A Preliminary Formalism for Heterogeneous Value Conversion in the Generative Relational Framework

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Abstract

This discussion paper develops a preliminary formal language for heterogeneous value conversion within the Generative Relational framework. The motivating problem is the apparent simplicity of a conversion written as $A \to B$. In many economic, legal, institutional, and political settings, the values, value-bearing entities, conversion procedures, and conversion nomos are themselves historically generated within wider relational structures. A conversion therefore need not be understood as a direct mapping between two already constituted objects. It may instead be represented as a history in which entities are identified, relations become institutionally legible, operations extend or terminate relational structures, and later operations act on configurations already changed by earlier ones.

The paper develops this representation in stages. It introduces higher-order event structures and a partial-order account of relational time; distinguishes accumulated history from active relational boundaries; and models observation, entity identification, generative transformation, and conversion through typed maps and partially defined operators. Concepts from 2-complexes, operator composition, field theory, and spin-foam models provide mathematical resources while the analysis maintains ontological suspension regarding their ultimate physical interpretation. Conversion is represented as a partially composable history of state-dependent operations, making room for branching and merging histories, co-generated value, attribution, order sensitivity, and compositional noncommutativity. The framework also treats conversion nomos as historically emergent and revisable, and separates descriptive stabilization from normative justification. A further layer accommodates theory-relative normative evaluation, judgment-induced operators, epistemic truncation, and an illustrative topological account of epistemic fidelity.

The objective is representational rather than foundational. The paper does not propose a complete ontology of value, a universal algebra of conversion, or a substantive theory of conversion justice. It offers one preliminary architecture for making some of the relational, temporal, epistemic, institutional, and normative structure hidden by $A \to B$ explicit. Empirical parameterization, stronger operator-algebraic or categorical formulations, alternative higher-order representations, and domain-specific theories of justice remain open for further work.

Keywords: heterogeneous value; value conversion; relational formalism; 2-complex; operator composition

Discussion Paper Note

This manuscript develops a preliminary formal representation of heterogeneous value conversion within the Generative Relational framework. Its immediate analytical object is the apparently simple transformation $A \to B$. The paper examines the additional relational, historical, epistemic, institutional, and normative structures that may be required when A and B are heterogeneous value forms whose constitution and conversion depend on temporally extended relations.

The formal language is exploratory. Higher-order complexes, partial-order event structures, operator composition, field-theoretic vocabulary, and spin-foam models are used as mathematical resources. Their inclusion does not establish that economic or social reality possesses the same ontology as the physical systems from which some of these formalisms originate. The manuscript adopts ontological suspension and distinguishes structural representation, formal analogy, and empirically parameterized modeling.

The paper is also deliberately limited in its normative ambition. It provides interfaces through which different theories of justice may evaluate conversion histories, entity identification, procedural ordering, and resulting relational changes. It does not define a fixed GR criterion of just conversion. Nomos, recognition, evaluative judgment, and the admissibility of conversion procedures are treated as historically and relationally situated objects of inquiry.

The present draft develops the formalism incrementally. Definitions and conventions are introduced only when they perform a specific analytical role. Alternative representations, including hypergraph, simplicial, categorical, operator-algebraic, and field-theoretic approaches, remain open where they can represent the relevant structure more adequately.

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Notices

Status. This working draft records an evolving stage of a preliminary formal inquiry into heterogeneous value conversion and is circulated for discussion. Formal objects, notation, section structure, and the relation among 2-complex, operator-algebraic, field-theoretic, and categorical representations remain subject to revision. The manuscript does not present a complete formalization of the Generative Relational framework or a settled substantive theory of conversion justice.

Statement on the use of language models. The exploratory discussions and preparation of this paper involved OpenAI's ChatGPT. ChatGPT supported exploratory dialogue, formal reconstruction, argumentative criticism, source discovery followed by verification, and drafting in LaTeX. The author selected the research questions, directed and approved the theoretical commitments and epistemic status of the claims, and bears sole responsibility for the manuscript, including its definitions, formal constructions, arguments, conclusions, and errors. Authorship credit remains with the human author.

1 Introduction

This section establishes the analytical problem, scope, and methodological position of the paper. The discussion introduces heterogeneous value conversion as a process whose apparent representation as $A \to B$ can compress substantial relational and historical structure. It then states the role of the proposed formalism, identifies the mathematical resources developed in later sections, and specifies the limits of the present inquiry.

Heterogeneous value conversion occurs when values represented through different forms, scales, institutional languages, or relational structures enter a common procedure of comparison, exchange, compensation, transfer, or transformation. Labor may be converted into wages, ecological loss into monetary compensation, knowledge into intellectual-property claims, one currency into another, and historically generated contributions into present forms of recognition or entitlement. The visible conversion can be compact, while the conditions that make the conversion possible may include prior classification, recognition, attribution, measurement, institutional stabilization, and normative authorization.

The present paper develops a preliminary formal language for representing this structure within the Generative Relational framework. Its objective is methodological. The formalism makes explicit some of the relational and historical structures hidden when a conversion is written only as $A \to B$. The analysis proceeds through higher-order relational histories, partial-order event structures, relational entity identification, composable transformations, conversion histories, state-dependent operator families, conversion nomos, and interfaces for theory-relative normative evaluation.

The values denoted by A and B are treated as relationally situated manifestations rather than isolated substances whose relevant properties are fully contained within the symbols themselves. A day of labor, a monetary amount, a credit claim, a scientific contribution, a parcel of land, or an ecological resource can depend on temporally extended relations involving persons, institutions, material infrastructures, technological systems, ecological conditions, symbolic orders, and prior historical events. The subjects associated with these values may likewise require history-bearing representations. A conversion procedure therefore operates within a broader relational structure in which values, participating entities, institutional interfaces, and conversion norms can themselves be generated and transformed.

A static graph offers one useful representation of relational adjacency, yet its ordinary form provides limited resources for representing the higher-order and temporally extended processes emphasized in the present inquiry. The paper therefore introduces 2-complexes and related higher-order structures as one candidate representation. Vertices, edges, and faces can encode event-like occurrences, continuing relational connections, and higher-order processes. A partial order over events provides a model-relative representation of generative precedence. This construction permits ordinary chronological time to be introduced later as a compatible coordinate where an empirical application requires it, while avoiding the need to make a universal external clock primitive to the formal language.

Spin networks and spin-foam histories provide one physical source of inspiration for this history-bearing representation. Their inclusion is structural and pedagogical. The manuscript does not infer that heterogeneous value conversion instantiates quantum-gravitational dynamics. The 2-complex, partial-order, and history-weight language is used because it offers a compact way to represent events, boundaries, higher-order relations, and evolving histories. Alternative mathematical structures remain available where they represent the intended relational organization more adequately.

The same restraint applies to operator and field language. The paper separates observation maps from generative transformations. An observation map extracts a finite, context-dependent representation from a broader relational configuration. A generative operator changes the configuration itself and may create new events, relations, claims, contracts, symbols, monetary positions, or other structures. Conversion can then be represented as a partially composable history of such transformations. Earlier operations may alter the configuration on which later operations act, allowing sequential order to have formal and procedural significance. Operator-algebraic language is used to clarify composition and order sensitivity, while stronger algebraic claims are reserved for cases in which the required mathematical structure has been explicitly established.

Entity identification occupies a distinct place in the formalism. Institutions and evaluators do not ordinarily access the complete relational history of a person, community, value, or contribution. They operate through finite records, categories, measurements, evidentiary practices, and historically situated interpretive structures. The entity selected for valuation or conversion is therefore modeled as a bounded relational-historical identification. The identification may omit, fragment, aggregate, or otherwise distort relations that remain relevant to the broader ontology. Epistemic questions consequently enter conversion before a numerical conversion ratio or settlement rule is applied.

Conversion nomos is likewise represented within the evolving relational configuration. Rules concerning admissible ownership, recognized value forms, valid settlement, permissible conversion, and acceptable compensation can be stabilized through institutions, repeated practices, legal decisions, power relations, material constraints, and prior conversion histories. The formalism therefore allows nomos to affect the space of available operations while also remaining subject to later transformation. This representation separates the historical emergence of a norm from its normative justification.

Normative evaluation enters as a further layer. The core formalism does not supply a fixed substantive criterion of justice. Different theories may select different relational entities, histories, observables, and evaluative relations. Procedural theories may emphasize standing, sequence, and contestability; epistemic theories may emphasize recognition and structural fidelity; capability-oriented approaches may emphasize the future possibilities available to affected subjects; distributive and relational approaches may emphasize allocation, dependency, status, or power. The same conversion history can therefore support several theory-relative evaluations without requiring those theories to collapse into a single normative metric.

The paper also permits normative judgment to be represented as dynamically consequential. A judgment may generate an operator-like influence that changes which subsequent transformations become accessible, institutionally supported, inhibited, or comparatively weighted. Such a judgment is itself situated within the relational history and may contribute to subsequent normogenesis. This representation supports analysis of evaluative reflexivity while retaining the distinction between a theory's substantive normative commitments and the formal language used to represent their consequences.

The present construction remains preliminary. The 2-complex representation is one candidate for history-bearing higher-order structure. The partial-order representation of generative precedence and the historical-persistence convention are adopted for analytical tractability within the current model and do not establish a final metaphysics of time or physical spacetime. Operator, field-theoretic, and spin-foam vocabulary is introduced only where it performs a specific representational role. Empirical parameterization, stronger algebraic structures, computational realizations, and substantive theories of conversion justice remain open research directions.

The paper therefore pursues a limited objective: to construct a coherent preliminary representation of heterogeneous conversion in which historical structure, entity identification, observation, transformation, nomos, and normative evaluation remain analytically distinguishable while participating in a common process language. The following sections will first develop the conceptual and mathematical background required for readers without specialist training in advanced algebra or quantum gravity, then construct the conversion formalism step by step before examining illustrative applications and open problems.

2 Conceptual Background

This section establishes the conceptual vocabulary required before the mathematical construction begins. Its role is to separate several operations that are frequently compressed into the language of conversion: the constitution of heterogeneous values, their commensuration, the attribution of value to relationally generated entities, the stabilization of conversion nomos, and the historical conditions under which these operations become possible. The discussion is conceptual rather than empirical. Definitions introduced here specify how the terms are used within the present formalism and prepare the mathematical background developed in the following sections on higher-order structures, relational time, operators, fields, and spin-foam-inspired histories.

2.1 Heterogeneous Value

The present inquiry begins from cases in which the two sides of a conversion cannot be assumed to be instances of one already established homogeneous quantity. A wage payment and a day of labor, a monetary payment and an ecological loss, a currency and another currency, or a legal entitlement and a historically generated contribution can each be compared or converted under particular institutional arrangements. The existence of a practical conversion does not by itself establish that the converted items share a single intrinsic value substance or a complete common metric.

Definition 2.1 (Heterogeneous value)**.** Within the present formalism, two value forms are *heterogeneous* when their relevant properties, modes of manifestation, or conditions of generation cannot be assumed in advance to lie in one fully specified common value space. Heterogeneity may concern ontology, measurement, institutional recognition, temporal structure, normative significance, or combinations of these dimensions.

Definition 2.1 is intentionally broad. Heterogeneity does not require absolute incomparability. Two value forms may become comparable for a limited purpose while remaining heterogeneous in other respects. A court may assign monetary damages to an injury without thereby exhausting the injury's bodily, relational, historical, or symbolic significance. A labor market may express work through an hourly wage while leaving many dimensions of effort, care, risk, training, and social reproduction outside the conversion interface. A foreign-exchange market may provide a numerical ratio between currencies whose institutional histories and international functions remain different.

The distinction between heterogeneity and incomparability is therefore important. The paper treats heterogeneity as a property of the analytical situation prior to the imposition or emergence of a particular conversion interface. A conversion may create a local form of comparability without proving that every relevant feature of the converted values has become commensurable.

2.2 Commensuration and Conversion

Commensuration and conversion play different analytical roles. Commensuration establishes a representational basis on which heterogeneous objects can be compared according to selected dimensions. Conversion performs or recognizes a transformation, exchange, compensation, transfer, settlement, or other relation using such a basis. The two operations frequently occur together in practice, yet separating them helps identify where epistemic and normative choices enter.

Definition 2.2 (Commensuration)**.** *Commensuration* is the construction, selection, or stabilization of a representational structure through which heterogeneous value forms become comparable with respect to specified features or purposes.

Definition 2.3 (Conversion)**.** A *conversion* is a historically situated process through which one or more value forms, claims, rights, resources, obligations, or relational positions are transformed, exchanged, compensated, settled, or otherwise connected under an operative conversion structure.

Definitions 2.2 and 2.3 leave open several distinct modes of conversion. Exchange, compensation, substitution, representation, settlement, transfer of rights, and translation need not possess identical structures. A payment can settle a contractual obligation without making the payment ontologically identical to the labor previously performed. Monetary compensation can create a legally recognized response to an environmental loss without reproducing the lost ecological structure. A currency exchange can transform an asset position while leaving the institutional histories of the two currencies distinct.

The notation $A \to B$ will therefore be used as a coarse symbolic entry point rather than as a complete account of conversion. The paper progressively reconstructs the structures suppressed by this shorthand. In particular, the formalism distinguishes the histories through which A and B become identifiable, the entities associated with them, the operations composing the conversion process, and the nomos under which the conversion becomes institutionally or normatively admissible.

Remark 2.4 (Local comparability)**.** A conversion interface may establish comparability only within a restricted context. The existence of a locally operative ratio or settlement rule does not entail a global equivalence relation across every property of the converted values.

Remark 2.4 will become important when the formalism introduces relationally scoped domains and theory-relative evaluation. It permits the same pair of value forms to be comparable under one institutional purpose and only partially comparable, or differently represented, under another.

2.3 Value Attribution and Co-Generation

Conversion often presupposes an answer to a prior question: which entity is treated as bearing, generating, owning, contributing, receiving, or being affected by the relevant value? The present framework treats that answer as analytically constructed rather than automatically supplied by the ontology. A legal owner, a productive contributor, an experiential bearer, an affected community, and a representative institution may overlap, yet they need not coincide.

Definition 2.5 (Co-generated value)**.** A value is *co-generated* when its emergence depends materially on a plurality of relational processes whose contributions cannot be adequately represented by assigning the full generative history to one isolated point-like entity.

Co-generation does not imply that every relation in the entire historical universe must receive equal analytical weight. It indicates that the attribution of value to a single visible actor can be a coarse-graining operation whose adequacy depends on the purpose of analysis. Productive activity may rely on prior knowledge, education, care, infrastructure, institutions, ecological conditions, technologies, and contributions from other actors. The practical problem is therefore to identify a finite relational structure relevant to the conversion without pretending to recover an unlimited total history.

Definition 2.6 (Attribution)**.** *Attribution* is the operation through which contribution, ownership, authorship, responsibility, entitlement, burden, or another value-relevant status is assigned to an identified relational entity or set of entities.

Definition 2.6 separates attribution from generation. A relational process may contribute to the generation of a value without receiving institutional attribution, while an entity may receive legal ownership or accounting credit that only partially corresponds to the wider generative history. This distinction creates an interface between the conversion formalism and later questions of epistemic justice, distributive justice, and power over recognition.

The same distinction applies to what is conventionally called a value holder. The notation h_A will not denote a primitive owner located at a single graph vertex. It will serve, when useful, as shorthand for a history-bearing relational entity identified under a particular analytical or institutional context. The mathematical treatment of this identification is developed later in the dedicated section on observation and entity identification.

2.4 Conversion Nomos

A conversion is rarely governed only by a numerical ratio. It also depends on rules, conventions, permissions, expectations, institutional recognitions, and background practices that determine which entities can enter the conversion, which value forms count as admissible, what constitutes valid settlement, and which procedures can revise or contest the outcome. The paper uses *nomos* as a compact term for this historically stabilized normative-institutional structure.

Definition 2.7 (Conversion nomos)**.** *Conversion nomos* denotes the historically situated structure of norms, conventions, institutional rules, recognized permissions, and stabilized expectations that conditions the admissibility, interpretation, performance, and revision of a conversion process.

Definition 2.7 treats nomos as broader than a formal legal rule and narrower than the total relational ontology. It may include law, contract conventions, accounting practices, professional standards, market expectations, customary understandings, technological protocols, and other stabilized structures that shape conversion. The formalism later associates nomos with the space of operations that are currently available or admissible within a relational configuration.

The present framework treats nomos as historically generated and dynamically consequential. A conversion rule can emerge through repeated practice, institutional design, contestation, technological change, political power, environmental constraint, or combinations of these processes. Once stabilized, it can shape later conversion possibilities and participate in generating further relational changes. Historical emergence and normative justification remain analytically distinct. A conversion nomos can be socially effective without thereby receiving a complete justification under every theory of justice.

Remark 2.8 (Normogenesis)**.** The term *normogenesis* refers in this paper to processes through which conversion-relevant normative structures emerge, stabilize, transform, or lose operative force within the wider relational history.

Remark 2.8 establishes only a descriptive category. The paper does not assume that an emergent norm is normatively valid because it has emerged. A later section on normative evaluation separates the representation of normogenesis from theory-relative normative evaluation.

2.5 Relational Emergence and Historical Dependence

The formalism adopts a process-oriented representation in which values, subjects, institutions, and conversion nomos can depend on temporally extended relational histories. The term *relational emergence* is used here in a limited modeling sense: an analytically identified structure can depend on relations and prior events that are not reducible to a list of intrinsic properties assigned to an isolated object at one moment.

This commitment has two consequences for conversion. First, a present conversion may depend on historical structure that is invisible in a snapshot representation. Two entities with similar observable present characteristics can possess different generative histories, institutional positions, obligations, or vulnerabilities. Second, conversion itself can alter later relational structure. A contract can create obligations, a credit operation can create a debt relation, a legal recognition can establish standing, a monetary conversion can change asset positions, and a judgment can modify future institutional possibilities.

The paper therefore distinguishes accumulated history from currently active relational structure. A relation that has terminated can remain part of the realized history, while a newly created claim can become active at the current relational boundary. This distinction motivates the partial-order event representation developed later in the paper and the separation between historical ontology and active relational boundaries.

Historical dependence also motivates the treatment of procedural order as analytically significant. If an earlier operation changes the relational configuration on which a later operation acts, reversing the order can yield a different history. The formalism does not assume in advance that every conversion-related operation commutes with every other. The stronger consequences of this order sensitivity are developed later in the paper.

2.6 Formal Scope and Ontological Suspension

The mathematical constructions developed in this paper are intended as representational tools for heterogeneous conversion. They are not presented as proofs of an ultimate ontology of economic, social, legal, or physical reality. The distinction is especially important because the manuscript draws on mathematical languages whose original domains include graph theory, topology, operator methods, field theory, and spin-foam approaches to quantum gravity.

Convention 2.9 (Ontological suspension)**.**

The present paper treats the mathematical structures introduced here as candidate formal representations. Relational, higher-order, field-theoretic, operator-algebraic, and spin-foam-inspired language receives a stronger ontological interpretation only when that interpretation is explicitly stated and independently justified.

Convention 2.9 permits the formalism to borrow useful mathematical organization while preserving disciplinary and metaphysical restraint. A 2-complex can be useful because it represents vertices, edges, faces, boundaries, and extended histories. This usefulness does not establish that value conversion is a quantum-gravitational process. Operator language can express partial transformations, composition, and order sensitivity. This role does not by itself establish a complete operator algebra. Field language can represent distributed quantities or local structures. Its use does not imply that every social quantity is a physical field.

The paper therefore distinguishes three levels of formal use. A *structural representation* specifies mathematical objects used directly in the proposed conversion model. A *formal analogy* highlights a transferable structural resemblance while retaining differences between domains. An *empirically parameterized model* would additionally require observable definitions, estimation procedures, and empirical validation. Most of the present manuscript operates at the first two levels. Empirical parameterization remains an open direction for later development.

The preliminary status of the formalism also leaves alternative representations open. Hypergraphs, simplicial complexes, higher categories, process algebras, temporal networks, stochastic processes, and other mathematical structures may provide better representations for particular conversion problems. The framework developed here should therefore be evaluated by its analytical usefulness, internal consistency, and capacity to expose otherwise hidden assumptions in heterogeneous conversion rather than by the breadth of its mathematical vocabulary alone.

3 Mathematical Background I: Relations and Higher-Order Structures

This section introduces the mathematical structures required to represent heterogeneous conversion without assuming prior training in advanced algebra, topology, or mathematical physics. Its role is preparatory. The discussion begins with sets, relations, and maps, then moves through graphs, hypergraphs, simplicial complexes, and cell complexes before specifying the 2-complex representation used later in the manuscript. The objective is to explain what each structure can represent, what information it suppresses, and why a history-bearing conversion formalism may require more than a static graph. The presentation remains elementary in mathematical technique while preserving the distinctions needed by the later formal construction.

Figure 1 previews the progression developed in this section. The distinctions are important because a mathematical representation determines which relational features can be expressed directly and which must be encoded indirectly through labels, auxiliary variables, or additional structure.

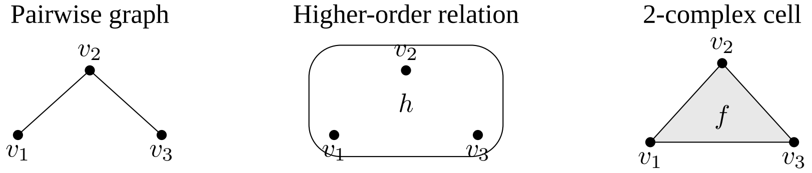


Figure 1: Three levels of relational representation used as background for the formalism. A graph records pairwise edges, a higher-order relation can connect several vertices in one relation, and a 2-complex can represent a face whose boundary is composed of edges and vertices. The figure is schematic and does not assert that these structures are interchangeable.

3.1 Sets, Relations, and Maps

A *set* is a collection of distinguishable elements. In the present paper, a set may contain events, identified entities, labels, admissible operations, or other formal objects. If X is a set and x is one of its elements, the notation $x \in X$ records membership. The internal meaning of an element is supplied by the model in which the set is used.

A binary relation records which ordered pairs of elements satisfy a specified relational condition. For sets X and Y, a relation from X to Y is a subset of their Cartesian product. This standard representation is stated in Equation 1.

$$R \subseteq X \times Y. \tag{1}$$

If $(x, y) \in R$, the formalism records that x stands in relation R to y. The equation itself does not specify whether the relation means ownership, temporal precedence, contractual obligation, causal dependence, institutional recognition, or another domain-specific relation. Such semantics enter through definitions and labels.

Definition 3.1 (Map)**.** A *map* $f : X \to Y$ assigns to each element $x \in X$ a specified element $f(x) \in Y$. A map therefore adds a functional requirement beyond the more general notion of a relation.

Definition 3.1 supplies the map concept used later in several distinct roles. Observation maps extract a finite representation from a larger relational configuration. Identification maps construct a bounded relational entity for analysis. Generative operations transform one admissible configuration into another. These roles should remain distinct even when the same mathematical language of maps is used to express them.

Composition expresses sequential application. If $f : X \to Y$ and $g : Y \to Z$, their composition $g \circ f : X \to Z$ first applies f and then applies g. The order is semantically important because the first map determines the input received by the second. This elementary observation later becomes central when conversion-related operations are only partially composable or when reversing their order changes the resulting relational history.

Remark 3.2 (Formal typing)**.** A symbol should be interpreted together with its domain and codomain. Two expressions that use the same letter can represent mathematically different objects when they act on different spaces. The later formalism therefore states the type of observation maps, generative operations, and evaluation operators explicitly.

Remark 3.2 is especially important because the manuscript uses several layers of representation. A value manifestation, an ontological transformation, and a normative evaluation should not be treated as interchangeable outputs merely because each can be represented by a map.

3.2 Graphs and Relational Representation

Graphs provide a familiar first representation of relational structure. A graph consists of vertices together with edges connecting selected pairs of vertices. For the purposes of this paper, a graph can be written as a pair $G = (V, E)$, where V is a vertex set and E is an edge set. Directed graphs distinguish the ordered pair (v_i, v_j) from (v_j, v_i), while undirected graphs treat the connection symmetrically.

Graphs are useful when the primary analytical information concerns pairwise relations. A vertex might represent an institutionally identified actor, an event, or a value-bearing object, while an edge might represent a transaction, dependency, contractual connection, or flow. The representation can be enriched through weights, directions, labels, multiple edge types, or time indices.

Example 3.3 (Pairwise conversion network)**.** A currency-conversion network can represent currencies as vertices and available exchange interfaces as edges. Edge labels may contain quoted exchange ratios, transaction costs, settlement conditions, or institutional restrictions. Such a graph is useful for questions concerning connectivity and conversion paths, even though it does not by itself represent the historical emergence of the currencies or the higher-order institutions supporting the conversion system.

Example 3.3 illustrates both the usefulness and the limitation of graph representation. A graph can describe current relational accessibility while leaving the generative history of its vertices and edges implicit. The present paper therefore treats graphs as an important local representation rather than as a sufficient representation of every conversion process.

3.3 Higher-Order Relations

Some relational processes involve several components jointly. Their structure can become awkward when represented only as a collection of pairwise edges. A committee decision, a multilateral contract, a collaborative production process, or an institutional rule may depend on a configuration that cannot be decomposed into independent pairwise relations without losing information about joint participation.

The term *higher-order relation* is used here for a relation whose elementary instance can involve more than two participating objects. A ternary relation, for example, can be represented as a subset of $X \times Y \times Z$. More generally, an n-ary relation associates an ordered n-tuple with a relational condition. Higher-order structures provide alternative ways to represent such interactions directly.

The distinction is relevant to value conversion because co-generation, attribution, and institutional authorization can depend on collective configurations. A value may emerge through a coordinated process involving several relations simultaneously. Representing that process only through pairwise connections can obscure the fact that the relevant unit of analysis is the joint configuration itself.

3.4 Hypergraphs and Simplicial Complexes

A hypergraph extends a graph by allowing one hyperedge to connect an arbitrary finite set of vertices. A hypergraph can therefore represent a higher-order relation directly. If V is a set of vertices, a hyperedge h is a nonempty subset of V, and a hypergraph is a collection of such hyperedges together with the vertex set.

Hypergraphs are useful when a relation among several participants should be treated as one relational unit. A collaborative production episode involving four identified entities can be represented by one four-vertex hyperedge rather than by six pairwise edges. The representation does not require every subset of that relation to count as an independent relation.

A simplicial complex introduces a stronger closure condition. Informally, if a simplex belongs to the complex, all of its faces also belong to the complex. Thus, if a filled triangle with vertices v_1, v_2, v_3 is included as a two-dimensional simplex, its three edges and three vertices are included as lower-dimensional simplices. This closure property provides a systematic nested representation of higher-order structure.

Definition 3.4 (Abstract simplicial complex)**.** Let V be a vertex set. An *abstract simplicial complex* K is a collection of nonempty finite subsets of V such that whenever $\sigma \in K$ and τ is a nonempty subset of σ, then $\tau \in K$.

Definition 3.4 makes the closure condition explicit. Hypergraphs and simplicial complexes therefore encode higher-order relations differently. The choice between them depends on whether the analytical problem requires lower-dimensional faces to be treated as constitutive parts of every higher-dimensional relation.

For the present paper, simplicial complexes provide useful background but are not adopted as the sole representation of conversion histories. The formalism later requires faces whose boundaries can be assembled from edges in ways that need not correspond to a simplex with a fixed number of vertices. Cell-complex language provides this greater flexibility.

3.5 Cell Complexes

A cell complex is built by attaching cells of different dimensions. Zero-dimensional cells are points or vertices. One-dimensional cells can be interpreted as edges. Two-dimensional cells can be attached along closed paths built from one-dimensional cells. Higher-dimensional versions continue the same construction.

The advantage of cell-complex language is representational flexibility. A two-dimensional cell need not be restricted to a triangle. Its boundary may involve several edges, and different cells can be glued along shared lower-dimensional cells. This structure is useful when a higher-order process has an extended boundary assembled from several relational segments.

The manuscript uses cell complexes primarily as combinatorial history-bearing structures. Geometric shape, length, angle, and embedding in ordinary Euclidean space are not required unless a later application introduces them explicitly. A face drawn as a polygon in a figure is therefore a visual aid for incidence and boundary structure, not a claim that the represented social or economic process possesses that physical geometry.

3.6 Two-Dimensional Complexes

A 2-complex is a cell complex whose cells have dimension at most two. The present formalism uses vertices, edges, and faces as the basic combinatorial ingredients. A concise representation is stated in Equation 2.

$$\mathcal{K} = (V, E, F). \tag{2}$$

Here V denotes the set of vertices, E the set of edges, and F the set of faces. Equation 2 is schematic because a complete specification also requires incidence and attachment data describing how edges meet vertices and how face boundaries are built from edges.

Definition 3.5 (Combinatorial 2-complex)**.** Within the present paper, a *combinatorial 2-complex* is a collection of vertices, edges, and faces equipped with incidence data sufficient to identify the endpoints or boundary vertices of each edge and the ordered or oriented edge sequence forming the boundary of each face.

Definition 3.5 is deliberately operational. The manuscript does not require the full machinery of algebraic topology in order to use 2-complexes as history-bearing relational structures. Later sections add partial order, labels, active boundaries, and state-dependent operations according to the needs of the conversion model.

The central motivation for introducing faces is that a face can encode an extended relational process whose internal significance is not exhausted by its boundary edges considered separately. A face may later represent, depending on the application, a conversion episode, a higher-order institutional interaction, a jointly generated process, or another temporally extended relation. The precise semantics remain model-dependent.

Table 1 summarizes the representational progression from graphs toward 2-complexes. The comparison is pragmatic rather than exclusive. Different applications can retain simpler structures when those structures preserve the relations relevant to the analytical question.

3.7 Vertices, Edges, Faces, and Incidence

The three cell types play different formal roles. A vertex is a zero-dimensional event location in the complex. An edge connects or relates vertices according to the incidence structure. A face is attached along an edge path and records a two-dimensional relational unit. These interpretations remain intentionally general because later applications may assign more specific meanings.

An incidence relation records which cells participate in the boundary of another cell. For example, if an oriented edge e begins at vertex $s(e)$ and ends at vertex $t(e)$, the functions s and t identify its source and target vertices. A face f can then possess an oriented boundary built from edges. A schematic boundary expression is stated in Equation 3.

$$\partial f = e_1^{\epsilon_1} e_2^{\epsilon_2} \cdots e_m^{\epsilon_m}, \qquad \epsilon_k \in \{-1, +1\}. \tag{3}$$

Table 1: Relational structures used as mathematical background. The table compares their basic representational roles without treating them as mutually reducible.

Structure	Basic element	Direct representational strength	Limitation relevant here
Graph	Pairwise edge	Pairwise connectivity, paths, weighted or directed relations	Higher-order interaction and extended history often require additional encoding
Hypergraph	Hyperedge over several vertices	Joint participation of several entities in one relation	Does not by itself impose nested boundary structure
Simplicial complex	Simplex with all faces	Nested higher-order relations with systematic face closure	Higher cells are constrained to simplex structure
Cell complex	Cells attached along boundaries	Flexible higher-order boundary and incidence structure	Requires explicit semantics and attachment data
2-complex	Vertices, edges, and faces	History-bearing higher-order structure with explicit face boundaries	Temporal order and substantive labels require additional structure

Equation 3 records the ordered traversal of the face boundary, with ϵ_k indicating whether the traversal follows or opposes the chosen orientation of edge e_k. The notation is combinatorial. It is introduced because orientation can later help represent directional histories, transfers, or process order without requiring every relation to be reduced to a scalar flow.

Remark 3.6 (Cell semantics)**.** Vertices, edges, and faces receive their domain-specific interpretation only after a model specifies what kinds of events and relational processes they represent. The dimensional language describes formal incidence structure and should not be read as an automatic ranking of substantive importance.

Remark 3.6 prevents a common source of confusion. A face is mathematically higher-dimensional than an edge within the complex, yet this does not imply that the represented phenomenon is normatively more important or empirically larger.

