A Preliminary Formalism of Pre-Asymmetry in Generative Relational Theory: On the Formal Conditions Preceding Relational Asymmetry

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Abstract

Generative Relational Theory (GR) treats relational structures as historically situated, relationally constituted, and generatively evolving. This orientation repeatedly requires distinctions among difference, asymmetry, relational differentiation, and the conditions under which such distinctions become meaningful. This paper develops a preliminary formalism for *pre-asymmetry*: the conditional domain in which the antecedents of a determinate relational asymmetry are analyzed without assigning them a single ontological origin. Three regimes are kept open: antecedent symmetry, asymmetry that is primitive relative to the selected model domain, and a pre-differentiated regime in which the transformations required for a symmetry judgment are only partially defined. The third regime motivates partial operations, inverse-semigroup structures, and restriction-categorical representations of definedness. Once symmetry is sufficiently articulated, group actions, orbits, stabilizers, and residual subgroups provide a precise language for relational interchangeability and for one route to symmetry-reduction-generated differentiation. The paper keeps structural formalization distinct from dynamical explanation: perturbative reduction, bifurcation, stochastic selection, high-susceptibility amplification, and historical accumulation are formulated as candidate mechanism families. A bounded application to relational power further shows that typed asymmetry requires an additional trajectory-shaping condition before it becomes power-relevant. The framework preserves ontological suspension while making the formal conditions of symmetry and asymmetry available for conditional analysis.

Keywords: Generative Relational Theory; pre-asymmetry; partial symmetry; relational differentiation; symmetry breaking

Preliminary Research Note

This paper belongs to the broader Generative Relational (GR) research programme. It develops a limited formal question that arises when relational analysis uses asymmetry as an explanatory object. GR treats relational beings, relations, and their possible transformations as historically situated and generative. The present inquiry therefore asks which formal conditions make symmetry and asymmetry meaningful within an articulated relational domain, and which alternative conditions remain admissible before such an articulation is available.

The paper adopts ontological suspension as a methodological discipline. It does not assign ultimate priority to symmetry, asymmetry, or a pre-differentiated condition. It keeps these possibilities open and studies their conditional formal consequences. The resulting mathematical language should therefore be read as a family of structural representations rather than as a direct proof of an ultimate social, physical, or metaphysical ontology.

The paper also distinguishes structural formalization from dynamical explanation. Partial algebras, partial symmetry, inverse-semigroup structures, restriction categories, groups, orbit structure, and related constructions may clarify what can be represented once specified relational conditions are met. The mechanisms through which a relational system acquires or loses those conditions remain more uncertain. Perturbative, nonlinear, stochastic, critical, and historically cumulative mechanisms are therefore introduced only where their provisional epistemic status can remain explicit.

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Notices

This page records the manuscript's publication status, licence, development disclosure, and relation to the broader Generative Relational research programme.

Status. This preliminary discussion paper develops an early formal treatment of pre-asymmetry within Generative Relational Theory. Definitions, mathematical constructions, examples, and the proposed relations among partial symmetry, articulated symmetry, symmetry reduction, and relational differentiation remain open to revision. The paper does not present a completed general dynamics of relational systems and does not select a final ontology of symmetry or asymmetry.

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Statement on the use of language models. The exploratory discussions and preparation of this paper involved OpenAI's ChatGPT. ChatGPT supported exploratory dialogue, formal reconstruction, argumentative criticism, drafting in LaTeX, and technical checking. The author selected the research questions, directed and approved the theoretical commitments and epistemic status of the claims, and bears sole responsibility for the manuscript, including its definitions, formal constructions, arguments, conclusions, and errors. Authorship credit remains with the human author.

1 Introduction

Generative Relational Theory (GR) approaches social, epistemic, normative, and other complex structures through relations that are historically situated and capable of generating further relations, distinctions, constraints, and possibilities. Within this orientation, a relational being is not exhausted by a list of intrinsic properties at a single moment. Its position is partly constituted by the relations through which it has emerged, the relations that currently sustain it, and the transformations that remain possible from its present configuration. Historical trajectory, relational embedding, and generative consequence therefore enter the analysis together. This orientation creates a recurring need to describe directional differences among relational positions, including differences in dependency, access, epistemic position, transformative capacity, generative capacity, and the ability to shape future possibilities.

GR distinguishes among ontological reality, relational reality, and manifestation reality. The present paper works primarily at the relational level: it asks which stable relational distinctions, transformations, and invariances can be formally represented once a domain of analysis has been specified. Claims about ultimate ontological reality remain suspended, while manifestation is treated only where later sections need to distinguish a relational structure from the context-dependent forms through which that structure becomes accessible or actionable. This division of labor keeps the formalism tied to the level at which its objects are defined.

Relational asymmetry provides one possible language for these differences, yet its own formal status requires clarification. A difference between two relational beings does not by itself establish an asymmetry in the sense needed for GR. Two beings can differ in many properties while occupying equivalent positions under a specified relational transformation. Conversely, two beings that appear similar under a limited description can occupy different relational positions once their dependencies, histories, accessible transformations, or future reachability are taken into account. A formal account of asymmetry must therefore specify the relational structure with respect to which a difference becomes directional, non-exchangeable, or otherwise consequential.

The need for such an account becomes visible in several prospective GR applications. A later theory of power, for example, may need to explain how unequal relational positions shape the feasible trajectories of different subjects. Analyses of heterogeneous value conversion may need to distinguish mere difference among value forms from directional conversion asymmetries. Epistemic analysis may need to distinguish different bodies of knowledge from asymmetric access, dependency, or authority. These applications do not require a single universal definition of asymmetry, but they share a prior formal problem: the conditions under which relational positions can be compared as symmetric, asymmetric, or not yet subject to that distinction. Power is thus one illustrative motivation among several, rather than the defining object of the present paper.

A direct appeal to symmetry breaking offers an initially attractive route. Once a relational configuration admits a well-defined symmetry structure, a reduction in that structure can differentiate positions that were previously exchangeable. Ordinary group-theoretic language can then characterize transformations that preserve a configuration, the orbits generated by those transformations, and the residual symmetries that remain after a structural change. This route can make a class of relational asymmetries formally legible. It also reveals a prior assumption: symmetry breaking can operate only after the relations, positions, transformations, and equivalences required to formulate a symmetry have already become sufficiently articulated.

The present inquiry therefore places pre-asymmetry before symmetry breaking as its primary analytical object. Pre-asymmetry is used here as a provisional term for the formal conditions that may precede a determinate relational asymmetry. The term does not designate a single ontological state. It names a problem domain in which several possibilities remain open. One possibility is an antecedent symmetric regime in which relevant positions are already related by a well-defined symmetry and later become differentiated. A second possibility is a regime in which asymmetry is already primitive relative to the available relational description, so that no prior symmetric state is required for the analysis. A third possibility is a pre-differentiated regime in which the operations and equivalence relations required to ask whether a

configuration is symmetric or asymmetric have not yet acquired a complete domain of applicability.

The third possibility requires particular care. Treating it as a third truth value would preserve the assumption that the predicate of symmetry is already well formed and merely awaits evaluation. Treating it as a superposition of a symmetric and an asymmetric state would similarly presuppose that both terms of the superposition are already defined. The formal issue considered here occurs one level earlier. A relational operation can be only partially defined, and a candidate exchange or transformation may have no application to a configuration that lacks the differentiation required to specify its operands, domain, or preserved structure. Under such conditions, the relevant formal task concerns the domain in which the symmetry question becomes meaningful. Partial operations, partial group actions, inverse-semigroup structures, and restriction-categorical representations of definedness become natural candidates for representing this regime.

This extension also introduces a distinction between symmetry formation and symmetry breaking. A relational system may first acquire sufficiently stable positions, admissible transformations, and preserved relations for a symmetry structure to become articulated. Partial or local symmetries may thereby become organized into a more global structure. Symmetry breaking concerns a subsequent change in an already articulated symmetry structure. The two processes need not share the same dynamics. Their separation helps prevent a group-theoretic formalism from silently assuming the very relational differentiation that the paper seeks to explain.

The paper adopts an explicitly conditional attitude toward these possibilities. GR does not require the present inquiry to decide whether ultimate reality is fundamentally symmetric, fundamentally asymmetric, or prior to the applicability of that distinction. The methodological commitment can be stated directly: we do not foreclose ontological possibilities in advance. While suspending final ontological commitment, we conditionally, formally, and generatively analyze the different structures that may be possible. Ontological suspension thereby functions as a discipline of inquiry. It preserves the distinction between a useful formal representation and a claim about ultimate ontology while allowing substantive analysis of the relational conditions under which particular predicates, transformations, and dynamical descriptions become meaningful.

The same discipline applies to the mathematical tools used in the paper. A group, inverse semigroup, restriction category, groupoid, or dynamical system is not treated as a context-free description of what relational reality ultimately is. Each formalism is introduced for a specified analytical role. Partial algebraic and categorical structures are considered where definedness and local applicability are central. Group-theoretic structures are considered where symmetry has become sufficiently articulated to support global or specified automorphisms. Dynamical tools are introduced only when the analysis turns from the structure of symmetry to hypotheses concerning its formation or loss. This formal pluralism follows the relational orientation of GR itself: the adequacy of a mathematical representation depends on the relational object, scale, question, and invariants that the analysis seeks to preserve.

The structural part of the paper therefore proceeds through three connected problems. The first concerns the formal conditions under which symmetry can be posed as a meaningful relational predicate. The second concerns partial symmetry and the possible articulation of more global symmetry from relational operations with limited domains. The third concerns the differentiation of positions once an articulated symmetry structure changes. In the latter case, group actions, orbit structure, stabilizers, and residual symmetry provide a candidate language for describing how previously exchangeable relational positions can cease to remain equivalent under the transformations that still preserve the configuration.

The dynamical part of the inquiry is intentionally more tentative. A structural account can characterize the symmetry content of an initial and a later configuration without yet possessing a general theory of the process that connects them. Perturbative symmetry breaking provides one candidate hypothesis, particularly where a small structured disturbance changes a configuration's residual symmetry or is amplified by an unstable relational response. Nonlinear amplification, bifurcation, stochastic selection, critical sensitivity, mediator-induced differentiation, and historical accumulation provide additional possibilities. The present paper treats these mechanisms as hypotheses and heuristics whose adequacy depends on future work on GR dynamics.

The resulting framework has a limited objective. It neither derives relational asymmetry universally from prior symmetry nor proposes a general dynamical law of relational differentiation. It makes three distinct regimes formally available within a common GR inquiry: antecedent symmetry, relational asymmetry without a postulated symmetric antecedent, and pre-differentiated structure represented through partiality and restricted definedness. Their separation provides a formal basis for later GR analyses of power, heterogeneous conversion, epistemic relations, and other domains in which the origin and structure of relational asymmetry affect the analysis.

