces all four responses to be positive. A negative response at 11 forces all four to be negative. In the remaining cases, 00 is negative and 11 positive, while the two middle responses can take their four possible combinations. Those possibilities give the conjunctive, A -responsive, B -responsive and disjunctive rows.

The conjunctive profile responds only when both components are present. The disjunctive profile responds when either component is present. These are useful descriptions of the intervention responses. They do not prove that a person’s psychology contains a literal conjunction or disjunction of two independent internal conditions. Different processes can produce the same four responses. Establishing a mechanism requires further evidence about those processes.

14.3 Marginal Means and Aggregate Interaction

Let $\mu_{ab} = \mathbb{E}[Y_i(a, b)]$ be the mean response under each complete offer. Under an appropriate randomized comparison, the four means can be identified for the declared population even though each person is observed under only one condition. The joint profile distribution remains a separate object.

The profile table gives the marginal identities

$$\begin{aligned}
\mu_{00} &= q_1, \\
\mu_{10} &= q_1 + q_A + q_{\vee}, \\
\mu_{01} &= q_1 + q_B + q_{\vee}, \\
\mu_{11} &= 1 - q_0.
\end{aligned} \tag{25}$$

Define the additive aggregate interaction

$$\delta = \mu_{11} - \mu_{10} - \mu_{01} + \mu_{00}. \tag{26}$$

Substituting the profile identities yields

$$\delta = q_{\wedge} - q_{\vee}. \tag{27}$$

The cancellation has a substantive interpretation. The always-positive, always-negative, and single-component profiles contribute zero to this contrast. A conjunctive response contributes positive one, while a disjunctive response contributes negative one. The aggregate interaction therefore records the

balance between those two profiles under individual monotonicity. It does not separately identify their prevalence.

An average interaction of zero can consequently coexist with substantial joint dependence at the individual-response level. Equal proportions of conjunctive and disjunctive profiles cancel in the contrast. This possibility is important for barrier interpretation: a zero interaction does not establish that two conditions operate independently for every participant. The contrast concerns an aggregate combination of potential responses, and its meaning depends on the profile model.

14.4 Sharp Bounds for Conjunctive Responses

The marginal identities leave one degree of freedom in the six-profile distribution. The next result gives its exact range and constructs every compatible distribution.

Proposition 14.2 (Conjunctive-response bounds). *Assume individual component monotonicity and compatible marginal means satisfying $0 \leq \mu_{00} \leq \min\{\mu_{10}, \mu_{01}\}$ and $\max\{\mu_{10}, \mu_{01}\} \leq \mu_{11} \leq 1$. Then the sharp probability-law interval for the conjunctive-response proportion is*

$$q_{\wedge} \in [\max\{0, \delta\}, \min\{\mu_{11} - \mu_{10}, \mu_{11} - \mu_{01}\}]. \quad (28)$$

For a finite population with integer marginal totals, retain the corresponding attainable profile-count lattice.

Proof. Write $z = q_{\wedge}$. Equation (27) gives $q_{\vee} = z - \delta$. Nonnegativity of these two weights requires $z \geq \max\{0, \delta\}$. The marginal identities give $q_A = \mu_{10} - \mu_{00} - z + \delta$ and $q_B = \mu_{01} - \mu_{00} - z + \delta$. Their nonnegativity requires $z \leq \mu_{11} - \mu_{01}$ and $z \leq \mu_{11} - \mu_{10}$ respectively. These inequalities prove necessity.

For sufficiency, choose any z in the displayed interval and define the four weights just described, together with $q_1 = \mu_{00}$ and $q_0 = 1 - \mu_{11}$. Every weight is nonnegative under the interval and marginal-order conditions. Direct addition gives total weight one, and substitution reproduces all four marginal means. Thus every retained value belongs to an admissible profile distribution. For a finite population of size N , choose Nz integer; integer marginal totals then make the reconstructed profile counts integers. Assigning the corresponding numbers of units to the profiles attains every retained lattice point. $\square$

The proof identifies where the bounds come from. The lower endpoint prevents negative conjunctive or disjunctive proportions. The upper endpoint preserves nonnegative single-component proportions. The always-positive and always-negative weights are already fixed by the extreme offer means. This explicit reconstruction is more informative than reporting an interval without showing which possible populations attain it.

The result presumes exact marginal means. A field study estimates them with sampling and assignment uncertainty. Substituting four noisy estimates into the formula produces a numerical summary but does not by itself create a confidence procedure for the profile proportion. Estimated means can even violate the model's order restrictions through sampling variation. A real analysis must distinguish evidence against monotonicity from ordinary estimation uncertainty and use an uncertainty method suited to the actual design.

14.5 A Zero-Interaction Mixture with Different Response Compositions

A constructed example makes the remaining ambiguity concrete. Let the four means in the order $(00, 10, 01, 11)$ be $(0.1, 0.4, 0.4, 0.7)$. The aggregate interaction is zero, and the conjunctive-response interval is $[0, 0.3]$. The disjunctive proportion equals the conjunctive proportion throughout that interval.

One compatible population assigns proportions 0.3 to each single-component profile, 0.1 to the always-positive profile, and 0.3 to the always-negative profile. It contains no conjunctive or disjunctive responses. A second compatible population assigns 0.3 to each of the conjunctive and disjunctive

profiles, with the same extreme-profile weights and no single-component profiles. Both populations produce exactly the four specified means.

For a finite population of ten people, the first composition contains three *A*-responsive people, three *B*-responsive people, one always-positive person and three always-negative people. The second contains three conjunctive people, three disjunctive people, and the same one and three extreme-profile people. Intermediate compositions with one or two conjunctive people also exist, paired with the same number of disjunctive people and correspondingly fewer single-component people.

The ambiguity can matter for implementation even when the four average policy responses are known. A policy maker concerned only with average uptake under one of the four offers can compare the means directly. A researcher claiming to know how many people require both components needs the profile bounds. A facilitator trying to understand why a particular group failed needs further case evidence. The same experiment can answer the first question while leaving the latter two partly unresolved.

The example also helps distinguish a barrier from its ranking. Someone with a conjunctive response may regard one condition as more salient while requiring both. Someone with a disjunctive response may identify two serious obstacles while being able to use either of two alternative solutions. Their stated comparisons can be informative about experience, but the response composition concerns the outcomes under the complete offers. The study should investigate the relation between those descriptions rather than presuming identity.

14.6 Ordered Means and Failed Individual Monotonicity

The individual restriction can fail while all observed mean comparisons look favorable. Two constructed counterexamples show why a check of marginal order cannot validate the response-profile inference.

In the first example, assign proportion 0.2 to profile $(1, 0, 0, 1)$, 0.2 to the *A*-responsive profile, 0.2 to the *B*-responsive profile, and 0.4 to the always-negative profile. The first profile violates individual monotonicity: either component alone changes a positive response to a negative one, while their joint presence restores it. The marginal means are $(0.2, 0.2, 0.2, 0.6)$, which satisfy every required aggregate order. The aggregate interaction is 0.4, yet the conjunctive profile has proportion zero. Applying the conditional lower bound would incorrectly require at least 0.4 conjunctive responses.

The second example assigns one half of the population the conjunctive profile and the other half the exclusive-or profile $(0, 1, 1, 0)$. The latter responds to either component alone but refuses their combination. The means are $(0, 0.5, 0.5, 0.5)$, again ordered. The actual conjunctive fraction is one half, while the conditional upper endpoint would be zero. This time dropping individual monotonicity defeats the upper inference.

These profiles can be interpreted as possible response patterns without inventing empirical motives. A combined offer might create an acceptable compromise that neither single change provides, or two individually useful changes might conflict when combined. Capacity, timing, group composition and institutional meaning could contribute. Establishing any such mechanism requires evidence. The counterexamples perform the narrower logical task of showing that aggregate order leaves the individual restriction unverified.

A study can investigate the plausibility of monotonicity through process evidence and carefully defined intervention components. It can also present results with and without the restriction. Where the stronger model materially narrows the conclusion, the report should make the dependence prominent. A convenient interpretation of every component as an improvement is an insufficient basis for a strong individual claim.

14.7 Factorial Assignment and the Population Represented

A four-condition comparison can identify the marginal response means under appropriate assignment assumptions. The design should specify whether it assigns people, groups, branches or time periods, because that choice determines the interference and population interpretation.

Assigning individuals independently may be unsuitable when participants need companions or share capacity. Assigning groups can preserve some coordination, but groups may differ in size and membership. Assigning branches can represent a policy package while requiring a justified boundary for cross-branch effects. The earlier population-weighted assignment analysis supplies a guide: the estimand should follow the people or systems the study intends to represent, and weighting should match that target.

All four combinations must be feasible enough to deliver. An evening schedule may require staffing that also supports the accessible booking route, making independent component variation difficult. The study could then compare complete feasible packages rather than claiming a factorial decomposition that the institution cannot implement. The mathematical attractiveness of four cells does not create four real policy options.

The common prior-history requirement also affects timing. Parallel assignment from a baseline cohort can compare offers after comparable histories under randomization. A repeated within-person sequence requires attention to learning, carryover, anticipation and changing relationships. An order that is randomized can support a sequential question, but it does not automatically restore the fixed-history profile. The research plan should choose the target before selecting a design that appears convenient.

14.8 The Decision Role of Component Analysis

Component analysis can guide a policy decision without recovering a complete individual response taxonomy. If the institution can implement the joint offer and the relevant mean outcome is credibly estimated, that mean can inform the decision. The profile analysis matters when the reasoning extends to claims about necessary combinations, barrier mechanisms, targeted provision or individual needs.

The bounds can also identify when more detailed research is useful. A wide range of conjunctive responses may motivate interviews about failed single-component offers or a study of how groups coordinate. Such work should be designed to distinguish the relevant profiles or mechanisms, not merely collect more general expressions of satisfaction. Its evidential contribution should be stated before the additional burden is imposed.

The final policy judgment still includes costs, feasibility and competing uses. A joint offer can maximize uptake while consuming resources needed elsewhere. A single component can provide a smaller attendance increase but a more accessible or sustainable opportunity. These comparisons require a public objective beyond the interaction coefficient. The coefficient and profile bounds clarify the response evidence; they do not determine which outcome the institution should prioritize.

The local contribution is therefore a disciplined bridge between barrier language and policy response. The analysis permits genuine complementarity, substitutes and offsetting patterns, while preserving the assumptions required to infer their prevalence. It also explains why a successful factorial study can leave retrospective interest and underlying mechanism questions open. Those limits sharpen the study's usefulness by preventing one well-measured contrast from carrying several unsupported meanings.

15 Group Composition and Collective Participation

An offer for a collective activity involves the composition of the group that might use it. This section examines that composition as part of the demand target and, in some cases, as an outcome of the offer. It develops a constructed case of overlapping groups, distinguishes membership from representation, and considers how changing roles and purposes affect a claim about continuity. The objective is to preserve collective conditions without treating a group label as a complete description of a social unit.

A group can submit one request while containing several people with different expectations, constraints and degrees of commitment. An institution can count that request accurately and still know little about the conditions under which the proposed activity could occur. Conversely, a set of individual expressions of interest may lack the coordination needed to become a workable group. The study should examine the relations connecting these levels before transferring evidence between them.

15.1 Proposed Membership and Actual Commitment

A membership list can serve several purposes. It can identify the people an organizer hopes to involve, those who have received an invitation, those who have agreed to a particular meeting, or those who have participated before. These are different states of the proposal. The record should identify which state the list represents and when it was established.

Suppose an organizer names six people when asking for a room. Three have agreed to the proposed time, two have expressed interest in the general activity, and one has yet to respond. The list supplies evidence of the organizer's proposed composition. It supplies three reported commitments to that time, two broader reports of interest and one unresolved response. Coding the list as six completed individual requests would silently strengthen the evidence. Coding it as a single person's private preference would erase the coordination already undertaken.

The study can preserve both levels. It can record one group request, the proposed membership, each member's known response status and the conditions the organizer regards as necessary. A later update can distinguish a confirmed booking from actual attendance. The statuses should remain time indexed because an agreement can change when the venue, cost or membership changes. A person who accepted an earlier version may need to consider the revised offer.

The cost of securing these confirmations also matters. Organizers may spend time contacting members, explaining rules and reconciling schedules before the institution receives a formal request. That work can remain invisible in the administrative record. A demand assessment that begins at submitted applications can miss attempts that ended during coordination. The relevant inquiry should preserve failed and incomplete attempts where they bear on the specified target, with clear limits on retrospective recall.

15.2 Overlapping Membership and the Unit of a Count

Consider a constructed case with two groups. Group A contains persons 1 through 6. Group B contains persons 5 through 9. The two membership lists contain eleven entries, while their union contains nine unique people. Each group requests one two-hour meeting. The numbers are stipulated to reveal the relationships among counting units.

If the meetings occur at different workable periods and every listed member attends, the result contains eleven person-event attendances among nine people. Both figures can be useful. The first describes participation occasions; the second describes reach across unique people. The two group requests supply a third count, while the four reserved room-hours supply a fourth. Each total answers a different question about the same arrangement.

Suppose the only available rooms permit the meetings at the same time. Persons 5 and 6 cannot be physically present for both full meetings. Their membership in both lists does not create two simultaneous capacities to attend. The institution and groups must consider a choice, a schedule revision, a substitute or a changed activity. A total of eleven expressions of membership would conceal this conflict if treated as an achievable simultaneous attendance count.

The target should also specify whether participation in two groups represents two valuable opportunities or repeated service to the same people for the decision at hand. A reach objective may place weight on the nine-person union. A practice-development objective may care about both group activities. A resource question may focus on room-hours or staff support. These objectives can coexist, but a report should distinguish them rather than select the count that makes a preferred arrangement appear largest.

The simple arithmetic supplies a practical audit. The analyst should be able to reconcile group requests, listed memberships, unique people, attendance occasions and resource use. A discrepancy can reveal duplicate identifiers, revised lists or a changed unit. It should be investigated before being called an error in behavior. An accurate membership count can differ legitimately from an accurate unique-person count.

15.3 Shared Members and Different Activity Outcomes

The same case can show why identical aggregate attendance totals leave group feasibility unresolved. Suppose A's initial task requires at least four participants, including a reading role supplied by person 5. Suppose B requires at least four, including a facilitation role supplied by person 6. These requirements are stipulated for the example and remain open to justified substitution or revision.

If the meetings overlap and person 5 chooses A while person 6 chooses B, A can contain persons 1 through 5 and B persons 6 through 9. Both proposed tasks pass the minimum-count and role tests conditionally. There are nine person-event attendances among nine people. Willingness, actual role performance and the other delivery conditions still require evidence.

If both shared members choose A, A has six attendees and B has three. The total remains nine attendances among nine people. B now fails both its minimum-count condition and its facilitation-role condition. The unchanged aggregate conceals the difference between two conditionally workable activities and one workable activity alongside an incomplete proposal. The relevant evidence includes the distribution of people and capacities across groups.

The example extends the coordination problem without assigning permanent value rankings to people. Person 6 matters to B's current task through a particular role. Another acceptable facilitator could change the comparison. The group might also revise its task so three members can proceed usefully. Those alternatives should be investigated. The original task and the revised activity remain distinct targets, even if both are described informally as the same group meeting.

An evaluation can therefore record both achieved attendance and task compatibility. The latter requires explicit criteria and observation. A role listed on a form may be present only nominally: a designated reader may lack access to the materials, or a facilitator may be unable to perform the agreed function. Counting the person in the room supplies evidence of presence. Evidence of the relevant contribution requires a separate account of what occurred.

15.4 Group Formation through an Offer

An offer can help create the group whose later demand it appears to reveal. This subsection connects group composition to the formation analysis while keeping the actual-offer response target intact. It concerns a change in coordination and relationships, which may occur alongside changes in individual interest.

A credible room offer can give an organizer a concrete time and place around which to contact others. People who previously supported the general idea may commit once the arrangement becomes definite. Some may invite friends, and a person initially reluctant to attend alone may join when a familiar participant agrees. The resulting group response depends on the sequence of communication and commitments. It cannot be reconstructed from isolated initial answers without a model of that sequence.

The observation may still support a clear prospective claim. Under the specified offer and recruitment process, a group formed and participated. A comparison with another assigned process can investigate how that process affects group formation or attendance. The result need not be translated into a claim that the same organized group existed in latent form before the offer. Earlier membership, individual willingness and later collective organization are distinguishable historical objects.

This distinction changes the study unit. If the intervention helps groups form, restricting analysis to groups that existed before assignment may omit part of its effect. If the analysis includes only groups that formed afterward, it selects on an outcome of the intervention. A baseline person or household cohort can sometimes retain the broader target, with group formation recorded as one result. A policy-level outcome can also count new groups under a specified counting rule. Each design needs a clear population and a rule for identifying the groups it counts.

The rule should address splits, mergers and renaming. Two meetings organized by the same person can represent one continuing group, two distinct activities, or a changing practice. A new name does not settle the matter. The study should identify the continuity features relevant to its question, such

as overlapping membership, a shared task, acknowledged succession or an ongoing commitment. The chosen criterion should remain revisable when the practice changes in ways the initial definition missed.

15.5 Membership Revision and Continuity of Purpose

Groups often change membership without simply ending. This subsection examines how a demand claim can remain intelligible through such changes and where a new target is required. The analysis treats continuity as a question supported by specified relations, not an all-purpose identity assumption.

Suppose the original six-member group loses two members after the first meeting and gains two others. The group name and venue remain unchanged. Attendance stays at six. The new members may bring capacities that improve the activity, while the departing members may have left because of scheduling, disagreement or dissatisfaction. A constant headcount describes one feature of the sequence. It leaves continuity of participation, reasons for exit and the evolving purpose open.

For a baseline-cohort effect, the original members remain the relevant people even after departure. Their later response or experience needs an interpretable target and appropriate follow-up. For an activity-level question, the continuing meetings and their changing composition may be central. For a public reach question, entrants may be a positive outcome. The population-change analysis supplies distinct calculations for these perspectives; a group narrative should preserve the corresponding distinction in words.

Purpose can change with composition. New members may prefer practical preparation to open discussion, or a departing member may have supplied the knowledge that made the earlier task possible. The group can reasonably adopt a new purpose. The evaluation should record the transition and assess the later practice on a stated basis. Describing every later meeting as fulfillment of the original demand would hide the revision.

A useful continuity record can be modest. It can state the meeting sequence, major membership changes, task revisions, who proposed them and whether the participants understood the activity as continuing. These facts do not mechanically decide continuity, but they make the judgment answerable to evidence. The record should also allow dissent: some members may regard the later practice as a replacement while others regard it as development.

15.6 Representation within a Group Request

A group request usually has a contact person, but administrative contact and authority to speak for every member are different relations. This subsection identifies the evidence needed when an evaluation attributes preferences, reasons or endorsement to a group.

An organizer can accurately describe the booking process and still be uncertain about why another member declined. A spokesperson may have authority to accept a room reservation while lacking authority to change the group's purpose or impose a fee. The study should identify the scope of the role relevant to each statement. A single interview can supply substantial information without becoming a complete account of every participant's judgment.

The practical response need not involve interviewing every member about every matter. The research can seek direct accounts where disagreement is plausible or consequential, use existing group decisions where their process is clear, and report limits where broader endorsement remains unverified. The choice should follow the claim. A statement that the organizer accepted the offer requires different evidence from a statement that all members regarded it as suitable.

Private disagreement can be difficult to express when access depends on the organizer's effort. A participant may support the activity while objecting to its timing or format. The study should provide a route for such distinctions where they matter, with appropriate care for the relationships involved. The existence of a confidential route does not guarantee complete disclosure. It supplies one condition under which a wider range of accounts may become available.

The institution also needs a workable decision process. It cannot always wait for unanimous agreement before allocating a room. The relevant public question is how the chosen procedure treats disagree-

ment and which decisions it authorizes. An evaluation can document that process, identify whose views were represented and examine the consequences. It should avoid converting a practical decision rule into evidence that every participant held the same preference.

15.7 The Wider Constituency beyond Organized Groups

Organized groups have capacities that dispersed individuals may lack. They can articulate a proposal, prepare a membership list and sustain communication with staff. This subsection considers how that difference affects the interpretation of demand evidence and the distribution of research attention.

Two equally numerous sets of interested people can generate different administrative traces if one has an organizer and the other lacks a connection through which to coordinate. The difference can matter to feasibility: an organized group may be closer to a workable activity. It can also matter to fairness if the institution treats the volume of organized requests as a complete measure of public interest. The evaluator should distinguish evidence of present organizational capacity from a population-wide distribution of possible response.

A formative intervention can include help with coordination, such as a clearly described invitation, a common meeting time or a route for finding others with a related interest. Such support becomes part of the evaluated offer. It can reveal or create possibilities for collective activity. The study should assess its resource cost, voluntariness and effect on the activity without pretending that organization is a neutral preliminary outside the intervention.

The wider constituency includes people who prefer individual use, existing groups with competing requests, people who might join later and staff who sustain the arrangement. A demand assessment focused on the proposing group can provide valuable local evidence while leaving those other claims unresolved. A public allocation decision needs a declared way to bring the relevant consequences into view. The number of signatures on one request cannot stand in for that entire process.

Attention to the wider constituency also constrains recruitment. Recruiting only through existing organizers can overrepresent people already connected to institutions. A supplementary route through the defined public population may reveal different activity ideas or constraints. The study should record which routes generated which participants and avoid interpreting a convenience network as an exhaustive map of the community.

15.8 Incentives Created by Group Counting Rules

The institution's counting and allocation rules can affect how groups present themselves. This subsection examines possible responses to those rules as part of the observation process. The examples are analytic possibilities requiring evidence in any actual setting.

If each registered group receives one reservation, participants may have a reason to create several nominal groups around closely related activity. If a minimum membership is required, organizers may include people with weak commitments to reach the threshold. If large groups receive priority, several distinct practices may be presented as one association. These responses can be legitimate adaptations, strategic presentation or administrative misunderstanding. The count alone cannot distinguish them.

The study should examine what the rule asks applicants to demonstrate and whether the evidence corresponds to the intended public purpose. A rule meant to ensure workable activity may need verified task and attendance conditions more than a nominal membership threshold. A rule meant to distribute scarce slots may need to consider overlapping membership or repeated use. The appropriate revision depends on the problem the rule is intended to solve.

The evaluation itself can create similar incentives. If success is measured by the number of newly formed groups, a program may divide one continuing practice into several reported units. If success is measured only by unique people, valuable repeated collaboration may appear less productive than one-time reach. These are possible effects of indicator selection, not reasons to abandon measurement. They are reasons to retain multiple relevant units and the process that connects them.

The chapter's contribution is a disciplined account of collective response. A group request is a social and administrative accomplishment with a membership, a history and a proposed task. Its evidential value becomes clearer when those conditions are retained. The resulting analysis can recognize organized activity, emerging coordination and disagreement without turning a group name or membership total into an unsupported claim about prior demand or public entitlement.

16 Policy Assignment and Causal Comparisons

This section develops a finite-population assignment construction for a prospective library study. It defines the intervention and population, permits interaction within branches, proves a population-weighted estimator unbiased, and examines the practical assumptions needed for interpretation. The proof uses established randomization reasoning. Its role is to make the target and the weighting transparent, while leaving uncertainty calculations dependent on the actual study design.

16.1 Branch Packages and Baseline Cohorts

A branch-level intervention can include a revised schedule, facilitated activity, information provision and a defined allocation rule. This subsection states how such a package becomes a potential-outcome object. The package must be sufficiently specified that its versions can be recognized during implementation.

Let branches be indexed by $j = 1, \dots, J$. Before assignment, define disjoint cohorts I_j of sizes $N_j > 0$, with $N = \sum_j N_j$. Assign each branch $A_j \in \{0, 1\}$, where zero and one denote two fully described packages. The package can be the formative exposure followed by a common offer from Section 11, or a different explicitly stated policy comparison. Let $V_i(a)$ denote the declared response of cohort member i when their branch receives package a .

Within a branch, participants can interact arbitrarily in this construction. Their responses may depend on the full branch package, shared capacity, facilitation and other cohort members. The simplified index $V_i(a)$ assumes that assignments to other branches have no effect on the response. If cross-branch interaction matters, the response requires a vector of branch assignments or another justified exposure description. The study should investigate this boundary before treating the simplified notation as an empirical fact.

Define the potential branch total $T_j(a) = \sum_{i \in I_j} V_i(a)$. Cohort membership precedes treatment. A person who ceases to visit remains a cohort member for this target, subject to the declared follow-up rule. If the population were defined by post-treatment visitors, the intervention could change who enters the denominator. That would produce a different comparison and complicate the interpretation of a change in average participation.

16.2 The Population-Weighted Target

The primary target in this construction gives each baseline person equal weight. This subsection states that target and separates it from a comparison that weights branches equally. Both targets can be legitimate, but their distinction matters whenever cohort sizes differ.

Definition 16.1 (Baseline-population policy contrast). The finite-population average contrast is

$$\tau = \frac{1}{N} \sum_{j=1}^J [T_j(1) - T_j(0)]. \quad (29)$$

It is the average change in the declared response over the fixed baseline population when comparing the two branch packages under the stipulated interference boundary.

Equal weighting of branch means produces

$$\tau_{\text{branch}} = \frac{1}{J} \sum_{j=1}^J \left[\frac{T_j(1)}{N_j} - \frac{T_j(0)}{N_j} \right]. \quad (30)$$

If one branch has ten cohort members and another has ninety, an effect of one for every person in the smaller cohort and zero in the larger yields $\tau = 0.1$ and $\tau_{\text{branch}} = 0.5$. The difference is arithmetical, not an estimator defect. It reflects which units receive equal importance in the target. A study intended to inform the average effect on residents should state why branch weighting, if used, answers that question.

There can also be a substantive reason to weight branches equally, such as evaluating an average administrative site. Such a target should be described in those terms. The public significance of giving equal weight to institutions or to people is a separate evaluative issue. A transparent report can present both quantities and explain their uses without allowing one to stand in for the other.

16.3 A Randomization Expectation Calculation

The estimator below uses known assignment probabilities. This subsection proves its expectation directly, including the case where assignments across branches are dependent. The calculation establishes unbiasedness under the design; it supplies no automatic guarantee of useful precision.

Let $p_j = \Pr(A_j = 1)$, with $0 < p_j < 1$ for every branch. The assignment mechanism may be balanced or otherwise dependent. Assume complete accurate observation and consistency, so

$$T_j^{\text{obs}} = A_j T_j(1) + (1 - A_j) T_j(0).$$

Consider

$$\hat{\tau} = \frac{1}{N} \sum_{j=1}^J \left[\frac{A_j T_j^{\text{obs}}}{p_j} - \frac{(1 - A_j) T_j^{\text{obs}}}{1 - p_j} \right]. \quad (31)$$

Proposition 16.2 (Design expectation of the branch estimator). *Under the fixed-cohort, consistency, complete-observation and interference assumptions above, expectation over the assignment mechanism gives $\mathbb{E}[\hat{\tau}] = \tau$.*

Proof. Because A_j is binary, $A_j T_j^{\text{obs}} = A_j T_j(1)$ and $(1 - A_j) T_j^{\text{obs}} = (1 - A_j) T_j(0)$. The potential totals are fixed within the finite-population design. Therefore

$$\mathbb{E} \left[\frac{A_j T_j^{\text{obs}}}{p_j} \right] = \frac{\mathbb{E}[A_j]}{p_j} T_j(1) = T_j(1),$$

and the corresponding control expectation equals $T_j(0)$. Linearity of expectation permits subtraction and summation regardless of dependence among the A_j . Dividing by N gives Equation 29. $\square$

The elementary proof is useful precisely because its limits are visible. Dependence among assignments leaves the expectation calculation intact but affects the estimator's variance. Few branches can yield unstable estimates, and unequal probabilities can increase variability. A real study needs a design-specific uncertainty analysis, an appropriate randomization or sampling distribution, and a feasibility assessment of the number and diversity of branches. The manuscript's finite calculations verify the displayed algebra; they cannot certify a study's power or precision before its design is chosen.

16.4 Implementation, Attrition, and Recording

Assignment concerns an intended package, while the field study observes its implementation. This subsection distinguishes the effect of assignment from stronger claims about actual receipt and explains how missing responses alter the proof.

Suppose a branch is assigned an additional session but staff illness prevents its delivery. An assignment-based comparison can still estimate the effect of being assigned the package as implemented in the study, provided the outcome definition and follow-up remain consistent. A claim about the effect of actually attending the session is different. People who attend may differ from those who decline or

whose session is canceled. Replacing assignment by receipt in the same formula requires new assumptions. The reason for retaining assignment in the primary analysis is to preserve the comparison created by the design.

Incomplete follow-up also changes the estimator's meaning. If some outcomes are unobserved, the recorded branch total may differ from $T_j(a)$. A complete-case analysis can condition on a post-assignment selection process. The coverage bounds from Section 8 provide one way to retain the missing contribution without assuming it resembles observed responses. Additional models may narrow those bounds, but their assumptions should be listed separately from the randomization itself.

Recording can respond to treatment. A staffed pilot may produce better records than ordinary service. A difference in recorded attendance would then combine a behavioral change with a measurement change. Independent observations or a common recording protocol can investigate that difference. The comparison should retain the cost and practical feasibility of the enhanced recording system, especially if the institution could not sustain it outside the study.

16.5 Cross-branch Exposure and Policy Versions

The branch boundary is a working restriction that requires scrutiny. This subsection identifies several ways it can fail and the kinds of revised target that would follow.

A participant can attend a branch in another arm, hear about the activity from a friend assigned elsewhere, or shift use because another branch becomes crowded. These pathways connect the assignments. If they matter, $T_j(a)$ should be replaced by a total indexed by a broader assignment vector or a validated exposure mapping. The additional complexity is justified when it changes the policy question. An evaluation of system-wide expansion may need to include substitution among branches, while an evaluation of one local offer may seek a narrower contrast.

Policy versions require similar attention. Two branches assigned the same package might implement different hours, publicity, facilitation or capacity. A broad package definition can deliberately include local adaptation, making the target an effect of that adaptive implementation strategy. A narrower claim about a standardized intervention needs stronger fidelity. The study should specify the permissible variation in advance and document actual variation. Otherwise the shared label can conceal different interventions whose effects are being averaged without an intelligible common object.

16.6 Interpretive Scope of Randomization

Random assignment is valuable because it creates a comparison with a known design. It does not decide which target deserves study or settle the meaning of a response. This final subsection connects the statistical construction to the paper's wider argument.

An estimated positive effect on attendance concerns the declared response window and population. Its public importance depends on what participation enables, who benefits, what it costs and which opportunities are displaced. A common-offer contrast can establish that earlier exposure changes later uptake while leaving the latent history of interest unresolved. A branch-package contrast can establish an average effect while allowing different individual changes. These are useful findings with specific limits.

The appropriate role of the experiment is to add warranted distinctions to the policy discussion. Its results should be joined to evidence about implementation, nonparticipants, adverse experiences and competing uses. The decision can then rely on a clearly described effect rather than on an unqualified claim that demand has been measured once and for all.

17 Population Change and Policy Comparison

A policy can change participation and also change which people appear in the population being measured. This section separates those changes through an exact finite decomposition. It then considers fixed baseline cohorts, people who leave the study population, and the public meaning of an eligibility boundary.

The objective is to prevent a change in the observed group from being silently interpreted as a change in the responses of the same people.

The issue extends beyond residential movement. Registration rules, recruitment routes, branch assignment, and definitions of an active user can all change membership in an analytical population. A successful activity may attract people absent from an earlier file. A restrictive arrangement may lead people to seek another institution. These possibilities are empirical questions in an actual study. The decomposition shows what can be concluded when membership and outcomes are observed accurately, before adding a causal explanation for the changes.

17.1 Repeated Populations and Their Shared Members

Let I_0 and I_1 be the finite populations measured on two occasions, with positive sizes N_0 and N_1 . Their shared members form the set $S = I_0 \cap I_1$. Leavers form $L = I_0 \setminus I_1$, and entrants form $E = I_1 \setminus I_0$. These terms describe membership in the declared study population. They need not describe residential movement or a person's relationship to the library in every other sense.

For a specified binary outcome Y_{it} , define the stayer totals $T_{S0} = \sum_{i \in S} Y_{i0}$ and $T_{S1} = \sum_{i \in S} Y_{i1}$. The observed leaver baseline total is $T_{L0} = \sum_{i \in L} Y_{i0}$, and the observed entrant follow-up total is $T_{E1} = \sum_{i \in E} Y_{i1}$. An empty group contributes zero. This use of totals avoids defining an average for an empty group. The two observed population means are

$$\mu_0 = \frac{T_{S0} + T_{L0}}{N_0}, \quad \mu_1 = \frac{T_{S1} + T_{E1}}{N_1}. \quad (32)$$

Proposition 17.1 (Membership decomposition of observed change). *The difference between the two population means satisfies*

$$\mu_1 - \mu_0 = \frac{T_{S1} - T_{S0}}{N_1} + T_{S0} \left(\frac{1}{N_1} - \frac{1}{N_0} \right) + \frac{T_{E1}}{N_1} - \frac{T_{L0}}{N_0}. \quad (33)$$

This is a descriptive identity and imposes no policy-assignment model.

Proof. Subtract the two expressions in (32). Add and subtract T_{S0}/N_1 . The first added term combines with T_{S1}/N_1 to produce the stayer-change term. The remaining occurrence of T_{S0} combines with its baseline contribution to produce the denominator term. The entrant and leaver contributions remain as displayed. Every step is an algebraic identity using the disjoint population partitions. $\square$

The four terms answer a limited explanatory question: where in the observed counts does the change occur? They separate changes among shared members, the reweighting induced by a changed population size, the follow-up contribution of entrants, and the baseline contribution of leavers. They leave open why any component changed. A policy, a local event, personal circumstances, or a recording revision could contribute. The causal question requires additional evidence and assumptions.

The representation also makes its denominator choice visible. The stayer-change term uses N_1 because of the chosen algebraic decomposition. Other exact decompositions can use another reference denominator and distribute the arithmetic interaction differently. The existence of a clean formula should not make its terms appear uniquely natural causal components. The population means themselves and the underlying group totals remain the primary descriptive facts.

17.2 Aggregate and Stayer Trends in a Constructed Population

A constructed example shows how a headline increase can coexist with a decrease among the people observed on both occasions. Suppose each occasion's population contains one hundred people. Eighty are shared members, twenty leave, and twenty enter. At baseline, thirty-two shared members participate and none of the leavers participates. At follow-up, twenty-four shared members and all twenty entrants participate.

The observed population mean rises from 0.32 to 0.44. Among the shared eighty people, the mean falls from 0.40 to 0.30. The decomposition gives a stayer contribution of -0.08 , a zero denominator-change term, an entrant contribution of 0.20, and a zero leaver contribution. Their sum is the aggregate increase of 0.12. Both descriptions are correct under their stated populations.

The example supports several possible stories but establishes none of them. A new program might attract a distinct group while becoming less suited to existing users. An unrelated population change might bring in people with greater propensity to participate. A revised registration process might make entrants easier to observe. Investigating these stories requires the histories and observation conditions behind the group totals. The arithmetic identifies where to look.

The example also constrains language about success. A report that states only that participation increased can describe the current population mean accurately. A claim that existing users became more engaged would contradict the shared-member pattern in this constructed case. A broader evaluation could judge the expansion valuable despite that decline, or judge the loss to prior users serious. Either judgment requires reasons beyond the aggregate increase.

The appropriate reporting practice presents the population boundary and a small set of group transitions when linkage is feasible. This is particularly useful when the intervention itself aims to reach previously unrecorded people. Entrants then represent an important possible outcome, not a nuisance to be removed. The analysis can describe that reach while separately assessing what happened to the baseline cohort.

17.3 The Fixed Baseline-Cohort Target

A fixed cohort avoids one kind of denominator change by retaining the same people in the target across occasions. It introduces a follow-up obligation for people who leave the later administrative population. This subsection derives the resulting finite bounds when those follow-up outcomes are missing.

Suppose the outcome at occasion one is well defined for every member of I_0 , including leavers. All baseline outcomes are known, as are the follow-up outcomes of the shared members. Write $T_0 = T_{S0} + T_{L0}$. The baseline-cohort mean change is

$$\Delta_0 = \frac{T_{S1} + \sum_{i \in L} Y_{i1} - T_0}{N_0}. \quad (34)$$

The leaver follow-up labels are the unresolved component.

Proposition 17.2 (Bounds for missing leaver follow-up). *If the n_L leaver follow-up labels are unrestricted binary values, the sharp bounds are*

$$\Delta_0 \in \left[\frac{T_{S1} - T_0}{N_0}, \frac{T_{S1} + n_L - T_0}{N_0} \right]. \quad (35)$$

The exact finite set contains the corresponding lattice points with spacing $1/N_0$.

Proof. The missing sum in (34) can take every integer from zero through n_L . Each value is attained by assigning that many leavers a positive follow-up outcome. The remaining terms are fixed. Subtracting T_0 and dividing by N_0 yields the displayed endpoints and lattice. $\square$

In the constructed example above, the baseline total is thirty-two, the shared-member follow-up total is twenty-four, and twenty follow-up labels are missing. The baseline-cohort mean change therefore ranges from -0.08 to 0.12. The observed current population increase of 0.12 is the upper endpoint of a different target's range only by the particular numbers chosen. It provides no general substitute for measuring the leavers' outcomes.

Recovering follow-up for leavers can narrow the range, provided the recovered measure matches the target. If the outcome is participation in a specified library activity, a report of participation elsewhere requires an explicit decision about whether the target includes that alternative. If the outcome is access

to a suitable discussion opportunity more broadly, the alternative may be directly relevant. The measurement should follow the declared question rather than whichever follow-up information proves easiest to obtain.

17.4 Outcome Interpretability after Leaving

The binary completion argument assumes that every missing follow-up label has a meaningful value. Some forms of leaving challenge that assumption. This subsection distinguishes an unknown outcome from an outcome whose definition has become inapplicable.

A person who moves away can still have a defined outcome such as participation in any suitable local discussion activity. The outcome of accepting a particular in-person offer at the original branch may be different: travel conditions now form part of the offer, and eligibility may have changed. The study must decide which outcome it intends. A missing value code cannot settle that substantive choice.

A person can also leave an administrative population because the definition of an active member changes. Their relationship to the library may remain otherwise similar. In that case, the outcome may be perfectly well defined even though the record disappears from the later file. Linking the files can recover the person for the fixed-cohort analysis. Treating the administrative disappearance as a negative outcome would turn a membership rule into an outcome value.

Eligibility can itself be a policy effect. A new rule may broaden or narrow who can apply for a room. A fixed baseline-cohort target can describe the effect on initially eligible people, while a current-public target can describe the newly eligible population. These are complementary questions. The study should avoid letting a policy improve its apparent mean merely by removing low-response people from the denominator without reporting that exclusion.

Some events require a different outcome structure altogether. If participation becomes structurally inapplicable, the study can preserve an explicit state representing that condition and define the policy question accordingly. Converting every such state to zero imposes a substantive ordering. Dropping every such person changes the population. Both choices can sometimes be justified, but their consequences should be visible and compared when they materially affect the conclusion.

17.5 Stayer Comparisons and Post-Policy Selection

Analyzing only shared members can appear to solve the problem of changing composition. It does preserve the identity of the people being compared. When staying is affected by the policy, however, the selected group can differ across policy conditions. This subsection explains why a stayer comparison needs its own causal interpretation.

Suppose a policy encourages some people to remain registered or return for follow-up. The observed stayers in that policy condition then include people who might have left under another condition. Selecting stayers after assignment conditions the analysis on a potential policy consequence. The original randomization does not automatically make the selected groups comparable. The fixed-cohort assignment estimand avoids this selection by retaining the baseline denominator and addressing missing outcomes directly.

The stayer analysis can still provide useful descriptive evidence. It can show how the observed shared members changed, identify experiences associated with continuation, and generate hypotheses about retention. Its label should reflect that role. A causal effect among people who would stay under either policy would be a different potential-membership target, requiring additional assumptions or bounds beyond the descriptive identity.

The distinction matters for the substantive interpretation of demand. People who remain visible after repeated difficulties may be unusually persistent, unusually well resourced, or especially committed. These are possible selection mechanisms, not established characteristics. Their possibility is enough to question an automatic extrapolation from remaining users to everyone who encountered the earlier arrangement. The study should investigate departures where feasible and preserve the uncertainty where they remain unobserved.

17.6 Entrants, Reach and Formed Participation

Entrants can represent an intended consequence of an intervention. A new offer may make the library relevant to people previously absent from its records. This subsection considers how to study that reach without confusing entry with a recovered measure of earlier demand.

A person can enter after learning that a space is available, joining a newly formed group, or acquiring an interest through an introductory event. The entry record can establish when the person became visible and what participation followed. It leaves open how much of the response would have occurred under an earlier alternative offer. A retrospective account can add evidence while retaining the history and interpretation limits already discussed.

The study can define reach through several distinct outcomes: new contacts, new requests, first participation, or sustained involvement. Each requires a different observation window and record. A high number of new contacts can coexist with low attendance if the offer remains impractical. A small number of first attendances can support a continuing practice. The selected outcome should follow the institutional purpose.

Entry can also create capacity effects for existing users. The allocation record should therefore connect reach with the distribution of available places and times. An increase in the number of distinct participants can accompany less frequent participation for a continuing group. The evaluation can describe both effects and ask whether the resulting distribution is publicly defensible. The aggregate increase alone leaves that judgment open.

17.7 The Public Boundary of a Population Claim

The population used for analysis is also part of the public meaning of the research. This subsection examines how an evaluation can state that boundary without allowing an administrative convenience to decide who matters.

An institution may have a defined service area, while its practical users and affected people extend across that boundary. A study can focus on residents when the decision concerns resident access. It can focus on a baseline applicant cohort when the decision concerns the application process. Each choice can be useful if the resulting claim names its scope.

The difficulty arises when a narrow result is presented as an account of the whole public. A study of registered adults cannot automatically settle the opportunities of unregistered young people. A study of room applicants cannot establish the absence of interest among people who never found the application route. These are population-transfer questions, separate from the accuracy of the original measurements.

A defensible report can present several related population views without collapsing them. The baseline cohort supports a stable comparison; the current eligible population describes present provision; the entrant record describes reach; and accounts from outside the frame can identify missed conditions or questions. The views remain connected through explicit membership rules and target definitions.

The broader contribution is an accountable relationship between counts and people. Population change can be a real policy consequence, a selection process, an administrative revision, or several of these together. The analysis should preserve those possibilities and investigate the mechanisms that matter for the decision under review.

18 Outcomes among Participants

An expanded offer can change both participation and the composition of the people who report an experience afterward. This section examines the resulting selection problem. It distinguishes an average among observed attendees from a comparison within a common potential-participation group, derives sharp bounds when only participation shares and bounded outcome means are retained, and considers the interpretation of outcomes that are inapplicable outside participation.

The setting is a library evaluation that asks attendees about a learning experience, usefulness, satisfaction, or another bounded feature of an event. Such information can improve understanding of the event.

Its meaning for the original public population depends on who attended, who answered, and which experience the question describes. Randomizing the offer does not automatically remove selection created after that assignment.

18.1 Principal Strata and the Common Comparison Group

Frangakis and Rubin define principal strata through joint potential values of a posttreatment variable and explain why conditioning separately on its observed value can compare different groups. Their discussion also distinguishes an unobserved outcome from one lacking an applicable value [2]. Lee develops sharp selection bounds using outcome distributions under independence and individual monotonicity of selection [11]. These are established antecedents; the present result retains only bounded means and participation shares.

The distinction is relevant when an offer brings in people who would not have attended under the comparison condition. The treated attendee mean includes those newly participating people. The control attendee mean can concern a smaller, differently composed group. Comparing the two means therefore combines a change in experience with a change in who supplies an experience to evaluate.

The analysis below uses a common potential-participation group to define a narrower causal target. That target is useful for some questions, but it is not inherently the most important public objective. An institution may reasonably care about the newly participating group or the total reach of an activity. The purpose of the common-group analysis is to prevent a selected mean from being misinterpreted as an effect on the same people.

18.2 Participation and an Applicable Experience Outcome

Compare two complete offers, indexed by $a = 0, 1$, after a declared baseline history. Let $S_i(a) \in \{0, 1\}$ indicate whether person i would participate under offer a . Let $Z_i(a) \in [0, 1]$ be an outcome meaningful when $S_i(a) = 1$, such as a bounded assessment of the experienced activity. A nonparticipant has no experienced-event score under that offer in this model. The value is inapplicable, rather than a zero waiting to be inserted.

The offer and interference environment must make these potential responses interpretable. If the policy changes the people present, that change belongs in the complete offer's consequences. A suitable group or branch model may be needed. Random assignment, positive assignment probabilities, consistency and adequate measurement are assumed for the comparison of the marginal participation and attendee outcome distributions.

Assumption 18.1 (Individual monotone participation). For each person, $S_i(1) \geq S_i(0)$. The new offer therefore preserves participation for everyone who would participate under the comparison offer, while possibly inducing additional participation.

This assumption can fail through capacity displacement, schedule changes, altered peers or a less attractive activity version. An increase in the aggregate attendance rate supplies a weaker fact. As in the component analysis, individual monotonicity needs a substantive argument and cannot be inferred merely from a favorable average.

Under monotonicity, the possible participation pairs are $(1, 1)$, $(0, 1)$ and $(0, 0)$. Call these the always-participating, induced and never-participating strata, with the labels understood relative to these two offers. Write $p_a = \Pr\{S_i(a) = 1\}$ and $m_a = \mathbb{E}[Z_i(a) \mid S_i(a) = 1]$. Assume $p_0 > 0$ and $p_1 \geq p_0$. The stratum shares are then p_0 , $p_1 - p_0$ and $1 - p_1$.

The common-group target is

$$\tau_A = \mathbb{E}[Z_i(1) - Z_i(0) \mid S_i(0) = S_i(1) = 1]. \quad (36)$$

Both experience outcomes are meaningful in this stratum. Its membership remains partly unobserved because each person receives only one offer.

18.3 The Attendee Mixture and Its Mean Bounds

All control participants belong to the always-participating stratum under the monotonicity assumption. Their observed mean therefore identifies that stratum's control mean, m_0 . Treated participants combine the always and induced strata. Let their treated means be μ_{A1} and μ_{I1} respectively. Their score totals per population unit obey

$$p_1 m_1 = p_0 \mu_{A1} + (p_1 - p_0) \mu_{I1}. \quad (37)$$

When the induced stratum has zero mass, its contribution is zero without requiring a defined conditional mean.

Proposition 18.2 (Mean-summary bounds for always-participant outcomes). *Under the stated assumptions, the sharp bounds given only the participation shares and attendee means are*

$$\begin{aligned} L_A &= \max \left\{ 0, \frac{p_1 m_1 - (p_1 - p_0)}{p_0} \right\}, \\ U_A &= \min \left\{ 1, \frac{p_1 m_1}{p_0} \right\}, \end{aligned} \quad (38)$$

for μ_{A1} . The target effect lies in $[L_A - m_0, U_A - m_0]$.

Proof. The induced-stratum mean is bounded by zero and one. Rearranging equation (37) therefore gives $(p_1 m_1 - (p_1 - p_0))/p_0 \leq \mu_{A1} \leq p_1 m_1/p_0$. Intersecting with the mean's own $[0, 1]$ range gives the displayed bounds.

For sharpness when $p_1 > p_0$, choose any retained μ_{A1} and define $\mu_{I1} = (p_1 m_1 - p_0 \mu_{A1})/(p_1 - p_0)$. The bounds guarantee that this value lies in $[0, 1]$. Outcome distributions within the two strata can be chosen to have these means, while the always-stratum control distribution has mean m_0 . This constructs every retained target value under the stated summary information. When $p_1 = p_0$, the mixture contains only the always stratum and its treated mean is m_1 . Subtracting the fixed control mean gives the effect interval. $\square$

The result is sharp relative to the information it retains. A full distribution of participant scores can restrict which values the common stratum can contain. Lee's distributional trimming analysis uses such additional information. The simple interval here should therefore be treated as a mean-summary result, not renamed as a full-distribution bound or presented as a new general theorem.