3.8 Subcomplexes and Boundaries

A large relational history can contain far more structure than a particular conversion analysis can use directly. The formalism therefore requires a way to select a relationally relevant domain while preserving its internal incidence structure. A *subcomplex* provides this role.

Definition 3.7 (Subcomplex)**.** A *subcomplex* $U \subseteq \mathcal{K}$ is a collection of cells from $\mathcal{K}$ that is closed under the boundary relations required by the chosen complex structure. In practical terms, when a cell is retained in U, the lower-dimensional cells needed to represent its boundary are retained as well.

Definition 3.7 supplies the structural basis for the notation U_C, which later denotes a conversion-relevant subcomplex. The symbol does not imply geographic locality. A conversion performed at one physical location may depend on institutions, monetary infrastructures, ecological conditions, legal relations, or historical processes distributed across many places. The role of U_C is analytical scoping: it identifies the part of the modeled relational history treated as relevant to conversion C.

Boundary language will also become important when the paper distinguishes accumulated history from currently active relational structure. In a history-bearing representation, the boundary of a selected subcomplex can identify where a process connects to structures outside the selected domain or where one relational stage interfaces with another. Later sections refine this idea through relational slices and active boundaries.

3.9 Labeled 2-Complexes

A bare 2-complex records incidence but does not record the substantive attributes needed for value conversion. The formalism therefore allows labels on cells. A label may encode a type, institutional status, value manifestation, relational category, confidence level, resource quantity, or another model-specific attribute.

Let L_V, L_E, and L_F denote label spaces for vertices, edges, and faces. Label maps can then be written as $\lambda_V : V \to L_V$, $\lambda_E : E \to L_E$, and $\lambda_F : F \to L_F$. The labeled complex is summarized in Equation 4.

$$\mathfrak{K} = \big(V, E, F, \lambda_V, \lambda_E, \lambda_F\big). \tag{4}$$

Equation 4 introduces only the minimum structure needed at this stage. Partial ordering, relational slices, normos, observation maps, and generative operators are added in later sections rather than being packed into the definition of a labeled 2-complex.

The next section adds relational temporality. In particular, it introduces a partial order over events, antichains as relational slices, and order ideals as accumulated historical domains. These additions permit the 2-complex to represent an ordered history without assuming that the primitive temporal variable must be an external scalar clock.

4 Mathematical Background II: Partial Order and Relational Time

This section introduces the temporal structure used later to represent conversion histories. Its role is to provide an event-ordering language that can express historical dependence without treating an external scalar clock as a primitive component of the formalism. The discussion proceeds from partial orders to generative precedence, relational slices, accumulated historical domains, optional clock coordinates, and a historical-persistence convention adopted for tractability. The construction is mathematical and representational. It does not assert that physical spacetime, social history, or every empirical conversion process possesses the same underlying temporal ontology.

4.1 Events and Event Structures

The higher-order structures introduced in Section 3 acquire historical content only after some form of ordering is added. The present paper therefore treats selected vertices as *events*: occurrences at which a modeled relational configuration is generated, modified, recognized, terminated, or otherwise made relevant to the subsequent history. An event is a formal element of the representation. Its interpretation may correspond to a transaction, recognition, legal decision, production episode, institutional intervention, material transformation, or another occurrence specified by an application.

Let V denote the event set of a conversion-relevant complex. The paper does not require every vertex to represent an instantaneous physical event. A vertex may summarize an occurrence at the resolution adopted by the model. What matters is that the vertex can participate in an ordering relation expressing historical or generative precedence.

Definition 4.1 (Event structure)**.** An *event structure* in the present formalism is a pair $(V, \preceq)$ consisting of an event set V and a partial order $\preceq$ on V. The induced strict relation $\prec$ is defined by $v_i \prec v_j$ when $v_i \preceq v_j$ and $v_i \neq v_j$.

Definition 4.1 intentionally separates event incidence from event order. The 2-complex introduced in Section 3 records which cells are attached to which other cells. The order relation records which events are treated as lying earlier in the modeled generative history. Incidence and precedence may interact, yet they are not identified automatically.

Figure 2 provides a small illustrative event structure. The diagram should be read as a Hasse-style representation of precedence rather than as a metric spacetime diagram. Vertical placement is used only to aid interpretation.

4.2 Partial Orders

A partial order permits some events to be comparable while leaving others unordered. This feature is useful when a history contains several processes that can develop without requiring one global sequence. A total ordering would force every pair of events into an earlier–later relation even when the model has no basis for doing so.

Definition 4.2 (Partial order)**.** A binary relation $\preceq$ on a set V is a *partial order* when it is reflexive, antisymmetric, and transitive. These conditions require $v \preceq v$ for every $v \in V$; require $v_i = v_j$ whenever both $v_i \preceq v_j$ and $v_j \preceq v_i$ hold; and require $v_i \preceq v_k$ whenever $v_i \preceq v_j$ and $v_j \preceq v_k$ hold.

The strict relation $\prec$ derived from Definition 4.2 is irreflexive and transitive. When $v_i \prec v_j$, the present model treats v_i as preceding v_j in the relevant event history. When neither $v_i \preceq v_j$ nor $v_j \preceq v_i$ holds, the two events are incomparable within the adopted representation.

Incomparability does not necessarily mean physical simultaneity, causal independence, or empirical irrelevance. It means only that the formal model does not impose an order between the two events. This restraint matters when the available evidence identifies several branches of a relational history without determining a unique sequence among them.

Remark 4.3 (Order and information)**.** The relation $\preceq$ records only the precedence commitments represented by the model. Additional empirical or theoretical information can refine the order in a later model version without implying that the earlier representation was a complete description of temporal reality.

Remark 4.3 aligns the partial-order construction with the paper's broader ontological suspension. The formalism requires sufficient order to compose histories and define historically accumulated domains, while leaving stronger claims about physical causality or metaphysical time open.

4.3 Causal and Generative Precedence

The order relation used in this paper is primarily interpreted as *generative precedence*. The term is chosen because conversion histories may contain legal, institutional, symbolic, material, ecological, and epistemic relations whose ordering cannot always be reduced to one physical causal vocabulary.

Definition 4.4 (Generative precedence)**.** For events $v_i, v_j \in V$, the relation $v_i \prec v_j$ expresses *generative precedence* when the formal model treats the realized occurrence or relational consequences of v_i as belonging to the historical domain on which the representation of v_j depends.

Definition 4.4 is intentionally weaker than a universal claim of physical causation. A prior legal registration may be generatively relevant to a later transfer of ownership. A recognition event may be generatively relevant to a later compensation claim. A production event may be generatively relevant to a later monetary conversion. A drought, technological change, or institutional decision can enter the same event history when the application treats it as part of the generative structure of later events.

Generative precedence can therefore combine several modes of dependence within one formal ordering while preserving their substantive labels elsewhere in the complex. The order relation states that a historical dependency is represented; the labels and higher-order cells state what kind of dependency it is.

Caution 4.5 (Causal interpretation)**.** The notation $v_i \prec v_j$ should not be read automatically as a claim that v_i is a sufficient physical cause of v_j. The relation encodes model-relative generative precedence. Stronger causal semantics require additional assumptions supplied by the application.

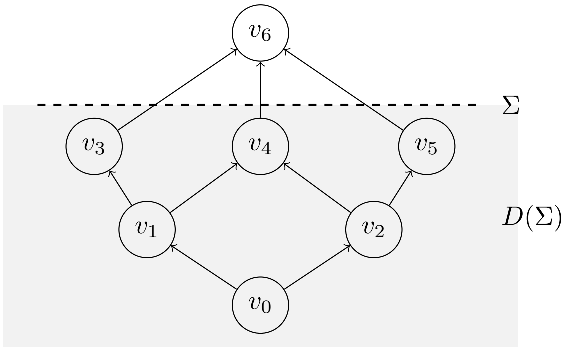


Figure 2: Illustrative partially ordered event structure. The dashed line indicates a relational slice Σ positioned above the antichain $\{v_3, v_4, v_5\}$. The shaded region represents the accumulated order ideal $D(\Sigma)$ associated with that slice. Vertical placement is illustrative and does not define a metric or universal clock.

Caution 4.5 allows the same formal machinery to support later social, economic, legal, ecological, or physical applications without silently identifying their causal vocabularies.

4.4 Antichains and Relational Slices

A conversion analysis often requires a finite representation of a system "at" some stage of its history. Introducing an external scalar time immediately would impose more temporal structure than the present formalism needs. Antichains provide a weaker construction.

Definition 4.6 (Antichain)**.** An *antichain* $\Sigma \subseteq V$ is a set of events such that distinct events in Σ are pairwise incomparable under $\preceq$.

Definition 4.6 supports the use of selected antichains as *relational slices*. A slice collects events that the model treats as mutually unordered while separating an accumulated historical domain from possible later extensions. In Figure 2, the events v_3, v_4, and v_5 form such an antichain.

Definition 4.7 (Relational slice)**.** A *relational slice* is an antichain Σ selected to index a modeled stage of an event history. Its analytical role is to support historical restriction, active-boundary construction, and comparison between successive modeled configurations.

Definition 4.7 does not require every maximal antichain to be used, nor does it identify an antichain with an objective physical simultaneity surface. The slice is a modeling device whose adequacy depends on the resolution and purpose of the conversion analysis.

When two selected slices Σ and Σ' are used to represent successive stages, the notation $\Sigma \prec \Sigma'$ will be used informally to indicate that the relevant events of Σ lie within the realized historical domain preceding Σ'. The exact lifting of the event order to a relation between slices can be specified more strongly when a particular application requires it.

4.5 Order Ideals and Historical Domains

A relational slice by itself does not contain the history that generated it. The formalism therefore associates each slice with an accumulated downward-closed set of preceding events.

Definition 4.8 (Order ideal)**.** Let $(V, \preceq)$ be a partially ordered set. A subset $D \subseteq V$ is an *order ideal* when $v \in D$ and $u \preceq v$ imply $u \in D$.

Definition 4.8 supplies the downward-closure condition used to define the accumulated historical domain of a selected slice Σ in Equation 5.

$$D(\Sigma) = \{v \in V \mid v \preceq s \text{ for at least one } s \in \Sigma\}\,. \tag{5}$$

Equation 5 collects the events represented as belonging to the realized history of the selected slice. The corresponding restriction of the wider relational structure will later be written as $\mathfrak{G}|_{D(\Sigma)}$.

The order ideal is particularly important for relational entities. A person, institution, currency, contract, or value-bearing structure observed near a slice may depend on events distributed throughout $D(\Sigma)$. Representing such an entity only by cells intersecting the current slice would suppress the history through which its present relational position emerged.

Remark 4.9 (Finite epistemic access)**.** Equation 5 defines a modeled historical domain and does not imply that an observer can recover every event it contains. Later entity-identification operators act on finite and potentially biased epistemic access to this history.

Remark 4.9 separates ontological representation from practical observability. The distinction becomes central when the paper introduces epistemic truncation and theory-relative evaluation.

4.6 Order-Compatible Time Coordinates

Empirical economics and institutional analysis frequently require dates, durations, lags, rates, and time series. The absence of primitive scalar time in the foundational representation therefore does not imply that clock coordinates are excluded. The paper treats them as additional coordinatizations compatible with the event order when such a representation is useful.

Definition 4.10 (Order-compatible time coordinate)**.** An *order-compatible time coordinate* on an event domain is a map $\tau : V \to \mathbb{R}$ satisfying the monotonicity condition stated in Equation 6 for all ordered event pairs in its domain.

Definition 4.10 is completed by the compatibility condition stated in Equation 6.

$$v_i \prec v_j \quad \Longrightarrow \quad \tau(v_i) < \tau(v_j). \tag{6}$$

Equation 6 ensures that the coordinate respects modeled precedence. It does not require the converse: two events can receive different clock coordinates even when the partial order leaves them incomparable. A specific empirical application may also employ several coordinates, calendars, institutional periods, or coarse-grained time scales.

This distinction allows the formalism to retain ordinary economic time when needed. Exchange rates, inflation, contract duration, production cycles, and policy lags can all be indexed by clock time after a suitable coordinate is chosen. The partial order remains the more primitive representational structure within the present model because it records precedence without forcing every event into a single total sequence.

Remark 4.11 (Time-scale extensions)**.** Fast–slow dynamics and other multi-timescale models can be introduced after an order-compatible coordinate or family of coordinates has been specified. Their later use does not require the partial-order representation to be replaced by scalar time at the foundational level.

Remark 4.11 prepares the connection with later dynamical applications while keeping the present section limited to temporal representation.

4.7 Historical Persistence as a Modeling Convention

The formalism needs a tractable rule for relating an accumulated history at one slice to the accumulated history at a later slice. The present paper adopts a persistence convention for this purpose. The convention supports provenance, responsibility, and procedural comparison by treating realized historical structure as retained when a model is extended.

Convention 4.12 (Historical Persistence Convention)**.** For the class of conversion histories modeled in this paper, a later realized history is represented as an extension of the earlier realized history. If Σ precedes Σ', the model uses an order- and incidence-preserving embedding of the earlier historical substructure into the later one. The convention is adopted for analytical traceability and does not assert an ultimate physical or metaphysical law of spacetime.

A schematic extension under Convention 4.12 is stated in Equation 7.

$$\mathfrak{G}_{\leq\Sigma'} = \mathfrak{G}_{\leq\Sigma} \cup \Delta\mathfrak{G}_{\Sigma\rightsquigarrow\Sigma'}. \tag{7}$$

Equation 7 represents later history as the retained earlier structure together with newly generated structure $\Delta\mathfrak{G}_{\Sigma\rightsquigarrow\Sigma'}$. The union symbol is schematic: a complete implementation must preserve the required incidence, labels, and order relations among retained and newly attached cells.

The convention permits later events to create new relations to earlier events. A retrospective recognition, for example, can be represented by a new event connected to earlier production or contribution events. The model does not need to rewrite those earlier events in order to represent a changed present interpretation of them. Historical occurrence and current relational significance therefore remain analytically distinguishable.

Caution 4.13 (Physical scope)**.** Convention 4.12 is a convenience of the present conversion formalism. A future GR model concerned with fundamental physical spacetime could adopt a more general history-transformation rule if the relevant physical theory required it.

Caution 4.13 prevents a practical bookkeeping assumption from being elevated into a foundational claim. The present manuscript requires only a stable representation of realized histories sufficient to compare conversion procedures and their later consequences.

The structures introduced in this section provide the temporal scaffolding for the operator language developed next. A conversion can now be represented over a history-bearing complex whose events are partially ordered, whose selected slices index modeled stages, and whose order ideals preserve accumulated relational history. The following section introduces maps and operators acting on such configurations and distinguishes observation, transformation, composition, partial composability, and noncommutative ordering.

5 Mathematical Background III: Operators and Operator Algebra

This section introduces the transformation language used later to represent heterogeneous conversion as a history of composable operations. Its role is preparatory: it distinguishes ordinary maps, partial maps, endomorphisms, operators, composition, state-dependent operator families, and operator algebras before those notions are specialized to relational conversion. The discussion assumes only elementary set notation and basic familiarity with functions. Particular care is given to type consistency because the later formalism distinguishes epistemic readout from ontological transformation and allows some transformations to be defined only on restricted relational configurations.

5.1 Functions, Partial Maps, and Transformations

A function assigns to each element of a domain exactly one element of a codomain. If X and Y are sets, a map $f : X \to Y$ therefore associates every $x \in X$ with a value $f(x) \in Y$. The domain and codomain are part of the mathematical type of the map. This typing becomes important when several maps are composed, because the output of one map must be admissible as the input of the next.

Many institutional and relational operations are not available for every possible configuration. A transfer may require a recognized title; a legal remedy may require standing; a conversion may require an accepted currency or an existing contractual relation. A partial map provides a simple representation of this restricted availability.

Definition 5.1 (Partial map)**.** Let X and Y be sets. A *partial map* from X to Y is a function $f : D_f \to Y$ defined on a subset $D_f \subseteq X$. The notation $f : X \rightharpoonup Y$ records that the map may be undefined for some elements of X.

Definition 5.1 allows admissibility conditions to remain inside the formal structure. The statement that $f(x)$ is undefined is then different from assigning an exceptional output to x. Later sections use this distinction when one procedural ordering enables an operation that another ordering leaves unavailable.

A transformation is used here as a general term for a map whose application changes the represented configuration. The word does not, by itself, imply linearity, invertibility, continuity, or preservation of any particular structure. These stronger properties require separate assumptions.

Table 2 anticipates a distinction used throughout the later formalism. Observation maps and state transformations can interact in one empirical process, yet their mathematical types remain different unless an explicit combined construction is introduced.

5.2 State Spaces and Endomorphisms

A state space collects the configurations that a model treats as admissible. The term *state* is used broadly here. It need not denote a physical microstate or a vector in a Hilbert space. In a relational application, one state may encode an accumulated event history, a currently active relational boundary, institutional structures, and an operative nomos.

Let Ω denote an abstract state space. A transformation that maps admissible states back into Ω is an endomorphism.

Definition 5.2 (Endomorphism)**.** An *endomorphism* of a set Ω is a map $\Phi : \Omega \to \Omega$. A *partial endomorphism* is a partial map $\Phi : \Omega \rightharpoonup \Omega$.

Definition 5.2 provides the minimal type later used for generative transformations through partial endomorphisms. This choice provides type consistency: a transformation begins from a relational configuration and yields another relational configuration. Additional outputs, such as measured values or institutional records, are represented separately rather than being inserted into the codomain of the ontological transformation itself.

The identity transformation on Ω is denoted by id_Ω. It satisfies Equation 8 whenever the relevant compositions are defined.

$$\mathrm{id}_\Omega(\omega) = \omega. \tag{8}$$

Equation 8 provides a useful reference case. A purely passive readout can be paired with the identity state transformation, whereas a performative observation can be paired with a transformation that changes the subsequent relational configuration.

5.3 Operators

The word *operator* has several established meanings across mathematics and physics. In linear algebra and functional analysis, an operator commonly denotes a map acting on a vector space or function space, often with linearity assumed. In quantum theory, observables and dynamical transformations are represented through operators on suitable state spaces. In broader applied mathematics, the term is also used for transformations acting on structured objects.

The present paper adopts the broader transformation sense at the preliminary level. A symbol such as Φ denotes an operator-like transformation only after its domain, codomain, and substantive role have been specified. The core conversion formalism therefore does not infer linearity from operator notation.

Caution 5.3 (Operator terminology)**.** The notation $\Phi : \Omega \rightharpoonup \Omega$ describes a partial transformation of relational configurations. Calling Φ an operator does not establish that Ω is a vector space, that Φ is linear, or that the collection of such transformations already forms an operator algebra in the functional-analytic sense.

Table 2: Basic map types used in the manuscript. The table separates the mathematical type of a map from the substantive interpretation assigned later.

Object	**Type**	**Role in the present formal background**
Map	$f : X \to Y$	Produces one output in Y for every input in X.
Partial map	$f : X \rightharpoonup Y$	Represents an operation available only on an admissible subset of X.
Endomorphism	$f : X \to X$	Transforms a configuration while remaining within the same state space.
Partial endomorphism	$f : X \rightharpoonup X$	Represents a state transformation whose admissibility depends on the current state.
Observation map	$\pi : X \to Z$	Produces a readout or representation in another space without, by definition, specifying a state transformation.

Caution 5.3 is important for the manuscript's later use of algebra-inspired language. Composition of transformations is already sufficient to study order sensitivity. Stronger algebraic structure is introduced only when the relevant closure and linearity properties are explicitly supplied.

An observation map has a different type. If X_i is an observation space, an epistemic readout can be written as $\pi_i : \Omega \to X_i$. A process that both produces a readout and changes the state can be represented by two associated maps rather than by an ambiguously typed single operator. A later section develops this distinction in the conversion setting.

5.4 Composition of Operators

Composition represents sequential application. Let $f : X \to Y$ and $g : Y \to Z$. The composite map $g \circ f : X \to Z$ applies f first and g second. Equation 9 defines the composite.

$$(g \circ f)(x) = g\big(f(x)\big). \tag{9}$$

The order shown in Equation 9 is conventional: the rightmost map acts first. This convention will be retained when conversion histories are written as compositions of relational transformations.

Composition is associative whenever the maps are composable. For maps of compatible type, Equation 10 holds.

$$(h \circ g) \circ f = h \circ (g \circ f). \tag{10}$$

Equation 10 allows a long history to be grouped in different ways without changing the resulting composite map. Associativity concerns grouping; it does not imply that the order of different operators can be exchanged.

Figure 3 illustrates ordinary composition and a partially composable alternative. The lower path emphasizes that a second operation may become available only after the first operation has moved the state into its domain.

5.5 Partial Composability

For partial maps, composition requires more than matching codomains. Let $\Phi_1 : \Omega \rightharpoonup \Omega$ and $\Phi_2 : \Omega \rightharpoonup \Omega$. The composite $\Phi_2 \circ \Phi_1$ is defined at a state ω only when ω belongs to the domain of Φ_1 and the resulting state $\Phi_1(\omega)$ belongs to the domain of Φ_2. Equation 11 states this condition.

$$\omega \in \mathrm{Dom}(\Phi_2 \circ \Phi_1) \quad \Longleftrightarrow \quad \omega \in \mathrm{Dom}(\Phi_1) \text{ and } \Phi_1(\omega) \in \mathrm{Dom}(\Phi_2). \tag{11}$$

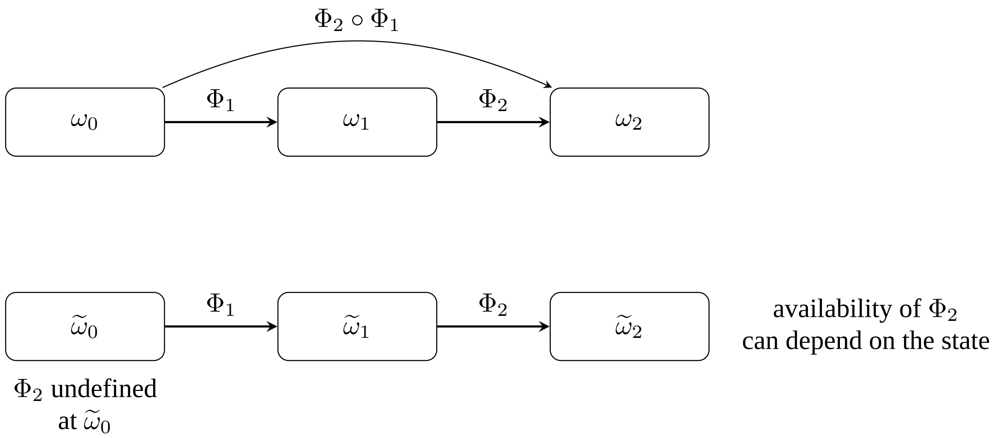


Figure 3: Composition and partial composability. The upper path shows a conventional composition $\Phi_2 \circ \Phi_1$. The lower path illustrates a situation in which Φ_2 is unavailable at the initial state but becomes admissible after Φ_1. The diagram is schematic and does not yet assign conversion-specific semantics to the operators.

Equation 11 gives procedural accessibility a formal location. An earlier operation can create the conditions under which a later operation becomes available. The reverse ordering may remain undefined. This situation is stronger than ordinary noncommutativity because there may be no pair of competing outputs to compare.

Example 5.4 (Procedural accessibility)**.** Suppose an operation Φ_R establishes a recognized status and a later operation Φ_C requires that status. A state ω may satisfy $\omega \notin \mathrm{Dom}(\Phi_C)$ while $\Phi_R(\omega) \in \mathrm{Dom}(\Phi_C)$. The composition $\Phi_C \circ \Phi_R$ is then defined at ω, whereas the reversed sequence need not be defined there.

Example 5.4 remains deliberately abstract. Later sections interpret analogous structures through recognition, valuation, conversion, settlement, and revision processes.

5.6 Commutative and Noncommutative Composition

Two operators commute when exchanging their order leaves the composite transformation unchanged wherever the comparison is well typed and both composites are defined. For endomorphisms $\Phi_1, \Phi_2 : \Omega \to \Omega$, commutativity is stated in Equation 12.

$$\Phi_2 \circ \Phi_1 = \Phi_1 \circ \Phi_2. \tag{12}$$

When Equation 12 fails, the pair is noncommutative under composition. The present formalism primarily uses this compositional notion. It does not initially define a commutator by subtraction because the state space Ω has not been assumed to possess vector-space structure and the transformations have not been assumed to admit addition or subtraction.

Definition 5.5 (Compositional noncommutativity)**.** Two transformations Φ_1 and Φ_2 are *compositionally noncommutative* on a domain D when both $\Phi_2 \circ \Phi_1$ and $\Phi_1 \circ \Phi_2$ are defined on D and there exists at least one $\omega \in D$ for which their outputs differ.

Definition 5.5 separates order sensitivity from the stronger algebraic structure associated with a commutator $[A, B] = AB - BA$. If a later representation embeds the relevant transformations into a linear operator algebra, the conventional commutator may become available. The preliminary conversion model does not require that additional step.

Order sensitivity is substantive when an earlier operation changes the state on which a later operation acts. If $\Phi_1(\omega) = \omega_1$ and $\Phi_2(\omega) = \widetilde{\omega}_1$, the two second-stage transformations generally act on different intermediate configurations. A later section develops the conversion consequences of this observation.

5.7 Operator Families

A single operator rarely represents an entire conversion process. It is more useful to consider a family of available transformations. Let $\mathfrak{O}$ denote a collection of partial endomorphisms on Ω. Different elements may represent recognition, classification, valuation, contractual modification, settlement, revision, or other operations once substantive meanings are introduced.

The family need not be closed under every algebraic operation. At the present stage, the minimal requirement is that composable members can be sequentially applied and that their domains are explicit. This permits conversion histories to be represented as paths through the space of admissible transformations.

Definition 5.6 (Admissible operator family)**.** An *admissible operator family* on Ω is a specified collection $\mathfrak{O}$ of partial endomorphisms $\Phi : \Omega \rightharpoonup \Omega$ together with the domain information required to determine when finite compositions are defined.

Definition 5.6 is intentionally weaker than the definition of an algebra. It captures the process structure needed later without introducing addition, scalar multiplication, or topological closure before those operations have an analytical role.

5.8 State-Dependent Operators

The set of available transformations can depend on the current relational configuration. A new legal status may create new claims; a new currency may enable new settlement operations; a contract may create enforcement possibilities; a terminated relation may remove previously available actions. The operator family is therefore allowed to vary with the state.

Let $\mathfrak{O}(\omega)$ denote the admissible operator family at $\omega \in \Omega$. Equation 13 records this assignment.

$$\mathfrak{O} : \Omega \longrightarrow \mathcal{P}(\mathrm{PEnd}(\Omega)), \qquad \omega \longmapsto \mathfrak{O}(\omega), \tag{13}$$

where $\mathrm{PEnd}(\Omega)$ denotes the class of partial endomorphisms on Ω and $\mathcal{P}$ denotes the power-set construction. Equation 13 is a bookkeeping representation rather than a commitment to a particular algebraic category.

State dependence adds a second form of history sensitivity. The first operation changes the state; that changed state can alter which second operation exists or how it is parameterized. A sequence should therefore be read as a history of transformations selected relative to successive configurations rather than as repeated action by a globally fixed operator set.

Remark 5.7 (Contextual operator selection)**.** The notation $\Phi^{[\omega]}$ can be used when the operative form of a transformation depends materially on the state ω. This notation emphasizes context dependence without asserting that every operator must vary with every state.

Remark 5.7 prepares the later notation for ontology-sensitive conversion histories. It also allows a distinction between a general operation type, such as recognition, and the concrete state-dependent transformation realized in a particular history.

5.9 Operator Algebra as a Representational Language

An algebra is, in one standard form, a vector space equipped with a bilinear multiplication. An operator algebra is an algebra whose elements are operators and whose multiplication is commonly given by operator composition. When the underlying space is linear, addition and scalar multiplication of operators can also be defined pointwise. The resulting structure supports conventional constructions such as commutators, spectra, norms, and closure properties when the required assumptions are available.

For a vector space V, the set of linear endomorphisms $\mathrm{End}(V)$ provides a familiar example. If $A, B \in \mathrm{End}(V)$ and λ is a scalar, then $A + B$, λA, and $A \circ B$ are again linear endomorphisms. This closure gives $\mathrm{End}(V)$ an algebraic structure under the usual operations.

The heterogeneous-conversion formalism developed later begins at a weaker level. Its configuration space need not be linear, and its generative transformations may be partial and state-dependent. Operator-algebraic language nevertheless remains useful in two ways. First, composition supplies a disciplined notation for procedural order. Second, a later representation may act on a linear space of functions, weights, or observables associated with relational histories, thereby lifting nonlinear state transformations into genuine linear operators on that auxiliary space.

Remark 5.8 (Prospective operator representation)**.** A later formal extension may represent a relational transformation Φ through an operator $\rho(\Phi)$ acting on a suitable function or history space. Conventional operator-algebraic constructions should then be applied to $\rho(\Phi)$ only after the representation space and algebraic assumptions have been specified.

Remark 5.8 keeps the algebraic aspiration separate from the minimal conversion formalism. The paper can therefore use operator composition without implying that every relational transformation already belongs to a C^*-algebra, a von Neumann algebra, or another specialized operator-algebraic structure.

5.10 Algebraic Scope and Mathematical Restraint

The operator language introduced in this section is intended to make conversion histories type-consistent, composable, and sensitive to procedural order. Three levels should remain distinct throughout the manuscript. At the first level, partial maps and compositions provide the minimal process language. At the second level, operator-inspired notation organizes families of state transformations and observations. At the third level, a genuine operator algebra may be introduced if a later representation supplies linear spaces, closure properties, and the additional structure required by the intended analysis.

Caution 5.9 (Algebraic scope)**.** The present use of operators and noncommutative composition is a formal representation of relational processes. It does not establish that heterogeneous value conversion is fundamentally governed by the same algebraic structures used in quantum mechanics, quantum field theory, or operator-algebraic physics.

Caution 5.9 preserves the manuscript's ontological suspension while retaining the mathematical benefits of compositional reasoning. The next section introduces field-theoretic language and clarifies the relation between fields, operators, and operator-valued structures. This background will later support a careful distinction between distributed relational structure and transformations acting on that structure.

6 Mathematical Background IV: Fields and Operator-Valued Structures

This section introduces field-theoretic language needed later to distinguish distributed relational structure from transformations acting on relational configurations. Its role is preparatory. The discussion begins with ordinary scalar and vector fields, develops the distinction between local field values and global field configurations, and then explains how operators act on spaces of fields or become indexed by spacetime locations. The final subsections clarify the limited sense in which field-theoretic language is used in the present manuscript. No knowledge of quantum field theory is assumed, and the section does not identify heterogeneous value conversion with a physical field theory.

6.1 Scalar and Vector Fields

A field assigns a mathematical value to each point of a specified domain. In the simplest case, let M be a domain whose elements are interpreted as locations, events, or positions in some background space. A scalar field assigns one scalar to each point. Equation 14 states the corresponding map.

$$\phi : M \longrightarrow \mathbb{R}. \tag{14}$$

Equation 14 can represent temperature over a region, density over a material body, or another quantity described by one real number at each location. The symbol $\phi(x)$ denotes the field value at $x \in M$. The field itself is the entire assignment ϕ, rather than any one local value.

A vector field assigns a vector to each point. If the vectors belong to $\mathbb{R}^n$ in a chosen coordinate representation, the simplest form is stated in Equation 15.

$$\mathbf{v} : M \longrightarrow \mathbb{R}^n. \tag{15}$$

Equation 15 can describe, for example, a velocity field in continuum mechanics. At each point x, the value $\mathbf{v}(x)$ specifies a local direction and magnitude. More generally, the value assigned at x may belong to a vector space or another mathematical structure associated with that point.

Definition 6.1 (Field)**.** Within this manuscript, a *field* is a structured assignment of values to locations or elements of a base domain. The codomain and compatibility conditions are specified by the application under consideration.

Definition 6.1 is intentionally broad. Differential geometry refines this idea through sections of bundles, while physical field theories impose additional smoothness, symmetry, dynamical, and variational structures. Those refinements are introduced only when they have a direct analytical role.