2 Relational Asymmetry in Generative Relational Theory

This section establishes the conceptual role of relational asymmetry within Generative Relational Theory before the paper turns to pre-asymmetry and its formal representation. The objective is to identify what kind of difference requires an asymmetry analysis, which features of GR make that analysis history- and configuration-dependent, and why the concept should remain typed by the relational dimension under consideration. The discussion proceeds from relational position to directional consequence, then to historical and generative dependence, and finally to the scope of later applications. Power is used as an illustrative case because it makes the consequences of relational asymmetry especially visible, while the concept developed here remains broader than power.

2.1 Relational Position and Analytical Scope

GR treats a relational being through the configuration in which it participates, its trajectory of formation, and the transformations that its present relations make possible. A comparison between two beings therefore requires more than a comparison between two inventories of attributes. Two beings can differ in intrinsic or descriptive properties while occupying equivalent relational positions for the purpose of a specified analysis. Conversely, two beings can share many observable properties while differing substantially in dependency, access, responsibility, exposure, or transformative capacity. Relational asymmetry concerns the latter class of differences: distinctions whose significance arises from the organization of relations rather than from non-identity alone.

Let $\mathcal{C}$ denote a relational configuration and let a and b denote two positions within the part of that configuration under analysis. The term *position* is used provisionally. It refers to the relational place occupied by a being within $\mathcal{C}$, including the relevant incoming and outgoing relations, dependencies, permissions, constraints, and available transformations. This formulation leaves open the eventual mathematical type of $\mathcal{C}$. Later sections consider partial maps, inverse-semigroup structures, restriction categories, and group actions precisely because the appropriate representation depends on which relational features have become sufficiently articulated.

Definition 2.1 (Relational difference)**.** A relational difference between positions a and b in a configuration $\mathcal{C}$ is a difference in one or more specified relations, relational capacities, dependencies, or admissible transformations associated with those positions. Relational difference records non-equivalence under the selected description; it does not by itself assign a direction of consequence or a normative status to that difference.

Definition 2.1 keeps the analytical threshold low. A relational difference may matter descriptively while remaining irrelevant to the particular process being studied. For example, two positions may differ in one relation and remain equivalent with respect to access to a resource or the ability to transform a shared configuration. GR therefore requires an additional criterion before such difference is treated as asymmetry.

2.2 Directional Consequence in Relational Asymmetry

Relational asymmetry becomes analytically salient when a relational difference acquires a direction of consequence. Direction here does not require a spatial orientation or a scalar ordering. It means that the

relational organization distinguishes the consequences associated with exchanging, reversing, or otherwise comparing the relevant positions. The exact comparison operation is left open at this stage because the paper later distinguishes total symmetry operations from partial operations whose domains of applicability may still be forming.

A useful preliminary representation is to index asymmetry by a relational dimension α. We write $\Delta_\alpha(a, b \mid \mathcal{C})$ as a placeholder for the structured difference between positions a and b with respect to that dimension. The descriptor is intentionally non-scalar at this stage. It may later encode a structural defect of exchangeability, a difference in directed dependence, a difference in reachability, or another typed relation appropriate to the domain. Treating it as a number before the relevant relational structure has been defined would compress distinctions that later sections need to preserve.

Definition 2.2 (Relational asymmetry, preliminary)**.** Within a specified relational dimension α, positions a and b are provisionally described as relationally asymmetric when their relational organization gives them directionally different consequences, capacities, or constraints under the comparison relevant to α. The description is relative to $\mathcal{C}$ and to the specified dimension; it carries no presumption that asymmetry is primitive, globally defined, or normatively objectionable.

The preliminary status of Definition 2.2 is deliberate. The later analysis of the conditions of relational symmetry will specify the structural conditions under which an exchange or invariance test becomes meaningful, and subsequent sections will replace part of this verbal description with partial and group-theoretic constructions. The present definition supplies the conceptual criterion needed to distinguish asymmetry from generic heterogeneity before those constructions are available.

Several dimensions can therefore exhibit different asymmetry structures within the same configuration. A pair of positions can be comparatively symmetric in access while strongly asymmetric in dependency. They can be similar in formal permission while differing in practical reachability. They can share an institutional role while carrying different historical obligations. This typed view prevents a local asymmetry from being silently generalized into a claim about the relation as a whole.

2.3 Historical and Generative Dependence of Relational Position

GR adds two constraints that make relational asymmetry more than a static comparison. First, relational positions are historically constituted. Second, their significance is partly generative: a present relation matters through the future distinctions, constraints, and possibilities that it helps produce. These constraints make asymmetry dependent on trajectory as well as on current configuration.

A static snapshot can therefore underdescribe an asymmetric relation. Suppose positions a and b presently appear to have comparable formal access to a resource. Their histories can still differ in ways that change the set of credible actions available from the present state. One position may possess accumulated recognition, institutional memory, established channels of coordination, or obligations inherited from prior events. The other may lack those historically generated relations even where an instantaneous inventory of formal rights looks similar. The relevant asymmetry lies partly in the trajectory that has made the present configuration possible.

The same point applies prospectively. A relational difference becomes more important for GR when it changes which further relations can be generated, how readily particular transitions can occur, or how strongly later states depend on actions at the current position. This does not require the paper to adopt a single measure of generative capacity. It requires the formalism to preserve the possibility that asymmetry is expressed through differentiated future reachability rather than only through unequal present holdings.

For this reason, the descriptor $\Delta_\alpha(a, b \mid \mathcal{C})$ should eventually be read together with a trajectory-sensitive relational configuration. A later dynamics may represent histories, event partial orders, or other structures of becoming. The present paper does not settle that choice. Its narrower requirement is that a formal account of asymmetry should remain compatible with historical constitution and generative consequence instead of treating relational positions as timeless labels.

This temporal dependence also clarifies why asymmetry can emerge without a large initial difference. A small event can alter a relation that later becomes self-reinforcing, while repeated interactions can stabilize

a difference that was initially contingent. Such possibilities motivate the later discussion of dynamical hypotheses. They do not establish a general symmetry-breaking mechanism. At the conceptual level, they show why the origin of an asymmetry and its later magnitude can belong to different parts of a relational history.

2.4 Application Scope and the Power Case

The concept developed in this section is intended to travel across several GR domains without collapsing their differences. Epistemic relations may involve asymmetry in access to evidence, credibility, or interpretive capacity. Value conversion may involve asymmetry in the ability to define conversion interfaces, retain value, or impose commensurability. Institutional relations may involve asymmetry in permission, dependence, or reachability. Human–AI relations may exhibit manifestation-level similarity while retaining different relational histories, dependencies, and forms of replaceability. Each case requires its own relational dimension and comparison structure.

Power provides a particularly useful illustration because it makes the limits of an undifferentiated asymmetry concept visible. A later GR theory of power may represent power as a relationally emergent structure that changes the effective possibilities of different subjects. Such a theory would require some account of asymmetry, since equivalent relational positions would provide little basis for explaining differential trajectory-shaping capacity. The converse does not follow. Many asymmetries have no substantial power effect, and an asymmetry can remain descriptively important without producing sustained influence over another subject's feasible trajectories.

The power case therefore supplies a boundary condition for the present paper. Relational asymmetry must be defined broadly enough to support later analyses of power while remaining conceptually independent of power. This separation also prevents a normative conclusion from entering the definition of asymmetry. Asymmetry can support cooperation, specialization, care, expertise, division of responsibility, or domination depending on the relational configuration and its dynamics. Normative evaluation requires additional criteria concerning the formation, mediation, consequences, contestability, and possible transformation of the relation.

The conceptual distinctions established here prepare the transition to pre-asymmetry. Once relational asymmetry is understood as a typed, historically situated, directionally consequential feature of a relational configuration, a prior question becomes unavoidable: the structures needed to identify such an asymmetry may themselves have different antecedent conditions. A comparison may begin from articulated symmetry, from an already asymmetric configuration, or from a regime in which the relevant exchange and equivalence operations have not yet become fully applicable. The next section develops these possibilities as the paper's central pre-asymmetry classification.

3 Ontological Possibilities of Pre-Asymmetry

This section establishes the ontological and analytical status of *pre-asymmetry* before the paper introduces a more specific formal language for symmetry. Its role is to prevent a group-theoretic treatment from silently assuming that symmetry is always the antecedent condition of relational asymmetry. The objective is therefore classificatory rather than dynamical: three regimes are distinguished according to the status of symmetry, asymmetry, and the applicability of the distinction between them. The discussion also states the form of ontological suspension used throughout the paper. These regimes are presented as conditional possibilities within the current GR research programme, rather than as an exhaustive metaphysical taxonomy. Figure 1 previews the three branches and the distinct formal roles they receive in the analysis.

Definition 3.1 (Pre-asymmetry)**.** In this paper, *pre-asymmetry* denotes the conditional analytical domain in which the antecedent formal conditions of a determinate relational asymmetry are examined. The prefix *pre-* marks logical or model-relative precedence and does not by itself assert temporal priority, ontological fundamentality, or a universal developmental sequence.

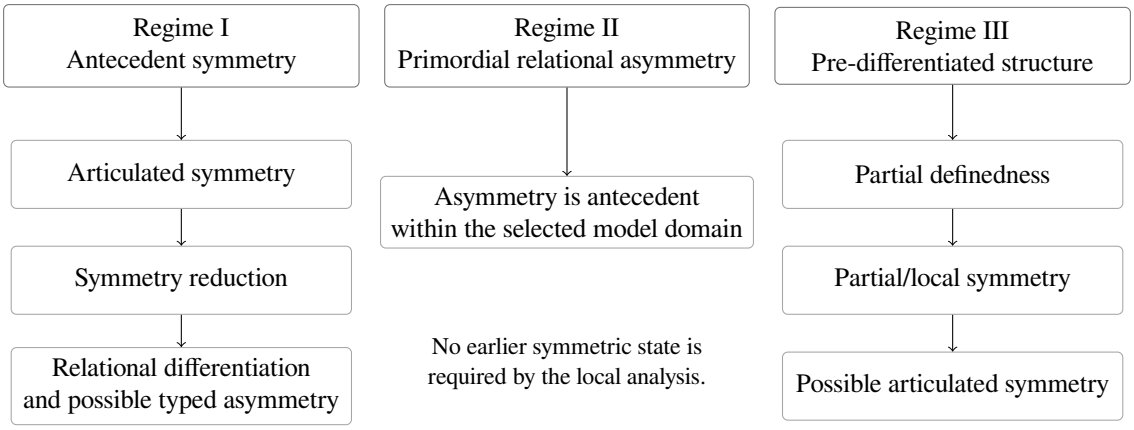

Figure 1: Three conditional pre-asymmetry regimes and the formal routes considered in the paper. The left and right branches admit further structural development; the middle branch preserves asymmetry as antecedent relative to the selected model domain.