The distinction can be decisive. Suppose every treated participant has exactly the score 0.5. Every subset of those participants has the same mean, so the always-participant treated mean is also 0.5 whenever that stratum is nonempty. A calculation retaining only the overall mean and participation shares may permit a much wider interval. Discarding the individual score distribution creates that additional ambiguity; the data themselves may contain more information.

18.4 A Falling Attendee Mean with an Ambiguous Common-Group Effect

A constructed example shows the difference between the observed attendee contrast and the common-stratum target. Let $p_0 = 0.4$, $p_1 = 0.6$, $m_0 = 0.7$ and $m_1 = 0.6$. Participation rises by twenty percentage points while the average reported experience among attendees falls by one tenth.

The treated score total per population unit is 0.36, and the induced stratum has mass 0.2. The treated always-participant mean can range from $(0.36 - 0.2)/0.4 = 0.4$ to $0.36/0.4 = 0.9$. Its effect relative to the control mean therefore lies between -0.3 and 0.2 . The observed attendee-mean difference of -0.1 does not determine the sign of the effect on the common stratum.

At the lower endpoint, the induced participants have mean one and the always participants have mean 0.4. Their weighted mixture gives the observed 0.6. At the upper endpoint, the induced participants have mean zero and the always participants have mean 0.9. Their mixture gives the same observed mean. The two possible populations support very different interpretations of the experience of people who would attend under either offer.

This ambiguity should not lead the evaluator to ignore the induced participants. Their experience is part of the policy's consequence. The point is to preserve the distinction between a mixed attendee mean and a common-group effect. The public decision can consider both groups, including whether the expanded offer produces a suitable experience for people newly reached. A favorable effect on existing participants would not compensate automatically for a poor experience among newcomers, and the reverse judgment also requires explicit reasons.

For a binary positive assessment, a separate population event is well defined: participation together with a positive reported assessment. In the example, its probability rises from $0.4 \times 0.7 = 0.28$ to $0.6 \times 0.6 = 0.36$. That event can be useful for a particular evaluation purpose. It is distinct from the always-participant effect and from a general measure of welfare. Its zero value for nonparticipants follows from the event's definition, rather than an imputed experience score.

18.5 Equal Participation Shares and a Failed Selection Assumption

Equal participation rates can appear to eliminate the selection problem. Under individual monotonicity they do eliminate the induced stratum, because $p_1 - p_0 = 0$. Without that assumption, equal rates can conceal offsetting entry and exit. This subsection gives a direct counterexample.

Suppose the always, induced, leaving and never participation strata each have share one quarter. The leaving stratum has participation pair $(1, 0)$ and therefore violates monotonicity. Both observed participation shares equal one half. In one world, always participants have control outcome zero and treated outcome one; leavers have control outcome one; induced participants have treated outcome zero. Both observed attendee means equal one half, while the always-stratum effect is positive one.

In a second world, reverse those defined binary outcomes. Again both attendee means equal one half, but the always-stratum effect is negative one. The observed participation shares and means coincide across the worlds. The equality of aggregate participation rates therefore cannot justify the point-identification conclusion available under individual monotonicity.

The example is relevant when an access change alters who finds an activity suitable. Some earlier participants may leave because of congestion or format changes while new participants arrive. The resulting total can remain stable. An evaluation should investigate those transitions where feasible and preserve the uncertainty when individual potential membership remains unobserved. Describing a new rule as more inclusive does not mathematically exclude a leaving stratum.

18.6 Undefined Experience and a New Composite Target

The distinction between an inapplicable experience and a missing response deserves practical attention. A person who did not attend cannot report how useful that event was to them in the same sense as an attendee. A person who attended but skipped the questionnaire has an applicable experience whose report is missing. The two absences require different treatment.

Assigning both people a zero score would combine nonparticipation, unanswered questions and negative experience. The resulting variable can be defined, but it answers a new composite question. Its interpretation should be stated openly. If the purpose is to count the proportion of the original public who both participate and report a positive outcome, a conjunction event can be meaningful. If the purpose is to assess the quality of the experience among participants, the zero assignment would distort that target.

A continuous participation-weighted score is also possible, but its interpretation requires more care. Multiplying a bounded experience scale by participation attaches a value of zero outside the event and treats the scale's distances as comparable contributions. Those are target and measurement choices. They should not be presented as a purely technical solution to missingness or as an automatic measure of public benefit.

The study should therefore retain separate fields for participation, questionnaire applicability, questionnaire response and the reported outcome. The analysis can construct a composite when it serves an explicit question, while the original fields preserve other interpretations. This record design keeps later evaluators from having to infer whether a zero meant absence, silence or dissatisfaction.

18.7 Missing Questionnaires after Participation

Participation selection can be followed by a second observation process: some attendees answer the experience questionnaire and others do not. The experience is meaningful for both groups, but only one group's report is observed. This subsection combines the participation-mixture bound with the earlier bounded-missingness argument.

Let $R_i^q(a)$ indicate questionnaire completion among participants under offer a . Write $r_a = \Pr\{R_i^q(a) = 1 \mid S_i(a) = 1\}$ and let s_a be the respondent mean when $r_a > 0$. If the response share is zero, the observed score contribution is zero without defining a mean for the absent respondent group. The actual participant mean satisfies

$$m_a \in [\ell_a, u_a], \quad \ell_a = r_a s_a, \quad u_a = r_a s_a + 1 - r_a. \quad (39)$$

Here $r_a s_a$ denotes the observed score contribution, including its zero-response convention.

The decomposition follows by assigning the missing attendees any bounded mean between zero and one. It requires no assumption that questionnaire completion is independent of experience. A respondent who enjoyed the event may be more willing to answer, while a dissatisfied participant may be especially motivated to comment. The study should investigate such processes when possible. The interval preserves their unresolved effect when the available information consists only of shares and means.

Proposition 18.3 (Always-participant bounds with missing questionnaires). *Retain individual monotone participation, $p_0 > 0$, and the other target conditions. Given only participation shares, participant response shares and respondent means, the sharp probability-law interval for the always-participant effect is*

$$\tau_A \in \left[\max \left\{ 0, \frac{p_1 \ell_1 - (p_1 - p_0)}{p_0} \right\} - u_0, \min \left\{ 1, \frac{p_1 u_1}{p_0} \right\} - \ell_0 \right]. \quad (40)$$

Questionnaire response may depend arbitrarily on the experience and participation stratum. Additional distributional or response restrictions can narrow the interval.

Proof. The lower and upper treated always-participant endpoints in (38) are nondecreasing in m_1 . Their extremes therefore use $m_1 = \ell_1$ and $m_1 = u_1$. The control always-participant mean equals m_0 , whose extremes are ℓ_0 and u_0 . Subtracting the largest control mean from the smallest treated mean gives the lower endpoint; the reverse choices give the upper endpoint.

For attainability under the retained summaries, construct binary respondent scores with positive proportion s_a . Assign missing scores all zero or all one to attain the required participant mean endpoint. Within the treated participants, allocate the always-stratum mass first to negative-score units for its minimum, or positive-score units for its maximum. The available score masses give the same mixture endpoints as before. Questionnaire membership adds no restriction because its dependence on stratum is unrestricted. The control and treated potential marginals can be coupled within the always stratum without further cross-offer restrictions. Finally, mixtures of the two endpoint laws preserve the participation shares, response masses and observed score totals. Their always-stratum share remains the same positive p_0 , so the effect varies linearly and attains every intermediate value. $\square$

For a constructed example, let $p_0 = 0.4$, $p_1 = 0.6$, $r_0 = 0.8$, $s_0 = 0.7$, $r_1 = 0.5$ and $s_1 = 0.6$. The control participant mean lies in $[0.56, 0.76]$, while the treated participant mean lies in $[0.30, 0.80]$. The treated always-participant mean can then range over all of $[0, 1]$, and its effect lies in $[-0.76, 0.44]$. The respondent means alone would conceal much of this uncertainty if substituted directly for the full participant means.

The width can guide further work. Improving questionnaire coverage can reduce the observation uncertainty, while information about the composition of participation addresses the stratum mixture. A richer outcome distribution can also narrow what is compatible. These are different forms of evidence. The study should choose among them according to the inference it needs and the burden of collecting the information.

The result also preserves the distinction between missing and inapplicable outcomes. A missing attendee score belongs in this bounded completion argument because an experience occurred. A nonattendee's experience score remains inapplicable under the original target. Combining the two absences would erase the very selection structure the model is intended to clarify.

18.8 Outcome Meaning across Changed Offers

A common question can change its practical meaning when the offer changes. This subsection examines the measurement bridge required for comparing participant outcomes across the two conditions.

An expanded activity may introduce more demanding tasks or more ambitious purposes. Participants can judge the event against those changed expectations. A lower usefulness rating might then reflect a more demanding reference point rather than a worse experience in every relevant sense. Conversely, a simpler activity may receive a high rating while supporting a narrower purpose. The numerical response remains a real answer to the question; its evaluative interpretation requires the context.

The study can address this issue by preserving a short statement of the participant's purpose and the specific experience being rated. It can also test whether the scale is understood similarly across offer versions. These steps provide evidence for comparability; they do not guarantee it. Where meanings diverge materially, separate descriptive accounts may be more appropriate than a single pooled score.

Interest formation can also change the purpose itself. A participant may begin by seeking a room for one task and discover another valued activity through the offer. The post-event outcome then concerns the experienced sequence. A causal contrast of reported scores can be well defined while its interpretation as an effect on one stable personal goal remains doubtful. The report should name which claim it intends.

18.9 The Role of Participant Outcomes in a Demand Evaluation

Participant reports contribute evidence about what an offered activity was like for the people who encountered it. They can identify difficulties, valued features and possible improvements. Their selection conditions limit some population and causal claims while leaving this practical contribution intact.

An unmet-demand inquiry should therefore connect participant outcomes with the earlier stages of the participation route. Requests, allocation and attendance describe who reached the experience. The participant report describes an aspect of that experience. A follow-up can examine continuation and changed interest. Each stage can inform the others without replacing them.

The final report should state whether a participant-only comparison is descriptive, a bounded common-stratum effect, or part of a separately defined population outcome. This clarity is especially important when an expanded offer reaches people with different histories and needs. A changing mean can reveal a changing public, a changing experience, a changing measure, or several of these together. The proposed framework provides a disciplined way to distinguish those possibilities and to identify the evidence still needed.

19 Feedback between Access and Evidence

This section examines a policy rule that uses existing participation to decide whether an opportunity remains available. It constructs a simple feedback process, derives its possible trajectories, and studies a conditional exploratory remedy. The subsequent interpretation considers the formative effects of access, the costs of producing evidence, and the institutional conditions under which findings can actually revise a decision.

19.1 A Threshold Rule for Continued Access

The construction concerns an activity offered to a stable population. Its purpose is to show how a decision procedure can preserve an uninformative state even when additional participation would occur under access. The rule is hypothetical and carries no attribution to a named library.

Let $d \in [0, 1]$ be a fixed response proportion under an enabled offer, $g_t \in \{0, 1\}$ indicate whether the offer is enabled at time t , and $y_t = g_t d$ be perfectly recorded participation. Suppose the next period's opening is determined by a positive threshold η :

$$g_{t+1} = \mathbf{1}\{y_t \geq \eta\}, \quad 0 < \eta \leq 1. \quad (41)$$

The rule retains access only when the current participation proportion reaches the threshold. It ignores attempted participation, expressed requests, comparative offers and other evidence.

Proposition 19.1 (Closed and open trajectories). *Under Equation 41, the closed state $g_t = 0$ is absorbing for every d . An open state remains open exactly when $d \geq \eta$. If $d < \eta$, an open state becomes closed after one period and remains closed thereafter.*

Proof. When $g_t = 0$, the recorded value is $y_t = 0 < \eta$, so $g_{t+1} = 0$. Repeating the same substitution proves absorption. When $g_t = 1$, the recorded value is d , so the next gate is one precisely when $d \geq \eta$. If the next gate is zero, the first part applies. $\square$

For $d \geq \eta$, the model therefore permits two stable histories: continued opening and continued closure. The initial availability can determine which is observed. The closed history supplies no participation evidence that would distinguish a large d from a small one. A decision procedure can thus maintain its own absence of discriminating evidence. The problem lies in the combination of the restrictive arrangement, the narrow observation channel and the threshold rule.

This mechanism differs from a generic claim that policy affects behavior. It identifies a specific inferential dependency: the evidence used to authorize access can occur only after access is authorized. The circularity is practical and evidential. It can be broken by changing the evidence channel, the decision rule, or the opportunity for a bounded test. Which remedy is appropriate depends on the costs, obligations and affected uses in the actual case.

19.2 Exploratory Opening under Stable Responses

A limited exploratory rule can produce information within the fixed-response model. This subsection derives its effect and states the assumptions under which the first opening becomes informative. The calculation concerns a probability of exposure, not a general guarantee of policy improvement.

Suppose a closed activity independently receives a test opening with probability $\epsilon \in (0, 1]$ in each period. Assume complete effective access and measurement during a test. If $d \geq \eta$, the first opening reveals the fixed participation proportion and the threshold rule then retains access. For T test opportunities,

$$\Pr(\text{at least one test opening by } T) = 1 - (1 - \epsilon)^T. \quad (42)$$

Independence makes the probability of remaining closed throughout the tests the product $(1 - \epsilon)^T$; taking its complement proves the expression.

Several qualifications define the result's scope. The test must offer the relevant conditions rather than merely announce formal permission. The response proportion must remain stable through the waiting periods and the test. The measurement must detect the response. The threshold rule must act on the resulting evidence. If any condition fails, the displayed probability still describes the occurrence of a randomized opening under its own assignment assumptions, but the interpretation as successful learning or continued access requires revision.

A pilot can fail to supply a fair test if it opens at a highly inconvenient time, provides little notice, lacks a required facilitator or allocates too few places. Low uptake in that package is evidence about that

package. It does not automatically settle the response to a better-specified alternative. At the same time, every negative result should not be dismissed by inventing another missing condition after the fact. The study should specify plausible barriers, implementation checks and revision rules before interpreting its outcomes.

19.3 Exploration When Access Forms Interest

The fixed-response remedy becomes more complex when access changes interest. This subsection transfers the timing distinction from Section 11 into the feedback process. It separates the value of learning about an implemented opportunity from a retrospective claim about an initial latent state.

In the two-world construction, any first opening produces participation even though the pre-exposure dispositions differ. An exploratory policy therefore breaks the closed activity pattern while leaving the latent historical distinction unresolved. It can demonstrate a response to the actual offer and support further investigation of its consequences. It cannot, through that interface alone, determine whether an already positive disposition was uncovered or formed immediately before measurement.

This result should change the report's language. A study might find that a specified pilot generated sustained participation, with documented accounts of how participants understood the opportunity. That finding can be valuable. Describing it as proof that administrators had previously ignored a known latent demand adds a historical claim. Evidence of earlier requests, prior experiences or a suitable controlled comparison would be needed to support that stronger conclusion.

The distinction also protects participants' interpretations. Some may describe the pilot as the realization of something long desired; others may describe it as a discovery or a new possibility. An evaluation should preserve those differences. A single origin story can erase the diversity of trajectories through which a public activity becomes meaningful.

19.4 Evidence Burdens and Repeated Testing

Exploratory learning consumes resources and can impose burdens. This subsection examines how repeated demands for proof can themselves affect the opportunity being evaluated. The argument is a proposed institutional analysis whose application requires local evidence.

If a group must repeatedly demonstrate demand before receiving a stable room allocation, organizing the demonstrations may become a condition of access. People with time, organizational experience and confidence in institutional procedures may meet that condition more easily. Others may appear to show little demand because the proof requirement is difficult to satisfy. The investigation should therefore include the work needed to produce evidence, who performs it, and whether the required evidence can realistically change the allocation.

A short pilot also changes expectations. Participants may invest in an activity they believe will continue; staff may spend time building trust; a group may rearrange other commitments. Ending the pilot is then an event with consequences, even if the initial decision was reversible on paper. A study should communicate its duration and decision process clearly, preserve a route for feedback, and consider the continuity requirements of the practice. These are design obligations arising from the specific proposed intervention.

Testing can remain valuable without becoming indefinite. A protocol can state what evidence would support continuation, what would call for redesign, and what would justify ending the tested package. It can also identify questions that the pilot cannot settle. The objective is an inquiry whose findings can alter a decision, rather than a sequence of observations whose criteria move whenever an unwanted result appears.

19.5 Institutional Revision and Accountability

The feedback model becomes practically significant when connected to actual decision authority. This final subsection specifies the evidence needed to investigate that connection and the limits of a formal

diagnosis.

An empirical audit should collect the current rule, the data used to assess it, the stated decision criterion and the record of resulting action. It should examine whether a finding can trigger revision and who has standing to challenge the interpretation. A rule may formally invite feedback while providing little time, information or authority for a response to affect the outcome. Conversely, staff may already revise practice through informal learning that a formal document fails to capture. Both possibilities require observation.

The formal construction helps an investigator ask whether the opportunity needed to generate evidence is conditional on that same evidence. It leaves the actual frequency and importance of such loops open. A sound application would show the mechanism in a particular case, identify alternative evidence channels, and assess the costs and consequences of a remedy. The result would be a bounded account of institutional learning, with uncertainty and public responsibility preserved.

20 Sequential Offers and Policy Evaluation

A library often changes its offer after observing earlier participation. An introductory event may lead to a recurring group, low attendance may prompt a revised schedule, and a successful activity may receive additional resources. This section defines a prospective policy target for such sequences. It derives a finite-history evaluation identity, explains the assignment and support assumptions required by that identity, and examines the information cost of a policy whose successful history is rarely observed.

The analysis permits interest formation within the sequence. Earlier participation can change later confidence, relationships, skills, requests, and institutional expectations. The target is the outcome under a specified rule for making successive offers through that changing history. It remains different from a retrospective claim about a disposition before the sequence began. A dynamic evaluation method can accommodate formation while leaving that earlier target unresolved.

20.1 Established Methods and the Local Application

Sequential policy evaluation has established methodological antecedents. Jiang and Li present trajectory importance weighting through products of target-to-behavior action-probability ratios, discuss substantial variance over long horizons, and distinguish policy selection from evaluation [10]. Robins, Hernán, and Brumback analyze treatment histories in which changing covariates can respond to earlier treatment and influence later treatment, developing inverse-probability weighting under stated causal assumptions [16].

These antecedents constrain the present contribution. The difficulty of policy-dependent history belongs within existing causal and policy-evaluation methodology. The library application develops an explicit finite reconstruction for a particular demand question. Its contribution concerns the interpretation of the offer sequence, the observation regime, and the distinction between prospective value and earlier latent interest. The derivation supplies its own assumptions and imports no uninspected uncertainty procedure from the sources.

The finite-history formulation also avoids an unnecessary claim that a small present-state description captures everything relevant. The full recorded history can enter the policy and transition law. This is mathematically permissive and practically demanding: a larger history can make support sparse and data collection burdensome. A useful application therefore needs a justified representation of the history, with sensitivity to consequential information that the record omits.

20.2 Histories, Actions and Terminal Outcomes

Fix a finite horizon $T \geq 1$. At each stage t , the institution chooses an action A_t from a finite action set. An action can specify whether a room opens, which introduction is offered, or which allocation procedure is used. After the action, a new observation X_{t+1} is recorded. The history available before

the next action is

$$H_t = (X_0, A_0, X_1, \dots, A_{t-1}, X_t). \quad (43)$$

The initial observation X_0 includes the declared baseline information.

The analysis treats the unit as an entire interacting system, or as a cluster whose interference boundary has been justified. Within that unit, one person's participation may affect others. The history can include group formation, requests, resource availability, and the observation process. An individual-level outcome could also be studied, but its dependence on the system's actions and other participants would need to remain explicit.

Let $Y = y(H_T)$ be a terminal outcome in $[0, 1]$. It can be a normalized participation total for a fixed baseline population, an indicator that a declared activity occurred, or another bounded outcome with a specified interpretation. The terminal form is chosen for a transparent proof. It can incorporate a predeclared summary of earlier outcomes contained in the history. The value assigned to that summary remains separate from the probability calculation.

A policy is a rule for choosing actions from histories. The logging policy $b_t(a | h)$ describes how the observed actions were assigned. The target policy $\pi_t(a | h)$ describes the alternative rule to be evaluated. Both can adapt to the observed history. The word fixed in "fixed target policy" means that the rule is specified independently of the evaluation trajectories; it permits the rule to respond to new observations according to its stated instructions.

20.3 Common Transition Laws across Policies

The central comparison assumes that the two regimes share an initial population law and a history-indexed response process. This subsection states that assumption directly because it carries the substantive causal burden of the weighting result.

Let $p_0(x_0)$ be the common initial law. Let $K_t(x' | h, a)$ be the probability of the next observation after history h and action a . The kernel can include learning and formation: different earlier histories can produce different responses to the same current action. Common kernels across policies therefore permit change within the history. They require that the same specified history and action have the same transition interpretation in the two regimes.

Assumption 20.1 (Sequential regime comparison). The horizon and history spaces are finite. The logging action probabilities are known. The target policy is fixed independently of the evaluation trajectories. The two regimes share p_0 and the kernels K_t , with consistent action and outcome definitions. Every terminal history with positive target probability has positive logging probability.

Random assignment conditional on the logged history can provide a design route to the common-kernel comparison in an appropriate causal model. If staff use unrecorded information to choose actions, the observed conditional response can reflect that selection. For example, staff may offer additional support when they privately know that a group faces difficulty. The later outcome then depends on information absent from the analysis history. Treating the observed action frequency as if it arose from the declared randomization would misrepresent the assignment process.

The assumption also requires stable intervention meaning. An action labeled open may include facilitation in the logging data and omit it in the target policy. The two actions then have different versions even if their labels match. Similarly, a target policy may change staff workload so substantially that the response to a nominally identical offer differs. The history or action definition must represent the consequential difference, or the comparison requires a narrower scope.

The common initial law defines the population to which the target value applies. Evaluating a policy for a different population requires a transport argument beyond replacing the action probabilities. A model that changes both the initial public and the policy can still be studied, but the weighting identity below addresses the policy change under a common initial law. This restriction connects the sequential analysis to the population distinctions in Section 17.

20.4 The Finite Trajectory Identity

The proof compares complete trajectory probabilities. It uses the symbol Ω for the importance weight, keeping it distinct from the earlier latent disposition W . Under the regime comparison assumption, the logging and target trajectory laws are

$$P_b(h_T) = p_0(x_0) \prod_{t=0}^{T-1} b_t(a_t | h_t) K_t(x_{t+1} | h_t, a_t), \quad (44)$$

and

$$P_\pi(h_T) = p_0(x_0) \prod_{t=0}^{T-1} \pi_t(a_t | h_t) K_t(x_{t+1} | h_t, a_t). \quad (45)$$

The factors representing assignment differ; the initial and transition factors agree by assumption.

For a logging-supported trajectory, define

$$\Omega(h_T) = \prod_{t=0}^{T-1} \frac{\pi_t(a_t | h_t)}{b_t(a_t | h_t)}. \quad (46)$$

A zero target numerator gives a zero product. The support condition ensures that every target-relevant denominator is positive along the trajectories needed by the target.

Proposition 20.2 (Evaluation of a fixed sequential policy). *Under the sequential regime comparison,*

$$V(\pi) = \mathbb{E}_\pi[Y] = \mathbb{E}_b[\Omega Y]. \quad (47)$$

The result concerns the declared terminal outcome under the actual target policy history.

Proof. For any target-supported trajectory, multiply equation (44) by equation (46). Each logging action factor cancels with its denominator. The remaining expression is equation (45). A logging-supported trajectory with zero target probability contributes zero after weighting. The support assumption excludes a target-supported trajectory missing from the logging law. Summing $y(h_T)P_b(h_T)\Omega(h_T)$ over the finite history space therefore equals the sum of $y(h_T)P_\pi(h_T)$, which is the target expectation. $\square$

For two fixed supported policies, subtracting their identities gives a difference in prospective values. The outcome definition and population must remain comparable across the targets. A numerical difference in differently defined outcomes would have no automatic policy interpretation. The proof's simplicity makes this conceptual work especially important: cancellation can be exact while the objects being compared are poorly chosen.

20.5 Formation within a Shared History Process

The transition model can represent the formation of interest without treating that formation as an observation error. This subsection explains the resulting target and the retrospective limit that remains.

Suppose an initial introduction helps participants understand a possible group activity. A first meeting creates relationships, and those relationships affect requests for a second meeting. The policy then allocates a later slot according to those requests. A history containing the introduction and first meeting can have a different next-response distribution from a history without them. The kernels K_t permit exactly that difference.

The target policy value asks what outcome the specified rule would produce through these successive changes. It does not require interests to remain fixed. It does require the response process under each history and action to transfer appropriately between the logging and target regimes. An intervention that changes the meaning of the institution in an unrepresented way can challenge that transfer.

Latent states can exist within the modeled process. A person's confidence or prior disposition may affect response even when it is not directly recorded. The formal assumption is about the observable

trajectory laws under the specified assignment regimes. Random assignment can support a comparison without measuring every latent cause, provided the relevant design and consistency conditions hold. An observational assignment that selects on those latent causes requires additional justification.

The earlier nonidentification example still applies to retrospective disposition. Two worlds can yield the same observed responses after each opening while differing in the latent state before the first opening. The policy-value identity can agree in those worlds because its target concerns the actual sequence. Thus success in prospective evaluation supplies no automatic resolution of the distinct historical question.

20.6 Support and the Policies the Data Can Reach

The support condition determines which policy histories the logging data can inform through the stated identity. This subsection relates that condition to a library that repeatedly makes future access depend on past recorded activity.

If a closed state always remains closed in the logging process, a target policy that opens from that state can place positive probability on histories absent from the data. The required action-probability ratio then has a zero logging denominator. The weighting identity cannot recover the missing response. The failure reflects an absence of relevant support, not a numerical inconvenience that can be repaired by replacing zero with an arbitrary small number.

Exploratory offers can create support. A logging rule that occasionally opens after low recorded activity can observe histories excluded by the deterministic closure rule. The opportunity has practical effects, including possible interest formation, costs and displacement. Those effects belong to the study's public justification. Statistical support alone does not authorize every exploratory action.

Local positivity should be assessed at target-reachable histories. It is unnecessary to demand every action at every logically imaginable history if the target policy could never reach that history. Conversely, an action observed somewhere in the dataset may remain unsupported at the history where the target needs it. Opening a room after a successful group formed does not by itself reveal opening's effect before any group exists.

The distinction guides the design of exploration. The study should identify which missing histories matter for the policy question and whether they can be reached through feasible, appropriate offers. A general desire for more variation can create burden without resolving the relevant gap. Targeted exploration links the intervention to an explicit identification need.

20.7 Long Horizons and Rare Supported Sequences

Positive support can coexist with very limited information. The following constructed example derives that difference exactly. It concerns an action sequence requiring uninterrupted opening; random state transitions can still occur within that action event.

Suppose the logging policy independently opens at each of T stages with probability $\epsilon \in (0, 1]$. The target policy always opens. Let the terminal success outcome be one exactly when every action opens and zero otherwise. This success rule is stipulated for the example. A real practice's dependence on uninterrupted access would require evidence.

The all-open action event has logging probability ϵ^T . Every logging-supported trajectory in that event has importance weight ϵ^{-T} , while a trajectory containing a closed action receives zero target weight. The weighted observation is therefore

$$Z = \Omega Y = \epsilon^{-T} \mathbf{1}\{\text{every action opens}\}. \quad (48)$$

Proposition 20.3 (Variance for the constructed rare sequence). *The weighted observation satisfies*

$$\mathbb{E}_b[Z] = 1, \quad \text{Var}_b(Z) = \epsilon^{-T} - 1. \quad (49)$$

For n independent identically distributed whole-system trajectories, the variance of their sample mean is $(\epsilon^{-T} - 1)/n$.

Proof. The nonzero value ϵ^{-T} occurs with probability ϵ^T , giving expectation one. Its square has expectation $\epsilon^{-2T}\epsilon^T = \epsilon^{-T}$. Subtracting the squared mean gives the variance. For independent copies, covariance terms vanish when computing the variance of the average, producing the division by n . $\square$

With $\epsilon = 1/2$ and $T = 6$, the successful action sequence occurs with probability $1/64$, and the weighted observation has variance 63 . The target value is exactly one in the example, yet many logging trajectories produce a zero weighted observation. A rare successful sequence receives the large weight needed for the expectation to remain correct.

The example establishes a possible information cost of a long required sequence under sparse exploration. It does not establish the sample size needed for a library study or the optimal exploration probability. Those questions depend on the actual outcome process, dependence among systems, policy costs, and the decision's precision requirements. A formally unbiased estimator can remain practically unhelpful with the available data.

Independent whole-system trajectories are a demanding resource. Six successive periods in one library do not constitute six independent systems when each period changes the next history. Overlapping windows share participants, resources and prior events. Treating them as independent replicates would understate the relevant dependence. A real uncertainty analysis must use the actual assignment and observation structure.

20.8 Selecting a Policy from the Same Evaluation Record

The fixed-target requirement has a simple counterexample. This subsection shows why a policy selected after observing an evaluation outcome needs a different argument from the fixed-policy identity.

Consider one fair binary action A , with logging probabilities one half for each action. Let the outcome be one under either action. After observing A , choose a deterministic target policy that always selects the action just observed. Every selected policy has true value one. Yet reusing the same observation in the fixed-policy formula assigns weight $1/(1/2) = 2$, so the weighted outcome is always two.

The contradiction is informative because the outcome process is otherwise trivial. The error comes from using the same action realization to choose the target and evaluate it while retaining the original unconditional comparison. The fixed-target proof no longer describes that operation. A data-adaptive evaluation can be developed, but it needs its own assumptions and analysis.

An independent training observation provides one transparent repair in this example. Use it to choose the target, then evaluate on a fresh fair action. Conditional on the selected policy, the new action matches with probability one half. The weighted outcome is two on a match and zero otherwise, giving conditional expectation one. The example demonstrates the role of separation; it does not establish sample splitting as the only possible method.

For the library study, this concern arises when staff select the apparently best sequence after reviewing the same outcomes used to report its expected success. The selection can be sensible for learning, while the resulting estimate requires correction for its evaluation role. A staged design can use exploratory data to propose a policy and later data to assess that fixed proposal. The study should state which stage supports which claim.

20.9 Policy Learning and Public Responsibility

The sequential analysis concludes with the practical interpretation of a learning policy. A rule can adapt to evidence while remaining explicit about the people, outcomes and values guiding its adaptation. That explicitness is necessary for both causal interpretation and public scrutiny.

A policy that allocates future access only where recent attendance is high serves a particular learning rule. It can overlook groups whose formation requires continuity before attendance becomes substantial. A policy that continues every experiment indefinitely can consume scarce resources and displace established uses. Comparing such rules requires a horizon, an outcome, a feasible action set, and a value account. The trajectory method addresses the outcome distribution conditional on those choices.

The institution should also state how participants can understand and challenge the rule. A temporary offer can create expectations, relationships and organizational work. Ending it affects more than a final attendance count. These consequences can enter the outcome record or remain explicit considerations in the decision discussion. Their importance should not depend on whether they fit one scalar value function.

The resulting methodological position is constructive. A library can evaluate and improve sequences of opportunities while allowing interests to develop through them. It can distinguish supported policy histories from unsupported extrapolations, identify when exploration creates useful evidence, and recognize when that evidence remains too sparse for a precise conclusion. The earlier latent-demand question retains its separate status throughout.

21 Transport across Library Populations and Settings

A response identified in one study can inform another setting only through a comparison between the two targets. This section makes that comparison explicit. It derives a finite-stratum transport calculation, allows bounded differences between source and target responses, and examines unsupported target groups. The discussion then considers formative recruitment, peer conditions and the resource changes that accompany expansion.

Conditional transfer and standardization have established methodological antecedents. Dahabreh and colleagues separate identification within a randomized study from extending potential-outcome means to a new population. Their sufficient conditions include conditional mean comparability, positivity and consistency; source conditional means are then averaged over the target covariate distribution [1]. The present construction gives an elementary finite-stratum application with explicit discrepancy allowances. Its library-specific argument concerns the offer and history bridges needed before that calculation has the intended meaning.

21.1 The Source Response and the Target Response

The source population is the population for which the study identifies a response under a specified offer. The target population is the population to which a later claim applies. The two populations can differ in location, recruitment, prior exposure, relationships and available alternatives. Naming these differences begins the transport argument; it leaves their relevance to the response open to investigation.

The outcome should retain the same practical question. A source estimate of attending one supported discussion under a guaranteed place cannot be transferred directly into a target prediction of attending a recurring program with limited capacity. The activity, horizon and allocation have changed. A useful comparison can still be constructed, but it must define the target offer and state which source evidence bears on its response.

History is part of this comparison. Source participants may have encountered the activity through recruitment interviews, a formative demonstration or earlier meetings. Target residents may encounter the eventual offer through an ordinary notice. These histories can affect comprehension, confidence, trust and coordination. A transport claim that conditions only on a few demographic categories can leave these consequential differences unresolved.

The target also needs an interpretable population boundary. A claim about the next cohort at the same branch differs from a claim about all residents, all potential groups or all branches in a system. Each boundary implies a distribution of relevant conditions and a denominator. A convenient source sample does not determine the appropriate public target by itself.

21.2 Finite Strata and Conditional Comparability

The following notation separates the population weights from the response comparison. Let X take values in a finite shared stratum vocabulary. A stratum can include specified prior history and partici-

pation conditions. Let $q_x = \Pr_T(X = x)$ be the target population weights, treated initially as known, nonnegative and summing to one. Subscripts S and T refer to source and target populations.

Fix a reference offer a that is interpretable in both populations. The binary potential response Y^a includes any formation produced by that actual offer. Define $\mu_T(x) = E_T[Y^a \mid X = x]$, and let O be the supported strata in which the source conditional mean $\mu_S(x) = E_S[Y^a \mid X = x]$ is identified. Identification in the source requires its own assignment, recording and interference assumptions.

The target is $\theta_T = E_T[Y^a]$. Partitioning its expectation gives

$$\theta_T = \sum_x q_x \mu_T(x). \quad (50)$$

This equation is an identity inside the target population. It contains no source information until a bridge connects $\mu_T(x)$ to $\mu_S(x)$. Replacing the target mean by the source mean is the substantive transport step.

Exact conditional comparability would assert $\mu_T(x) = \mu_S(x)$ on supported strata. Matching the recorded stratum labels supplies a necessary descriptive alignment for that assertion; it does not establish equality of responses. Two people with the same recorded history category can face different actual costs, peer conditions or interpretations of the offer. The study should explain why the retained strata are adequate for the particular transfer being proposed.

The comparison need not require every person in a stratum to respond identically across populations. It concerns conditional means for the stated target. That weaker requirement can still be demanding. A mean equality can fail when the composition of unrecorded response-relevant conditions changes within a stratum. The analysis should identify plausible sources of such change before relying on the equality.

21.3 Bounds Allowing Conditional Differences

A sensitivity model can retain uncertainty about conditional comparability. For each supported stratum, posit a discrepancy allowance $\eta_x \in [0, 1]$ satisfying

$$|\mu_T(x) - \mu_S(x)| \leq \eta_x.$$

Outside O , leave the target mean unrestricted in $[0, 1]$. The allowances describe exact restrictions in this model. They are not estimated confidence limits or empirically established tolerances merely because the analyst assigns numerical values.

Define

$$\ell_x = \max\{0, \mu_S(x) - \eta_x\}, \quad u_x = \min\{1, \mu_S(x) + \eta_x\} \quad (x \in O).$$

Assume that every combination of stratum means in the permitted intervals is jointly admissible. This rectangular assumption excludes additional shared-capacity or cross-stratum restrictions from the current model. Such restrictions may be relevant in a library system and would need explicit representation.

Proposition 21.1 (Transport bounds with unsupported target strata). *With known target weights, identified source means on O , the stated discrepancy allowances and joint admissibility, the sharp probability-model interval is*

$$\theta_T \in \left[\sum_{x \in O} q_x \ell_x, \sum_{x \in O} q_x u_x + \sum_{x \notin O} q_x \right]. \quad (51)$$

Under exact conditional comparability on supported strata, the remaining interval width equals the unsupported target mass. If that mass is zero, the interval reduces to the standardization value $\sum_x q_x \mu_S(x)$.

Proof. Equation (50) expresses the target as a weighted sum with nonnegative weights. For each supported stratum, intersecting the discrepancy restriction with the binary-response range gives $[\ell_x, u_x]$. Unsupported means remain between zero and one. The smallest weighted sum assigns every supported mean its lower endpoint and every unsupported mean zero. The largest assigns every supported mean its upper endpoint and every unsupported mean one.

The joint-admissibility assumption permits both assignments. Within each target stratum, construct a Bernoulli potential response with the chosen mean and mix the strata with weights q_x . These probability laws attain the endpoints while preserving the retained source information and restrictions. Convex mixtures of the endpoint target laws preserve the weights and all conditional intervals, and vary the target mean linearly. Every intermediate value is therefore attainable in this probability model. Setting every supported allowance to zero leaves only the unsupported contribution; its maximum minus minimum is its population mass. $\square$

The probability-model statement should be distinguished from exact finite-population completion. A finite target with fixed person counts can impose a lattice and additional compatibility conditions. The endpoint derivation remains useful, but unrestricted probability-mixture attainment should not be described as proof that every real-valued point is attainable by a fixed finite list of people. The target domain matters to the meaning of sharpness.

21.4 A Constructed Three-Stratum Transfer

Consider a target population with stratum weights 0.5, 0.3 and 0.2. The source identifies response means 0.2 and 0.6 in the first two strata. It contains no identified response for the third. These are stipulated probability quantities for an example, not estimates from actual libraries.

Under exact conditional comparability in the supported strata, the known target contribution is

$$0.5(0.2) + 0.3(0.6) = 0.28.$$

The unsupported third stratum contributes between zero and 0.2. The target response therefore lies in $[0.28, 0.48]$. The source can provide exact information about its two strata while leaving a substantial target range because the target contains a group outside that support.

Now allow conditional discrepancies of 0.1 in the first stratum and 0.2 in the second. The permitted target means become $[0.1, 0.3]$ and $[0.4, 0.8]$. The lower target endpoint is $0.5(0.1) + 0.3(0.4) = 0.17$. The upper is $0.5(0.3) + 0.3(0.8) + 0.2 = 0.59$. Thus the interval becomes $[0.17, 0.59]$.

The widening records weaker comparison information. It is not caused by a new unfavorable observation. The analyst has allowed the supported target strata to differ from the source in ways the initial exact-transfer assumption excluded. A report should state the substantive reasons for considering those allowances and show how the conclusion changes across them.

The source's overall mean can be different from every value relevant to the target comparison. Suppose the source population places weight 0.25 on its first stratum and 0.75 on its second. Its overall response is 0.50. That number uses a different composition. Under exact supported transfer, the target range remains $[0.28, 0.48]$. Copying the source average would therefore ignore both the changed weights and the unsupported target group.

This example also clarifies what additional research could accomplish. Better measurement of target weights would address uncertainty in composition, which the illustration initially treats as known. Observing a credible offer in the third stratum could reduce unsupported mass. Comparative evidence about the first two strata could justify narrower discrepancy allowances. These are different research tasks; collecting more source observations in an already well-measured stratum may leave the central transfer uncertainty largely unchanged.

21.5 Recruitment and Research Participation in the Transfer

The source study can itself change the response it measures. This possibility matters when a target policy will be delivered without the same research process. The transport argument should identify whether recruitment, questioning and repeated contact belong to the source offer, its history or an additional effect requiring a separate assumption.

Suppose source participants first discuss possible activities in an interview. They then receive a demonstration and finally the common offer. Target residents later receive only the final invitation.

Even when the invitation text is identical, the histories differ. Source participants may have developed a vocabulary for their interests, learned how the activity works or formed connections with others. Those changes can affect response under the invitation.

The verified methodological antecedent makes clear that consistency and research-participation conditions matter to transfer. In the library application, an additional assumption that participation in the study has no effect beyond the assigned offer requires evidence. It should not be inherited silently when the study's own purpose includes elicitation and formation. The present model can instead define the complete source and target histories or retain a discrepancy allowance for their unresolved difference.

One practical approach is to compare delivery versions explicitly. A later study can investigate the full research-supported package and a version intended for routine service. Another can use a recruitment procedure that more closely matches the target implementation. Either approach changes the research design and may have costs. The important point is that the difference becomes a named object of inquiry rather than an invisible qualification to the transported estimate.

Baseline questions deserve the same attention. A questionnaire about possible discussion activities may prompt consideration that had not previously occurred. The response to a later offer can then depend partly on that question. A study can still identify a useful target including the questionnaire history. It should avoid claiming that the source represents an unprompted target population unless that bridge is supported.

21.6 Stratum Definitions and Post-Offer Selection

The stratum vocabulary can improve or undermine a transport comparison. This subsection examines the timing and interpretation of the variables used to align populations. A useful label must refer to a sufficiently comparable condition at the relevant point in the causal sequence.

Prior participation can be a baseline history variable when measured before the evaluated offer. Participation after the offer is an outcome or a post-offer condition. Stratifying on the latter can change the target and select different people under different policies. The study should identify the time at which every proposed transfer variable is defined and explain its role.

Reported confidence can be especially ambiguous. A baseline confidence item may capture an existing self-assessment, an effect of prior institutional contact, or a response to the questionnaire's framing. A later confidence item may be affected by the formative exposure. Treating all such items as the same stable characteristic would conceal changes in both history and measurement. The instrument version and timing should accompany the stratum definition.

Very detailed strata can improve descriptive comparability while reducing source support. A source study may contain few or no people with a particular combination of history, schedule, language and peer conditions. A coarser classification restores apparent overlap by combining those people with others, but it may require a stronger within-stratum comparability assumption. The tradeoff should be assessed substantively. A larger overlap percentage is not automatically a better transport design.

The target weights also require evidence. An administrative population count may cover registered members while the target claim concerns all residents. A survey may estimate the distribution with nonresponse and measurement uncertainty. The initial formula treats q_x as known to isolate the transfer step. An empirical application must carry uncertainty about those weights or state a narrower target for which the distribution is adequately established.

21.7 Peer Conditions and Expansion of the Offer

Transport can fail because the intervention changes when more people receive it. This subsection connects the population calculation to capacity and interference. It distinguishes a comparable individual reference response from the attendance total under an expanded joint policy.

A pilot may guarantee space for every invited person because its cohort is small. A system-wide offer may face a capacity constraint, waiting lists or competition between groups. The sum of individual responses to hypothetical guaranteed places can then exceed attendance that the system can jointly

accommodate. The reference-response target remains interpretable, but it should not be reported as an achievable rollout total without an allocation model.

Peer composition can also change. A pilot may bring together people with a shared interest or an established organizer. A broader invitation may involve strangers with different purposes, or may enable new groups to form. The interaction can improve or reduce participation. The complete target offer should state which peer process it includes. Conditioning on a demographic stratum leaves that process unresolved unless the model supplies a justified bridge.

Staffing can create another difference. An initial study may provide intensive facilitation, rapid responses to questions and flexible booking. Routine provision may offer a different level of support. A transported estimate for the original package says little about the reduced-support version unless the response comparison is investigated. The institution should identify which components it can sustain before treating the pilot as evidence for scale.

The rectangular sharpness assumption in (51) is particularly important here. If one stratum's participation uses capacity that another stratum needs, their means may be jointly restricted. Setting every stratum to its upper endpoint can describe individually possible responses that cannot occur together under the joint policy. The analyst should either retain the reference-offer interpretation or add the shared constraint and derive the appropriate joint set. An attractive upper total should not erase that distinction.

21.8 Transport of Effects and Public Conclusions

The construction has focused on a response under one offer. A causal effect compares two offer responses, and an evaluative conclusion compares consequences against public purposes. This subsection identifies the additional steps required for those broader claims.

Transporting an effect requires interpretable source and target versions of both offers. Exact equality of an average effect can sometimes be weaker than equality of both response means, but it is a different assumption and should be stated directly. Applying separate mean bounds and subtracting endpoints gives a conservative contrast; calling it sharp requires the retained joint admissibility of the two response systems. Shared monotonicity or other cross-offer restrictions can change the attainable set.

The target effect can also differ from a judgment about provision. A transported attendance gain leaves the quality of the activity, distribution of opportunities, displaced uses and cost open. Evidence about those outcomes may require different transport bridges. A setting can have similar attendance responses and very different resource consequences. The public conclusion should identify which elements have been transferred and which remain local judgments.

A useful transport report therefore contains a comparison dossier rather than only a weighted estimate. It names the source and target populations, the complete offers and histories, the source identification conditions, the shared strata, the unsupported target mass and the evidence for conditional comparability. It then presents the calculation with its uncertainty and the public scope of the resulting claim.

The method permits a productive restricted conclusion. A source study can support an expectation for a defined family of target settings while leaving other populations or delivery versions open. The restriction is informative when it follows from explicit conditions and evidence. It becomes evasive only if the target is repeatedly narrowed to preserve a desired result without independent justification. Retaining the comparison history keeps transport answerable to both the source study and the population for which the policy is being considered.

22 Outcome Categories and Historical Comparison

This section examines what an evaluation can compare when its descriptive categories change. It distinguishes a revised label from a new activity, constructs a compatibility bridge between old and new records, and derives finite bounds on a recoded outcome. The discussion then considers participant language and archival practices needed to preserve meaningful revision.

22.1 Category Revision within an Evaluation

An evaluation may begin with categories such as study, discussion and program attendance, then encounter activity that cuts across them. This subsection explains why a revised vocabulary can improve inquiry while complicating a comparison over time.

A group might describe its activity as mutual learning, while the initial recording system places it under conversation. The difference can concern a new practice, a previously overlooked feature of an existing practice, a change in institutional recognition, or a change in terminology. Those possibilities require evidence. Renaming an entry cannot by itself establish that a new kind of social phenomenon has come into existence. Conversely, retaining an old label can conceal meaningful changes in purpose and organization.

Reporting categories also change for practical reasons. The IMLS official survey repository records a FY2026 proposal revising or removing several program-modality and attendance fields [5]. This is a documented measurement revision. The example supports the importance of retaining instrument versions; it supplies no automatic equivalence between old and new fields and no proof of newly formed demand.

A cross-period comparison should consequently specify whether it concerns a stable activity under revised labels, a stable evaluative purpose measured by different instruments, or a changing practice that requires a new target. These are different comparison tasks. A single percentage-change figure can be misleading if its numerator is defined differently in the two periods. The appropriate response may be a bridge, a restricted common category, a range of possible comparisons, or an explicit decision to leave some records uncomparing.

22.2 A Finite Compatibility Bridge

The following construction describes one limited form of recoding uncertainty. It begins with old-category counts and a justified set of possible new labels for each old category. The assumptions make explicit what is required to call the resulting bounds sharp.

Let Z be the finite old category set and Z' the new set. A retained archive contains n_z records with old label z , where $N = \sum_z n_z > 0$. Let $B(z) \subseteq Z'$ be the nonempty set of new labels compatible with a record known only to have old label z . Let $T \subseteq Z'$ be the new target category or group of categories. Compatibility is an interpretive claim justified by the category definitions and retained evidence; the notation alone never establishes it.

Assumption 22.1 (Independent record compatibility). Every option in $B(z)$ is admissible for each record with old label z , and compatible choices for different records can be made jointly without additional cross-record restrictions.

Proposition 22.2 (Bounds under a category bridge). *Under independent record compatibility, the fraction assigned to T has the sharp lower and upper bounds*

$$L = \frac{1}{N} \sum_z n_z \mathbf{1}\{B(z) \subseteq T\}, \quad (52)$$

$$U = \frac{1}{N} \sum_z n_z \mathbf{1}\{B(z) \cap T \neq \emptyset\}. \quad (53)$$

The finite attainable set respects the record-count lattice.