6.2 Fields over Spacetime

Physical field theories commonly define fields over spacetime. If M denotes a spacetime manifold with local coordinates x^μ, a scalar field is written as $\phi(x)$ and a vector field as $V^\mu(x)$. The coordinates provide a representation of where field values are evaluated; they do not by themselves determine the field dynamics.

A familiar schematic distinction separates the domain, the local value, and the complete field configuration. Equation 16 represents one field configuration as the collection of its local values.

$$\Phi = \{\phi(x) \mid x \in M\}. \tag{16}$$

Equation 16 is set-like notation for intuition rather than a definition of a function space. In a more developed theory, admissible configurations form a space $\mathcal{F}(M)$ whose elements satisfy the regularity and boundary conditions required by the model.

The distinction between a spacetime field and the partial-order event structures introduced in Section 4 should remain explicit. The present paper does not require the base domain M to be a fixed continuum spacetime. Later GR-inspired constructions can place field-like labels on vertices, edges, faces, or other cells of a history-bearing complex. Field language is therefore used at a level compatible with both continuum and discrete representations.

6.3 Local and Global Field Structure

Fields provide a useful language for separating local information from global configuration. A local value $\phi(x)$ describes one location, while the complete field ϕ describes the coordinated assignment across the domain. Local modifications can therefore change global structure without requiring every point to change directly.

For a subdomain $U \subseteq M$, the restriction of a field to U is denoted $\phi|_U$. Equation 17 states the restriction map at the level of configurations.

$$\mathrm{res}_U : \mathcal{F}(M) \longrightarrow \mathcal{F}(U), \qquad \phi \longmapsto \phi|_U. \tag{17}$$

Equation 17 illustrates a general idea that later becomes important for relational conversion: practical analysis usually accesses only a scoped part of a much larger structure. The restriction operation itself is mathematically simple; the difficult question in a social or institutional setting concerns how the relevant scope U is identified and which relations become invisible outside that scope.

Field configurations also permit compatibility questions across overlapping regions. If U and V overlap, locally specified fields can represent one global field only when their assignments agree on the overlap under the relevant compatibility rule. This local-to-global perspective later motivates caution when multiple institutions identify the same relational entity through different observational cuts.

Remark 6.2 (Discrete relational fields)**.** For a labeled 2-complex $\mathcal{K} = (V, E, F)$, a field-like assignment can be defined directly on cells, for example $\lambda_V : V \to S_V$, $\lambda_E : E \to S_E$, and $\lambda_F : F \to S_F$. The terminology "field" then refers to a distributed assignment over the complex rather than to a continuum field on a manifold.

Remark 6.2 provides a bridge to the spin-network and spin-foam background developed in the following part of the manuscript. It also clarifies why the present manuscript can use field language without assuming a continuum model of social or economic spacetime.

6.4 Fields and Dynamical Evolution

A field theory becomes dynamical when a rule determines how admissible field configurations vary across the relevant evolution parameter or event structure. In elementary continuum notation, a field equation may take the schematic form stated in Equation 18.

$$\mathcal{D}[\phi] = J, \tag{18}$$

where $\mathcal{D}$ is a differential or other dynamical operator and J represents a source term. Equation 18 is intentionally generic. Different physical theories specify different operators, source structures, boundary conditions, and symmetries.

An alternative formulation treats evolution as a map on a configuration space. If $\mathcal{F}$ denotes the space of admissible field configurations, an evolution transformation may be represented by Equation 19.

$$\mathcal{U} : \mathcal{F} \rightharpoonup \mathcal{F}. \tag{19}$$

Equation 19 connects directly with the partial-endomorphism language of Section 5. The distinction between a field configuration and the operator that transforms it is central: the field supplies distributed state information, while the operator supplies an evolution or intervention rule.

6.5 Classical Fields and Operators

A classical field and an operator occupy different mathematical roles. A field such as ϕ is an element of a configuration space, while an operator such as $\mathcal{U}$ acts on one or more configurations. Equation 20 states this distinction in the simplest typed form.

$$\phi \in \mathcal{F}, \qquad \mathcal{U} : \mathcal{F} \rightharpoonup \mathcal{F}, \qquad \phi' = \mathcal{U}(\phi). \tag{20}$$

Equation 20 is the main conceptual distinction required later. A distributed normative, institutional, or relational pattern may be represented in field-like language, while a judgment, conversion, recognition, or intervention can be represented through an operator when its analytical role is to transform a configuration or alter the space of subsequent operations.

Table 3 summarizes the distinction without imposing one mathematical representation on every later concept. The manuscript will generally prefer operator language when the analytical object is a transformation and field language when the analytical object is a distributed structure.

Table 3: Field and operator roles in the present mathematical background.

Object	Typical mathematical role	Later relevance to conversion formalism
Field ϕ	Distributed assignment over a base domain or complex	Represents spatially or relationally distributed structure when such a representation is useful
Field configuration Φ	One complete admissible assignment	Provides a state-like object on which transformations may act
Operator $\mathcal{U}$	Map or partial map between configurations	Represents evolution, intervention, recognition effects, or conversion-related transformations
Operator family	Collection of admissible transformations	Represents the transformations available under a relational configuration or nomos
Operator-valued field	Location-indexed family of operators	Provides a possible representation when local operations depend on position or event

6.6 Operator-Valued Fields

The distinction between fields and operators does not prevent them from being combined. An operator-valued field associates an operator with each point or location. In schematic notation, Equation 21 represents such an assignment.

$$\widehat{\phi} : M \longrightarrow \mathrm{Op}(\mathcal{H}), \qquad x \longmapsto \widehat{\phi}(x), \tag{21}$$

where $\mathrm{Op}(\mathcal{H})$ denotes a specified class of operators on a state space $\mathcal{H}$. Equation 21 is sufficient for the conceptual distinction needed here. In quantum field theory, additional mathematical care is usually required, and quantum fields are commonly treated as operator-valued distributions rather than ordinary pointwise functions.

The phrase "promoting a field to an operator" often appears informally in discussions of quantization. The underlying mathematical construction is more demanding than a direct change of notation. Quantization requires a choice of state space, observables, commutation structures, and other theory-specific ingredients. The present manuscript therefore does not assume that every classical field possesses a unique or automatic operator-valued counterpart.

Caution 6.3 (Quantization analogy)**.** Field-to-operator language in this paper is representational. The later use of operator-like normative or relational transformations does not assert that social or economic fields have been physically quantized.

Caution 6.3 preserves a useful distinction for later sections. A field-like description can be introduced when relational influence is distributed across a complex, while an operator representation can be adopted directly when the object of interest is how that structure modifies subsequent evolution.

6.7 Field-Theoretic Language in Relational Modeling

Field language can be useful for relational modeling when a quantity or influence is distributed across many parts of a system. Examples include spatially varying resource access, institutionally varying conversion conditions, or labels assigned across the cells of a history-bearing complex. A relational field may therefore be written abstractly as Equation 22.

$$\eta : \mathcal{C}(\mathcal{K}) \longrightarrow S, \tag{22}$$

where $\mathcal{C}(\mathcal{K})$ denotes a selected collection of cells of the complex $\mathcal{K}$ and S is an application-specific value space. Equation 22 can represent local labels or distributed influences without committing the paper to a continuum geometry.

For some applications, the distributed field may parameterize a transformation. If η is a field-like assignment and $\Phi[\eta]$ is an operator constructed from that assignment, the relation can be represented by Equation 23.

$$\eta \longmapsto \Phi[\eta]. \tag{23}$$

Equation 23 does not require a physical quantization procedure. It simply states that a distributed structure can determine or parameterize a transformation. Later normative sections may use this distinction when a judgment is represented directly by an operator, while leaving open the possibility that a more detailed model first represents a distributed evaluative structure and then constructs an operator from it.

Figure 4 summarizes the layered representation. The intermediate field can be omitted when the analysis has no need to resolve distributed local influence. In that case a transformation can be represented directly by an operator on the relevant configuration space.

6.8 Field Language and Ontological Restraint

The field concepts introduced in this section are analytical resources. Their usefulness lies in distinguishing distributed assignments, local-to-global structure, configuration spaces, and transformations. These distinctions can later clarify whether a proposed GR object is best treated as a spatially distributed structure, a history-bearing cell assignment, an operator on relational configurations, or an operator-valued family indexed by events or cells.

Several levels of commitment should remain separate. A field can be used as a descriptive representation without asserting a fundamental field ontology. A discrete cell assignment can be called field-like without presupposing a continuum spacetime. An operator-valued field can provide a mathematical analogy without implying physical quantization. A field theory in the stronger physical sense requires explicit dynamical equations, state spaces, symmetries, observables, and empirical interpretation.

Remark 6.4 (Representational hierarchy)**.** The later conversion formalism uses the weakest mathematical structure adequate to the local problem. Field language is introduced when distributed structure matters; operator language is introduced when transformation matters; operator-valued or field-theoretic extensions remain available when a later model requires both simultaneously.

Remark 6.4 establishes the scope used in the remainder of the manuscript. The following spin-network and spin-foam background develops a more specific physical source of inspiration for history-bearing 2-complexes, boundary states, and locally weighted histories.

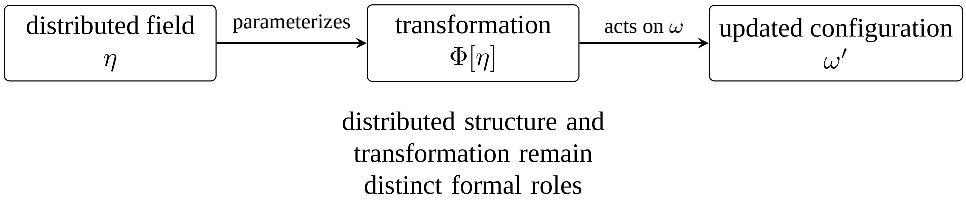


Figure 4: A field-like assignment can parameterize an operator without being identified with the operator itself. The distinction permits distributed relational structure and transformational action to be modeled separately.

7 Physical Background: Spin Networks and Spin Foams

This section introduces the physical source of inspiration for the history-bearing 2-complex representation used later in the manuscript. Its role is explanatory rather than foundational. The discussion begins with event-based and relational descriptions of spacetime, introduces spin networks as labeled graphs, develops spin foams as labeled 2-complex histories between boundary states, and then explains the local-weight structure used in spin-foam models. The final subsections isolate the structural features borrowed by the present conversion formalism and state the limits of the analogy. Only elementary knowledge of graphs, vector spaces, and operators from Sections 3–6 is assumed.

7.1 Event-Based and Relational Spacetime Representations

General relativity already weakens the role of a fixed external stage by treating spacetime geometry as dynamical. Approaches to quantum gravity motivated by general covariance go further by seeking descriptions that do not presuppose a preferred external time or fixed background geometry. Covariant loop quantum gravity and spin-foam models belong to this family of approaches and formulate transition processes through relationally specified boundary data and intervening histories [3, 4].

The present manuscript borrows only a structural lesson from this setting. A history can be represented through relations among events and higher-order cells without taking a single external clock as the primitive organizing object. The partial-order structure introduced in Section 4 is therefore compatible with an event-oriented representation, although it is an additional modeling choice of the present framework. Standard spin-foam constructions do not, by themselves, provide the specific generative partial order adopted here.

Remark 7.1 (Physical inspiration and model-relative time)**.** The physical literature motivates a history-based representation, while the relation $\prec$ used in this manuscript remains a model-relative order of generative precedence. The manuscript does not identify this order with a fundamental causal order of quantum spacetime.

Remark 7.1 preserves the separation between physical inspiration and the conversion model's analytical commitments.

7.2 Spin Networks

A spin network is, at the structural level required here, a graph enriched by representation-theoretic labels. In the standard gauge-theoretic construction, graph edges carry irreducible representations of a group and vertices carry intertwining operators that combine the representation spaces associated with incident edges [1]. A compact schematic description is stated in Equation 24.

$$\mathcal{S} = \left(\Gamma, \{\rho_e\}_{e\in E(\Gamma)}, \{\iota_v\}_{v\in V(\Gamma)}\right). \tag{24}$$

In Equation 24, Γ is the underlying graph, ρ_e denotes the representation assigned to edge e, and ι_v denotes the intertwiner assigned to vertex v. An intertwiner can be understood, at an introductory level, as a structure-preserving multilinear map that combines incoming and outgoing representation data consistently at a vertex.

For the present manuscript, the important point is methodological. A graph can carry more information than adjacency alone when its cells possess labels with defined transformation and compatibility rules. The later conversion formalism uses this principle without importing the physical interpretation of spin labels, angular momentum, or quantum geometry.

Figure 5 provides a schematic transition from graph-like boundary descriptions to an intervening higher-dimensional combinatorial history. The physical spin-foam interpretation is introduced in the following subsections; the later conversion formalism borrows only the representational architecture.

7.3 Spin-Foam Histories

A spin foam extends the graph-based idea from a state-like boundary object to a history represented by a 2-complex. Baez introduced spin foams as 2-dimensional complexes interpolating between spin networks, with faces labeled by group representations and edges labeled by intertwiners [2]. The subsequent spin-foam literature develops this construction as a covariant approach to quantum-gravitational transition amplitudes [3].

A schematic labeled spin foam can be written as Equation 25.

$$\mathcal{F} = \left(\mathcal{K}, \{\rho_f\}_{f\in F(\mathcal{K})}, \{\iota_e\}_{e\in E(\mathcal{K})}\right), \qquad \mathcal{K} = (V, E, F). \tag{25}$$

Equation 25 separates the combinatorial carrier $\mathcal{K}$ from the labels attached to its cells. The faces provide a genuinely higher-order structure absent from an ordinary graph. A history therefore contains information about how graph-like boundary structures are related through intervening two-dimensional cells.

This feature motivates the present use of 2-complexes for heterogeneous value conversion. A conversion history may contain branching, merging, institutional mediation, co-generation, and extended relations that are difficult to represent as a single pairwise edge between A and B.

7.4 Boundary States and Interpolating Histories

Spin-foam models distinguish boundary states from the history connecting them. A spin foam can have an initial boundary spin network and a final boundary spin network; the intervening 2-complex supplies a candidate history relating the two [2, 4]. In schematic notation, Equation 26 records this structure.

$$\partial\mathcal{F} = \mathcal{S}_{\text{in}} \sqcup \mathcal{S}_{\text{out}}. \tag{26}$$

Equation 26 suppresses orientation conventions and model-specific details in order to emphasize the boundary-history distinction. A state description records relational data on a boundary; a history description records how one boundary configuration can be related to another through an intervening complex.

The later conversion formalism adopts the same structural distinction in a nonphysical domain. A conversion is represented as a history between relational configurations rather than as a bare numerical equality between two endpoints. The analogy concerns representational architecture, not physical equivalence.

7.5 Vertices, Edges, and Faces in Spin-Foam Models

The interpretation of cells in a spin foam differs from the interpretation adopted later for conversion histories, but the cell hierarchy is instructive. In common spin-foam formulations, faces carry representation labels, edges carry intertwiner data, and vertices are associated with local interaction amplitudes or elementary transition events [2, 3]. The incidence relations among these cells determine how local data are composed into a complete history.

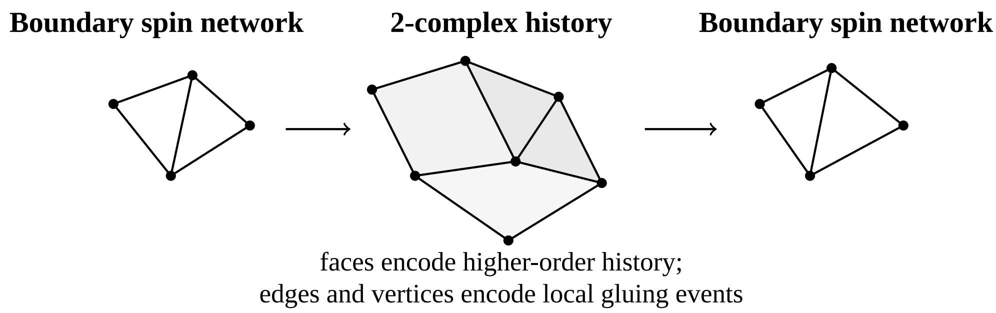


Figure 5: Schematic relation between boundary spin networks and an intervening 2-complex history. The drawing is pedagogical and does not represent a specific spin-foam model or physical triangulation.

Table 4: Physical spin-foam structures and their limited representational use in the present manuscript.

Structure	**Spin-foam role**	**Representational lesson used later**
Boundary graph	Spin-network state with representation and intertwiner labels	A relational configuration can be represented on a lower-dimensional boundary
Face	Representation-labeled 2-cell	Higher-order historical structure can extend beyond pairwise adjacency
Edge	Gluing structure carrying intertwiner data	Intermediate relations can mediate compatibility among neighboring higher-order cells
Vertex	Local interaction or transition contribution	Local events can alter how several relational processes meet
Whole foam	Candidate interpolating history	Conversion can be represented as a history between configurations rather than as one edge $A \to B$

Table 4 deliberately avoids assigning social meanings directly to physical spin-foam labels. The later formalism defines its own semantics for events, relations, higher-order processes, values, subjects, institutions, and nomos.

7.6 Local Amplitudes and History Amplitudes

A further useful feature of spin-foam models is the construction of a history amplitude from contributions associated with local cells. In a common schematic form, a fixed foam $\mathcal{F}$ receives face, edge, and vertex amplitudes whose product contributes to the weight of that history [2, 3]. Equation 27 records this structure without committing to a particular physical model.

$$\mathcal{A}(\mathcal{F}) = \prod_{f \in F(\mathcal{F})} A_f \prod_{e \in E(\mathcal{F})} A_e \prod_{v \in V(\mathcal{F})} A_v. \tag{27}$$

Equation 27 is schematic because actual spin-foam models specify model-dependent amplitudes, measures, constraints, and convergence conditions. Transition amplitudes can then involve sums over compatible foams. Equation 28 states the corresponding structural idea.

$$\mathcal{A}\left(\mathcal{S}_{\text{in}} \to \mathcal{S}_{\text{out}}\right) = \sum_{\mathcal{F} :\, \partial\mathcal{F} = \mathcal{S}_{\text{in}} \sqcup \mathcal{S}_{\text{out}}} \mathcal{A}(\mathcal{F}). \tag{28}$$

Equation 28 motivates, at most, a later possibility of assigning weights to alternative conversion histories. The present manuscript initially uses the weaker term *history weight*. A probabilistic or complex-amplitude interpretation requires additional mathematical and empirical structure and is left open.

7.7 Two-Complexes as History-Bearing Structures

The principal feature borrowed from spin-foam models is the use of a 2-complex as a history-bearing object. A graph describes pairwise incidence at one level. A 2-complex adds faces whose boundaries are composed of edges, thereby representing structured relations among relations. When boundary subcomplexes are distinguished, the same object can represent an interpolating history connecting relational configurations.

The conversion formalism later supplements this structure with a partial order $\prec$ on relevant events. This addition permits model-relative generative precedence, antichains, and historical domains as developed in Section 4. The combined structure is therefore inspired by spin foams while remaining mathematically distinct from standard spin-foam models.

Definition 7.2 (History-bearing relational 2-complex)**.** For the purposes of this manuscript, a *history-bearing relational 2-complex* is a labeled 2-complex $\mathcal{K} = (V, E, F)$ equipped, where required by the application, with additional structures such as a partial order on selected events, boundary subcomplexes, cell labels, and admissibility conditions.

Definition 7.2 is representational. It does not assert that every conversion process is intrinsically two-dimensional or that a 2-complex is the unique adequate higher-order representation.

7.8 Spin-Foam Inspiration for Generative Relational Modeling

The spin-foam analogy contributes four specific ideas to the later conversion model. First, a relational state can be distinguished from a history connecting states. Second, higher-order cells can represent relational structure that cannot be reduced conveniently to pairwise adjacency. Third, local labels and compatibility data can be composed across a larger history. Fourth, alternative histories can, in principle, receive different weights without identifying one coarse-grained endpoint with the entire process.

These ideas help clarify why the apparent conversion $A \to B$ is insufficient as a complete representation. The values A and B, the entities identified around them, and the conversion procedure can all be embedded in an extended history with intermediate events and higher-order relations. Later sections use this history to define conversion configurations and composable operations without assigning physical spin labels to social or economic entities.

7.9 Structural Analogy and Physical Restraint

The relationship between spin-foam physics and heterogeneous value conversion is a structural analogy. The paper does not propose that currency, labor, legal claims, or normative judgments are quantum-geometric excitations. It also does not infer quantum interference, Hilbert-space structure, or physical amplitudes from the presence of a 2-complex representation.

Caution 7.3 (Scope of the spin-foam analogy)**.** The use of spin networks, spin foams, and local history weights supplies mathematical inspiration for representing relational states and histories. Physical claims require the additional structures and empirical interpretation of the relevant physical theory. The conversion formalism introduced later defines independent meanings for its cells, labels, operators, and weights.

Caution 7.3 is essential to the methodological stance of the manuscript. The physical literature provides a mature example in which graph-like boundary data, higher-order histories, local labels, and weighted transitions coexist [2, 3, 4]. The present paper uses these features as disciplined sources of formal vocabulary while maintaining ontological suspension regarding the underlying structure of economic, social, normative, and physical reality.

8 Relational Configuration Space for Value Conversion

This section assembles the mathematical ingredients introduced in Sections 3–7 into the configuration language used by the conversion formalism. Its role is integrative: the higher-order historical complex, partial-order event structure, active relational boundary, conversion-relevant scope, and provisional nomos are collected into a single typed state description. The objective is to specify what later operators act on while preserving the distinction between a modeled relational history and the partial values, subjects, institutions, or environments manifested from it. The construction remains representational. It supplies a working state space for heterogeneous value conversion and does not claim to exhaust the ontology of the systems represented.

Figure 6 summarizes the architecture developed in this section. The nested regions indicate analytical scope rather than physical enclosure. In particular, a conversion-relevant domain can include spatially distant institutions, infrastructures, ecological processes, or historical relations when they are relevant to the conversion under study.

8.1 Generative Relational Configuration

The formalism requires a state object on which later transformations can act. Let $\mathfrak{G}$ denote a modeled generative-relational history carried by a labeled higher-order complex equipped with the event-order structure introduced in Section 4. At a selected relational slice Σ, the realized historical restriction is written $\mathfrak{G}_{\leq\Sigma}$. The configuration also records an active relational boundary $\mathcal{B}_\Sigma$ and a currently effective nomos structure $\mathfrak{N}_\Sigma$.

Definition 8.1 introduces the state object used throughout the remaining formalism.

Definition 8.1 (Generative relational configuration)**.** A *generative relational configuration* at a relational slice Σ is a structured state

$$\omega_\Sigma = (\mathfrak{G}_{\leq\Sigma}, \mathcal{B}_\Sigma, \mathfrak{N}_\Sigma)\,, \tag{29}$$

where $\mathfrak{G}_{\leq\Sigma}$ is the modeled realized history, $\mathcal{B}_\Sigma$ is the active relational boundary supported by that history, and $\mathfrak{N}_\Sigma$ is the nomos structure represented as effective at the slice.

Equation 29 is intentionally compact. The internal structure of its components can be refined for a particular application. A model may attach labels, measures, institutional states, material variables, or additional higher-order cells without changing the type of the configuration. The configuration therefore serves as a carrier for later conversion operators rather than as an exhaustive inventory of reality.

Let Ω denote the class or state space of configurations admitted by a chosen model. Equation 30 records this notation.

$$\Omega = \{\omega_\Sigma \mid \omega_\Sigma \text{ satisfies the structural and admissibility conditions of the model}\}\,. \tag{30}$$

Equation 30 does not require Ω to be a vector space, manifold, or Hilbert space. Those stronger structures may be introduced in later extensions when their assumptions are justified. At the present stage, Ω provides a typed domain and codomain for the partial transformations developed later in the manuscript.

8.2 Historical Structure

The historical component of Definition 8.1 integrates the 2-complex representation of Section 7 with the partial-order structure of Section 4. Let $D(\Sigma)$ denote the order ideal associated with the relational slice. The modeled realized history is the restriction stated in Equation 31.

$$\mathfrak{G}_{\leq\Sigma} = \mathfrak{G}\big|_{D(\Sigma)}. \tag{31}$$

Equation 31 permits a subject, institution, value manifestation, or conversion rule represented at Σ to depend on an extended history rather than only on structures incident to the current slice. Historical

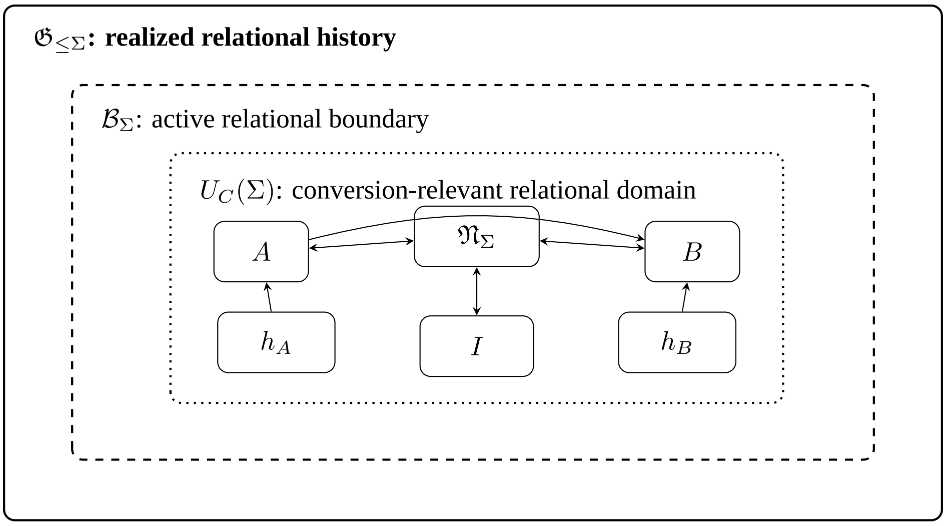


Figure 6: Schematic architecture of a conversion configuration. The realized history $\mathfrak{G}_{\leq\Sigma}$ contains an active relational boundary $\mathcal{B}_\Sigma$ and a conversion-relevant relational domain $U_C(\Sigma)$. The symbols A, B, h_A, h_B, I, and $\mathfrak{N}_\Sigma$ indicate context-dependent manifested structures used in a particular analysis. Their placement does not imply that they are primitive or point-like entities.

depth can nevertheless be truncated in practical analysis. A later section treats such truncation as an epistemic operation whose adequacy may itself require evaluation.

The Historical Persistence Convention from Section 4 is retained for the present formal model. When a later slice Σ' extends the represented history of Σ, the earlier realized structure is embedded into the later one. This convention supports provenance, retrospective analysis, and composition of conversion histories. It remains a tractability assumption rather than a claim about the ultimate physics of time.

8.3 Active Relational Boundaries

The realized history can contain relations whose active consequences have ended. A discharged debt, expired contract, completed employment relation, or terminated institutional authorization can remain historically represented while ceasing to structure currently admissible actions in the same way. The component $\mathcal{B}_\Sigma$ therefore records the relational structures treated as active at the selected slice.

Definition 8.2 (Active relational boundary)**.** An *active relational boundary* $\mathcal{B}_\Sigma$ is a model-specified structured collection of cells, labels, states, or relational conditions supported by $\mathfrak{G}_{\leq\Sigma}$ that remain relevant to admissible continuation from the slice Σ.

Definition 8.2 uses the word *boundary* in a process sense. It should be distinguished from the topological boundary operator ∂ of a cell complex. Depending on the application, $\mathcal{B}_\Sigma$ may be represented by a subcomplex, a labeled collection of active relations, or another boundary-state structure compatible with the chosen model.

A later operation may modify the active boundary while preserving the prior history. Equation 32 gives a schematic update in which D_+ denotes structures becoming active and D_- denotes structures ceasing to remain active.

$$\mathcal{B}_{\Sigma'} = (\mathcal{B}_\Sigma \setminus D_-) \cup D_+. \tag{32}$$

Equation 32 is set-theoretic shorthand for a potentially richer structured update. The subsequent operator construction distinguishes ontological extension from relational termination and supplies the corresponding transformation language.

8.4 Relationally Scoped Conversion Domains

The history $\mathfrak{G}_{\leq\Sigma}$ can be much larger than the structure required for one conversion analysis. A practical model therefore specifies a conversion-relevant relational domain $U_C(\Sigma)$. The domain collects the historical and active structures treated as relevant to a conversion problem C at the selected slice.

Definition 8.3 (Conversion-relevant relational domain)**.** For a conversion problem C represented at slice Σ, a *conversion-relevant relational domain* $U_C(\Sigma)$ is a model-selected substructure of $\mathfrak{G}_{\leq\Sigma}$ containing the events, relations, higher-order cells, labels, institutions, material conditions, and other structures retained for analysis of C.

The scope in Definition 8.3 is relational rather than merely geographic. A currency conversion carried out in one city can depend on distant monetary authorities, payment infrastructures, legal jurisdictions, international markets, and historical institutional arrangements. An environmental compensation process can depend on ecological structures extending beyond the administrative jurisdiction in which the payment is made.

The selected domain can change as the conversion develops. New information, institutional recognition, or material consequences may justify enlarging or revising $U_C(\Sigma)$. The formalism therefore treats scope selection as part of the modeling procedure rather than as a fixed ontological partition of $\mathfrak{G}$.

Table 5 separates the formal state components from the context-dependent manifestations used in a particular conversion analysis. The latter should not be read as additional primitive substances appended to the state.

8.5 Values as Relational Manifestations

The symbols A and B are used throughout the manuscript as shorthand for heterogeneous value forms involved in a conversion. Within the present configuration model, they are represented as partial manifestations of a larger relational state. A provisional notation is stated in Equation 33; the following section develops the observation maps more carefully.

$$A_\Sigma^{(c)} = \pi_A^{(c)}(\omega_\Sigma), \qquad B_\Sigma^{(c)} = \pi_B^{(c)}(\omega_\Sigma), \tag{33}$$

where c records the observational, institutional, or analytical context relevant to the manifestation. Equation 33 expresses a limited claim: the value forms used in a conversion model are obtained through a context-sensitive representation of the relational configuration. It does not imply that every substantive theory of value reduces to an observation map.

This formulation is useful for heterogeneous conversion because the same relational history can admit several value manifestations. A labor process may be represented through hours, wages, skill classifications, produced outputs, legal obligations, capabilities, or other value-related descriptions. The conversion formalism can compare such representations while preserving their dependence on a larger history.

8.6 Subjects and Value Holders as History-Bearing Structures

The same representational principle applies to subjects and value holders. A symbol such as h_A should not be interpreted automatically as a point-like owner attached to a value token. The modeled holder or bearer can include historically extended relations through which agency, standing, ownership, responsibility, dependence, or participation becomes relevant to the conversion.

A provisional manifestation notation for a holder and a more general subject is stated in Equation 34.

$$h_{A,\Sigma}^{(c)} = \pi_{h_A}^{(c)}(\omega_\Sigma), \qquad S_{i,\Sigma}^{(c)} = \pi_{S_i}^{(c)}(\omega_\Sigma). \tag{34}$$

Equation 34 permits the identified entity to include a finite relational-historical substructure rather than one vertex. A person's present position can depend on prior education, care relations, contracts,

Table 5: Principal structures in a relational configuration for heterogeneous value conversion.

Symbol	**Formal role**	**Interpretive scope**
$\mathfrak{G}_{\leq\Sigma}$	Realized history	Higher-order event and relational structure retained up to the selected slice
$\mathcal{B}_{\Sigma}$	Active boundary	Structures treated as currently active for admissible continuation
$\mathfrak{N}_{\Sigma}$	Nomos structure	Currently effective normative, legal, institutional, or conventional constraints represented by the model
$U_C(\Sigma)$	Relational scope	Conversion-relevant substructure selected for analysis
A, B	Manifested values	Context-dependent value forms identified from the relational configuration
h_A, h_B	Manifested holders or bearers	History-bearing relational entities identified in relation to the value forms
I, E	Institutional and material manifestations	Institutional, infrastructural, ecological, or other environmental structures retained by the analysis

infrastructures, legal recognition, material conditions, and many other relations represented within the selected historical depth. A collective subject can likewise be represented through a structured history rather than through aggregation into one featureless node.