Definition 3.1 allows the paper to treat antecedent symmetry, model-relative primordial asymmetry, and pre-differentiated structure within one problem domain without identifying them.

3.1 Antecedent Symmetry

The first regime begins from a relational configuration in which a relevant symmetry has already become meaningful. Positions, transformations, and a criterion of relational equivalence are sufficiently articulated for the configuration to admit an invariance analysis. Two positions can then be compared by asking whether an admissible exchange, reversal, or other transformation preserves the relational structure selected for analysis. In this regime, relational asymmetry may arise when a previously available invariance ceases to hold.

This possibility provides the most direct setting for the later group-theoretic construction. If a configuration admits a symmetry structure, a subsequent reduction of that structure can differentiate positions that were previously exchangeable under the specified transformation family. The resulting asymmetry is therefore generated relative to an antecedent symmetry. The explanatory content of this account depends on the symmetry being defined before the breaking process is invoked. A claim of symmetry breaking presumes that the relevant transformations, invariants, and positions have already acquired enough relational articulation to support a symmetry judgment.

Antecedent symmetry is consequently one possible condition preceding relational asymmetry, rather than the definition of pre-asymmetry itself. GR has no current basis for extending this regime into a universal origin story. Some relational descriptions may begin after symmetry is already established; others may begin with an asymmetry for which no prior symmetric state is available or required; still others may concern conditions in which the symmetry–asymmetry distinction has not yet become fully applicable. Keeping these possibilities separate is necessary for the later formalism to preserve its intended scope.

3.2 Primordial Relational Asymmetry

The second regime treats relational asymmetry as antecedent relative to the available description. The term *primordial* is used here in a restricted sense. It identifies an asymmetry that the model does not derive from an earlier symmetry within the domain under analysis. It does not establish that asymmetry is metaphysically ultimate, temporally first in an absolute sense, or fundamental to ontological reality.

This regime matters because a symmetry-breaking formalism can otherwise create an artificial demand for an earlier symmetric condition. Consider a directed relation whose earliest available and theoretically

relevant description already distinguishes two positions. A model can represent that directed difference without reconstructing an unobserved exchange-symmetric predecessor. Requiring such a predecessor would add an ontological and historical claim that does not follow from the relational description itself.

Primordial relational asymmetry therefore marks a limit on the explanatory scope of symmetry breaking. A later symmetry can still emerge within an already asymmetric configuration, and that symmetry can subsequently change or break. The earlier asymmetry remains antecedent relative to the model unless a further account supports its derivation. This possibility preserves a distinction between explaining the transformation of an asymmetry and explaining the origin of asymmetry as such.

The regime also helps maintain the paper's treatment of history. A theory can begin from the earliest relational structure that is warranted by its evidence or by its formal assumptions without converting that starting point into a claim about ultimate beginnings. In this sense, *primordial* denotes relative primitiveness within a specified analysis. The term keeps open the possibility that a different scale, a longer historical reconstruction, or a different ontology may provide a deeper antecedent structure.

3.3 Pre-Differentiated Relational Regimes

The third regime concerns a condition in which symmetry and asymmetry have not yet acquired a complete domain of applicability. This possibility differs from both antecedent symmetry and primordial asymmetry because the analytical difficulty arises before a symmetry judgment can be made. The relevant relational positions, transformations, comparison rules, or equivalence conditions may remain only partially articulated. Under such conditions, the statement that a configuration is symmetric or asymmetric can exceed what the relational structure presently supports.

The distinction is structural rather than merely epistemic. An observer can lack knowledge about whether a well-defined relation is symmetric, in which case the symmetry predicate remains applicable even though its truth is unknown. A pre-differentiated regime presents a different situation: the operation required to perform the comparison may itself lack a complete domain of definition. The formal question therefore concerns the conditions under which a symmetry judgment becomes well-formed for the relational system under study.

This regime gives positive analytical content to apparent non-applicability. The research object includes the emergence of distinguishable positions, the formation of admissible transformations, the stabilization of comparison criteria, and the development of domains on which relational operations become defined. The paper later uses partial operations, partial symmetry, inverse semigroups, and restriction-categorical structures to represent aspects of this condition. Those tools do not resolve the ontology of the pre-differentiated regime. They provide a formal language for studying relations whose operations are defined only on articulated parts of a larger relational domain.

The pre-differentiated regime also motivates a distinction between *symmetry formation* and *symmetry breaking*. Symmetry formation concerns the emergence of a sufficiently articulated relational structure in which a family of transformations and invariants can be meaningfully specified. Symmetry breaking concerns a later change within such an articulated structure. A relational history may contain one process, both processes, or neither. Their separation avoids treating global symmetry as an automatic starting point for generative relational analysis.

This third regime should not be represented as a simple third truth value alongside "symmetric" and "asymmetric." Such a representation would retain the assumption that a common predicate already applies to all three cases. Likewise, assigning amplitudes to symmetric and asymmetric alternatives would presuppose that those alternatives have already been formed as meaningful states. The present proposal locates the difficulty earlier: the relational grammar required to state the alternatives may itself be only partially available.

3.4 Conditional Formal Inquiry under Ontological Suspension

The three regimes require a methodological stance that can analyze distinct formal possibilities without converting one of them into an unearned ontological commitment. GR adopts ontological suspension for

this purpose. In the present paper, suspension has an active methodological meaning: the analysis keeps multiple ontological possibilities open while examining the relational structures, formal conditions, and generative consequences that would follow under each possibility.

Remark 3.2 (Positive ontological suspension)**.** We do not foreclose ontological possibilities in advance. While suspending final ontological commitment, we conditionally, formally, and generatively analyze the different structures that may be possible.

Remark 3.2 distinguishes ontological restraint from analytical refusal. A refusal would terminate inquiry once an ultimate ontology could not be established. Positive suspension permits a different procedure. A formal construction can be developed conditionally, its assumptions can be stated, its consequences can be examined, and its domain of adequacy can be compared with alternative constructions. The resulting analysis can be substantive even when it does not identify one formalism with reality as such.

This stance also constrains the use of mathematical tools in the remainder of the paper. Group theory is appropriate where a sufficiently articulated symmetry structure exists. Partial algebras, partial group actions, inverse-semigroup structures, and restriction categories become relevant where the definedness of relational operations is itself part of the problem. Dynamical-system tools can be used to formulate hypotheses about the formation or breaking of symmetry. Their mathematical usefulness does not establish that relational reality is ontologically identical to a group, semigroup, category, or dynamical system.

The resulting formal pluralism is relationally constrained. A representation earns its role by preserving the structure relevant to the local research question and by making its assumptions and omissions visible. Different regimes can therefore require different mathematical languages without turning their plurality into arbitrariness. The adequacy of a formalism depends on the relational distinctions it can express and on the transformations it allows the analysis to track.

The classification developed in this section sets the boundary conditions for the formal work that follows. Antecedent symmetry permits an invariance-based analysis and later symmetry reduction. Primordial relational asymmetry remains available where no prior symmetry is warranted within the chosen domain. Pre-differentiated regimes require an account of how transformations and comparison operations acquire domains of applicability before symmetry can be assessed. The next formal task is therefore to specify the relational conditions under which symmetry becomes a meaningful structural property.

4 Conditions of Relational Symmetry

This section specifies the minimum relational structure required before symmetry can function as a well-defined analytical property in Generative Relational Theory (GR). Its role is intermediate: Section 3 kept open several possible conditions preceding relational asymmetry, whereas the present section identifies the structural commitments required by the branch in which symmetry is meaningful. The analysis proceeds through relational positions, admissible transformations, equivalence criteria, and applicability domains. These components are then assembled into a preliminary symmetry-admissible relational frame. The construction is intentionally weaker than a group action. It establishes the conditions under which symmetry can be asked before later sections introduce partial symmetries and globally articulated group-theoretic structure.

4.1 Relational Positions and Comparison Structure

A symmetry judgment requires more than the presence of distinguishable beings. GR treats a relational position as a place within a configuration of dependencies, capacities, constraints, histories, and possible interactions. Two beings may therefore differ in many intrinsic or descriptive respects while occupying equivalent positions relative to the relational structure retained for a particular analysis. Conversely, beings that appear similar under a coarse description may occupy distinct relational positions once their dependencies, histories, or available transformations are taken into account.

The relevant unit of comparison is consequently a relational position within a configuration rather than an isolated object. Let $\mathfrak{X}$ denote a class of relational configurations considered by a local analysis. For each configuration $\mathcal{C} \in \mathfrak{X}$, let $P(\mathcal{C})$ denote the set or structured collection of positions that the analysis is able to distinguish in $\mathcal{C}$. The notation does not imply that positions are metaphysically prior to relations. It records that a symmetry analysis has already reached a level of articulation at which certain relational locations can be tracked across admissible transformations.

This condition is substantive in GR because individuation can itself be historically generated. A pre-differentiated regime may fail to provide a stable pair of positions to exchange, compare, or transform. A later configuration may supply that articulation through repeated interaction, institutional stabilization, material differentiation, or another generative process. The existence of $P(\mathcal{C})$ should therefore be interpreted locally: it states what positions are available to the present formal description, not what ultimate ontology must contain.

Relational comparison also requires a specification of what aspects of a configuration are being compared. Exchangeability with respect to access need not imply exchangeability with respect to historical trajectory, dependency, generative capacity, or responsibility. A symmetry judgment is consequently indexed by a retained relational structure. This indexing prevents the formalism from turning a local invariance into a claim of unrestricted identity.

4.2 Admissible Transformations

Relational positions become symmetry-relevant only when the analysis specifies transformations under which their structural status can be tested. Let $\mathcal{T}$ denote a family of transformations admitted by the local model. A transformation may exchange positions, relabel structurally equivalent roles, reverse an orientation, transport a local configuration, or perform another operation whose preservation properties are analytically meaningful. The choice of $\mathcal{T}$ therefore belongs to the model and must be justified by the relational question under study.

Admissibility has two components. First, a transformation must have a coherent interpretation within the relational domain. An operation that exchanges a teacher and a student may be meaningful in a role-symmetry analysis while being inappropriate in an analysis that fixes historically accumulated obligations. Second, the transformation must be defined on the configuration to which it is applied. These conditions separate a meaningful symmetry test from a purely syntactic permutation of labels.