Proof. If $B(z) \subseteq T$, every admissible recoding counts the record. If $B(z) \cap T = \emptyset$, every recoding excludes it. Each remaining record has at least one option in T and at least one outside T . Adding the forced positives gives the lower bound; adding every possible positive gives the upper bound. To attain the lower endpoint, choose a non-target option for each flexible record. To attain the upper endpoint, choose a target option for each flexible record. The compatibility assumption ensures that these record-level choices are jointly admissible. $\square$

Suppose a constructed archive contains twenty records definitely classifiable as the new target, thirty definitely outside it, and fifty whose old labels permit either classification. The target fraction lies between 0.2 and 0.7. Assigning every ambiguous record to the target because the new category is considered valuable would replace evidence with a preference for one endpoint. The bridge requires the ambiguity to remain visible until additional information justifies a narrower assignment.

22.3 Refinement through Richer Records

Retaining descriptions richer than the original category can improve later comparison. This subsection derives the direction of that improvement and identifies its evidential condition. More detailed records help only when they validly restrict the compatible interpretations.

Suppose new evidence yields nonempty sets $B'(z) \subseteq B(z)$ for the same records and target. A previously forced positive remains forced, because every newly permitted label was already a target label. A previously impossible positive remains impossible, because refinement adds no target option. Thus $L' \geq L$ and $U' \leq U$. The interval narrows weakly under a valid refinement. A refinement may leave it unchanged if the removed options are irrelevant to membership in T .

The construction can be applied at the individual record level when records contain different details. An activity description may identify collaborative problem solving even when the old category says only meeting. A time-stamped field note may distinguish a planned program from spontaneous use. Preserving those descriptions allows future evaluators to revisit the classification. The raw record itself still requires interpretation and can contain errors; retention expands the available evidence rather than guaranteeing a final answer.

An empty compatibility set signals an unresolved comparison. Silently dropping such records changes the denominator and can bias the apparent trajectory. The analyst should preserve an unresolved category, investigate the mismatch, or state the narrower population to which the comparison applies. If recodings are coupled, such as when several records must be assigned consistently to a single underlying event, the independent-choice proof also requires revision. The endpoints may remain valid bounds while ceasing to be jointly attainable.

22.4 Participant Language and Interpretive Authority

Categories can be generated through the practices they describe. This subsection considers how an evaluation can incorporate participant language without treating every newly proposed label as a verified analytical finding.

Participants may identify distinctions that an administrative form misses. They can explain why an activity counted as casual conversation involved mutual support, interpretation of a local issue, or collective learning. Such accounts are evidence about participants' descriptions and meanings. Their relationship to other accounts, observed practices and evaluative judgments requires inquiry. The evaluation should preserve the original wording, identify who used it, and record how it influenced the category revision.

Staff may also supply important distinctions concerning resource use, accessibility, preparation and conflict. An adequate category process should permit these perspectives to confront each other. A participant-defined category can be insightful while overlooking costs borne by others. An administrative category can be operationally useful while concealing the purpose of a practice. The proposed method treats classification as a contestable part of inquiry, with reasons and revision history preserved.

22.5 Comparison and Evaluation

A category bridge connects descriptions. It leaves the value assigned to those descriptions open. This final subsection connects that distinction to the broader demand question.

An increase in records classified as community discussion may reflect more events, broader recording coverage, a revised coding rule, or some combination. Even a verified increase leaves questions

about quality, voluntariness, distribution and displaced uses. An evaluator should first establish which descriptive comparison is warranted, then state the evaluative premises that give it policy significance. Combining those stages into a single favorable indicator would obscure both the evidence and the judgment.

The archive should therefore retain original records, instrument versions, coding decisions, bridge assumptions and unresolved cases. This creates a history through which later inquiry can correct interpretations without erasing earlier evidence. Such preservation supports institutional learning because revised categories can improve understanding while remaining accountable to the records and people from which they were developed.

23 Linked Events and the Meaning of Archive Counts

Historical comparison requires a decision about what a record represents. This section develops the compatibility analysis when several records describe the same event. It distinguishes event counts from record counts, derives exact bounds under a coupled classification constraint, and follows the result through a constructed archive. The later discussion examines uncertain linking, conflicting descriptions and the institutional incentives that can change the density of a record without changing the underlying activity.

The issue arises before any judgment about value. A booking, a revised booking, a staff note, an attendance sheet and a participant account can concern one meeting. Treating them as five observations of five meetings changes the unit. Treating the five descriptions as interchangeable can lose the different evidence each supplies. A useful archive preserves both the event relation and the distinct epistemic roles of the records.

23.1 Events, Episodes, Persons, and Records

The first step is to name the unit that the comparison intends to count. An event can be a scheduled meeting. An episode can be a bounded period of activity within that meeting. A person can participate in several episodes or events. A record can describe any of these units, sometimes incompletely. These relationships should be represented explicitly before totals are combined.

Suppose a two-hour meeting contains a discussion of historical materials and a later planning conversation. An evaluator interested in meetings may count one event. An evaluator interested in forms of activity may distinguish two episodes. A participant register may contain six people, while an administrative system contains three entries because the booking changed twice. Each count can be accurate for its stated unit. The danger appears when a conclusion about one unit is inferred from another without a bridge.

The unit should follow the question rather than the convenience of the largest available table. A room-utilization question may need occupied time. A participation question may need unique people and the conditions of their involvement. A continuity question may need a sequence of meetings connected to a practice. An unmet-demand question may need responses to specified offers among a defined population. A raw number of database rows acquires one of these meanings only through the recording process and a justified relation to the target.

Administrative precision can disguise this problem. A system may assign exact timestamps and identifiers to every update. Those identifiers establish distinct entries in the system. They do not establish distinct activities outside it. Conversely, several activities can be summarized in one entry. The archive should retain the level at which an identifier is unique and the rule used to connect it to the activity being studied.

23.2 Known Event Links and a Coupled Category Constraint

The following model adds two kinds of information to the earlier independent-record bridge. First, the archive identifies which records belong to each event. Second, separate evidence fixes the number of

ambiguous events that belong to the target category. The result concerns the consequences of that information; its premises require empirical justification.

Let there be $J > 0$ distinct events. Event j has m_j records, where every m_j is a positive integer and $N = \sum_j m_j$. The event partition is known and correct. The target definition is fixed. Write $X_j \in \{0, 1\}$ for membership of event j in that target. Every record attached to an event inherits the same event-level target membership for this particular analysis. A record can retain additional categories for its own content.

Partition the events into forced-positive events P , forced-outside events O , and ambiguous events A . Events in P satisfy $X_j = 1$; events in O satisfy $X_j = 0$. Put $a = |A|$. Suppose reliable additional evidence establishes that exactly k ambiguous events belong to the target, with $0 \leq k \leq a$. The identities of those events remain unresolved.

Assumption 23.1 (Admissible event subsets). Every subset of k events in A is jointly compatible with the retained evidence. The multiplicities and event links are correct, and there are no additional restrictions connecting ambiguous membership to record multiplicity.

The assumption is stronger than saying that each ambiguous event might individually belong to the target. The exact-count condition couples the choices: assigning one event to the target reduces the number that can be selected from the rest. The admissible-subset statement describes the remaining joint possibilities after that coupling is imposed.

Let $M_P = \sum_{j \in P} m_j$, and order the ambiguous multiplicities as $m_{(1)} \leq \dots \leq m_{(a)}$. Empty sums equal zero. The event proportion and the record-weighted proportion are

$$\theta_{\text{event}} = \frac{1}{J} \sum_j X_j, \quad \theta_{\text{record}} = \frac{1}{N} \sum_j m_j X_j.$$

The second quantity is explicitly weighted by the number of records attached to an event. Its interpretation differs from the first even when both use the same target category.

Proposition 23.2 (Bounds with a fixed number of target events). *Under the stated event links and admissible-subset assumption,*

$$\theta_{\text{event}} = \frac{|P| + k}{J}. \quad (54)$$

The sharp endpoints for the record-weighted proportion are

$$L_{\text{record}} = \frac{M_P + \sum_{r=1}^k m_{(r)}}{N}, \quad (55)$$

$$U_{\text{record}} = \frac{M_P + \sum_{r=a-k+1}^a m_{(r)}}{N}. \quad (56)$$

The exact finite set consists of $(M_P + \sum_{j \in S} m_j)/N$ for all subsets $S \subseteq A$ with $|S| = k$. It can contain gaps between the endpoints.

Proof. Every admissible classification contains the $|P|$ forced-positive events and exactly k selected ambiguous events. This fixes the numerator in (54). The positive record total equals the fixed contribution M_P plus the multiplicities of the selected ambiguous events. Hence the stated subset sums give the exact finite set.

To minimize a selected sum, replace a selected multiplicity by a smaller unselected multiplicity whenever such a pair exists. Each exchange preserves the number selected and weakly decreases the sum. Repeating the exchanges reaches the k smallest multiplicities. The admissible-subset assumption makes that selection feasible. The symmetric exchange toward larger unselected multiplicities reaches the k largest values and maximizes the total. Division by the same positive N gives both endpoints. The empty-sum convention handles $k = 0$, while $k = a$ selects every ambiguous event. $\square$

The result is an elementary counting consequence. Its contribution here is to expose the unit and joint-classification assumptions that an archive comparison must carry. A narrower record interval supplies stronger information only if the event links and exact-count evidence are warranted. Formal sharpness cannot compensate for an incorrect link.

23.3 A Constructed Archive with Unequal Record Density

Consider four ambiguous meetings with respectively one, two, four and seven records. There are fourteen records in total, no forced-positive or forced-outside events, and independent evidence establishes that exactly two meetings belong to the target category. All six possible pairs remain compatible with that evidence. These numbers are constructed to display the counting problem.

The event proportion is exactly one half. The possible positive record totals are the six pair sums: three, five, eight, six, nine and eleven. Sorting them yields

$$\theta_{\text{record}} \in \left\{ \frac{3}{14}, \frac{5}{14}, \frac{6}{14}, \frac{8}{14}, \frac{9}{14}, \frac{11}{14} \right\}.$$

The endpoint interval runs from $3/14$ to $11/14$. Values such as $4/14$ and $7/14$ lie between those endpoints yet remain unattainable under the retained information. Reporting the interval is permissible as a summary if the exact finite limitation is stated.

The wide record range coexists with complete knowledge of the event proportion. This is possible because the unresolved question concerns which events generated more records. If the two sparsely documented meetings belong to the target, record prevalence is low. If the two heavily documented meetings belong to it, record prevalence is high. The uncertainty concerns the association between event category and recording density; it leaves the known count of target events unchanged.

Suppose the seven-record meeting generated several booking revisions and two staff notes about setup. Those additional entries can be valuable evidence about process. They do not supply seven independent meetings or seven independent participants. A report that describes the archive as containing eleven target activities in the upper-endpoint world would silently change record multiplicity into activity frequency. The corrected statement is that eleven records concern two target meetings.

The same distinction affects a comparison over time. If a later system creates a new record whenever a booking changes, the number of records can rise while the number of meetings remains fixed. If the target meetings require more changes than other meetings, their record-weighted share can rise as well. This is a possible data-generating mechanism, not an allegation about any actual system. The investigator should examine record creation rules before interpreting the rise as expanded activity.

23.4 Evidence for the Event Partition

The formal result conditions on correct links. A practical study must state how those links were established and which uncertainties remain. This subsection separates administrative matching from substantive identity of events.

A shared booking identifier can support a link among revisions to one reservation. A common date, room and organizer can support a candidate link between a staff note and an attendance sheet. Those fields can also recur across separate meetings. Matching should therefore use the available combination of fields and the purpose of the relation. A link that is sufficiently reliable for locating a file may require further checking before it determines a count of distinct events.

The study should distinguish definite links, definite separations and unresolved candidate links. Definite links can be retained with their supporting fields. Definite separations need a reason when superficial features overlap, such as two activities using the same room consecutively. Unresolved candidates should remain visible. Silently choosing the most convenient partition can create false precision in both the numerator and denominator.

An uncertain partition can change J , the multiplicities and the category evidence simultaneously. Two records might describe one meeting with mixed activity or two meetings with different purposes.

The formula above can be applied conditionally to each defensible partition, but the resulting conditional answers need a further comparison. A union of supported possibilities may be appropriate for a deliberately conservative report. Selecting a partition because it produces a desired prevalence would reverse the evidential direction.

There are practical ways to improve the link evidence. The archive can preserve the original event identifier when a record is corrected, record whether a new entry replaces or supplements an earlier one, and retain a reason for merges or splits. Participants and staff can help clarify ambiguous sequences. Their accounts should be recorded as evidence for a link, with the same attention to uncertainty used for other testimony. An authoritative role in the institution does not automatically make a recollection an error-free identifier.

23.5 Conflicting Descriptions of One Event

Linking records can reveal disagreement that row-level counting conceals. One meeting may be described as discussion in a booking, instruction in a staff note, and mutual learning in a participant account. The link establishes that the descriptions concern the same event; it leaves their relationship to the target category open.

The descriptions may address different aspects. A booking category can concern the permitted use of a room. A staff description can concern the organization of speaking turns. A participant description can concern the experienced purpose. These categories need not be mutually exclusive. Forcing a single label can erase the distinction between administrative function, interaction form and participant meaning.

A target that genuinely requires one event-level binary classification should state the rule connecting those aspects to membership. If the target is any meeting involving collaborative interpretation, a permitted-use label of instruction may leave membership ambiguous. If the target is a formally registered library program, a participant's account of learning may establish something valuable while leaving formal registration unchanged. The analyst should preserve that difference in the compatibility set.

Disagreement can also concern the same aspect. Participants may disagree about whether the meeting allowed reciprocal discussion. A staff note may report a largely one-directional presentation, while an organizer describes it as participatory. The study can compare the accounts with retained observations, speaking opportunities and the intended task. The disagreement itself belongs in the record. Its resolution should depend on a justified interpretation of the target, not a fixed rule that one class of speaker always has final authority.

The event-level model does not require every attached record to use the same descriptive vocabulary. It requires consistency only in the derived event-membership variable used for the specified analysis. Maintaining that distinction allows the archive to support several legitimate future questions without pretending that a single classification exhausts the event.

23.6 Category Revision and the Direction of Inquiry

A new category can emerge because researchers learn something important from these conflicting records. The revision should begin with the interpretive problem and its evidence, then state what the new category distinguishes. Its value depends on the work it performs in the inquiry.

Suppose repeated cases reveal activities that combine participant-led reading with institutional support. The current categories may force a choice between informal use and organized program. A revised category could preserve the joint arrangement more accurately. The study should explain which features the new distinction captures, how those features are observed, and whether older records contain enough information to recover them. A useful new concept can coexist with limited retrospective comparability.

The revision can also change the evaluative question. A category initially used to count occupied rooms may become a measure of collective learning. That transition requires a new bridge from observed activity to the claimed outcome. Retaining the same variable name would conceal the change. The archive should preserve the earlier purpose and identify the point at which the variable acquired a different interpretation.

Participant language is especially valuable when it identifies a feature absent from the original instrument. Its preservation should include the context of use, the people who endorsed or contested the description, and the analyst's later translation into a category. A polished label can otherwise appear to have been a shared understanding throughout the activity when it was proposed only afterward. Historical accuracy requires keeping the sequence.

The same care applies to abandoning a category. A category may prove too heterogeneous for the target, redundant with a better distinction, or dependent on information the study cannot obtain. Recording that reason allows later researchers to understand the change. Deleting the category and recoding every earlier event into a new scheme without a bridge would obscure the limitation that prompted the revision.

23.7 Recording Incentives and Evidential Independence

Record density can respond to institutional attention. An activity under review may receive more notes, more follow-up questions and more carefully preserved correspondence than routine activity. This can improve knowledge of that activity while making an unadjusted record count unsuitable for comparing prevalence across categories.

The extra records are often dependent. Several entries can repeat one organizer's account, and a later report can summarize earlier notes. Counting agreement among those entries as independent corroboration would overstate the evidence. The study should trace whether records arise from separate observations, shared sources or administrative copying. Agreement can still be useful, but its evidential role depends on that relationship.

An institution can also create fewer records for an activity that falls outside recognized categories. Requests may be handled informally, inquiries may end before an application, or participants may lack a route to describe the proposed use. Those possibilities connect the archive analysis to access-evidence feedback. A low count can reflect the recording route as well as the activity itself. Investigating the route is therefore part of interpreting the count.

The appropriate archival aim is traceable sufficiency. The study should preserve enough information to identify the unit, reconstruct material decisions and revisit consequential classifications. More records can improve that capacity, but an unrestricted accumulation of entries is an unreliable proxy for it. A concise event record with explicit links and a reasoned classification can be more useful for a target than many disconnected repetitions.

The section's formal and practical results converge on a specific reporting rule. Name the unit, retain the links, state the classification constraint, and distinguish exact possibilities from an endpoint summary. These steps make historical comparison answerable to the archive that supports it. They also preserve room for conceptual revision without allowing a revised vocabulary or a denser record to become evidence of unmet demand by itself.

24 A Prospective Library Study

This section turns the conceptual and formal distinctions into a prospective research design. It specifies a bounded activity, a documentary stage, a population assessment, an opportunity audit, and possible intervention comparisons. The design is a proposal. Its implementation requires a library partner, appropriate research review, a feasible sampling frame and an agreement about how participants and staff can influence the inquiry. No fieldwork or empirical findings are claimed here.

24.1 Activity Selection and the Decision Question

The study should begin with an actual decision that a library expects to make. This subsection proposes a bounded example and explains why the decision context must precede instrument selection.

A suitable candidate would be the allocation of scheduled space for a particular form of small-group learning or discussion. The study would name the branches, population, activity, times and relevant alternative uses. It would identify the decision under review: adding a session, changing a booking procedure, offering facilitation, revising a group-size requirement, or some other specified modification. Combining all possible service improvements in one package could make the initial study hard to interpret. The choice should be guided by the actual institutional question and the conditions that users and staff regard as consequential.

The proposal should distinguish three initial claims. First, an access claim states which conditions currently prevent or facilitate participation. Second, a demand claim states the response expected under a specified alternative. Third, a decision claim states why that response would matter relative to costs, competing uses and institutional purposes. Each claim receives its own evidence requirements. An administrator may have a strong reason to preserve quiet study even if a discussion activity would attract participants; a group may have a strong reason to request discussion space even if aggregate room use is already high.

24.2 Documentary and Implementation Audit

The first empirical stage reconstructs the current opportunity and its record production. This subsection specifies the documents and observations needed to distinguish a published rule from actual access.

The research team would collect the relevant policy versions, public instructions, booking interfaces, schedules, capacity limits and staff procedures. It would trace how a person learns about the activity, requests a place, receives a response, and reaches the event. The audit would record the point at which a failed attempt leaves a trace or disappears from the administrative system. It would also distinguish library decisions from conditions beyond the library's immediate control, such as transport or a caregiver's availability, while examining possible interaction between them.

Implementation evidence would include observations of selected request and allocation processes, interviews with staff responsible for different stages, and appropriately consented participant accounts. The aim would be to identify the ordinary procedure and meaningful variations. A staff member's account could clarify a written rule while remaining one perspective on implementation. A user's difficult experience could identify a possible barrier while leaving its prevalence unresolved. The study would preserve both the detail of the case and the population question it generates.

To investigate the central inferential concern, the audit would also seek actual decision records. These might show how attendance, requests, complaints or waiting lists were interpreted. The research team would identify the target claimed by the decision-maker and the evidence offered for it. An inference would be assessed against its own stated purpose. A record used simply to allocate staff for next week's known bookings should receive a different assessment from the same record used to dismiss the possibility of an untried activity.

24.3 Population Coverage and Recruitment

The second stage defines who is represented in the study. This subsection proposes a combination of a stable cohort and supplementary outreach, with their targets reported separately.

A baseline cohort should be defined before any intervention assignment. Its frame might be a geographically specified resident sample or another justified population associated with the selected branches. Recruitment should include people who do not currently visit, since a visitor-only sample conditions on existing access. The study would document language coverage, accessibility of contact routes, nonresponse, exclusions and uncertain eligibility. Where a complete frame is infeasible, the report should state the directly observed population and retain transport to a wider public as a separate assumption.

Supplementary outreach can reach people whose relationship to the library is poorly represented in the main frame. It may provide valuable accounts of barriers and unanticipated uses even when its inclusion probabilities are unknown. Such evidence should be analyzed for the questions it can answer:

identifying a mechanism, refining an offer, or documenting a particular experience. Combining it with a probability sample through an unexplained pooled percentage would obscure both sources. A joint report can preserve qualitative and quantitative contributions without forcing them into one denominator.

Recruitment itself is an exposure. An invitation to discuss a possible library activity may make the activity salient or communicate that the institution welcomes participation. The protocol should record the information supplied at recruitment and keep it consistent where required by the comparison. A claim about people entirely unexposed to that information would then require a different design. The study population can still be informative, provided the target includes the recruitment conditions that actually occurred.

24.4 Baseline Description and Opportunity Assessment

Before assigning a new package, the study should measure the relevant existing conditions. This subsection identifies the baseline information needed to interpret later differences while avoiding a presumption that baseline questions neutrally reveal a fixed preference.

The baseline instrument would distinguish prior participation, known opportunities, attempted access, reasons for declining particular offers, desired activities and uncertainty about what a proposed activity involves. It would preserve respondents' own descriptions before presenting a closed list where feasible. The instrument should record whether an answer refers to a present intention, a conditional response or a public recommendation. Different forms of uncertainty should remain available as legitimate answers.

The opportunity audit would separately record formal eligibility, practical opening hours, communication, booking routes, available places, actual allocation, cancellations and required supporting conditions. Staff resources would be included explicitly. These measures would allow a zero event to be examined against several possible mechanisms rather than automatically classified as refusal. The baseline would also document uses that could be displaced by the proposed modification, including people who would prefer the current arrangement.

Repeated baseline observation could reveal routine temporal variation. The appropriate duration depends on the activity's schedule and local conditions; the present paper supplies no numerical duration or sample size without a site-specific feasibility assessment. The protocol should specify why the chosen window represents the decision's relevant conditions and which seasonal or exceptional circumstances remain outside it.

24.5 A Formative-Exposure Comparison

Where feasible, one comparison would assign a formative experience before a common later offer. This subsection specifies its purpose, implementation and interpretive limit.

The exposure might be a bounded introductory session that allows people to understand a group activity, meet a facilitator and try its basic form. The comparison condition would retain a specified alternative experience or ordinary service. At the later observation stage, both arms would receive the same stated offer. The primary outcome would be a declared response to that common offer, with a fixed observation window and common recording procedure. The target is Equation 19, incorporating the effects of the common offer itself.

The protocol would record all material components of the exposure, including staff attention, peer contact, time and information. If the research question concerns a particular component, the study should include a comparison capable of separating it. An assignment to an introductory group experience can estimate the effect of that package while leaving its internal causal mechanisms open. Qualitative follow-up can investigate participants' interpretations, but it does not by itself identify a formal mediation effect.

The study would also track declined invitations and incomplete follow-up. Participation in the exposure is voluntary. Assignment and actual receipt would be recorded separately, and the primary comparison would retain the assignment-based target. The report would include coverage uncertainty where the

common-offer response remains unobserved. The design would never require an individual to demonstrate a preferred activity or to adopt the institution's proposed interest.

24.6 A Branch-Level Opportunity Comparison

A second possible comparison would assign a revised opportunity package across branches. This subsection connects the design to Section 16 and states the conditions that would determine whether it is feasible.

The package could combine a specified schedule, room allocation, communication procedure and facilitation. The baseline cohort would remain fixed while the study measured individual responses, branch totals, staff resources and displaced uses. Within-branch interaction would be part of the package's effect. The design would investigate movement and communication across branches before relying on a restricted interference assumption.

The assignment scheme should reflect the number and comparability of available branches. Stratification or balance may be useful, but the actual procedure must be recorded so that assignment probabilities and uncertainty calculations match it. The estimator in Equation 31 demonstrates the meaning of population weighting. A final analysis plan would specify the appropriate estimator and interval procedure for the chosen design, rather than assuming that the displayed expectation proof settles every statistical choice.

If random assignment is infeasible, the study can still conduct the documentary, coverage and implementation stages. A before-and-after comparison or comparison with another branch would need additional assumptions about concurrent changes and selection. Its evidential status should be stated accordingly. The value of the inquiry should not depend on calling every empirical comparison an experiment.

24.7 Study Gates and Revision Decisions

The final design stage specifies conditions for proceeding, redesigning or stopping. These gates make the proposal reviewable before participants invest time in it and before investigators encounter results that could influence their methodological preferences.

The study should proceed only when the activity and alternative are clear, the population can be described, the observation procedure is feasible, and the intervention has an appropriate institutional and participant basis. It should be redesigned when the intended offer cannot be delivered, the common measurement differs across arms, the interference boundary is untenable, or the burden of observation threatens the activity being studied. A serious conflict among uses may call for a deliberative redesign before further assignment.

After the study, a decision record would separate the descriptive findings, causal claims, remaining uncertainty and evaluative reasons. It would state what evidence supports continuation, what motivates modification, and which questions require a different investigation. Participants and staff should be able to contest factual descriptions and explain consequences that the original instrument overlooked. That process makes the study a source of publicly examinable learning while preserving the limits of the evidence it produces.

24.8 Offer Specification and Operational Rehearsal

Before recruitment, the research team and library staff should rehearse the offer from the participant's point of view. This subsection describes that rehearsal in detail because a nominally well-defined intervention can still contain gaps between its written description and its practical delivery. The rehearsal is a proposed preparation activity, with results to be recorded before the main study begins.

The team would first prepare a single offer document identifying the activity, the intended participants, the schedule, the place, the expected interaction, the duration and the participation conditions. It would state whether a person must request a place, whether a companion can join, how allocation works,

and what happens if the activity is canceled. It would describe the information that participants receive at each stage. The team would then inspect the offer through each intended communication route, including any accessible or translated version, to identify consequential differences. A shortened poster and a detailed registration page can communicate different expectations; the study needs to know which version a person encountered.

Next, staff would work through several declared request scenarios. One person requests early, another arrives after capacity is reached, another needs more information, and another requests a condition absent from the initial form. The exercise would record how each request is handled, which rule applies, what information the person receives, and which administrative trace remains. The scenarios are tests of the procedure, not invented empirical cases. Any failure discovered would lead to a documented revision of the offer or its recording before the main comparison.

The rehearsal should also include cancellation and partial participation. Suppose a person receives a place but arrives late, attends only the beginning, or leaves before the planned group activity. The study must decide which outcome each event satisfies. A definition of any attendance can legitimately count a brief arrival, while a definition of participation in the activity may require more. The event rule should be justified by the research question and applied consistently. Investigators should preserve enough detail to distinguish the outcome definitions where doing so is feasible, rather than changing the rule after observing which version produces a more favorable result.

Capacity procedures deserve particular attention. The team would identify whether applications remain open after places are filled, whether the waitlist has a limit, and whether unsuccessful applicants receive another offer. Different procedures produce different evidence about attempted demand. A closed application form can yield no additional requests by design. A waitlist can reveal some interest while discouraging people who expect little chance of admission. The study should therefore record the interface state and the information shown at the time of a request. A count of submitted applications needs that context before it can be interpreted as the full number of people who wanted a place.

The team would then rehearse the common-offer comparison. The description, allocation and recording of the final offer should be the same in the ways required by the target. If the earlier exposure changes what staff know about a participant, staff may unintentionally provide different encouragement or help at the final stage. The design can either include that difference in the exposure package or use a procedure that limits it. The choice should be explicit. A formally identical invitation does not establish a common effective offer if the surrounding assistance differs systematically.

The rehearsal should document who can make implementation changes during the study. A facilitator may need to respond to an unexpected situation, and staff should retain appropriate authority to manage the space. The protocol would specify which changes require a version note, which alter the primary target, and how deviations are reported. Preserving practical flexibility is compatible with rigorous interpretation when the study retains the actual sequence of changes. Concealing adaptation under a single unchanged program label would make the final comparison difficult to assess.

24.9 A Staged Sequence of Evidence Decisions

The study can proceed through stages that each answer a specific decision about the next inquiry. This subsection develops that sequence without prescribing arbitrary dates or sample sizes. The duration and scale of each stage depend on the selected activity and the available evidence.

The first stage would settle whether the central inferential issue actually arises. The documentary audit might reveal that administrators already distinguish unavailable periods from unrequested periods and use additional community evidence before changing a service. In that case the study should examine whether the proposed method adds anything to the existing procedure. It should avoid searching indefinitely for a failure simply because the initial theoretical example made one seem likely. A finding that the institution already handles the distinction well is a legitimate result and can inform the method's revision.

If the audit identifies a material ambiguity, the second stage would choose the least burdensome evidence capable of addressing it. A missing log field might be repaired through a short recording pilot.

Uncertainty about how people interpret an offer might be addressed through interviews. Uncertainty about a counterfactual response might justify a feasible offer comparison. The study should state why the selected source can distinguish the relevant explanations. Collecting all possible data before identifying the unresolved claim would increase burden without ensuring a stronger inference.

The third stage would test the proposed instrument and implementation. The team would examine whether participants understand the questions, whether staff can maintain the records, and whether the procedure changes the activity in a material way. A failure at this stage is an opportunity to redesign. It should remain distinct from a negative result about the activity itself. If a booking form is unusable, low pilot registration primarily tests that form and its surrounding procedure. The study can preserve the finding while clarifying the next offer to be investigated.

The fourth stage would implement the principal comparison under the agreed analysis plan. The report should retain the original target, the delivered versions, the population reached, and any deviations. An intermediate finding that reveals a serious burden or an unanticipated conflict may justify changing the study. The reason for the change should be recorded before its effect on the preferred conclusion is assessed. The resulting analysis can then distinguish the initial comparison from the revised phase.

The fifth stage would combine the findings for the actual policy decision. This is a synthesis of evidential roles rather than a vote among datasets. The assignment contrast may identify a response change; interviews may explain how participants understood it; resource records may reveal the work required to sustain it; and accounts from other users may identify displaced opportunities. A decision record should show how each source bears on the choice. If the main uncertainty has shifted from a factual question to a value disagreement, the process should identify that transition explicitly.

Finally, the study should plan for an outcome in which the evidence remains inconclusive. A broad bound can reveal that a visitor sample was insufficient, or an interference problem can limit a causal claim. The institution can still decide using stated reasons and a revision process. The report should identify what was learned about the limits of the original evidence, which new questions are material, and whether further inquiry is proportionate. An inconclusive study becomes wasteful when it conceals its uncertainty or fails to change the understanding of the decision; it can be useful when it clarifies what the evidence can and cannot warrant.

25 Fieldwork Preparation and Intervention Delivery

The prospective design becomes testable only when its offers, populations, records, and implementation responsibilities are concrete. This section specifies the preparation and delivery work needed before a library field study can begin. It develops a staged protocol for one focal activity, explains how to preserve the comparison when ordinary operational changes occur, and identifies the decisions that require substantive review before data collection. The protocol remains a proposal; its feasibility depends on agreement with the participating institution and affected public.

The study should begin with a limited activity whose participation process can be reconstructed. A shared-room discussion group is one possible case. Other cases may be more suitable after local consultation. The choice should be governed by a consequential inferential question and a feasible observation design, rather than by the convenience of a readily available attendance field. A clear activity boundary makes it possible to identify the offer that changes, the people who encounter it, and the records that can support a conclusion.

25.1 Activity Selection and Target Review

The preparation team first writes a short account of the activity and the decision it is intended to inform. The account should name the current arrangement, the proposed alternative, the affected population, and the meaning of successful participation. These entries form the basis for review by staff and potential participants before they become variables in an analysis plan.

For a room-based activity, the current arrangement includes more than whether applications are formally accepted. It includes available times, maximum group size, application lead time, charges if any, staff support, accessibility arrangements, cancellation rules, and the allocation of scarce slots. The alternative offer identifies which of these components changes. Describing the intervention only as “more access” would leave too many versions compatible with the label and make a later effect difficult to interpret.

The target review asks whether the outcome matches the decision. If the decision concerns making a group activity possible, counting individual visits can be insufficient. If the decision concerns wider participation, counting sessions alone can conceal repeated use by a small group. If it concerns the quality of a shared activity, a headcount can omit the central purpose. The review can select more than one outcome, provided it states their distinct roles and avoids treating a long list as a substitute for a central research question.

The review should also identify an interpretable negative case. What would count as a person declining a genuinely available opportunity? What would count as a group failing to form despite adequate access? What would count as a policy being implemented correctly while producing little uptake? Defining such cases in advance protects the study against reinterpreting every disappointing result as evidence of another hidden barrier. Remaining barriers can be investigated, but their existence requires evidence.

Finally, the target review records which retrospective questions the design can address. A later participant’s account of earlier interest is a retrospective report, and a response after formative exposure is a history-indexed outcome. The preparation document should preserve those names. If a proposed conclusion depends on a latent earlier disposition, the team must specify an operational bridge or acknowledge that the disposition remains a modeling variable. A fieldwork schedule cannot resolve that conceptual requirement by itself.

25.2 Documenting the Existing Participation Route

The next preparation stage reconstructs the route from an initial idea to an actual event. Its purpose is to identify the stages at which the current record begins, the stages it omits, and the points where staff or participants make discretionary judgments. The reconstruction combines documents with direct process review; a published policy supplies only part of the evidence.

A practical exercise follows a hypothetical application through the current system. The team records where a person learns that rooms are available, what information the application requires, how staff respond, what happens when the preferred time is unavailable, and how a successful event enters the administrative record. A second exercise follows an unsuccessful application, and a third follows a person who considers applying but stops earlier. The latter case can expose a route that has no administrative endpoint to count.

The exercise should retain the difference between a rule and an observed practice. A form may permit several alternatives, while staff routinely offer only one because of workload. Staff may provide useful informal assistance that the written process omits. Participants may learn a practical workaround from other users. These possibilities remain open until observed. Recording them can reveal resources already supporting access as well as difficulties that a policy change might address.

The process account should include ordinary exceptions. A group may change its date, reduce its size, move to another branch, or proceed without formal booking. An application can be withdrawn for reasons unrelated to the policy. A complete route therefore contains exits and substitutes alongside the successful path. Treating every exit as suppression would predetermine the study’s answer. The classification should follow the evidence available for the particular episode.

This reconstruction produces a baseline description of the observation regime. It supports later interpretation of a change in records because the evaluator can ask which route components changed with the policy. It also identifies low-burden additions to data collection, such as recording whether an unavailable requested time led to an accepted alternative. Such additions should be justified by an inferential use, with unused detail excluded from the routine workload.

25.3 Recruitment Routes and Inclusion Decisions

Recruitment translates the target population into a reachable study population. The protocol should specify several routes when the research question extends beyond registered users, while retaining the selection limitations of each route. The objective is to improve coverage and document its boundaries.

An administrative route can contact people with recent room requests or relevant registrations. A residential or other population frame can reach people with no recorded activity. Community organizations can support contact with people who encounter barriers to the first two routes. Each route should have a defined purpose. Recruitment through an organization may provide depth about a particular experience while leaving population prevalence unresolved. That contribution remains valuable when reported in its proper form.

The inclusion decision should precede assignment to the intervention conditions whenever the estimand concerns a fixed cohort. Otherwise, the policy can change who becomes eligible or visible in each condition, and a comparison of participants can mix compositional change with response change. A fixed baseline cohort permits the population-weighted comparison developed earlier. A separate record can describe people who arrive later and the policy's possible effect on public reach.

Duplicate recruitment requires careful handling. A person may appear in the membership list, respond to a residential invitation, and be referred by an organization. The study should count that person once for a person-level target while preserving all routes through which contact occurred. The linkage method should collect only what is necessary to resolve the duplication and maintain the agreed follow-up. A demand study has little reason to build an extensive personal history unrelated to its question.

The recruitment record should distinguish ineligibility, failed contact, declined participation, and an unanswered item. These categories describe different observation processes. A person who declines the study may still be eligible for the library offer, subject to the institution's ordinary arrangements. The protocol should clarify that relation so that the evaluation does not make research participation an unintended condition of public access. Any narrower arrangement would require a specific justification within the agreed design.

25.4 Constructing Credible Offers

An intervention offer must be sufficiently concrete for a potential participant to understand what accepting it would involve. This subsection specifies the information and operational commitments that make an offer interpretable. It also preserves the offer's possible formative effects within the target.

A room offer should identify the available period, location, capacity, expected activity, applicable conduct conditions, booking action, support available, and the consequences of declining or cancelling. Where the group must provide its own organizer or materials, that requirement belongs in the offer. Where staff provide facilitation, that assistance is part of the intervention. Participants should not have to infer essential conditions from informal knowledge unavailable to the comparison group.

Credibility requires practical follow-through. An invitation to a room that is repeatedly unavailable at confirmation measures a different opportunity from a reliably reservable room. The protocol should therefore verify capacity before invitations are issued and record changes promptly. A cancelled session remains an event in the study's history even when it produces no attendance. Removing it from the analysis because the offer failed would condition the sample on an implementation outcome.

The offer can include an introduction to the activity. The preparation team should decide whether that introduction is common to all conditions or part of the formative exposure being compared. A brief explanation, a demonstration, and a facilitated first meeting can have different effects. The study should preserve those versions without assuming that one of them is a neutral measurement occasion. The observed response belongs to the version actually delivered.

Participants may interpret the same text differently because of their histories. A formal invitation can be reassuring to one person and unconvincing to another after earlier failed attempts. Standardizing the text controls one component of delivery; it does not make the received meaning identical. Short

comprehension and credibility questions can help document that difference. Their use should be limited to what the analysis needs, and their own role in clarifying the offer should be recorded.

25.5 Allocation under Finite Capacity

Scarce places require an allocation rule before the offer is delivered. The rule affects who can participate, how participants interpret their chance of access, and whether one person's gain displaces another. This subsection connects the capacity model to a feasible fieldwork procedure.

The study should distinguish an invitation to request a place from a guaranteed place. If more requests than places are possible, the invitation explains how allocation works. Requests can then be measured separately from admission and attendance. Treating a request as attendance would merge willingness with the allocation outcome. Treating a denied request as a voluntary refusal would conceal the capacity restriction.

The allocation rule can be fixed priority, a lottery, a rotation, or another publicly defensible procedure. The evaluation records the rule actually used, including exceptions and tie handling. Different rules can produce different distributions even when total occupancy is the same. A total effect on attendance therefore leaves the distributional assessment open. The study can report who was admitted, who waited, and which alternatives were available without claiming that one procedure is automatically fair.

Capacity should be measured in the units that constrain the activity. A room-hour limit differs from a person-place limit, and a staff-hour limit can bind before the room is full. A group requiring a continuous two-hour period cannot necessarily use two disconnected one-hour slots. Recording only the total nominal hours would miss that constraint. The relevant capacity description follows the feasible activity, rather than forcing the activity into the easiest aggregate resource measure.

The protocol should anticipate unused allocations. A person can receive a place and later cancel; a group can fall below the size required for its activity. A waiting-list rule may fill the place if sufficient notice exists. The resulting attendance depends on this sequence, not only on the initial admission count. Recording the sequence helps separate low request volume, cancellation, communication delay, and failures of replacement. Each suggests a different possible implementation change.

25.6 Delivery Fidelity and Meaningful Variation

Fidelity concerns which features of the planned intervention were delivered. It should preserve the components needed to interpret the comparison while allowing ordinary responsiveness to participants. This subsection distinguishes consequential deviations from harmless variation in presentation.

The team should name the intervention's essential components. These may include a specific access rule, a guaranteed response time, available facilitation, or the common final offer in an exposure comparison. A deviation in an essential component changes the version of the treatment received. Other variations, such as choosing among equivalent wording that improves comprehension, may be permissible within the protocol. The distinction should be agreed before delivery and revisited if fieldwork reveals that a supposedly minor feature affects the opportunity materially.

A fidelity record can remain short. For each offer or session, staff record whether the essential components were available, whether a planned element changed, and the reason for any consequential change. This record supports interpretation of the assigned-policy effect and a separate description of actual delivery. It should avoid becoming a burdensome parallel bureaucracy whose demands reduce the service being evaluated.

The primary assignment analysis should preserve the original assignment when the target is the effect of assigning a policy. Reclassifying people according to the version they successfully received can create selection because receipt depends on implementation and participant circumstances. A descriptive account of receipt is still useful. A causal effect of receipt requires additional assumptions and an appropriate design, which the assignment result does not provide automatically.

Meaningful variation can also generate new research questions. If participants repeatedly ask for a different time or format, the requests may reveal a mismatch in the planned offer. The study can preserve

those observations and propose a later revision. Changing the offer mid-comparison without recording the change would blur the estimand. Keeping every feature fixed despite clear practical failure can waste effort. A staged protocol resolves this tension by distinguishing a rehearsal phase from a declared comparison phase and by specifying how a necessary mid-study revision will be reported.

25.7 Observation Windows and Follow-Up

The observation window determines which events belong to the outcome. This choice is particularly important when participation requires planning, repeated meetings, or learning through an initial experience. The protocol should connect each window to a substantive process rather than selecting it solely for administrative convenience.

A short window can measure immediate uptake of a concrete offer. A later window can measure continuation, return, or participation in a newly chosen activity. These outcomes respond to different questions. A person who attends once and then stops may have sampled an activity and made an informed refusal. Another may want to continue but lose access. Follow-up should distinguish these possibilities through a proportionate combination of records and accounts.

Repeated measurement also changes the history. Reminder messages can increase salience, interviews can encourage reflection, and a follow-up offer can create an additional opportunity. These actions belong in the intervention record. The design can compare response histories containing them, provided it does not describe the resulting sequence as passive observation of an untouched preference.

Attrition should be reported by stage and outcome. Missing follow-up answers differ from verified absence of activity. A person can continue participating while declining further interviews, or stop attending while remaining willing to discuss the experience. The protocol should preserve these distinctions in its data structure. If the main outcome is available independently of interview participation under the agreed arrangements, that fact can reduce one source of missingness while leaving the interpretation of absent testimony open.

25.8 Contingencies and Study Revision

Ordinary operations can disrupt a field comparison. The protocol should anticipate a limited set of consequential contingencies and state how their occurrence changes the analysis. This preparation protects interpretability while allowing the institution to respond to actual needs.

A room closure can remove capacity from one branch. Staff absence can reduce facilitation. A local event can change attendance opportunities across several branches. The study records the timing, affected offers, and practical response. The assignment effect can still describe the policy under the realized implementation environment, but a claim about a reliably delivered version may then require a separate analysis or later study. The report should state which target remains supported.

Cross-branch use can challenge the assumed interference boundary. A participant assigned through one branch may attend another branch's event, and staff may share the intervention practice. The protocol should collect the minimum routing information needed to detect consequential cross-boundary exposure. If such exposure is common, the branch-isolated potential-outcome model needs revision. Calling the exposure contamination describes a design problem; it need not imply that participants acted improperly by using an available public service.

The study should have an explicit revision record when a central assumption fails. That record states the assumption, the evidence of failure, the conclusions affected, and the proposed replacement. The replacement can be a narrower target, a revised comparison, or a descriptive analysis when a causal one is no longer defensible. Preserving the failed route is part of the research result because it explains why the final inference has its actual scope.

25.9 Readiness for a Defensible Comparison

The final preparation decision concerns whether the study is ready to produce interpretable evidence. Readiness requires a concrete offer, a justified population, an implementable assignment rule, a measurement process, and a way to record consequential departures. It also requires a clear account of the decision the evidence will inform.

A rehearsal can test the application route, delivery record, observation window, and interview burden without being presented as an effect estimate. Its findings may show that the activity boundary is unclear or that a planned common offer cannot be delivered consistently. Resolving those issues before the comparison is more informative than collecting a larger dataset around an unstable target. The rehearsal's own observations remain useful process evidence with their limited status preserved.

The resulting preparation file should allow an independent reader to reconstruct what each study condition made possible. That file connects the formal estimand to the actual institution. A well-derived estimator then has a defined object to estimate, and a qualitative account has a documented sequence against which its interpretations can be assessed. This connection is the practical standard by which the proposed identification framework should be judged.

26 A Constructed Study Dossier

This section carries the proposed inquiry through one complete constructed case. It connects the population record, branch assignment, common offer, attendance count, questionnaire follow-up and final decision statement. Every branch, participant count and outcome in the dossier is fictional. The case tests the coherence of the analysis and reporting procedure; it supplies no empirical evidence about a public library or its users.

The dossier is deliberately modest in the number of assigned units. This makes the difference between a computable estimate and a precise conclusion visible. It also permits a direct reconciliation of records without claiming that a small pilot can settle every population, historical or evaluative question. The same case remains in view as the analysis moves from assignment to experience and public interpretation.

This dossier is separate from the eighty-person classification-audit example in Section 31. The equal denominators are a numerical coincidence. The present initial calculation stipulates complete accurate attendance follow-up, while the earlier audit begins with incomplete labels and error budgets. Their records and assumptions should not be combined.

26.1 The Population Record and Its Boundary

Eight branches each establish a baseline cohort of ten eligible people before assignment. The total cohort contains eighty unique people. Each belongs to one branch cohort for the study, and the record preserves a rule for that assignment. The target concerns these eighty people under the specified branch policies. The cohort is a constructed research population, not a representative sample of an entire city by stipulation.

Eligibility concerns the ability to receive and consider the specified common offer under the study protocol. It should not be defined by later willingness to participate, since that willingness is part of the response being investigated. The baseline record identifies recruitment routes, consent to the relevant research procedures, existing connections to the branch and the status of follow-up information. A person can decline the activity while remaining a member of the outcome cohort where the approved observation procedure permits that distinction.

The ten-person branch size is chosen for transparent arithmetic. A real study would justify its cohort and sample sizes through the policy question, design and practical constraints. Equal sizes simplify the present calculation but do not remove the need to preserve unique identities across branches. A person listed in two cohorts would change both the population count and the assumed separation of branch responses.

The population record also identifies what lies outside the target. People recruited after assignment, visitors who hear about the activity later and other branch users can be relevant to the broader policy.

Branch	Assigned formative exposure	Common-offer attendees
1	1	5
2	0	2
3	1	8
4	0	6
5	0	3
6	1	4
7	0	5
8	1	7

Table 3: One fictional realization of independent branch assignment. Each branch has ten baseline members. These totals illustrate an analysis and represent no observed library study.

Their experiences should be recorded through appropriate additional channels. They should not be silently added to the eighty-person denominator after the outcomes become visible. A broader public conclusion requires a separate population and transport argument.

26.2 Assignment and the Two Stages of the Offer

Each branch receives an independent assignment with probability one half to a specified formative exposure. The exposure provides an opportunity to encounter the proposed activity before the common observation offer. The comparison branches follow the specified comparison process. The intervention includes its invitation, materials, staffing, duration and group conditions. A program name alone would be insufficient to define the contrast.

After the exposure period, every branch cohort receives the common offer. Its description fixes the activity, time window, location, booking route, ability to decline and observation procedure. The common offer itself may form interest, confidence or coordination. The target is therefore the effect of prior assigned exposure on response under that later offer, with its formative possibilities included.

For the initial calculation, each branch can accommodate all ten cohort members under the common offer. This capacity stipulation ensures that an observed absence is not produced by a study-imposed lottery for fewer places. It leaves other reasons for nonparticipation open. Within a branch, people may affect one another through invitations, shared expectations or coordination. Those relations belong inside the branch potential total. The initial model stipulates that one branch's assignment has no effect on another branch cohort's response.

The design uses independent Bernoulli assignment. It can therefore produce unequal numbers of exposed and comparison branches. The constructed realization happens to contain four of each. This coincidence should not be rewritten as a design that fixed four treated branches: the assignment variance calculation later in the dossier uses independence. A realized balance and a restriction imposed by the randomization procedure are different facts.