The manifestation h_A is context-sensitive. Legal ownership, experiential bearing, generative contribution, beneficial receipt, and exposure to loss can select different relational structures around the same value process. The present paper keeps these distinctions available for later analysis rather than identifying them through a single holder relation.

8.7 Institutions and Material Environments

Institutions and material environments enter the same configuration rather than appearing only as external parameters. A legal regime, payment network, energy infrastructure, ecosystem, technological platform, or material resource can participate in the historical generation and subsequent convertibility of a value form. The formalism therefore permits institutional and material structures to occupy cells and labels within $\mathfrak{G}_{\leq\Sigma}$, to appear within $\mathcal{B}_{\Sigma}$, and to enter the relational scope $U_C(\Sigma)$ when relevant.

This treatment does not require institutions and ecosystems to possess the same semantics as human subjects. It requires only that the chosen representation preserve the relations through which they constrain, enable, mediate, or undergo change in the conversion process. Application-specific labels can distinguish human agency, legal authority, material dependence, ecological process, symbolic mediation, and other modes of participation.

The inclusion of environmental structure is particularly important for avoiding an implicit background model in which human actors interact while material conditions remain fixed outside the ontology. Within the present formal representation, what an application calls the "environment" can itself be part of the evolving relational history. Unmodeled influences can still be represented statistically or as unresolved inputs at an effective modeling level without being declared ontologically external to reality.

8.8 Nomos within Relational Configurations

The component $\mathfrak{N}_{\Sigma}$ records the conversion-relevant nomos treated as effective at a relational slice. The term includes, depending on the application, legal rules, institutional permissions, market conventions, accounting categories, recognized settlement forms, professional standards, and other normative structures that shape which transformations are currently intelligible or admissible.

Definition 8.4 (Conversion-relevant nomos)**.** A *conversion-relevant nomos* $\mathfrak{N}_\Sigma$ is the modeled normative and institutional structure that contributes to determining the admissibility, interpretation, or recognized consequence of conversion-related operations at the slice Σ.

Definition 8.4 does not treat nomos as a timeless parameter. Its historical support lies within $\mathfrak{G}_{\leq\Sigma}$, and later operations can participate in its stabilization, contestation, or revision. A later section develops this endogenous evolution in detail.

At the configuration level, the principal formal effect of nomos is to restrict or structure the operator family available from a state. If $\mathfrak{O}(\omega_\Sigma)$ denotes candidate transformations associated with the configuration, the subset represented as admissible under the current nomos can be written as Equation 35.

$$\mathfrak{O}_\Sigma^{\mathrm{adm}} = \{\Phi \in \mathfrak{O}(\omega_\Sigma) \mid \Phi \text{ satisfies the admissibility conditions represented by } \mathfrak{N}_\Sigma\}\,. \tag{35}$$

Equation 35 is deliberately broad. A legal prohibition, institutional prerequisite, technical standard, or conventional rule can restrict operators in different ways. Later sections distinguish the descriptive emergence of such constraints from their normative evaluation.

The configuration model developed in this section therefore provides a common state type for the remainder of the paper. Historical structure, active relations, nomos, and relational scope remain distinguishable, while values, subjects, institutions, and environments can be represented as context-dependent manifestations of the same history-bearing configuration. The following section specifies the observation and entity-identification maps that make these manifestations operational.

9 Observation and Entity Identification

This section specifies how finite entities and observations are obtained from the relational configurations introduced in Section 8. Its role is epistemic and representational. The objective is to distinguish four operations that can otherwise be conflated in conversion analysis: producing an observational readout, identifying a relational entity, granting that entity institutional recognition, and allowing an observation to alter the subsequent relational configuration. The discussion proceeds from typed observation maps to history-sensitive entity identification, then introduces historical-depth control, epistemic truncation, context dependence, identification bias, institutional legibility, and performative observation. These distinctions prepare the later treatment of generative operators and epistemic justice.

Figure 7 summarizes the architecture. The upper branch represents an epistemic readout that leaves ontological transformation unspecified. The lower branch represents a state-changing recognition process that acts on the configuration after an entity has been identified. The two branches are related analytically, while their mathematical types remain distinct.

9.1 Observation Maps

The configuration ω_Σ is intentionally richer than any one quantity used in a conversion procedure. A wage record, ownership status, currency amount, legal classification, ecological indicator, or testimony is therefore represented as a readout from the configuration rather than as an additional primitive component of the ontology. Let $X_i^{(c)}$ denote an observation space associated with observable i under context c. The corresponding observation map is stated in Equation 36.

$$\pi_i^{(c)} : \Omega \longrightarrow X_i^{(c)}. \tag{36}$$

Equation 36 specifies a readout type. It does not, by definition, specify an ontological transformation of ω_Σ. The context superscript can encode an institutional perspective, measurement protocol, legal vocabulary, analytical purpose, scale, or other conditions under which the readout is constructed.

For a particular configuration, the manifested value of observable i is given by Equation 37.

$$x_{i,\Sigma}^{(c)} = \pi_i^{(c)}(\omega_\Sigma). \tag{37}$$

Equation 37 includes the value and holder manifestations introduced provisionally in Equations 33 and 34. The formalism therefore treats "A," "B," "holder," "wage," "price," and comparable descriptors as context-sensitive observations when an application represents them in this way.

An observation can be coarse or rich. A scalar price may compress a large relational history into one number, while a legal record or relational profile may retain a larger structured object. The codomain $X_i^{(c)}$ is therefore left general. It can be numerical, categorical, graphical, topological, textual, or otherwise structured when the application supplies the relevant semantics.

Remark 9.1 (Observation and modeled reality)**.** The distinction between ω_Σ and $\pi_i^{(c)}(\omega_\Sigma)$ is internal to the present model. It expresses a separation between a modeled relational configuration and a particular readout of that configuration. It does not establish direct access to an observer-independent ultimate ontology.

Remark 9.1 preserves the ontological suspension adopted throughout the manuscript while still permitting explicit analysis of information loss and representational dependence.

9.2 Relational Entity Identification

Many conversion procedures require an entity before they can assign a value, entitlement, obligation, or conversion relation. The entity may be a person, organization, community, asset, contribution, ecosystem, institutional role, or another structured object. Within the present framework, such an entity is represented as a finite relational-historical construction selected from a much larger configuration.

Let $\mathfrak{E}$ denote a class of admissible entity representations. For a focal referent a, context c, and historical-resolution parameter λ, an entity-identification map is defined in Definition 9.2.

Definition 9.2 (Relational entity-identification map)**.** A *relational entity-identification map* is a typed map

$$\mathcal{I}_a^{(c,\lambda)} : \Omega \longrightarrow \mathfrak{E} \tag{38}$$

that associates a relational configuration with a finite structured representation of the entity identified around the focal referent a under context c and historical resolution λ.

The identified entity corresponding to Definition 9.2 is stated in Equation 39.

$$\mathcal{E}_{a,\Sigma}^{(c,\lambda)} = \mathcal{I}_a^{(c,\lambda)}(\omega_\Sigma). \tag{39}$$

Equation 39 permits $\mathcal{E}_{a,\Sigma}^{(c,\lambda)}$ to contain several vertices, edges, faces, labels, and historically ordered events. The entity can therefore preserve a relational history rather than collapsing automatically to one node. The focal referent a identifies the analytical center of the construction without implying that every relation in the resulting entity is reducible to a property internal to a.

The term *operator* can be used informally for $\mathcal{I}_a^{(c,\lambda)}$ in the broad transformation sense introduced in Section 5. Its type nevertheless differs from the generative partial endomorphisms developed later: entity identification maps Ω into an entity-representation space $\mathfrak{E}$, whereas a generative operator maps configurations to configurations. Preserving this distinction avoids treating epistemic construction and ontological evolution as the same mathematical operation.

9.3 Historical Depth

An identified entity cannot ordinarily contain the entire relational ancestry represented in $\mathfrak{G}_{\leq\Sigma}$. Practical identification therefore requires a rule for historical depth or relational resolution. The parameter λ in Definition 9.2 records this dependence without assuming one universal metric of historical distance.

A useful representation is a nested family of historical domains associated with the focal referent a. For ordered resolution parameters $\lambda_1 \preceq \lambda_2$, the family satisfies Equation 40.

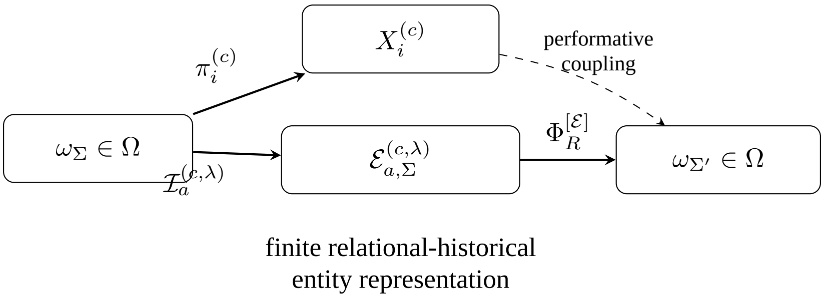


Figure 7: Typed distinction among observation, entity identification, and recognition. Observation maps and entity-identification maps produce representations in dedicated codomains. Recognition is represented separately as a partial state transformation. A performative observation may couple a readout to a later state-changing operation without changing the type of the readout map itself.

$$D_a^{(\lambda_1)}(\Sigma) \subseteq D_a^{(\lambda_2)}(\Sigma) \subseteq D(\Sigma). \tag{40}$$

Equation 40 is a filtration-like construction. Increasing λ can represent deeper historical reach, broader relational distance, weaker relevance thresholds, or a combination specified by the application. The formalism does not require λ to be chronological time.

The entity-identification map can then be understood as operating on information retained from $D_a^{(\lambda)}(\Sigma)$. A short-depth identification may represent a worker through a present contract and current task. A deeper identification may include training, prior institutional relations, collaborative history, material infrastructure, or other relations that an evaluative theory treats as relevant. Neither depth is automatically appropriate for every purpose.

Remark 9.3 (Finite historical access)**.** The historical-depth parameter expresses an epistemic and computational limitation as well as an analytical choice. The present formalism does not require an evaluator to reconstruct an indefinitely extended relational ancestry before an entity can be identified.

Remark 9.3 is important for later normative analysis. Justice criteria can evaluate the adequacy of a chosen historical depth without requiring complete recovery of the entire modeled past.

9.4 Epistemic Truncation

Historical-depth selection, relational scope, available evidence, and institutional categories jointly produce epistemic truncation. Truncation is broader than simple loss of data. It can remove relational structure, merge distinctions, break higher-order interactions into pairwise fragments, or omit events whose relevance becomes visible only under a different analytical scale.

The formalism treats truncation as a relation between a richer reference structure and an identified representation when a suitable reference is available. Let $U_a^{\mathrm{ref}}(\Sigma)$ denote a model-specified reference domain for the focal referent. The support retained by an identified entity is written $\mathrm{supp}(\mathcal{E}_{a,\Sigma}^{(c,\lambda)})$. The omitted support can then be represented schematically by Equation 41.

$$\mathcal{O}_{a,\Sigma}^{(c,\lambda)} = U_a^{\mathrm{ref}}(\Sigma) \setminus \mathrm{supp}\left(\mathcal{E}_{a,\Sigma}^{(c,\lambda)}\right). \tag{41}$$

Equation 41 is available only when the reference domain and support relation are defined within the same model. It should not be interpreted as direct measurement of an unknowable complete ontology. More general distortion measures can compare structural invariants, incidence patterns, historical relations, or other features without requiring literal set inclusion.

Table 6 distinguishes several forms of epistemic distortion that become relevant in later justice analysis.

Table 6: Illustrative structural distortions in relational entity identification.

Distortion	Formal tendency	Illustrative consequence
Omission	Relevant cells or labels fall outside the identified support	A contribution, dependency, or historical condition becomes institutionally invisible
Fragmentation	One connected or higher-order structure is represented as disconnected parts	Co-generation can appear as several isolated contributions
Conflation	Distinct structures are mapped into one category or entity	Different forms of burden, agency, or contribution become indistinguishable
False connection	A relation is introduced or strengthened by the identification procedure	Institutional categories can imply dependence or equivalence absent from the selected reference model
Historical truncation	Earlier ordered events are excluded by the chosen depth	Present status is detached from the history through which it emerged
Scale distortion	Structure visible at one resolution is treated as invariant across scales	Local regularity can be generalized beyond the relational range in which it is supported

Table 6 does not classify every difference between an entity representation and a richer relational domain as injustice. It supplies structural vocabulary for later evaluative theories that may judge some distortions normatively significant.

9.5 Context-Dependent Identification

Entity identification is indexed by context because different conversion procedures can require different relational cuts. A labor court, statistical office, employer, family, bank, and ecological assessment can identify different structures around the same focal referent without thereby referring to wholly different realities. The difference lies partly in the operators, evidence, scales, and purposes used to construct the represented entity.

For two contexts c_1 and c_2, context dependence is expressed in Equation 42.

$$\mathcal{I}_a^{(c_1,\lambda_1)}(\omega_\Sigma) \neq \mathcal{I}_a^{(c_2,\lambda_2)}(\omega_\Sigma) \quad \text{in general.} \tag{42}$$

Equation 42 states structural plurality rather than error. A legal identification may emphasize standing and title, while a capability-oriented evaluation may retain dependencies and accessible options. An economic accounting system may select measurable transactions, while an ecological model may preserve material flows that the accounting system omits.

The same plurality creates a translation problem. When one conversion process combines observations produced under several contexts, the formalism should record the maps through which the corresponding entity representations are related. Treating apparently identical labels as automatically equivalent can hide substantial changes in relational content.

9.6 Identification Bias

Context dependence becomes bias when the selection procedure exhibits a systematic distortion relative to a specified epistemic or normative standard. The qualifier is important. The formalism does not infer bias merely from the existence of different entity representations. A bias claim requires a reference criterion that states which structures should be retained, weighted, or made contestable for the purpose at hand.

Let Q denote such a criterion and let δ_Q be a discrepancy functional defined on admissible entity representations. A model can then assign the identification discrepancy in Equation 43.

$$\delta_Q\left(\mathcal{E}^{(c,\lambda)}_{a,\Sigma}, U^{\mathrm{ref}}_a(\Sigma)\right) \geq 0. \tag{43}$$

Equation 43 is intentionally schematic. The discrepancy can be topological, statistical, historical, semantic, or institution-specific. A later section develops topological fidelity as one illustrative possibility. The choice of Q and δ_Q introduces substantive assumptions and therefore belongs to an explicit evaluative layer rather than to the bare ontology of conversion.

Bias can also be endogenous to institutional practice. Records may be more complete for actors with greater administrative capacity; recognized categories may fit some forms of work better than others; historical archives can preserve some contributions more readily than others. The resulting asymmetry can be reproduced each time the same identification procedure is applied. The operator language is useful here because a repeated epistemic procedure can become part of the mechanism through which later conversion relations are structured.

9.7 Recognition and Institutional Legibility

Identification and recognition should remain distinct. Identification produces a representation in $\mathfrak{E}$. Recognition gives some identified entity an effective status within the active relational configuration or nomos. A community can be identified by an analyst without possessing legal standing; a contribution can be recorded without generating an entitlement; an asset can be physically present without being accepted as eligible collateral.

Recognition is therefore modeled as a generative partial transformation parameterized by the identified entity. Equation 44 states the minimal type.

$$\Phi^{[\mathcal{E}]}_R : \Omega \rightharpoonup \Omega. \tag{44}$$

Equation 44 allows recognition to change $\mathcal{B}_\Sigma$, $\mathfrak{N}_\Sigma$, or other structures represented by the configuration. The map can be partial because recognition may require standing, evidence, jurisdiction, or other prerequisites.

The distinction between Equations 38 and 44 is analytically important. The former answers which relational entity is represented. The latter answers what happens when a system grants that representation operative status. Conversion procedures can fail at either stage: an entity can be misidentified, or it can be identified adequately while remaining institutionally illegible.

9.8 Performative Observation

Some observations participate in changing the configuration they describe. A credit rating can alter borrowing conditions; an official classification can change eligibility; a public valuation can influence bargaining; a legal determination can create or extinguish recognized claims. The present framework treats this performativity through coupled maps with distinct types rather than through one ambiguously typed operator.

Let the readout be produced by $\pi^{(c)}_i$ as in Equation 36. When the readout x_i conditions a state-changing transformation, the associated update is represented by Equation 45.

$$\omega_{\Sigma'} = \Phi^{[x_i]}_i(\omega_\Sigma), \qquad x_i = \pi^{(c)}_i(\omega_\Sigma). \tag{45}$$

Equation 45 preserves type consistency. The observation map produces an element of $X^{(c)}_i$, while the associated generative operator remains a partial endomorphism on Ω. The superscript $[x_i]$ records that the concrete transformation can depend on the observation produced from the initial state.

Performative observation creates a feedback relation between epistemology and ontology. Once $\Phi^{[x_i]}_i$ produces $\omega_{\Sigma'}$, a later observation is evaluated on the changed configuration. For a second readout π_j, Equation 46 makes this dependence explicit.

$$x_{j,\Sigma'} = \pi_j^{(c')}\left(\Phi_i^{[x_i]}(\omega_\Sigma)\right). \tag{46}$$

Equation 46 states the mechanism that later produces order sensitivity in conversion histories. The second observation is drawn from an ontology already modified by the first observation-associated transformation. Reversing the order can therefore yield a different relational history even when the same observation types appear in both procedures.

The distinctions developed in this section establish the epistemic interface of the conversion formalism. Observation maps provide typed readouts; entity-identification maps construct finite relational-historical entities; historical-depth parameters make bounded access explicit; recognition operators grant operative status; and performative observations couple readout to subsequent state change without collapsing those operations into one type. The next section develops the state-changing operators themselves, including ontological extension, event creation, relation generation, and termination.

10 Generative Operators

This section develops the state-changing operations used by the heterogeneous-conversion formalism. Its role is dynamical: Sections 8 and 9 specified what a relational configuration contains and how finite observations or entities are identified from it; the present section specifies how such configurations can be extended, reconfigured, or terminated through typed partial transformations. The objective is to preserve a single state type while allowing operators to create new events, relations, higher-order cells, active statuses, and future possibilities. The discussion proceeds from a general definition of a generative operator to ontological extension, event creation, edge and face generation, relational termination, observation–intervention coupling, state-dependent operator families, and changes in the space of future operations.

Figure 8 summarizes the architecture. A generative operator acts on a relational configuration in Ω and returns another configuration in Ω. The historical component, active boundary, and nomos can change together, while the Historical Persistence Convention in Section 4 remains a modeling convention for retaining already-realized history in the present version of the formalism.

10.1 Partial Transformations of Relational Configurations

The minimal dynamical object is a partial endomorphism on the configuration space Ω. Partiality is important because many operations require prerequisites such as recognized standing, available resources, institutional authority, compatible technical infrastructure, or a pre-existing relation. Definition 10.1 introduces the general operator type used in the remainder of the paper.

Definition 10.1 (Generative operator)**.** A *generative operator* is a partial transformation

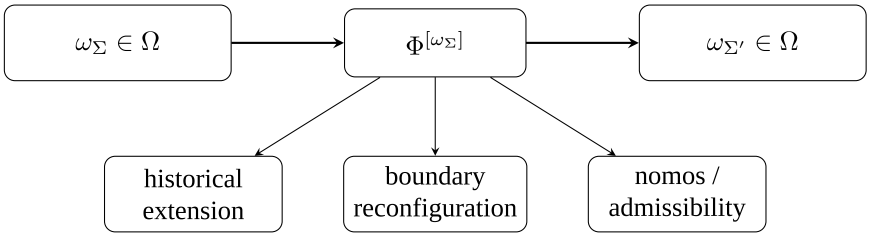
$$\Phi : \mathrm{Dom}(\Phi) \subseteq \Omega \longrightarrow \Omega \tag{47}$$

whose application can modify the modeled relational configuration by extending realized history, changing the active relational boundary, changing the effective nomos, or producing a combination of these effects.

Equation 47 preserves type consistency: the input and output are both elements of Ω. Observation maps and entity-identification maps retain their distinct codomains from Section 9. The adjective *generative* refers to the possibility that applying Φ creates new modeled structure. It does not imply that every application increases welfare, complexity, value, or any other normatively favored quantity.

For a state-dependent realization, the notation introduced in Remark 5.7 can be retained. Equation 48 records the resulting update.

$$\omega_{\Sigma'} = \Phi^{[\omega_\Sigma]}(\omega_\Sigma). \tag{48}$$



A concrete transformation may update one, two, or all three components.
The operator type remains $\Omega \rightharpoonup \Omega$.

Figure 8: Typed architecture of a generative operator. A partial transformation maps one admissible relational configuration to another while potentially extending realized history, reconfiguring the active relational boundary, and modifying the represented nomos or admissible-operation structure. The figure indicates possible effects rather than requiring every operator to modify every component.

Equation 48 permits the concrete form of the operation to depend on the configuration from which it is applied. The superscript is therefore contextual notation, not an additional argument in the mathematical type.

10.2 Ontological Extension

A central operation in the present model is extension of the history-bearing complex. Under the Historical Persistence Convention, a later modeled history contains an embedded copy of the already-realized history together with additional structure. Let $\Delta\mathfrak{G}_{\Sigma\rightsquigarrow\Sigma'}$ denote the newly represented historical structure introduced between two relational slices. The extension relation is stated in Equation 49.

$$\mathfrak{G}_{\leq\Sigma'} = \iota_{\Sigma\Sigma'}(\mathfrak{G}_{\leq\Sigma}) \cup \Delta\mathfrak{G}_{\Sigma\rightsquigarrow\Sigma'}. \tag{49}$$

In Equation 49, $\iota_{\Sigma\Sigma'}$ denotes the order-preserving embedding associated with the Historical Persistence Convention. The union symbol is schematic: the precise gluing operation depends on the chosen complex category and on how the boundary of the new cells attaches to the retained history.

The extension component of a generative operator can be denoted $\phi_+^{[\Delta\mathfrak{G}]}$. Definition 10.2 records its intended role without imposing a specialized creation-operator algebra.

Definition 10.2 (Ontological extension operator)**.** An *ontological extension operator* $\phi_+^{[\Delta\mathfrak{G}]}$ is a generative operator whose historical component adds an admissible new subcomplex $\Delta\mathfrak{G}$ to the represented realized history while retaining the prior historical subcomplex according to the adopted persistence convention.

The notation ϕ_+ is intentionally analogical. It marks constructive extension and should not be identified with a quantum creation operator unless a later representation explicitly supplies the required linear state space and algebraic structure.

10.3 Event Creation

The simplest historical extension adds a new event vertex. Let v_+ be an event not already contained in the current vertex set $V_{\leq\Sigma}$. Event creation requires more than set insertion: the new event must receive labels, incidence data where relevant, and a partial-order relation compatible with the existing event structure.

Let V' denote the vertex set after extension. The vertex update is stated in Equation 50.

$$V' = V_{\leq\Sigma} \cup \{v_+\}. \tag{50}$$

Equation 50 becomes a valid event extension only after the order relation is extended consistently. Let $\prec'$ denote the resulting order. A minimal compatibility condition is stated in Equation 51.

$$\prec \subseteq \prec', \qquad \prec' \text{ remains a partial order on } V'. \tag{51}$$

Equation 51 preserves previously modeled precedence relations while allowing new relations involving v_+. A production event, contract formation, payment, institutional recognition, publication, judgment, or ecological disturbance can each be represented through an application-specific event type. The formalism does not require these examples to share the same semantics; it requires only that their modeled event structure be explicit.

Remark 10.3 (Event creation and physical time)**.** Event creation is defined relative to the partial-order representation adopted by the model. It should not be interpreted as a claim that fundamental physical spacetime literally grows by discrete social events.

Remark 10.3 maintains the separation between the history-bearing formal representation and the unresolved physical ontology discussed in Section 7.

10.4 Edge and Face Generation

Many conversion-relevant changes cannot be represented by adding a vertex alone. A new event may establish a continuing relation, and several relations may jointly participate in a higher-order process. The 2-complex representation therefore permits operators to generate edges and faces together with any required labels or orientations.

Let $E_{\leq\Sigma}$ and $F_{\leq\Sigma}$ denote the existing edge and face sets. A schematic cell extension is stated in Equation 52.

$$E' = E_{\leq\Sigma} \cup \Delta E, \qquad F' = F_{\leq\Sigma} \cup \Delta F. \tag{52}$$

Equation 52 is subject to the incidence and boundary conditions of the chosen 2-complex. The added cells therefore cannot be selected independently of the complex structure. For a new face $f_+ \in \Delta F$, its boundary must be represented by an admissible cycle or chain of edges. Equation 53 states this requirement schematically.

$$\partial f_+ \subseteq E'. \tag{53}$$

Equation 53 is deliberately minimal. A more specialized cellular, simplicial, or categorical formulation can replace it when the application requires stronger orientation or gluing data.

Figure 9 illustrates a simple extension in which a new event and two new relations complete a higher-order face. The visual should be read as a structural example rather than as a universal semantics for value conversion.

The ability to add faces is important when the conversion-relevant process depends on higher-order co-participation. A jointly produced claim, collective agreement, multi-party settlement, or institutional authorization may require a structure that cannot be faithfully represented by one isolated dyadic edge.

10.5 Relational Termination

A history-bearing formalism also requires a representation of relations that cease to remain active. Literal deletion from the realized historical complex would conflict with the current Historical Persistence Convention. The present model therefore distinguishes historical retention from active termination.

Let $D_- \subseteq \mathcal{B}_\Sigma$ denote the portion of the active relational boundary that ceases to remain active, and let D_+ denote structure that becomes active through the same transformation. The boundary update is stated in Equation 54.

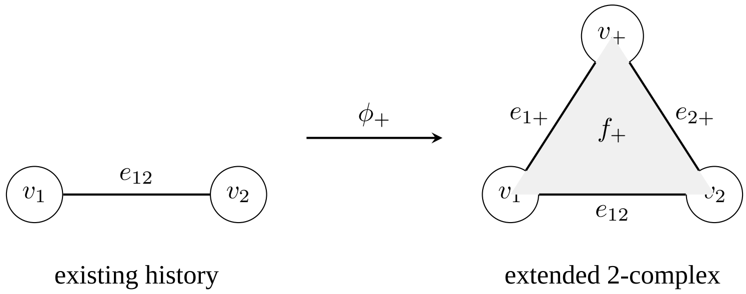


Figure 9: Illustrative generation of a vertex, edges, and a face. An extension operator adds a new event v_+ and relations that support a new higher-order cell f_+. The diagram illustrates one admissible structural pattern; application-specific semantics determine what the added cells represent.

$$\mathcal{B}_{\Sigma'} = (\mathcal{B}_\Sigma \setminus D_-) \cup D_+. \tag{54}$$

Equation 54 changes the currently active structure without erasing the events or relations through which the terminated structure previously existed. A contract can expire, a debt can be discharged, an institutional status can be revoked, or a market relation can cease while the corresponding historical process remains represented in $\mathfrak{G}_{\leq \Sigma'}$.

Definition 10.4 (Relational termination operator)**.** A *relational termination operator* ϕ_{term} is a generative operator whose principal effect is to remove specified relational structure from the active boundary or future continuation while retaining its realized historical representation under the adopted persistence convention.

Definition 10.4 uses *termination* in preference to *annihilation*. The latter term may suggest historical erasure or a stronger physical analogy that the present formalism does not require.

10.6 Observation–Intervention Instruments

Section 9.8 represented performative observation through a readout $x_i = \pi_i^{(c)}(\omega)$ followed by a state-changing operator $\Phi_i^{[x_i]}$. For some applications it is convenient to package these two typed components into a single instrument while retaining their distinct outputs.

Let $X_i^{(c)}$ be the readout space associated with observation i. Definition 10.5 introduces the corresponding instrument.

Definition 10.5 (Observation–intervention instrument)**.** The type of an *observation–intervention instrument* is stated in Equation 55.

$$\mathcal{M}_i^{(c)} : \text{Dom}(\mathcal{M}_i^{(c)}) \subseteq \Omega \longrightarrow X_i^{(c)} \times \Omega \tag{55}$$

whose value at ω records both the observation readout and the resulting configuration update.

The factorization of Definition 10.5 is stated in Equation 56.

$$\mathcal{M}_i^{(c)}(\omega) = \left(\pi_i^{(c)}(\omega), \Phi_i^{[\pi_i^{(c)}(\omega)]}(\omega)\right). \tag{56}$$

Equation 56 should be read as a packaging device rather than as a replacement for the distinction between observation and ontological transformation. The first component lies in $X_i^{(c)}$; the second lies in Ω. This instrument type is useful when a measurement, classification, recognition, or public valuation simultaneously records information and conditions a subsequent state change.

10.7 State-Dependent Operator Families

The abstract state-dependent family $\mathfrak{O}(\omega)$ introduced in Equation 13 can now be interpreted in terms of generative transformations. Let $\mathfrak{O}^{\mathrm{gen}}(\omega)$ denote the candidate generative operators available from ω, and let $\mathfrak{O}^{\mathrm{adm}}(\omega)$ denote those represented as admissible under the current configuration and nomos.

The inclusion relation is stated in Equation 57.

$$\mathfrak{O}^{\mathrm{adm}}(\omega) \subseteq \mathfrak{O}^{\mathrm{gen}}(\omega) \subseteq \mathrm{PEnd}(\Omega). \tag{57}$$

Equation 57 separates mathematical availability from modeled admissibility. A technically possible transformation can remain institutionally unavailable, legally impermissible, materially unsupported, or undefined because a prerequisite relation is absent.

The distinction is summarized in Table 7.

Table 7 also makes clear that $\mathcal{M}_i^{(c)}$ is not itself an endomorphism on Ω. It is an instrument whose second component is generated by an endomorphic state transformation. Maintaining this type distinction prevents the observation layer from being absorbed accidentally into the ontology-changing layer.

10.8 Evolution of Available Operations

A generative transformation can alter which transformations are available afterward. This effect is important for conversion because a new contract can create enforcement operations, a recognition event can create standing, a payment can discharge one claim while enabling another, and a new currency or platform can create conversion possibilities that did not previously exist. The operator family therefore co-evolves with the relational configuration.

Let $\omega' = \Phi(\omega)$. The induced change in the candidate operator family is stated schematically in Equation 58.

$$\mathfrak{O}^{\mathrm{gen}}(\omega) \longmapsto \mathfrak{O}^{\mathrm{gen}}(\Phi(\omega))\,. \tag{58}$$

Equation 58 does not require a separate fixed operator that maps one power set into another. It records the state dependence already built into $\mathfrak{O}^{\mathrm{gen}}$. The changed configuration supplies a changed domain of available transformations.

This structure also clarifies why later operations can be ontology-dependent. Suppose Φ_1 takes ω_0 to ω_1. A second operation selected from $\mathfrak{O}^{\mathrm{gen}}(\omega_1)$ acts on a configuration that did not exist before Φ_1. Reversing the procedural order can therefore change both the intermediate ontology and the set of operations from which the subsequent step is selected.

Proposition 10.6 (Configuration-dependent operational availability)**.** *Within the present formalism, if* $\mathfrak{O}^{\mathrm{gen}}(\omega_0) \neq \mathfrak{O}^{\mathrm{gen}}(\Phi_1(\omega_0))$*, then the first transformation can change the availability or parameterization of later transformations even when the later operation types retain the same descriptive names.*

Proposition 10.6 follows from the state-dependent operator-family construction rather than from a substantive empirical claim about every conversion system. It provides the formal basis for the order-sensitive histories developed in the following sections on conversion histories and order sensitivity.

The generative layer developed in this section therefore supplies the state-changing vocabulary required for heterogeneous conversion. Partial transformations preserve a common configuration type; extension operators add events and higher-order cells; termination operators modify active continuation without historical erasure under the current modeling convention; instruments package readouts with updates while preserving type distinctions; and state-dependent operator families allow the set of future operations to evolve with the ontology. The next section uses these components to define conversion histories as composite, potentially branching sequences of relational transformations.