The distinction also clarifies why object permutation alone is insufficient for GR. If a and b are exchanged, the transformation must specify what happens to the relations incident on those positions. A relational exchange therefore acts on an articulated portion of the configuration, including the relations required by the selected comparison structure. The paper does not yet require every transformation to be globally defined or composable with every other transformation. Those stronger conditions belong to the later development of partial actions and group-theoretic structure.

The family $\mathcal{T}$ may accordingly contain transformations with different domains of applicability. This feature is essential for the pre-differentiated regime described in Section 3. A configuration may support one class of transformations while leaving another class undefined. Symmetry can thus become meaningful unevenly across relational dimensions rather than appearing as a single global property at one instant.

4.3 Relational Equivalence and Preserved Structure

A transformation has symmetry significance only relative to a criterion for deciding whether the transformed configuration preserves the structure relevant to the analysis. GR therefore requires a relational equivalence relation or, more generally, an explicitly stated criterion of structural equivalence. Denote this criterion by $\simeq_\kappa$, where the index κ records the relational content retained by the local model.

The index κ may encode a family of relations, incidence patterns, dependency structures, role assignments, capacities, accessibility conditions, or other features that the analysis treats as invariant-relevant. The notation is deliberately abstract because different GR applications may preserve different structures.

A legal analysis, an epistemic network, a value-conversion system, and a biological interaction system need not share the same invariants even when each admits a meaningful symmetry analysis.

Structural equivalence is weaker than numerical or object identity. A transformation can alter labels, coordinates, or other representational details while preserving the relational structure designated by κ. The distinction is necessary if symmetry is to capture interchangeability of relational positions rather than literal sameness. At the same time, $\simeq_\kappa$ cannot be chosen arbitrarily after observing the desired result. Its retained structure must be fixed by the analytical purpose and stated before the symmetry judgment is evaluated.

The role of invariants can then be understood through equivalence rather than introduced as an independent collection of quantities in every case. If a transformed configuration remains equivalent under $\simeq_\kappa$, the transformation preserves the relational content selected by κ. In applications where explicit invariant functions are useful, they can be added as a more concrete representation of the same preservation requirement. The present framework leaves that choice open so that a qualitative relational structure need not be forced prematurely into a scalar encoding.

4.4 Applicability Domains and Definedness

The symmetry question also depends on whether a transformation is defined for a given configuration. For each transformation $\tau \in \mathcal{T}$, let $D_\tau \subseteq \mathfrak{X}$ denote the configurations on which τ is applicable. The domain D_τ records a structural condition, not merely a computational convenience. Membership in D_τ means that the relational description contains enough articulation for τ to have a coherent action on the configuration.

This definedness condition distinguishes two situations that are easily conflated. In the first, τ is meaningful for $\mathcal{C}$, while available information is insufficient to determine whether the transformed configuration preserves the selected structure. This is an epistemic limitation concerning a well-formed symmetry question. In the second, $\mathcal{C} \notin D_\tau$, so the transformation itself lacks an applicable relational interpretation. The second situation belongs to the pre-differentiated regime and motivates the partial structures developed in the next formal section.

Definedness can be transformation-specific. A configuration may belong to D_{τ_1} while remaining outside D_{τ_2}. This permits a relational system to be sufficiently articulated for one symmetry question and insufficiently articulated for another. It also allows the applicability domains to change over a historical trajectory. A later section will consider the possibility that relational development enlarges, contracts, or reorganizes such domains as positions and operations become stabilized.

The emphasis on definedness gives positive structure to the third pre-asymmetry regime. The formal problem is no longer expressed only as an absence of a truth value. It becomes a study of which relational conditions place a configuration inside the domain of a transformation and how those domains themselves may be generated. Partial algebra and restriction-categorical machinery become relevant precisely because they can represent this domain structure without converting every relational operation into a total map.

4.5 Symmetry-Admissible Relational Frames

The preceding components can be assembled into a preliminary structure that is sufficient for stating a local symmetry judgment while remaining weaker than a group action. Table 1 summarizes the notation used at this layer, and Definition 4.1 assembles those objects into a single frame. The construction is a formal representation for the present paper rather than an ontological definition of relational reality.

Definition 4.1 (Symmetry-admissible relational frame)**.** A *symmetry-admissible relational frame* is a tuple of the form stated in Equation 1.

$$\mathfrak{F} = (\mathfrak{X}, P, \mathcal{T}, \{D_\tau\}_{\tau \in \mathcal{T}}, \simeq_\kappa). \tag{1}$$

Here $\mathfrak{X}$ is a class of relational configurations; P assigns the relational positions available within each configuration; $\mathcal{T}$ is a family of admissible transformations; D_τ is the applicability domain of each $\tau \in \mathcal{T}$; and $\simeq_\kappa$ is the structural equivalence criterion associated with the relational content κ retained by the analysis.

Table 1: Core notation for the symmetry-admissible relational frame.

Formal object	Analytical role
$\mathfrak{X}$	Class of relational configurations retained by the local analysis.
$\mathcal{C}$	A relational configuration within $\mathfrak{X}$.
$P(\mathcal{C})$	Relational positions distinguishable in $\mathcal{C}$ at the selected level of articulation.
$\mathcal{T}$	Family of admissible transformations considered by the local model.
D_τ	Applicability domain of transformation τ.
κ	Index for the relational content or resolution retained by the comparison.
$\simeq_\kappa$	Criterion of structural equivalence relative to the retained content κ.
$\mathfrak{F}$	Symmetry-admissible relational frame collecting the preceding data.

Definition 4.1 does not assume closure of $\mathcal{T}$, global invertibility, the existence of identity transformations on every relevant domain, or associativity of a composition law. Those properties may arise in stronger structures, including partial actions, inverse semigroups, groupoids, and groups. Their absence at this stage is intentional because the paper seeks a framework that can approach symmetry before global symmetry structure has been established.

Within a symmetry-admissible frame, a transformation τ is a symmetry of a configuration $\mathcal{C}$ relative to κ when both definedness and structural preservation hold. The local condition is stated in Equation 2.

$$\mathcal{C} \in D_\tau \qquad \text{and} \qquad \tau(\mathcal{C}) \simeq_\kappa \mathcal{C}. \tag{2}$$

Equation 2 separates the applicability of a transformation from the invariance judgment made after the transformation is applied. If $\mathcal{C} \in D_\tau$ and the second condition fails, the configuration is asymmetric with respect to τ and κ. If $\mathcal{C} \notin D_\tau$, the present frame does not assign either outcome because the transformation required for that symmetry test is unavailable on the configuration.

The definition also makes symmetry explicitly relational and indexed. A configuration may be symmetric relative to one retained structure κ_1 and asymmetric relative to another κ_2. Likewise, two positions may be exchangeable under one family of transformations and distinguished under another. GR therefore treats symmetry as a property of an articulated relational frame, not as an unqualified predicate attached to an isolated object.

The notation in Table 1 is intentionally local to the symmetry-admissible layer; later sections introduce additional objects only when their assumptions become available.

4.6 Structural Threshold for Symmetry

The symmetry-admissible frame identifies a structural threshold at which symmetry becomes formally meaningful. The threshold requires distinguishable relational positions, transformations with interpretable domains, and an equivalence criterion specifying the relational content to be preserved. These elements permit a symmetry judgment without assuming that the transformations already form a group or that the configuration has reached a globally articulated regime.

The threshold also clarifies the three pre-asymmetry possibilities. An antecedent symmetric regime contains relevant transformations that are defined and preserve the selected structure. A primordial asymmetric regime may contain the same basic comparison structure while one or more relevant transformations fail to preserve it from the earliest stage represented by the model. In a pre-differentiated regime, at least one analytically relevant transformation is unavailable because the configuration has not entered its domain of applicability.

5 Partial Symmetry

This section develops a formal language for symmetry in relational regimes where admissible transformations are meaningful only on parts of the configuration space. Its role is to give mathematical content to the pre-differentiated regime introduced in Section 3 and to refine the applicability domains introduced in Section 4. The analysis proceeds in five stages. Partial relational transformations first make definedness explicit. Partial group actions then show how group-indexed transformations can retain a weakened compositional structure without becoming globally defined. Inverse semigroups provide an algebraic language for systems of partial reversible transformations. Restriction categories encode domains of definition directly at the categorical level. A final subsection considers the narrower role that groupoids may play when local reversible relations are organized between articulated domains. The section treats these constructions as complementary representations rather than as competing ontologies.

5.1 Partial Relational Transformations

The symmetry-admissible frame of Definition 4.1 already associates each transformation $\tau \in \mathcal{T}$ with an applicability domain $D_\tau \subseteq \mathfrak{X}$. At the broadest level, this is a partial-algebraic stance: the operations used by the model need not be total on their nominal carrier, and definedness is retained as part of the structure. The present paper does not posit one universal partial algebra for GR, because the arities, carriers, and compositions of relational operations remain application-dependent. It begins with the weaker data needed here, namely individual partial transformations. A natural formal refinement is to represent τ as a partial map whose domain is part of the mathematical data. Equation 3 gives this representation.

$$\tau : D_\tau \longrightarrow \mathfrak{X}, \qquad D_\tau \subseteq \mathfrak{X}. \tag{3}$$

Equation 3 means that applying τ to a configuration outside D_τ is undefined within the present model. Undefinedness here carries a structural interpretation: the relational description has not supplied the articulation required for that transformation to act coherently on that configuration.

A partial transformation need not be reversible. Symmetry, however, concerns a stronger class of transformations for which a local reversal is available. Let $D_\tau^-, D_\tau^+ \subseteq \mathfrak{X}$. A *partial relational bijection* is a bijection $\tau : D_\tau^- \to D_\tau^+$. Its inverse $\tau^{-1} : D_\tau^+ \to D_\tau^-$ is again a partial map. The collection of all partial bijections of a set, composed wherever the usual composition is defined, forms the symmetric inverse monoid on that set; inverse-semigroup theory develops this structure as an algebra of partial symmetries [6].

The local symmetry condition from Equation 2 can therefore be specialized to a partial relational bijection. For a configuration $\mathcal{C} \in D_\tau^-$, the transformation is symmetry-preserving relative to κ when $\tau(\mathcal{C}) \simeq_\kappa \mathcal{C}$. A configuration outside D_τ^- remains outside the applicability domain of that symmetry test. This formal separation keeps three cases distinct: a defined transformation that preserves the retained structure, a defined transformation that fails to preserve it, and a transformation whose action is not defined on the configuration under consideration.

The distinction also prevents partiality from being interpreted as a numerical degree of symmetry. A configuration can support many partial transformations with overlapping but non-identical domains. Its relational articulation is then represented by the pattern of available operations rather than by a single scalar measuring how symmetric the configuration is. This feature will matter when the formalism later considers symmetry formation, because enlargement or reorganization of applicability domains can alter which comparisons become available before any global group action is present.