26.3 The Realized Attendance Record

Table 3 gives the fictional branch totals under the common offer. The initial illustration assumes complete, accurate observation of attendance in the fixed cohort, consistency with the delivered policy and the stated interference boundary. Those conditions isolate assignment uncertainty for the first calculation. Later parts of the dossier reintroduce the separate selection problem in experience questionnaires.

The exposed branches contain twenty-four attendees among forty baseline members. The comparison branches contain sixteen among forty. The observed attendance proportions are therefore 0.60 and 0.40. These are descriptive summaries of the realized arms. Their causal interpretation depends on the assignment model and the other conditions just stated.

The record should permit a reader to trace each total to unique baseline members. It should distinguish an actual attendance event from an application, an allocated place, a stated intention and a ques-

tionnaire response. If a person attends two sessions within the common-offer window, the person-level outcome can remain one when the target is any attendance. A count of attendance occasions would be another outcome and should be named separately.

Implementation records remain important even under the illustrative stipulation of accurate attendance. They identify which offer version each cohort received, whether capacity was actually available and whether the activity was delivered within the promised window. A later discovery that one branch received a materially different offer would require revisiting the target and the consistency claim. The arithmetic would still compute a number, but the interpretation of that number could change.

26.4 The Assignment Estimate and Its Precision

The estimator from Section 16 weights each observed branch total by the inverse probability of its assignment. With $p_j = 1/2$ and $N = 80$, the constructed estimate is

$$\hat{\tau} = \frac{2(24) - 2(16)}{80} = 0.20.$$

Under the model, this estimator is unbiased over the assignment distribution for the fixed-cohort effect of the assigned exposure on common-offer attendance. Unbiasedness describes repeated assignment relative to a fixed potential-total schedule. It does not make the realized difference equal to that schedule's effect.

The conservative uncertainty calculation makes this distinction concrete. Under the stronger independent-branch design, equal cohort sizes and one-half assignment probabilities, the bound in Section 32 is $B = 4/J$. With eight branches, $B = 0.5$. Taking $\alpha = 0.05$ gives radius $\sqrt{B/\alpha} = \sqrt{10}$, which exceeds three. Intersecting the resulting interval around 0.20 with the known effect range $[-1, 1]$ leaves the full range.

This outcome reflects the conservatism of the elementary guarantee and the small number of independently assigned branches. It is not an estimate that the true effect is equally likely everywhere in that range. It is also not a proof that every better justified analysis must be uninformative. A more informative method could use additional structure or a different design, but its assumptions and guarantee would need their own verification.

The dossier therefore supports a limited report: the observed assigned-arm difference is positive, while this deliberately conservative assignment procedure supplies no directional precision in the example. The report should resist converting the attractive point estimate into a claim of demonstrated improvement. It should also resist treating the broad interval as evidence that the activity failed. The study has produced a realization and a clear account of the uncertainty that this procedure leaves.

The number of people cannot substitute mechanically for the number of independently assigned units. Eighty individual records improve the description of cohort attendance, but assignment occurs through eight branch packages. Within-branch coordination is permitted and can make responses dependent. An analysis that treats the eighty people as independently randomized would use a different design from the one actually specified.

26.5 Experience Questionnaires after Attendance

The dossier next adds an experience outcome defined for attendees. Suppose eighteen of the twenty-four exposed-arm attendees return a questionnaire, and twelve respondents report the specified favorable experience. In the comparison arm, twelve of sixteen attendees return it, and nine report that experience. These additional counts are fictional and concern a particular question whose wording and interpretation would need validation.

Among respondents, the favorable fractions are $12/18 = 2/3$ and $9/12 = 3/4$. The exposed arm has a higher observed attendance proportion and a lower observed favorable fraction among questionnaire respondents. Those facts concern different denominators. Their coexistence creates no mathematical contradiction and supplies no automatic causal account of participant experience.

For the realized exposed-arm attendees, six questionnaire outcomes are missing. If the item is binary and the twelve observed favorable reports are accurate, the full-attendee favorable count can range from twelve to eighteen. Its proportion lies between $12/24 = 0.50$ and $18/24 = 0.75$. In the comparison arm, four responses are missing, so the corresponding count ranges from nine to thirteen and the proportion from $9/16$ to $13/16$.

These are completion statements for the realized attendee samples. They follow from assigning possible values to missing questionnaire outcomes. They should not be substituted directly for exact population potential-outcome means in the always-participant bounds. Doing so would discard assignment uncertainty, selection into attendance and the distinction between observed samples and the probability quantities in that model. The formal results clarify the required target; they do not authorize treating every calculated sample fraction as an identified causal parameter.

The interpretation of the questionnaire also matters. A favorable answer can concern enjoyment, perceived learning, confidence or another specified experience. Those outcomes should remain distinct unless a justified composite is defined. The item may be meaningful only after participation. Coding every nonattendee as having zero enjoyment would add a convention about an otherwise undefined experience. A separate event such as attending and reporting a favorable experience can be defined, but it answers a different question.

26.6 Process Accounts and Rival Interpretations

The qualitative component can investigate how the realized pattern arose. It should follow the same records and target distinctions rather than supplying a favorable narrative around the point estimate. This subsection identifies several possible accounts and the evidence each would require.

One possibility is that the formative exposure helped people understand the activity and coordinate attendance. Another is that it increased familiarity with staff or made the invitation more credible. A third is that the exposure attracted additional attendees with uncertain expectations, lowering the favorable fraction among those who later responded. Each account is compatible with some versions of the observed totals. The counts alone do not choose among them.

The instruments can obtain accounts of the invitation sequence, what people expected before the common offer, actual scheduling decisions and experiences during the activity. An account that the exposure introduced a new possibility supports a reported interpretation of the experience. A documented application immediately after a group conversation supplies evidence of sequence. A claim that the conversation caused the application requires the corresponding causal argument. The integration should preserve these different strengths of inference.

The study should also seek adverse implementation evidence. A comparison branch might have received a less intelligible invitation, or an exposed branch might have received extra assistance outside the intended protocol. Such findings can alter the evaluated package or challenge fidelity. The record should distinguish a planned component from an unintended delivery difference. The response can be to revise the intervention description, restrict a conclusion or redesign the next phase, depending on the nature of the deviation.

Rival interpretations of experience deserve equal attention. A demanding activity can produce lower immediate enjoyment while serving a participant's valued purpose. It can also produce frustration without a compensating benefit. The study needs the relevant accounts and outcome definitions before drawing an evaluative conclusion. The agency and public-justification analysis explains why a favorable questionnaire percentage cannot carry the entire judgment.

26.7 A Decision Record Consistent with the Dossier

The fictional results permit a concrete decision discussion even while major uncertainty remains. The record should state the target, the observed summaries, the uncertainty calculation, the unresolved experience evidence and the public purposes at stake. It should also distinguish an experimental next step from a finding that justifies broad permanent expansion.

One possible decision is to conduct a revised, adequately resourced comparison that improves precision and addresses the implementation uncertainties revealed by the first phase. The reason would concern the value of learning about a plausible opportunity, the feasibility of a better study and the consequences of waiting. The positive observed attendance difference can motivate further investigation without being treated as a confirmed population effect.

Another possible decision is to continue a limited offer where the activity is acceptable and the resource conflict is manageable, while retaining explicit uncertainty about its wider effects. That is a practical choice under incomplete evidence. Its justification should identify the benefits or opportunities considered sufficient for continuation, the costs borne by others and the conditions for review. The decision should not borrow a stronger causal claim than the study supports merely to make continuation sound scientific.

A third decision could end the particular pilot because its costs exceed the institution's available resources or because participants identify problems that remain unresolved. Ending the pilot would not establish an absence of demand for every related offer. The decision record should state the actual reason and preserve the evidence for later reconsideration. The continuity analysis addresses how reliance and commitments affect that process.

The dossier demonstrates a useful division of work. The protocol defines the offer and population. The administrative record tracks actual stages. Assignment supports a conditional causal comparison. Completion bounds preserve missing questionnaire outcomes. Accounts help interpret implementation and experience. Public judgment determines how the remaining evidence and uncertainty bear on provision. Keeping those roles connected gives the study a coherent result even when a single decisive demand estimate remains unavailable.

27 Interviews, Observation, and Record Design

This section specifies proposed instruments for the prospective study. It distinguishes the purpose of interviews, structured questions, direct observation and administrative records, then describes how the sources can be linked without losing their different meanings. The instruments require local testing. Their wording is a starting proposal for review, not a validated scale or a set of questions known to recover latent demand.

27.1 Interviewing about Particular Attempts

An interview can reconstruct an attempted activity in greater detail than an aggregate record. This subsection proposes an event-centered sequence that reduces the burden of asking respondents to provide an abstract theory of their own demand.

The interviewer could begin by asking the person to describe a recent occasion when they wanted to use a library space with others. Follow-up questions would identify the intended activity, how the person learned about the available options, what they attempted, which response they received, and what happened next. The interviewer would distinguish firsthand experience from an expectation based on other people's accounts. Dates, branches and documents would be recorded where the respondent is comfortable providing them, while uncertainty would remain explicit.

A different sequence would address a person who never attempted access. The interviewer could ask what kinds of activities they associate with the library, which opportunities they know about, and whether any arrangement has seemed unsuitable or difficult to approach. The question should permit the person to say that the activity holds little interest. Presenting every nonuse as a barrier problem would pressure respondents toward the study's preferred explanation. The inquiry should learn whether a barrier account is appropriate, alongside other possibilities.

An account of a failed attempt is evidence about that account and may support further factual claims when corroborated. It should not automatically be generalized to every person in a category. Conversely, a lack of corroborating administrative records should be interpreted in light of the recording procedure.

If unsuccessful requests leave no entry, absence from the database has a different evidential role from absence in a system designed to log every request.

27.2 Questions about Specified Offers

Hypothetical-offer questions can clarify conditions that matter to a response. This subsection proposes a structure that preserves uncertainty and separates stated intentions from observed uptake.

A question should describe an offer in ordinary language: activity, location, time, duration, expected participation, any cost and the way a place would be allocated. It could ask whether the respondent would request a place, would expect to attend if allocated one, would want further information, or would prefer a different arrangement. These alternatives correspond to different stages of the process. A single yes-or-no question about interest may collapse them before the study can identify the relevant barrier.

The follow-up should ask which conditions are essential to the answer. A respondent may say they would attend only with a known companion, only when the activity is in a particular language, or only if they can leave early. Those qualifications belong to the response. Recoding them as unconditional positive demand would create a stronger claim than the answer supplies. The study can use the qualifications to define a more suitable offer and then state that the target has changed.

Where alternative offers are presented, their order and framing should be recorded. Asking first about a facilitated session may alter the interpretation of a later question about independent room use. Introducing examples may help respondents understand the activity while also shaping what they consider. Instrument testing should investigate these effects. The study can include the actual elicitation procedure in its target while leaving broader generalization conditional on further evidence.

27.3 Observation of Access Stages

Direct observation can identify where an activity advances or stops. This subsection proposes a stage-based record that keeps the event sequence visible. The observer's role and the practical intrusiveness of observation must be considered in the local design.

The proposed stages are information encounter, request, eligibility decision, allocation, arrival, participation, departure and follow-up. A particular activity may require fewer stages or a different order. The record would identify the stage observed, the relevant time, the unit, the observation method and the level of certainty. It would distinguish an observed event from an inference based on a later account. A missed stage should remain missing rather than filled with the most convenient explanation.

For capacity-constrained activity, allocations and attendance should be separate. A place offered to a person who declines differs from a place never offered, and both differ from an allocated place lost because the session is canceled. Recording the sequence permits analysis of where an intended opportunity failed. It also helps explain why a policy that increases formal eligibility may leave attendance unchanged.

Observation can alter behavior. Staff may apply rules differently while being observed, and participants may experience observation as scrutiny. The study should use an appropriate consent and information process, minimize unnecessary detail, and assess whether the observation procedure changes the activity. A claim about ordinary service outside the study requires attention to those differences. The goal is useful evidence with a clear account of how its production may affect what is seen.

27.4 A Linked Record Structure

A linked record can preserve different evidence stages without requiring every source to use the same descriptive categories. Table 4 sets out a proposed structure and the inferential limit of each component. The table organizes a design; it supplies no empirical result.

An event identifier can connect records concerning the same occurrence while preserving their separate sources. A person-level identifier can support longitudinal analysis where justified and appropriately managed. The study should collect only the linkage detail needed for the research question and consider

Table 4: Proposed record components and their inferential roles

Component	Question supported	Additional bridge required
Offer version	Conditions presented to a person	Actual understanding and implementation
Request record	Attempt to obtain a place	Attendance if admitted; coverage of unrecorded attempts
Allocation record	A place offered or denied	Acceptance, arrival and realized access
Event observation	Activity within a stated window	Completeness, unit consistency and observer effects
Participant account	Experience and interpretation	Transfer to other people or causal attribution
Follow-up response	Later use or assessment	Attrition, changed circumstances and target continuity
Decision record	Evidence and reasons used in action	Actual implementation and opportunities for challenge

whether less identifying records can answer it. Public reporting can preserve aggregate and qualitative findings without exposing participants' private histories.

Version fields are necessary for offers, policies and instruments. A response to an early pilot description should remain linked to that version even if the final program changes. A later recoding should retain the original label and the reason for revision. This structure supports the category bridges in Section 22 and prevents a polished final dataset from erasing the history of the inquiry.

27.5 Staff Accounts and Resource Records

Staff evidence is essential to understanding how an opportunity is delivered. This subsection proposes a focused account of workload and discretion, with the same epistemic care applied to participant accounts.

Interviews should ask staff to describe the tasks required to provide the activity, the conditions under which requests can be accommodated, and the conflicts they encounter. Preparation, communication, setup, facilitation, accessibility support and follow-up can all consume resources. An attendance total alone may conceal these requirements. Resource records should therefore use a declared unit and distinguish routine work from exceptional effort during the study.

Staff discretion may help users navigate a rigid procedure or create uneven access. The study should examine particular decisions and their reasons, including the information available at the time. A retrospective account formed after an outcome is known can differ from the original decision context. Preserving contemporaneous records where available helps distinguish those perspectives. The purpose is to understand implementation and possible improvement, with responsibility assessed against actual conditions.

27.6 Instrument Validation and Integration

The proposed instruments become useful only through testing and revision. This final subsection identifies the validation tasks that must precede strong claims about demand.

Participants should be able to explain what they understand each offer and question to mean. Staff should assess whether the record structure can be maintained within ordinary work. Investigators should compare observations and administrative entries for a sample of events, inspect disagreements in coding, and assess missingness across groups and stages. A successful pilot of an instrument should establish that it measures the intended object with an acceptable burden; attractive response rates alone are insufficient.

Integration should proceed through explicit questions. An interview may suggest a barrier; an observation may document it; an intervention may test a response to its modification; and a follow-up may reveal how that response persists. Each step adds a different kind of support. The final account should retain the chain and its unresolved links, allowing readers and participants to see how the conclusion was

constructed and where further inquiry could change it.

27.7 Proposed Interview Sequences for Contrasting Histories

This subsection supplies concrete interview sequences for three different histories: an unsuccessful attempt, a considered refusal and a newly developed interest. The sequences are proposed prompts for local testing. Their purpose is to preserve differences among histories before converting accounts into comparative codes.

For an unsuccessful attempt, an opening prompt could be: “Please describe the activity you hoped to do and what you did to find a place for it.” The next prompts would ask what information was available, which branch or contact route was used, whether a specific time was requested, and what response was received. The interviewer would ask what the person understood that response to mean and whether another arrangement was offered. A final prompt would ask what the person did afterward, including whether the activity occurred elsewhere, changed form or was abandoned. The sequence separates desire, attempt, institutional response and subsequent action.

The interviewer should then check the interpretation with the respondent. For example, a summary might distinguish a refused request from a request the person never submitted because the website showed no suitable time. Both can indicate constrained opportunity, but they produce different administrative evidence. The respondent should be able to correct the summary and retain uncertainty about details. A later code such as blocked request should cite the relevant account and its limitations rather than replacing the account with an apparently self-evident fact.

For a considered refusal, an opening prompt could be: “What did you understand the offered activity would involve, and how did you decide whether to take part?” Follow-up would distinguish disinterest in the activity from an inconvenient condition, an attractive alternative, uncertainty, or an experience that made the setting unsuitable. The interviewer could ask whether a specified change would matter while making clear that the person may still decline. The sequence should avoid implying that the investigator expects to discover a hidden barrier behind every negative answer.

The record should preserve the target of the refusal. A person may reject the proposed group format while valuing the underlying topic, or reject a particular time while wanting the activity on another day. Another may say that they simply prefer to use the library alone. Those accounts warrant different interpretations. The study can report a distribution of responses to the specified offer while retaining qualitative differences that explain why modifying one component might matter. It should never convert every qualified refusal into a positive demand for the investigator’s preferred redesign.

For a newly developed interest, the opening prompt could ask: “How did your understanding of this activity change from first hearing about it to taking part?” The interviewer would ask what was unfamiliar, which experience mattered, and how the person now describes their reasons for continuing or leaving. A separate prompt would ask whether the activity connects to earlier concerns or practices. This allows the person to describe continuity and novelty together. It avoids forcing them to choose a single story in which the interest either existed completely beforehand or was created entirely by the program.

Retrospective accounts can be influenced by the later experience. The study should preserve them as present interpretations of a history, alongside any earlier records. A contemporaneous request may support an earlier articulated interest. A later recollection may add meaning while leaving its exact timing uncertain. Comparing the records can reveal agreement, revision or tension. The purpose is to understand the trajectory, not to rank every current account below an earlier form or to assume that the latest account is the final truth about the past.

27.8 Proposed Questions for Staff and Decision-makers

Staff and decision-maker interviews need their own sequence because the institutional question concerns how evidence becomes action. This subsection proposes prompts that reconstruct a decision’s context and distinguish a formal rule from its interpretation.

An opening prompt could ask: “Please describe a recent decision about allocating this space or activity and the information available when it was made.” Follow-up would identify the options considered, the relevant policy, the role of existing use, the consideration of unsuccessful requests and the conditions that could have changed the decision. The interviewer would ask whether the records distinguished unavailable opportunities from available but unused ones. Where a decision document exists, the interview would clarify its interpretation while preserving the document as a separate source.

A second sequence would focus on implementation: “What work is required to make this offer available, and where do requests usually encounter difficulty?” Staff could describe routine tasks, exceptional cases, coordination with other services, and the support needed to sustain the activity. The interviewer would ask how cancellations, capacity limits and informal requests are recorded. The sequence should permit staff to identify limitations in the proposed study itself, including unrealistic reporting requirements or an intervention that omits necessary work.

A third sequence would examine revision: “What kind of finding would lead you to change the current arrangement, and who would take part in that decision?” This prompt distinguishes an inquiry whose results can matter from one that collects evidence without a usable decision route. A response can also reveal that a factual question is secondary to a resource or value constraint. If additional demand would leave the decision unchanged because the room is required for another obligation, the evaluation should state that constraint rather than suggesting that demand measurement alone will resolve the allocation.

27.9 Coding Disagreement and Respondent Review

The final instrument extension concerns interpretation after collection. This subsection proposes a process for retaining disagreement among coders and respondents when it affects the claim under investigation.

The coding guide should define each category through its role in the inference. A permission barrier, an allocation loss, a logistical constraint, a considered refusal and an uncertain account should have distinguishable criteria. Coders should be able to preserve multiple relevant conditions when they coexist. Forcing each case into one exclusive cause can obscure the joint barriers identified by the feasibility analysis. A code should represent what the evidence supports at that stage, with ambiguous cases retained for review.

Disagreement should be examined substantively. Two coders may interpret the same account differently because the category is unclear, the evidence is incomplete, or the account combines several stages. The response may be to revise a definition, seek clarification or retain an unresolved case. A single agreement statistic can help describe a coding exercise but cannot settle whether the category captures the intended concept. The study should record consequential revisions and assess their effect on the final comparison.

Respondent review can help correct factual errors and identify a misleading summary. It should be offered in an accessible form suited to the study’s burden and sensitivity. A respondent’s disagreement with an interpretation is itself important evidence, though it need not determine every analytic judgment. The final account can preserve the person’s description, the investigator’s interpretation and the reasons for any remaining difference. That separation supports a rigorous inquiry in which participants’ voices remain available without making the analysis depend on a fictional unanimity.

27.10 Observation of Elicitation Effects

The baseline instrument can itself change a later response. This subsection proposes a bounded way to investigate that possibility when it is material to the study. The design applies the common-offer target already developed in Section 11, treating the questionnaire as a potential exposure rather than assuming its neutrality.

Where feasible and appropriate, one group could receive the substantive baseline questions and another a specified minimal-contact procedure. Both would later receive the same offer and response

measurement. The comparison would concern the effect of the actual elicitation package on that later response. The package includes the questions, their descriptions of possible activities, interviewer contact and any information communicated through the process. A difference would demonstrate a consequence of that package under the assignment assumptions; it would not automatically identify which question or psychological mechanism produced the change.

The study could hold the activity exposure fixed while comparing elicitation procedures, or independently assign both components if resources and the interference structure permit. The resulting combinations would separate questions received before experience from questions received after it. Each comparison should have a stated purpose. A design with additional arms is justified when it resolves a material uncertainty about the measurement process, not simply because a more elaborate design appears more rigorous. The actual number of independent assignment units would determine whether the comparison is informative enough to undertake.

This proposal also makes a practical limitation visible. People in the minimal-contact condition still encounter recruitment, consent and the final offer. The comparison therefore never creates a person wholly untouched by the study. Its target is the incremental effect of one described elicitation procedure relative to another. Claims about a completely unexposed public would require a separate transport argument. Keeping the comparison local allows the study to learn something useful without treating neutral measurement as an established fact.

The analysis should consider both positive and negative effects. Questions may make a possibility intelligible and increase uptake, or may make the activity seem demanding and reduce it. They may also change who participates without altering the total. A follow-up account can help identify how people interpreted the contact, while the assignment comparison retains the primary causal target. If the effect is small within a sufficiently informative design, that finding can support a limited measurement-stability claim for the tested procedure and population. It should be accompanied by the actual uncertainty and the range of effects the design could meaningfully detect.

The study must weigh the value of this additional evidence against burden and lost baseline information. A person who receives fewer initial questions provides less historical description, and the comparison may complicate the main protocol. A smaller validation study or qualitative instrument testing may sometimes be more appropriate. The central requirement is to make the possible formative role of questioning an explicit research choice, with its costs, target and limitations open to review.

28 Interview Interpretation and Case Adjudication

Interview evidence can explain how a person encountered an opportunity, understood a restriction, or developed an interest. Its contribution depends on the inference drawn from the account and the evidence available to support that inference. This section develops a case-adjudication procedure through constructed examples. It separates faithful recording, episode reconstruction, alternative interpretations, and the limited conclusion warranted by each account.

Every quoted sentence in the examples below is invented for methodological illustration. The examples represent possible reasoning situations, not reported fieldwork, actual participants, or empirical findings. Their purpose is to make interpretive choices reviewable before interviews are conducted. A real study would preserve participants' own words, document the elicitation context, and revise these preliminary categories when the evidence requires it.

28.1 Retrospective Reports of Unexpressed Interest

The first case concerns a retrospective claim about a desire that left little contemporaneous trace. Imagine an interviewee saying, "I wanted to hold a discussion here before, but I never asked because I thought the room was unavailable." The statement contains an account of earlier interest, a belief about availability, and an explanation of nonapplication. These components should be distinguished before the analyst assigns a causal interpretation.

Faithful recording retains the whole sentence and the question that preceded it. An initial code might identify retrospective interest, perceived unavailability, and a decision against applying. These codes describe what the person reports. A stronger code such as “demand suppressed by policy” adds a claim about the actual policy and its counterfactual effect. That stronger claim requires evidence about the rule, the person’s practical conditions, and the offer under which the activity would have occurred.

The follow-up can ask for a concrete episode: the approximate time, the intended activity, who would have participated, and what led to the belief about room availability. A remembered conversation, an earlier message to friends, or an unsuccessful search for booking information can provide additional evidence. The study should avoid pressuring the person to produce documentation that ordinary life gave them little reason to preserve. Absence of corroboration leaves the account less constrained; it does not establish that the account is false.

Several rival interpretations remain possible. The person may have held a specific earlier intention. The earlier interest may have been vague and become more definite during the later interview. The person’s current positive experience may influence the way the earlier period is remembered. These possibilities concern the history and articulation of the account. They can coexist in part: a genuine earlier attraction can acquire a more precise form through later experience.

A defensible case conclusion therefore distinguishes levels. The interview directly supports the claim that the person now reports earlier interest and perceived unavailability. Corroborating contemporaneous evidence can strengthen a claim about the earlier episode. A conclusion about actual participation under an earlier alternative offer remains conditional on other requirements being satisfied. The analyst can preserve the insight about a possible deterrent without presenting the interview as a complete recovery of a latent disposition.

28.2 Reports of Uncertain Permission

The second case concerns a person whose understanding of the rule differs from its written terms. Imagine the statement, “I did not know ordinary groups could ask for the room.” The immediate evidence concerns awareness of an application route. The investigator should distinguish lack of information, ambiguous communication, and a practical expectation that an application would fail.

A written policy permitting applications can coexist with limited public awareness. It can also coexist with a history in which applications of a particular kind rarely receive usable offers. The interview alone cannot determine which situation applies. The follow-up asks how the person learned about the library’s spaces, what uses they associated with the room, and whether they encountered any message that discouraged an application. The process reconstruction then checks the actual information route available at that time.

The distinction affects the proposed intervention. A clear explanation can address an information gap when the underlying opportunity is already feasible. If the application would still be rejected for capacity or scheduling reasons, information alone may change requests without changing attendance. Both outcomes can teach the institution something. More requests can reveal interest in a specified process while also increasing staff work and disappointment when capacity remains unchanged. The evaluation should retain all of those consequences.

An invitation can make the rule credible in a way that a generic policy page did not. A person may believe that an offer directed to their group signals practical acceptance. The ensuing uptake then reflects information and institutional reassurance together. The study can document this mechanism through the account and the delivered offer. It should avoid reducing the effect to the transfer of a neutral fact if the institutional relationship also changed.

The case conclusion can identify perceived permission as a plausible participation condition and propose an informative comparison. It should preserve the uncertainty between a communication problem and a practically justified expectation of exclusion. That distinction matters for both causal interpretation and institutional responsibility. An institution may need to improve its information, its actual offer, or both, depending on what the additional evidence establishes.

28.3 Unsuccessful Booking Sequences

The third case concerns a documented attempt that fails during scheduling. Imagine a group organizer saying, “We asked for three evenings, but each available room was at a time our members could not attend.” Unlike an unexpressed interest, this account can be linked to a concrete sequence of requests and responses. The sequence still requires interpretation.

The investigator reconstructs the requested times, group requirements, alternatives offered, and reason each alternative was unusable. The record should distinguish an unavailable room from an available room outside the group’s feasible schedule. Both constrain the activity, but they suggest different possible changes. Adding a room at an unsuitable time may leave the activity unrealized; changing the schedule may be sufficient when a suitable room already exists.

The organizer’s account can also conceal variation within the group. Some members may be able to attend an offered time while others cannot. The group may require particular people, a minimum number, or a commitment to include everyone. These are different coordination structures. A headcount of willing individuals cannot determine which structure applies. A proportionate follow-up asks what made the group activity viable and how the group chose among possible compromises.

The analyst should consider alternatives outside the library without assuming that they are equivalent. The group may have met elsewhere at a cost, moved online, shortened the activity, or abandoned it. Each outcome informs the consequence of the failed booking. A substitute can preserve some benefits while losing accessibility, continuity, privacy, or a valued public setting. The study should record the group’s reasons and any corroborating conditions before assessing substitutability.

A supported conclusion might state that a specified booking attempt failed because the available times did not overlap the group’s reported feasible schedule. A further claim that one additional evening slot would have produced the activity requires confidence that the remaining conditions were adequate. The counterfactual can be investigated through a later credible offer, while retaining the history change between the earlier attempt and the later response.

28.4 Participation Shaped by Companionship

The fourth case concerns a participant whose choice depends on another person. Imagine the statement, “I came because my friend wanted someone to join them, and then I found the activity interesting.” This account combines a social reason for initial attendance with a later evaluation of the experience. It challenges a measurement scheme that assumes each person’s uptake expresses an isolated, stable preference.

The initial reason can be an autonomous choice to support a friend. It can also involve pressure, obligation, curiosity, or several motives. The interview should allow the person to describe that relation without imposing a binary distinction between authentic interest and externally caused behavior. Social conditions can contribute to meaningful agency, while particular forms of pressure can warrant criticism.

For the causal design, the friend’s assignment or invitation may affect the participant’s outcome. An individual comparison that indexes only the participant’s own exposure would then omit a relevant treatment component. The branch-level design can accommodate some of this interference when the relationship falls inside the declared branch boundary. Cross-boundary friendships require a broader exposure account or a narrower causal conclusion.

The later interest is also a substantive outcome. The participant may want to return independently, continue with the friend, or choose a related activity. These responses can guide future provision. They remain consequences of the experienced sequence, including companionship and the event. The account supports a possible formation process without proving that interest was entirely absent before the first attendance.

A careful case conclusion therefore records the social route into participation, the person’s evaluation of that route, and the later response. It keeps mechanism hypotheses separate from the person’s own account. The study can then ask whether offers designed for pairs or existing groups produce different

outcomes from individual offers, with the relevant coordination conditions built into the intervention definition.

28.5 Continuation Requests after Initial Participation

The fifth case concerns enthusiasm after a newly available activity. Imagine a participant saying, “I had never thought of doing this here, and now I would like the group to meet every month.” The account can support a current request for continuation and a reported change in the person’s imagined uses of the library. It also introduces a new demand on future resources.

The analyst should distinguish the positive experience, the current preference for continuation, and the feasibility of the requested schedule. An enjoyable pilot does not guarantee that every participant can attend monthly. A group can express a shared aspiration while disagreeing about who will organize it. Follow-up can examine these practical conditions without treating the original enthusiasm as insincere.

The request can be a legitimate input into public deliberation even if the interest was formed through the pilot. Its value depends on the activity, participants’ endorsement, available alternatives, and competing claims. A library can take the request seriously while explaining resource limits. The fact of formation supplies neither automatic entitlement to continuation nor a reason to dismiss the request as manufactured.

The institution’s role in creating expectations matters. An offer presented as an exploratory session differs from a promise of an ongoing program. Participants may reasonably organize their plans around the latter. The study should preserve what was communicated and how participants understood it. A later disagreement about continuation may concern a commitment as well as the quantity of current interest.

The supported conclusion can identify a current desire for continued provision under specified conditions. A claim about unmet demand before the pilot remains separate. The case also supports a normative question about responsible initiation: how should an institution introduce an opportunity when its future capacity is uncertain? The answer requires a public account of commitments and fair allocation alongside the evidence about uptake.

28.6 Refusal after a Credible Opportunity

The sixth case concerns a negative response that deserves recognition in its own terms. Imagine the statement, “The room and time were fine, but I tried the activity and decided it was not for me.” The account can indicate an informed refusal after experience. The study should preserve that possibility when assessing a formative offer.

Follow-up can ask what the person found unsuitable and whether another activity would better serve the same purpose. The objective is to understand the response, not to find a hidden barrier that invalidates every refusal. A participant may dislike the activity, prefer another use of time, or feel that its benefits are adequately available elsewhere. Those reasons can represent successful exploration rather than a failed policy.

The account also helps distinguish refusal from displacement. A person may leave because the group format is unsuitable, because another participant dominates, or because a conflict makes the space unwelcoming. These experiences concern different components of the delivered offer. Respect for refusal permits careful inquiry into those components; it does not require the analyst to treat every exit as an unanalyzable private preference.

A credible negative case constrains the paper’s central argument. Once a specified opportunity has been delivered and the person’s response measured under adequate conditions, a negative result is evidence about that offer-indexed response. The possibility of another imaginable offer does not erase it. The study can retain the negative finding while remaining precise about its scope.

28.7 Staff Accounts and Institutional Constraints

Staff testimony offers evidence about implementation, resource use, discretion and the interpretation of records. Its analysis should apply the same discipline used for participant accounts. An institutional role can provide important access to information without making every generalization from that role correct.

Imagine a staff member saying, “We usually have space, but groups ask for the same two evenings.” The statement suggests a distributional constraint in time rather than a simple shortage of total hours. The follow-up examines requested periods, available alternatives, and the reasons groups cannot shift. A calendar record can corroborate part of the account while leaving the groups’ practical flexibility open.

Another staff member might say, “Few groups ask, so expanding the hours would waste staff time.” This combines a descriptive claim about requests, a counterfactual claim about response to expanded hours, and a value judgment about resource use. The analysis should separate those components. The request record may support the first under its collection rules. The second requires an offer bridge. The third requires a standard for the value and cost of the resulting activity.

The procedure should also preserve reasons supporting the existing rule. Staff may identify a binding workload, a conflict with an existing service, or a history of cancellations that affects feasibility. Those claims deserve evidence and review, just as claims of unmet demand do. A study organized only to discover grounds for expansion would weaken its ability to assess the actual tradeoff.

28.8 Elicitation and Changes in the Account

An interview can help a participant articulate an interest that was previously indefinite. The analysis should preserve the sequence of questions and responses when that sequence changes the available account. This subsection treats such change as an observable feature of inquiry.

Suppose a person initially says that they have little use for a library room. After discussing several possible activities, they identify a project they would like to try. The later answer can be sincere and useful. It belongs to a history containing the discussion. Recording only the final positive answer would conceal that history and invite a mistaken retrospective interpretation.

The investigator can ask whether the person had considered the activity before, what part of the discussion made it relevant, and how tentative the new idea remains. The answers help describe articulation and formation. They do not create direct access to every earlier mental state. A later confident description can remain compatible with several earlier dispositions.

The instrument should therefore separate an initial open question from later examples or prompts. The coding record notes when a response follows a supplied possibility. This distinction allows the study to report both unprompted accounts and responses to a deliberate introduction. It also supports the proposed experiment on elicitation effects, should that experiment be feasible and relevant.

28.9 Adjudication, Disagreement and Revision

The final procedure brings the cases together without reducing them to one binary code. For each episode, the analyst preserves the account, the target claim under consideration, the supporting evidence, and the remaining alternatives. A reviewer can then assess the inference at the point where it is made.

Disagreement between analysts should be recorded through its reason. One analyst may interpret an account as a specific earlier intention, while another regards it as later reconstruction. The record can preserve both interpretations and identify what additional evidence would distinguish them. Forcing consensus can remove an uncertainty that matters to the research question.

Participant review can clarify whether the research account represents the person’s meaning. It does not make the participant responsible for certifying the analyst’s whole causal argument. A person can endorse a summary of their experience while remaining unable to determine what would have happened under every alternative offer. The review should respect that distinction.

The resulting case analysis can support several kinds of conclusion: a documented failed attempt, a current reported interest, a plausible formation process, an informed refusal, or an unresolved interpretive

tension. These conclusions can be integrated with quantitative results when their targets and populations align. Their value lies in the specific distinctions they establish, including the uncertainties they preserve for further inquiry.

29 Proposed Instrument Package

The methodological argument requires instruments that preserve its distinctions in ordinary fieldwork. This section supplies a concrete proposed package for a shared-room activity. It includes an initial contact module, an episode account, a specified-offer module, participation and follow-up questions, and a staff event record. The wording is original to this proposed design. Its comprehension, burden and measurement properties require local piloting before use.

The package is modular because different questions need different respondents and observation occasions. A person who declines the activity may still contribute an account of the offer. A person who attends may decline the interview. Staff can record an unavailable slot without knowing the applicant's earlier interests. The data structure should preserve those partial contributions rather than require every person to complete every module before their evidence is usable.

29.1 Contact and Population Membership

The contact module establishes the respondent's relation to the declared study population and records how the study reached them. Its purpose is coverage assessment and routing. It should avoid presenting current library use as a prerequisite for having a relevant perspective on the offer.

Contact opening. A proposed opening is: "We are studying whether the library's arrangements make this shared activity usable for the people it is intended to serve. We are interested in successful experiences, unsuccessful attempts, and reasons the activity may be unsuitable. Taking part in the study is optional. The activity offer and the research questions are separate; we will explain the arrangements that apply to each." The final wording should reflect the actual relationship between service access and research participation, rather than promise a separation the institution cannot deliver.

Population relation. The first item asks: "Which of the following describes your connection with the area served by this study?" The response categories should follow the approved population definition, such as residence, work, study, regular service use, or another described connection. A free-description route is needed for a relationship the initial list misses. The analyst retains the original description when deciding eligibility.

Recruitment route. The next item asks: "How did you receive this invitation?" The record distinguishes direct study contact, a library communication, an organization, another person, and another described route. Multiple routes can be recorded. The item supports assessment of recruitment and duplicate contact; it should not be used to infer the respondent's degree of interest.

Recent contact with the library. A proposed item asks: "During the study's stated reference period, did you use a library service, visit a library space, or try to arrange an activity through a library?" Separate follow-ups identify use, visit and attempt. A response of none routes the person to an open account of relevance and alternatives. It does not omit the later question about the specified new offer.

Access to the instrument. Before the substantive module, the interviewer offers the available modes of response and asks whether another format would make the conversation easier. The record identifies the mode actually used and any assistance involved. This information supports measurement review. It should describe the collection process without treating the respondent's need for a particular format as an explanation of their demand.

29.2 An Initial Account before Examples

The initial account precedes the study's detailed activity examples. Its role is to preserve what the person brings to the conversation before the instrument supplies additional possibilities. The order matters because later prompts can help form or articulate an interest.

Current uses and concerns. The opening substantive question is: "When you think about using the library's spaces, what activities or concerns come to mind for you?" The interviewer allows the respondent's own categories and avoids immediately translating the answer into the focal program's terms. A person may discuss quiet study, companionship, information, a practical project, or a use outside the investigator's initial list.

Concrete episode. The next question asks: "Can you describe a particular occasion when you used, tried to use, or considered using a library space for something that mattered to you?" If the person has no such occasion, the interviewer records that answer and moves to their present assessment. A detailed episode should not be manufactured through repeated prompting when the respondent has none to report.

Sequence and conditions. For a relevant episode, the interviewer asks: "What did you hope to do, what steps did you take, and what happened next?" Follow-up questions identify the time, other people involved, information available, requests made, alternatives offered, and the eventual activity. These are components of one narrative rather than a demand to complete an administrative checklist. The analyst later codes the stages while preserving the sequence.

Reason for stopping or changing course. When an attempt ends or changes, the interviewer asks: "What led you to stop, postpone, or do something else at that point?" A further question asks: "If that particular difficulty had been resolved, was there anything else that would still have affected whether the activity could happen?" The second question investigates joint conditions. It remains a retrospective report and should not be treated as a verified counterfactual outcome.

Alternative route. The module ends by asking: "Did the activity happen in another place or form, and how did that compare with what you had hoped to do?" This distinguishes abandonment from substitution. The comparison invites the respondent's account of practical differences without assuming that another venue was either equivalent or inadequate.

29.3 The Specified-Offer Card

The offer card makes the prospective target concrete. Each study condition uses a card that accurately describes the version the institution can deliver. The card is part of the intervention and its wording should be retained with the resulting responses.

The card should state the activity, location, available period, expected duration, group size, application action, cost if any, support available, and relevant participation conditions. It should also distinguish a request for a place from a guaranteed place. If a lottery or priority rule allocates scarce places, the card explains that rule in language a potential participant can use. If the offer is a single exploratory session, its limited duration should be clear.

Comprehension. After the card, the interviewer asks: "In your own words, what would you be able to do through this offer, and what would you need to arrange?" This item can reveal a misunderstood condition. The interviewer can clarify factual misunderstandings, but the record should retain that clarification as part of the exposure. The final response belongs to the clarified offer history.

Credibility. The next item asks: “How confident are you that the described arrangement would actually be available if you followed the stated steps?” A short response scale can be accompanied by an explanation in the respondent’s own words. Confidence in availability is distinct from interest in the activity. A low answer can motivate investigation of prior experience or communication without establishing that the institution would in fact fail to deliver.

Current response. A concrete response item asks: “Given the arrangement described here, would you want to request a place, decline this offer, or leave the decision open for now?” This answer is recorded as reported willingness to request, decline, or remain undecided. It does not itself establish that a request was submitted. An actual request is recorded only through the separately specified application action or a corresponding administrative record. If the card permits immediate submission, the record distinguishes the answer, submission, allocation and attendance. Transfer from the expressed answer to request willingness or actual-offer participation requires its own interpretation bridge. Leaving the decision open should not be automatically coded as refusal.

Conditions still unresolved. The follow-up asks: “What, if anything, would you still need to know or arrange before deciding?” Responses can include schedule, companions, transport, accessibility, confidence about the activity, or another issue. The initial analysis should preserve a free-description field. Replacing it immediately with a fixed barrier list could conceal a condition the design failed to anticipate.

Reason for declining. If the person declines, the interviewer asks once: “Would you like to say what makes this offer unsuitable or unnecessary for you?” The response is optional. The interviewer should accept a simple preference for another activity or use of time. The purpose is to understand refusal where the person wishes to explain it, not to keep searching until a participation barrier is found.

29.4 Requests, Allocation and Actual Participation

The next module records the route from a response to an actual event. Much of it can be completed through the agreed administrative process, reducing repeated questions to participants. The module should maintain separate fields for each stage.

A request record identifies the offer version, requested slot, relevant group or person unit, and whether the request was complete enough to process. An allocation record identifies whether a place was offered, which rule determined the result, and whether another option was available. An acceptance record identifies whether the offered place was taken. A participation record identifies the event and the agreed unit of attendance. These records can be linked while remaining conceptually separate.

Unsuccessful request. When a request produces no usable place, a short follow-up asks: “What arrangement, if any, was offered after your request, and was it usable for the activity you intended?” The answer should be checked against the administrative sequence where possible. A mismatch between the accounts is a case for review, not a reason to assume one party’s explanation is automatically complete.

Cancellation or absence. When an accepted place produces no attendance, the optional question is: “What changed between accepting the place and the planned event?” The response can distinguish a personal change, a group coordination difficulty, an implementation change, or a decision that the activity was unsuitable. The analyst should preserve multiple reasons when they interact.

Event occurrence. For attendance, the record states whether the participant took part in the defined activity and what observation supports that classification. A booking confirmation alone is insufficient for this field. Where informal participation cannot be classified confidently, the record retains an unresolved state for later review. This connects the instrument directly to the finite classification model.

Alternative activity. A brief follow-up asks whether the person or group used another place or format after an unsuccessful or changed arrangement. The question should be linked to the same episode. A general report of other library use does not establish that the focal activity was successfully substituted.

29.5 Experience after Participation

The experience module applies to people who participated in the relevant activity. Its items concern an experienced event and should not be assigned fabricated values for people who never encountered that event. The selection analysis later distinguishes the meaning of these participant-only responses from a population-level effect.

Purpose and fit. The first question asks: “What did you hope to get from this activity, and how well did the experience fit that purpose?” The response can be followed by a bounded rating if the study has a reason to use one. The purpose statement is retained because the same rating can refer to different goals. A single scale cannot make those goals comparable by itself.

Valued and difficult elements. The next questions ask: “Which parts of the experience were useful or worthwhile to you?” and “Which parts were difficult, unsuitable, or something you would change?” Asking both allows mixed evaluations. It also avoids treating criticism as incompatible with valuing the opportunity.

Conditions of participation. A further item asks: “What made it possible for you to take part this time?” Follow-up can identify the offer, help from others, schedule changes, staff support, prior skills, or chance circumstances. The account can inform mechanism hypotheses, while the analyst retains its status as an explanation offered after the event.

Freedom to revise involvement. The module asks: “If you had wanted to change how you took part or leave the activity, what options did you understand you had?” The question concerns the person’s perceived and experienced room for revision. It does not certify a general property of autonomy from one answer. Concrete examples of attempted revision can provide stronger process evidence.

Future interest. The final experience item asks: “After this experience, what would you like to do next, if anything?” A response can concern continuation, a different activity, independent organization, or no further involvement. The question deliberately locates the answer after experience. The analyst should not transfer it backward as an unchanged measure of prior interest.

29.6 Follow-Up and Formation

Follow-up examines continuity and change after the initial event. Its timing should match the activity’s practical rhythm and be specified before the relevant comparison. The module separates what people now report from what records establish about later activity.

Subsequent action. The first question asks: “Since the activity, have you taken any steps to continue it, try something related, or use another opportunity?” Follow-up identifies concrete actions, including requests, conversations, planning, attendance, and decisions against continuation. An intention and an action remain separate fields.

Interpretation of change. The next item asks: “Has the experience changed what you think the library could be useful for, or what you would like to do there?” The interviewer invites both change and continuity. A participant may have confirmed an earlier concern, developed a new one, or decided that the activity holds little value. The account supports analysis of formation without requiring a complete reconstruction of the earlier mental state.

Earlier consideration. When the person describes a new interest, a follow-up asks: “Had you considered anything like this before the study, and how was it similar or different?” The answer can reveal continuity, articulation or a contrast with the earlier period. It remains retrospective testimony. The interviewer should preserve uncertainty in the respondent’s account rather than insist on a clean origin date for the interest.

Continuation conditions. The final question asks: “What would need to be available for the next step you described to be workable?” This can reveal resources, companionship, time, confidence, or organizational work needed for continuation. A favorable response to one event does not automatically establish a feasible recurring practice. The follow-up connects current interest to a specified possible next offer.

29.7 Staff Delivery and Decision Record

The staff record documents the opportunity actually produced. It should be short enough to complete during normal delivery while retaining the fields necessary for interpretation. The following structure is a proposal to be tested in the operational rehearsal.

For each offer, staff record the planned and delivered versions, the available resource unit, the time the offer became known, and any consequential change. For each request, staff record its outcome and a concise reason linked to the applicable procedure. For each event, staff record occurrence, capacity used, relevant support, and any departure that affected participation or observation. These fields describe a process; they do not require staff to infer a participant’s latent motives.

The decision record adds a separate layer. It states the decision under consideration, the evidence used, the population to which the evidence refers, the target claim drawn from it, and the public reasons for the resulting choice. A short entry can distinguish a low request count from a forecast of low response under changed access. It can also state when the institution relies on other reasons, such as continuity of an existing use or a resource constraint.

The record should preserve alternative explanations considered at a consequential decision. A low count might reflect low interest in the specific offer, limited awareness, a remaining practical barrier, scarce capacity, or incomplete recording. The decision maker need not investigate every imaginable explanation. The record should identify the serious alternatives relevant to the choice and explain why the available evidence favors or leaves open each one.

29.8 Coding Rules and Unresolved Responses

The coding procedure translates answers into analytical fields while retaining their scope. It should be documented before the main comparison and revised transparently when piloting reveals a mismatch between the categories and participants’ accounts.

A positive interest statement, a request, an accepted allocation and an attendance each receive a distinct code. A respondent’s account of a prior failed attempt receives an episode code and an evidence status. A description supplied after the offer card is marked as occurring after that exposure. A reported reason for refusal remains a reason attributed to the respondent unless additional evidence supports a stronger causal claim.

The codebook includes explicit states for unanswered, inapplicable, uncertain and unresolved. These states have different meanings. An experience rating is inapplicable to a person who never participated;

an omitted answer by a participant is missing; a disputed attendance label is unresolved; an undecided response to an offer is a current decision state. Combining them into zero would make the instrument contradict the paper's target distinctions.

Coding disagreement should be resolved through the reason for disagreement. If analysts disagree about whether an episode counts as participation, the issue may concern evidence or the definition. If they disagree about whether a person had earlier interest, the issue may concern the interpretation of retrospective testimony. The record can retain alternative codes or a compatibility set when the disagreement remains material. A forced single label is justified only when the available evidence and coding rule support it.