Table 7: Principal operator types used in the generative layer of the formalism.

Operator type	Typed role	Principal modeled effect
Φ	$\Omega \rightharpoonup \Omega$	General partial transformation of a relational configuration
$\phi_{+}^{[\Delta\mathfrak{G}]}$	$\Omega \rightharpoonup \Omega$	Extends realized history by admissible new cells and labels
ϕ_{term}	$\Omega \rightharpoonup \Omega$	Terminates active relational continuation while retaining modeled history
$\Phi_{R}^{[\mathcal{E}]}$	$\Omega \rightharpoonup \Omega$	Grants operative recognition or institutional legibility to an identified entity
$\mathcal{M}_i^{(c)}$	$\Omega \rightharpoonup X_i^{(c)} \times \Omega$	Packages a typed readout with its associated configuration update

11 Conversion Histories

This section assembles the configuration and operator layers into a formal representation of heterogeneous conversion. Its role is processual: Section 10 defined typed transformations of relational configurations, while the present section specifies how a finite, partially composable family of such transformations constitutes a conversion history. The objective is to make explicit the intermediate operations, branching structures, co-generative relations, and attribution steps that can be hidden by a coarse notation such as $A \to B$. The discussion proceeds from a basic history definition to operator sequences, branching and merging structures, higher-order processes, co-generated value, attribution, and the final coarse-graining through which a complex history becomes visible as an ordinary conversion.

Figure 10 summarizes the central distinction. A conversion history is represented at the configuration level by a typed sequence of partial transformations. The familiar statement $A \to B$ appears only after an observation or coarse-graining map selects a restricted manifestation of that history.

11.1 Conversion as Relational Transformation

A heterogeneous conversion is represented first as a transformation history on the configuration space Ω. Let $\omega_0 \in \Omega$ be an initial configuration and let $\Phi_1, \dots, \Phi_n$ be generative operators whose domains are satisfied successively. The intermediate configurations are defined recursively in Equation 59.

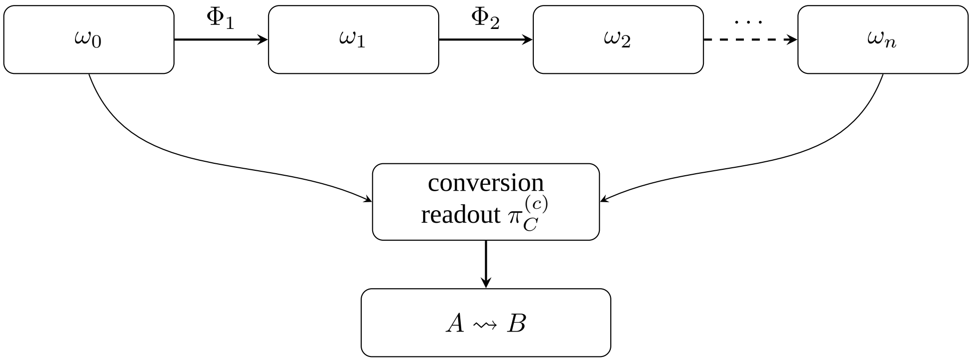
$$\omega_k = \Phi_k(\omega_{k-1}), \qquad k = 1, \dots, n. \tag{59}$$

Equation 59 places the history at the same typed level used throughout Section 10: each step maps an admissible relational configuration to another relational configuration. The visible value forms A and B can be observed at selected stages through maps of the type introduced in Section 9; they do not determine the full state type.

Definition 11.1 (Conversion history)**.** A *conversion history* is a finite composable family of generative operators $\Phi_1, \dots, \Phi_n$ together with an initial configuration $\omega_0 \in \Omega$ such that Equation 59 is defined at every step. Its composite transformation is stated in Equation 60.

$$\Gamma_C = \Phi_n \circ \cdots \circ \Phi_2 \circ \Phi_1 : \mathrm{Dom}(\Gamma_C) \subseteq \Omega \longrightarrow \Omega. \tag{60}$$

Equation 60 uses Γ_C for the composite conversion transformation. The associated history includes the intermediate configurations from Equation 59, because later questions of procedure, attribution, and justice can depend on those intermediate structures even when the same initial and final coarse-grained values are observed.



The displayed conversion is a context-dependent manifestation of a richer history. Intermediate states may contain recognition, attribution, authorization, settlement, institutional, material, and other relational changes.

Figure 10: Conversion history and coarse-grained conversion readout. The upper sequence represents typed transformations among relational configurations. A context-dependent observation map extracts a simplified conversion manifestation such as $A \rightsquigarrow B$. The figure does not require every conversion to contain the same intermediate operations.

11.2 Composite Conversion Histories

The compositional representation permits a conversion to contain several analytically distinct stages without fixing one universal sequence. A wage payment, for example, can involve prior entity identification, contractual authorization, measurement of time or output, monetary transfer, settlement, and changes in active obligations. A currency conversion can involve quotation, order submission, matching, settlement, balance-sheet updates, and institutional reporting. These examples motivate composition while leaving the local semantics application-specific.

For a given history, let the ordered trace of configurations be denoted by $\mathrm{Tr}(\Gamma_C; \omega_0)$. Equation 61 defines this trace.

$$\mathrm{Tr}(\Gamma_C; \omega_0) = (\omega_0, \omega_1, \dots, \omega_n). \tag{61}$$

Equation 61 retains process information that would disappear if the history were represented only by the pair (ω_0, ω_n). The trace is still a coarse formal object: it records represented configurations rather than every feature of the wider ontology. Its analytical value lies in preserving the sequence on which later order-sensitive arguments can operate.

Remark 11.2 (History granularity)**.** The decomposition of a conversion history into operators is model-dependent. A process represented by one operator at one analytical scale may be decomposed into several operators at another scale. The present formalism therefore requires explicit granularity rather than a uniquely privileged decomposition.

Remark 11.2 is important when different disciplines describe the same conversion at different resolutions. Legal analysis may separate authorization and settlement, an accounting model may aggregate them, and an engineering implementation may decompose settlement into several technical events.

11.3 Conversion-Relevant Operator Sequences

A conversion history should include operations because they are relevant to the represented conversion, not simply because they occur nearby in clock time or network distance. The present framework therefore treats conversion relevance as a scoped modeling relation. Let U_C denote the relational domain introduced in Section 8. A sequence $\Phi_1, \dots, \Phi_n$ is conversion-relevant when the modeled effects of those operators contribute to the constitution, admissibility, execution, interpretation, settlement, or subsequent status of the conversion within U_C.

This criterion admits heterogeneous operator types. A recognition operator can be relevant because it grants standing to an entity; an extension operator can be relevant because it creates a contract or credit relation; a termination operator can be relevant because payment discharges an obligation. The operators need not share one substantive meaning to participate in one conversion history.

The distinction between candidate and admissible operators from Equation 57 also remains active. At an intermediate state ω_k, the next step must be selected from an operator family that is defined for that configuration. The admissibility condition is stated in Equation 62.

$$\Phi_{k+1} \in \mathfrak{O}^{\text{adm}}(\omega_k) \subseteq \mathfrak{O}^{\text{gen}}(\omega_k). \tag{62}$$

Equation 62 makes the history state-dependent. An earlier transformation can create, remove, or reparameterize the possibilities from which a later transformation is selected. This dependence prepares the noncommutative analysis developed in the following section.

11.4 Branching Histories

A single realized conversion history is linear only at the level of its chosen operator sequence. The space of admissible continuations can branch because more than one next transformation may be defined from an intermediate configuration. Let $\text{Succ}(\omega_k)$ denote the set of modeled successor configurations reachable by one admissible generative operation. Equation 63 defines this set.

$$\text{Succ}(\omega_k) = \left\{ \Phi(\omega_k) \;\middle|\; \Phi \in \mathfrak{O}^{\text{adm}}(\omega_k) \right\}. \tag{63}$$

Equation 63 supplies a local branching structure without assigning probabilities or amplitudes to the branches. One branch may correspond to settlement, another to contestation, another to termination, and another to a delayed or revised conversion. A later empirical model can add transition weights when evidence supports them.

Figure 11 distinguishes branching of possible histories from the coexistence of multiple generative lineages inside one realized history. The two structures should not be conflated.

11.5 Merging Histories

The term *merging* requires care because two mutually exclusive alternative futures cannot simply be added as though both had occurred. In the present formalism, merging refers instead to multiple realized relational lineages that are jointly represented in the historical domain of a later configuration. Their coexistence can be encoded by incidence relations, shared boundary structure, or higher-order cells inside $\mathfrak{G}_{\leq\Sigma}$.

Let H_1 and H_2 be realized subcomplexes contained in the historical domain of a later configuration ω_Σ. A later operator can act on a state whose history contains both subcomplexes, as expressed in Equation 64.

$$H_1 \cup H_2 \subseteq \mathfrak{G}_{\leq\Sigma}. \tag{64}$$

Equation 64 is deliberately weaker than defining a binary operator on two ontological states. The state type remains $\Omega \rightharpoonup \Omega$: the later transformation acts on one configuration whose represented history already contains several lineages. This choice preserves the typing discipline established in Sections 5 and 10.

A multi-party contract, collectively produced artifact, or conversion involving several prior claims can therefore be modeled by allowing the relevant historical domain to contain several incoming lineages that meet in later events or faces. The model can retain their distinct provenance even when a later coarse-graining presents the result as one converted value.

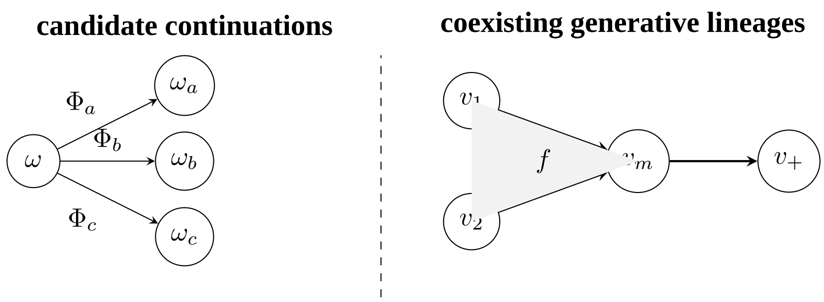


Figure 11: Two distinct structures relevant to conversion histories. The left panel shows alternative admissible continuations from one configuration. The right panel shows multiple generative lineages represented inside one realized higher-order history and participating jointly in a later event. Branching alternatives and higher-order co-generation therefore require different interpretations.

11.6 Higher-Order Conversion Processes

Some conversions depend on relations among several entities that are jointly constitutive of the process. Pairwise edges can record bilateral connections, while a face or another higher-order cell can represent that a set of relations participates in one conversion-relevant process. This structure is useful when authorization, production, settlement, or recognition depends on a collective configuration.

Let $f_C \in F$ be a face designated as conversion-relevant under the local labeling scheme. Its boundary contains the relations jointly implicated in the higher-order process. Equation 65 records the structural requirement.

$$\partial f_C \subseteq E(U_C). \tag{65}$$

Equation 65 does not assign a universal semantic meaning to faces. In one model a face may represent a jointly authorized transaction; in another it may represent a production process, institutional settlement, or a higher-order dependency among several relations. The application must specify the labeling semantics and explain why a higher-order cell improves on a pairwise graph representation.

Remark 11.3 (Higher-order representation)**.** A 2-complex is one candidate representation of higher-order conversion histories. Hypergraphs, simplicial complexes, temporal networks, event structures, or categorical constructions may be preferable when their mathematical properties better match the application.

Remark 11.3 maintains the representational pluralism established in Sections 3 and 7.

11.7 Co-Generated Value

Definition 2.5 introduced co-generation conceptually. The history formalism now permits a more explicit representation: a value manifestation can depend on several realized relational lineages or higher-order cells within the conversion-relevant historical domain. Let A be observed from configuration ω_Σ through a context-dependent map $\pi_A^{(c)}$. Equation 66 states the readout.

$$A^{(c)} = \pi_A^{(c)}(\omega_\Sigma). \tag{66}$$

Equation 66 does not assign A to one point-like creator. The observation may depend on a relationally identified subcomplex $\mathcal{E}_A^{(c,\lambda)}$ produced by the entity-identification machinery in Section 9. When several historical lineages materially contribute to the manifestation, the represented generative support can be written schematically as a family of subcomplexes $H_1, \ldots, H_m$ contained in the relevant historical domain.

The formalism therefore separates three questions: which relational histories participated in generating the manifestation, which of those histories are epistemically identified by the model, and which identified entities receive attribution. These questions can produce different structures even when they concern the same observed value A.

11.8 Attribution within Conversion Histories

Attribution is represented after entity identification because an attribution requires an identified bearer, contributor, owner, recipient, or responsible party. Let $\mathfrak{E}_C$ denote the family of entities identified within the conversion-relevant domain. An attribution map under context c is introduced in Equation 67.

$$\alpha^{(c)} : \mathfrak{E}_C \times \mathcal{V}_C \longrightarrow \mathcal{L}_C. \tag{67}$$

In Equation 67, $\mathcal{V}_C$ denotes the value manifestations represented in the local model and $\mathcal{L}_C$ denotes the space of attribution labels or statuses. Such labels can represent contribution, ownership, authorship, entitlement, burden, responsibility, or another application-specific status. The map is intentionally general because these statuses obey different substantive rules.

Attribution can itself become historically consequential. If an institutional attribution changes rights, standing, payment, recognition, or future operator availability, the attribution procedure must be coupled to a generative operator of the type developed in Section 10. The map $\alpha^{(c)}$ then records the attribution readout or assignment, while a corresponding state-changing operator records its performative effect. This preserves the separation between epistemic/institutional labeling and ontological transformation.

11.9 Observed Conversion as Coarse-Graining

The ordinary representation $A \to B$ can now be treated as a coarse-grained observation of a conversion history. Let $\mathscr{H}_C$ denote the class of conversion histories admitted by the model, and let $Y_C^{(c)}$ be a context-specific conversion readout space. Definition 11.4 introduces the corresponding map. Table 8 summarizes the representational levels that the readout can compress.

Definition 11.4 (Conversion readout)**.** A *conversion readout* is a context-dependent map of the form stated in Equation 68 that extracts a restricted observable representation from a conversion history.

$$\pi_C^{(c)} : \mathscr{H}_C \longrightarrow Y_C^{(c)}. \tag{68}$$

A readout space can contain a pair of manifested values, a conversion ratio, a settlement record, an accounting entry, or another context-specific representation. For a simple value-pair readout, Equation 69 gives the schematic form.

$$\pi_C^{(c)}(\Gamma_C) = \left(A^{(c)}, B^{(c)}\right). \tag{69}$$

Equation 69 explains the limited role of the familiar notation $A \to B$: it can summarize a conversion interface while suppressing intermediate configurations, co-generative histories, attribution, nomos, material dependencies, and procedural order. This suppression is sometimes analytically useful. The formalism requires only that the loss of structure be recognized when questions depend on what the coarse-graining removes.

Proposition 11.5 (History-sensitive representation)**.** *Within the present formalism, two conversion histories can share the same conversion readout under $\pi_C^{(c)}$ while differing in their traces, intermediate relational configurations, operator availability, attribution structures, or higher-order historical organization.*

Table 8: Representational levels in a heterogeneous conversion history.

Level	Principal object	Analytical role
Relational configuration	$\omega_k \in \Omega$	Records modeled history, active relational structure, and nomos at an intermediate stage
Generative operation	$\Phi_k : \Omega \rightharpoonup \Omega$	Transforms one admissible configuration into another
Conversion history	$\Gamma_C = \Phi_n \circ \cdots \circ \Phi_1$	Represents the composite transformation and its intermediate process trace
Higher-order history	subcomplexes, faces, and lineages	Preserves co-generation and collective relational organization when pairwise representation is too thin
Attribution layer	$\alpha^{(c)}$	Assigns application-specific statuses to identified relational entities
Conversion readout	$\pi_C^{(c)}(\Gamma_C)$	Produces a restricted observable summary such as a value pair, ratio, or settlement record

Proposition 11.5 follows directly from the many-to-one character permitted for $\pi_C^{(c)}$. It does not assert that every practically relevant conversion requires reconstruction of its full represented history. It identifies the formal possibility that endpoint-equivalent readouts can conceal procedurally or normatively relevant differences.

Table 8 makes explicit that the formalism does not require every application to reconstruct every layer at maximal resolution. The appropriate representation depends on the analytical question. The next section focuses on one consequence of retaining the intermediate history: the order of conversion-relevant operations can change the ontology on which subsequent operations act, producing compositional noncommutativity and partial asymmetry between procedural orderings.

12 Order Sensitivity and Noncommutative Conversion

This section isolates the consequences of procedural order within the conversion formalism. Its role is narrower than the general operator background of Section 5: the objective here is to determine when two conversion-relevant operations generate different histories because they are applied in different orders. The analysis proceeds through sequential dependence, compositional noncommutativity, state-dependent subsequent operations, partial composability, three representative ordering problems, and historical irreversibility. The resulting structure is descriptive. It establishes that ordering can matter within the model; it does not by itself determine which ordering is normatively preferable.

Figure 12 gives the basic architecture. Both paths begin from the same configuration ω_0, but the first operation changes the intermediate configuration on which the second operation acts. The two paths can therefore terminate in different configurations even when the operation types carry the same descriptive names.

12.1 Sequential Dependence

A conversion history is sequentially dependent when the output of one operation supplies part of the state on which a later operation is defined or interpreted. Let Φ_i and Φ_j be conversion-relevant partial transformations. Starting from ω_0, the first ordering produces the intermediate and final configurations stated in Equation 70.

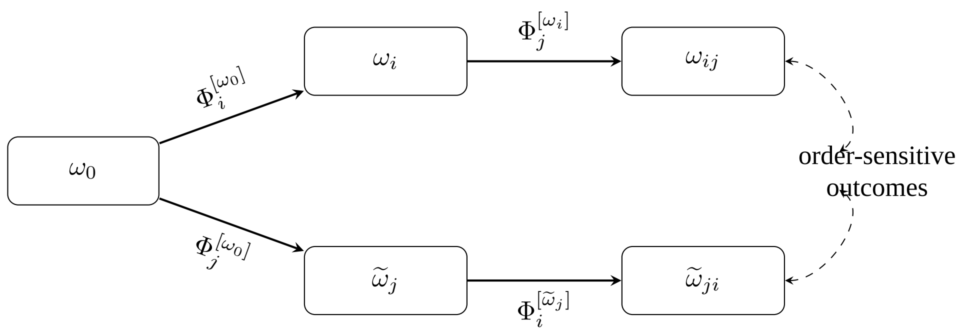


Figure 12: Two orderings of conversion-relevant operations. Each first operation changes the intermediate relational configuration, so the second operation is applied relative to a different state. The diagram represents ontology-sensitive ordering and does not assume that the two second-stage operators have identical parameterization.

$$\begin{aligned}\omega_i &= \Phi_i^{[\omega_0]}(\omega_0), \\ \omega_{ij} &= \Phi_j^{[\omega_i]}(\omega_i).\end{aligned} \tag{70}$$

The reversed ordering produces the configurations in Equation 71.

$$\begin{aligned}\widetilde{\omega}_j &= \Phi_j^{[\omega_0]}(\omega_0), \\ \widetilde{\omega}_{ji} &= \Phi_i^{[\widetilde{\omega}_j]}(\widetilde{\omega}_j).\end{aligned} \tag{71}$$

Equations 70 and 71 make the state dependence explicit. The superscripts identify the configuration relative to which the concrete transformation is selected or parameterized. The notation allows an operation type to persist across histories while its realized form changes with the ontology on which it acts.

Definition 12.1 (Sequential dependence)**.** A pair of operation types (Φ_i, Φ_j) exhibits *sequential dependence* at ω_0 when the realized second-stage transformation depends materially on the intermediate configuration generated by the first-stage transformation.

Definition 12.1 has a broader scope than noncommutativity. Sequential dependence can occur even when two particular orderings happen to produce the same final configuration. It identifies the mechanism through which order can become consequential: the second operation is evaluated against an ontology already modified by the first.

12.2 Noncommutative Operator Composition

Section 5.6 introduced compositional noncommutativity for partial transformations on Ω. The conversion setting adds state-dependent parameterization. When both orderings in Figure 12 are defined at ω_0, order sensitivity is recorded by Equation 72.

$$\Phi_j^{[\omega_i]} \circ \Phi_i^{[\omega_0]}(\omega_0) \neq \Phi_i^{[\widetilde{\omega}_j]} \circ \Phi_j^{[\omega_0]}(\omega_0). \tag{72}$$

Equation 72 is the principal noncommutative relation used in this section. It compares two realized conversion histories and requires only composition of partial transformations. No subtraction of operators is used because Ω has not been assigned the linear structure required for a conventional commutator.

Definition 12.2 (Ontology-sensitive noncommutativity)**.** Two conversion-relevant operation types are *ontology-sensitive noncommutative* at ω_0 when both orderings are defined, the first operation in each ordering generates a potentially different intermediate configuration, and the resulting final configurations differ as in Equation 72.

Definition 12.2 gives noncommutativity a process interpretation. The difference between the two histories can arise from changed labels, active relations, institutional status, admissible operator families, or higher-order cells added during the first transformation. A final coarse readout may hide some of these differences. For this reason, noncommutativity can occur at the configuration level even when a restricted observable reports the same apparent amount of value transferred.

Remark 12.3 (Algebraic representation)**.** If a later construction represents the relevant transformations by linear operators $\rho(\Phi_i)$ on a specified function, observable, or history space, the conventional commutator $[\rho(\Phi_i), \rho(\Phi_j)]$ can be studied. The present section requires only the compositional relation in Equation 72.

Remark 12.3 preserves the distinction between an algebra-inspired conversion language and a fully specified operator algebra. The latter remains an optional extension discussed in the later section on formal scope and open problems.

12.3 State-Dependent Subsequent Operations

Order sensitivity can be stronger than a difference between two fixed transformations. An earlier operation can change the family of operations that is available afterward. Let $\mathfrak{O}^{\mathrm{adm}}(\omega)$ be the admissible family introduced in Equation 57. After Φ_i acts, the admissible family becomes $\mathfrak{O}^{\mathrm{adm}}(\omega_i)$. The corresponding change is stated in Equation 73.

$$\mathfrak{O}^{\mathrm{adm}}(\omega_0) \longmapsto \mathfrak{O}^{\mathrm{adm}}(\omega_i). \tag{73}$$

Equation 73 means that the second stage can differ in two ways. Its input configuration has changed, and the set from which the later operation is selected can also have changed. A recognition operation may create standing; a contract may create enforcement procedures; a payment may remove one claim while creating a receipt or discharge relation; a classification may activate a regulatory category. The formalism places these changes within the same state-dependent operator architecture.

Proposition 12.4 (Order-induced operational change)**.** *Let Φ_i be defined at ω_0. If applying Φ_i changes the admissible operator family, then placing Φ_i before a later conversion-relevant operation can change that later operation's availability or realized parameterization.*

Proposition 12.4 is formal rather than empirical. Whether the admissible family actually changes in a concrete wage, currency, environmental, or legal conversion requires application-specific modeling.

12.4 Partial Composability and Procedural Accessibility

A second source of order sensitivity occurs when only one ordering is defined. This case is represented by partial composability rather than by Equation 72. Let Φ_i create a prerequisite for Φ_j. The accessibility relation is stated in Equation 74.

$$\omega_0 \in \mathrm{Dom}(\Phi_j \circ \Phi_i), \qquad \omega_0 \notin \mathrm{Dom}(\Phi_i \circ \Phi_j). \tag{74}$$

Equation 74 describes a one-way procedural sequence. The difference from noncommutativity is categorical: there are not two defined final configurations to compare. One path is available and the reverse path is absent from the modeled procedure.

Figure 13 illustrates this stronger form of order dependence.

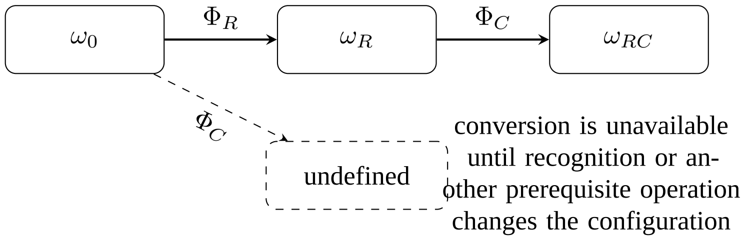


Figure 13: One-way procedural accessibility. The upper sequence is defined because the first operation creates conditions required by the second. The dashed lower path represents an ordering whose first step is undefined in the initial configuration.

This structure is relevant when procedural standing, authorization, ownership, classification, or evidentiary recognition is a prerequisite for conversion. It also prevents every ordering problem from being described loosely as noncommutativity. The formal distinction between unequal outputs and undefined reverse composition is retained throughout the manuscript.

12.5 Recognition and Conversion Order

Recognition provides a simple conversion-specific illustration because Section 9.7 already distinguishes entity identification from state-changing institutional recognition. Let Φ_R denote a recognition transformation and Φ_C a conversion transformation. When both orderings are available, the two histories are stated in Equation 75.

$$
\begin{aligned}
\Gamma_{RC} &= \Phi_C^{[\omega_R]} \circ \Phi_R^{[\omega_0]}, \\
\Gamma_{CR} &= \Phi_R^{[\omega_C]} \circ \Phi_C^{[\omega_0]}.
\end{aligned}
\tag{75}
$$

Equation 75 does not assign normative priority to either sequence. It identifies two different histories. Recognition before conversion can alter standing, labels, admissible contracts, evidentiary status, bargaining relations, or institutional categories before the conversion is executed. Recognition after conversion acts on a configuration in which ownership, obligations, prices, or other relational structures may already have changed.

Example 12.5 (Delayed recognition)**.** Suppose a contribution is converted into a monetary claim before an institution recognizes a relational feature relevant to its valuation. The subsequent recognition event acts on a configuration that already contains the completed conversion and its consequences. A model in which recognition precedes conversion therefore traces a different history even if both histories later contain the same recognition label and the same nominal monetary amount.

Example 12.5 illustrates why endpoint comparison can be insufficient. The two histories can differ in opportunity, information, bargaining structure, or revision possibilities even when a coarse terminal readout appears similar.

12.6 Valuation and Conversion Order

Valuation introduces another order-sensitive pair. Let Φ_V denote a state-changing valuation process when public valuation, pricing, appraisal, rating, or another evaluative act has modeled consequences for the relational configuration. The relevant histories are stated in Equation 76.

$$
\begin{aligned}
\Gamma_{VC} &= \Phi_C^{[\omega_V]} \circ \Phi_V^{[\omega_0]}, \\
\Gamma_{CV} &= \Phi_V^{[\omega_C]} \circ \Phi_C^{[\omega_0]}.
\end{aligned}
\tag{76}
$$

The distinction in Equation 76 is particularly important when valuation is performative. A valuation can alter expectations, create reference points, affect bargaining positions, activate institutional thresholds, or change the information available to the participants. Conversion before valuation can in turn alter the object, rights, or relational context subsequently evaluated.

The formalism also accommodates a passive valuation readout. If valuation is represented only by an observation map $\pi_V : \Omega \to X_V$ and produces no state change, then it should not be inserted into the conversion history as a generative operator. The history may still record when the readout was obtained, but noncommutativity at the ontological level requires a transformation acting on Ω.

Caution 12.6 (Evaluation and valuation)**.** Economic or institutional valuation and normative evaluation are distinct operations in the present framework. A valuation can assign a price, rating, category, or other conversion-relevant representation. Normative evaluation is introduced later as a theory-relative assessment of relational histories and should not be inferred from the valuation operation alone.

Caution 12.6 prevents order-sensitive pricing or appraisal from being treated as a complete account of justice.

12.7 Settlement and Revision Order

Settlement and revision provide a third ordering problem because settlement can terminate active obligations or close procedural pathways. Let Φ_S denote settlement and Φ_Q a contestation or revision transformation. When both orderings are defined, Equation 77 distinguishes revision before settlement from revision after settlement.

$$\begin{aligned}\Gamma_{QS} &= \Phi_S^{[\omega_Q]} \circ \Phi_Q^{[\omega_0]},\\ \Gamma_{SQ} &= \Phi_Q^{[\omega_S]} \circ \Phi_S^{[\omega_0]}.\end{aligned} \tag{77}$$

Settlement can change the active boundary $\mathcal{B}_\Sigma$ by terminating obligations represented as active before settlement. Revision after settlement therefore need not act on the same claim structure that existed before settlement. In some modeled institutions, the revision operator can remain available through appeal, restitution, reopening, or retrospective adjustment. In others, it may become undefined or materially restricted. Equation 74 accommodates the latter case.

Table 9 summarizes the three representative operator pairs without treating them as an exhaustive taxonomy.

Table 9 highlights the analytical role of the intermediate configuration. The formal concern is the transformation of the relational conditions supporting the second operation, rather than a general assertion that one temporal ordering is preferable.

12.8 Historical Dependence and Irreversibility

Order sensitivity becomes especially consequential when earlier operations generate changes that cannot be undone within the modeled operator family. The present framework uses the Historical Persistence Convention from Section 4.7: realized history is retained for analytical purposes while later operations can alter active relations and future continuations. Irreversibility therefore refers to the absence of an admissible transformation that restores a prior configuration at the chosen level of representation.

Let Φ take ω to ω'. Reversibility relative to an admissible family is defined in Equation 78.

$$\Phi \text{ is reversible at } \omega \quad \Longleftrightarrow \quad \exists\, \Psi \in \mathfrak{O}^{\mathrm{adm}}(\omega') \text{ such that } \Psi(\Phi(\omega)) = \omega. \tag{78}$$

Equation 78 defines reversibility relative to the represented configuration and admissible operator family. It does not assert microscopic physical reversibility or irreversibility. A conversion can be irreversible at one analytical granularity while admitting partial rectification at another.

Table 9: Representative order-sensitive operator pairs in heterogeneous conversion.

Operator pair	**Intermediate structure affected by the first operation**	**Potential source of order sensitivity**
Recognition / conversion	Standing, institutional legibility, labels, admissible claims	Conversion can occur under different recognized entity structures or may require prior recognition
Valuation / conversion	Reference values, informational conditions, bargaining context, thresholds	Conversion terms and later valuation can depend on which operation first changes the relational configuration
Settlement / revision	Active obligations, claim status, procedural accessibility	Settlement can terminate or transform structures on which later contestation or revision would act

Definition 12.7 (Order-sensitive irreversibility)**.** A pair (Φ_i, Φ_j) exhibits *order-sensitive irreversibility* at ω_0 when at least one ordering generates a configuration from which the other ordering's relevant prior relational structure cannot be restored by any represented admissible transformation.

Definition 12.7 gives later procedural and normative analysis a precise object. The issue is not merely that two orderings differ; one sequence can close possibilities that remain available in the other. This can occur through termination of active relations, transfer of rights, destruction or transformation of material objects, depletion of resources, expiration of procedural standing, or other modeled changes.

The concepts introduced in this section form a simple hierarchy of scope. Sequential dependence is the broad mechanism through which a first operation changes the conditions of a later one. Compositional noncommutativity applies when both orderings are defined and produce different outputs. Partial composability records the stronger accessibility case in which one ordering is defined and the reverse ordering is unavailable. These categories should therefore remain analytically distinct even when they occur within the same conversion system.

Caution 12.8 (Normative scope)**.** Order sensitivity, noncommutativity, partial composability, and irreversibility are structural properties of represented histories. They do not by themselves determine whether a conversion is just, legitimate, efficient, or desirable. Normative evaluation requires additional theory-specific commitments developed in the subsequent sections on conversion nomos, normative evaluation, and epistemic justice.

Caution 12.8 preserves the limited role of the present section. The formalism identifies where procedural order can alter the ontology, operational possibilities, and historical trace of conversion. The following section turns from ordering to conversion nomos and examines how admissibility structures can themselves emerge, stabilize, and change within relational histories.