5.2 Partial Group Actions

A family of partial bijections acquires additional structure when its elements are indexed by a group and retain a weakened form of the group multiplication law. Partial group actions provide a standard mathematical framework for this situation. They replace the globally defined action map of an ordinary group

action with partially defined transformations while preserving identity, inversion, and composition on the domains where those operations are meaningful [2, 4].

Let G be a group with identity e, and let $\mathcal{I}(\mathfrak{X})$ denote the inverse monoid of partial bijections on $\mathfrak{X}$. A convenient formulation of a partial action is a map $\alpha : G \to \mathcal{I}(\mathfrak{X})$ satisfying the relations in Equation 4.

$$\alpha(e) = \mathrm{id}_{\mathfrak{X}}, \qquad \alpha(g^{-1}) = \alpha(g)^{-1}, \qquad \alpha(g)\alpha(h) \subseteq \alpha(gh) \quad (g, h \in G). \tag{4}$$

In Equation 4, inclusion means extension of partial maps: whenever the composite $\alpha(g)\alpha(h)$ is defined, $\alpha(gh)$ is defined there and agrees with the composite. This formulation captures the weakening emphasized in the mathematical literature: the group multiplication remains informative, while the action of a group element need not be defined on the whole carrier set [4]. The algebraic definition supplies a partial action, not yet a symmetry judgment. Within the present GR framework, an element g functions as a partial symmetry at a configuration $\mathcal{C}$ only when $\alpha(g)$ is defined on $\mathcal{C}$ and the transformed configuration satisfies the retained invariance criterion $\alpha(g)(\mathcal{C}) \simeq_\kappa \mathcal{C}$.

For GR, the significance of a partial action is methodological. A candidate transformation family may already possess a coherent algebraic relation among its locally available operations even when relational articulation is incomplete. The group G then indexes possible transformation types, whereas the domains of $\alpha(g)$ record where each type has become applicable. This permits a model to distinguish the algebraic availability of a transformation type from its realized domain in a particular relational regime.

The partial-action representation should nevertheless be introduced only when its assumptions are warranted. Section 4 deliberately allowed a family $\mathcal{T}$ with no closure or composition law. A partial action adds substantial structure: an identity transformation is globally defined, every indexed transformation has a partial inverse, and composition must agree with group multiplication wherever the local composition exists. Relational systems that lack these properties may still admit meaningful partial operations without supporting a partial group action. The present paper therefore treats partial group actions as one formal layer inside the broader pre-asymmetry framework rather than as its universal foundation.

5.3 Inverse-Semigroup Structure

Inverse semigroups provide a second algebraic description of partial reversible transformations. An inverse semigroup is a semigroup S in which each element s has a unique inverse-like element $s^\dagger$ satisfying the relations in Equation 5. This definition generalizes the algebraic reversibility of groups while allowing nontrivial idempotents associated with partial domains [6].

$$ss^\dagger s = s, \qquad s^\dagger s s^\dagger = s^\dagger. \tag{5}$$

Equation 5 does not make every element globally invertible. For a partial bijection $s : D_s^- \to D_s^+$, the element $s^\dagger$ is its inverse bijection on the image. The composites $s^\dagger s$ and $ss^\dagger$ are partial identities on the domain and image, respectively. Equation 6 records these two idempotents.

$$e_s = s^\dagger s, \qquad r_s = ss^\dagger. \tag{6}$$

In the partial-bijection representation, Equation 6 gives e_s as the identity restricted to D_s^- and r_s as the identity restricted to D_s^+. This feature is especially useful for GR because the algebra retains information about where a symmetry operation is defined. The idempotents can therefore represent articulated subdomains without replacing those domains by an external annotation.

Let $\mathcal{S}_0 \subseteq \mathcal{I}(\mathfrak{X})$ be a chosen family of admissible partial relational bijections, closed under neither composition nor inversion at the outset. To interpret these bijections as partial symmetries, the local model must additionally specify that the retained relational content is preserved wherever a generator is applied. If this preservation condition is stable under the compositions and inversions retained by the model, the inverse subsemigroup generated by $\mathcal{S}_0$, denoted $\langle \mathcal{S}_0 \rangle_{\mathrm{inv}}$, provides a candidate algebra of partial relational symmetries. This is a GR construction proposed for the present framework. Its analytical meaning depends on the choice of $\mathcal{S}_0$: the generators must correspond to relational transformations justified by the local

model, and their compositions should be interpreted only within the domains generated by partial-map composition.

This construction also clarifies the relation between groups and partial symmetries. A group can appear as a special regime in which the relevant symmetry transformations are total and their domain idempotents coincide with the global identity. The inverse-semigroup layer therefore allows articulated global symmetry to be represented as a stronger case without requiring globality at the pre-differentiated stage. The paper uses this inclusion as a formal possibility, not as a claim that every relational system must historically develop from an inverse semigroup into a group.

The connection between partial group actions and inverse semigroups is established in the mathematical literature. Exel associated partial group actions with actions of a universal inverse semigroup, and later treatments make the correspondence and globalization structure explicit [2, 4]. Within the present paper, this result supports the compatibility of the two algebraic languages. It does not determine which language should be primary in a given GR application. Partial actions emphasize group-indexed transformation types; inverse semigroups emphasize the algebra generated by the partial symmetries themselves.

5.4 Restriction-Categorical Definedness

Partial maps can also be organized categorically. Restriction categories were developed to abstract the structure of categories of partial maps by assigning each arrow a restriction idempotent that records the portion of its domain on which it is defined [1]. The elementary example is the category of sets and partial functions. For a partial map $f : X \rightharpoonup Y$, its restriction $\overline{f} : X \rightharpoonup X$ acts as the identity exactly where f is defined and is undefined elsewhere [1].

Applied to the partial relational transformation in Equation 3, the corresponding restriction map can be represented as in Equation 7.

$$\overline{\tau}(\mathcal{C}) = \begin{cases} \mathcal{C}, & \mathcal{C} \in D_\tau, \\ \text{undefined}, & \mathcal{C} \notin D_\tau. \end{cases} \tag{7}$$

Equation 7 turns applicability into an internal piece of the formal structure. In the category of partial maps, the restriction satisfies $\tau\,\overline{\tau} = \tau$, expressing that applying the domain restriction before τ does not change the partial transformation on its defined region. Cockett and Lack abstract this behavior, together with compatible axioms for composition, into the notion of a restriction category [1].

For the present GR programme, the main value of restriction structure is conceptual precision about definedness. The restriction idempotent $\overline{\tau}$ can encode the articulated region on which a relational operation has meaning. This yields a formal separation between the existence of an applicability domain and the later invariance judgment evaluated inside that domain. Section 4 expressed the separation externally through D_τ; restriction structure makes the same distinction available internally through a morphism associated with τ.

The categorical language also permits different objects to carry different local domains and transformation structures. This flexibility may become useful when the paper moves beyond a single configuration class toward heterogeneous relational systems. The current section does not construct a full GR restriction category because the objects, morphisms, and composition laws required by such a category depend on the intended application. The result established here is narrower: restriction-categorical definedness supplies an appropriate formal model for the claim that applicability itself can be structured and compositional.

5.5 Local Reversibility and Groupoid Interfaces

Groupoid language becomes potentially useful when locally articulated domains are treated as separate objects and reversible transformations are permitted between them. A groupoid is a category in which every arrow is invertible. If articulated relational subdomains $U, V, \ldots$ are taken as objects and an admissible bijection $\tau : U \to V$ is taken as an arrow, then locally reversible transformations can be composed whenever codomain and domain match. Under suitable closure conditions, such data form a groupoid.

The groupoid perspective differs from the inverse-semigroup perspective in emphasis. An inverse semigroup keeps partial transformations as elements of a single algebra and records their local domains through idempotents. A groupoid makes the local domains explicit as objects and treats reversible passages among them as arrows. The two perspectives can therefore illuminate different relational questions even when they encode closely related partial-symmetry information.

The present paper leaves the exact groupoid construction open. Introducing one prematurely would require decisions about what counts as an object, whether objects represent configuration subspaces or relational position types, and which local transformations deserve compositional closure. These choices are application-dependent. Groupoids are therefore retained as a candidate interface for local symmetry rather than incorporated into the minimal formalism developed in this section.

5.6 Scope of the Partial-Symmetry Layer

The partial-symmetry layer gives the pre-differentiated regime a positive formal structure. Partial maps represent transformation-specific applicability. Partial group actions add a weakened group-indexed composition law when such structure is justified. Inverse semigroups organize systems of partial reversible transformations and retain domain information through idempotents. Restriction categories internalize definedness at the categorical level. Groupoids provide a possible representation when local reversible domains deserve explicit object status.

These formalisms answer a limited but important question: how can relational symmetry be represented when globally defined symmetry transformations are not yet available? They do not establish a unique process through which partial symmetry becomes global symmetry, nor do they establish that every pre-differentiated regime admits all of these structures. Their common analytical contribution is to make partiality mathematically explicit while preserving the distinction between undefined transformations and defined transformations that fail an invariance test.

The next section therefore turns to a separate problem. Once partial transformations and their domains are available, relational development may alter those domains, stabilize positions, and support a more globally articulated symmetry structure. The formation of such a structure requires additional assumptions about relational history and dynamics. It cannot be inferred from partiality alone.

6 Formation of Articulated Symmetry

This section develops a structural account of how a relational regime may acquire a more articulated symmetry structure after partial transformations have become available. Its role is to connect the partial-symmetry layer of Section 5 with the group-theoretic treatment developed in the next section. The objective is limited: the analysis identifies forms of relational stabilization that can make broader symmetry judgments possible, while leaving the underlying dynamics open. Historical articulation, transformation domains, compositional coherence, and the relation between local and more global symmetry are treated as distinct components. The resulting picture is a candidate formation scheme within GR rather than a universal developmental sequence.

6.1 Historical Articulation of Relational Positions

GR treats relational positions as historically constituted. The set or structured collection $P(\mathcal{C})$ introduced in Section 4 therefore need not remain fixed across a relational history. Repeated interaction can stabilize a distinction that was previously transient, a role can acquire continuity across successive configurations, and a previously available distinction can lose its relevance when the relations supporting it dissolve. Symmetry formation accordingly requires more than the appearance of additional transformations. It also requires enough persistence in the positions on which those transformations act for comparison across configurations to be meaningful.