29.9 Piloting and Instrument Revision

The package requires a practical trial before being treated as a usable study instrument. Piloting should assess whether people understand the offer, whether the questions preserve distinctions meaningful to them, and whether the record can be completed without excessive burden. It should also examine how the sequence changes the conversation.

The pilot can ask respondents to explain what they understand a question to be asking. A future-use item may be interpreted as a promise, a general attitude, or a forecast. A request item may be interpreted as a guaranteed booking. Such differences should lead to wording or target revision before the main study. Their discovery is an instrument-development result, not evidence that respondents answered incorrectly.

Translation and alternative formats require the same target review. A literal word match can fail to preserve the practical distinction between requesting, being allocated a place and attending. The pilot should assess how the relevant concepts are understood in each format. A stable coding scheme should emerge from that work rather than be assumed from identical field names.

The final instrument record retains the version used, its revision history, the reasons for changes, and the studies or pilots supporting its interpretation. The present package supplies a concrete starting point. Its scientific value depends on the work that connects its questions to the target responses and experiences they are intended to represent.

30 Analysis and Decision Reporting

This section develops an analysis and reporting procedure for the proposed study. It works through constructed numerical examples, distinguishes several uncertainty layers, and specifies the findings that different evidence patterns could support. It then introduces a deliberately simple decision model to show where evaluative premises enter. The calculations illustrate a method; they are neither observed library results nor forecasts for an identified site.

30.1 A Worked Gate and Coverage Record

The first constructed example separates permission, response and population coverage. Its purpose is to show how a report can preserve an informative partial conclusion without silently filling unknown values.

Suppose the declared population contains 100 people, the alternative offer is fixed, and the strong gate assumptions hold. Permission is granted to 40 people, of whom 24 participate. The other 60 are prevented from participating through the route under study. The observed participation proportion is 0.24. Among permitted people the proportion is 0.6. The population demand target lies in $[0.24, 0.84]$, and the blocked response proportion lies in $[0, 0.6]$. The record alone is compatible with every integer count of blocked positive responses from zero through sixty.

This example supports several accurate sentences with different meanings. One can say that twenty-four people participated through the observed route, that sixty percent of permitted people participated

under the gate model, and that the model leaves the responses of sixty blocked people unresolved. One cannot infer from these data alone that sixty percent of the full population would attend the alternative offer. Nor can one infer that all blocked people would participate. The lower and upper endpoints preserve both possibilities in a form that can inform further evidence collection.

Now suppose an appropriate additional instrument identifies the same target for thirty of the blocked people, including nine positive responses. The known positive total becomes thirty-three, leaving thirty unknown people. The demand bound becomes $[0.33, 0.63]$. The blocked response proportion becomes bounded by $[0.09, 0.39]$. These calculations use the same population and target. If the additional instrument merely records a different form of stated interest, the report should present the two targets separately or state the bridge under which the combined bound would be valid.

The example also shows how to choose a useful next inquiry. An additional survey of already observed permitted users may explain why they attended, while a suitable inquiry reaching unresolved blocked people can narrow the population demand interval. The latter is more informative for this target, provided its cost, burden and validity are acceptable. Method choice should follow the unresolved part of the inference rather than the ease of collecting another familiar metric.

30.2 Recording Loss and False Precision

The second example examines a different uncertainty layer. It begins with a recorded participation probability and a justified sensitivity bound. The calculation concerns actual participation, leaving the connection to demand under an alternative offer separate.

Suppose $r = 0.24$, recorded positives correspond to actual events, and an independent assessment warrants $\lambda = 0.8$. Equation 7 gives $p \in [0.24, 0.30]$. If the warranted sensitivity lower bound is only 0.4, the interval becomes $[0.24, 0.60]$. The observed record is the same; the different intervals reflect different additional information about the recording process. A report should state the basis for choosing the sensitivity bound and show how the conclusion changes across plausible values.

These intervals are structural compatibility bounds at the population-law level. If 0.24 is estimated from a sample, its sampling uncertainty must also enter the analysis. A narrow confidence interval around the recorded rate cannot remove uncertainty about sensitivity. Conversely, excellent knowledge of sensitivity cannot eliminate sampling variability in a small observation sample. The report should preserve both layers so that precision about one quantity does not appear to establish precision about another.

The assumption of accurate recorded positives is also substantive. If a booking counts as attendance even when nobody arrives, false positives occur relative to the attendance target. The one-sided bound then requires a different model. An investigator should examine the event definition before adding a generic measurement-error parameter. Sometimes an apparent error is a target mismatch: the database correctly records reservations, while the analyst has interpreted the field as participation.

Table 5 separates the principal uncertainty layers used in this paper. Its purpose is to prevent a statistical interval from carrying a broader meaning than its construction warrants.

30.3 A Worked Assignment and Weighting Example

The third constructed example illustrates what the branch estimator averages. It uses two branches only to make the arithmetic visible. A two-branch study would have substantial practical inferential limits; the calculation offers no claim that such a design would be sufficient for an empirical evaluation.

Let cohort sizes be $N_1 = 10$ and $N_2 = 90$. Set potential totals $T_1(0) = 0$, $T_1(1) = 10$, and $T_2(0) = T_2(1) = 18$. The first branch's entire cohort changes response, while the second branch's total remains unchanged. The population-weighted effect is $10/100 = 0.1$. The equally weighted average of branch-level proportion changes is $(1 + 0)/2 = 0.5$. These are distinct targets.

Suppose exactly one branch is randomly assigned treatment, with either assignment equally likely. Then $p_1 = p_2 = 0.5$, although the assignments are dependent. If branch one receives treatment, the observed totals are ten and eighteen; Equation 31 gives $(20 - 36)/100 = -0.16$. If branch two receives

Table 5: Uncertainty layers and the evidence needed to address them

Layer	Unresolved object	Relevant additional work
Target definition	The activity, offer, history or population being compared	Clarify the research question and offer specification
Coverage	Responses of people outside the observed set	Extend appropriate observation or report bounds
Recording	Actual events missing from or misrepresented in records	Independent validation of the recording channel
Sampling	Variation between a sample and its target observation law	Design-specific statistical inference
Assignment	Causal attribution to a package	A justified comparison and implementation account
Historical formation	Relation of a later response to an earlier disposition	New historical evidence or explicit controlled-target assumptions
Evaluation	Public significance of the identified outcomes	Declared values, competing claims and legitimate decision process

treatment, the observed totals are zero and eighteen; the estimator gives $(36 - 0)/100 = 0.36$. Their equally weighted expectation is 0.1, as the proof requires.

The negative estimate in one assignment does not contradict the positive finite-population effect. It illustrates design variability in a very small example with unequal branches. Unbiasedness is a statement about expectation over assignment, not a promise that every realized estimate lies near the target or has its sign. A study needs sufficient information and a suitable uncertainty procedure to support a practical conclusion. The example is useful because it makes that limitation concrete rather than hiding it behind a formal estimator name.

30.4 Common-Offer Results and Historical Claims

The fourth example shows how a prospective finding should be reported when interest can form through exposure. This subsection uses hypothetical population rates to distinguish an average effect from an individual or historical conclusion.

Suppose the common-offer response rates under two exposure histories are $p_0 = 0.3$ and $p_1 = 0.5$, with a justified design supporting their causal interpretation. The average contrast is 0.2. The report can state that assignment to the specified earlier exposure increases the response to the common offer by twenty percentage points for the declared target population, subject to the design and measurement assumptions. It should include the uncertainty appropriate to the actual observations; the present example treats the rates as stipulated population values.

The positive-change proportion is bounded by $[0.2, 0.5]$, with a possible negative-change proportion between zero and 0.3. The data therefore leave open whether a relatively small group changes positively with nobody changing negatively, or whether larger positive and negative changes coexist. Additional subgroup observations can reveal differences in marginal effects, but they do not automatically identify each person's pair of potential responses. An individual-level claim should retain this distinction.

The finding also leaves the latent initial disposition from Section 11 unresolved. The common offer is part of the observed process and may be formative. A useful report would describe what the exposure and offer enabled, how participants interpreted their experience, and whether later participation persisted. It would reserve a statement about ignored pre-existing wishes for evidence that actually concerns those wishes.

30.5 A Conditional Decision Illustration

Evidence supports a decision only in relation to evaluative premises and constraints. This subsection uses a simple affine comparison to display that dependence. The model is intentionally limited and should be

replaced when the decision requires distributional or plural criteria.

Suppose $\theta \in [L, U]$ is an appropriately defined beneficial-outcome quantity. Let $b > 0$ represent the value assigned to one unit of that quantity and c the net cost of the proposed change within a declared comparison. Define

$$\Delta(\theta) = b\theta - c. \quad (57)$$

The equation states the chosen value model. It cannot infer b , c or the public meaning of θ from an attendance count.

Proposition 30.1 (Robust ranking in the affine illustration). *The change is weakly preferred for every compatible θ exactly when $bL \geq c$. Retaining the comparison arrangement is weakly preferred throughout the evidence set exactly when $bU \leq c$. When $bL < c < bU$, compatible values support opposing strict rankings.*

Proof. Since $b > 0$, Δ increases with θ . Its minimum over the interval is $bL - c$, and its maximum is $bU - c$. Their signs give the three cases. Equality at an endpoint produces a weak ranking and a possible tie. $\square$

For a constructed example with $b = 1$, $c = 0.4$ and $\theta \in [0.3, 0.6]$, the evidence permits both signs of the gain. More evidence might resolve the ranking under that value model, while a different valuation could change which information is decision-relevant. If the lower bound rises to 0.5, the change is weakly preferred throughout the new interval. If the supposed benefit is simply participation count, however, the evaluation has already assumed that each counted response deserves the same positive value. That premise needs justification, particularly where other uses are displaced.

Public decisions may require several outcomes, minimum access guarantees or constraints protecting particular uses. The affine illustration then becomes inadequate. Its role is to show the logical location of values, not to reduce the institution's responsibilities to one number. A report should state when the selected model is merely a sensitivity exercise and when a decision-maker has explicitly adopted its premises.

30.6 Finding Categories and Revision Triggers

A final report should distinguish what the inquiry observed, what it inferred, and what remains a proposal. This subsection sets out a practical way to make those distinctions accessible to readers who did not follow the study's development.

A descriptive finding should identify the source, unit, period and coverage. A conditional identification finding should name the target and assumptions, report the compatible set, and distinguish structural uncertainty from sampling uncertainty. A causal finding should specify the treatment versions, comparison, population and interference conditions. A historical finding should identify evidence concerning the earlier event or disposition. A decision recommendation should state its evaluative premises, resource constraints and responsible authority.

Each finding should also identify a revision trigger. Discovery of an unrecorded request channel might change the interpretation of the application data. An implementation failure might narrow the intervention to which a causal estimate applies. New follow-up evidence might alter an assessment of persistence. A participant challenge might reveal that a category combines activities with different meanings. These revisions should preserve the earlier record and state why the conclusion changed.

The report's central achievement would be a clear inferential path from policy-conditioned observations to a bounded conclusion. That path should be short enough to inspect and detailed enough to challenge. It enables evidence to play a serious role in public decision-making while keeping the assumptions and judgments that connect evidence to action visible.

30.7 Rival Explanations for a Rise in Recorded Use

A rise in recorded use after a policy change can support several explanations. This subsection develops an explicit rival-explanation analysis for the prospective study. The task is to identify observations that bear differently on the explanations, rather than treating a plausible narrative as a causal finding.

One explanation is removal of a practical barrier. Evidence supporting it would connect earlier unsuccessful attempts to the specific condition changed by the intervention and show that the resulting offer became usable. A comparison with people facing different conditions could add information, subject to its design assumptions. The explanation becomes weaker if the relevant barrier remained unchanged or if the people who began participating never encountered it. An increase in attendance alone cannot establish this mechanism.

A second explanation is greater awareness of an existing opportunity. The study would inspect what information people received and whether the offer's practical conditions changed. An announcement can alter expectations as well as knowledge. Participants may interpret the communication as a sign that their activity is welcome. The mechanism should therefore be described at the level supported by the evidence. If information and institutional encouragement cannot be separated, the report can identify the effect of the communication package while leaving its components open.

A third explanation is formation through experience. The common-offer comparison can establish whether a specified earlier exposure changes a later response under the study assumptions. Participant accounts can describe learning, relationships and changing interpretation. The report should avoid claiming a particular psychological mechanism solely because the average contrast is positive. The formative package may also reduce logistical uncertainty or introduce a necessary companion. Those pathways can coexist and may require a more detailed design to separate.

A fourth explanation is improved recording. The intervention may introduce staff attendance checks or a booking system that captures previously informal events. Independent observations can investigate whether actual activity rose alongside the recorded count. If both measures were altered at the same time, the study should retain the resulting ambiguity. A well-documented measurement improvement is still useful, but its numerical increase should not be reported as a behavioral effect without a bridge.

A fifth explanation is substitution. People may move from another branch, another time or another venue to the new activity. Local attendance then rises while total participation across the relevant system changes less. The study should define whether substitution is a problem for the target or part of the policy's intended benefit. A more suitable location can improve access even if the number of events is unchanged. Evidence about prior and alternative uses can clarify that question without assuming that every new local attendance is an additional activity in the wider population.

A sixth explanation is a concurrent change outside the intervention. A schedule change in the surrounding community, a related program or an unusual event could alter participation. Random assignment can protect a specified comparison under its assumptions, while a simple before-and-after record requires stronger reasoning about such changes. The report should identify which rival explanations the design addresses and which remain possible. A list of conceivable confounders is less useful than an account of the mechanisms relevant to the actual setting.

30.8 Rival Explanations for Stable or Falling Use

An unchanged or lower count deserves the same analytical care. This subsection develops the symmetrical inquiry so that the method remains open to unfavorable findings about the proposed activity.

Stable use can reflect low additional demand, incomplete implementation, remaining barriers, capacity saturation, displacement between activity types, or a delay before a group can coordinate. These explanations imply different evidence needs. Capacity saturation calls for application and allocation records; incomplete implementation calls for offer-version and resource records; coordination calls for information about the required group and timing. The analyst should inspect the planned measures before adding a new post-hoc explanation.

Falling use can indicate that the revised offer is less suitable, that some participants dislike the new format, or that another opportunity has become more attractive. Such findings can support redesign or closure of the tested package. The study should preserve negative experiences and reasons for withdrawal, including accounts that challenge its initial interpretation of unmet demand. A research program committed to understanding generative opportunities must also investigate when a proposed opportunity fails to produce something people value.

The final report should state which explanations remain compatible and which have been weakened by evidence. It should identify a practical next decision without pretending that every mechanism has been point identified. This approach makes both favorable and unfavorable results usable. The inquiry retains its direction through a clear target and a disciplined response to evidence, rather than through a commitment to a predetermined policy outcome.

31 Operational Validation Cases

The preceding models identify distinct ways in which a record can remain uncertain. This section develops worked cases that show how those distinctions change an evaluation's next action. It considers the allocation of audit effort, duplicate and missing records, calibration that fails to transport, a zero count after recruitment, and the interpretation of a successful formative offer. Every numerical case is constructed for analysis. The calculations describe conditional possibilities rather than measured library outcomes.

The organizing question in each case is what additional work would resolve. An audit can correct existing labels, contact can expand coverage, an experiment can identify a policy contrast, and deliberation can clarify the value assigned to an outcome. These activities should be chosen according to the uncertainty that matters for the decision. A more precise attendance count has limited value for a question whose central ambiguity concerns a different offer or a disputed public purpose.

31.1 Allocating Effort between Existing Records and Missing People

The first case compares two forms of additional observation within a fixed target population. Suppose a completed pilot concerns eighty people, and the target is their actual binary participation during the declared pilot period. Records cover fifty people, with eighteen positive labels. The analysis allows at most two false positives and four false negatives among those covered records. The remaining thirty people have unresolved participation labels.

The finite error-budget result gives a lowest possible population total of sixteen and a highest possible total of fifty-two. The corresponding proportion ranges from 0.20 to 0.65. The lower endpoint removes two possible false-positive records. The upper endpoint adds four possible false negatives to the eighteen positive labels, then permits all thirty uncovered people to have participated. The interval is the convex hull of the attainable lattice with step $1/80$.

Consider an audit of ten covered records, six of which were originally positive. Suppose the audit verifies one false positive and two false negatives. Its actual positive total is seven. Forty unaudited covered records remain, with twelve positive labels. The residual caps are one false positive and two false negatives. The updated total ranges from $7 + 12 - 1 = 18$ to $7 + 12 + 2 + 30 = 51$, giving proportions from 0.225 to 0.6375. The audit improves the result while leaving most of the coverage uncertainty intact.

Now suppose a separate recovery effort resolves twenty of the thirty previously uncovered labels and verifies five positives among them. The known actual positive total becomes twelve: seven from the original audit and five from the recovered records. Ten labels remain uncovered. The unaudited covered records and residual error caps remain unchanged. The population total now ranges from $12 + 12 - 1 = 23$ to $12 + 12 + 2 + 10 = 36$, or from 0.2875 to 0.45.

The comparison shows why coverage recovery can matter more than repeated review of already constrained labels for this particular target. It also shows why that conclusion is conditional. The recovered

labels must refer to the same pilot period and participation definition. If the recovery interview instead elicits current interest in a future activity, its five positive answers cannot be inserted into the participation total. The arithmetic would remain executable while the target identity failed.

Cost and burden also matter. Recovering twenty missing labels may require much more effort than auditing ten existing records. Some recovery routes may be intrusive or impractical. The decision about further work therefore combines expected information with cost and participant burden. The calculation makes the information component visible. It does not provide the full allocation objective or establish that population precision should override every other consideration.

31.2 Duplicate Entries and the Unit Being Counted

The second case concerns a record whose positive labels include duplicate entries. A duplicate can inflate a count, but whether it constitutes an error depends on the target unit. This subsection distinguishes person, visit, event, and group counts before proposing a correction.

Suppose one person attends three sessions and appears once in each session log. For a count of attendances, all three records can be correct. For a count of distinct participants, the records must be linked so that the person contributes once. The same file therefore supports two legitimate totals under different counting rules. Calling two of the entries duplicates without naming the target would treat a unit conversion as a clerical error.

Now suppose a single attendance is entered twice because a paper register and a digital form are both imported. For either a person count or an attendance count, the duplicate can create an overcount unless the records are reconciled. The evidence needed for reconciliation includes the event, time, and linkage information sufficient to establish that the entries refer to the same occurrence. A matching name alone can be insufficient when names are shared or entered inconsistently.

A group creates a further unit problem. One organizer may register a meeting for six people, while only four attend. A file can contain one booking, six expected participants, four actual attendances, and perhaps one continuing group. Each measure describes a different feature. The evaluation should retain these distinctions when they matter to the question. A single generic field called participation invites accidental substitution among them.

The proposed correction process begins by specifying the target unit and linking records within that unit. It then assigns each suspected duplicate a resolution status: confirmed common event, confirmed distinct event, or unresolved. Unresolved pairs can be carried through a sensitivity analysis when their number matters. Automatically merging every similar record can undercount legitimate activity; automatically retaining every entry can overcount. The review should state which evidence supports the linkage decision.

This work also affects denominator construction. A person appearing in several recruitment routes should normally contribute once to a person-level cohort. An event appearing in several reporting systems should contribute according to the selected event unit. A consistent numerator with an inconsistent denominator can still produce a misleading proportion. The validation record therefore reviews both sides of the ratio rather than correcting only the visible positive count.

31.3 Calibration Transport across Activity Types

The third case examines an apparently useful recording correction whose transport fails. Suppose a validation exercise establishes a credible recording channel for scheduled, staff-supervised sessions. The evaluation then considers applying it to informal activity in an open area. The question is whether the two observation processes support the same parameters.

The scheduled session may have a register, a named facilitator, and a defined start and end. Informal activity may have none of these features. Participants may join and leave gradually, and staff may reasonably avoid interrupting them to collect data. The difference concerns how the record is generated. A shared institutional location and a common label of library use do not establish channel stability.

Applying the scheduled-session sensitivity to the informal activity can produce a narrow corrected interval. That interval would be conditional on an unsupported transport assumption. The appropriate next step is to investigate the informal observation process directly or use a wider sensitivity range that reflects the unresolved channel. The more conservative range can be scientifically preferable because it preserves the actual uncertainty rather than importing precision from a different event.

The study can also revise the target. If reliable person-level counts would impose disproportionate observation on informal activity, an event-level description or a qualitative account of use may be more appropriate. Such a revision changes what can be concluded. It can still provide useful evidence about the existence and character of an activity without pretending to estimate its population prevalence precisely.

This case illustrates a general validation principle: the audit standard and recording channel should be suited to the activity, rather than requiring every activity to resemble the most easily counted program. Otherwise, evaluation incentives can encourage institutions to privilege forms of participation that produce tidy records. That possibility is a conditional institutional concern, requiring empirical investigation in an actual setting. The design can reduce it by preserving multiple evidential forms with clearly stated roles.

31.4 Zero Responses after Selective Recruitment

The fourth case concerns a survey that receives no positive answers to its focal question. Suppose the original invitations were sampled through a credible population frame, but only some recipients completed the instrument. The reported zero belongs to the completed-response set. Its population interpretation depends on the selection and measurement processes after invitation.

The evaluator first checks whether every completed respondent encountered the focal question. A skip rule may have omitted it for people who reported no recent library use. If the target is interest in a new activity among nonusers, that skip rule could exclude the people most relevant to the question. The zero would then summarize a restricted question path. The appropriate repair concerns questionnaire routing, not a more elaborate estimator applied to the same empty field.

Next, the evaluator distinguishes a negative answer from an unanswered item. A form that records both as zero creates a classification problem. Recovering the original response state can reveal whether the observed zero reflects actual negative answers or missing data. If the original state was never retained, the analyst must preserve that uncertainty rather than reconstructing responses from an administrative default.

The response stage then requires assessment. People who completed a long survey may differ from those who declined or stopped early. A probability invitation design alone does not establish that the final respondents form a simple random sample of the target labels. The exact zero-count calculation from Section 10 applies only when its selection and measurement conditions hold. An alternative response model may support inference, but it must be stated and justified.

Finally, the content of the question matters. A zero count of favorable answers to an abstract description can coexist with uptake after a concrete introduction, or with a different response to another offer. The survey result remains evidence about the stated question under its elicitation conditions. The next research step depends on whether the decision concerns that expressed response, a more specific offer, or a formative intervention. The number zero alone cannot select the next step.

31.5 Exposure Effects and Retrospective Interpretation

The fifth case concerns a result that appears encouraging for a new activity. Suppose a study randomly assigns a formative introduction, then gives both groups the same final offer. Assume the assignment, observation, and interference conditions required for the declared contrast are satisfied. The treated group shows higher uptake under the common offer. The result supports an effect of the exposure history on that response.

The interpretation should retain the common offer in the estimand. Both groups encounter its description, practical conditions, and any interest it creates before participation. The contrast therefore

compares two histories that share a final intervention. It does not compare a formed interest with an untouched state of mind. This precision leaves the effect useful: the library can learn whether the additional introduction improves participation under an offer it can actually deliver.

The result leaves individual transitions partially identified when each person appears in only one exposure condition. Higher aggregate uptake can combine newly positive responses with some newly negative responses. The marginal-response bounds specify the compatible proportions. An assumption that the introduction can only increase response would sharpen the result, but it needs substantive support. A more detailed introduction might reasonably deter someone who learns that the activity does not suit them.

The retrospective claim remains different. The higher uptake does not establish how much interest existed before the study began. The earlier countermodel permits identical participation after opening while pre-exposure dispositions differ. Qualitative accounts and historical records can add evidence about that earlier period, but their interpretation requires the separate bridge already discussed. The experiment's success should not be used to bypass that requirement.

The practical recommendation can therefore focus on the supported prospective target. If the introduction produces valued participation at an acceptable cost and respects the relevant public commitments, it may be worth continuing. Those evaluative conditions need assessment beyond the uptake contrast. The policy can be justified through its contribution to present and future opportunities even while the prevalence of an earlier latent interest remains unresolved.

31.6 Evidence Resolution and Decision Resolution

The final case separates learning more about an outcome from settling a policy choice. Suppose additional validation narrows the participation range substantially. The remaining question is whether that improvement changes the decision-relevant uncertainty or merely refines a quantity whose value is still disputed.

If the choice follows an explicitly justified benefit-and-cost threshold, a narrowed range can move entirely to one side of that threshold. The earlier conditional decision result then supplies a robust conclusion within the model. If the range still crosses the threshold, further information may matter. The calculation should use the quantity to which the benefit premise actually applies. Substituting attendance for beneficial participation requires an evaluative and empirical bridge.

Some choices remain disputed even with exact counts. One group may prioritize a quiet environment, another a shared discussion space, and another accessible hours for people with constrained schedules. An attendance total cannot determine how those claims should be weighed. Additional measurement can describe the consequences of alternatives, while deliberation addresses the public reasons for choosing among them. Treating the disagreement as merely a lack of demand data would mislocate the issue.

The evaluation can make this distinction visible in its final record. It states which empirical claims became better supported, which counterfactual claims remain conditional, and which value judgments still determine the recommendation. This structure allows a decision to proceed under uncertainty without pretending that every uncertainty has been eliminated. It also identifies what later evidence could reasonably trigger reconsideration.

The worked cases give the formal analysis a practical discipline. Each proposed data improvement should name the target it improves, the assumption it tests, and the decision it can inform. Work that cannot answer those questions may still have exploratory value, but it should be labeled accordingly. The strongest evaluation connects its effort to a consequential inference while preserving the distinction between an observed record, an interpreted demand claim, and a publicly justified choice.

32 Analysis Specifications for a Prospective Study

A prospective study needs an analysis record that connects its fields to the claims it intends to make. This section specifies that connection for the proposed library comparison. It separates target registration,

data reconciliation, assignment analysis, missingness, participant outcomes, qualitative integration and decision reporting. An elementary finite assignment calculation makes one form of statistical uncertainty explicit while preserving the other uncertainties that the study must address.

The specification is conditional on a feasible study design. Actual branch participation, sample size, outcome distributions and recording quality remain empirical obligations. The purpose is to make the planned analysis concrete enough to review before data are collected, including the circumstances under which a preferred comparison would need to be narrowed or abandoned.

32.1 The Pre-Analysis Target Record

Before assignment, the study records the focal population, the offer versions, the prior-history boundary, the primary outcome, the observation window and the policy contrast. It also records the implementation conditions required to interpret that contrast. These entries should be understandable independently of a statistical formula.

For a branch comparison, the population record identifies the baseline cohort attached to each branch and explains how cross-branch membership is handled. The offer record specifies the conditions available under each policy. The outcome record distinguishes a request, allocation, attendance and any participant-only assessment. The history record identifies which introductions, questionnaires or prior experiences occur before the comparison begins.

The primary outcome should follow the decision the study is meant to inform. A count of distinct participants can assess reach under a specified offer. A group-occurrence indicator can assess whether a coordinated activity becomes possible. A participant-experience score addresses another question and should not silently replace the primary participation target when its result is more favorable. Secondary outcomes can be retained with their purposes stated in advance.

The target record also identifies the unit of assignment and the interference model. If branches share a binding staff pool, treating their outcomes as isolated may be untenable. If participants move freely among branches, the study needs a justified exposure boundary or a system-level target. These issues should be resolved through the operational review before the analysis claims a branch-specific causal effect.

32.2 Population and Record Reconciliation

The first analytical step reconstructs the target cohort and the observation route. It checks that each person or group is represented according to the intended unit, that duplicate recruitment routes are linked, and that later entrants remain distinguishable from baseline members. The objective is a defensible denominator rather than the largest possible count.

The record should retain the reason for every population exclusion. A person outside the declared eligibility rule differs from an eligible person who could not be contacted. A person who declined the research differs from a person who declined the activity. An incomplete questionnaire differs from an incomplete application. These distinctions determine which missingness model applies and which target remains measurable.

The outcome reconciliation then checks the event unit. Multiple records can refer to one attendance, while one group booking can involve several people and several meetings. The analysis preserves the mapping from source records to the derived unit. A correction should record its reason and the evidence used, allowing another reviewer to assess whether the change concerned duplication, classification or a revised target.

Unresolved records remain visible. The study can report the range of counts compatible with them or use a justified validation model. A clean final spreadsheet is not a sufficient reason to resolve every ambiguity through an arbitrary default. The earlier finite bounds provide a transparent account of what remains unknown when the target labels cannot be fully recovered.

32.3 The Primary Assigned-Policy Comparison

When the fixed-cohort branch design is supported, the primary comparison uses the population-weighted target from Section 16. Each branch contributes according to its cohort size and known assignment probability. The analysis should retain the actual assignment design, including any dependence induced by balancing or resource constraints.

The assigned-policy effect includes the implementation delivered under assignment. A branch that fails to provide part of the planned offer remains assigned to its original condition for this target. The delivery record explains what occurred. Reclassifying branches only after observing successful implementation can select a different comparison and requires a separate causal argument.

The report should present the policy-specific totals and the weighted contrast with their population meaning. An equal-weight average across branches can be included when an average-branch target is useful, but it should receive that label. Different cohort sizes make the two targets different even when both calculations are correct. The choice should follow the question rather than the result that produces the largest estimated effect.

The analysis should also distinguish a fixed baseline cohort from the current population at follow-up. Entry can be a valuable outcome of expanded access, while missing follow-up among baseline members creates another uncertainty. The membership decomposition can describe these changes alongside the primary fixed-cohort comparison. Combining them into one denominator would conceal the distinction the study was designed to preserve.

32.4 A Conservative Finite Assignment Calculation

The expectation result alone does not show how precise an estimate will be. The following calculation provides a deliberately conservative guarantee under a stronger design than the general branch comparison. Its purpose is to make that distinction explicit through a complete derivation.

Suppose branch assignments are independent Bernoulli draws with fixed known probabilities $p_j \in (0, 1)$. Potential branch totals $T_j(a)$ are fixed under the design and satisfy $0 \leq T_j(a) \leq N_j$, with $N = \sum_j N_j$. Outcomes are accurately recorded, and the earlier interference restriction holds. Define the branch contribution

$$Z_j = \frac{A_j T_j(1)}{p_j} - \frac{(1 - A_j) T_j(0)}{1 - p_j}, \quad \hat{\tau} = \frac{1}{N} \sum_j Z_j. \quad (58)$$

These are the contributions to the previously derived unbiased estimator.

The two possible values of Z_j differ by $T_j(1)/p_j + T_j(0)/(1 - p_j)$. Direct expansion of its second moment gives

$$\begin{aligned} \text{Var}(Z_j) &= p_j(1 - p_j) \left[\frac{T_j(1)}{p_j} + \frac{T_j(0)}{1 - p_j} \right]^2 \\ &\leq \frac{N_j^2}{p_j(1 - p_j)}. \end{aligned} \quad (59)$$

The inequality substitutes the maximum possible total in each policy condition. It is a worst-case bound and uses no estimated outcome variance.

Proposition 32.1 (Conservative finite assignment coverage). *Under the independent-branch conditions, let*

$$B = \frac{1}{N^2} \sum_j \frac{N_j^2}{p_j(1 - p_j)}. \quad (60)$$

For $0 < \alpha < 1$, the set

$$\left[\hat{\tau} - \sqrt{B/\alpha}, \hat{\tau} + \sqrt{B/\alpha} \right] \cap [-1, 1] \quad (61)$$

contains the fixed population effect τ with assignment probability at least $1 - \alpha$.

Proof. Independent assignments make the branch contributions independent conditional on the fixed potential-total schedule. Summing equation (59) and dividing by N^2 gives $\text{Var}(\hat{\tau}) \leq B$. The estimator is unbiased, so its mean squared error equals its variance. For any $r > 0$, $(\hat{\tau} - \tau)^2 \geq r^2 \mathbf{1}\{|\hat{\tau} - \tau| \geq r\}$. Taking expectations yields a tail probability at most B/r^2 . Set $r = \sqrt{B/\alpha}$. The true effect lies in $[-1, 1]$, so intersecting with that known range preserves the coverage event. $\square$

For equal branch sizes and assignment probabilities one half, the variance bound becomes $B = 4/J$, and the radius is $2/\sqrt{J\alpha}$. With few branches, this worst-case radius can be so wide that the interval conveys little beyond the known effect range. The result remains mathematically valid while demonstrating poor guaranteed precision. It should not be presented as an efficient analysis or a sample-size recommendation.

The independence restriction is essential to the variance addition. A design that assigns a fixed number of branches to treatment generally induces dependence. The earlier unbiasedness result can still apply, but covariance terms enter its variance. The current confidence calculation therefore cannot be transferred by retaining the same marginal probabilities and ignoring the assignment relation.

The guarantee also concerns assignment uncertainty alone. Misclassified attendance, incomplete cohort coverage, unsupported intervention versions or cross-branch effects can invalidate the target connection even when the probability calculation is correct. Those uncertainties remain separate parts of the analysis. A narrow statistical interval should never be presented as if it had resolved them automatically.

32.5 Measurement Sensitivity and Assumption Revision

The measurement analysis should identify which observations are verified, which remain uncertain, and which restrictions are being imposed on the uncertain records. It can then calculate the corresponding finite or probability-law bounds. The choice of restriction should have an evidential explanation.

An error-cap analysis can display several plausible finite caps when the data do not justify one exact value. A channel analysis can display a justified range of sensitivities and false-positive rates. A coverage analysis can retain unknown labels explicitly. These are conditional analyses. Their numerical precision should reflect the strength of the evidence supporting their inputs, not the ease with which an analyst can select a convenient parameter.

Validation can contradict a proposed restriction. An audit that finds more errors than a cap permits makes that model incompatible with the audit. The analysis should record the failure and explain the revised restriction. It should not silently truncate a residual budget or delete the contradictory record to preserve the earlier result. The revision itself is part of the study's learning.

Measurement sensitivity should also be evaluated in relation to the decision. A range may leave a recommendation unchanged under a declared value model, or it may permit competing choices. The report should state that relationship without treating it as a reason to suppress uncertainty. A decision robust to the current measurement range can still depend on an unresolved target or value premise outside that range.

32.6 Participant Outcomes and Questionnaire Completion

The participant-outcome analysis should report participation shares, questionnaire response shares and respondent outcomes separately. The common-stratum bounds in Section 18 can then be applied under their individual monotonicity and measurement assumptions. The raw attendee mean remains available as a descriptive result with its selected-population label.

If the full outcome distribution is retained and a verified distributional method is appropriate, it may provide more information than the mean-only bounds. The analysis should state which information and assumptions produce the narrower result. It should also preserve the possibility that the outcome scale has different meanings across offer versions. A technically refined bound cannot repair an undefined or poorly interpreted score.

Missing questionnaires create uncertainty among people who actually experienced the activity. Non-participation creates a different outcome-applicability issue. The data structure should retain this distinction through every derived table. A composite event such as participation with a positive assessment can be reported when useful, but it should not replace the experience target without an explicit change in the question.

The analysis should permit effects on different groups to differ. A new offer can support previously participating people while giving newcomers an unsuitable experience, or improve newcomers' opportunities while disrupting an established group. The common-stratum result, the induced participation share, and qualitative accounts can illuminate different parts of that pattern. None alone supplies a complete public evaluation.

32.7 Integration of Episodes and Quantitative Comparisons

Qualitative episodes can explain what an intervention component meant, identify a missing stage, or reveal an alternative mechanism. Their integration should preserve the population and target each episode concerns. A vivid account should not be treated as an estimate of prevalence unless the sampling and measurement design supports that use.

The analysis can organize episodes around the participation route: awareness, consideration, request, allocation, attendance, experience and continuation. Within each stage, it records the supported description and serious rival interpretations. This organization connects the accounts to the quantitative fields without forcing every experience into a single numerical code.

An episode can challenge a model assumption. A documented displacement can weaken individual monotonicity. A cross-branch invitation can challenge the interference boundary. A participant's explanation of a misunderstood question can challenge a measurement bridge. The appropriate response is to identify which result depends on the challenged assumption and assess whether its scope should change. The episode's value lies in that inferential role, not only in whether it illustrates the main narrative.

The integration can also support the model. An account may support the interpretation that an unavailable time prevented a particular attempt. The analysis should distinguish a documented schedule incompatibility, the participant's stated reason, and the further claim that a feasible time would have produced participation. Records can verify delivery of a common offer when they document its specified components; an account of refusal can support an interpretation of considered preference. These forms of evidence narrow different uncertainties. The report should preserve supportive and adverse evidence with comparable attention to their limits.

32.8 Exploration, Confirmatory Claims and Policy Selection

The study can generate useful questions beyond its original comparison. A new group, an unexpected barrier or a different activity format may emerge. The analysis should distinguish these exploratory findings from the target and assumptions specified before assignment.

Exploration can guide a later offer or a revised study. It can also reveal that the original question was poorly chosen. Those are substantive contributions. The distinction becomes important when the same data are used to select the most favorable policy and to claim that its expected outcome has been estimated without selection effects. The sequential counterexample shows why a fixed-policy evaluation argument cannot simply be reused after such selection.

A staged research program can use early data to formulate a target policy and later independent data to evaluate it. Other approaches may be possible with suitable methods. The report should state which route it uses and what uncertainty remains. The purpose is to preserve the evidential meaning of learning, rather than discourage adaptation when the initial plan proves incomplete.

32.9 Decision Reporting and the Scope of the Conclusion

The final report should connect the supported empirical claims to the actual decision. It states what offer was evaluated, whom the result represents, which outcomes were measured, which assumptions were supported or challenged, and how uncertainty affects the proposed action. It then states the public reasons that convert those findings into a recommendation.

A useful conclusion can be narrower than the original ambition. The study may establish an effect of an introduction under a common offer while leaving earlier latent demand unresolved. It may identify a recording problem while remaining unable to estimate a population prevalence. It may show that an alternative venue is feasible for one group while leaving another group's opportunity unmet. Each result can inform a concrete action when its scope is clear.

The report should also identify the conditions for reconsideration. A changed population, a revised offer, new capacity, a better measurement bridge or evidence against a value premise can alter the decision. Preserving those conditions makes the evaluation part of an ongoing public inquiry. It prevents the completed study from becoming a permanent verdict detached from the circumstances that gave its findings meaning.

33 Consequences, Decision Criteria, and Further Inquiry

An identified response or a bounded interval becomes relevant to provision through a further account of consequences and public reasons. This section develops that transition explicitly. It begins with a deliberately simple value model, derives deterministic and randomized regret under a declared criterion, and examines the limits of that criterion for public decisions. It then distinguishes learning about a fixed response from an inquiry that changes the history and the response itself.

The calculations are conditional decision illustrations. Their numbers represent stipulated values, not measured benefits, costs or public preferences. They show what follows after an evaluative mapping and a decision rule have been adopted. They cannot establish that the mapping captures everything that matters or that the institution should use the selected rule. Keeping those stages separate is central to a responsible application of the demand analysis.

33.1 The Decision Alternatives and Their Consequences

Suppose the immediate choice is between retaining a specified current arrangement and expanding through a specified offer. The alternatives need the same level of detail used in the response model. Expansion includes its activity, population, capacity, staffing, allocation and history. Retention includes the existing opportunities and consequences that continue. A comparison between a concrete expansion and an undescribed status quo would conceal relevant costs and benefits.

For the scalar illustration, normalize the net value of retention to zero and define the net gain from expansion as $\Delta(\theta) = b\theta - c$, with $b > 0$. The response target satisfies $\theta \in [L, U] \subseteq [0, 1]$, where the interval is nonempty and closed. The constant b is the total value coefficient in the chosen scale, and c is the cost difference represented in that scale. A statement that $b = 100$ means that a unit change in the response proportion changes the stipulated index by one hundred units. It is not automatically a benefit of one hundred units per participant.

The model requires a bridge from response to value. If θ is attendance, the linear coefficient implicitly assigns a consequence to that attendance and treats the relevant difference as adequately summarized by the proportion. Heterogeneous experiences, displaced uses, staff burdens and distribution may require a richer representation. The present calculation should therefore be read as an intentionally limited decision model whose evaluative premises are visible.

The interval also has a specific meaning. It can represent values compatible with a complete finite record and a model, or an interval in a population-probability model. It is not a probability distribution over θ . A midpoint, endpoint or average of endpoints acquires a decision role only through an additional criterion. The evidence set alone leaves that choice open.

33.2 Robust Rankings and a Mixed Decision Interval

The earlier threshold model identifies cases in which the retained evidence supports the same ranking throughout the interval. Since $b > 0$, expansion's smallest gain is $bL - c$ and its largest is $bU - c$. Expansion is weakly preferred throughout when $bL \geq c$; retention is weakly preferred throughout when $bU \leq c$. Equality allows a tie at the relevant endpoint.

When $bL < c < bU$, both rankings remain compatible with the retained information. This result is a substantive finding about the combination of evidence and evaluative assumptions. It need not be described as a failure of analysis. The report has identified why the decision remains sensitive and where additional evidence or a further criterion would matter.

Consider the separate constructed values $b = 100$, $c = 35$, $L = 0.2$ and $U = 0.6$. At the lower response endpoint, expansion has gain -15 ; at the upper endpoint, it has gain 25 . The threshold response is 0.35 , inside the interval. A decision based solely on the interval and a requirement of uniform superiority cannot select one of the two policies.

The same evidence interval could support a robust ranking under a different cost or value premise. This sensitivity should be reported as a difference in the decision model, not a change in the observed demand evidence. Conversely, a narrower response interval can produce a robust ranking while the value assumptions remain fixed. The decision record should identify which premise drives any revision.

33.3 Deterministic Worst-Case Regret

A declared regret criterion supplies one way to choose within a mixed interval. At each compatible θ , regret is the best value available from the two policies minus the value of the selected policy. Maximum regret is the largest such loss over the retained interval. The criterion chooses the policy with the smaller maximum regret. This definition introduces a decision rule beyond the identification result.

The balancing argument has an established antecedent in Manski's treatment-choice analysis, which derives a two-treatment minimax-regret solution under an explicit welfare criterion [12]. The following proof reconstructs that reasoning for two complete library policies. Its lottery interpretation is specified separately from the source's allocation of population fractions to treatments. The calculation carries no claim of a new generic decision rule.

Let $V_r(\theta) = 0$ for retention and $V_e(\theta) = \Delta(\theta)$ for expansion. The best value at a state is $\max\{0, \Delta(\theta)\}$. Expansion regret is therefore

$$\max\{0, \Delta(\theta)\} - \Delta(\theta) = \max\{0, c - b\theta\}.$$

This expression weakly decreases with θ , so its maximum occurs at L . Retention regret is $\max\{0, b\theta - c\}$, which weakly increases and reaches its maximum at U . Thus

$$\mathcal{R}_e = \max\{0, c - bL\}, \quad \mathcal{R}_r = \max\{0, bU - c\}. \quad (62)$$

The constructed example gives $\mathcal{R}_e = 15$ and $\mathcal{R}_r = 25$. Under this criterion, deterministic expansion is preferred because its largest forgone value is smaller. The result differs from a requirement that expansion be better in every compatible state. It accepts the possibility of a loss at some states in order to limit the largest loss relative to the better policy at each state.

The criterion treats the interval as a set of relevant possibilities and measures losses through the stipulated index. It supplies no probability that the worst state will occur. It also does not establish that avoiding the largest regret is the only defensible institutional objective. A decision maker may have other public reasons or constraints. The calculation is useful because it makes one criterion's implications inspectable.

33.4 A Lottery over Complete Policies

The decision set can be enlarged to include a lottery that selects expansion with probability $p \in [0, 1]$ and retention otherwise. This is a lottery over complete policies. It differs from random assignment used

to estimate an effect and from a lottery allocating individual seats. The present model assumes that the lottery leaves each selected policy's conditional consequences unchanged and introduces no additional cost or legitimacy effect.

In the strict mixed case, define $r_L = c - bL > 0$ and $r_U = bU - c > 0$. Expected value under the policy lottery is $p\Delta(\theta)$. When expansion is better at a state, expected regret is $(1 - p)\Delta(\theta)$. When retention is better, expected regret is $-p\Delta(\theta)$. The largest losses on the two sides occur at the endpoints, giving

$$\mathcal{R}(p) = \max\{pr_L, (1 - p)r_U\}. \quad (63)$$

Proposition 33.1 (Minimum maximum expected regret). *Under the strict mixed interval and the stated complete-policy lottery assumptions, the unique minimizing probability and its maximum expected regret are*

$$p^* = \frac{r_U}{r_L + r_U}, \quad \mathcal{R}(p^*) = \frac{r_L r_U}{r_L + r_U}. \quad (64)$$

For $r_L, r_U > 0$, this value is strictly below the smaller deterministic maximum regret.

Proof. The term pr_L strictly increases with p , while $(1 - p)r_U$ strictly decreases. They are equal when $pr_L = (1 - p)r_U$, which yields $p = r_U/(r_L + r_U)$. To the left of that point, the decreasing term exceeds its value at the intersection. To the right, the increasing term exceeds its intersection value. The intersection uniquely minimizes their maximum. Substituting gives $r_L r_U/(r_L + r_U)$. Since both r_L and r_U are positive, this quantity is smaller than either of them. $\square$

In the example, $r_L = 15$ and $r_U = 25$, so $p^* = 25/40 = 5/8$. Maximum expected regret is $375/40 = 9.375$, below the better deterministic value of fifteen. The improvement concerns expected regret over the policy lottery. A realized expansion at the lower endpoint can still incur regret fifteen, and a realized retention at the upper endpoint can still incur regret twenty-five.

The boundary cases should be handled directly. If expansion weakly dominates throughout the interval, choosing it gives zero regret. If retention weakly dominates, choosing retention gives zero. If both values tie throughout the interval, every lottery has zero regret. The interior formula should not be extended by dividing through quantities that cease to be positive outside its stated domain.

33.5 Public Admissibility of Randomized Decisions

The mathematical improvement under expected regret leaves the public admissibility of the lottery open. A lottery can alter expectations, planning, trust or the distribution of consequences. If those effects matter, the conditional values in the model require revision before the calculation applies.

A complete-policy lottery may be difficult to justify when participants have organized around a promised opportunity or when one alternative fails an institutional obligation. An obligation can operate as a constraint on the feasible decision set, rather than a small cost that can be averaged away. Removing an inadmissible policy changes the comparator and can make the lottery irrelevant. The study should state such constraints before optimizing the scalar index.

The lottery may also have a different meaning from a fair allocation procedure. Randomizing among equally eligible applicants can serve a particular distributional purpose. Randomizing whether a whole activity will exist serves another purpose and affects different people. The regret proof establishes nothing about equal treatment among applicants because its units are complete policies and compatible states of the response target.

Public explanation matters even where randomization is permissible. People may reasonably ask why a favorable policy can lose a draw or why uncertainty is handled through chance rather than additional inquiry. The response would need to identify the criterion, alternatives, consequences and institutional authority. A formula that minimizes an expected loss cannot supply that explanation on its own.

The example is therefore a test of the separation between proof and recommendation. The proof can be correct, the scalar model can be internally coherent and the lottery can still be unsuitable for the institution. That possibility is part of the analysis. Mathematical optimization identifies a consequence

of a criterion; public judgment assesses the criterion, its representation of the situation and the permitted actions.

33.6 The Response-to-Value Bridge

The coefficient b deserves scrutiny because it carries much of the evaluative work in the simple model. Attendance can be associated with different experiences and purposes, and the same attendance proportion can have different public significance across offers. A response estimate becomes a value estimate only through a supported or explicitly stipulated relationship.

If the activity's value depends on learning, agency or a collective contribution, the study needs evidence about those outcomes. A high response can coexist with modest benefit, while a small recurring group can achieve a purpose participants regard as important. The experience and agency analyses explain why the number of attendees alone supplies an incomplete ranking.

The bridge can also vary across people. Additional participation by people who previously lacked a workable opportunity may have a different distributional significance from repeated attendance among those already well served. A scalar average can represent such a judgment only after weights or constraints are declared. The analyst should avoid allowing a convenient equal-count measure to decide the public priority implicitly.