13 Conversion Nomos and Normogenesis

This section develops the endogenous normative component of the conversion formalism. Its role is to specify how a conversion-relevant nomos can be represented as part of a relational configuration, how it structures the admissibility and interpretation of conversion operations, and how that nomos can itself stabilize, become institutionalized, be contested, and change. The discussion is formal and descriptive. It does not infer normative legitimacy from historical emergence, nor does it supply a fixed substantive criterion of justice. The analysis proceeds from nomos extraction and operator admissibility to stabilization, institutionalization, interface power, contestation, revision, and relational governance of normogenesis.

Figure 14 summarizes the architecture. A nomos structures the currently admissible conversion operations. Realized operations extend relational history and can reinforce, weaken, or modify the institutional support of that nomos. Contestation and revision occur within the same history-bearing configuration, so a later nomos can differ from its predecessor without being treated as an external rule imposed from outside the model.

13.1 Nomos as an Emergent Relational Structure

Section 8.8 introduced $\mathfrak{N}_\Sigma$ as the conversion-relevant nomos contained in the relational configuration ω_Σ. The present section makes its historical status explicit. Let $\mathscr{N}$ denote the class of nomos structures admitted by the model. The nomos component can be extracted from a configuration by the map stated in Equation 79.

$$\mathrm{nom} : \Omega \longrightarrow \mathscr{N}, \qquad \mathrm{nom}(\omega_\Sigma) = \mathfrak{N}_\Sigma. \tag{79}$$

Equation 79 is a typed readout of the third component of the configuration defined in Equation 29. It does not imply that nomos is epistemically transparent or reducible to a single written rule. Depending on the application, $\mathfrak{N}_\Sigma$ can represent a structured combination of legal rules, recognized conventions, procedural standards, institutional permissions, default expectations, accounting categories, settlement practices, and other conversion-relevant normative relations.

A conversion history Γ_C therefore carries a corresponding nomos trace. Let its configuration trace be the sequence defined in Equation 61. The associated nomos trace is stated in Equation 80.

$$\mathrm{Tr}_{\mathfrak{N}}(\Gamma_C; \omega_0) = \big(\mathrm{nom}(\omega_0), \ldots, \mathrm{nom}(\omega_n)\big). \tag{80}$$

Equation 80 permits a conversion history to be studied together with changes in its effective normative structure. A stable nomos appears as persistence across part of the trace; a revision appears as a change in one or more conversion-relevant normative relations. The formalism therefore treats normogenesis as history-bearing evolution within Ω rather than as the assignment of a timeless external parameter.

Definition 13.1 (Normogenesis)**.** Within the present formalism, *normogenesis* is the modeled historical process through which the nomos component of relational configurations is generated, reproduced, stabilized, contested, or revised along admissible relational histories.

Definition 13.1 is intentionally broad. The definition identifies a formal locus for normative evolution while leaving the empirical mechanisms and substantive legitimacy of particular nomoi to application-specific analysis.

13.2 Nomos and Admissible Operations

The most direct operational role of nomos is to structure the set of transformations that can count as admissible at a configuration. Equation 35 represented this role by the subset $\mathfrak{O}^{\mathrm{adm}}(\omega_\Sigma)$. For a specified candidate transformation Φ, the same relation can be written through the admissibility indicator in Equation 81.

$$\mathrm{Adm}_{\mathfrak{N}_\Sigma}(\Phi \mid \omega_\Sigma) = \begin{cases} 1, & \Phi \in \mathfrak{O}^{\mathrm{adm}}(\omega_\Sigma), \\ 0, & \Phi \notin \mathfrak{O}^{\mathrm{adm}}(\omega_\Sigma). \end{cases} \tag{81}$$

Equation 81 represents only a minimal binary layer of admissibility. Richer applications can distinguish permissions, obligations, defaults, sanctions, procedural prerequisites, degrees of institutional support, or context-dependent interpretations. The binary indicator is useful because it makes one structural consequence visible without presupposing a complete deontic logic.

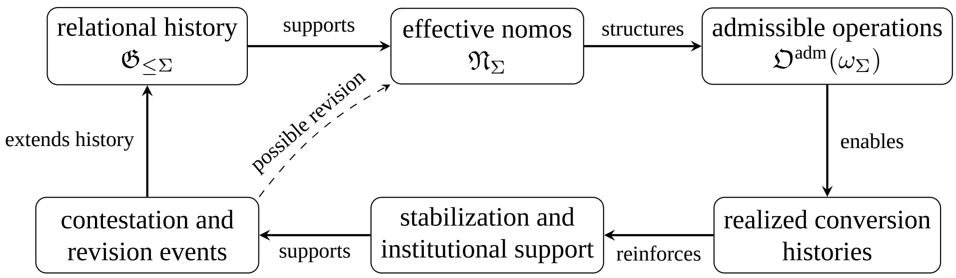


Figure 14: Conversion nomos and normogenesis within the relational-history formalism. The diagram separates the effective nomos from the histories and institutions that support it while allowing realized operations, stabilization, contestation, and revision to alter later normative configurations.

Nomos can also affect the meaning and consequence of a transformation that remains mathematically available. A payment performed through the same technical transfer mechanism can have different institutional significance depending on whether it is recognized as settlement, deposit, gift, collateral, tax payment, or another legally and socially structured form. The formalism therefore permits $\mathfrak{N}_\Sigma$ to affect both operator admissibility and the interpretation of the resulting configuration.

Remark 13.2 (Admissibility and legitimacy)**.** The condition $\mathrm{Adm}_{\mathfrak{N}_\Sigma}(\Phi \mid \omega_\Sigma) = 1$ records admissibility relative to the represented nomos. It does not establish that the operation is normatively justified under an external or theory-relative account of justice.

Remark 13.2 preserves the separation between descriptive normogenesis and normative evaluation. A historically effective rule can be modeled accurately even when later sections subject it to criticism under a particular evaluative theory.

13.3 Repeated Practice and Stabilization

Nomos can become stable without remaining textually or institutionally identical at every slice. The relevant feature may be persistence of an operational pattern across repeated conversion situations. Let $\mathcal{Q} = \{\Phi_1, \ldots, \Phi_m\}$ be a finite family of conversion-relevant test operations. The admissibility profile of $\mathfrak{N}_\Sigma$ relative to $\mathcal{Q}$ is defined in Equation 82.

$$\mathbf{a}_{\mathcal{Q}}(\omega_\Sigma) = \big(\mathrm{Adm}_{\mathfrak{N}_\Sigma}(\Phi_1 \mid \omega_\Sigma), \ldots, \mathrm{Adm}_{\mathfrak{N}_\Sigma}(\Phi_m \mid \omega_\Sigma)\big). \tag{82}$$

Equation 82 provides one deliberately coarse way to identify operational persistence. If repeated histories preserve the same profile for the selected operator family, the conversion-relevant nomos is operationally stable relative to that family even when other aspects of the configuration continue to evolve.

Definition 13.3 (Operational stabilization of nomos)**.** A nomos is *operationally stabilized relative to* $\mathcal{Q}$ over a sequence of relational slices when the admissibility profile in Equation 82 remains invariant, or remains within a specified application-dependent tolerance, across those slices.

Definition 13.3 does not identify repetition with justification. Repeated practice may reproduce a rule because participants endorse it, because institutions support it, because alternatives are costly, because one actor controls a relevant interface, or because material conditions make alternative procedures unavailable. The formalism records stabilization while leaving its causal interpretation open.

Repeated conversion can also be self-reinforcing. Once a settlement method, unit of account, classification, or valuation procedure is repeatedly recognized, later actors can coordinate around the resulting expectations and infrastructures. In the present framework, such reinforcement appears as a history in which realized operators extend $\mathfrak{G}_{\leq\Sigma}$ in ways that preserve or expand the future domain of similar operations. This mechanism can produce path dependence without requiring that the stabilized practice be optimal or uniquely determined.

13.4 Institutionalization

Institutionalization gives stabilized nomos a more persistent material, symbolic, and procedural support. A conversion norm can become encoded in contracts, accounting systems, technical standards, legal categories, professional routines, organizational roles, databases, payment infrastructures, or other history-bearing structures. These supports are represented inside $\mathfrak{G}_{\leq\Sigma}$ and can remain active in $\mathcal{B}_{\Sigma}$.

For analytical purposes, let $\mathrm{Supp}(\mathfrak{N}_{\Sigma}) \subseteq \mathfrak{G}_{\leq\Sigma}$ denote a selected support subcomplex containing the modeled events, relations, and higher-order structures that materially or institutionally sustain the effective nomos. The relation is stated in Equation 83.

$$\mathrm{Supp}(\mathfrak{N}_{\Sigma}) \subseteq \mathfrak{G}_{\leq\Sigma}. \tag{83}$$

Equation 83 is an analytical selection rather than a claim that every nomos has a uniquely identifiable minimal support. Different applications can select different levels of historical and institutional granularity. The construction is useful because it prevents nomos from appearing as an abstract rule detached from the infrastructures and relations through which the rule remains effective.

Institutionalization can alter operator availability in a durable way. A rule encoded simultaneously in legal procedure, technical infrastructure, organizational routines, and accounting practice may persist even when particular participants change. The 2-complex representation makes this persistence compatible with historical replacement of individual actors: the nomos can remain supported by a changing collection of events, edges, and faces rather than by one permanent central node.

13.5 Power over Conversion Interfaces

Conversion procedures are mediated by interfaces at which values are identified, classified, made commensurable, transferred, settled, or revised. Power can therefore enter the formalism through differential access to transformations that alter these interfaces. The present paper does not reduce power to a scalar quantity. It represents several distinct forms of conversion-relevant influence through access to state-changing operators.

Table 10 summarizes principal interface locations that can be represented without presupposing a single theory of power.

Table 10 treats power as relationally distributed across several formal loci. A participant can have substantial influence over one interface and little influence over another. A technically decentralized conversion procedure can also retain concentrated control over classification, infrastructure, or revision. Conversely, a central coordinating node can possess substantial operational capacity while exercising limited authority over the substantive nomos. These distinctions become important when the formalism is later used for political-economic or justice-oriented analysis.

Remark 13.4 (Power and normogenesis)**.** A norm can be relationally emergent while its emergence remains strongly conditioned by unequal access to conversion interfaces. Emergence therefore describes a mode of historical production; it does not imply symmetry among the relations participating in that production.

Remark 13.4 is important for interpreting bottom-up normogenesis. A decentralized appearance at the level of final rules does not, by itself, establish that the underlying processes of classification, participation, settlement, or revision were equally accessible.

13.6 Contestation

Contestation introduces events through which an existing conversion rule, interpretation, classification, or procedure becomes an explicit object of challenge. Within the present formalism, contestation is represented by an ordinary generative transformation rather than by an external critique. Let Φ_{con} denote a contestation operator whose domain contains configurations in which the relevant standing, channel, or material possibility for contestation exists. Its action is stated in Equation 84.

Table 10: Illustrative locations of conversion-interface influence in the formalism.

Interface location	**Formal object affected**	**Illustrative consequence**
Entity identification	$\mathcal{I}_a^{(c,\lambda)}$ or its context parameters	Changes which relational structures become visible as the relevant entity
Classification and valuation	observation maps and conversion-relevant operator parameters	Changes the categories or scales through which heterogeneous values become comparable
Operational admissibility	$\mathfrak{O}^{\mathrm{adm}}(\omega)$	Enables, conditions, or excludes particular conversion procedures
Settlement infrastructure	settlement-related generative operators	Changes which transfers acquire recognized institutional consequences
Contestation and revision	domains of contestation or revision operators	Changes who can reopen, challenge, or modify an established conversion relation
Nomos support	$\mathrm{Supp}(\mathfrak{N}_\Sigma)$	Reinforces or weakens the structures through which a conversion rule remains effective

$$\Phi_{\text{con}} : \mathrm{Dom}(\Phi_{\text{con}}) \subseteq \Omega \longrightarrow \Omega. \tag{84}$$

Equation 84 preserves the common state type used throughout the paper. The operator can add a challenge event, create new evidentiary or deliberative relations, alter standing, activate a review procedure, or modify the active support of a nomos. Contestation therefore extends relational history even when the effective nomos remains unchanged.

The domain of Φ_{con} is analytically significant. A conversion system can provide formal rules for objection while leaving the corresponding operator practically undefined for some identified entities because access, standing, information, resources, or procedural channels are absent. This distinction connects contestation to the partial-composability analysis in Section 12.4.

13.7 Normative Revision

Revision occurs when contestation, institutional change, material transformation, repeated practice, or another generative history changes the effective conversion nomos. The formalism identifies revision through change in the nomos trace and, when useful, through change in an operational admissibility profile.

Let ω_Σ and $\omega_{\Sigma'}$ be configurations before and after a revision-relevant history. Relative to a finite test family $\mathcal{Q}$, an operational revision is present when Equation 85 holds.

$$\mathbf{a}_{\mathcal{Q}}(\omega_\Sigma) \neq \mathbf{a}_{\mathcal{Q}}(\omega_{\Sigma'}). \tag{85}$$

Equation 85 detects revision only at the selected operational resolution. A nomos can change semantically while leaving the chosen test family unaffected, and an admissibility profile can change because other components of the configuration changed. A full application should therefore examine the nomos trace, relevant institutional support, and the conversion domain together rather than interpreting Equation 85 as a complete criterion of normative change.

Revision can also be prospective. A new norm can create operator families that were previously unavailable, alter prerequisites for conversion, or establish new routes for recognition and settlement. The resulting change in $\mathfrak{O}^{\mathrm{adm}}(\omega)$ makes normative evolution part of the generative dynamics of the system rather than a commentary added after conversion has occurred.

13.8 Relational Governance of Normogenesis

The formalism can represent different governance relations to normogenesis without selecting one as universally preferable. A direct governance transformation can specify a target nomos and update the configuration so that the target becomes effective. An indirect relational transformation can instead modify participation, information, infrastructure, operator access, or contestability and allow a later nomos to emerge through subsequent histories.

The indirect pattern can be represented by the two-stage history in Equation 86.

$$\omega_0 \xrightarrow{\Phi_G} \widetilde{\omega}_0 \xrightarrow{\Gamma_{\mathfrak{N}}} \omega_1, \qquad \mathrm{nom}(\omega_1) = \mathfrak{N}_1. \tag{86}$$

In Equation 86, Φ_G changes relational conditions relevant to norm formation, while $\Gamma_{\mathfrak{N}}$ denotes the subsequent history through which an effective nomos is generated or stabilized. The final $\mathfrak{N}_1$ is not fixed by the notation itself. This representation can therefore model governance that acts on the conditions of normogenesis as well as governance that acts directly on substantive rules.

The distinction is analytically useful for multi-level conversion systems. A central coordinating node may maintain interoperability, provide common information, preserve review channels, or prevent an interface from becoming unavailable while leaving substantial portions of valuation or conversion practice to distributed interactions. Another system may centralize both infrastructure and substantive norm setting. The present formalism represents these architectures through different histories and operator-access structures; their comparative desirability remains a separate normative question.

The nomos layer developed in this section completes the endogenous conversion architecture required for the subsequent normative analysis. Nomos is represented as a history-supported component of relational configurations, its operational effects appear in admissible operator families, stabilization and institutionalization can be traced without being equated with legitimacy, interface power can shape normogenesis, and contestation or governance can generate later normative structures. The following section introduces theory-relative evaluation over these histories while preserving the distinction between an emergent nomos and a normative judgment about that nomos.

14 Normative Evaluation

This section introduces a theory-relative normative layer over the conversion histories developed in Sections 11–13. Its role is to specify how a justice theory or other normative framework can select relationally relevant entities, extract evaluative information from a conversion history, and generate a judgment that may subsequently influence relational evolution. The objective is integrative rather than substantive: the formalism supplies a common process language while leaving the content of justice to the evaluative theory being represented. The discussion therefore separates evaluative structure, entity identification, evaluation, judgment generation, judgment-induced evolution, optional history-weight deformation, and reflexive interaction with emergent nomos.

Figure 15 summarizes the architecture. A conversion history is evaluated only after a theory-relative identification of the relational structures regarded as normatively relevant. The evaluative result does not automatically alter the ontology. A subsequent judgment-generation map can produce a state-changing operator, and the application of that operator can modify later relational configurations, operator availability, and nomos.

14.1 Theory-Relative Evaluative Structures

The formalism does not contain a fixed substantive criterion that assigns justice independently of an evaluative theory. Let $\mathfrak{T}$ denote a class of evaluative theories or normative frameworks admitted for analysis, and let $T \in \mathfrak{T}$ denote one such theory. Theories can differ in the entities they identify, the information they regard as relevant, the relations they compare, and the kinds of normative conclusions they permit.

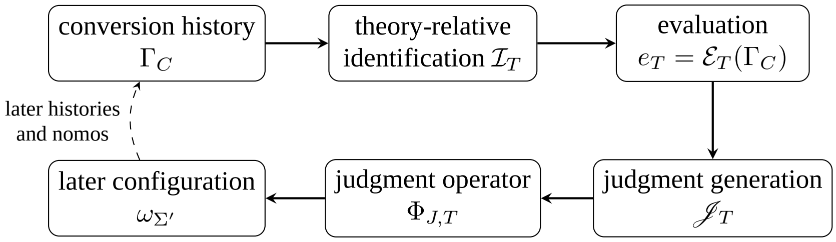


Figure 15: Theory-relative normative evaluation and subsequent judgment-induced evolution. Evaluation and ontological transformation are separated by type: $\mathcal{E}_T$ produces an evaluative object, whereas $\mathscr{J}_T$ can generate a partial endomorphism that acts on a relational configuration.

Let $\mathscr{H}_C(\Omega)$ denote the class of finite conversion histories generated by Definition 11.1. The history domain used for normative evaluation is stated in Equation 87.

$$\mathscr{H}_C(\Omega) = \{\Gamma_C : \omega_0 \rightharpoonup \omega_n \mid \Gamma_C \text{ is a finite admissible conversion history}\}\,. \tag{87}$$

Equation 87 provides a common process domain without prescribing what a theory must value. For each T, let E_T denote an evaluative space whose elements can be scalar, vectorial, ordered, set-valued, topological, logical, or otherwise structured. A theory can also provide a comparison relation $\succeq_T$ on a subset of E_T. The relation is not required to be total.

Definition 14.1 (Theory-relative evaluative structure)**.** A *theory-relative evaluative structure* has the typed components stated in Equation 88.

$$\mathbb{E}_T = \big(E_T, \succeq_T, \mathcal{C}_T\big), \tag{88}$$

where E_T is the theory's evaluative codomain, $\succeq_T$ is an optional comparison relation on evaluative objects, and $\mathcal{C}_T$ is an optional family of theory-specific constraints, requirements, or prohibitions.

Definition 14.1 permits different normative traditions to retain different logical forms. A theory that compares capabilities can use a partially ordered multidimensional space. A procedural theory can encode conditions on participation or ordering. A rights-oriented theory can emphasize constraints in $\mathcal{C}_T$ without reducing them to a single welfare score. A theory that declines to compare some heterogeneous values can leave the corresponding elements incomparable under $\succeq_T$.

Caution 14.2 (Normative pluralism and formal neutrality)**.** The common domain $\mathscr{H}_C(\Omega)$ does not make different justice theories equivalent. It supplies a shared object of analysis while preserving theory-specific entities, observables, constraints, and comparison relations.

Caution 14.2 is especially important for heterogeneous conversion, where disagreement can concern the legitimacy of commensuration itself rather than only the value assigned after commensuration.

14.2 Entity Identification within Evaluation

Normative evaluation ordinarily acts on a finite relational object rather than on the entirety of $\mathfrak{G}_{\leq\Sigma}$. Section 9 introduced entity-identification maps $\mathcal{I}_a^{(c,\lambda)}$. A normative theory can assemble one or more such identifications along a conversion history in order to construct its evaluative domain.

Let Γ_C have configuration trace $\mathrm{Tr}(\Gamma_C;\omega_0) = (\omega_0,\ldots,\omega_n)$. A theory-relative identification map can be represented by Equation 89.

$$\mathcal{I}_T : \mathscr{H}_C(\Omega) \longrightarrow \mathfrak{E}_T, \qquad \mathcal{I}_T(\Gamma_C) = \mathcal{E}_T^{\mathrm{id}}, \tag{89}$$

where $\mathfrak{E}_T$ denotes the class of theory-relevant entity configurations and $\mathcal{E}_T^{\mathrm{id}}$ denotes the identified relational-historical material supplied to evaluation. Equation 89 can incorporate several focal entities, several slices, and several historical depths. It therefore generalizes a single entity-identification map without requiring that the evaluated subject be represented as one vertex.

The identification stage is normatively consequential because a theory can only evaluate relations that enter its evaluative representation. A conversion history may contain labor, care, infrastructure, ecological contribution, legal standing, monetary claims, or historical dependencies whose visibility depends on the identification procedure. The following section develops the justice implications of this limitation in greater detail.

Remark 14.3 (Identification before evaluation)**.** Within the present architecture, a difference in normative assessment can arise either from different evaluative criteria applied to the same identified structure or from different identifications of the relational structure supplied to otherwise similar evaluative criteria.

Remark 14.3 locates one important source of disagreement without reducing it to error. Different theories can legitimately choose different evaluative domains, while systematic omission or distortion remains available for separate epistemic criticism.

14.3 Evaluation Operators

After theory-relative entity identification, the evaluative layer maps a conversion history and its identified relational material into E_T. The corresponding evaluation operator is defined in Definition 14.4.

Definition 14.4 (Evaluation operator)**.** For a theory T, an *evaluation operator* has the type stated in Equation 90.

$$\mathcal{E}_T : \mathscr{H}_C(\Omega) \longrightarrow E_T \tag{90}$$

whose value is determined from the conversion history together with the theory-relative identification $\mathcal{I}_T(\Gamma_C)$.

The evaluative output for a particular conversion history is stated in Equation 91.

$$e_T(\Gamma_C) = \mathcal{E}_T(\Gamma_C) \in E_T. \tag{91}$$

Equation 91 is intentionally general. The evaluative object e_T can be a scalar when a theory supplies a scalar criterion, a vector when several dimensions remain irreducible, a set of violated constraints, a relational profile, or another structured object. The formalism does not require a common cardinal scale across justice theories.

If $\succeq_T$ is defined for the relevant outputs, two histories can be compared according to Equation 92.

$$\Gamma_1 \succeq_T \Gamma_2 \quad \Longleftrightarrow \quad \mathcal{E}_T(\Gamma_1) \succeq_T \mathcal{E}_T(\Gamma_2). \tag{92}$$

Equation 92 expresses a theory-relative comparison rather than a universal ordering over conversion histories. Histories can remain incomparable, and a theory can reject a conversion because of a constraint in $\mathcal{C}_T$ even when no scalar comparison is supplied.

Table 11 summarizes the principal objects introduced in the normative layer and separates them from the effective nomos developed in Section 13.

14.4 Judgment as Operator Generation

Evaluation and judgment are separated because an evaluative representation need not by itself change the modeled ontology. A judgment becomes dynamically relevant when an evaluative object is taken up by a subject, institution, procedure, or other relational structure and generates an operator that can act on the current configuration.

Let $\mathrm{PEnd}(\Omega)$ denote the class of partial endomorphisms introduced in Section 5. A theory-relative judgment-generation map is stated in Equation 93.

Table 11: Principal objects in the normative-evaluation layer.

Object	**Type or domain**	**Analytical role**
$\mathfrak{N}_\Sigma$	component of ω_Σ	Effective conversion nomos represented as part of the relational configuration
$\mathcal{I}_T$	$\mathscr{H}_C(\Omega) \to \mathfrak{E}_T$	Identifies the relational-historical material regarded as evaluatively relevant by theory T
$\mathcal{E}_T$	$\mathscr{H}_C(\Omega) \to E_T$	Produces a theory-relative evaluative object without requiring a state change
$\mathscr{J}_T$	$(\omega, e_T) \mapsto \Phi_{J,T}$	Generates a judgment-induced state operator from a situated evaluation
$\Phi_{J,T}$	$\Omega \rightharpoonup \Omega$	Participates in subsequent relational evolution when the judgment is enacted

$$\mathscr{J}_T : \mathrm{Dom}(\mathscr{J}_T) \subseteq \Omega \times E_T \longrightarrow \mathrm{PEnd}(\Omega). \tag{93}$$

For a situated configuration ω and evaluative object e_T, the generated judgment operator is defined by Equation 94.

$$\Phi_{J,T}^{[\omega,e_T]} = \mathscr{J}_T(\omega, e_T), \qquad \omega \in \mathrm{Dom}\left(\Phi_{J,T}^{[\omega,e_T]}\right). \tag{94}$$

Equation 94 represents judgment as operator generation rather than as a binary label such as "good" or "bad." The substantive theory enters through e_T and through the situated map $\mathscr{J}_T$. The output operator can preserve a relation, create a claim, initiate review, redistribute attention, alter access, trigger contestation, or generate another application-specific transformation. The formalism leaves these substantive effects open.

Definition 14.5 (Judgment-generated operator)**.** A *judgment-generated operator* is a partial endomorphism of Ω produced by a theory-relative judgment-generation map and applicable to the configuration in which that judgment is enacted.

Definition 14.5 keeps the operator type consistent with the generative transformations introduced in Section 10. The judgment event can therefore enter the same conversion-history formalism as other state-changing operations.

14.5 Judgment-Induced Changes in Future Evolution

Once a judgment-generated operator is enacted, it becomes part of relational history and can change the configuration on which later operations act. The immediate update is stated in Equation 95.

$$\omega_{\Sigma'} = \Phi_{J,T}^{[\omega_\Sigma,e_T]}(\omega_\Sigma)\,. \tag{95}$$

Equation 95 has the same state type as Equation 48. A normative judgment can therefore affect the historical complex, active relational boundary, effective nomos, or a combination of these components when the application represents the corresponding effects.

The changed configuration can also change future operational availability. Combining Equation 95 with Equation 58 yields the schematic change stated in Equation 96.

$$\mathfrak{O}^{\mathrm{gen}}(\omega_\Sigma) \longmapsto \mathfrak{O}^{\mathrm{gen}}\left(\Phi_{J,T}^{[\omega_\Sigma,e_T]}(\omega_\Sigma)\right). \tag{96}$$

Equation 96 formalizes a limited but important sense in which normative judgment can direct later evolution. The judgment need not determine one future state. It can create or remove procedural possibilities, alter standing, change which transformations are institutionally available, or modify the conditions under which later conversions occur.

Remark 14.6 (Judgment and determinism)**.** A judgment-generated operator need not determine a unique long-run history. It records one state-changing contribution to a relational process whose subsequent evolution can remain branching, stochastic, contested, or underdetermined.

Remark 14.6 preserves the distinction between normative influence and complete control of the future relational trajectory.

14.6 Normative Operators and History Weights

Some applications may represent future extensions by a family of candidate histories carrying transition or history weights. This representation is optional. It is useful when normative judgment changes the relative accessibility or support of several futures without selecting a single deterministic transformation.

For a configuration ω, let $\mathrm{Ext}(\omega)$ denote a modeled class of candidate future histories and let $\mathscr{W}_\omega$ denote a space of real-valued nonnegative weight functions on that class. A baseline weight function has the type stated in Equation 97.

$$W_\omega : \mathrm{Ext}(\omega) \longrightarrow \mathbb{R}_{\geq 0}. \tag{97}$$

A judgment can be represented by a deformation operator acting on history weights. One possible type is stated in Equation 98.

$$\mathcal{D}_{J,T}^{[\omega, e_T]} : \mathscr{W}_\omega \longrightarrow \mathscr{W}_{\omega'}. \tag{98}$$

Equation 98 permits normative influence to be distributed across several candidate futures. A simple multiplicative realization is given in Equation 99.

$$W_{\omega'}^{(J,T)}(\Gamma) = \kappa_T(\Gamma \mid \omega, e_T) W_\omega(\Gamma), \qquad \kappa_T(\Gamma \mid \omega, e_T) \geq 0. \tag{99}$$

Equation 99 is an illustrative parameterization rather than a substantive theory of justice. The factor κ_T can encode support, inhibition, institutional accessibility, or another application-specific effect. The paper does not interpret W_ω as a quantum amplitude, and no probability interpretation follows unless an application supplies normalization and empirical semantics.

Caution 14.7 (History weights and physical amplitudes)**.** The history-weight representation is inspired by path- and history-based mathematical formalisms. Within the present paper, W_ω denotes a generic transition or history weight. Complex quantum amplitudes, interference, and physical quantization require additional assumptions that are outside the present formal scope.

Caution 14.7 preserves the representational restraint established in Sections 6 and 7.

14.7 Emergent Nomos and Evaluative Reflexivity

The effective nomos $\mathfrak{N}_\Sigma$ and an evaluative theory T occupy different formal roles. Nomos is represented as part of the configuration and structures currently admissible or recognized conversion practices. An evaluative theory supplies an analytical or practical criterion for assessing histories. The two can nevertheless interact when evaluative judgments are enacted within the modeled relational system.

For a fixed analytical use, T can remain external to the state description while operating on Γ_C. For an endogenous use, the articulation, institutional adoption, or contestation of T can itself be represented by events and generative operators inside $\mathfrak{G}_{\leq \Sigma}$. The feedback architecture is stated schematically in Equation 100.

$$\omega_k \xrightarrow{\mathcal{E}_T} e_{T,k} \xrightarrow{\mathscr{J}_T} \Phi_{J,T}^{[k]} \xrightarrow{\Phi_{J,T}^{[k]}} \omega_{k+1}, \qquad \mathfrak{N}_{k+1} = \mathrm{nom}(\omega_{k+1}). \tag{100}$$

Equation 100 expresses evaluative reflexivity without identifying normative emergence with normative truth. The current relational configuration conditions what can be observed and evaluated; an enacted judgment can alter later relations; and the resulting history can contribute to a later nomos. Environmental conditions, technologies, institutions, historical relations, material constraints, and unresolved contingencies can all enter this process through the configuration rather than being reduced to human–human interaction alone.

Remark 14.8 (Situated evaluation)**.** When an evaluative theory is itself modeled as historically articulated and institutionally instantiated, its judgments become part of the relational history they evaluate. The present formalism treats this reflexivity as compatible with normative criticism; historical emergence explains the location of an evaluative structure without establishing its substantive validity.

Remark 14.8 preserves the separation developed in Section 13 between genealogy and justification. A theory can be historically situated while still criticizing the nomos from which it emerged, and different evaluative theories can remain in unresolved disagreement over the same conversion history.

The normative layer developed in this section therefore adds evaluation without installing a fixed universal nomos. Theory-relative identification selects the relational structures supplied to evaluation; $\mathcal{E}_T$ produces evaluative objects in a theory-specific codomain; $\mathscr{J}_T$ can generate state-changing judgment operators; and optional history-weight deformations provide one representation of distributed normative influence over future histories. The following section uses this architecture to examine epistemic justice more closely, with particular attention to relational identification, structural distortion, and topological fidelity.

15 Epistemic Justice within the Formalism

This section develops one epistemic-justice layer that can be placed over the observation and entity-identification architecture introduced in Section 9. Its role is to make explicit how a conversion procedure can depend on which relational structures become visible, how those structures are represented, and which historical relations remain available to subsequent evaluation. The objective is not to define epistemic justice universally. The section first relates the formalism to established concerns about epistemic injustice, then introduces representational discrepancy, historical visibility, structural distortion, topological fidelity, and persistence across relational scales. The final subsection formulates a topological epistemic-justice criterion as one illustrative theory-relative construction rather than as a fixed commitment of the broader framework.

The term *epistemic injustice* is used in contemporary philosophy for wrongs connected to persons in their capacities as knowers. Fricker's influential account distinguishes testimonial injustice from hermeneutical injustice and connects both to relations of social power and epistemic participation [5]. The present formalism does not attempt to reduce these established categories to topology or entity identification. It addresses a narrower structural question: when a conversion process depends on a finite relational representation, what can be said about systematic loss or deformation of the relational structures that the chosen evaluative theory regards as epistemically relevant?