A local formal representation can index symmetry-admissible frames by a historical ordering. Let $(I, \preceq)$ be a partially ordered index set representing the event or historical ordering relevant to a particular analysis. For a local modeling window in which the configuration class $\mathfrak{X}$ and candidate transformation family $\mathcal{T}$ can be held fixed, the evolving frame can be represented as in Equation 8.

$$\mathfrak{F}_i = (\mathfrak{X}, P_i, \mathcal{T}, \{D_{\tau,i}\}_{\tau \in \mathcal{T}}, \simeq_{\kappa,i}), \qquad i \in I. \tag{8}$$

Equation 8 is a bookkeeping device for relational development. The index i need not be clock time, and the partial order permits histories in which some events are ordered while others remain incomparable. The representation allows the available positions, applicability domains, and equivalence criterion to vary along the history without requiring the paper to identify one universal temporal parameter.

Historical comparability introduces a further modeling obligation. If a position at index i is said to persist to index j, the analysis must specify what supports that persistence. Depending on the application, continuity may be supplied by an institutional role, a causal trajectory, a record of interaction, a material carrier, a legal identity, or another relation that permits the position to be tracked. GR does not infer persistence from a repeated label alone. This condition is especially important when a later symmetry appears to exchange positions that did not exist as stable relational locations at an earlier stage.

The family in Equation 8 also permits symmetry formation to be path-dependent. Two histories can reach configurations with similar observable position structures while differing in the relations that stabilized those positions. A later symmetry judgment may therefore preserve a present relational pattern while leaving distinct historical structures outside the retained content κ. The choice of κ must make this scope explicit when historical trajectory matters to the intended comparison.

6.2 Applicability-Domain Development

Partial symmetry makes transformation-specific domains D_τ explicit. Symmetry formation can therefore be studied through changes in those domains. A transformation that is initially meaningful only on a narrow class of configurations may later acquire a broader applicability domain as relational positions, comparison rules, and supporting structures become stabilized. Domain development can also proceed in the opposite direction when articulation is lost or when historical changes invalidate an earlier operation.

A simple monotone expansion model can be useful in cases where articulation accumulates over the selected historical interval. For $i \preceq j$, the candidate condition is stated in Equation 9.

$$D_{\tau,i} \subseteq D_{\tau,j}. \tag{9}$$

Equation 9 is a special-case representation rather than a general law of GR. Relational histories can contract, split, or reorganize applicability domains, and two transformations can develop in different directions. The analytical value of the inclusion is narrower: it gives a precise representation of one way in which an initially partial operation can become applicable across a larger relational regime.

Domain development can be heterogeneous across transformations. A configuration may enter the domains of several local exchanges while remaining outside the domain of a transformation that requires a broader relational comparison. This permits symmetry structure to form unevenly. Local exchangeability in one subsystem, for example, does not by itself establish exchangeability across the whole configuration. A later globally articulated symmetry therefore requires coordination among applicability domains rather than the isolated expansion of a single domain.

The same point applies to the retained structure κ. A transformation can become defined before the analysis has a stable criterion for the relational content it is expected to preserve. Conversely, an equivalence criterion can be available while the transformations needed to test it remain partial. Formation of articulated symmetry requires these components to become mutually usable within a common analytical regime.

6.3 Compositional Stabilization

A collection of individually defined partial transformations does not yet provide a coherent symmetry structure. Their compositions must also become interpretable on appropriate relational domains. For partial

transformations τ and σ, the domain of the ordinary composite is stated in Equation 10.

$$D_{\sigma\circ\tau} = \{\mathcal{C} \in D_\tau : \tau(\mathcal{C}) \in D_\sigma\}. \tag{10}$$

Equation 10 shows why expansion of the individual domains does not automatically produce compositional coherence. A configuration can lie in D_τ and D_σ while falling outside $D_{\sigma\circ\tau}$ because the image under τ is not a configuration on which σ is defined.

Compositional stabilization refers to a relational regime in which a selected family of transformations can be applied in sufficiently consistent sequences for their algebraic relations to become meaningful. This can involve enlargement of composition domains, stabilization of the images of transformations, and the appearance of identity-like or inverse-like operations on the selected regime. The present section does not require these properties to form a group. It identifies them as structural conditions that make the stronger group-theoretic representation developed in the next section available.

The distinction between individual definedness and compositional stabilization is important for the interpretation of pre-differentiated regimes. A system can support several meaningful local exchanges without supporting a coherent algebra of exchanges. In that case, symmetry exists locally relative to particular operations while the broader transformation structure remains only partially articulated. The development of composition is therefore one possible dimension of symmetry formation alongside the stabilization of positions and applicability domains.

Inverse-semigroup and restriction-categorical representations from Section 5 remain useful during this stage. Their idempotent and restriction structure retains information about where transformations and composites are defined. As articulation develops, these domain structures can reveal whether a selected family is approaching a regime in which the same transformations act throughout a common relational region. The interpretation remains structural: the algebra records the organization of partiality, while a separate dynamical theory would be required to explain why the organization changes along a particular history.

6.4 Global Articulation of Local Symmetries

The term *global* is used here relative to a selected relational regime rather than to the whole of relational reality. Let $\mathfrak{X}_* \subseteq \mathfrak{X}$ be the configurations retained for a local analysis and let $\mathcal{T}_* \subseteq \mathcal{T}$ be a selected transformation family. A basic condition for global definedness on that regime is stated in Equation 11.

$$\mathfrak{X}_* \subseteq \bigcap_{\tau\in\mathcal{T}_*} D_\tau, \qquad \tau(\mathfrak{X}_*) \subseteq \mathfrak{X}_* \quad \text{for each } \tau \in \mathcal{T}_*. \tag{11}$$

Equation 11 states that every selected transformation is defined throughout $\mathfrak{X}_*$ and maps the selected regime back into itself. It establishes a common domain on which the transformations can be treated as total maps relative to that regime. Additional requirements concerning composition, identities, inverses, and preservation of $\simeq_\kappa$ are needed before the transformations constitute the group-theoretic symmetry structure developed in the next section.

The passage from partial to global action has established mathematical treatments in the theory of partial group actions, where suitable partial actions can admit globalizations or enveloping actions under specified conditions [4]. The GR notion of symmetry formation is broader than this mathematical globalization problem. A globalization begins with an already specified partial action and asks whether it can be represented within a suitable global action. GR also permits the transformation domains, relational positions, and equivalence criteria themselves to emerge or reorganize historically. Mathematical globalization can therefore serve as one formal component when its assumptions are satisfied, while the historical formation of the structure to be globalized remains a separate relational question.

This distinction helps avoid a retrospective reading of partial symmetry. A later global action does not imply that the same group structure existed latently at every earlier stage. Earlier configurations may have supported only a smaller family of partial transformations, or may have lacked some of the positions required by the later action. The later symmetry can be treated as genuinely articulated within the model

even when a mathematical embedding permits the completed structure to be represented in a larger global space.

Articulated symmetry consequently has two senses that should remain distinct. The first concerns *formal articulation*: transformations are sufficiently defined and compositionally coherent to support a stronger algebraic structure. The second concerns *historical formation*: the relations required by that formal structure have become stable along a trajectory of the system. A formal globalization may establish the first sense without explaining the second.

6.5 Formal Scope of Symmetry Formation

The formation picture developed in this section is deliberately conditional. It identifies several dimensions along which a relational regime can become more suitable for global symmetry analysis: positions can acquire historical persistence, applicability domains can expand or reorganize, compositions can become coherent, and a selected family of transformations can become total on a common relational regime. None of these developments is assumed to be monotone, irreversible, or universal.

The framework also leaves open histories in which articulated global symmetry never forms. A relational regime may remain locally structured through partial symmetries; different regions may support incompatible transformation families; or primordial asymmetries may constrain the available operations from the earliest stage represented by the model. These possibilities remain consistent with the three-regime classification in Section 3.

Most importantly, the present section supplies structural conditions rather than a settled dynamics. The history-indexed family in Equation 8 organizes possible changes in relational articulation, while Equations 9–11 identify candidate patterns within that change. They do not specify the causal law that produces the transition. Perturbative, nonlinear, stochastic, critical, and historically cumulative mechanisms will later be treated as provisional dynamical hypotheses with distinct assumptions and scopes.

Once a selected regime supports globally defined, compositionally coherent transformations, the remaining task is to specify the algebraic structure of those transformations and the equivalence they preserve. The next section develops the group-theoretic representation appropriate to that articulated case. This progression keeps symmetry formation conceptually separate from symmetry breaking: the former concerns the constitution of a symmetry-capable relational regime, while the latter concerns a change in an already articulated symmetry structure.

7 Relational Symmetry

This section develops the group-theoretic layer appropriate to a relational regime in which symmetry has become sufficiently articulated for globally defined, compositionally coherent transformations to act on a common domain. Its role is to formalize the stronger symmetry structure prepared by Sections 4–6. The objective is to define relational symmetry, interchangeability, orbit structure, and position-specific stabilizers without identifying structural equivalence with ontological identity. The construction remains indexed by the relational content retained through κ. Group structure therefore represents symmetry within a specified relational description rather than an unrestricted symmetry of a being as such.

7.1 Group Actions on an Articulated Relational Regime

Let $\mathfrak{X}_* \subseteq \mathfrak{X}$ be a selected articulated regime of relational configurations, and let G be a group. The formation conditions considered in Section 6 make it meaningful, in the present branch of the framework, to consider a total action of G on $\mathfrak{X}_*$. The action is stated in Equation 12.

$$\alpha : G \times \mathfrak{X}_* \longrightarrow \mathfrak{X}_*, \qquad \alpha(e, \mathcal{C}) = \mathcal{C}, \qquad \alpha(g, \alpha(h, \mathcal{C})) = \alpha(gh, \mathcal{C}). \tag{12}$$

Equation 12 introduces ordinary group action only after the selected transformations have acquired common definedness and the required composition law. The identity $e \in G$ acts trivially, and group multiplication

represents composition of the corresponding relational transformations on the selected regime. Nothing in this definition requires G to exhaust the admissible transformations of relational reality. It organizes the transformation family retained by the local model.

The equivalence criterion $\simeq_\kappa$ introduced in Section 4 must interact coherently with the action if symmetry is to be defined on equivalence classes rather than on arbitrary representational details. The required compatibility is stated in Assumption 7.1.

Assumption 7.1 (Retained equivalence and action compatibility)**.** Within the articulated regime $\mathfrak{X}_*$, the relation $\simeq_\kappa$ is an equivalence relation. For configurations $\mathcal{C}, \mathcal{C}' \in \mathfrak{X}_*$ and $g \in G$, it is also compatible with the group action in the sense stated in Equation 13.

$$\mathcal{C} \simeq_\kappa \mathcal{C}' \quad \Longrightarrow \quad \alpha(g, \mathcal{C}) \simeq_\kappa \alpha(g, \mathcal{C}'). \tag{13}$$

Assumption 7.1 states the equivalence properties used below and the compatibility condition in Equation 13. The group action therefore respects the relational distinctions represented by $\simeq_\kappa$, and the action descends to the quotient of $\mathfrak{X}_*$ by the retained equivalence. The assumption is substantive: if a candidate transformation fails to preserve the meaning of the equivalence criterion itself, the transformation and the criterion do not jointly define the symmetry structure considered here.