Costs need comparable care. The institution's budget can omit participants' travel, organizational work or effects on other users. Some consequences may be difficult to express in the chosen money or value units. The response should be to preserve them as additional considerations or constraints, rather than assume that an unpriced consequence is zero. A more complex representation can be warranted when the omitted dimension changes the decision.

The linear model remains useful as a transparent illustration. Its simplicity makes the boundary between evidence and evaluative premise visible. A real application may replace it with a vector of consequences, a set of admissibility conditions or a deliberative comparison. Such a change is an improvement when it represents a material feature that the scalar model concealed.

33.7 Distribution and Constraints beyond a Total Index

Two policies can have the same total value index while distributing opportunity and burden differently. This subsection develops the implication for the regret calculation. A decision criterion applied to a scalar total can miss distinctions that the public process regards as decisive.

Suppose an expansion uses a room at the only workable time for a small existing group. Another arrangement produces the same total attendance at a higher institutional cost while preserving that group's access. A total index can favor the cheaper arrangement if it assigns insufficient weight to the lost opportunity. The conflict concerns the representation of consequences and public priorities, not an error in the attendance arithmetic.

A constraint can require that an existing essential service remain available, that a promised arrangement be honored for a stated period, or that a participation condition meet an agreed standard. Whether such a constraint is justified is a public question. Once adopted, it changes the feasible policy set. A regret calculation over policies that violate the constraint would answer a different decision problem.

Some disagreements can be represented through several admissible evaluative specifications. The report can show which policies are preferred under each and identify where they agree. Agreement across plausible value specifications can strengthen a practical recommendation. Disagreement should remain explicit rather than being hidden by an unexplained average of weights.

The task is proportionality. A minor difference may not justify a complicated multidimensional analysis. A consequential exclusion or commitment may require it. The evaluator should identify which dimensions can change the decision and explain why the chosen representation is adequate for those dimensions. The simplicity of the final report should be earned through that assessment.

33.8 Identification Ranges and Assignment Confidence

The paper contains several kinds of interval. Their roles in a decision must remain distinct. An identified set records compatibility with a model and retained information. A confidence procedure has a repeated-sampling or assignment guarantee. A sensitivity interval varies an explicitly uncertain assumption. Combining them requires an account of the joint target and guarantee.

The conservative branch interval in the constructed study addresses assignment uncertainty under accurate outcomes and a fixed cohort. It leaves recording error, transport and the response-to-value bridge outside its guarantee. Treating that interval as if it covered every uncertainty would overstate what the design established. Conversely, a sharp finite error-budget interval contains no confidence level unless a sampling model has been added.

A report can present these layers separately when a fully combined procedure has not been justified. It can state the assignment estimate under validated records, show how recording assumptions affect the target range and examine transport or value sensitivity. Such a presentation may be less compact than one final interval, but it preserves the reasons the conclusion changes.

The choice of decision criterion can depend on the status of the interval. Applying worst-case regret to a confidence set is a distinct procedure from applying it to a set known to contain the target under exact model restrictions. The resulting decision guarantee would require analysis of the confidence procedure and losses. The static proof above does not supply that guarantee by itself.

This separation also guides communication. A reader should be able to tell whether a wide range comes from few assigned branches, unobserved people, uncertain classification, unsupported population transfer or disagreement about value. Those causes suggest different responses. A single undifferentiated statement of uncertainty can obscure the most useful next step.

33.9 Additional Evidence and Decision Sensitivity

Further inquiry is valuable when it can change a consequential conclusion or improve the basis for action at an acceptable cost. This subsection examines that relation without assuming a probability distribution over the unknown response or claiming a general optimal research design.

If the retained interval lies wholly above the expansion threshold under the accepted value model, more precise response information may leave the ranking unchanged. It can still matter for capacity planning, distribution or public accountability, but the reason for collecting it should be stated. Precision alone is an incomplete measure of research usefulness.

If the interval crosses the threshold, additional information may resolve the ranking. The useful information depends on why the interval is wide. An audit of record errors can narrow one component; broader population coverage can narrow another; a credible comparative offer can address a causal response; and direct discussion can clarify an evaluative premise. These tasks should be compared through the uncertainty they can actually reduce.

The analyst should distinguish the possible benefit of a test from its expected benefit. A test can in principle produce a decisive result while usually leaving the decision unchanged. An expected value of information requires a model of possible test outcomes and a decision criterion, including research cost. The current interval alone supplies neither. A transparent proposal can state the possible resolution and its assumptions without inventing a probability of success.

Research can also be stopped for good reasons while some uncertainty remains. The remaining range may have little practical consequence, the required data may be too burdensome, or the institution may have a defensible provisional action with a review mechanism. The stopping decision should preserve the unresolved claim and the reason further inquiry was judged disproportionate. It should not turn incomplete information into certainty merely to close the project.

33.10 Inquiry that Changes the Response History

An investigation can change the opportunity and response it is intended to study. In that case its value cannot be described entirely as information about a fixed state. This subsection develops a separate constructed example using the actual-offer history target. The numbers and value index here differ from the preceding regret example.

Fix a later actual offer a . In the stipulated model, its response proportion is $\theta(h_0; a) = 0.1$ under the initial history and $\theta(h_1; a) = 0.5$ after a demonstration. The demonstration can supply information, familiarity and coordination; these effects are included in h_1 . The net value of subsequently expanding is stipulated as $100\theta - 30$, and the demonstration costs five additional units. Retaining immediately has value zero in this illustration.

Immediate expansion has value $100(0.1) - 30 = -20$. Demonstration followed by expansion has value $100(0.5) - 30 - 5 = 15$. These are consequences of the specified policy sequences under a deliberately complete assumed value model. The positive value of the second sequence cannot be called pure information value about a fixed θ , because the demonstration changes the history-indexed response in the stipulated construction.

The example leaves the actual-offer definition intact. The later offer's physical description can remain the same while its prior history changes. Its response includes any further formation produced by the offer itself. The comparison concerns two history-indexed potential responses, not a measured pre-existing essence of interest. A different model could assign response 0.5 under both histories and produce the same post-demonstration observation. Observing the later response alone would not establish the formative change.

The practical implication is that research design and service design can become connected. A demonstration intended to clarify preferences may also create a worthwhile opportunity. A survey can prompt a question; a group interview can establish contacts; a common offer can produce confidence through experience. The evaluation should record these effects as parts of the intervention sequence when they matter. Treating them as contamination to be ignored can misdescribe the policy actually delivered.

The reverse possibility also deserves attention. Research can impose burden, create expectations that cannot be met or discourage participation through a poor experience. A complete comparison of inquiry sequences should include those consequences. The fact that inquiry can form interest supplies neither a general reason to maximize exposure nor a guarantee that the resulting formation is desirable.

33.11 Learning Policies and the Choice to Wait

A decision can specify how the institution will act after receiving new evidence. This creates a policy over histories rather than a single static action. The sequential analysis provides a formal route for evaluating such policies under its stated support and common-process assumptions. This subsection connects that route to the decision to delay or stage provision.

A staged policy might offer a bounded demonstration, review attendance and participant accounts, then continue, revise or end the activity according to specified rules. The target includes the initial offer, the information gathered, the possible revisions and the consequences of each path. A favorable estimate for one final action does not identify the value of the entire staged policy unless the relevant histories are represented.

Waiting can have benefits and costs. It can permit better preparation, avoid a poorly understood resource commitment or allow a more informative comparison. It can also postpone a valued opportunity, lose organizational momentum or impose continued uncertainty on applicants. The evaluation should identify these consequences instead of treating delay as a neutral baseline outside the policy choice.

The earlier access-evidence feedback example shows a further danger. If evidence is generated only after provision, a decision to wait for evidence can preserve the absence that made the question unresolved. A staged exploratory offer can change that process. Its justification should include both the learning opportunity and the direct consequences of opening, with formative effects and resource burdens represented.

A policy selected after seeing the evaluation data requires care. The sequential counterexample demonstrates why a target chosen to match observed experience cannot automatically be evaluated with a formula derived for a fixed target. Independent evaluation, a justified adaptive design or an explicitly exploratory status may be needed. The decision record should identify when a policy was chosen and which evidence was used to choose it.

33.12 A Reviewable Decision Record

The final report should allow a reader to reconstruct the decision without repeating the entire study. This subsection specifies a concise structure that follows from the preceding analysis. The structure is a proposed reporting practice, with its usefulness subject to the method assessment.

The record first states the decision alternatives and the public purpose. It identifies the population, the complete offers and the consequences regarded as material. It then distinguishes verified facts, conditional estimates, identified sets, sensitivity assumptions and unresolved interpretations. The language should make clear which type of evidence supports each statement.

The next part states the decision criterion and constraints. If a scalar index is used, its coefficients and omitted dimensions should be visible. If a robust ranking is claimed, the record should show that the ranking holds across the retained uncertainty. If a criterion such as regret selects among mixed possibilities, the additional normative choice should be acknowledged. If legitimate disagreement remains, the record should identify its content and the authority responsible for resolving it.

The recommendation then follows from those premises. It can be to expand, retain, revise the offer, support a substitute, conduct further inquiry or make a bounded commitment with review. The record should specify what would prompt reconsideration: a changed cost, a new feasible alternative, evidence about an unsupported population or an observed consequence outside the current model. This makes revision a planned part of responsible action rather than an admission that the earlier decision lacked a basis.

The record should also preserve the distinction between the decision and the strongest empirical claim. An institution can reasonably act under uncertainty, but its action does not prove the uncertain proposition. Continuing a pilot does not establish high prior demand; ending one does not establish universal lack of interest. Accurate language allows practical judgment and epistemic restraint to coexist.

The decision analysis completes the paper's progression from records to public action. A demand target can be defined, a conditional result proved and a study designed without settling every value question. Additional criteria can support action, provided their premises remain visible. The most useful outcome is a decision whose evidence, uncertainties, consequences and reasons can be examined and revised by the people entitled to take part in that judgment.

34 Philosophical Interpretation of Demand

This section examines the philosophical commitments that remain after the formal distinctions have been made. It considers the contextual meaning of demand, the relation between formation and agency, the authority of expressed preferences, and the role of public institutions in enabling possibilities. The discussion develops the paper's own arguments and objections. Ontological questions remain suspended; the models organize inquiries into social facts without claiming to settle the ultimate nature of persons, preferences or institutions.

34.1 Contextual Responses and Stable Dispositions

An offer-indexed response is both more precise and more limited than an unqualified claim that someone possesses demand. This subsection examines the philosophical value of that limitation. It asks what stability an inquiry needs and what it loses by presuming stability too early.

Every practical desire has some conditions of expression. A person may want to join a discussion whose language they understand, whose time they can manage and whose participants they trust. Specifying those conditions helps identify the object of their response. It need not imply that the response is arbitrary or that the person has no enduring commitments. A stable commitment can express itself differently as opportunities change. Conversely, similar attendance patterns can arise from different purposes. The observable event and the continuity of a person's concern belong to distinct explanatory levels.

An investigator may reasonably seek a stable disposition when it supports a useful explanation or prediction. The burden is to show which variations in offer and history leave the relevant response structure stable. A latent variable earns its role through such work. It cannot obtain explanatory authority merely by receiving a name that sounds deeper than the observed event. The binary *W* construction in Section 11 therefore retains an interpretation gate: the model's initial values distinguish worlds, but a substantive account must explain what those values mean for actual people and how evidence could constrain them.

This stance avoids an unnecessary metaphysical choice. The study can examine stable patterns where evidence supports them and history-sensitive responses where the distinction matters. It can treat a person's account as meaningful while still investigating how an offer is understood. The object is a defensible inquiry into a particular practice, with its concepts open to revision when they cease to illuminate that practice.

34.2 Articulation and the Formation of a Question

A person may be unable to state a demand before encountering a language for the opportunity. This subsection distinguishes articulation from a simple recovery of an already complete inner preference and from the stronger claim that an institution creates every interest it elicits.

Suppose a resident initially says that the library offers nothing relevant to them. During conversation, they describe helping neighbors interpret documents and express interest in a place where that activity could occur collectively. The exchange may clarify a concern that existed in another form. It may also develop a new idea whose specific organization was absent before the conversation. Both descriptions can be plausible, and the person's own account can change as the possibility becomes more concrete. Treating the final proposal as a complete preference hidden behind the initial answer may erase the work of articulation.

The opposite claim would also be excessive. The fact that a question helped articulate a concern does not make the concern merely the investigator's creation. Participants bring histories, experiences, frustrations, hopes and relationships that constrain what the proposed activity can mean to them. An evaluation should examine the interaction between those histories and the elicitation process. It should preserve moments of disagreement or refusal that reveal where the investigator's categories fail.

This has a methodological consequence. Open descriptions and later structured questions can serve different functions. The former can reveal the language through which people understand an activity; the latter can support a defined comparison after the offer is clarified. Their order and influence should be recorded. A more articulate later answer can improve the inquiry without being retrospectively substituted for the earlier answer as though the conversation had simply removed noise from a fixed value.

34.3 Formation and Practical Agency

Interests often develop through participation. This subsection considers why that fact can be compatible with agency and identifies the additional conditions that make formation ethically and politically significant.

A person can learn to enjoy an activity, acquire the competence needed to take part, or discover that a shared practice supports an existing concern. Such formation can expand their practical repertoire. The resulting interest need not have existed in a fully articulated form beforehand to deserve respect. A library that provides an introductory experience can therefore produce a valuable opportunity even if its

effect is partly formative. The question is how the person participates in that formation and what choices remain available.

Agency requires more than a later positive response. A participant should have an intelligible account of the offer, room to decline or reinterpret it, and some practical ability to revise their involvement. The institution's role in forming an interest becomes more troubling when essential resources depend on displaying the desired response, when alternatives are concealed, or when withdrawal carries an unrelated penalty. These are proposed conditions for evaluating the process. Their presence in an actual case requires evidence.

The criterion should also avoid an impossible demand for influence-free choice. Every activity has a setting, language and history. The relevant distinction concerns forms of influence that participants can understand, question and respond to, and forms that undermine those possibilities. The study should investigate who controls the offer, how the purpose is explained, which alternatives are accessible and how participants describe their freedom to refuse. Those questions give the philosophical concern an empirical form.

34.4 Refusal and the Limits of Participation Goals

An evaluation organized around unmet demand can unintentionally treat every nonparticipant as a problem to be solved. This subsection argues for preserving refusal as a possible exercise of agency, while retaining the distinction between refusal and constraint.

A person may understand an accessible offer and decide that it holds little value for them. They may prefer solitary reading, another institution, private time or an activity outside the library's role. That response should be available within the study's categories. If the instrument assumes that every decline reflects misinformation, anxiety or exclusion, it converts the investigator's participation goal into a theory of the person. A serious demand inquiry must permit the hypothesis of low demand to receive support when the evidence warrants it.

Refusal can also coexist with constraint. Someone may decline the offered session because its schedule is incompatible while wanting a different offer. Another may decline because a prior experience makes the institution unwelcome. A third may prefer an alternative that is available only at a high cost. The interpretation depends on which offer is under review and what the person's reasons concern. A binary response becomes useful only when the qualifications needed for that interpretation remain available.

The public institution's responsibility is not exhausted by maximizing the number of positive responses. It may support a diverse range of opportunities so that different people can exercise different preferences, including the preference to use a space without joining an organized activity. An evaluation should therefore ask whether the policy improves the suitability and contestability of opportunities, alongside any change in participation. Those outcomes require their own concepts and evidence rather than being silently assumed from attendance.

34.5 Demand, Need, and Public Reasons

Demand and need can overlap without being identical. This subsection considers why a public-library decision may require reasons that a behavioral response alone cannot supply. It keeps those reasons explicit so that the formal model does not acquire an unintended normative authority.

A highly popular activity can consume resources needed for a less frequent but important use. A person may have a serious information need while lacking the confidence or knowledge required to request help. A group may have strong demand for a use that conflicts with other people's ability to access the space. These possibilities show why realized preference cannot be the sole criterion of institutional priority. They also show why an investigator should avoid relabeling every desired activity as a need merely to strengthen its claim on resources.

The term need should be connected to an articulated standard: what function or opportunity is required, for whom, under which circumstances, and why its absence matters. A public decision can then

compare that standard with the institution's role, available resources and other obligations. Demand evidence may reveal whether a proposed response would be used, while need evidence may explain why an opportunity deserves attention even when initial uptake is low. The relationship between the two is a substantive argument.

The affine decision illustration in Section 30 makes the location of values visible but leaves their justification open. Assigning a positive value to every additional participant already selects a criterion. Adding weights for different groups selects further priorities. Mathematical transparency is useful because it allows these choices to be examined; it cannot make them legitimate by displaying them in an equation. Public reasons must remain available in language that affected people can assess.

34.6 Recognition and the Status of an Activity

An institution can recognize an activity and thereby change its practical conditions. This subsection separates recognition of an existing practice from the factual claim that the practice previously occurred and from the decision to support its continuation.

An informal group may already meet near a library without appearing in a program database. Recognizing its activity can make space, information or support available. That institutional change may help the activity persist or transform. The earlier occurrence remains a historical question supported by appropriate evidence. The later recognition is another event with its own effects. Treating recognition as the sole point at which the activity becomes real would erase practices outside the institution's record; treating recognition as a neutral label would overlook the resources and authority it can add.

Recognition can also reshape a practice. A group may need a formal organizer, fixed schedule or reportable outcome to enter an institutional category. Those requirements can help coordination while excluding spontaneous or loosely organized uses. A study should examine what participants gain and what the activity must change to become legible to the institution. The question is particularly relevant when low recorded participation is then used to assess activities that never fit the reporting form.

The proposed category bridge preserves this distinction. It permits an evaluator to revise descriptions while documenting the assumptions required for historical comparison. It also makes room for unresolved categories and participant language. A practice can deserve further inquiry before it has a stable administrative name. That position supports conceptual openness while retaining the requirement that substantive claims be supported by evidence.

34.7 Adaptation to Restricted Opportunities

People can adjust their expectations to the opportunities they encounter. This subsection examines the epistemic and normative consequences of that possibility without assuming that every low stated demand reflects institutionally induced resignation.

Repeated inability to obtain a room may lead a group to stop applying. A person may learn that a particular use is unwelcome and cease considering it. Those responses could reduce both observed attempts and expressed expectations. If such a history occurs, present low demand can be partly an effect of earlier arrangements. Establishing that explanation requires historical and comparative evidence. It cannot be inferred solely from a disagreement between the evaluator's preferred activity and the person's current answer.

The normative temptation is to discount the current response as an adaptation to deprivation and replace it with the analyst's judgment. That move can reproduce the very loss of agency it seeks to criticize. A better inquiry preserves the person's present account, investigates the history and conditions that shape it, and makes suitable alternatives intelligible without demanding a favorable response. A newly available option may support further learning; the person's choice after encountering it remains relevant.

The controlled-target distinction is useful here. A response to an actual alternative offer can be studied prospectively. A claim about what the person would have wanted under an entirely different past is more demanding and may remain weakly constrained. Public reasons for improving an opportunity can

sometimes be established without recovering that historical counterfactual. The investigation should ask which historical claim the decision truly needs and which can remain an open philosophical or empirical question.

34.8 Evidence Production and Unequal Burdens

Evidence does not appear without work. This subsection examines how requiring proof of demand can distribute burdens unevenly and how a study can make that distribution visible.

An organized group may submit a detailed proposal, recruit members and attend meetings to demonstrate support. A person seeking an unfamiliar or unrecognized use may lack the language, network or time required to do the same. If the institution treats the quantity of organized evidence as a direct measure of underlying importance, it may reward the capacity to produce evidence rather than the importance of the opportunity alone. The distinction is a hypothesis about an institutional mechanism; it needs observation of actual procedures and their consequences.

The remedy cannot simply abolish evidential requirements. Public resources need accountable reasons for allocation, and unsupported claims can also misrepresent other people's interests. The proposed method asks whether the required evidence is appropriate to the claim, whether affected people can reasonably provide it, and whether the institution can help investigate questions that private applicants cannot settle. A burden can be justified while still requiring accommodation or shared institutional effort.

Evidence production can itself become an outcome of the policy. A facilitated process may help previously unorganized participants articulate and document an activity. That effect should be recorded separately from attendance. It can improve the conditions for future public deliberation, but it can also favor the categories and formats supplied by the facilitator. Evaluating the process requires attention to who can challenge those formats and what remains excluded from the final account.

34.9 Institutional Responsibility under Uncertainty

Partial identification leaves several factual possibilities open. This subsection considers what public responsibility requires when evidence cannot select one of them. The argument distinguishes uncertainty from an automatic reason for either inaction or expansion.

If current records are compatible with both low and substantial demand, the institution still faces a decision. It can seek more information, conduct a bounded pilot, redesign the offer, retain the current arrangement, or allocate resources using other reasons. Each choice has consequences. Waiting for certainty can preserve a restrictive arrangement; acting immediately can consume resources or displace existing uses. The evidence set describes what is unresolved. The decision should state how the institution weighs the costs of these different errors and who bears them.

The distribution of uncertainty matters. A small average effect can conceal a consequential benefit for a group with few alternatives. A broad interval can coexist with strong evidence that one documented barrier affects a particular person. A public decision can act on such limited findings while remaining explicit about their scope. Conversely, a dramatic anecdote should not become a population estimate without a suitable bridge. Responsible judgment connects the scale of the conclusion to the scale of the evidence.

Revisability provides one practical response. A decision can name the evidence that would trigger reconsideration, the period of review, and the route through which participants and staff can challenge a description. Reversibility should be assessed in terms of actual consequences, including relationships and expectations formed by a pilot. A formally temporary policy can have lasting effects, so institutional responsibility extends to how an experiment is introduced, sustained and concluded.

34.10 The Scope of a Generative Interpretation

The final philosophical issue concerns the sense in which opportunities can be generative. This subsection uses the term to describe a question about formation and possibility while preserving its evaluative

openness.

An arrangement can help generate an activity, a skill, an interest, a group or a new way of describing a shared concern. Those effects can alter what participants subsequently find possible. The paper's models represent only selected parts of that process: an actual-offer response, a history-dependent update, a coordination condition or a revised category. They provide analytical handles rather than a complete theory of social becoming. An empirical application should show which transformation occurred and how the proposed description improves understanding of it.

Generative effects are also evaluatively open. A policy can generate dependence, unwanted commitments, exclusionary group boundaries or burdens that participants did not anticipate. Reproductive effects can preserve valuable continuity, predictable access and an activity's accumulated knowledge. The distinction between generating and reproducing relations therefore carries no automatic ranking. The evaluation should examine the actual relations formed or maintained and the reasons for supporting or revising them.

The philosophical contribution of the demand analysis is a disciplined openness to such formation. It permits inquiry into interests and activities that existing records underrepresent, while preserving the right to question the offered activity and the authority behind its evaluation. Evidence can then support a public conversation about possible arrangements without pretending that either present attendance or an imagined latent preference has already settled that conversation.

35 Public Justification for Formative Offers

An institution can help people develop interests through the opportunities it provides. The resulting participation raises a public question that an earlier-demand estimate cannot settle: under what conditions is it appropriate to initiate and sustain that formative opportunity? This section develops the paper's own argument through explicit premises, comparative cases, and objections. It distinguishes justification before an offer, participants' evaluations after experience, and the claims of people affected without participating.

The scope is a public-library offer involving a shared activity or space. The argument concerns the reasons an institution can give for choosing and evaluating that offer. It leaves broad ontological questions about the ultimate constitution of persons and preferences suspended. It also keeps a normative proposal separate from a statement of existing law or an empirical description of a named library's conduct.

35.1 Prior Interest and the Authority to Offer

The first issue is whether a library must demonstrate a substantial pre-existing preference before making an activity available. Such evidence can be a good reason to act, particularly when people have already articulated an unmet request. Its absence can have several meanings, including limited interest, limited information, or an opportunity that people have had little occasion to consider.

Consider a person who has never participated in a neighborhood discussion group. A clearly described introductory meeting gives them a chance to try the activity. They understand that attendance is optional, can leave without an unrelated penalty, and later judge the experience valuable. The institution can explain the offer through its public purpose, modest resource use, and fair relation to other uses. In this constructed case, the absence of an earlier fully articulated preference need not defeat the justification for making the offer.

The argument depends on explicit premises. First, a public institution can have a justified role in providing opportunities through which people learn what they find worthwhile. Second, a person's informed evaluation of an experience is relevant to that opportunity's value, even when the evaluation develops through participation. Third, the offer must remain answerable to feasibility, distribution and the institution's public role. Given these premises, a prior preference is one possible reason for provision rather than a universal prerequisite.

The conclusion is deliberately narrower than permission to create any interest an institution prefers. The premises require an account of the opportunity and its effects. An institution that selects an activity for unrelated private advantage, conceals material conditions, or ignores competing claims would face objections that later attendance alone cannot answer. The formative character of an offer changes the evidential question; it does not remove the need for justification.

This distinction also clarifies the role of the retrospective latent variable. The earlier disposition can matter historically and causally. It need not be the sole source of authority for a present offer. Treating its unknown value as an automatic barrier to every exploratory activity would give an often unidentifiable quantity excessive normative control. Public reasons can sometimes support an opportunity while the historical question remains open.

35.2 Evaluation before and after Experience

Justification before an offer and endorsement after experience have different informational bases. This subsection examines how they can inform each other without allowing either to replace the other completely.

Before the activity, participants and staff can assess its description, likely burdens, alternatives, and relationship to the library's purpose. Some aspects remain uncertain because the activity has yet to occur. A small exploratory offer can be appropriate when its uncertainty is openly acknowledged and its consequences are proportionate. The prospective assessment concerns whether trying the opportunity is justified under those conditions.

After the activity, participants can evaluate an experience they previously lacked. They may endorse it, reject it, reinterpret its purpose, or propose a different continuation. Staff can report delivery demands and effects on other services. These observations can change the judgment about future provision. The later evaluation is valuable precisely because the experience has added information.

A favorable later evaluation does not retroactively validate every feature of the original process. Suppose participants were given a misleading description but happened to enjoy the event. Their enjoyment provides a reason to consider a similar activity under clearer terms. The misleading invitation remains a feature requiring correction. Outcome value and procedural justification can diverge.

A disappointing result can likewise leave the initial offer reasonably justified. An appropriately limited pilot may test a plausible opportunity and reveal that participants prefer something else. The institution can learn from that result without treating the experiment as a failure of public responsibility. The evaluation should ask whether the uncertainty was handled responsibly, not merely whether the final response was positive.

This separation supports a two-stage decision record. The initial record states why an exploratory offer was reasonable and what would be learned. The later record states what happened, how participants and affected others assessed it, and why continuation or revision is justified. Maintaining both records avoids rewriting the original uncertainty as though the outcome had always been predictable.

35.3 Endorsement and Dependence on the Institution

Participants' endorsement is important evidence about an activity's meaning. Its interpretation can be complicated when the institution also controls resources that participants need. This subsection identifies the relevant dependence without assuming that every institutional relationship invalidates a positive response.

A participant may praise an activity because they enjoyed it. They may also worry that criticism will reduce future access to a room, assistance, or a valued relationship with staff. These motives can coexist. A positive response should therefore be interpreted within the conditions under which it was requested and the person's understanding of its consequences.

The evaluation can reduce this ambiguity by separating service eligibility from favorable research responses, explaining how criticism will be used, and providing a credible route for declining further dis-

cussion. These are proposed design practices, whose adequacy depends on how participants experience them. A formal assurance alone cannot establish that people feel able to disagree.

The institution should also avoid defining success through expressions that participants learn to reproduce. If continued support depends on reporting a particular kind of transformation, the evaluation can shape the language of endorsement. The result may still contain sincere accounts, but its interpretation requires attention to the incentive and the available forms of dissent.

The constructive response is to invite reasons and specific experiences alongside overall ratings. A participant can value the opportunity while criticizing its schedule, facilitation or continuation rules. Preserving that mixed account provides more useful evidence than requiring an unqualified positive or negative verdict. It also allows participants to remain authors of their interpretation within the inquiry's limited scope.

35.4 Refusal, Revision and the Shape of an Invitation

A formative offer should make room for a person to revise their involvement. This subsection examines what that room requires in practice and how it differs from an impossible demand for an influence-free choice.

Every invitation presents an activity through some description, location and social setting. A well-chosen example can make the activity understandable; an enthusiastic facilitator can make it attractive. The mere presence of influence does not settle the normative issue. The relevant questions concern what participants can understand, which alternatives remain available, and how the process responds to their judgments.

A person may initially accept and later decide that the activity is unsuitable. The design should permit that change without turning it into evidence of deficiency or disloyalty. A group may want to retain the shared space while changing the activity proposed by the institution. The possibility of such reinterpretation can indicate that the offer supports agency beyond a predetermined behavioral response.

Revision has practical limits. A booked room cannot always be reassigned at the last moment, and other people may rely on a group's commitment. Respect for withdrawal therefore coexists with reasonable coordination conditions. Those conditions should be understandable and connected to the activity's needs. An unrelated penalty for declining the institution's preferred activity would require a different and much stronger justification.

The evaluation can investigate revision through episodes. What happened when a participant requested a change? Could the group explain why the original format was unsuitable? Did staff offer alternatives, clarify constraints, or treat the request as a rejection of the whole program? These observations give the philosophical concern a concrete empirical form.

35.5 Public Reasons for Affected Nonparticipants

A formative offer can affect people who never attend. They may share the space, depend on staff time, or contribute to the public resources supporting the activity. This subsection argues that their position belongs in the justification even when the focal participants report a valuable experience.

Suppose a discussion group flourishes during a period previously available for quiet study. Its participants' endorsement supports a claim about the group's value. Quiet users may still experience a loss that the attendance count omits. The public question concerns the relationship between those uses, the available alternatives, and the reasons for the chosen allocation.

An institution can sometimes justify supporting an activity that many people will never use. A diverse public facility serves different interests at different times. The justification need not require every person to benefit directly from every event. It should explain how the activity fits the institution's role and how its resource claim stands alongside other legitimate uses.

The affected public can also have an interest in the possibility of future use. A person may value knowing that a suitable meeting space will be available if a need arises, despite having little current

demand. That account concerns an option and an institutional expectation. Its value requires articulation and assessment; it is not already contained in present attendance.

The evaluation should therefore avoid treating nonparticipants as a single residual group. Some are uninterested and unaffected, some face constraints, some prefer competing uses, and some value the availability of an option. Their different relationships to the offer call for different evidence and different public reasons.

35.6 New Expectations and Institutional Commitments

An introductory offer can generate expectations about future provision. This subsection distinguishes a newly formed interest from a commitment the institution has made or reasonably encouraged people to rely on.

Participants may invest effort in organizing a group, recruiting members, preparing materials, or arranging their schedules. If the institution presents the activity as an ongoing opportunity, that work can create reliance on continued access. The later request for continuation then concerns both current interest and the meaning of the earlier communication.

A clearly limited pilot can also produce relationships that matter after the formal period ends. The fact that its endpoint was announced helps define the institution's commitment. It does not make every consequence of closure irrelevant. A responsible decision can consider transition arrangements, alternative spaces, or the preservation of the group's ability to organize independently.

The distinction prevents two opposite simplifications. A newly formed interest does not automatically require indefinite public support. Conversely, labeling an offer experimental does not eliminate every responsibility for the relationships it helped create. The appropriate response depends on what was offered, what participants reasonably understood, what reliance developed, and what alternatives are feasible.

The research record should therefore preserve the language and conditions of the initial offer. An attendance trajectory alone cannot resolve a later disagreement about promises or expectations. Interviews, communications and implementation records can clarify the sequence, while the normative assessment considers the public reasons for honoring, revising or ending the arrangement.

35.7 Participant Contribution to the Activity's Purpose

Participants can help shape what the activity becomes. This subsection distinguishes their contribution to its purpose from a narrow role as sources of feedback about an institutionally fixed design.

An institution may initially offer a discussion format, while participants discover that they need a place to work together on a practical project. Their proposal can reveal a use the initial design overlooked. The evaluation can examine whether the revised activity remains within the library's role and feasible conditions, then assess it on its own terms.

Allowing participants to influence purpose does not require the institution to accept every proposal. Shared resources, other users and staff responsibilities remain relevant. The important distinction is whether a proposal can receive a reasoned response, or whether the evaluation recognizes only the original behavioral target as a legitimate outcome.

This issue connects category formation with public authority. If a new activity is forced into an existing outcome label, its distinctive purpose may disappear from the report. If every new description is accepted without analysis, comparison and accountability can become difficult. The category bridge offers a middle task: preserve the account, state the mapping, and retain unresolved differences that matter to the judgment.

The participant's contribution also changes the meaning of formation. The institution can provide a setting within which people develop purposes that exceed the initial invitation. That development can be valuable without being entirely attributable to the institution. The evaluation should describe the interaction and preserve the participants' work in the resulting account.

35.8 Limits of a Demand-Centered Justification

The argument closes by identifying the work that a demand inquiry can perform within a broader public justification. It can clarify responses, constraints, formation and competing uses. It cannot make the institution's purpose legitimate simply by measuring those quantities precisely.

A demand-centered analysis can give excessive priority to people who already know how to articulate a request. A formation-centered analysis can give excessive discretion to institutions that claim to know which interests people should develop. The proposed approach constrains both tendencies through explicit targets, visible assumptions, participant interpretation and public reasons for allocation.

These requirements remain demanding. Institutions have limited time, participants disagree, and some consequences resist measurement. The response is proportionate inquiry: identify the decision's central uncertainties, collect evidence suited to them, and make the remaining judgments intelligible. Comprehensive analysis should improve that practical connection rather than multiplying requirements without a purpose.

The resulting position permits an institution to initiate an opportunity, learn from its formative effects, and revise its provision. It also requires the institution to respect refusal, account for affected nonparticipants, and explain the conditions of continuation. Earlier demand remains a relevant question among these questions, with its evidential and normative role stated explicitly.

36 Agency and the Evaluation of Library Outcomes

Participation can support a person's purposes without exhausting the value of the opportunity through which it occurs. This section examines the difference between uptake, experienced benefit, and the conditions under which people can pursue and revise their purposes. It places a bounded primary philosophical comparison beside the paper's own library cases, then develops the implications for measurement and public judgment.

The central problem is practical. An evaluation may know how many people attended and how positively they rated an event, yet remain uncertain whether the arrangement supported the purposes they regarded as important. Conversely, a person may value the opportunity while finding the activity demanding or choosing not to use it. These possibilities require separate evaluative questions rather than a single more elaborate attendance score.

36.1 Agency, Well-Being, and Evaluative Scope

Sen distinguishes well-being from agency and achievement from freedom. His account of agency concerns goals and values a person judges important, extending beyond personal advantage, while retaining a role for responsible assessment. The resulting plurality can leave some comparisons incompletely ordered [18]. These distinctions provide a specific interlocutor for the present analysis; they supply neither a library allocation rule nor evidence that a particular program improves agency.

The following cases develop the present paper's own application. They examine what an evaluation would need to observe, how an outcome could be interpreted, and where public judgment remains necessary. The analysis avoids equating agency with the number of menu options or treating every reported preference as a conclusive value judgment. It also avoids using a philosophical distinction to authorize officials to replace participants' purposes with the institution's preferred activities.

Section 35 develops the fuller cases concerning institutional dependence, later endorsement and the value of an unused option. The present section concentrates on distinct questions: contribution beyond personal enjoyment, collective achievement alongside unequal voice, incomplete rankings, and the evaluative commitments introduced by selecting an indicator.

36.2 Valued Contribution with Modest Personal Enjoyment

Consider a participant who helps organize a reading or discussion group because they regard the shared activity as worthwhile. They may find the preparation tiring and the sessions less personally enjoyable than another use of their time. A low enjoyment rating could therefore coexist with a positive judgment about participating. The activity supports a purpose that extends beyond immediate personal pleasure.

An evaluation focused only on enjoyment might classify the outcome as disappointing. An evaluation focused only on attendance might classify it as successful. Both summaries would omit part of the person's account. A better inquiry asks what the participant sought to accomplish, whether the activity helped them pursue that purpose, and whether the burdens were acceptable on terms they could assess and revise.

The person's endorsement still requires interpretation. They may feel a valued responsibility, face social pressure, or experience both. The study should preserve the distinction between choosing a demanding contribution and being unable to refuse an expectation. Concrete questions about how the role was taken up, what alternatives were available, and what happened when the person sought help can illuminate that distinction. A general declaration of commitment cannot resolve it by itself.

The institution also has a responsibility to assess its dependence on such contributions. A program can appear inexpensive because participants absorb organizational work that the evaluation never records. Recognizing the value of contribution should not make its burden invisible. The public decision can consider support, rotation, compensation where appropriate to the institution's arrangements, or a less demanding format. The relevant point is to evaluate the conditions sustaining the contribution alongside its achievements.

A further distinction concerns the purpose against which the participant evaluates success. At the beginning, the organizer may hope to complete a particular project. Through the activity, they may come to value maintaining a group capable of addressing future concerns. A later statement that the activity achieved its purpose can then refer to a revised purpose. The evaluation should preserve the initial and later accounts where this change matters, rather than treating one as the definitive intention that explains the whole sequence.

This revision can be a valuable part of participation. It can also make an apparently simple success measure difficult to compare across people or periods. One participant evaluates completion of a task, another the continuity of a relationship, and another the opportunity to explore. A shared numerical scale can collect their judgments while leaving their evaluative objects different. The researcher should decide whether the study seeks to represent those varied judgments or assess achievement against one separately defended criterion.

The distinction becomes particularly important when a participant sacrifices an immediate preference for a longer-term purpose. A person may prefer to avoid an organizational task yet consider undertaking it worthwhile. Recording only the avoided preference would miss their reason; recording only the declared commitment would miss the cost. An adequate account can preserve both, allowing the public decision to consider support for the purpose without romanticizing the burden through which it is pursued.

36.3 Refusal after Learning

An introductory offer may improve a person's understanding of an activity and lead them to decline further participation. An attendance-centered evaluation can regard this as lost engagement. An agency-oriented inquiry asks whether the person gained an opportunity to make an informed judgment and whether the resulting refusal is respected.

Suppose a participant expected a casual conversation but discovers that the group requires sustained preparation. They try one session, decide the commitment is unsuitable, and choose another activity. The initial experience can have served a useful exploratory purpose even though continuation is zero. The outcome should be described through that sequence. It would be misleading to count the refusal as evidence that the original introduction necessarily failed.

The same pattern can also reveal a problem. The invitation may have omitted the preparation requirement, leaving the participant unable to assess the offer beforehand. In that case, the later informed refusal coexists with a preventable communication failure. The study should distinguish learning that reasonably required experience from learning information that could have been supplied earlier. This distinction affects how the offer should be revised.

The institution's response to refusal is part of the outcome. A person who can leave respectfully may remain willing to explore other opportunities. A person whose refusal is interpreted as a deficit may withdraw from the institution more broadly. These are empirical possibilities requiring follow-up. Their relevance is clear before their magnitude is known: a public institution's participation goal should leave room for people to judge an activity unsuitable.

36.4 Collective Purposes and Unequal Voice

A group can pursue a shared purpose while its members have unequal influence over the activity. This subsection examines why group success and individual opportunity should remain distinguishable in the evaluation.

An organizer may define the schedule, speaking format and direction of discussion. Other members may value the group while finding those choices difficult to question. A group-level success indicator can accurately record that meetings occurred and a project was completed. It leaves open how different members contributed to the purpose and whether their concerns could shape the practice.

The study can investigate this issue through concrete decisions. Who proposed the activity? How were changes discussed? What happened when a member requested a different time or format? Did the group have a way to redistribute organizational work? These questions concern the activity's internal process. They should be asked proportionately and with attention to the risk that an intrusive evaluation alters the relationships it studies.

The library's role also needs definition. Providing a room does not make staff responsible for directing every internal group decision. Facilitation can support participation, but it can also impose an institutional model of how the group should operate. A public justification should explain which conditions the institution undertakes to support and which decisions remain with the group. The evaluation can then assess that limited role rather than assume either total responsibility or complete detachment.

The resulting account can preserve several levels of value. The group may achieve a shared purpose, some members may gain practical confidence, and others may experience limited voice. A single positive or negative label would conceal that pattern. The public decision can respond to the specific concern while retaining the aspects participants value.

Group purposes also emerge through a process that determines which proposals receive attention. A confident organizer may introduce the initial idea, while a quieter member's alternative appears only after several meetings. The final shared purpose can therefore reflect the sequence of participation as well as an aggregation of prior individual preferences. A study that asks only whether the group achieved its announced goal may miss how that goal was selected and whose possibilities were excluded before the announcement.

The investigator should avoid assuming that every unequal contribution is unjustified. Expertise, commitment and the willingness to perform organizational work can give some members a reasonable coordinating role. The relevant question is whether that role remains answerable to the group's purposes and whether others have a practical route to raise a concern. Equal speaking time is one possible arrangement, but it is not a complete measure of influence or an automatically suitable rule for every activity.

The library's reporting requirements can affect this internal process. A requirement for one named organizer and one stable outcome can make administration feasible while encouraging the group to present a simpler purpose than its members actually share. The evaluation should investigate the effects of those requirements. It can preserve the administrative entry and a richer account of the group's activity, allowing practical coordination without treating the entry as a complete representation of collective agency.

36.5 Measurement without a Complete Ranking

The evaluation may identify several relevant outcomes without obtaining an agreed scalar ordering of them. This subsection examines how such a result can remain useful for a decision.

Suppose one offer produces greater participation, another supports more continuity, and a third gives participants more influence over the activity's format. The evidence can clarify those differences. It may also establish that one option is less costly or more accessible for a particular group. A decision requires judgments about which differences matter most in the institution's circumstances. The analysis should preserve the evidence separately from any weights used to combine it.

Some comparisons can be resolved without a complete ranking of every value. An offer may be clearly infeasible, an apparent advantage may disappear after a recording correction, or a modest revision may improve several conditions at once. Identifying such relationships can narrow the decision set. Remaining disagreement can then focus on the tradeoff that actually persists, rather than on a diffuse dispute about whether the program is generally good.

Where a numerical decision model is useful, the report should state what each term represents and which values it leaves outside the model. A participation coefficient can express a declared objective; it cannot silently absorb agency, fairness, enjoyment, continuity and public purpose. Keeping the model limited can make it more informative because readers can see which judgment would change the recommendation.

The study should also distinguish unresolved value disagreement from empirical uncertainty. More interviews may clarify participants' reasons, and a better experiment may estimate a policy effect. Neither guarantees agreement about the proper allocation of scarce public space. A responsible evaluation identifies what additional evidence could resolve and what remains a matter for authorized public judgment.

36.6 Indicator Selection and Evaluative Targets

Selecting an indicator can change the question an evaluation appears to answer. This subsection examines that transition through several constructed contrasts. The aim is to make the value assumptions in measurement visible before the resulting numbers guide allocation.

Suppose a program uses the number of people reporting that they learned something new as its central success measure. The indicator can be appropriate when the offer aims to introduce unfamiliar material or skills. A group organized to maintain an existing language practice, preserve shared knowledge, or provide a reliable opportunity for competent activity may achieve its purpose with less reported novelty. A low score on the new-learning item would then describe one aspect of the experience while leaving the central purpose undermeasured.

The distinction connects generative and reproductive effects without ranking them in advance. An activity can create a new interest, and another can sustain a valued practice through changing circumstances. Either can be beneficial or burdensome depending on its content and conditions. A novelty-centered indicator gives one kind of effect an evidential advantage if the evaluation treats its score as general public value. The appropriate repair is to align the indicator with the activity's justified purpose or retain several explicitly differentiated outcomes.

Confidence provides another case. A participant can leave an introductory event more confident because they gained useful competence. They can also become less confident after discovering that a task is more complex than they assumed. The latter response may represent improved understanding, a poorly supported experience, or another process. A confidence score measures a reported state; its direction alone does not establish the quality of the learning. Additional evidence about the task, the person's aims, and the practical abilities involved is needed for that interpretation.

Frequency of attendance can similarly reward one form of use. A person who returns repeatedly may find an ongoing activity valuable. Another may obtain the information or connection needed in one visit and then act independently. Treating more visits as inherently better can make successful independence appear inferior to continued reliance. The evaluation should state whether it values continuity, efficient

resolution of a need, a sustained community practice, or another outcome. The attendance pattern acquires meaning through that purpose.

Measures of expressed satisfaction can also encourage a focus on experiences that are easy to enjoy immediately. Some worthwhile activities require effort, disagreement or an initial period of uncertainty. This possibility should not be used to excuse an activity that participants find harmful or pointless. It means that satisfaction is one piece of evidence whose relation to the public purpose needs an argument. A demanding experience earns support through its content, conditions and participants' reasons, rather than through the evaluator's assertion that difficulty is good for them.

Indicator selection can affect future offers through the access-evidence feedback process. If only easily recorded positive outcomes support renewal, activities whose benefits are slower or differently expressed can lose access before adequate evidence appears. Conversely, a vague claim of long-term value can shield an ineffective activity from scrutiny. The evaluation needs a credible account of the expected outcome, the period in which evidence should become available, and the conditions that would count against continuation. That account should be specific enough to permit learning in either direction.

The practical consequence is a target review for every consequential indicator. The review asks what the field measures, whose perspective it represents, what period it covers, and which public conclusion it is being asked to support. It also asks what valuable or adverse effect the field would systematically miss. This is a conceptual and empirical task, not merely a choice of labels in a spreadsheet. The formal demand models become more useful when their outcome variables have passed that review.

36.7 Implications for the Demand Question

The philosophical comparison returns the paper to its central target. Demand under a specified offer is useful evidence about a possible response. Its public meaning depends on the purposes pursued, the conditions of participation, the effects on others, and the institution's reasons for provision.

The analysis supports two cautions with positive practical consequences. First, an attendance or satisfaction measure should be used for the question it can answer. Second, a broader agency or public-value claim should identify its own evidence and evaluative premises. These requirements make it possible to appreciate a formative opportunity without pretending that uptake proves every aspect of its worth.

The proposed instruments can implement part of this distinction by asking about purpose, experience, revision and future plans. Their answers remain reports whose interpretation requires care. Administrative records can establish some conditions and events; participant accounts can explain their meaning; a public decision can assess the resulting claims. The forms of evidence can work together while retaining their different authority.

The outcome is a richer evaluation of an opportunity, with the demand estimate occupying a defined place within it. The estimate can reveal a response the earlier regime concealed, show an effect of a new offer, or remain bounded by missing information. The public justification then considers what that response means for the people and purposes involved, including the people who decline, remain outside the record, or bear the costs of the arrangement.

37 Public Claims and the Burden of Evidence

An institution can require evidence before allocating scarce resources, while the opportunity to produce that evidence is itself unevenly distributed. This section examines that tension. It distinguishes the strength of a factual claim, the practical work required to support it, and the authority to make a decision under uncertainty. The analysis develops concrete cases involving requests, petitions, group representation and the review of an existing restriction.

The argument does not remove evidential standards from public decisions. Unsupported claims can misrepresent people, divert resources, or conceal costs borne by others. The objective is to ask whether a particular evidential demand is appropriate to the claim and whether its production conditions are

understood. This extends the paper's central method from participation records to the process through which people demonstrate demand.

37.1 The Strength of a Claim and Its Required Support

A report that few requests were recorded during a stated period makes a narrower claim than a report that few people would participate under a changed offer. A report that nobody wants the activity is stronger still. The evidence required should track that change in scope.

The request count can be checked against the recording process. The alternative-offer claim requires a target and a bridge from existing observations. A universal claim about everyone in the population requires evidence capable of excluding the relevant counterexamples. The finite bounds show why the same small record cannot automatically support all three statements. The difference concerns what the claim asserts, not whether the speaker supports expansion or retention.

An applicant's claim also needs a defined scope. Saying that several named people want to meet can support a limited proposal. Saying that the whole neighborhood needs a recurring program introduces a larger population and a stronger evaluative term. The study should preserve the original insight while examining the additional evidence required for the broader claim. Sympathy with the proposed activity does not justify silently enlarging its evidential support.