Figure 16 summarizes the basic architecture. A relational reference domain and an identified entity can differ even when both belong to the same modeled history. An epistemic-justice theory specifies which features of that difference matter and how they should be evaluated.

15.1 Relational Recognition

Epistemic evaluation enters the conversion framework after a relational structure has been selected for observation or identification. Section 9.2 represented this step through $\mathcal{I}_a^{(c,\lambda)}$. The resulting entity $\mathcal{E}_{a,\Sigma}^{(c,\lambda)}$ can be adequate for one task and inadequate for another because adequacy depends on which relations the evaluative theory regards as relevant.

Let Q denote an explicit epistemic criterion associated with a theory T. Within the model, Q can specify a relational reference domain $U_{a,Q}^{\mathrm{ref}}(\Sigma)$ against which the identified entity is assessed. The identified representation used by the procedure is stated in Equation 101.

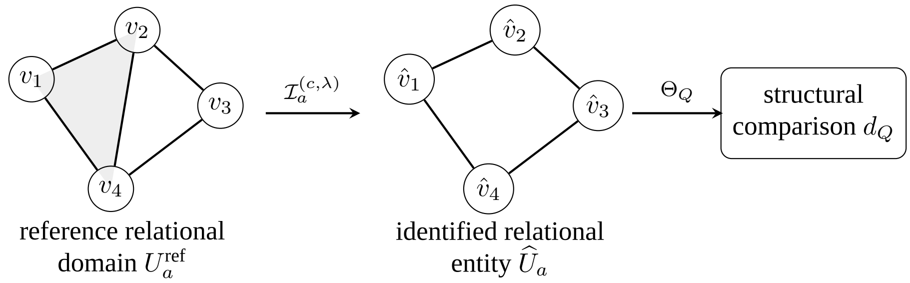


Figure 16: Epistemic comparison between a modeled relational reference domain and an identified entity. The illustration shows a case in which the identified representation retains the same visible vertices while losing an internal relation and a higher-order cell. A theory-relative descriptor Θ_Q determines whether such changes are epistemically significant for the purpose at hand.

$$\widehat{U}_{a,Q}^{(c,\lambda)}(\Sigma) = \mathcal{I}_a^{(c,\lambda)}(\omega_\Sigma). \tag{101}$$

Equation 101 introduces the hat notation only for the present epistemic comparison. The reference object $U_{a,Q}^{\mathrm{ref}}(\Sigma)$ remains model-relative: it is the richer relational structure that criterion Q asks the analysis to preserve or represent, not an assertion of complete access to an observer-independent ontology.

A theory can therefore distinguish *recognition of a focal referent* from *adequate relational recognition*. The first requires that an entity be represented at all. The second concerns whether relations judged relevant under Q remain visible in the representation. A worker can be recognized as a contractual party while relations of training, dependency, collective production, or historical contribution remain absent from the representation used for conversion. The formalism locates that difference at the identification layer rather than treating it as a defect in the later numerical conversion ratio alone.

Remark 15.1 (Recognition and normative scope)**.** Relational recognition is theory-relative in this section. The criterion Q determines which relations are epistemically relevant to the evaluation. A different theory can select a different reference domain without the formalism treating one selection as universally privileged.

Remark 15.1 preserves the normative pluralism established in Section 14 while permitting explicit criticism of a particular identification procedure under a specified criterion.

15.2 Epistemic Truncation and Omission

Finite observation requires truncation. The historical-depth family in Equation 40 already represents this limitation by restricting the portion of $D(\Sigma)$ supplied to entity identification. Epistemic justice becomes relevant when a theory claims that some omitted relations are important for understanding the entity or conversion under evaluation.

When the identified support and reference domain are subcomplexes of the same modeled history, the omitted support can be represented by Equation 102.

$$O_{a,Q}^{(c,\lambda)}(\Sigma) = U_{a,Q}^{\mathrm{ref}}(\Sigma) \setminus \operatorname{supp}\left(\widehat{U}_{a,Q}^{(c,\lambda)}(\Sigma)\right). \tag{102}$$

Equation 102 is available only when set-theoretic subtraction is meaningful for the chosen representations. It records omission inside the model and does not imply that every omitted cell is equally significant. A theory can attach different importance to omitted vertices, edges, faces, labels, or ordered historical relations.

Historical truncation can also be represented through a depth discrepancy. For two depths $\lambda_1 \preceq \lambda_2$, Equation 103 compares the structures available to identification at the two resolutions.

$$\Delta_a^{(\lambda_1,\lambda_2)}(\Sigma) = D_a^{(\lambda_2)}(\Sigma) \setminus D_a^{(\lambda_1)}(\Sigma). \tag{103}$$

Equation 103 makes a practical limitation explicit: a conversion institution can only process a finite historical representation, while the choice of historical depth can determine whether particular relations become visible. Epistemic criticism therefore concerns the rule by which the truncation is chosen as well as the amount of information omitted.

15.3 Attribution and Historical Visibility

Attribution assigns contribution, responsibility, ownership, authorship, or another conversion-relevant status to identified entities. Section 11.8 treated attribution as a relationally situated operation within conversion history. The epistemic question concerns whether the historical structures that support an attribution remain visible when that operation is performed.

Let $\mathcal{A}_Q$ denote an attribution rule interpreted under epistemic criterion Q. The attribution input is constructed from the identified entity and the conversion history. Equation 104 states this dependence.

$$\alpha_Q = \mathcal{A}_Q\left(\widehat{U}_{a,Q}^{(c,\lambda)}(\Sigma), \Gamma_C\right). \tag{104}$$

Equation 104 does not define a particular attribution metric. It makes visible the dependence of attribution on prior identification. If a co-generative historical relation is absent from $\widehat{U}_{a,Q}^{(c,\lambda)}$, a later attribution procedure may reproduce the omission even when the attribution rule is internally consistent on its supplied input.

Historical visibility can therefore be treated as a precondition for some forms of attribution justice. A later recognition event can add a relation to earlier events under the Historical Persistence Convention, but it does not make the earlier conversion history identical to a history in which the relation was recognized before conversion. The order-sensitive analysis of Section 12 therefore remains relevant to epistemic repair: retrospective recognition can alter later configurations while leaving the realized ordering of earlier operations intact within the present model.

Caution 15.2 (Visibility and contribution)**.** Visibility of a relation does not by itself establish that the relation carries a claim to compensation, ownership, or another substantive entitlement. Epistemic identification determines what enters evaluation; the normative significance of the identified relation belongs to the evaluative theory.

Caution 15.2 prevents the epistemic layer from silently deciding distributive or proprietary questions.

15.4 Structural Distortion in Entity Identification

Omission is only one form of epistemic deformation. An identification procedure can preserve the same number of represented components while changing how those components are related. The present subsection therefore introduces a theory-relative structural descriptor that can compare relational form rather than only set membership.

Let Θ_Q map admissible relational structures into a descriptor space Z_Q. The structural descriptor is stated in Equation 105.

$$\Theta_Q : \mathfrak{C}_Q \longrightarrow Z_Q. \tag{105}$$

Equation 105 can encode incidence patterns, connectivity, higher-order cells, partial-order relations, labels, or combinations selected by Q. Given a discrepancy function d_Q on Z_Q, structural distortion is represented by Equation 106.

$$\delta_Q\left(U_{a,Q}^{\mathrm{ref}}, \widehat{U}_{a,Q}\right) = d_Q\left(\Theta_Q(U_{a,Q}^{\mathrm{ref}}), \Theta_Q(\widehat{U}_{a,Q})\right). \tag{106}$$

Equation 106 generalizes the schematic discrepancy introduced in Equation 43. It permits fragmentation, conflation, false connection, higher-order collapse, and historical-order distortion to be represented even when the same visible entities appear in both structures.

Table 12 summarizes several possible structural tests. The table is illustrative; an application can select only the tests justified by its epistemic theory and data.

15.5 Topological Fidelity

Topology supplies one possible descriptor family for relational structures because it focuses on features preserved under continuous deformation and can distinguish connectivity, cycles, cavities, and related structural properties. In the present paper, topology is used as an illustrative epistemic language rather than as a complete theory of meaning, testimony, or recognition.

Let $|U|$ denote a chosen topological realization of a relational complex U. For a criterion Q, let $\mathcal{T}_Q$ denote a finite family of selected topological invariants or summaries. Equation 107 defines the corresponding descriptor vector.

$$\mathbf{T}_Q(U) = \big(\tau_1(|U|), \ldots, \tau_m(|U|)\big). \tag{107}$$

Equation 107 leaves the invariants open. For introductory applications, τ_i can include Betti numbers. The zeroth Betti number β_0 counts connected components, while β_1 records independent one-dimensional cycles. Higher-dimensional invariants can be included when the representation and interpretation justify them.

A topological discrepancy is then defined in Equation 108.

$$d_{\mathcal{T},Q}\left(U^{\mathrm{ref}}_{a,Q}, \widehat{U}_{a,Q}\right) = d_Q\left(\mathbf{T}_Q(U^{\mathrm{ref}}_{a,Q}), \mathbf{T}_Q(\widehat{U}_{a,Q})\right). \tag{108}$$

Equation 108 gives a theory-relative notion of *topological fidelity*. A small discrepancy means that the chosen topological features are similar under the specified comparison. It does not imply complete representational adequacy. Two structures can share the same Betti numbers while differing in labels, order, geometry, causal interpretation, power relations, or other features that matter normatively.

Remark 15.3 (Topological fidelity and semantic adequacy)**.** Topological fidelity is neither necessary nor sufficient for epistemic justice in general. It is one possible structural criterion inside a richer evaluative theory. The criterion becomes meaningful only after the application justifies the topological realization, the selected invariants, and their normative relevance.

Remark 15.3 prevents the illustrative formalization from turning a structural analogy into a universal account of epistemic justice.

15.6 Persistent Relational Features

Entity identification depends on historical depth, relation thresholds, scale, and other resolution parameters. A feature that appears at only one arbitrary resolution can therefore be less informative than a feature that persists across a range of resolutions. Persistent homology provides a mathematical language for tracking topological features across a filtration and representing their durations through barcodes or persistence diagrams [6].

Let a relational filtration associated with focal entity a be given by Equation 109.

$$K_a^{(0)} \subseteq K_a^{(1)} \subseteq \cdots \subseteq K_a^{(r)}. \tag{109}$$

Equation 109 can be generated by increasing historical depth, lowering a relation-strength threshold, enlarging relational radius, or another monotone resolution rule. Persistent homology associates each filtration with topological features that are born and later disappear as the scale changes. Let $\mathrm{Dgm}_k(K_a)$ denote the persistence diagram in homological degree k.

Table 12: Illustrative structural tests for epistemic evaluation of entity identification.

Structural test	Compared feature	Potential epistemic concern
Support retention	Cells or labels retained from a reference domain	Relevant historical or relational material becomes absent from evaluation
Connectivity	Connected components and paths	A co-dependent structure is represented as isolated actors or fragments
Higher-order incidence	Faces or multi-relation cells	Co-generation or collective processes collapse into pairwise relations
Order preservation	Partial-order relations among events	Historical precedence or procedural sequence is misrepresented
Cycle structure	Loops or recurrent relational pathways	Recirculation, mutual dependence, or repeated interaction disappears from representation
Scale persistence	Structural features across a filtration	A feature visible only at one arbitrary resolution is treated as structurally robust

For a reference filtration and an identified filtration, one possible discrepancy measure is the bottleneck distance stated in Equation 110.

$$\delta_{\text{pers},k} = d_B\left(\text{Dgm}_k(K^{\text{ref}}_{a,Q}), \text{Dgm}_k(\widehat{K}_{a,Q})\right). \tag{110}$$

Equation 110 illustrates how epistemic comparison can be made less dependent on one arbitrarily chosen scale. The interpretation remains application-specific. Persistence of a topological feature indicates structural stability across the selected filtration; it does not by itself establish social importance, causal relevance, or normative entitlement.

15.7 Topological Epistemic Justice as an Illustrative Theory

The preceding constructions permit one explicit example of a justice theory that can be represented inside the broader normative architecture of Section 14. The example is intentionally limited. It treats preservation of selected relational-topological features as one epistemically relevant consideration when a conversion institution identifies an entity from a richer modeled relational history.

Let T_{TopEJ} denote this illustrative theory. Its evaluative output is the vector of theory-selected discrepancies stated in Equation 111.

$$\mathcal{E}_{\text{TopEJ}}(\Gamma_C) = \left(d_{\mathcal{T},Q}, \delta_{\text{pers},0}, \ldots, \delta_{\text{pers},p}\right) \in E_{\text{TopEJ}}. \tag{111}$$

Equation 111 leaves the evaluative codomain multidimensional. A theory can define acceptable tolerances without converting every discrepancy into one cardinal score. For a tolerance vector $\boldsymbol{\varepsilon}$, one possible admissibility condition is stated in Equation 112.

$$\mathcal{E}_{\text{TopEJ}}(\Gamma_C) \preceq \boldsymbol{\varepsilon}. \tag{112}$$

Equation 112 should be read only within the componentwise or otherwise specified preorder chosen by T_{TopEJ}. The tolerances, invariant family, filtration, reference domain, and significance of deviations are substantive commitments of the illustrative theory. They are not supplied automatically by the GR conversion formalism.

Caution 15.4 (Illustrative status of topological epistemic justice)**.** The topological construction in this subsection is a proposed example of how one epistemic-justice theory can be represented within the formalism. It is not presented as a reconstruction of Fricker's account, as a complete theory of epistemic injustice, or as a fixed normative criterion of the Generative Relational framework.

Caution 15.4 marks the distinction between a shared formal language and a substantive theory instantiated within that language. A different epistemic theory can replace Θ_Q, d_Q, or the topological descriptors while retaining the same conversion-history and entity-identification architecture.

The section therefore locates epistemic justice at the interface between relational ontology and finite identification. Conversion procedures operate on representations whose historical depth, support, connectivity, higher-order relations, and scale dependence can differ from the richer modeled structures they are intended to represent. The topological example shows how one theory can make a subset of these differences formally evaluable while preserving normative underdetermination at the level of the general framework. The following section turns from the abstract architecture to illustrative heterogeneous conversion structures in labor, currency, collective production, knowledge, environmental contribution, and retrospective evaluation.

16 Illustrative Heterogeneous Conversion Structures

This section instantiates the preceding formalism in six stylized conversion settings. Its role is illustrative rather than empirical: the examples show how the same typed architecture can represent heterogeneous conversion without asserting that one operator sequence, one relational scope, or one evaluative theory is uniquely correct for the domain. The objective is to demonstrate what becomes visible when apparently simple conversions are expanded into relational configurations, entity identifications, attribution structures, conversion histories, and context-dependent readouts. The discussion proceeds from labor and currency cases to collective production, knowledge and symbolic value, environmental contribution, and retrospective evaluation. Table 13 summarizes the analytical emphasis of the six examples.

16.1 Labor and Monetary Conversion

Labor-to-money conversion provides a compact example of the difference between a conversion readout and a conversion history. Let $\omega_0 \in \Omega$ be a configuration in which a labor relation is active and let Γ_{LM} denote the represented history through which labor becomes associated with a monetary settlement. The visible labor and monetary manifestations are stated in Equation 113.

$$L = \pi_L^{(c_L)}(\omega_0), \qquad M = \pi_M^{(c_M)}(\Gamma_{LM}(\omega_0))\,. \tag{113}$$

Equation 113 leaves the ontological type of L and M application-specific. A labor readout can encode duration, output, service, contractual performance, or another institutional manifestation. A monetary readout can encode a payment amount, account balance, claim, or settlement record. The equation therefore represents two observations from a richer relational process rather than an identity between labor and money.

A minimal conversion history can be represented by Equation 114.

$$\Gamma_{LM} = \Phi_{\text{set}} \circ \Phi_{\text{pay}} \circ \Phi_{\text{meas}} \circ \Phi_{\text{auth}} \circ \Phi_{\text{rec}}. \tag{114}$$

The operators in Equation 114 are schematic names for recognition, authorization, measurement, payment, and settlement transformations. A particular institution can decompose or aggregate these steps differently. The formal point is that the coarse statement $L \rightsquigarrow M$ can conceal an ordered family of state-changing operations. Recognition can determine which person, task, or contribution enters the conversion; authorization can create contractual standing; measurement can construct the quantity to be priced; payment can alter monetary holdings; and settlement can terminate or modify active obligations.

The value holder is likewise represented through relational identification rather than as a point-like owner. For a focal worker a, Equation 115 defines an identified labor-relevant entity.

$$\widehat{U}_{a,L}^{(c,\lambda)} = \mathcal{I}_a^{(c,\lambda)}(\omega_0). \tag{115}$$

Table 13: Illustrative heterogeneous conversion structures. Each row identifies a coarse conversion manifestation and the additional relational structure emphasized by the formalism. The examples are schematic and do not supply empirical estimates or substantive justice verdicts.

Case	Coarse manifestation	Relational structure made explicit	Principal formal question
Labor and money	Labor $\rightsquigarrow$ wage	History-bearing worker and organization, contract, measurement, settlement, active obligations	Which intermediate operations and identifications are suppressed by the wage readout?
Currency conversion	$M_A \rightsquigarrow M_B$	Monetary institutions, exchange interface, settlement infrastructure, wider international relations	Which relational scope constitutes the observed conversion ratio?
Collective production	Joint activity $\rightsquigarrow$ attributed value	Co-generative subcomplex, identified contributors, attribution labels	How does attribution relate to the represented generative history?
Knowledge and symbols	Knowledge $\rightsquigarrow$ symbolic or monetary status	Historical knowledge lineages, recognition, classification, institutional legibility	Which relations become visible when symbolic value is assigned?
Environmental contribution	Material and ecological relation $\rightsquigarrow$ compensation or price	Ecological dependencies, temporal depth, material persistence, affected entities	Which environmental relations are retained in the conversion domain?
Retrospective evaluation	Earlier conversion $\rightsquigarrow$ later reassessment	Later recognition events, historical persistence, revised attribution and nomos	How can later recognition alter the present without rewriting the represented past?

Equation 115 can include selected relations to education, workplace organization, tools, care, infrastructure, risk, prior experience, or other historical structures when the modeling purpose requires them. The finite entity representation remains a partial cut through the wider history. A narrower identification can be appropriate for payroll execution while remaining inadequate for another question concerning attribution, capability, or historical contribution.

The labor example therefore illustrates three distinct analytical levels: a coarse wage readout, a conversion history, and an entity-identification layer. None of these levels alone provides a substantive criterion of wage justice. Their separation makes it possible for later evaluative theories to specify which parts of the history and which identified relations matter for their own normative analysis.

16.2 Currency and International Conversion

Currency conversion provides an example in which both sides of the conversion are themselves relationally generated institutional manifestations. Let M_A and M_B denote two currency readouts obtained from a relational configuration ω_Σ. Equation 116 states the readouts without treating either currency as an isolated substance.

$$M_A = \pi^{(c_A)}_{M_A}(\omega_\Sigma), \qquad M_B = \pi^{(c_B)}_{M_B}(\omega_\Sigma). \tag{116}$$

A quoted conversion ratio is then another context-dependent observation. Let q_{AB} denote the observed number of units of M_B associated with one unit of M_A under context c_q. Equation 117 introduces this quantity.

$$q^{(c_q)}_{AB} = \pi^{(c_q)}_{q_{AB}}(\Gamma_{AB}). \tag{117}$$

Equation 117 treats the quoted ratio as a readout of a conversion history rather than as a complete representation of monetary value. The corresponding conversion domain U_{AB} can include monetary institutions, market interfaces, contractual conventions, payment infrastructures, legal jurisdictions, reserve arrangements, material infrastructures, and relevant international relations when those structures contribute to the modeled conversion.

This representation also separates a numerical exchange relation from the historical conditions supporting convertibility. Two histories can yield the same coarse ratio under $\pi_{q_{AB}}^{(c_q)}$ while differing in settlement access, institutional support, risk allocation, legal restrictions, or the wider relations through which the currencies are accepted. Conversely, a change in the relational configuration can alter the conversion readout even when the nominal monetary units remain symbolically unchanged.

The example therefore illustrates a recursive feature of common equivalence. A currency can function as an equivalence medium for heterogeneous goods or obligations while itself entering a higher-level conversion relation with other currencies. The formalism does not require a universal hierarchy among currencies. It provides a representation in which the emergence, use, and transformation of such higher-level equivalence relations can be located inside a wider relational history.

16.3 Collective Production and Attribution

Collective production makes the distinction between generation and attribution especially visible. Let A be a value manifestation observed from configuration ω_Σ. The represented generative support of A can involve several relational lineages and higher-order cells inside a conversion-relevant domain. Let $U_A^{\text{gen}} \subseteq \mathfrak{G}_{\leq\Sigma}$ denote the finite subcomplex selected by the model as relevant to the generation of A.

The identification of contributors is a separate operation. Let $\mathfrak{C}_A$ be the set of entities identified from U_A^{gen} under context c. Attribution then assigns application-specific statuses through the map stated in Equation 118.

$$\alpha_A^{(c)} : \mathfrak{C}_A \times \{A\} \longrightarrow \mathcal{L}_A. \tag{118}$$

Equation 118 permits labels such as contributor, owner, author, beneficiary, responsible party, or another status. The codomain $\mathcal{L}_A$ does not imply that these labels are commensurable. It records that attribution is a distinct representational operation applied after relevant entities have been identified.

Figure 17 illustrates the separation between a co-generative history and a narrower attribution readout. Several historical lineages can contribute to a value manifestation while an institution records attribution to only a subset of the identified entities.

The distinction in Figure 17 allows attribution error to be located without requiring the formalism to specify one correct distributive rule. An evaluative theory can compare the identified and attributed structure with whatever historical or relational features it regards as normatively relevant. The formalism supplies the common process architecture while leaving substantive entitlement criteria open.

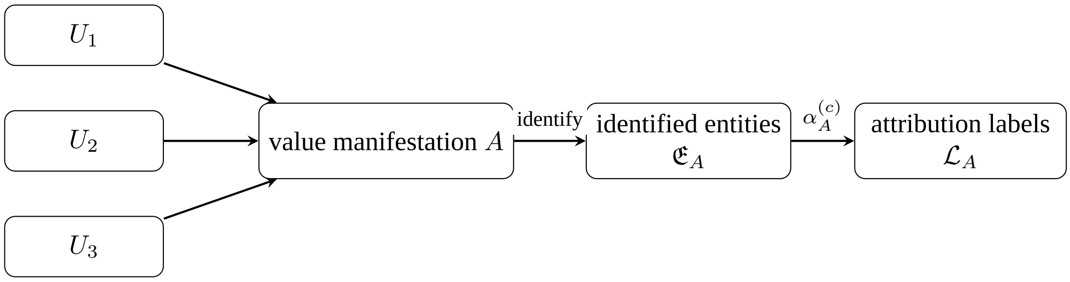
16.4 Knowledge and Symbolic Value

Knowledge-related conversion illustrates how recognition can participate in producing the object subsequently evaluated. Consider a manuscript, method, credential, classification, or symbolic designation whose social significance depends partly on institutional recognition. Let K be a knowledge-related manifestation identified from ω_Σ and let S be a later symbolic status. Equation 119 represents a stylized history.

$$\Gamma_{KS} = \Phi_{\text{stat}} \circ \Phi_{\text{rec}} \circ \Phi_{\text{id}}. \tag{119}$$

In Equation 119, Φ_{id} constructs or selects the relevant entity, Φ_{rec} grants a form of operative recognition, and Φ_{stat} creates or changes a symbolic status. The sequence is deliberately generic: a credentialing institution, archive, publication system, court, platform, or professional community can implement different concrete semantics.

The example demonstrates performativity. A symbolic status can be observed after it has been generated, but the act of recognition can also change future operator availability. A recognized credential can



Generative participation, epistemic identification, and institutional attribution are represented as distinct layers. A narrow attribution can therefore coexist with a wider represented co-generative history.

Figure 17: Collective production and attribution. The diagram separates several represented generative lineages from the value manifestation, the finite set of identified entities, and the later attribution of statuses. The labels U_1, U_2, U_3 denote illustrative relational substructures rather than isolated persons.

enable later applications; a recognized authorship relation can alter attribution; an institutional category can change which comparisons are admissible. The later state is therefore not merely a better-described version of the earlier state.

The history-bearing representation is important when symbolic value depends on prior relations that are not visible in the final label. A credential, citation, title, or classification can be a compact readout whose institutional usefulness depends on coarse-graining. The same coarse-graining can become epistemically problematic when the suppressed history is relevant to attribution, access, or later normative evaluation. Section 15 provides one way to represent such discrepancies without treating complete historical recovery as feasible.

16.5 Environmental and Material Contributions

Environmental conversion demonstrates why the wider ontology cannot be restricted to human–human relations. Let U_E denote a relationally scoped environmental-material subcomplex contributing to a conversion-relevant process. Its cells can represent ecological dependencies, material transformations, infrastructure, resource flows, or other application-specific relations. A monetary or administrative representation P_E is a readout from a later configuration rather than an ontological identity with U_E.

Equation 120 states this separation.

$$P_E = \pi_E^{(c_E)}(\Gamma_E(\omega_0)), \qquad U_E \subseteq U_C(\Sigma). \tag{120}$$

Equation 120 can represent a price, compensation amount, accounting entry, liability, or another institutional quantity. The conversion does not imply that the environmental-material relations contained in U_E are exhausted by P_E. The readout can be useful for settlement while retaining only a restricted set of features from the represented history.

Historical depth can be especially important in this example. A material condition visible at one slice can depend on processes that began much earlier than the conversion event, and the consequences of conversion can continue into later slices. The relational scope therefore determines which temporal and ecological dependencies enter the analysis. Increasing the historical depth parameter λ in an identification map can enlarge the represented environmental ancestry, while a narrower value may support a more local administrative purpose.

This example also clarifies the status of nonhuman and material structures within the formalism. Their inclusion in $\mathfrak{G}$ does not by itself assign moral standing, rights, or a fixed normative weight. It prevents the ontology from treating environmental conditions as an external background when they participate in the generation, persistence, or transformation of the values being converted. A substantive environmental-justice theory can then supply its own evaluative commitments over the represented history.

16.6 Historical Recognition and Retrospective Evaluation

Retrospective recognition provides a direct application of the Historical Persistence Convention and the order-sensitive formalism. Consider an earlier conversion history $\Gamma_C^{(0)}$ completed at relational slice Σ_0. At a later slice Σ_1, new evidence, categories, institutions, or judgments can generate a recognition operator $\Phi_R^{(1)}$ that changes the present relational configuration.

Equation 121 represents the later update.

$$\omega'_{\Sigma_1} = \Phi_R^{(1)}(\omega_{\Sigma_1}), \qquad \mathfrak{G}_{\leq \Sigma_0} \hookrightarrow \mathfrak{G}'_{\leq \Sigma_1}. \tag{121}$$

Equation 121 preserves the represented earlier history while allowing the later state to contain new relations to it. Retrospective recognition can therefore alter current attribution, institutional standing, interpretation, compensation possibilities, or future operator availability without making the earlier history identical to one in which recognition preceded conversion.

The distinction is stated formally in Equation 122. Let $\Phi_R^{(0)}$ denote a hypothetical recognition available before the original conversion and let $\Gamma_C^{(0)}$ denote the realized conversion. The two histories generally differ.

$$\Gamma_C^{(0)} \circ \Phi_R^{(0)} \not\equiv \Phi_R^{(1)} \circ \Gamma_C^{(0)}. \tag{122}$$

The symbol $\not\equiv$ in Equation 122 denotes non-equivalence of represented histories rather than numerical inequality. The later recognition operator acts on a configuration already transformed by the earlier conversion. Its available evidence, institutional context, nomos, and active relations can therefore differ from those of a recognition event placed before conversion.

Retrospective evaluation can be added as a theory-relative layer. Let T be an evaluative theory and let $\mathcal{E}_T$ act on the realized history together with the later identification structure. Equation 123 states the resulting evaluation.

$$e_T^{\text{retro}} = \mathcal{E}_T\left(\Gamma_C^{(0)}, \mathcal{I}_T(\omega'_{\Sigma_1})\right). \tag{123}$$

Equation 123 does not imply that later evaluators possess complete knowledge of the past. It allows later evidence and relational identifications to enter an evaluation while preserving the distinction between the realized historical process and its subsequent reassessment.

Remark 16.1 (Illustrative status of the examples)**.** The six cases in this section instantiate the formal vocabulary at a schematic level. They do not establish empirical models of wages, exchange rates, collective production, symbolic value, environmental compensation, or historical repair. Each domain requires its own data, institutional specification, and substantive theory before quantitative or policy conclusions can be drawn.

Remark 16.1 marks the boundary between formal illustration and domain-specific modeling. Across all six cases, the common analytical move is to replace an apparently simple value conversion with a history-bearing relational representation containing finite identification, attribution, transformation, nomos, and evaluation layers as required by the local question. The next section situates this preliminary formal language relative to existing research traditions and clarifies which components are inherited, reconstructed, or left open.

17 Interfaces with Existing Research

This section positions the preliminary conversion formalism relative to several established research traditions. Its role is comparative and disciplinary: the formal vocabulary developed in Sections 2–15 draws on ideas that already have substantial histories in economics, political economy, philosophy, complexity science, and mathematics. The objective is to identify which elements are inherited, which are reconstructed for the present conversion problem, and which interfaces remain open. The discussion proceeds across eight bodies of work and does not treat proximity of vocabulary as identity of theory. Table 14 summarizes the principal interfaces.

17.1 Economic Models of Value and Conversion

Economic theory already contains highly developed formal languages for representing heterogeneous goods, preferences, technologies, exchange, and prices. A canonical example is the Arrow–Debreu existence result for competitive equilibrium, which formalizes an integrated economy of production, exchange, and consumption under explicit assumptions on agents, technologies, preferences, and markets [7]. Within such frameworks, a price vector supplies a common numerical representation through which heterogeneous commodities can enter exchange relations. The present paper does not offer an alternative proof of equilibrium existence or a competing theory of price determination.

The interface is narrower. The conversion notation developed here asks what additional structures may be relevant when a visible price or transfer is treated as one readout from a history-bearing relational process. A price vector can remain the appropriate economic object for a market question while the conversion formalism separately represents entity identification, institutional authorization, historical attribution, settlement, and later revision. These additional structures do not invalidate ordinary price theory. They specify information that can be suppressed when the analytical question concerns the genealogy, procedure, or normative interpretation of a conversion rather than the equilibrium price alone.

Economic dynamics likewise provide substantial precedent for nonlinear and adaptive modeling with heterogeneous agents. Brock and Hommes, for example, analyze an asset-pricing model in which heterogeneous expectation rules are selected through evolutionary dynamics and can generate complicated or chaotic price behavior [8]. The present paper therefore does not treat nonlinearity, heterogeneous expectations, path dependence, or state-dependent economic evolution as GR discoveries. Their relevance here is methodological: established dynamical ideas demonstrate that economic conversion can already be modeled as a process, while the present formalism adds explicit interfaces to relational identification, higher-order histories, and conversion nomos.

17.2 Philosophy and Social Ontology of Money

The relational emergence of money also has substantial antecedents. Historical, institutional, credit-based, and social-ontological approaches have long resisted the treatment of money as a merely physical object carrying value by intrinsic substance. Desan's historical account reconstructs money as a political and institutional project whose design develops together with legal and governing practices [9]. Eich's political theory of money similarly treats monetary arrangements as politically constituted institutions and reconstructs debates over money through the history of political thought [10]. These bodies of work make it unnecessary, and inaccurate, for the present paper to claim novelty for the general proposition that monetary forms are historically and institutionally generated.

The value-conversion formalism uses this literature as an important boundary condition. A currency manifestation such as M_A or M_B can be represented as a context-dependent readout from a wider relational configuration without implying that the present paper has established the correct ontology of money. Section 16.2 therefore uses currency conversion to illustrate a recursive problem of equivalence: an institutional medium that supports comparison among heterogeneous goods can itself become an object of higher-level conversion. The formal interest lies in representing the histories and interfaces

Table 14: Interfaces between the preliminary conversion formalism and selected research traditions. The table records conceptual proximity and analytical boundaries rather than claims of theoretical equivalence.