For a fixed configuration $\mathcal{C} \in \mathfrak{X}_*$, define the relational symmetry group relative to κ by Equation 14.

$$G_\kappa(\mathcal{C}) = \{g \in G \mid \alpha(g, \mathcal{C}) \simeq_\kappa \mathcal{C}\}. \tag{14}$$

Equation 14 collects the transformations that preserve the retained relational structure of $\mathcal{C}$, allowing representational changes that remain inside its $\simeq_\kappa$-equivalence class. This construction is therefore more flexible than requiring literal equality $\alpha(g, \mathcal{C}) = \mathcal{C}$.

Proposition 7.2 (Subgroup of retained relational symmetries)**.** *Under Assumption 7.1, $G_\kappa(\mathcal{C})$ is a subgroup of G for each $\mathcal{C} \in \mathfrak{X}_*$.*

Proof. The identity belongs to $G_\kappa(\mathcal{C})$ because $\alpha(e, \mathcal{C}) = \mathcal{C}$. If $g, h \in G_\kappa(\mathcal{C})$, then $\alpha(h, \mathcal{C}) \simeq_\kappa \mathcal{C}$. Assumption 7.1 gives $\alpha(g, \alpha(h, \mathcal{C})) \simeq_\kappa \alpha(g, \mathcal{C})$, while $\alpha(g, \mathcal{C}) \simeq_\kappa \mathcal{C}$. Transitivity of $\simeq_\kappa$ therefore gives $\alpha(gh, \mathcal{C}) \simeq_\kappa \mathcal{C}$. Finally, if $g \in G_\kappa(\mathcal{C})$, compatibility applied to $\alpha(g, \mathcal{C}) \simeq_\kappa \mathcal{C}$ with g^{-1} yields $\mathcal{C} \simeq_\kappa \alpha(g^{-1}, \mathcal{C})$, and symmetry of $\simeq_\kappa$ gives $\alpha(g^{-1}, \mathcal{C}) \simeq_\kappa \mathcal{C}$. Thus $G_\kappa(\mathcal{C})$ contains identities and inverses and is closed under multiplication. □

Proposition 7.2 gives the articulated regime a precise group-theoretic symmetry object. The result depends on the retained equivalence and on the transformation group chosen by the model. A different index κ can therefore yield a different symmetry subgroup for the same underlying configuration. Relational symmetry remains typed by the structure whose preservation matters to the local analysis.

7.2 Position Actions and Relational Interchangeability

Configuration-level symmetry does not by itself specify how individual relational positions are transported. To discuss interchangeability, the model must also provide a coherent action of $G_\kappa(\mathcal{C})$ on the positions retained for the symmetry analysis. Let $P_\kappa(\mathcal{C})$ denote the relational positions distinguished at the resolution encoded by κ. A position action is a homomorphism of the form stated in Equation 15.

$$\rho_{\mathcal{C},\kappa} : G_\kappa(\mathcal{C}) \longrightarrow \mathrm{Bij}(P_\kappa(\mathcal{C})). \tag{15}$$

Equation 15 represents the induced transport of relational positions by symmetry transformations that preserve the retained configuration structure. The notation assumes that equivalent representatives of the configuration have been provided with a coherent position identification at the resolution κ. Where such transport is unavailable, a configuration-level symmetry does not yet justify a claim of position-level interchangeability.

Definition 7.3 (Relational interchangeability)**.** Let $a, b \in P_\kappa(\mathcal{C})$. The positions a and b are *relationally interchangeable relative to* $(\mathcal{C}, \kappa, G)$ when there exists $g \in G_\kappa(\mathcal{C})$ such that

$$\rho_{\mathcal{C},\kappa}(g)(a) = b. \tag{16}$$

Definition 7.3 makes interchangeability explicitly relative to an articulated relational structure. Equation 16 does not assert that a and b are identical beings, that their histories coincide, or that they are interchangeable under a different retained structure. It states that the selected symmetry group contains a transformation transporting one retained relational position to the other.

A literal exchange of two positions is a stronger condition than membership in the same symmetry orbit. For some applications, one may require an involutive symmetry $\sigma_{ab} \in G_\kappa(\mathcal{C})$ satisfying the relations in Equation 17.

$$\rho_{\mathcal{C},\kappa}(\sigma_{ab})(a) = b, \qquad \rho_{\mathcal{C},\kappa}(\sigma_{ab})(b) = a, \qquad \sigma_{ab}^2 = e. \tag{17}$$

Equation 17 defines a direct exchange symmetry when the application requires an explicit two-way exchange generated by one involution. Relational interchangeability in Definition 7.3 is broader: a sequence encoded by a general group element can relate two positions even when no single involution exchanges them. Keeping these notions separate prevents a transposition-specific intuition from being imposed on every symmetry structure.

7.3 Orbit Structure of Relational Positions

The group action in Equation 15 partitions retained positions into orbits. For $a \in P_\kappa(\mathcal{C})$, the orbit is defined in Equation 18.

$$\mathrm{Orb}_{\mathcal{C},\kappa}(a) = \{\rho_{\mathcal{C},\kappa}(g)(a) \mid g \in G_\kappa(\mathcal{C})\}. \tag{18}$$

Equation 18 collects the positions reachable from a by symmetries of the retained relational structure. The orbit therefore identifies a class of positions that are mutually interchangeable relative to $(\mathcal{C}, \kappa, G)$.

Proposition 7.4 (Orbit criterion for relational interchangeability)**.** *For $a, b \in P_\kappa(\mathcal{C})$, the positions are relationally interchangeable in the sense of Definition 7.3 if and only if*

$$\mathrm{Orb}_{\mathcal{C},\kappa}(a) = \mathrm{Orb}_{\mathcal{C},\kappa}(b). \tag{19}$$

Proof. If a and b are interchangeable, there exists $g \in G_\kappa(\mathcal{C})$ with $\rho_{\mathcal{C},\kappa}(g)(a) = b$. Group closure and invertibility then imply that every position reachable from a is reachable from b and conversely, so the two orbits coincide. If the orbits coincide, then $b \in \mathrm{Orb}_{\mathcal{C},\kappa}(a)$, and Equation 16 holds for some $g \in G_\kappa(\mathcal{C})$. □

Proposition 7.4 and Equation 19 give the paper a precise structural meaning for the phrase *equivalent relational position*. Same-orbit membership records symmetry-relative interchangeability. It does not establish equality of the relata outside the retained structure. A human and an artificial agent, two institutional offices, or two nodes in a network could occupy the same orbit under one deliberately coarse symmetry analysis while remaining strongly differentiated under another κ that retains history, embodiment, responsibility, dependency, or another relational feature. Orbit equivalence therefore remains indexed and revisable.

This orbit language is also the object needed by the subsequent analysis of relational differentiation. If a later symmetry structure is smaller than the one considered here, an orbit can separate into several orbits under the residual group. The next section will use that possibility to characterize one route by which previously interchangeable relational positions become asymmetric. The present section stops before making that dynamical or historical claim.

7.4 Stabilizers and Position-Specific Symmetry

Orbit structure records which positions can be related by the symmetry group. Stabilizers record the complementary information of which transformations leave a selected position fixed. For $a \in P_\kappa(\mathcal{C})$, define the stabilizer by Equation 20.

$$\mathrm{Stab}_{\mathcal{C},\kappa}(a) = \{g \in G_\kappa(\mathcal{C}) \mid \rho_{\mathcal{C},\kappa}(g)(a) = a\}\,. \tag{20}$$

Equation 20 identifies the subgroup of retained symmetries compatible with holding a fixed. It can therefore encode position-specific isotropy inside a configuration whose broader relational structure remains symmetric.

Positions in the same orbit have correspondingly related stabilizers. If $b = \rho_{\mathcal{C},\kappa}(g)(a)$ for some $g \in G_\kappa(\mathcal{C})$, their stabilizers satisfy Equation 21.

$$\mathrm{Stab}_{\mathcal{C},\kappa}(b) = g\ \mathrm{Stab}_{\mathcal{C},\kappa}(a)\, g^{-1}. \tag{21}$$

Equation 21 shows that same-orbit positions can have distinct stabilizer subgroups as literal subsets of $G_\kappa(\mathcal{C})$ while retaining the same stabilizer structure up to conjugation. This distinction becomes useful when later symmetry reduction changes which transformations remain available. Relational differentiation may then be visible both through orbit separation and through changes in the corresponding stabilizer structures.

The stabilizer formalism also reinforces the local meaning of symmetry. A position can possess a large stabilizer under one retained structure and a smaller one under another. Likewise, the same configuration can admit several relevant symmetry groups because different analyses preserve different relational content. The paper therefore treats orbit and stabilizer data as properties of an indexed formal representation, rather than as complete descriptions of the beings occupying those positions.

7.5 Scope of the Articulated Symmetry Layer

The articulated symmetry layer supplies a precise vocabulary for a branch of pre-asymmetry in which relational symmetry is already well-defined. A compatible group action on configurations gives rise to the subgroup $G_\kappa(\mathcal{C})$ of transformations preserving the retained relational structure. A coherent position action then permits interchangeability to be represented through group orbits and position-specific symmetry through stabilizers. These constructions make the later idea of orbit differentiation mathematically meaningful.

The layer remains conditional in several respects. It presupposes an articulated regime, a justified transformation group, a retained equivalence criterion, and coherent transport of relational positions. It applies most directly to the antecedent-symmetry branch of Section 3 and to later regimes in which symmetry has formed through processes such as those considered in Section 6. Primordial asymmetry does not require this group-theoretic antecedent, and pre-differentiated regimes may remain outside its domain until the relevant transformations become sufficiently defined.

The formalism also preserves GR's ontological restraint. Group action, orbit equivalence, and stabilizer structure describe invariances of a selected relational representation. They do not establish that the represented beings are ultimately identical, that the selected symmetry is exhaustive, or that relational reality possesses one privileged group structure. Their role is narrower: they identify when a specified family of transformations renders selected relational positions structurally interchangeable and thereby provide a reference structure against which later relational differentiation can be stated.

The next section uses this reference structure to analyze symmetry reduction. Once the symmetry group preserving a configuration is reduced, positions that shared an orbit under the earlier group may cease to be interchangeable under the residual group. That change supplies one formal route from articulated symmetry to generated relational asymmetry while leaving the other pre-asymmetry regimes open.