This symmetry is about inferential discipline, not identical practical burdens for every party. An institution may hold records and control the conditions needed to investigate a question, while an applicant has only an account of one unsuccessful attempt. Their capacities to provide evidence differ. A fair procedure can ask each to support the part of the claim they can reasonably investigate, while the decision record states what remains unresolved.

37.2 Demonstrating Demand through a Petition

A petition or list of interested participants can be useful evidence of an articulated request. Its meaning depends on what signatories understood, what action they endorsed, and how the list was assembled. This subsection treats the list as a specific observation rather than a direct census of latent demand.

Suppose a group submits twenty names in support of an evening discussion space. Some signatories intend to attend, some support the availability of the option, and some endorse the organizer's broader concern. These positions can all be sincere. The list should be interpreted through the wording presented to them. A signature supporting provision is not automatically a commitment to attend a particular session.

The collection process can form interest. An organizer explains the activity, introduces potential members to one another, and makes a shared possibility concrete. The resulting list can represent newly organized demand. That formation can be valuable and relevant to a prospective offer. It should not be described as a neutral extraction of a fully formed preference distribution that existed before the organizing work.

The petition also reflects the ability to mobilize. A group with an established network may gather names quickly, while people who do not yet know one another may struggle to demonstrate the same numerical support. This possibility does not invalidate the organized group's evidence. It challenges the inference that the quantity of organized evidence directly measures the relative importance of all possible uses.

A proportionate response can ask what the petition establishes and what a small exploratory offer could additionally reveal. The institution can distinguish support, stated intention, actual request and attendance in the resulting study. It can also investigate whether the evidential threshold would systematically exclude activities that require an initial opportunity before a group can form. The threshold should be justified in relation to that process rather than treated as neutral by definition.

37.3 A Group Spokesperson and the People Represented

An organizer often communicates a group's request. The evaluation should respect the practical necessity of representation while clarifying what the spokesperson is authorized or able to say. A group claim can contain several levels of agreement.

Members may agree that a room is needed while disagreeing about the schedule, activity format or organizer's proposed continuation. A statement that the group wants the program can compress those differences. The study need not interview every member about every detail, but it should investigate disagreements that materially affect the offer's feasibility or public justification.

The spokesperson's account can be corroborated through concrete group actions: jointly proposed dates, recorded discussions, completed requests, or members' descriptions of the activity. Such evidence supports a claim about the group's organization. It does not automatically establish that every member has the same individual response under every alternative offer. The coordination model and the individual demand model remain different objects.

The evaluation should also leave room for members whose views are less visible. A person may be unable to attend the consultation at which the group's position is presented, or may prefer to communicate a concern separately. Providing a proportionate alternative route can improve the account. The resulting disagreement should be preserved when it matters, rather than treated as a defect that must be removed to produce a single group preference.

The public decision can then respond to a more precise claim. It may support a trial arrangement while leaving the recurring schedule open, or ask the group to clarify a coordination requirement. The purpose is to make the offer workable and the representation accountable, not to demand unanimity before any collective activity is permitted.

37.4 Evidence Required to Review an Existing Restriction

An existing rule can appear to need less justification because it is already in force. A proposal to change it may then bear most of the evidential burden. This subsection examines the conditions under which that asymmetry is appropriate and when it can preserve a misleading inference.

Some continuity has practical value. Staff and users plan around established arrangements, and change can impose costs. A proposal should address those consequences. Yet the existing rule also produces outcomes and restricts alternatives. Its familiarity does not establish that its factual assumptions remain correct or that its effects require no review.

Suppose a rule limits a group activity, the resulting record contains little activity, and that record is then offered as the reason to retain the rule. The gate model shows the missing inference: activity under the restriction may not identify response under a feasible alternative. The institution can still retain the rule for other justified reasons, including capacity or competing uses. It should state those reasons rather than present the restricted count as if it had settled the alternative response.

A critic faces a corresponding obligation. Showing that the existing record leaves demand unidentified does not prove substantial demand or establish that expansion is the best use of resources. The critique creates an open empirical question and exposes an unsupported inference. A constructive proposal should identify feasible evidence, relevant alternatives and the value premises behind its preferred action.

The decision under uncertainty should therefore compare actual options: retain, revise, conduct a limited test, seek different evidence, or provide another route. Each option has consequences. The analysis can clarify which conclusions the record supports and which assumptions distinguish the options. It should not allow the mere presence or absence of a proposed change to determine the standard of truth applied to competing factual claims.

37.5 The Work of Producing Evidence

Evidence production consumes time, attention and organizational resources. This subsection examines how that work can affect whose interests become visible and how an institution can share the burden without abandoning scrutiny.

A detailed proposal may require familiarity with administrative language, access to records, a stable organizer and time to attend meetings. People seeking a loosely organized or unfamiliar use may lack those resources. Their difficulty producing the expected document can reflect the design of the evidential process as well as the strength of their interest. The study should investigate the process before treating an incomplete proposal as a negative demand signal.

Institutional assistance can improve the evidence. Staff can explain the required information, help applicants identify a feasible offer, or provide a format that separates purpose, participants and resource needs. This support can also shape the proposal. The record should preserve which elements came from applicants and which were developed through the assistance. Co-development need not invalidate the request, but it changes the account of how the request arose.

The institution should avoid requiring applicants to establish facts that only it can reasonably investigate. An applicant may describe why a requested time matters, while the library is better placed to document room availability and staff requirements. A shared inquiry can combine those forms of knowledge. The final decision should make the contributions and remaining uncertainties visible rather than place the whole burden on the party with less access to the relevant records.

The burden also affects nonparticipants in the evaluation. Repeated surveys and consultations can consume time without producing a visible decision. Participants may become reluctant to contribute further. That response can alter the evidence available in later rounds. The study should therefore explain how the collected information will be used and avoid repeating questions whose answers are already available without a clear reason for revision.

37.6 Different Evidence Forms and Comparable Scrutiny

A written application, an interview, an administrative record and an observed event provide different kinds of evidence. Comparable scrutiny means assessing each according to the claim it can support, rather than requiring every account to take one institutional form.

An oral account can provide detailed evidence of an unsuccessful attempt, including information absent from a booking database. The database can establish the recorded timing and official disposition of a request. A discrepancy between them should prompt inquiry into the record's coverage and the account's interpretation. Treating one form as inherently complete would prevent that inquiry.

The study can preserve evidence status explicitly. A statement may be a current report, a corroborated episode, a proposed explanation, or an unresolved disagreement. These labels help readers understand the inference without dismissing useful material. They also prevent the analyst from promoting an evocative account into an established population fact merely because it fits the paper's argument.

The same discipline applies to mathematical evidence. A correct bound establishes a consequence of its assumptions. The public validity of those assumptions depends on the target and evidence supporting them. A formula can clarify a dispute about missing information; it cannot supply the absent empirical premise by appearing rigorous. The model's role should remain open to challenge in language accessible to the people affected by the decision.

37.7 Thresholds, Risk and Public Judgment

The amount of evidence needed for an action can depend on its cost, reversibility and likely consequences. This subsection distinguishes that practical judgment from changing the meaning of a factual claim.

A modest exploratory offer may be reasonable under substantial uncertainty when its costs are limited and its potential information is useful. A permanent allocation that displaces an established use may require stronger evidence and a fuller account of alternatives. These are differences in the justification

for acting. They do not turn a weakly supported statement into a true one simply because the proposed action is small.

The decision record should therefore separate confidence in the factual claim from the threshold for action. It can say that demand remains uncertain but that a limited test is justified. It can say that a positive response is well supported but that the resource tradeoff favors another arrangement. This separation makes uncertainty usable without concealing it.

The burden of error can be distributed unevenly. An unnecessary pilot can consume staff time and displace another activity. A mistaken decision against provision can leave a group without a suitable opportunity and preserve the absence of evidence. The evaluation should describe those consequences and the public reasons used to compare them. A single demand estimate does not determine whose risk should receive priority.

Reconsideration can reduce some consequences of acting under uncertainty. A decision can state which evidence, change in circumstances or new proposal would trigger review. The route should be practical enough that affected people can use it. An abstract promise to remain open to evidence has limited value when the existing procedure prevents the relevant evidence from being produced.

37.8 The Authority of the Research Account

The researcher also exercises authority by selecting targets, categories, assumptions and examples. This final subsection applies the evidential-burden analysis to the paper itself.

The present framework can make an unsupported demand inference visible. It can also overstate its own contribution if it presents familiar methods as newly discovered, treats every restriction as suppression, or assumes that every formed interest deserves support. The verified literature and adversarial cases constrain those moves. The manuscript's claims should remain limited to the distinctions and conditional results it can defend.

In an empirical application, the researcher should preserve findings that favor the existing arrangement as carefully as findings that support change. A well-delivered offer can receive little interest. A proposed substitute can prove suitable. A staff constraint can be real. These outcomes can refine the research question and improve the public decision. They should not be hidden because the motivating example concerned an erroneous inference from restricted participation.

The public value of the analysis lies in making claims answerable to their evidence and assumptions. That includes the claims of applicants, administrators, participants and researchers. A comprehensive demand inquiry can then support a shared examination of possible arrangements while preserving uncertainty, disagreement and the distinct authority required for the final decision.

38 Competing Uses and Fair Opportunity

This section examines allocation when several library uses cannot be realized under the same conditions. It considers quiet and collective activity, distribution across time, staff resources, accessibility, priority and institutional discretion. The objective is to prevent the discovery of unmet demand from being treated as a complete decision rule. A defensible policy must consider how an opportunity for one use affects the opportunities of others.

38.1 Quiet Use and Collective Activity

Quiet study and discussion can each be valuable while requiring incompatible conditions in a particular room. This subsection develops that conflict as a concrete allocation problem. It leaves the actual acoustic and spatial facts to local investigation.

Suppose a small branch has one room that can serve either a discussion group or people seeking quiet study during the same period. A successful group session may increase the number of discussion participants while reducing the quality or availability of quiet use. Counting only the new session's

attendance would omit a relevant consequence. Counting only complaints could also misrepresent the effect, because willingness and ability to complain vary. The evaluation should identify both uses, the conditions each requires, and the ways the proposed arrangement affects them.

The alternatives need not be limited to permanent priority for one use. Different schedules, spatial changes, reservation procedures or additional support may permit a better combination. Each alternative has costs and practical limits. The study can investigate whether a proposed separation is effective, whether the available times are suitable for both groups, and whether the resulting allocation is intelligible to users. A claim that uses can coexist should be tested against the actual room and activity rather than assumed from a general preference for inclusion.

The conflict also reveals why a positive demand estimate is incomplete. Even if every proposed discussion participant would attend, the institution still needs reasons for choosing a schedule and allocating scarce space. Evidence can clarify the consequences of different options. The resulting decision requires an account of priority, distribution and institutional purpose that affected people can examine.

38.2 Time, Repetition, and Suitable Alternatives

An opportunity that exists at an unusable time can be formally present while remaining practically irrelevant to a person. This subsection examines temporal allocation and the meaning of an adequate alternative.

A branch might offer a group room during weekday mornings when some residents are at work or school. The existence of those hours does not establish that an evening request is already met. Conversely, an evening allocation can displace people whose quiet use is possible only then. The study should therefore collect time-specific responses and conditions, including the availability of substitutes. An undifferentiated weekly total can conceal the concentration of both demand and scarcity in a few periods.

Repeated access may also matter. A group that needs continuity to develop trust or complete a project can find occasional isolated slots unsuitable. Another group may need only one meeting. Allocating the same number of hours to both can produce different practical opportunities. A demand claim should state whether it concerns a single event, a recurring sequence or a sustained practice. The evaluation should then examine continuity, cancellation and the ability to plan, alongside aggregate hours.

An alternative outside the library may satisfy the activity under different costs or conditions. A commercial venue can require payment; a private home can exclude some participants; another branch can require travel. These differences are relevant evidence about the available routes. They do not automatically establish that the library must provide the preferred route. They help make the public decision concrete by showing what retaining the current arrangement would mean for the people concerned.

38.3 Staff Resources and the Production of Access

Access often depends on work that an attendance count fails to represent. This subsection identifies the role of staff resources in interpreting a successful pilot and judging its continuation.

An introductory activity can require planning, outreach, preparation, facilitation, conflict resolution and follow-up. Staff may also help people understand the booking process, arrange accommodations or coordinate a group. If those tasks are necessary to the observed response, they belong to the implemented offer. Treating the pilot's attendance as the likely response to an unsupported room opening would change the intervention while retaining its claimed effect. The study should identify which resources were supplied and which could be sustained.

Resource assessment should include the effect on other work. Staff time spent supporting one activity can reduce capacity elsewhere, even if the room itself is free. The comparison should record the relevant displaced tasks rather than treating every additional attendance as a costless gain. The appropriate unit may be time, staffing coverage, continuity or a qualitative account of work that resists a simple scalar measure. The form of the record should match the decision's purpose.

Staff accounts also deserve a careful interpretation. A claim that an activity is difficult to deliver can reflect a real resource constraint, an unfamiliar procedure, a remediable organizational problem or an expectation that experience may revise. The study should investigate these alternatives through concrete tasks and conditions. It should preserve staff expertise while allowing practices and assumptions to be questioned. An inquiry that ignores the work of access risks proposing opportunities that exist only in its model.

38.4 Accessibility and Heterogeneous Conditions

The same nominal offer can create different practical conditions for different people. This subsection examines how accessibility enters the target and the evaluation without reducing people to fixed deficit categories.

A person may require accessible transport, a particular communication format, a quieter environment, a companion or flexibility in attendance. Another may need information in a language absent from the initial announcement. The study should investigate the conditions relevant to the proposed activity and invite participants to describe what makes an offer usable. It should avoid assuming that everyone assigned the same program label received the same effective opportunity.

An accommodation can alter more than the probability of attendance. It can change the person's independence, confidence about future access, and ability to participate on terms they regard as appropriate. Those effects may be important even if attendance remains unchanged. An evaluation focused on use should therefore leave room for other explicitly defined outcomes. Their inclusion requires suitable measures and public reasons, rather than an assumption that every positive concept is already captured by participation.

Group averages can hide uneven implementation. A package could increase overall attendance while remaining inaccessible to a group with specific requirements. The study should investigate relevant heterogeneity through a justified sampling and analysis plan, while protecting confidentiality where small groups are involved. A subgroup label should serve the inquiry into conditions; it should never become an explanation by itself. The question is which features of the offer and environment produce the observed difference.

38.5 Priority Rules and Their Evidence

Priority rules allocate scarce opportunities among competing requests. This subsection considers several possible principles and explains what evidence would be needed to assess them. The discussion presents analytical alternatives without claiming that a single principle governs every library.

First-come allocation can be simple to administer while favoring people who receive information early or can respond quickly. Repeated-user priority can support continuity while limiting entry for new groups. Rotating access can broaden distribution while disrupting activities that need a stable schedule. A lottery can make a procedure transparent while leaving differences in need and the suitability of alternatives unaddressed. Each rule embodies a choice about what features should matter.

Demand evidence can inform these choices. Application histories can show whether some requests repeatedly lose access, while interviews can explain the consequence of irregular allocation. Capacity records can identify periods of acute scarcity. An evaluation should also inspect the cost of applying and the ease of understanding the rule. A formally equal procedure can create unequal practical burdens, and an exception designed to improve access can become opaque if its reasons are never recorded.

The decision record should state the principle used, the factual assumptions supporting it, and the conditions for revision. A group that disagrees can then challenge the relevant reason rather than confronting an unexplained result. This is a proposed accountability practice. Its feasibility depends on the scale and resources of the institution, but its core requirement is modest: the link between evidence, priority and allocation should remain intelligible.

38.6 Discretion, Consistency, and Context

Rules cannot anticipate every circumstance. This subsection considers how discretion can support contextual judgment while creating a need for evidence about consistency and unequal treatment.

A staff member may adapt a procedure to accommodate an unexpected situation. That adjustment can make a formally rigid opportunity usable. The same discretion can also lead to inconsistent decisions across similar cases. The evaluation should distinguish variation justified by relevant circumstances from variation whose reasons remain unclear. A demand model with one uniform gate may need refinement when practical permission depends on who is present, which information is available or how a request is articulated.

Documenting discretion requires proportionality. Recording every interaction in detail could impose an unacceptable burden and alter the relationship between staff and users. A focused sample of consequential decisions, a clear explanation of exception categories, and a route for users to question outcomes may provide useful evidence with less intrusion. The study should justify the chosen level of detail in relation to the inferential and public question it seeks to answer.

Consistency itself needs a relevant comparison. Applying the same procedure to people facing different conditions can reproduce unequal access. Applying different procedures without reasons can create arbitrariness. The evaluation should identify the differences that the institution treats as relevant and invite scrutiny of that judgment. This connects the formal demand target to the wider question of which conditions should be held equal, changed or accommodated in a public opportunity.

38.7 Participation in the Evaluation Process

People affected by the policy can contribute to its evaluation, but that participation has its own access conditions. This subsection extends the paper's method to the inquiry itself.

A public meeting about a library program can exclude people who cannot attend at that time, who lack confidence in the format, or who believe their concerns will have little effect. Counting meeting attendance as support for the final recommendation would reproduce the inference problem under study. The evaluation should identify whose perspectives it reached, whose remain missing, and what authority participants actually had to influence the questions, categories and decisions.

Participation can serve several roles: supplying evidence, interpreting an experience, proposing an alternative or sharing decision authority. These roles should be named. A consultation that collects accounts does not thereby transfer decision power, and an advisory group should not be described as representing everyone without a basis for that claim. At the same time, limited participation can provide valuable insights when its scope is stated honestly. The relevant standard is a clear account of what the process accomplishes and what it leaves open.

The study should preserve disagreement in its final record. Participants can agree that a barrier exists while disagreeing about the appropriate remedy. Staff can support an activity while identifying conditions necessary for its continuation. Reporting only the points that support the investigator's preferred recommendation would weaken the public value of the inquiry. A comprehensive account should show how the proposal responds to serious alternatives and which conflicts remain for an authorized decision.

38.8 Opportunity Judgments after the Study

The final allocation judgment combines findings about participation with a broader account of feasible and fair opportunities. This subsection states the practical conclusion supported by the preceding analysis.

A study can establish that a proposed offer attracts participants, that a particular restriction blocks requests, or that an introductory exposure changes later uptake. It can also document costs, competing uses and differences in access. Those findings create a more informed decision space. They do not collapse into an automatic rule that the activity should expand whenever attendance rises or disappear whenever it falls.

The institution should state which outcomes and obligations guide its choice, how uncertainty affects the decision, and what alternatives were considered. The decision should preserve a route for later evidence and changed circumstances to matter. Under those conditions, the demand inquiry can improve the public allocation of opportunities while remaining responsive to people whose preferred uses and practical conditions differ.

39 Continuity and the End of a Pilot

A pilot can create an activity whose continuation becomes a new public question. This section examines how to interpret and govern that transition. It develops the temporal implications of the earlier public-justification argument through concrete closure scenarios, distinguishing a limited initial offer, an emerging practice, a continuing commitment, and a revised decision under resource constraints. The scenarios are constructed for analysis and describe no actual library program.

The central difficulty is that a temporary arrangement can have lasting consequences. Participants may learn, organize, form relationships, or change their expectations before the evaluation period ends. These consequences can make the policy informative and valuable. They also mean that ending the pilot does more than return an unchanged population to its initial state. The evaluation should therefore treat continuation and closure as actions with their own targets and evidence.

39.1 A Limited Offer and Its Actual Sequence

Consider an introductory activity announced as four meetings over a stated period. The offer can be clear about its duration while leaving continuation open for later review. Participants then understand the immediate opportunity and the uncertainty surrounding a recurring program. The study's first outcome concerns what occurred within that limited sequence.

Suppose one meeting is cancelled because the room becomes unavailable. An evaluation that counts attendance only at the three completed meetings describes part of the delivered activity. It should also preserve the missing meeting and the consequence of its cancellation. If continuity was important for the group's purpose, the cancellation may change the offer materially. A claim about response to four reliable meetings would then require an additional bridge.

The cancellation can affect later attendance in several ways. Participants may lose confidence in the arrangement, encounter a new scheduling conflict, or use the interruption to reconsider their involvement. Others may remain committed and organize an alternative. These are possible mechanisms, not conclusions supplied by a declining count. The event sequence and participants' accounts are needed to distinguish them.

The temporal record should therefore name planned meetings, actual meetings, changes communicated, and the point at which participants learned of a change. A cancelled event known well in advance differs from an unexpected closure encountered after travel. The nominal absence of attendance is the same, while its practical meaning and relation to later response can differ substantially.

39.2 The Development of a Continuing Practice

A sequence of successful meetings can produce something more organized than the initial event. Members may adopt a shared project, distribute tasks, or develop a regular pattern of participation. The new practice creates a continuation question that should be specified rather than inferred from the first meeting's attendance.

The requested continuation can differ from the pilot. A group may need a stable monthly slot, less staff facilitation, more storage, or a format the original offer did not include. Its current request is evidence about that proposed arrangement. The pilot's success supplies relevant experience, while the new resource requirements and membership expectations need separate assessment.

A group can also become more capable of organizing independently. The library's support may have helped it develop a contact network, a workable format, or confidence in arranging meetings. Continuation at the same level of institutional support is then one option among several. The evaluation should investigate what the group can now do, what support remains necessary, and which transitions its members regard as workable.

The opposite development is possible. The group may discover that facilitation or a reliable room is more important than initially expected. A pilot delivered through exceptional staff effort can generate enthusiasm that a routine unsupported offer would not sustain. The continuation decision should therefore compare feasible future offers, not project the pilot's outcomes onto a cheaper version whose conditions have changed.

This distinction preserves the value of formation while making its consequences concrete. A new interest can support a justified request for further provision. It does not determine the exact form of that provision. The study's role is to clarify the relation among the developed practice, the resources it requires, and the alternatives available for sustaining its purpose.

39.3 Expectations, Reliance and Public Resources

Section 35 distinguishes an interest formed by a pilot from a commitment created through the institution's offer. The present subsection applies that distinction to the work participants may undertake in anticipation of continuation.

Participants can invest time in preparing materials, coordinating members, or planning a sequence of activities. Some of that work may be useful even if the pilot ends. Other work may depend specifically on access that the institution has encouraged them to expect. The decision should identify the relevant reliance rather than treat all prior effort as either irrelevant or an automatic claim to future resources.

A clear statement that continuation depends on review can limit some expectations. Its practical adequacy depends on how it was communicated and what other actions accompanied it. Repeated assurances of an ongoing program can undermine a brief formal qualification. Conversely, participants may fully understand that a pilot is exploratory and still reasonably hope it will continue. Hope, a reported expectation and an institutional commitment remain different objects of inquiry.

Public resources also have commitments attached to other uses. Extending one pilot can reduce the capacity available to an existing service whose users made their own plans. The new group's reliance is therefore one relevant consideration among several. A fair decision should explain how these claims are compared and which alternative arrangements were considered, rather than treating the most visible recent group as the whole affected public.

The study can support this judgment through a short commitment record: what the institution offered, what qualifications were stated, what participants understood, what work they undertook, and which consequences of ending are foreseeable. The record supplies evidence for a normative assessment. It should not present an automatically generated entitlement formula or infer obligations solely from the number of hours participants report investing.

39.4 Stopping Rules and the Meaning of Low Uptake

A pilot may begin with a rule that continuation requires a minimum level of attendance. Such a rule can make review expectations clear, but its interpretation depends on the activity's formation process and the conditions of delivery. This subsection examines what an attendance threshold can and cannot establish.

If a complete and credible offer is delivered to the declared population and uptake remains low, the result is evidence about that offer under the observed history. It can justify reconsidering the program when participation is central to its purpose. The analysis should permit this conclusion. A demand inquiry that explains every low count through an indefinitely postponed formation process would make its preferred activity difficult to evaluate critically.

The threshold can be poorly aligned with the activity. A group may require time to coordinate, a reliable sequence before members commit, or a period of learning before the activity becomes usable.

If the pilot is too short to observe the process it was meant to support, low early uptake may leave the relevant continuation target unresolved. That possibility needs an explicit account of the expected process and evidence capable of challenging it.

The distinction is between a testable developmental account and an open-ended excuse. A useful account identifies what should change, over what period, through which conditions, and what observations would weaken the expectation. It might predict more completed requests after information improves or more continuity after a regular slot is secured. The account should also recognize when the required conditions are infeasible or when the evidence favors a different activity.

The threshold's value premise should remain visible. A small activity can be important for a group with limited alternatives, while a large activity can impose substantial displacement. An attendance minimum may serve a practical resource rule, but it does not by itself establish the public value of the activity. The decision record should explain why that threshold is appropriate to the particular purpose and constraints.

39.5 Closure after a Positive Evaluation

A program can receive positive participant evaluations and still face a justified closure decision. Resources may be unavailable, a competing use may have a stronger claim, or the delivered version may depend on support that cannot be sustained. This subsection considers how to preserve the finding while explaining the decision.

The institution should distinguish “the activity produced a valued experience” from “the institution can continue this version under the current conditions.” The first can remain supported when the second fails. Rewriting the evaluation to portray the activity as unsuccessful merely to make closure easier would distort the evidence. A candid decision can recognize the value and explain the resource or priority judgment that prevents continuation.

The same discipline applies to uncertain benefits. A decision to end a pilot need not imply that the activity lacked value for every participant. It may reflect the scale of the evidence, the cost of further learning, or the availability of a more suitable offer. The report should state the actual reason. This makes later reconsideration possible if resources, evidence or alternatives change.

Transition arrangements can reduce some consequences of closure. The institution might help the group identify another feasible time, preserve access to materials, or clarify the route for a future request. The appropriateness of such support depends on the activity and available resources. The study should describe it as part of the closure offer, because participants' later responses can depend on whether the ending includes a practical path forward.

Closure can also generate new evidence. Participants may articulate which features were essential only when they confront the prospect of losing them. That account can inform future planning, while its retrospective and strategic context should remain visible. A strong request made at the end of a pilot is evidence of a current claim; it is not automatically a precise measure of the interest that existed before the pilot began.

39.6 Closure after an Adverse or Mixed Experience

Some pilots reveal substantial difficulties or outcomes that participants regard as unsuitable. Ending or redesigning the activity can then support the purposes of the evaluation. This subsection distinguishes such a response from a failure to appreciate formation or hidden demand.

A group may find the format unproductive, the schedule exhausting, or the organizational burden too high. Staff may identify conflicts that the initial design underestimated. These observations can justify revision even when attendance remains respectable. A count of bodies in a room should not outweigh a well-supported account that the experience fails the purpose for which the room was offered.

Mixed findings require a more specific response. One part of the activity may work well while another creates the difficulty. The component analysis can guide a question about changing the relevant condition, while interviews can clarify which feature participants would retain. A redesigned offer is

a new policy version whose response should be assessed accordingly. It should not inherit every claim made about the original pilot.

Participants may disagree about whether the activity should end. Some may value it strongly while others prefer an alternative. The institution can preserve that disagreement and consider whether a narrower or differently organized offer is feasible. The evaluation should avoid presenting a single average rating as consensus. It should also avoid granting an automatic veto to whichever account is most vivid or easiest to collect.

The decision should specify whether the adverse finding concerns the activity's purpose, its delivery, its allocation, or the measurement process. A data collection burden that disrupts a promising activity calls for a different remedy from a fundamentally unsuitable offer. Keeping those objects separate makes the response more proportionate and the research result more useful.

39.7 Reversibility and the History after a Pilot

A temporary policy can be administratively reversible while its social effects persist. This subsection clarifies why that fact matters for experimental interpretation and responsible revision.

People cannot necessarily return to the exact history they had before an introductory experience. They may have learned a skill, formed a relationship, discovered a possibility, or developed distrust after a failed offer. Reinstating the old room rule does not erase those changes. A before-and-after sequence therefore should not be treated as a simple return to the original state without a stability argument.

The formal history model accommodates this persistence by indexing later responses to the actual sequence. A policy that opens and then closes can have a different value from a policy that remains closed throughout, even if the final rule is the same. The distinction concerns the effects of the intervening opportunity. It also explains why repeated trials with the same people may answer a sequential question rather than reproduce an initial-offer comparison.

Responsible design should consider the likely persistence before beginning the pilot. This does not require certainty about every later effect. It requires a proportionate account of what could become difficult to undo and how the institution will respond if that consequence occurs. Relationships and expectations deserve attention alongside physical resources and administrative settings.

The result is a more concrete understanding of a bounded experiment. Its duration can be limited, its resource commitment can be specified, and its uncertainty can be acknowledged. Its history remains part of the public record and the participants' experience after the formal endpoint. The evaluation should preserve that continuity when interpreting subsequent demand.

39.8 Records for Later Reconsideration

An ended pilot can remain informative if its records preserve the question it actually tested. This subsection identifies what a later evaluator would need to distinguish a repeated opportunity from a materially changed one.

The archive should retain the complete offer, relevant population, delivered sequence, participation stages, outcome definitions, implementation departures, and the reasons for continuation or closure. It should also preserve serious disagreements and unresolved claims. These records allow a later proposal to identify which conditions it changes and which earlier findings remain relevant.

A later increase in requests may reflect new residents, a changed schedule, a different organizer, or interests formed through the earlier pilot. Treating the previous low attendance count as a permanent verdict would disregard those changed conditions. Conversely, restarting an offer with the same practical weaknesses should confront the earlier evidence rather than assume that a new title creates a new intervention.

The archive's purpose is accountable learning. It should preserve enough information to revise an interpretation without collecting unnecessary personal detail indefinitely. The appropriate level of retention depends on the research and institutional arrangements, and the study should state its actual plan.

The conceptual requirement is that the evidence, target and decision remain connected when the policy is later reconsidered.

The temporal analysis therefore extends the demand inquiry beyond the first uptake count. It asks what the offer made possible, what developed through it, what continuation would require, and how an ending changes the next history. Those questions make the policy's learning process reviewable while preserving the distinct authority of empirical evidence and public judgment.

40 Comparing Library Provision with Alternatives

An unmet-demand claim can concern a particular library offer or a broader opportunity that might be supplied elsewhere. This section distinguishes those targets and develops a procedure for assessing proposed substitutes. It examines location, cost, group coordination, continuity, public character and institutional responsibility through constructed cases. The purpose is to make the alternative concrete before treating it as a reason for provision or refusal.

A statement that another room exists supplies limited evidence. The room may be unavailable at the relevant time, unsuitable for some members, or governed by conditions that change the activity. Equally, a preference for the library does not by itself establish that every alternative is inadequate. The study should investigate the practical differences and connect them to the purpose of the proposed activity.

40.1 The Scope of the Opportunity Being Claimed

The first task is to identify what the demand claim concerns. A group may want to meet at a named branch because its members already use that location. It may want any accessible public room within a manageable distance. It may seek a setting that supports a particular kind of interaction, regardless of the institution providing it. These are different targets.

For a named-branch response, the offer includes that location and its associated conditions. A response to another venue is a different potential response. For a broader opportunity target, the analysis needs a criterion for which venues and formats count as suitable. The criterion should be connected to the activity and participants' circumstances. Calling every enclosed space a substitute would define suitability too weakly for many purposes.

The broader target can be useful for public coordination. Several institutions may together provide a workable range of spaces, reducing the need for each to supply every activity. The narrower target can remain important when the library has particular features that support access or the activity's meaning. The study should allow both possibilities and investigate which distinction matters to the decision.

The target should also specify the population. An alternative may be usable for most members while excluding one person whose participation is important to the group. A person-level attendance total can describe that distribution, but the group may regard the resulting activity as a different or unacceptable arrangement. The evaluator needs a justified account of whether suitability is assessed individually, collectively, or through several related criteria.

40.2 Nominal Availability and Practical Feasibility

A proposed alternative should be described through the same practical detail as the library offer. The comparison includes time, location, capacity, booking, accessibility, cost, support, group requirements and the reliability of access. This symmetry prevents the current offer from being evaluated concretely while the substitute remains an idealized possibility.

Suppose a nearby room is usually available during weekday afternoons, while the group can meet only in the evening. The existence of that room does not establish a feasible substitute for the evening activity. A weekly total of available hours can conceal the lack of overlap. The study should identify the actual feasible periods and whether the group can reasonably adapt its schedule.

The reverse case also matters. A group may initially prefer one evening but discover that another suitable period is workable after coordination. The alternative can then reduce the resource conflict. The evaluator should preserve the process through which the group assesses the change, rather than assume complete flexibility or complete inflexibility from the outset. A concrete offer can generate evidence that an abstract question about preferred times could not supply.

Practical feasibility can depend on reliability. An occasional room available at short notice may be sufficient for a one-time meeting but unsuitable for a recurring practice that requires advance planning. A nominally larger supply of irregular slots can provide less usable continuity than a smaller stable allocation. The comparison should follow the activity's temporal requirements, not only the quantity of resources listed.

40.3 Costs and Their Distribution within a Group

An alternative can shift costs from the institution to participants or distribute them unevenly among participants. This subsection examines those shifts as part of the opportunity comparison, without assuming that every private cost makes the alternative unacceptable.

A commercial venue may require a fee. A private room may require one member to host and prepare the space. A more distant branch may add travel time for some members. These conditions can alter who participates and who performs the work that makes the activity possible. The study should record the relevant costs in units suited to the decision, including time and organizational burden where a monetary total would omit important differences.

The average group cost can conceal a binding constraint. A modest fee for most members may be difficult for one person; a short journey for the organizer may be much longer for another member. A group can sometimes pool resources or adapt its format, but that possibility should be assessed rather than presumed. The public decision should be able to see whose participation the alternative makes more difficult.

Participants can also reasonably choose to bear some cost for an activity they value. The evaluator should respect that judgment while checking that the stated willingness corresponds to a credible arrangement. A hypothetical willingness to pay or travel is a report, not completed behavior. A later response to a real offer may differ because the cost becomes concrete, circumstances change, or the activity develops a different meaning.

The institutional question is broader than minimizing the library's own expenditure. Shifting a task to participants can preserve the activity efficiently or create burdens that undermine access. The appropriate judgment depends on the public purpose, available resources and distribution of the resulting conditions. A cost transfer should remain visible in the evaluation rather than disappear because it falls outside the library's budget record.

40.4 Private Settings and the Conditions of Participation

A private home or members-only setting can supply a room while changing who can enter and how the activity is governed. This subsection considers those differences as possible features of the offer, not automatic grounds for preferring a public institution in every case.

A home may provide comfort, familiarity and flexibility for some groups. It may also require a host to assume work, expose personal space, or decide who is welcome. Some members may find attendance difficult because of the location, the social relationship, or the conditions of access. The study should ask what the setting makes possible and what it changes for the particular activity.

A public library can offer a setting that participants understand as less dependent on one member's ownership or invitation. That feature may matter to the group's purpose. Its importance should be supported through participants' accounts and the actual access arrangements. Merely attaching the word public to a building does not establish that every person experiences equal or comfortable access to it.

The comparison can also affect continuity. A private host may be willing to support several meetings but unable to sustain them indefinitely. An institution may provide a more stable route while imposing

booking and conduct conditions. The group needs to assess the tradeoff. The evaluation should preserve the reasons for its judgment without assuming that institutional or private provision is inherently superior.

The resulting evidence concerns the offer's social and practical conditions. It can support a claim that a particular private substitute is unsuitable, adequate, or adequate only with additional support. It should not be generalized into a claim about all private settings from one example. The target remains the specified activity and population.

40.5 Remote Participation and a Changed Activity

A remote format can expand some opportunities while changing the activity for which space was requested. This subsection examines the conditions required to treat it as a substitute rather than simply another available channel.

Remote participation can remove travel and room constraints. It may require equipment, connectivity, a suitable private place, confidence with the platform, or a different form of coordination. These are possible conditions to investigate. Their presence and importance vary by activity and participant. A general claim that an online option solves access would be too broad without that inquiry.

The activity itself may change. A task involving shared materials, informal conversation, or a particular kind of collaboration can be affected by the format. Some groups may find the remote version equally useful or preferable; others may regard it as a partial substitute. The study should ask which purposes are preserved, which are changed, and whether the change is acceptable to the people concerned.

Hybrid provision can introduce further coordination problems. Participants in the room and participants joining remotely may have different access to conversation, materials or informal interaction. A headcount that combines them can accurately record total attendance while concealing unequal participation conditions. The evaluation should define what meaningful participation requires in the hybrid format and investigate whether it was delivered.

The measurement channel also changes. Digital systems can record logins or connection time with apparent precision, while in-person informal activity may leave fewer records. Greater record density does not establish greater activity value or better access. The classification and validation analysis applies to each format's actual observation process. A platform trace should be interpreted through the event it represents.

40.6 Several Institutions and a Shared Opportunity

An alternative may involve cooperation among institutions rather than a simple transfer from the library to another provider. This subsection examines what such an arrangement must specify for its effect to be evaluated.

A library might help a group identify a suitable room elsewhere, share information about available periods, or coordinate a recurring arrangement with another public facility. The resulting offer contains contributions from more than one institution. The evaluation should record those contributions and the points where a request can fail. A referral that never leads to usable access differs from a completed coordinated offer.

The responsibility boundary should also be clear. Participants need to know which institution handles booking, what conditions apply, and where to raise a problem. An arrangement can become harder to use when each institution assumes the other supplies essential information. The study can follow a request through the whole route and identify where uncertainty or delay arises.

Coordination can create a useful alternative without resolving every demand at the original branch. Some people may be unable to use the partner venue, and some activities may depend on resources available only at the library. The evaluation should preserve these remaining differences. A successful referral for one group does not establish universal substitutability.

The public decision can nevertheless learn from the arrangement. It may reveal that a modest coordination effort provides a suitable opportunity at lower cost or with less displacement. It may reveal

that the apparent alternative is fragile or inaccessible. Both findings are valuable when tied to the actual route and population, rather than inferred from the existence of a partnership label.

40.7 Comparative Offers and a Feasible Study

The alternative-provision question can sometimes be studied through credible comparative offers. The design should identify the contrast it can deliver and the population for which both options are meaningful. This subsection develops the evidential role of such a comparison.

Suppose a group can receive a library room or another described venue under comparable timing and support. Assignment or a carefully documented choice process can generate evidence about uptake and experience under those offers. The study should preserve the complete versions, including travel, costs, access rules and assistance. An apparent location effect can otherwise contain several unrecorded differences.

Random assignment can support a causal comparison when feasible and appropriate to the agreed study. A choice-based comparison can reveal preferences among available options while requiring additional assumptions for a causal effect. Participants who choose one venue may differ from those who choose another. The analysis should retain the role of the design rather than treat all observed venue differences as effects of location.

The population must also remain comparable. Restricting the study to people who can use either venue gives a clear common-feasibility target, but it may exclude precisely the people whose access makes the policy question important. A broader study can retain those people and define their available offers explicitly. The choice of target should be justified by the decision, not solely by the convenience of a balanced comparison.

The comparison can form preferences. Experiencing a different setting may reveal advantages or difficulties participants had not anticipated. A later preference for one venue therefore belongs to the history containing those experiences. The result can guide future provision without being described as a recovered preference that necessarily existed before the trial.

40.8 A Constructed Dossier of Four Offers

The following dossier applies the comparison to a single stipulated activity. Every person, arrangement and quantity in this case is constructed for analysis. The example supplies an inspectable chain from a listed room to a possible substitute; it represents neither an observed group nor actual venue prices. The same activity criterion remains in view until an explicit revision changes it.

Six members propose four meetings to compare two sets of local historical materials written in different languages. The proposed activity involves reading selected passages, comparing interpretations and producing a shared account of unresolved differences. Member R1 can facilitate the discussion, R2 can read language A, and R3 can read language B. Members R4, R5 and R6 contribute questions and interpretations. These assignments describe relevant capacities in the example. They confer no general hierarchy of participant worth.

For the first four meetings, the group proposes a minimum of four participants with all three roles covered: facilitation and reading in each language. The role condition has an intelligible relation to the task. Comparing two sets of materials requires access to both; a discussion of competing interpretations requires a workable way to organize contributions. The minimum of four preserves at least one contributor beyond the three initial role holders. That number remains a proposed activity criterion requiring agreement. The example neither proves that four is universally sufficient nor treats the named individuals as irreplaceable. An agreed substitute with the relevant capacity can cover a role.

Each meeting lasts two hours. The reservations in the dossier include additional access time for setup and departure, so the two hours describe usable activity time. All three physical rooms have capacity eight and permit the proposed activity. These stipulations remove room size and formal permission from the immediate comparison, allowing schedule, role coverage, distribution and purpose to become visible. A real assessment would need to verify those removed conditions as well.

Offer	Activity period	Schedule-compatible members	Conditional activity and cost assessment
Library	Tuesday, 18:00–20:00	R1–R6	Six members and all roles; zero incremental money cost
Community room	Wednesday, 18:00–20:00	R1, R2, R4, R5, R6	Five members; language-B role absent; zero incremental money cost
College room	Thursday, 18:00–20:00	R1, R2, R3, R4	Four members and all roles; fee 20 plus R3 travel 12 per meeting
Remote offer	Tuesday, 18:00–20:00	All six report connection availability	Delivery of the activity and private participation space remain untested

Table 6: Constructed alternative offers for one four-meeting activity. Costs are illustrative accounting units. Schedule compatibility and role coverage leave willingness, affordability and consent to be assessed.

Table 6 records schedule and travel compatibility, not completed willingness or all-inclusive feasibility. Affordability, consent and willingness remain separate inquiries. In particular, placing a member in the college column means that the time and journey fit that person's stated commitments. It leaves open whether the additional money cost makes attendance workable.

40.9 Person Coverage and Coverage of Essential Roles

The community room accommodates the schedules of five members, while the college room accommodates four. An assessment based solely on the number of compatible schedules would favor the community room. Under the stipulated first-phase task, however, the community offer lacks the language-B reader. Its larger person coverage fails the current role condition. The college offer covers every specified role and the minimum count, subject to the unresolved cost and willingness conditions.

This result is a conditional comparison of tasks and arrangements. It supplies no reason to regard R3 as more valuable than R5 or R6. A particular capacity is required by the current activity, and its distribution among participants makes one person's absence consequential for that activity. If another member can supply language-B reading, or if a willing substitute can join, the assessment changes. The evaluator should therefore document capacities and acceptable substitution rules instead of treating role labels as permanent personal attributes.

The capacity and coordination analysis in Section 13 explains the structural issue. A count of individually available people can omit the conjunction required for a joint activity. The present dossier adds a practical interpretation: the relevant conjunction concerns different roles, a shared period and a defined task. Multiplying six independent willingness probabilities would conceal these dependencies. Even a perfectly measured willingness total would leave the role condition unresolved unless it retained who can supply each contribution.

The task criterion also needs challenge. A group might overstate the need for its familiar facilitator, or assume that only one member can interpret particular materials. Inquiry can reveal a feasible substitute or a less demanding organization. That possibility justifies discussion of the criterion; it does not justify deleting it from the comparison before examination. The institution should be able to ask why a requirement matters and receive an answer connected to the task. The group should be able to revise a requirement without having its earlier account recoded as dishonest.

An immediate empirical implication follows. The relevant next step for the community offer is to investigate acceptable coverage of the missing role. Asking all five compatible members whether they like the room would address a different uncertainty. A favorable average venue rating cannot supply the absent reading capacity. A short discussion with the group about translation, substitution or a changed

first-phase task may be more informative than a larger satisfaction survey.

40.10 Costs of the College Offer

The college arrangement passes the stipulated schedule, role and minimum-size tests for four members. The remaining cost assessment must follow the same people and the same four meetings. This subsection calculates the stated money costs and explains what the calculation leaves open.

The room fee is twenty accounting units per meeting. R3 incurs twelve additional travel units per meeting relative to the library offer; the other three schedule-compatible members have zero additional travel cost in the stipulated comparison. Four meetings therefore create eighty units in room fees and forty-eight in additional travel, for a total of one hundred and twenty-eight units. These amounts describe the example only. They establish neither actual prices nor an affordability threshold.

If the four attendees divide the room fee equally, each pays five units per meeting. R3 pays seventeen units including travel, while each of the others pays five. An average of eight units per attendee per meeting accurately describes the total divided by four. It conceals the unequal payment required from the person whose reading role is necessary under the initial task criterion. The apparent availability of that role can consequently depend on a cost the person cannot or chooses not to bear.

Several arrangements could change this result. The group could distribute the fee differently, the institution could reimburse additional travel, or another provider could waive the room fee. Each arrangement changes who bears a cost and may change willingness. Reimbursement removes a participant's payment while retaining a resource cost somewhere in the arrangement. A complete decision record should identify that transfer. Treating a reimbursed cost as zero for the institution and zero for the participant would lose it from the comparison.

The group could also decide that the added burden is acceptable for four meetings. Such a judgment matters. The evaluator should record whether it was made with the actual amounts and arrangements understood, whether the person bearing the largest cost agreed, and whether the agreement covers the full period. A general expression of enthusiasm for the activity supplies weaker evidence than acceptance of the concrete college offer. Even that acceptance remains distinct from the completed sequence of payments and attendance.

Time and coordination costs may remain after the money total is settled. Someone must book the college room, distribute materials and maintain contact with members unable to attend. The dossier initially leaves those tasks unpriced because an invented price would add apparent precision without improving the case. The study can record who undertakes them, how much work they require and whether the arrangement is sustainable. A later resource comparison may use a justified valuation; the unvalued work should remain visible in the interim.

40.11 Task Revision and the Community Offer

The community arrangement can become workable through an explicit change in the activity. Suppose the group agrees to use the first four meetings to examine only the language-A materials, while seeking a later opportunity for comparison with language B. The missing language-B role then ceases to be necessary for that first phase. The community room covers five schedules and the revised role requirements. This is a plausible adaptation, with a changed target.

The revised task can preserve much of the group's purpose or postpone an essential part. The answer depends on why the two-language comparison mattered. If the initial four meetings were always intended as preparation, the revision may be minor. If the purpose was to confront conflicting accounts together from the beginning, the revision changes a central feature. The evaluation should record that distinction through the group's reasons and the materials involved, rather than infer equivalence from the shared label of local-history discussion.

The change also affects participation. R3 cannot attend the Wednesday meetings; R5 and R6 can. The revised arrangement includes more members by count while excluding the person who initially supplied one linguistic perspective. That distribution may be acceptable to the group, but it deserves an explicit

judgment. A numerical increase from four to five schedule-compatible members cannot settle whether the resulting conversation preserves the intended range of perspectives.

The study can follow the revision prospectively. It can record the agreed first-phase objective, the tasks actually attempted, the contributions available and the unresolved work deferred to the second phase. If the revised meetings succeed on their stated terms, the result supports that revised offer. It does not retrospectively establish that the original two-language task could have been performed in the community room without adaptation. Keeping the two targets distinct allows success to be recognized accurately.

The remote offer requires a related inquiry. Reported connection availability for all six members supplies one condition. The study still needs to investigate private space, access to materials, turn-taking and whether the proposed comparison can be carried out through the platform. A trial may establish that the group finds the format workable, or reveal changes it regards as unacceptable. Either finding concerns experience of the actual remote arrangement, including any learning the trial produces. The dossier therefore leaves the remote activity test open until that evidence exists.

40.12 Public Reasons for Accepting or Refusing Substitution

The dossier permits several defensible decisions, depending on verified conditions and declared public priorities. This subsection identifies the reasoning required for those decisions. It keeps the empirical availability of an arrangement separate from the authority to prefer it.

The library might coordinate the college offer, cover the additional travel burden and retain a route for R5 and R6 to contribute to later meetings. That decision could preserve the initial task while freeing the named library room for another use. Its justification should acknowledge the reduced inclusion and coordination work. The offer would be a supported partial substitute, with reasons for accepting the remaining differences. Describing it as equivalent to the six-member library offer would overstate the result.

The institution might support the community offer after the group adopts the revised task. The public reason could concern greater inclusion within the available schedules and a lower immediate money cost. The report should state that the two-language comparison has been deferred and identify how the deferred objective will be reconsidered. A decision that depends on future continuation should distinguish an actual commitment from an aspiration. Otherwise the apparent substitute can rely on an opportunity that nobody has undertaken to provide.