Research tradition	Element used or reconstructed in the present formalism	Boundary maintained in this paper
General equilibrium and price theory	Formal comparison through prices and conversion ratios	Price representation does not exhaust relational history, entity identification, or nomos
Social ontology and political theory of money	Money as institutionally and historically constituted	No new ontology of money is claimed
International monetary political economy	Currency hierarchy, monetary institutions, and power relations	Currency cases remain illustrative rather than a new theory of monetary rivalry
Procedural justice	Normative importance of interaction and procedure	Order sensitivity is a structural representation, not a complete procedural criterion
Epistemic justice	Wrongful exclusion or distortion in practices of knowing and recognition	Topological fidelity is introduced only as an illustrative GR-compatible criterion
Relational and process-oriented social theory	Processes, relations, and historical formation of entities	GR terminology is not treated as equivalent to any one relational tradition
Complexity and nonlinear economics	Adaptation, emergence, heterogeneous expectations, nonlinear dynamics, path dependence	Dynamical-systems language is not presented as a new contribution of GR
Higher-order and compositional mathematics	Beyond-pairwise interactions and compositional structure	2-complex and operator-inspired choices remain provisional representations

surrounding that conversion, while the substantive ontology of money remains open to established and competing monetary theories.

17.3 Institutional and Political-Economic Approaches

International conversion makes institutional and political-economic structure especially visible. Eichengreen's historical account of the international monetary system traces changes across the gold standard, Bretton Woods, and later monetary arrangements, showing that exchange-rate regimes and international finance are historically organized through changing institutions and political constraints [11]. Cohen's analysis of international currency power separately emphasizes the differentiated roles of international money and the relation between currency internationalization and state power [12]. These literatures already provide sophisticated accounts of monetary hierarchy, institutional persistence, and political asymmetry.

The present framework therefore uses the international-currency example as an interface rather than as a replacement theory. A conversion domain U_{AB} can include payment infrastructure, legal jurisdictions, reserve practices, financial markets, and international political relations when those structures matter to the local question. The advantage of the relational representation is that these structures can be retained in the same history that also contains the observed conversion readout. The framework does not infer from this representation that one particular institutional factor determines an exchange rate, nor does it rank currencies or predict monetary rivalry.

The institutional literature also clarifies why conversion nomos should not be treated as a free-standing scalar rule. Monetary convertibility, property claims, contractual settlement, and admissible payment forms are supported by legal and organizational arrangements that have histories of stabilization and revision. Section 13 reconstructs this point formally through state-dependent admissible operation families. The reconstruction supplies a typed language for conversion histories; the substantive explanation of how particular monetary institutions arise remains a domain-specific task for economics, history, law, and political economy.

17.4 Procedural Justice

Procedural theories of justice supply a second important interface. Ceva's account of interactive justice develops a proceduralist approach to persistent value conflict grounded in fair hearing and procedural equality [13]. The significance of this literature for heterogeneous conversion lies in its insistence that the normative quality of an interaction cannot be reduced to the final substantive outcome. Procedures can have an independent justice dimension even when disagreement over substantive values persists.

The order-sensitive formalism in Section 12 is compatible with that general insight while remaining much narrower. It represents the possibility that recognition, classification, valuation, conversion, settlement, and revision occur in different orders and that these orders can generate different relational histories. Compositional noncommutativity establishes a structural distinction among histories; it does not establish which ordering satisfies fair hearing, equality, consent, standing, or another procedural norm. A procedural theory must supply those evaluative commitments separately.

This distinction is particularly important for the planned use of the formalism. A history in which recognition precedes conversion can differ from one in which recognition occurs only after settlement, yet the earlier ordering does not become just merely because it is earlier. The formal language identifies where procedural differences enter the history and which later operations become possible or unavailable. Procedural justice supplies the normative criteria by which those differences can be assessed.

17.5 Epistemic Justice

The epistemic-justice literature provides the clearest established background for the identification problems developed in Section 15. Fricker's account distinguishes testimonial and hermeneutical forms of epistemic injustice and connects epistemic wrongs to social power and participation in practices of knowing [5]. The present formalism does not redefine those categories through topology, entity identification, or conversion operators.

Its narrower contribution is representational. When a conversion procedure requires a finite entity representation, the identification operator can omit historical relations, fragment a relational structure, conflate distinct structures, or introduce false connections. Such distortions can be analyzed before any substantive conversion ratio is evaluated. This makes epistemic identification part of the conversion architecture rather than a later correction added after the economic or legal transaction has already been specified.

The topological-fidelity construction in Section 15.7 should therefore be read as an illustrative theory layered onto the formalism. It asks whether theory-selected structural invariants of a reference relational domain are preserved to an acceptable extent by a finite identification. It is neither a reconstruction of Fricker's theory nor a proposed replacement for testimonial, hermeneutical, or other established epistemic-justice analyses.

17.6 Relational and Process-Oriented Philosophy

Relational and process-oriented approaches provide a broader intellectual background for representing subjects and values through histories rather than isolated substances. Emirbayer's relational sociology explicitly contrasts substantialist conceptions of the social world with approaches centered on dynamic, unfolding relations and identifies boundaries, entities, network dynamics, causality, and normative implications as problems for relational analysis [14]. The present paper shares the concern that an entity boundary can itself require explanation rather than being treated as an analytically innocent starting point.

The GR vocabulary nevertheless should not be collapsed into relational sociology or any single process philosophy. The present conversion formalism combines partial-order event history, higher-order cells, active relational boundaries, explicit observation maps, and state-changing operators for a specific analytical purpose. These choices are provisional mathematical representations. They do not establish that every concept developed in relational sociology has a direct counterpart in the 2-complex model, nor that relational ontology alone determines a theory of justice.

This boundary is especially relevant to value holders. Treating h_A or h_B as identified history-bearing structures does not imply that individual agency disappears into an undifferentiated network. The formalism permits local agency-bearing structures to be identified while retaining their relations to wider histories. The balance between individuation and relational embedding remains theory- and application-dependent.

17.7 Complexity and Dynamical Systems

Complexity economics already supplies a mature vocabulary for adaptation, emergence, historical dependence, and economies that evolve through the interactions of heterogeneous agents. Arthur's complexity framework describes economic processes as continually forming and adapting rather than merely moving toward a fixed equilibrium representation [15]. Heterogeneous-agent models such as Brock and Hommes provide more specific demonstrations of how nonlinear dynamics can arise from adaptive switching among expectation rules [8]. These precedents are directly relevant to the paper's use of state-dependent operators and evolving conversion structures.

The present formalism therefore treats nonlinear dynamics as an available analytical resource rather than a novelty claim. Its additional concern is the typing of transformations among relational configurations. A state-changing operator can alter the configuration on which later operators act, modify the currently active boundary, or change which operations are subsequently admissible. This typed process architecture can be combined with nonlinear or stochastic models when an application requires quantitative dynamics, but it does not itself specify a dynamical law or empirical parameterization.

The same restraint applies to criticality, attractors, and fast–slow dynamics. Those concepts may become useful in later work on conversion regimes, monetary transitions, or governance near critical states. Their presence in the broader GR research program does not make them necessary components of the preliminary conversion formalism developed here. The following discussion of formal scope therefore treats stronger dynamical specification as an open extension.

17.8 Higher-Order and Algebraic Modeling

The move beyond ordinary graphs is also grounded in established mathematical and complex-systems research. Battiston and collaborators survey higher-order network representations and show that interactions among three or more units can generate structural and dynamical phenomena that are not captured by pairwise links alone [16]. Their review covers hypergraphs, simplicial complexes, and related higher-order representations across biological, social, and technological systems. This literature supports the methodological decision to ask whether pairwise graphs are sufficient for a conversion problem before selecting a richer representation.

Compositional mathematics provides a complementary interface. Fong and Spivak develop applied category theory as a language for composing systems and translating among formal structures, with examples including resource theories, signal-flow graphs, circuits, databases, and dynamical behavior [17]. The present paper's use of partial maps, composable transformations, and typed conversion histories is compatible with this compositional perspective. A full categorical formulation, however, has not been established here.

The 2-complex representation used in this manuscript is therefore one candidate among several higher-order formalisms. Its advantage for the present problem is that vertices, edges, and faces can be combined with partial-order event structure and boundary histories in a way that remains visually and conceptually accessible. Operator-inspired language then represents state-changing transformations and order-sensitive composition. Future work may show that category-theoretic, hypergraph, simplicial, operator-algebraic, or other representations provide a cleaner foundation for particular parts of the model.

Remark 17.1 (Positioning of the preliminary formalism)**.** The present framework is best interpreted as a synthesis and typed reconstruction of several established analytical resources around the specific problem of heterogeneous value conversion. Relational emergence, institutional money, nonlinear economic dynamics, procedural justice, epistemic injustice, higher-order interactions, and compositional mathematics all have substantial antecedents. The paper's objective is to make their interfaces explicit within one preliminary conversion language while preserving disciplinary differences and open problems.

Remark 17.1 summarizes the methodological stance of this section. The formalism is intended to support translation across literatures without converting conceptual proximity into theoretical identity. The next section therefore turns from disciplinary positioning to the formal limits of the proposed representation and identifies the mathematical, empirical, normative, temporal, and computational questions that remain unresolved.

18 Formal Scope and Open Problems

This section records the formal limits of the preliminary conversion framework and identifies extensions that remain open. Its role is prospective rather than programmatic: Sections 2–17 introduced a usable representation of heterogeneous conversion, while several mathematical, empirical, computational, normative, and ontological choices remain deliberately underdetermined. The objective is to distinguish what the current formalism already supports from what would require an additional construction. The discussion proceeds from higher-order representations and algebraic extensions to empirical realization, normative underdetermination, temporal ontology, and the scalability of relational identification. Figure 18 and Table 15 summarize these extension directions.

18.1 Alternative Higher-Order Representations

The present manuscript uses a labeled 2-complex because vertices, edges, and faces provide a compact language for event-like occurrences, continuing relations, and higher-order processes. This choice is analytically convenient rather than uniquely implied by heterogeneous conversion. Hypergraphs can represent interactions among arbitrary subsets without requiring a cellular attachment structure. Simplicial complexes supply a different notion of higher-order closure. Temporal multilayer networks can preserve changing relation types while retaining graph-based algorithms. Event structures, causal sets, and process-based representations can foreground precedence or concurrency more directly than a 2-complex.

The open problem is therefore representational adequacy. A stronger theory should specify which empirical or conceptual features require a particular higher-order structure and which features are invariant under translation between representations. A conversion model that depends critically on a face in one representation should clarify what relational content that face encodes and whether the same content can be recovered in another formalism. The current framework leaves this choice local to the application.

Remark 18.1 (Representation choice)**.** A richer mathematical object is useful only when it preserves analytically relevant structure that a simpler representation loses. The preliminary use of 2-complexes therefore establishes a candidate language for history-bearing conversion processes rather than a general requirement that every conversion problem be modeled by a 2-complex.

Remark 18.1 is especially important for empirical work. The cost of reconstructing higher-order history can exceed its analytical benefit when the research question concerns a narrow transactional mechanism. Conversely, a pairwise graph can become inadequate when attribution, historical co-generation, or higher-order institutional relations are central to the problem.

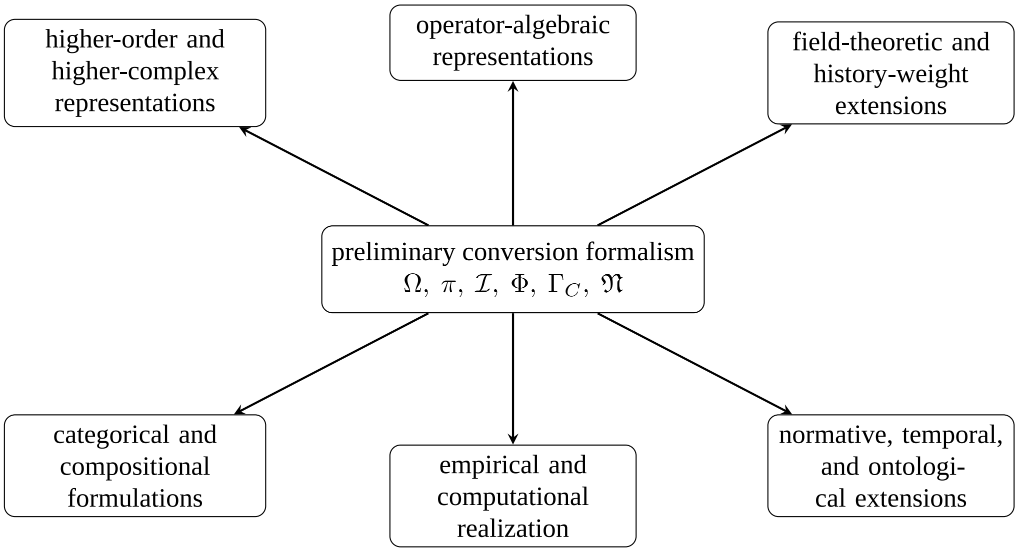


Figure 18: Extension map for the preliminary formalism. The central objects developed in this manuscript support several possible mathematical and applied extensions. The arrows indicate open research directions rather than a required sequence of development.

Table 15: Open-problem map for the preliminary conversion formalism. Each row identifies a current modeling choice and the additional work required for a stronger formulation.

Extension domain	Current representation	Open requirement
Higher-order structure	Labeled 2-complex with partial-order event structure	Criteria for selecting among 2-complexes, hypergraphs, simplicial structures, and higher complexes
Operator structure	Partial state-changing maps and state-dependent operator families	Conditions for a genuine algebraic or linear representation
Field representation	Optional fields and field-dependent transformations	Semantics linking local field values to conversion operators and evidence
Categorical structure	Typed composable histories	Explicit category, objects, morphisms, functors, and equivalence notions
History weighting	Optional nonnegative history weights	Empirical or theoretical basis for weights and any amplitude-like extension
Empirical realization	Application-specific readouts and domains	Measurement protocols, identifiable parameters, data interfaces, and validation
Computation	Finite scoped subcomplexes	Truncation rules, complexity bounds, scalable algorithms, and sensitivity analysis
Normative evaluation	Theory-relative evaluative structures	Substantive criteria supplied and defended by particular justice theories
Temporal ontology	Partial order plus Historical Persistence Convention	Comparison with alternative temporal and causal representations
Entity identification	Context-dependent finite relational cuts	Stability, uncertainty, scale selection, and epistemic-justice diagnostics

18.2 Two-Complex and Higher-Complex Extensions

The 2-complex representation developed in Sections 3 and 7 stops at faces. Some conversion processes may require relations among higher-order processes themselves. A legal conversion can involve a contract, an institutional rule, a review procedure, and a jurisdictional relation whose interaction is not naturally exhausted by a single face. A higher-dimensional cell complex or another higher-categorical structure may represent such nesting more directly.

A minimal extension would require an embedding of the present 2-complex representation into a richer complex. Equation 124 states the structural requirement abstractly.

$$\iota : \mathcal{K}_2 \hookrightarrow \mathcal{K}_{\geq 2}. \tag{124}$$

Equation 124 requires the currently represented vertices, edges, faces, incidence relations, and relevant order structure to remain recoverable inside the extended representation. The open task is to specify the semantics of higher cells. Adding dimensions without a corresponding analytical interpretation would increase notation while weakening explanatory transparency.

Higher-complex extensions also raise a boundary question. The active relational boundary $\mathcal{B}_\Sigma$ has been described at the level needed for a 2-complex history. A higher-dimensional model would require a compatible definition of boundary data, restriction to relational domains, and historical persistence across successive extensions. These constructions remain outside the present paper.

18.3 Operator-Algebraic Extensions

The current operator language is intentionally broad. A generative operation is a partial transformation $\Phi : \Omega \rightharpoonup \Omega$, and noncommutativity means that two compositions can be unequal when both are defined. This structure does not by itself provide addition, scalar multiplication, adjoints, norms, spectra, or the other ingredients associated with established operator algebras.

A stronger operator-algebraic formulation would therefore require a representation of conversion operations on a space carrying the necessary algebraic structure. One candidate is a state-dependent representation map of the form stated in Equation 125.

$$\rho_\omega : \mathcal{A}_\omega \longrightarrow \mathrm{End}(V_\omega). \tag{125}$$

In Equation 125, V_ω would be an application-specific linear or module-like space and ρ_ω would assign suitable conversion operations to endomorphisms. Only after such a representation is justified would expressions involving linear commutators, spectra, expectations, or operator norms have their standard mathematical meanings.

State dependence introduces an additional problem. If Φ transforms ω into ω', then V_ω and $V_{\omega'}$ may differ, and a fixed algebra acting on one common space may be inappropriate. A future theory may therefore require a bundle, field, or category of operator spaces indexed by relational configurations. The present manuscript keeps the weaker partial-map formulation because it remains well typed without resolving this additional structure.

18.4 Field-Theoretic Extensions

Section 6 distinguished fields from operators and treated field language as optional. Some conversion settings may nevertheless benefit from a distributed representation. Liquidity, institutional accessibility, ecological stress, recognition density, or another locally defined quantity can vary across a relational domain rather than being naturally attached to a single entity or event.

A field-theoretic extension would require a domain, a codomain, and a semantics for how field values influence conversion dynamics. Let η denote an application-specific field over a relational domain U_C. A field-dependent generative operation can then be represented abstractly by Equation 126.

$$\eta \longmapsto \Phi[\eta] : \Omega \rightharpoonup \Omega. \tag{126}$$

Equation 126 is only a dependency structure. A substantive field model must define how η is estimated, how its local values relate to vertices, edges, faces, or other cells, and how those values modify the transformation. The present formalism does not assume that every distributed influence should be represented as a field, nor that a field representation is superior to direct operator specification.

The same restraint applies to operator-valued fields. Such constructions have precise meanings in mathematical physics, while a conversion model would need its own domain, state space, regularity assumptions, and empirical interpretation. Their possible usefulness remains an open formal extension.

18.5 Categorical Formulations

The typed structure of conversion histories suggests a categorical reconstruction. Relational configurations can be treated as objects and admissible conversion operations as morphisms, with composition representing sequential transformation whenever domains and codomains match. Partial composability then becomes a structural feature rather than an exceptional case.

A categorical formulation would require an explicit process category, provisionally denoted $\mathbf{C}_{\text{conv}}$, together with clear definitions of identity morphisms and composition. Translation into another mathematical representation could then be expressed through a functor. Equation 127 states this possibility abstractly.

$$\mathcal{F} : \mathbf{C}_{\text{conv}} \longrightarrow \mathbf{D}. \tag{127}$$

The codomain $\mathbf{D}$ in Equation 127 could represent an operator model, a network representation, a database schema, or another application-specific formal system. Such a construction would be valuable if it clarifies which conversion properties are preserved under translation. A full categorical treatment would also make it possible to distinguish isomorphism, equivalence, coarse-graining, and loss of information more carefully than the current informal translation language.

The open difficulty is semantic discipline. Category-theoretic notation can make composition elegant while leaving the meaning of objects and morphisms underspecified. Any future categorical extension should therefore retain the paper's existing distinctions among observation, identification, state-changing transformation, nomos, and evaluation rather than collapsing them into undifferentiated arrows.

18.6 History Weights and Amplitude-Like Representations

The spin-foam background motivates an additional question concerning alternative histories. In the current manuscript, branching candidate histories can be represented without assigning them numerical weights. Some applications may require a measure of plausibility, support, frequency, institutional accessibility, or another transition tendency over the admissible history space.

A minimal history-weight construction can be written as a nonnegative functional. Equation 128 gives the corresponding type.

$$\mathcal{W}_\omega : \text{Hist}(\omega) \longrightarrow \mathbb{R}_{\geq 0}. \tag{128}$$

Equation 128 does not determine the interpretation or normalization of $\mathcal{W}_\omega$. A probability model would require a probability measure and statistical assumptions. An institutional-accessibility weight would require a different semantics. A normative history weight would require a theory-specific evaluative basis.

A complex-valued amplitude-like construction would be a substantially stronger step. It would require a justified state space, composition rule, interpretation of interference or phase, and an explanation of why complex amplitudes improve the conversion model. The spin-foam analogy alone does not supply those requirements. The present paper therefore uses the language of history weights when numerical tendencies are contemplated and leaves amplitude-like representations as a separate open problem.

18.7 Empirical Parameterization

The formalism is presently structural. It specifies types of objects and relations without identifying empirical parameters for a particular domain. Empirical use requires an explicit bridge from observable records to the modeled configuration, relational domain, operators, and history labels. Currency conversion, labor compensation, environmental valuation, and symbolic recognition would each require different measurement protocols.

A parameterized application should therefore state at least four elements: the data source, the identification rule for the relevant relational domain, the estimable parameters attached to the chosen representation, and the validation criterion. The entity-identification operator is especially important because the measured dataset is already a finite and institutionally produced representation of the wider relational history. Parameter estimation cannot repair relations that were excluded before measurement unless additional evidence is introduced.

The framework also creates an identification problem in the statistical sense. Distinct operator sequences or higher-order histories may produce the same observable conversion readout. Empirical work must therefore determine which latent structures can be distinguished from available data and which remain observationally equivalent. This question is application-specific and cannot be settled at the level of the present preliminary formalism.

18.8 Computational Realization

A computational implementation cannot store or traverse an unbounded relational history. Practical realization therefore requires finite truncation, sparse representation, indexing of event order, and algorithms for selecting conversion-relevant subcomplexes. The computational problem is not only one of size. The result of entity identification can depend on historical depth, relational radius, evidence thresholds, and the representation chosen for higher-order interactions.

One useful computational requirement is stability under controlled expansion of the observation window. Let $\widehat{U}_a^{(r)}$ denote the entity identified with relational or historical scope parameter r. A scalable method should report whether salient descriptors stabilize as r increases rather than silently treating one arbitrary truncation as complete. Section 15.6 developed an analogous logic for persistent relational features in epistemic analysis.

Algorithmic work must also accommodate partial composability. A conversion planner or simulator should check operator domains before composing transformations, record intermediate configurations, and distinguish impossible sequences from merely low-weight sequences. These requirements suggest a process-oriented software architecture rather than a single matrix calculation. Concrete data structures and complexity bounds remain open.

18.9 Normative Underdetermination

The formalism separates relational history from substantive normative evaluation. This separation creates deliberate normative underdetermination. A conversion history can be represented with considerable structural detail while different justice theories select different entities, observables, procedures, consequences, or relations as normatively salient. The formal language therefore does not contain a hidden rule that converts relational description into one universal justice verdict.

The open task for normative applications is to state the evaluative theory and its informational basis explicitly. A procedural theory can evaluate ordering, standing, participation, or contestability. A capability-oriented approach can evaluate changes in effective opportunities. A relational or non-domination theory can examine dependency and power. An epistemic theory can evaluate identification and recognition. These theories can disagree even when they operate on the same represented conversion history.

Nomos introduces further reflexivity because evaluative theories, legal concepts, and moral judgments can themselves become events within the relational history and can influence later conversion possibilities. The framework can represent this interaction through judgment-generated operators, but it does not infer that historically emergent normativity is self-justifying. The relation among normative emergence, critical reflection, and substantive justification remains a philosophical problem rather than a consequence of the formal syntax.

18.10 Temporal and Physical Ontology

The partial-order event structure and the Historical Persistence Convention provide a tractable representation of generative precedence. They should remain distinguished from claims about fundamental physical spacetime. The partial order is model-relative, and the convention that realized history is retained under later extensions was introduced to support provenance, responsibility, and procedural comparison.

Alternative temporal representations remain possible. Applications may require stochastic event order, interval orders, multiple incompatible clocks, cyclic process representations, or a model in which the effective historical structure itself can be revised. A future physical interpretation might also motivate structures that are incompatible with the present persistence convention. The current formalism leaves such possibilities open.

This restraint also applies to the spin-foam analogy. The use of boundaries, 2-complex histories, and local higher-order structure is mathematically suggestive, but the manuscript does not infer quantum gravity, physical amplitudes, or background independence in a technical physical sense from the conversion model. The formal analogy earns its place through representational usefulness rather than ontological identity.

18.11 Scalability of Relational Identification

Relational entity identification is one of the most consequential and difficult components of the framework. A deeply relational ontology can make the ancestry of an apparently local entity extremely large. Labor capacity can depend on education, care, infrastructure, institutional history, and material environments; a currency can depend on legal, financial, technological, and international relations; a knowledge object can inherit extended intellectual and organizational histories. Complete reconstruction is generally unavailable.

The practical question is therefore how to select a finite entity representation without converting convenience into an unnoticed ontological claim. Identification operators require explicit scope parameters, evidence rules, and uncertainty statements. Different applications may legitimately use different cuts, but comparisons among them should record the structural differences introduced by those cuts.

The topological and persistent descriptors introduced in Section 15 provide one possible family of diagnostics for structural loss across scales. Other diagnostics may be more appropriate for ordered, weighted, semantic, or institutional relations. The open problem is to develop identification procedures that are computationally feasible, empirically accountable, and normatively transparent while retaining enough historical structure for the conversion question under study.

Caution 18.2 (Open character of the formalism)**.** The extensions collected in this section should not be read as a roadmap toward one maximally elaborate formal system. Different conversion problems may require different combinations of higher-order topology, operator structure, fields, categories, statistical models, and normative theories. The preliminary framework is intended to support such specialization while keeping the type and scope of each added construction explicit.

Caution 18.2 states the governing principle of the open-problem program. Greater mathematical richness is warranted when it resolves a defined representational or empirical limitation. The concluding section returns to the narrower contribution of the manuscript: making the structure hidden by heterogeneous conversion explicit without treating one preliminary representation as a complete theory of value, justice, economics, or physical reality.

19 Conclusion

This concluding section consolidates the representational contribution of the manuscript and restates its limits. The paper began from a compact notation—heterogeneous conversion written as $A \to B$—and asked what additional structure becomes necessary when the values, entities, procedures, and normative conditions involved in that conversion are historically and relationally generated. The resulting formalism does not provide a complete theory of value, economics, justice, or Generative Relational ontology. It provides a preliminary language for making several layers of a conversion process explicit while preserving their analytical differences.

The first layer concerns relational and historical structure. Sections 3–7 introduced the mathematical and physical background needed to represent conversion through higher-order histories rather than through isolated endpoints. Graphs supply an accessible starting point for pairwise relations, while hypergraphs, simplicial complexes, cell complexes, and especially labeled 2-complexes provide richer resources for representing higher-order interaction. A partial order over events supplies a model-relative account of generative precedence, and relational slices provide finite locations from which historical domains can be described. The Historical Persistence Convention then supplies a tractable bookkeeping rule for the present manuscript: realized historical structure is retained under later extensions, while active relational status, interpretation, and future consequences may change. This convention supports provenance and procedural comparison without asserting a fundamental metaphysics of physical time.

The second layer concerns the configuration on which conversion operations act. Section 8 introduced a relational configuration containing realized history, an active relational boundary, and an effective nomos. Within that configuration, values, subjects, value holders, institutions, and material environments are represented through context-dependent manifestations rather than as isolated primitives. Section 9 then distinguished observation, entity identification, recognition, and performative effects. This distinction matters because a conversion procedure normally acts on a finite institutional representation of a much larger relational history. The entity supplied to valuation or exchange can therefore differ substantially from the relational structure relevant to the process under study.

The third layer concerns transformation. Section 10 represented state-changing processes by typed partial maps on the configuration space and separated these transformations from observational readouts. Ontological extension, event creation, edge and face generation, termination of active relations, and observation–intervention instruments can thereby be described within one typed architecture. Section 11 used these operations to represent a conversion as a composite history rather than as a single numerical correspondence. Branching candidate futures, merging lineages, co-generation, attribution, and coarse-grained conversion readouts can then be distinguished without requiring each application to use the same level of detail.

The fourth layer concerns order. Section 12 showed that conversion-related transformations may be order-sensitive because earlier operations can modify the configuration and the family of operations available afterward. Recognition followed by conversion can therefore define a different history from conversion followed by later recognition. In some cases both compositions exist and differ; in other cases one ordering is admissible while the reverse ordering is unavailable. The manuscript uses noncommutativity in this compositional sense. A conventional algebraic commutator requires additional linear structure and remains an optional future representation rather than a premise of the present formalism.

The fifth layer concerns nomos. Section 13 treated conversion nomos as part of the evolving relational configuration. Repeated practice, institutionalization, power over conversion interfaces, contestation, and revision can alter which transformations count as admissible or effective. This treatment allows historically stabilized conversion rules to be modeled without identifying stabilization with normative justification. It also permits governance to be represented at several levels, including direct intervention in conversion and indirect intervention in the relational conditions through which conversion norms emerge.

The sixth layer concerns normative evaluation. Sections 14 and 15 separated the formal description of a conversion history from the evaluative structures supplied by particular theories of justice. Entity identification can be theory-relative, evaluation can operate on different informational bases, and a judgment can generate an operator that influences subsequent relational evolution. The illustrative topological treatment of epistemic justice showed one possible use of the framework: a theory may evaluate whether a finite recognized structure preserves selected relational invariants of a reference domain across relevant scales. This example was intentionally presented as one possible normative construction rather than as a fixed GR criterion.

The examples in Section 16 demonstrated why these distinctions matter across different conversion settings. Labor and wage conversion can depend on how labor capacity and surrounding histories are identified. Currency conversion can depend on institutions and international relations that extend far beyond the immediate transaction. Collective production raises attribution questions because a coarse value-holder representation can obscure co-generation. Knowledge and symbolic value depend on recognition structures that can change after dissemination. Environmental conversions can omit material and ecological relations from the represented entity. Retrospective recognition can alter present obligations and judgments while leaving the modeled historical occurrence of earlier events intact. These examples support the paper's central representational point: similar endpoint notation can conceal substantially different relational histories.

Section 17 located this proposal alongside existing work in economics, monetary and political economy, procedural and epistemic justice, relational sociology, complexity research, higher-order networks, and compositional mathematics. The comparison is important for claim discipline. Relational accounts of money, nonlinear economic dynamics, process-oriented social theory, procedural justice, epistemic injustice, higher-order interaction, and compositional modeling all possess substantial prior literatures. The present paper does not claim novelty for those components individually. Its contribution is narrower: it assembles a typed preliminary language for the heterogeneous-conversion problem and uses that language to keep historical structure, entity identification, transformation, order, nomos, and normative evaluation in view at the same time.

The open problems collected in Section 18 also delimit the present result. The 2-complex representation is one candidate among several higher-order structures. The operator language remains a language of partial state-changing maps unless additional algebraic structure is supplied. Field-theoretic and spin-foam concepts remain representational resources whose physical interpretation is deliberately suspended. History weights require an empirical or theoretical semantics before they can function as probabilities, institutional tendencies, or amplitude-like quantities. Entity identification remains finite, scale-dependent, and epistemically vulnerable. Normative evaluation remains underdetermined until a substantive theory supplies its criteria. These limitations are part of the formalism's present scope rather than defects to be concealed.

The resulting proposal can therefore be stated compactly. A heterogeneous conversion is represented as a historically situated, partially observed, and potentially order-sensitive transformation of a relational configuration. The familiar expression $A \to B$ is retained as a useful coarse readout, while the richer history records the entities identified, the operations performed, the nomos under which they were admissible, and the relational configuration generated afterward. This representation creates a common interface through which economic, institutional, political, epistemic, and normative analyses can be compared without requiring their conceptual differences to disappear.

Further development can proceed in several directions without changing this limited conclusion. Specific applications can supply empirical identification rules and parameterized operators. Formal work can investigate categorical, operator-algebraic, field-theoretic, or higher-complex representations when they answer a defined analytical need. Normative work can instantiate procedural, epistemic, distributive, capability-oriented, relational, or other evaluative theories on the same conversion histories. Computational work can study finite truncation, stability across relational scales, and partial composability. The manuscript therefore closes with a preliminary formal proposition rather than a completed theory: heterogeneous value conversion can be studied more transparently when the conversion arrow is treated as the coarse manifestation of a richer relational history whose ontology, epistemic representation, transformation structure, nomos, and normative evaluation remain explicitly distinguishable.

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