The library might retain the Tuesday offer because full inclusion and the initial comparison are central to the supported purpose, and because competing uses can be accommodated on reasonable terms. That reason would be stronger than a bare preference for the familiar venue. It connects the allocation to concrete differences in the activity and population. The same reason could be outweighed where the Tuesday allocation substantially displaces another important use. The dossier makes the conflict assessable without supplying a universal priority ordering.

The group may refuse a substitute that passes the minimal role and count test. Refusal could reflect costs, reduced inclusion, a changed purpose or preference for the library's setting. The evaluator should investigate those reasons and retain their differing force. Passing a proposed minimum establishes one threshold of operational compatibility; it does not require participants to endorse every arrangement above it. Equally, a refusal does not by itself oblige the institution to provide the preferred alternative. The public decision must still weigh the supported purpose, distribution and resources.

The resulting decision record can be short because the dossier has already done the detailed work. It should identify the offer accepted or refused, the activity criterion, the people and roles affected, unresolved conditions, the cost distribution and the public reason for the choice. This makes subsequent revision possible. If an acceptable language-B substitute becomes available, if the college fee changes, or if the remote trial succeeds, the record shows which premise changed and which comparison should be reopened.

40.13 Institutional Role and the Limits of Substitution

Even a feasible substitute leaves a public question about the library's role. The institution may have reasons to provide a particular opportunity, to coordinate it elsewhere, or to prioritize another use. This subsection separates that judgment from the factual claim that an alternative exists.

A substitute can reduce the urgency of one resource claim when it preserves the relevant opportunity on reasonable terms. It can also leave important differences that justify continued library provision. The evaluation should make those differences concrete enough for public assessment. It should avoid treating a preferred institution as indispensable by definition or treating every nominal alternative as sufficient by definition.

The public character of the setting can be one relevant reason. A group may value access that is less dependent on private invitation, commercial purchase or organizational membership. That reason requires an account of the activity and the actual alternative conditions. It should be considered alongside resource limits and other uses rather than converted into an automatic priority rule.

The final decision should state which opportunity it intends to secure and how the chosen arrangement serves that purpose. If the institution retains a restriction because a suitable alternative is available, the record should identify the alternative and the evidence of suitability. If it expands provision despite an alternative, the record should explain which difference matters. This makes the recommendation answerable to facts and reasons that can later be reviewed.

The substitution analysis thus narrows the language of unmet demand. A group can have unmet demand for a named library offer while possessing another workable route to its activity, or it can face a broader unmet opportunity despite the existence of nominal substitutes. The distinction changes the empirical and normative argument. A comprehensive evaluation should preserve it rather than allowing the word alternative to settle the question before the relevant conditions have been examined.

41 Adversarial Assessment

This section tests the argument against objections that could undermine its concepts, derivations, empirical use or claimed contribution. It considers alternative explanations for low use, the interpretability of latent states, the value of interventions that form interests, and the limits of a complex evaluation design. The purpose is to identify conditions under which the proposed method should be revised, narrowed or abandoned.

41.1 Low Demand under a Suitable Offer

The strongest direct objection is that low recorded participation may reflect low demand. This subsection explains how the method permits that conclusion and identifies the evidence needed to distinguish it from other explanations.

If the offer is intelligible and suitable for the target question, practical conditions are sufficient, capacity is available, observation is accurate and the relevant population is represented, low participation can provide strong evidence of a low response to that offer. The gate critique never rules out this possibility. It challenges an inference made before those bridges are established. A study that treats every negative finding as another hidden barrier would cease to be open to evidence.

The design should therefore state its defeat conditions. It could specify that a well-implemented offer with adequate notice, verified practical access and repeated opportunities receives few requests or participants across the relevant observation window. Suitable interviews may show that people understand the activity and prefer other uses. Such evidence would support a narrow conclusion about that offer and population. It could motivate ending or redesigning the package while leaving other activities open.

The distinction between scope and evasion matters. A conclusion about one offer legitimately leaves other offers unresolved. It becomes evasive when the investigator repeatedly changes the target solely to avoid an unwanted answer. Revision should follow independent reasons, such as evidence of a specific

implementation failure or an offer feature that participants identify as consequential. The earlier target and its result should remain in the record so that the trajectory of the inquiry is visible.

41.2 The Interpretability of Latent Dispositions

The formation counterexample introduces a latent pre-exposure state. This subsection considers whether that state adds a useful question or merely creates an unobservable distinction by definition.

Two mathematically different state assignments can generate the same observations. That fact alone does not establish that the difference is scientifically meaningful. The investigator must explain what W^- represents, why it matters and which wider evidence or theory could constrain it. Without such an account, the counterexample remains a warning about an undefined historical claim rather than a discovery about human psychology. The paper retains that limitation explicitly.

The correction from latent W^- to actual-offer demand D is therefore substantive. In the constructed worlds, opening response demand is the same, while the latent initial state differs. If the practical decision concerns the actual opening, that latent ambiguity may be irrelevant. The counterexample becomes useful when an argument explicitly attributes later participation to an earlier disposition or claims that a formative effect is merely recovered prior demand. The critique then asks whether the stronger target has been defined and supported.

A controlled non-formative probe could make the historical distinction more operational, but it may be impossible to implement without changing the very state it seeks to hold fixed. The appropriate response is to retain the controlled target as hypothetical, investigate whether its assumptions are coherent, and prioritize directly interpretable targets where they answer the decision question. A formal possibility should not become a demand that every library evaluation solve an inaccessible psychological problem.

41.3 Useful Formation without Historical Recovery

A further objection is that a library need not distinguish discovery from formation if both produce a valuable activity. This subsection grants the force of that objection for some decisions and identifies situations where the distinction remains consequential.

For a prospective decision about continuing a program, evidence of meaningful participation and acceptable consequences can be sufficient without proving that the interest existed beforehand. The paper's common-offer design is constructed to preserve that useful target. A historical question should not be imposed simply because it is philosophically interesting. The evaluation should explain why the distinction matters to the claim under review.

The distinction becomes more important when an institution uses later participation to vindicate or condemn an earlier decision, when participants claim that prior requests were ignored, or when the process of formation raises concerns about dependence and authority. A program's success cannot by itself establish that earlier administrators knew or should have known the same response would occur. Conversely, a failure to identify prior latent interest does not erase a documented earlier request. Different claims require different historical evidence.

The framework should therefore be applied proportionally. It can identify which retrospective statements require support while allowing a policy to be evaluated through prospective outcomes. Its contribution is precision about claims, not the multiplication of philosophical obstacles to practical learning.

41.4 Selection after Assignment

Randomization can coexist with selective observation. This subsection examines one of the most serious threats to the proposed empirical design and states how its consequences should be reported.

Suppose the exposure arm receives more staff contact and therefore completes more follow-up surveys. A comparison of respondents can differ across arms even when the full-cohort response contrast is smaller or absent. The assignment mechanism protects the original comparison; it does not make

post-assignment respondents representative by itself. The study should retain the baseline cohort, report missing outcomes by arm and investigate the process that produces them.

Bounds can preserve the unresolved contribution. Additional follow-up or independent observation may narrow it. A model assuming that missing responses resemble observed ones can also yield a point estimate, but the assumption should be stated and tested against available information. The analysis should show whether the main conclusion survives plausible departures. If the conclusion changes across a broad range of credible missingness patterns, the appropriate finding is conditional.

The same concern applies to attendance in the formative exposure. Comparing actual participants with nonparticipants changes the question and reintroduces selection. The study may investigate those groups descriptively, but causal claims about receipt require a separate design or assumptions. Keeping the assignment contrast primary preserves a well-defined source of causal information even when uptake of the exposure is incomplete.

41.5 Interference beyond the Chosen Boundary

The branch construction assumes that a cohort's response depends only on its own branch's package. This subsection considers why that boundary may fail and how the study should respond.

Libraries belong to a wider system of places and relationships. People can change branches, bring friends from another area, or shift use in response to crowding. A treatment at one branch can therefore affect another cohort's outcomes. If the study ignores these pathways, its simple potential totals may lack a valid interpretation. The issue is especially important when the policy question concerns expansion across the whole system, since local gains can partly reflect substitution.

The design can collect limited information about cross-branch use and relevant communication, revise the assignment unit, or specify an exposure mapping that represents the important connections. Each remedy has costs and assumptions. An exposure mapping should be justified by the activity's actual relationships, not chosen solely because it permits a convenient estimator. If the boundary cannot be defended, the analysis should narrow its causal claims and preserve the descriptive findings.

The possibility of interference also constrains subgroup interpretation. An apparent effect for one group may arise through changed opportunities for another. The study should investigate plausible mechanisms of cooperation, competition and substitution. The relevant response system can be relational without requiring a complete model of every social connection.

41.6 Measurement Burden and Practical Feasibility

The proposed design contains several records and inquiry stages. This subsection asks whether that complexity could make the method unsuitable for ordinary library work.

A comprehensive research design can demand more time and expertise than a branch can supply. Requiring every library decision to include a full experiment, interviews, record linkage and category audit would be disproportionate. The framework should support a graduated approach. A simple decision may need only a clearly defined offer, a check of record units and a small amount of supplementary evidence. A consequential claim about a population or a major service change may justify a more extensive design.

The essential requirement is the connection between a claim and its evidence. The study should collect information because it resolves a material uncertainty or informs a decision. Data that add detail without changing the inferential or practical question can be omitted. Conversely, a convenient metric should not be treated as sufficient merely because a better inquiry would be costly. The report can state a limited conclusion that matches the available evidence.

A pilot of the research process should assess staff workload, participant burden, record quality and the feasibility of maintaining the procedure. If the method changes the activity excessively or consumes resources needed for the service itself, it should be redesigned. A theoretically elegant audit that institutions cannot use would need a different account of its practical contribution.

41.7 Existing Methods and the Novelty Gate

The paper combines familiar identification principles with a specific library question. This subsection assesses whether that combination warrants a standalone contribution.

Partial identification, causal assignment, interference, outcome evaluation and adaptive learning already have substantive antecedents. The manuscript acknowledges the verified sources on which its comparisons rely. The simple gate, expectation proof and finite bounds should not be presented as new general mathematical discoveries. Their role is to expose assumptions in a particular chain of demand inference and to make a proposed study internally coherent.

The standalone contribution would be stronger if the method identifies an actual inferential failure, supports a useful redesign, or clarifies an important demand claim that existing library evaluation practice leaves unclear. A broader literature confrontation remains necessary to assess that contribution. If the proposed distinctions are already fully represented in established library methods, the paper should state the relationship and reframe its contribution as synthesis or application. If the study adds no practical or conceptual value, its architecture should be narrowed.

The repository's earlier work also matters. General finite-trace nonidentification, historical records and feasible opportunities have already been developed in companion projects. This paper's value lies in the offer-history-population distinctions, the repaired formation countermodel, the connection to actual library records and the prospective design. Repeating general claims about unseen possibilities would fail its own contribution test.

41.8 Value Disagreement and Institutional Authority

The final objection concerns the possibility that better evidence leaves the main dispute unchanged. This subsection explains why that result can be informative and what the method should avoid claiming.

Two groups can agree about the likely attendance, costs and displaced uses while disagreeing about which opportunity deserves priority. The disagreement then concerns values, responsibilities or authority. Further measurement of the same outcomes may leave it unresolved. A report should identify that point rather than implying that more data will eventually select the proper end by themselves.

Evidence can still improve such a dispute by correcting false factual premises, clarifying feasible alternatives and showing who bears the consequences. The decision process must then provide reasons for the chosen priority and identify who has authority to decide. The mathematical framework supplies neither a mandate for researchers nor a substitute for institutional and public judgment.

The method succeeds when it makes the remaining disagreement clearer. It fails when its formulas conceal value choices, when its categories exclude serious alternatives without explanation, or when its findings are used to close a discussion that the evidence leaves open. These defeat conditions belong to the assessment of the method itself and should guide subsequent revision.

42 Rival Explanations of a Participation Record

The force of a demand argument becomes clearer when several explanations confront the same evidence. This section develops a separate constructed dossier in which registered participation rises after a new offer. It identifies what the stipulated record establishes, which rival interpretations remain compatible with it and what additional inquiry could discriminate among them. The final discussion sets limits on model revision so that the framework remains open to adverse findings.

Every quantity and event in this dossier is fictional. The case is independent of the eight-branch experiment and the classification-audit example. It represents a single descriptive policy sequence, with no random assignment assumed. Its purpose is to test the inferential architecture when a favorable-looking participation record invites a broader historical or public conclusion.

42.1 The Retained Record and Its Settled Content

For six weeks, a branch's formal register contains zero approved discussion meetings under the recorded route. The institution then introduces a clearly documented four-meeting pilot with a facilitator, a new application form and capacity for twenty unique participants. Fifty unique people submit actual applications under that limited-capacity offer. Twenty receive places and attend at least one meeting; thirty remain unallocated. After the four meetings, the pilot ends and the formal register again contains zero meetings of that type.

The example initially stipulates that the application identities, allocations and attendance events have been checked. The fifty applications are distinct submissions, and the twenty attendees are unique people. These stipulations prevent duplication and basic recording error from doing unexplained work in the first interpretation. A real study would have to establish their accuracy through its record process and validation.

Several narrow statements follow. The pilot received fifty actual applications under its specified terms. Twenty people attended within the stated window. Thirty applicants did not receive a place. The formal meeting register changed with the introduction and ending of the pilot. These are meaningful findings in the constructed record. They should remain visible even when stronger interpretations fail.

The thirty unallocated applications are unmet requests under that allocation process. Calling them thirty people who would certainly have attended if allocated would add a potential-response claim. Applicants can change plans, misunderstand conditions or respond differently after allocation. Likewise, the fifty actual applications should not be identified automatically with the unconstrained reference willingness variable in the capacity model. The known chance of receiving a place belongs to the actual application offer.

The record leaves several matters open. It contains no systematic baseline observation of informal discussions or other venues, no validated measure of pre-exposure dispositions, no comparison group for the new package, and no complete assessment of participants' experience or competing uses. These gaps define the scope of the rival explanations. They should be listed because they bear on particular claims, not because every possible unknown is equally relevant.

42.2 Restriction, Formation, and the Historical Target

One interpretation emphasizes a previously obstructed opportunity. People may have wanted to organize the relevant activity before the pilot, while the earlier route prevented formal participation. Another emphasizes formation: the invitation, facilitator and concrete possibility may have helped people develop an interest or a group that had not existed in the same form. Both can be compatible with the later applications and attendance.

The distinction should follow the repaired target analysis. The actual pilot offer can have a positive response in both histories. Its response includes what the offer does before application and attendance. A difference in a modeled pre-exposure disposition is therefore not automatically a difference in actual-offer demand. The dossier cannot recover a historical latent state simply by observing a successful opening.

Earlier articulated attempts would add relevant evidence. A dated request, a contemporaneous message organizing the activity or a documented refusal could support a narrower historical account of expressed intention and the institutional route. Such evidence would still need to be interpreted in relation to the later offer. A request for an unfacilitated room can differ from an application to the facilitated pilot. The comparison should identify which features persisted and which changed.

Retrospective accounts can also matter. A participant may report having wanted the activity for years, or describe the pilot as introducing something previously unimagined. These are evidence about present accounts of history and experience. Their relation to earlier behavior, wording and circumstances can strengthen or weaken a historical interpretation. The evaluation should preserve both kinds of account and avoid assigning each participant to a permanent discovery or formation type from one sentence.

The public decision may not require a final allocation between those explanations. Continuing a worthwhile activity can be justified through its present and prospective consequences. A claim that

an earlier restriction ignored known demand, however, requires historical evidence about the requests, information and alternatives available at the time. The stronger accusation should not be inferred from the pilot's success alone.

42.3 Record Creation and Previously Unregistered Activity

A second interpretation concerns the observation process. The new pilot introduced a registration form and a formal meeting category. Informal conversations may have occurred before it without appearing in that register. The rise from zero formal entries to twenty registered participants can therefore combine changes in activity and changes in recordability.

This possibility does not erase the twenty attendance events. It changes the comparison to the baseline. If similar activities were already occurring informally, the pilot may have formalized, supported or relocated them. Those changes can be valuable. A claim of entirely new activity requires evidence that the relevant baseline activity was absent or sufficiently different under the stated target.

The investigation should define the baseline comparison before searching for it. If the target concerns formal access to a supported recurring practice, informal brief conversations may be an imperfect substitute. If it concerns any discussion of the topic, those conversations may be central. The target determines which observations bear on the comparison. Selecting a broad or narrow activity definition after seeing the records can create an apparent effect through classification alone.

A useful additional inquiry could examine the record creation rules, retained room-use notes and contemporaneous accounts of informal activity. The study should distinguish evidence of occurrence from evidence of purpose and continuity. A note that people talked in the branch establishes less than a documented sequence of organized discussion, while a participant account may reveal a meaningful practice that the formal system never named. Neither source should receive an automatic interpretation without its context.

The linked-event analysis becomes relevant if the new program produces several records per meeting. Applications, booking revisions, attendance sheets and outcome questionnaires can describe the same activity. Their accumulation increases documentary detail. It does not multiply the number of meetings or unique people. A fair baseline comparison should preserve the relevant unit and the difference in recording density.

42.4 Substitution across Venues and Activities

A third interpretation is that the pilot attracted people who would otherwise have participated elsewhere. The named branch gains attendance, while total participation in the broader activity may change less or remain stable. The dossier initially contains no evidence about those alternative routes.

The relevance of substitution depends on the target. If the question concerns use of the named branch, the local attendance is directly relevant. If it concerns expanding public opportunity across the community, movement from another equally suitable venue may have a different significance. If the pilot reduces travel, cost or dependence on a private host, substitution can improve opportunity even without increasing the number of activity occasions. The evaluation should identify those consequences rather than treating every displaced alternative as a reason to discount the pilot.

Evidence about alternatives should be concrete. Participants can describe where the activity occurred previously, what conditions it required and how the pilot changed their choices. Records from another venue may corroborate an actual sequence where access and consent permit such inquiry. A hypothetical answer that a person could have met elsewhere is weaker evidence of a completed substitute. The alternative-offer dossier supplies the structure for assessing suitability.

Substitution can also occur among activities. A participant may use time previously devoted to solitary reading, another group or family responsibilities. The new attendance count does not describe the value of that tradeoff. Participants' judgments and the effects on others can be relevant. The public assessment should make consequential displacement visible without assuming that every change of activity represents harm.

At system scale, branches can affect one another. A pilot may draw established groups from nearby branches, relieve crowding there or create new competition for staff. A study claiming a system-wide effect should define an outcome that captures the relevant distribution. The branch model's no-between-branch-effect restriction would require review if these pathways were present.

42.5 Novelty, Support, and Persistence

A fourth interpretation concerns the temporary conditions of the pilot. The initial response may depend on novelty, intensive facilitation, a short commitment or unusual publicity. These components are part of the actual offer if delivered. Their role matters when the institution considers a continuing or less supported version.

The return to zero formal meetings after the pilot ends supplies little evidence about response to a continued offer, because the opportunity has been removed. It also supplies no proof that the same attendance would persist indefinitely if the pilot continued. The relevant future offer needs a duration, staffing arrangement, allocation process and expectation of continuity. A four-meeting response should be transported to that offer through an explicit comparison.

A follow-up can investigate several distinct trajectories. Participants may want to continue the same activity, develop a different task, move to another venue, take a break or leave. Each trajectory can reflect learning from the pilot. The study should distinguish reported intentions from actual subsequent arrangements and retain the reasons for change where they bear on provision.

Reducing support creates a component question. If the continuation removes the facilitator, the original pilot is an imperfect test of the new offer. A comparative design can investigate the role of support where it is feasible and appropriate. The joint-component analysis warns that a weak average interaction or response to one component may conceal dependence on combinations. A claim that the activity can sustain itself should therefore be tied to the actual conditions under which continuation is observed.

The institution can still make a bounded commitment under uncertainty. It can offer another defined period, state what support is available and establish a review process. The continuity analysis explains how such a decision creates expectations and responsibilities of its own. The evidence question and the commitment question should be considered together without allowing one to substitute for the other.

42.6 Participation, Experience, and Competing Consequences

The descriptive success of applications and attendance leaves the quality and public consequences of the pilot unresolved. This subsection develops rival evaluative readings while keeping them distinct from rival explanations of the attendance count.

Participants may find the activity useful and retain control over its direction. They may also experience uneven opportunities to contribute, frustration or pressure to report satisfaction. Staff may gain a workable program or face burdens that the attendance register omits. Other users may experience little effect or lose access to a valued space. These possibilities require evidence appropriate to each claim.

The same participant can report several consequences. A demanding discussion may be enjoyable in some respects and difficult in others. It may serve a valued purpose while creating a cost the person willingly bears. A one-dimensional favorable score can compress those judgments in ways that obscure the public question. The experience instruments should preserve the distinctions material to the decision.

Selection further complicates the comparison. People who attend are a selected group under the offer; those who complete a questionnaire are selected again. A favorable respondent fraction can be accurate for respondents while failing to describe all attendees or the original population. The participant-outcome bounds and nonresponse analysis state what additional assumptions would be needed for stronger claims. The dossier should not skip those gates because the initial attendance result looks promising.

An evaluative disagreement can remain after the facts are clarified. Participants may value the activity highly while other users reasonably prioritize a competing use of the room. The report can clarify the

extent and distribution of consequences, but it cannot derive the correct public priority from attendance alone. The decision needs reasons and an appropriate authority structure.

42.7 Evidence that Discriminates among the Explanations

The rival accounts identify different research needs. This subsection organizes those needs by the claim they can change. The purpose is to avoid collecting a large volume of evidence that repeatedly addresses one uncertainty while leaving the decisive alternatives untouched.

To assess a historical claim of earlier articulated demand, the study should seek contemporaneous requests and records of the route through which they were handled. To assess formation during the pilot, it can compare prospectively defined exposures followed by a common offer, with the relevant history and interference conditions. The first inquiry concerns historical expression and institutional knowledge; the second can identify an effect of exposure. They can inform one another while retaining different targets.

To distinguish increased activity from increased recording, the study should compare event definitions, recording coverage and evidence of baseline activity. Additional outcome questionnaires after the pilot may improve knowledge of experience while doing little to resolve baseline recording. A validation effort should therefore be directed at the channel and period relevant to the comparison.

To assess substitution, the study needs information about actual alternatives and changes in their use. A broader population or system outcome may be required. Asking only current attendees whether they liked the library leaves the alternative route unresolved. A concrete alternative-offer comparison can clarify suitability and tradeoffs, but it should preserve the selection or assignment process that generates the comparison.

To assess persistence under routine support, the study needs a specified continuation offer and follow-up under that offer. Repeating the original pilot with exceptional resources answers a replication question for the original package. It does not establish that a reduced-resource version will have the same response. The transport model clarifies the additional comparison assumptions.

The choice among these inquiries should follow the pending decision. If the institution is deciding whether to repeat the same bounded pilot, a complete history of earlier latent interest may be less useful than accurate information about present consequences and cost. If it is reviewing an allegation that earlier applications were ignored, historical records become central. A comprehensive framework should help choose the relevant inquiry, not require every inquiry in every case.

42.8 Revision, Refutation, and Preservation of Earlier Results

A flexible framework can become difficult to challenge if every adverse observation is explained by a new hidden condition. This subsection states how revision should remain accountable to the evidence. The distinction between a justified target refinement and an evasive change is central to the method's credibility.

A proposed mechanism should identify observations that would count against it within its scope. If the claim is that a specific application delay prevents participation, evidence that the relevant people receive timely, intelligible replies and still decline under otherwise specified conditions weakens that explanation for those cases. It may leave other mechanisms open, but the original delay claim should be revised accordingly.

If the claim is that facilitation increases common-offer attendance in the defined cohort, a well-implemented assigned comparison can supply adverse evidence. The analyst should report the estimated contrast and its uncertainty, including a small or negative result. Introducing a new outcome after seeing the attendance result can be justified as exploration; it should not replace the original result without a clear record of the change.

The framework should also accept evidence of low response to a credible offer. Such a finding can support ending or redesigning that package. The offer-history definition limits the conclusion to its proper scope. It should not be used as a device for asserting that some unspecified better offer must

reveal high demand. A possibility without a defined offer and evidential route remains a possibility, with correspondingly limited policy force.

Refutation can concern a bridge rather than the substantive opportunity. An audit can show that a registration count does not represent unique participants, or that a proposed history variable has inconsistent meanings across groups. The appropriate response is to repair or abandon that measurement use. The existence of a flawed indicator does not establish that the activity lacks value, just as a valuable activity does not validate its indicator.

Earlier results should remain in the record after revision. A new offer can produce a different response without making the earlier response false. A richer archive can narrow a historical category range while preserving why the earlier range was wider. A corrected count can replace an erroneous estimate with a documented explanation. This continuity of reasoning supports learning and makes later claims easier to assess.

The dossier thus provides a demanding standard for the paper's own argument. It should preserve narrow positive findings, expose the assumptions behind broader interpretations and identify evidence capable of changing its conclusions. The method earns its place through that discipline and through practical improvements in inquiry. Its complexity supplies no independent warrant for believing a favored account of demand.

43 Research Obligations and Extension

This section identifies the work required to turn the present construction into a stronger contribution. It separates conceptual validation, measurement, field implementation, historical inquiry and broader comparison. The obligations preserve the status of the manuscript's claims: conditional derivations are complete within their stated models, while empirical meaning, practical effectiveness and a comprehensive originality assessment remain open.

43.1 Validation of the Offer and Response Concepts

The first obligation concerns whether participants and staff can recognize the analytical objects used in the study. This subsection specifies the tests needed before the definitions are treated as adequate descriptions of a library practice.

Research Obligation 43.1 (Offer intelligibility). Test whether the proposed offer description communicates a sufficiently consistent activity and set of participation conditions across the intended population. Record consequential differences in interpretation and revise the target when a shared description conceals distinct practical offers.

The test should examine more than literal comprehension. People may understand the words while holding different expectations about interaction, welcome, privacy, obligation or the possibility of leaving. Those expectations can affect the response. The study should identify which differences matter to the decision and which variation can remain inside a broader package target. A description need not remove every ambiguity to be useful, but it must be adequate for the comparisons the paper makes.

The latent disposition W^- requires a separate assessment. A meaningful historical inquiry needs an empirical or theoretical bridge that gives the state a role beyond the counterexample. If such a bridge cannot be justified, the manuscript should retain the counterexample as a critique of an unsupported historical inference and concentrate empirical work on actual-offer responses and exposure effects.

43.2 Measurement Validation and Population Coverage

The second obligation concerns the link between actual events, records and the population to which a conclusion applies. This subsection identifies the minimum evidence needed to use the conditional bounds responsibly.

Research Obligation 43.2 (Record-channel validation). Compare a suitable independent observation sample with administrative records to assess event definitions, missed occurrences, false positives and assignment-dependent recording. State which activity and population the resulting calibration can represent.

Validation should preserve the distinction between occurrence and interpretation. Agreement between two systems that copy one source does not establish independent accuracy. A disagreement can reflect different event units rather than a simple error. The study should inspect the provenance of the records and the decisions by which they are matched. Any sensitivity bound used in the analysis should be justified for the relevant event type and conditions.

Population coverage requires its own record. The study should document the frame, recruitment routes, eligibility uncertainty and follow-up loss. A broader public claim may require additional sampling or a transparent transport argument. The finite-population calculations provide a way to state what remains unresolved; they cannot identify people absent from the researcher's population definition.

43.3 Historical and Longitudinal Inquiry

A single post-offer response leaves the persistence and history of interest open. This subsection proposes longitudinal questions that follow directly from the argument rather than treating repeated measurement as automatically valuable.

Follow-up could examine whether an activity becomes a stable practice, whether participants retain the ability to shape it, and whether interest changes when facilitation or publicity is reduced. It could distinguish continued participation, temporary experimentation, replacement by another activity and withdrawal caused by a changed constraint. These trajectories can matter to the decision about support even when the initial response contrast is already identified.

Historical inquiry should focus on claims that require it. Earlier requests, accounts of attempts, policy versions and evidence of alternative activities could clarify whether a present opportunity responds to an older articulated concern. The investigation should preserve uncertainty about dates, meanings and retrospective reconstruction. Later success can motivate looking for earlier evidence, but it should not determine in advance what that evidence must show.

43.4 Transport across Branches and Communities

An effect observed in one set of branches may change when transferred to a different setting. This subsection identifies the conditions that a transport argument would need to examine.

The relevant comparison should consider the offer, population, prior history, staff resources, alternative venues, allocation rule and group relationships. A shared program name supplies little evidence that these conditions match. The study can propose a restricted family of settings in which a result is expected to remain useful, then test the distinctions that might break that expectation. Qualitative comparison can help identify those distinctions, while quantitative transfer requires its own assumptions and uncertainty.

Scaling also changes the intervention. A small pilot may rely on exceptional staff attention, spare capacity or novelty. A system-wide program may face competition among groups and interactions across branches. The evaluation should distinguish replication of the same offer from expansion into a new resource and relational setting. The latter may deserve its own policy target even when it develops the same general activity.

43.5 Literature Confrontation and Method Comparison

The current source pass establishes several relevant antecedents and prevents a broad caricature of existing evaluation. A more comprehensive contribution requires a deeper comparison with public-library research on nonusers, community needs, participatory planning and the implementation of access rules. Each scholarly position must be verified before it enters the argument.

The comparison should ask what the proposed method changes in an actual investigation. Does it alter the target, reveal a missing denominator, distinguish a formative effect from a historical claim, or improve the decision record? A useful study could compare reports produced from the same evidence under different explicitly described analytic procedures, examining which conclusions remain warranted. Such work would test the method's value more directly than asserting that its combination of concepts is novel.

The research should also identify where the method is unnecessarily elaborate. If an existing procedure already supplies the required distinction, the paper can acknowledge it and use the established tool. The objective is a coherent and useful inquiry, with conceptual originality assessed separately from the importance of the problem.

43.6 Conditions for Further Development

Further development should follow unresolved questions with substantive consequences. Priority belongs to a documented case, instrument validation, an implementable comparison and a philosophical assessment of the resulting decision. Additional formalism is warranted when it exposes an assumption, sharpens a bound or clarifies a target that the existing model cannot handle. Additional prose should develop an argument or confront evidence rather than repeat the same caution in a new vocabulary.

The present architecture can support those extensions while preserving its core distinctions. An empirical finding may narrow or defeat a proposed mechanism. A participant account may require a new category. A literature comparison may reveal a stronger antecedent. Each revision should retain its reason and its relation to the earlier state. This makes the paper a revisable contribution to a research program whose claims remain accountable to both argument and experience.

44 Assessing the Proposed Method through Shared Evidence

The framework should be evaluated by what it enables investigators and institutions to do. This section proposes a study of the method itself. It specifies a comparison using shared evidence dossiers, distinguishes constructed cases from field records, and develops criteria for warranted conclusions, useful decisions and research burden. The proposal is an unimplemented research design. Its instruments and assessment rules require review and pilot testing before use.

The central question is whether the target-and-evidence architecture improves a concrete evaluation relative to a clearly described alternative procedure. A method can be mathematically coherent yet too burdensome, difficult to interpret or redundant with existing practice. Conversely, a modest clarification can be valuable if it prevents a consequential error or identifies a better next inquiry. The proposed comparison should be capable of finding either result.

44.1 The Unit of Method Assessment

The assessment should begin with a task that an evaluator could reasonably face. A dossier might contain a policy description, application records, attendance totals, a questionnaire, selected process accounts and a pending decision. The analyst's task is to produce a report answering specified questions from that evidence. The unit of assessment is the resulting set of claims and recommendations, with the work required to produce them.

The task should separate several kinds of claim. A descriptive question can ask how many unique people attended under the recorded offer. A causal question can ask whether assigned exposure changed common-offer attendance. A historical question can ask whether the later response establishes earlier articulated demand. A decision question can ask which alternatives remain defensible under stated public objectives. These questions should be explicit so that the assessment can identify which one an analyst actually answered.

The proposed framework may improve one kind of task more than another. It might be especially useful when the denominator changes or the outcome is selected after participation, while adding little to a simple count with complete accurate records. The evaluation should preserve that heterogeneity. An average performance score could conceal a narrow but important benefit or a broad burden with occasional gains.

The intended users also matter. A researcher with statistical training, a branch manager and a community participant may need different forms of the method. The study should identify which version is being assessed and what preparation it assumes. A comparison cannot establish the usability of a simplified decision sheet by testing only an expert's reading of the full mathematical manuscript.

44.2 The Comparator and Its Actual Competence

The alternative procedure should be specified fairly. It could be an institution's current evaluation process, an established reporting guide or another explicit analytic workflow. The comparator should include the safeguards and distinctions it actually uses. A deliberately weak procedure that treats every attendance count as demand would provide little evidence of the framework's practical contribution.

The verified library and evaluation sources already establish that nonuser research, outcome assessment, behavioral conditions and adaptive evaluation are substantive antecedents. The comparison should therefore ask what additional work the present architecture performs in a particular case. It may clarify an offer-history target, expose a selection denominator or preserve an unresolved historical claim. Those contributions should be demonstrated in the resulting analysis, not inferred from the framework's terminology.

The comparator can also outperform the proposed method. A familiar procedure may produce a correct, usable report with less staff time. It may incorporate local knowledge that the formal target specification initially misses. The study should record those strengths and use them to revise the framework. Method assessment should permit integration of useful components rather than force a competition in which only one complete approach can survive.

Before the main comparison, the research team should document the comparator through its actual materials and, where appropriate, observation of its use. The current proposal supplies no claim that any named institution follows a deficient procedure. A field comparison would need the institution's participation, appropriate permissions and a clear account of what is being evaluated.

44.3 Constructed Dossiers with Known Conditional Structure

Constructed cases can test whether analysts follow a stated model and distinguish its targets. The dossiers in this paper provide candidates, but they should be adapted and reviewed for the assessment rather than treated as validated test items. Each constructed case needs a complete generation record and a statement of what is known to the assessors.

For a finite gate case, the generator can specify a population, permission states and alternative-offer responses, then disclose only selected records to analysts. The assessors can know the full constructed schedule while asking what follows from the disclosed information. An analyst should be evaluated on the warranted identified set, not on whether a point guess happens to equal the hidden schedule's value. Guessing the truth from insufficient evidence differs from deriving a supported conclusion.

For a classification case, the generator can specify record errors and audit information. The disclosed error caps should be internally consistent in some dossiers and deliberately contradicted by a verified audit in others. The task can then test whether analysts detect incompatibility and revise the model. A method that always computes the formula without checking its premises would fail a central part of the assessment.

For a formation case, the dossier can contain the two observationally equivalent histories from the counterexample. Analysts can be asked separately about actual-offer response and the modeled pre-exposure state. A correct answer should preserve what is shared across the worlds and what remains

unresolved. Merely saying that nothing can be known would be too coarse, because actual-offer response can be identified in the constructed setting even when the latent historical state is not.

For a selected-outcome case, the generator can retain participation and questionnaire patterns. The assessment can distinguish a correct respondent summary, a warranted completion range and an unsupported causal claim about all participants. A useful report should explain the difference in language accessible to the intended decision maker. Formal correctness and communicative usefulness are related but separately assessable.

The limits of constructed cases should remain explicit. Their full information is known because the researchers create it. That advantage tests conditional reasoning and communication. It does not establish that the model captures actual library processes, that the instruments are valid or that the framework improves real decisions. Field cases are needed for those questions.

44.4 Field Dossiers and Unresolved Ground Truth

Field records rarely supply a complete causal or historical truth against which every report can be scored. The assessment should therefore distinguish verified process facts, justified model consequences and contested interpretations. A reference assessment can document reasons without pretending to possess certainty that the evidence lacks.

Some factual components can be checked directly. The study may verify policy versions, unique identifiers, actual timestamps, the assignment procedure and the relation between a booking and attendance record. These checks can identify clear errors in a report. Other components, such as the meaning of a participant's retrospective account, may remain open to more than one defensible interpretation.

An adjudication process should preserve that difference. Reviewers can state the evidence supporting each judgment, identify unresolved disagreement and distinguish an analyst's unsupported assertion from a reasoned interpretation that another reviewer rejects. A majority vote can be a practical way to complete an assessment, but it should not be described as creating the underlying truth of a contested philosophical or empirical claim.

The reference process should also be open to correction by the analysts and affected participants. An evaluator may identify a missing document or expose an ambiguity in the task itself. A participant may explain that a purportedly equivalent offer changed an important condition. Those contributions can improve the dossier. The study should record the revision and determine whether the original assessment remains comparable after the change.

Field dossiers should retain enough context to make the analytical question intelligible while protecting information that the comparison does not require. Removing context can make a case easier to distribute but can also erase the relationships needed to interpret it. The research design should justify what is retained, transformed or withheld and recognize the resulting scope of the assessment.

44.5 Assignment of Analysts and Learning Effects

A comparison of methods can be confounded by differences among analysts. This subsection outlines design choices for separating the procedure from the people who use it, without presenting one arrangement as universally appropriate.

One design assigns analysts to procedures before they receive comparable dossiers. Their preparation, experience and available resources should be recorded. Random assignment can support a comparison of the assigned analytic procedures under the study conditions, subject to its own implementation and outcome assumptions. The procedure includes training and materials supplied to the analyst, so the study should describe them as part of the intervention.

A second design asks the same analysts to use different procedures on different dossiers. This can reduce some between-person variation while introducing learning and order effects. Once an analyst learns the target distinctions, that knowledge may influence later use of the comparator. Counterbalancing order can address some sequence patterns but cannot make learning disappear. The study should define whether such transfer is part of the practical outcome or a threat to the intended comparison.

Time limits and support should be deliberate. Equal time can test what each procedure achieves under the same resource constraint. Allowing each procedure the time it requires can assess a different tradeoff between quality and burden. Neither comparison is sufficient for every institutional question. The report should identify which resource relation it studies and avoid interpreting one as the other.

Analysts may communicate, share templates or infer the study's favored approach. Such interactions can affect performance. The design should state the permitted collaboration and preserve relevant departures. It should also avoid framing one procedure as the expected superior method. A fair comparison needs a credible possibility that the proposed architecture adds little or creates new mistakes.

44.6 Claim-Level Assessment and Informative Restraint

The primary assessment can examine individual claims in the report. Each claim should identify a target, a stated conclusion, its evidence and any assumptions used to connect them. This provides a more interpretable basis than a single impression of whether the report is rigorous.

Possible categories include correct descriptive statements, mathematically valid conditional results, clearly marked exploratory interpretations, unsupported causal promotion, target substitution, denominator errors and omitted material uncertainty. These are proposed assessment categories. Their definitions should be tested with sample reports and revised when reviewers cannot apply them consistently.

The assessment should reward informative restraint rather than indiscriminate caution. A report that correctly identifies a narrow interval should receive more credit for warranted resolution than a report that leaves the entire unit interval open without reason. A report that establishes twenty actual attendees should retain that fact while limiting a historical claim. A method that produces only caveats would fail to use available evidence effectively.

The same principle applies to negative findings. A dossier with complete, accurate observation under a clearly specified offer can support a low-response conclusion. An analyst should state it when warranted. Reclassifying every such result as unknowable hidden demand would be an error of the framework's use. The assessment should include cases in which the proper conclusion is unfavorable to expansion.

Assumption transparency should be scored separately from assumption plausibility. An analyst can clearly state an implausible transfer restriction, or rely on a plausible restriction without making it visible. The first problem concerns the substantive argument; the second concerns reviewability. Both matter, but they require different remedies. A well-designed assessment should preserve the distinction.

44.7 Decision Usefulness and the Remaining Disagreement

A technically correct report can remain difficult to use. The assessment should therefore examine whether the report helps the intended decision maker understand the feasible alternatives, the consequences and the unresolved questions. This is a separate outcome from formal validity.

Reviewers can ask whether the report identifies which evidence would change the decision, which disagreements concern values and which concern facts, and which claims remain outside the study's scope. They can also examine whether the recommendation follows from the stated premises. A report that recommends expansion through an attendance effect while omitting its own broad uncertainty interval would fail this consistency test.

Decision usefulness should not be equated with producing a decisive recommendation in every case. A clear account of why two alternatives remain defensible can be useful, especially when the disagreement concerns legitimate public priorities. The assessment can examine whether the report makes that disagreement specific enough for an authorized process to address it. A false appearance of decisiveness should not receive a higher score merely because it is easier to act on.

The study can include structured discussions with intended users of the report. They can explain which parts they understood, what they believed the evidence established and which next action the report suggested. Those accounts can reveal miscommunication, such as reading a conditional bound as a probability interval or interpreting actual-offer response as prior latent demand. The research should

preserve the users' interpretations as data rather than assume that the author's intended meaning was successfully transmitted.

The report can also affect the decision process itself. It may invite new questions, change the proposed activity or make a previously hidden consequence salient. These effects belong to an evaluation of the method's practical role. They should be documented without treating every change as an improvement. A useful new question and an unnecessary diversion require different assessments.

44.8 Workload, Burden, and Implementation Failure

The comparison should record the resources needed to use the method. Analyst time, staff interruptions, participant contact, data preparation and revision effort can all matter. Their inclusion prevents a mathematically attractive procedure from receiving an implicit assumption of unlimited institutional capacity.

Workload should be described by task as well as total. A one-time investment in clarifying record units can improve later reports, while repeated manual linkage may create an ongoing burden. A difficult target discussion can prevent a costly mistaken study, or it can consume time without resolving a material ambiguity. The assessment should connect the work to its consequence where evidence permits.

Participant burden also deserves attention. Repeated questions about interests and refusals can be intrusive or tiring, and may alter the activity being studied. A method that requires more direct accounts should show why they are needed and whether a lighter procedure can support the same claim. The comparison should investigate response quality and participant experience alongside the amount of information collected.

Implementation failures can reveal design problems in the method. Analysts may confuse local symbols, apply a sharpness result outside its assumptions or treat every uncertainty as a separate interval that can be combined independently. Staff may find a proposed record category incompatible with their workflow. These findings should guide revisions to the materials and training. They should not be dismissed simply because the full manuscript contains a correct explanation somewhere.

The study should distinguish a failure of the method's concept from a failure of its presentation or delivery. If a concise guide omits a necessary assumption, the guide may require repair. If the distinction itself cannot be made intelligibly in the intended setting, the framework may need a narrower practical role. Both outcomes are possible and should remain available to the assessment.

44.9 Success, Defeat, and Revision of the Framework

The proposed assessment needs conditions under which the framework would be judged useful and conditions under which its role would shrink. These conditions should be specified before the comparison where feasible, while retaining a record of justified revisions prompted by pilot work.

Evidence of usefulness could include fewer consequential target substitutions, more accurate treatment of selected outcomes, clearer distinctions between process facts and causal claims, or a better-directed follow-up inquiry. The improvement should be assessed against its burden and the comparator's actual performance. A small gain can matter if it prevents a serious mistake at little cost; a large technical gain may have limited value if the procedure is impracticable.

Evidence against the framework could include repeated confusion caused by its vocabulary, correct comparator reports that already contain the same distinctions, unnecessary data collection, or recommendations that remain unchanged despite substantial added work. The appropriate response may be to simplify the method, restrict it to complex cases, integrate its useful components into an established procedure or abandon an unhelpful component.

The study should also assess whether the framework preserves the research object's openness. It should allow a participant account to reveal a missing category, a field observation to defeat a mechanism and a literature comparison to identify a stronger antecedent. A rigid checklist that forces every case into the initial model would conflict with the paper's account of evolving activities and evidence.

The method's eventual contribution will therefore be established through a combination of conditional reasoning, source confrontation and practical assessment. The present manuscript supplies the

definitions, derivations, constructed cases and a prospective design for that work. It leaves actual performance open. Keeping that status explicit makes the framework available for serious use and serious criticism on the same terms.

45 Conclusion

This paper has developed an account of unmet demand inference for participation in public libraries. It began with the production conditions of an activity record and followed the argument through offer specification, population coverage, recording, formation, capacity, assignment, feedback, category revision and public judgment. The resulting method connects each claim to a declared target and the assumptions under which its evidence can support it.

The governance gate establishes a narrow and important result. When a rule prevents an activity regardless of the response to a specified enabled offer, an empty activity record leaves that response unresolved. The finite bounds show how partial coverage can preserve useful information without filling in the unobserved population. The recording analysis separates actual events from the channel through which they appear in administrative data. These results require explicit event, population and measurement assumptions.

The temporal analysis adds a further distinction. A response to an actual offer includes the offer's effects before participation is recorded. A separately modeled pre-exposure disposition concerns another target. Two constructed worlds can agree on every opening response while differing in that latent initial state. The common-offer design preserves a useful prospective comparison: an earlier exposure can change later uptake under a shared offer, even while the historical disposition remains unresolved. Marginal response rates can identify an average contrast while leaving the pattern of individual positive and negative changes only partially identified.

Capacity and coordination make the social organization of participation explicit. Full occupancy can conceal additional candidates, expanded eligibility can displace an existing user, and fragmented offers can prevent a willing group from acting together. A branch-level comparison can include within-branch interaction, provided its interference boundary and baseline population are justified. Population-weighted and branch-weighted results answer different questions. The expectation calculation clarifies that distinction while leaving practical precision to the actual study design.

The analysis of feedback shows how continued access can become dependent on evidence that access itself is required to generate. Exploratory opening can alter that process under stated conditions, with its formative effects and burdens included in the evaluation. Category bridges provide another form of discipline: revised descriptions can improve understanding while preserving the limits of comparison with older records. Original descriptions, instrument versions and reasons for recoding remain part of the evidence.

The prospective study connects these distinctions to documents, interviews, observations and intervention comparisons. Its empirical implementation remains open. Published policies establish bounded institutional illustrations, and existing professional resources demonstrate that library evaluation already includes outcomes and adaptive learning. The paper's contribution must be judged by whether its target-specific analysis improves that work in a real case.

The expanded measurement analysis makes that practical test more demanding. An audited error budget can narrow a response range while revealing an incompatible assumption. A known recording channel can identify a probability under its conditions, while a changed population or observation process can break the transfer. Participant experience is selected through attendance and often through questionnaire completion; its causal interpretation requires a target that respects those selections and the possible inapplicability of experience outside participation. These distinctions preserve useful information without assigning it an unsupported population or historical meaning.

Collective participation also requires attention to composition and time. Two arrangements can produce the same attendance total while differing in whether the roles needed for the activity are available. A permission granted after a planning deadline can differ from a timely opportunity even when both

appear as approvals in the final register. An alternative venue can accommodate more schedules while changing the task or excluding a necessary perspective. These cases show why the complete offer and its history must remain attached to the demand claim.

The decision analysis makes the final bridge explicit. A response interval can support a robust ranking under declared value assumptions, or leave several policies defensible. A regret criterion can select among those possibilities and can favor a lottery within its model, while public admissibility remains a separate question. Further inquiry can narrow uncertainty, and it can also change the response by forming a new history. The value of a study-and-offer sequence therefore includes its actual consequences and cannot always be reduced to information about a fixed prior state.

The method's own evaluation remains equally important. The proposed comparison of analyses of shared evidence can examine whether these distinctions prevent consequential errors, improve the direction of inquiry and communicate uncertainty usefully. It can also reveal redundancy, excessive burden or concepts that need revision. A comprehensive theoretical construction becomes a stronger contribution when it identifies such defeat conditions and permits a simpler or better existing procedure to replace an unhelpful component.

Finally, demand evidence leaves public reasons visible. Participation can express a prior concern, develop a new interest or become unsuitable after experience. People retain reasons to decline, reinterpret or propose a different opportunity. Competing uses, staff resources, accessibility and institutional responsibilities require judgments beyond attendance. A responsible evaluation supports those judgments through clear evidence and explicit uncertainty, while preserving the practical authority of affected people to question how their opportunities and interests are represented.

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