8 Relational Differentiation under Symmetry Reduction

This section develops one formal route by which an articulated relational symmetry can become differentiated. Its role is to connect the group-theoretic symmetry structure of Section 7 with the preliminary concept of relational asymmetry introduced in Section 2. The analysis proceeds in three stages. First, it specifies the conditions under which symmetry groups associated with two historically related configurations can be compared as antecedent and residual structures. Second, it characterizes orbit refinement and orbit splitting under a residual subgroup. Third, it distinguishes the structural fact of lost interchangeability from the stronger, typed claim of relational asymmetry. The construction is kinematic in the present section: it describes a change in symmetry structure while leaving the mechanism that produces that change to the dynamical hypotheses considered later.

8.1 Residual Symmetry across Comparable Configurations

Symmetry reduction requires more than two configurations with symmetry groups of different sizes. The positions and transformations being compared must be connected across the relational history in a way that supports a meaningful antecedent–residual interpretation. Let $\mathcal{C}^-$ and $\mathcal{C}^+$ denote two configurations ordered within the historical window of analysis, with $\mathcal{C}^-$ antecedent to $\mathcal{C}^+$. Let $P_\kappa(\mathcal{C}^-)$ and $P_\kappa(\mathcal{C}^+)$ be their retained position sets. A historical tracking map is written as in Equation 22.

$$\iota : P_\kappa(\mathcal{C}^-) \rightharpoonup P_\kappa(\mathcal{C}^+). \tag{22}$$

Equation 22 is allowed to be partial because some earlier positions can disappear, merge, or lose the continuity required by the retained description. The map does not infer persistence from a repeated label. It records only the positions for which the application supplies a defensible historical identification of the kind required in Section 6.

Write the retained symmetry groups of the two configurations as $G^- = G_\kappa(\mathcal{C}^-)$ and $G^+ = G_\kappa(\mathcal{C}^+)$. Treating G^+ as a residual subgroup of G^- requires an additional comparison structure. The present paper uses the following assumption for the symmetry-reduction case.

Assumption 8.1 (Residual symmetry comparability)**.** There exists an injective homomorphism

$$\jmath : G^+ \hookrightarrow G^- \tag{23}$$

whose induced actions are compatible with the historically tracked positions on the domain of ι.

Under Assumption 8.1, the image $H = \jmath(G^+)$ can be identified with a subgroup of G^-. The symmetry change can then be represented by Equation 24.

$$H \leq G^-. \tag{24}$$

Equation 24 is the structural content of symmetry reduction used in this section. The notation does not state why the reduction occurred, how rapidly it occurred, or whether the change was caused by perturbation, endogenous instability, stochastic selection, institutional mediation, or another process. It also does not cover every possible transition between relational symmetry structures. Two historically related configurations can have symmetry groups that are not naturally comparable by subgroup inclusion; such cases require a different comparison construction.

For the remainder of this section, tracked positions are identified with their images under ι when no ambiguity results. This convention keeps the orbit notation readable while preserving the requirement that a historical bridge be supplied before a position is treated as persisting across the symmetry change.

8.2 Orbit Refinement under Residual Symmetry

A residual subgroup has fewer available transformations than its antecedent group. Its orbits can therefore remain equal to, or become finer than, the orbits generated by the antecedent action. This relation gives a precise meaning to the idea that a previously interchangeable relational position can become differentiated when some symmetries cease to be available.

Let P denote the tracked retained positions on which the antecedent group G^- and residual subgroup H act compatibly. For $a \in P$, write $\mathrm{Orb}_{G^-}(a)$ and $\mathrm{Orb}_H(a)$ for the corresponding orbits.

Proposition 8.2 (Orbit refinement under residual symmetry)**.** *If $H \leq G^-$ acts on P by restriction of the G^- action, then for each $a \in P$,*

$$\mathrm{Orb}_H(a) \subseteq \mathrm{Orb}_{G^-}(a). \tag{25}$$

Consequently, every G^--orbit is partitioned into one or more H-orbits.

Proof. Every $h \in H$ is also an element of G^-. Hence every position reached from a by the H action is also reached from a by the G^- action, which gives Equation 25. Orbits under any group action partition the acted-on set, so the H-orbits contained in a fixed G^--orbit form a partition of that antecedent orbit. □

Proposition 8.2 permits a structural definition of orbit splitting.

Definition 8.3 (Orbit splitting)**.** Let $a, b \in P$ be historically tracked positions. An antecedent orbit is said to *split between a and b under the residual symmetry H* when the conditions in Equation 26 hold.

$$\mathrm{Orb}_{G^-}(a) = \mathrm{Orb}_{G^-}(b), \qquad \mathrm{Orb}_H(a) \neq \mathrm{Orb}_H(b). \tag{26}$$

Definition 8.3 and Equation 26 express a loss of symmetry-relative interchangeability. Before the reduction, some element of G^- transports a to b; under the residual group, no element of H does so. The differentiation is therefore relative to the retained relational structure, the chosen historical tracking, and the antecedent–residual comparison. It does not state that the beings occupying the two positions have become ontologically distinct in a new sense.

Loss of one particular exchange transformation is weaker than orbit splitting. Suppose an antecedent involution $\sigma_{ab} \in G^-$ directly exchanges a and b in the sense of Equation 17. The exclusion of σ_{ab} from H removes that direct exchange, yet another element $h \in H$ may still transport a to b. In that case the direct exchange symmetry has been lost while relational interchangeability survives. Orbit splitting therefore captures the stronger structural change required for the present account of differentiation.

8.3 Relational Differentiation and Typed Asymmetry

Orbit splitting supplies a clear structural notion of relational differentiation, but the preliminary GR definition in Section 2 requires an additional directional consequence before the term *asymmetry* is applied. Two positions can become non-interchangeable without one thereby acquiring greater access, stronger dependency, broader reachability, or another directionally differentiated consequence. The distinction is important because symmetry reduction can generate heterogeneity that remains neutral with respect to the relational dimension being studied.

Definition 8.4 (Symmetry-reduction-generated relational differentiation)**.** Historically tracked positions a and b exhibit *symmetry-reduction-generated relational differentiation* when they satisfy the orbit-splitting condition in Equation 26 for an antecedent symmetry group G^- and residual subgroup H.

Definition 8.4 is deliberately structural. To connect it with the typed descriptor $\Delta_\alpha(a, b \mid \mathcal{C})$ introduced in Section 2, let $\mathsf{A}_\alpha(a, b \mid \mathcal{C}^+)$ denote the proposition that a and b satisfy the directional-consequence criterion for relational asymmetry in dimension α at the later configuration. The notation is a typed predicate, not a scalar measure of asymmetry.

Definition 8.5 (Symmetry-reduction-generated relational asymmetry)**.** A symmetry-reduction-generated relational differentiation between a and b constitutes a *symmetry-reduction-generated relational asymmetry in dimension* α when the typed predicate $\mathsf{A}_\alpha(a, b \mid \mathcal{C}^+)$ holds.

Definition 8.5 preserves the typed character of asymmetry. The same orbit split can matter differently across relational dimensions. A pair of positions can cease to be interchangeable in institutional role while remaining equivalent in access to a resource; a differentiation in epistemic authority can arise without a corresponding differentiation in formal permission; a reachability difference can appear while present holdings remain comparable. The domain-specific semantics of A_α must therefore be supplied by the application rather than by the group-theoretic structure alone.

This separation also clarifies the relation between Δ_α and the symmetry formalism. Orbit splitting can provide a structural source or witness for a later typed asymmetry descriptor, but it does not determine the mathematical type of Δ_α. Depending on the domain, the descriptor may later be represented by an ordered relation, a difference in admissible transformations, a reachability defect, a dependency structure, or another object suited to α. The present paper keeps that choice open.

8.4 Residual Stabilizers and Position-Specific Change

Orbit separation records whether positions remain mutually reachable by the residual symmetry. Stabilizers provide complementary information about the symmetries retained at each position. Under the subgroup identification in Equation 24, the residual stabilizer of a tracked position a is related to its antecedent stabilizer by Equation 27.

$$\mathrm{Stab}_H(a) = H \cap \mathrm{Stab}_{G^-}(a). \tag{27}$$

Equation 27 shows that a position can lose some of its local symmetry even when its orbit relation to another position does not change. Stabilizer change and orbit splitting are therefore related but non-identical signatures of symmetry reduction.

The distinction becomes useful when two positions remain in the same residual orbit. Their stabilizers remain conjugate within H, yet both may be smaller than their antecedent stabilizers in G^-. In such a case the relational regime has lost symmetry without differentiating the two positions from each other at the orbit level. Conversely, an antecedent orbit can split into several residual orbits whose stabilizers display different subgroup structures. A fuller application can use both orbit and stabilizer data to characterize which aspects of relational symmetry have been lost.

These structures remain indexed by the retained content κ. A symmetry reduction visible under one relational resolution can disappear under another if the latter excludes the relation that changed, while a finer resolution can reveal additional differentiations. The comparison therefore remains a statement about a specified relational representation rather than a complete account of the beings or configuration involved.

8.5 Scope of Symmetry-Reduction Explanations

The formalism developed in this section establishes a conditional route from articulated symmetry to relational differentiation. When historically comparable configurations support an antecedent group G^- and a residual subgroup H, the residual action refines the antecedent orbit partition. Positions that shared an antecedent orbit can separate into distinct residual orbits, producing a precise loss of relational interchangeability. Where that differentiation also satisfies a domain-specific directional-consequence predicate A_α, the paper describes the result as symmetry-reduction-generated relational asymmetry in the corresponding dimension.

Several limits are integral to this construction. Subgroup comparability is an assumption for the case analyzed here, not a property of every relational transition. Orbit splitting is sufficient for the defined structural differentiation but is not by itself sufficient for typed relational asymmetry. A reduction in symmetry can occur without orbit splitting, and relational asymmetry can be present without any earlier articulated symmetry. The primordial-asymmetry regime of Section 3 therefore remains outside the explanatory reach

of the present construction unless an additional historical model supplies an antecedent symmetry. Likewise, a pre-differentiated regime must first acquire enough relational articulation for the group-theoretic analysis to become applicable.

The section also remains neutral about dynamics. Equations 23–27 characterize relations between antecedent and residual symmetry structures; they do not specify the process that selects H, alters the configuration, or stabilizes the resulting differentiation. The next section turns to that missing level of explanation. It treats perturbative symmetry breaking, nonlinear amplification, stochastic selection, critical susceptibility, and historical accumulation as candidate dynamical hypotheses whose assumptions and scopes must be stated separately.

9 Dynamical Hypotheses for Symmetry Change

The preceding sections establish structural conditions under which partial symmetry, articulated symmetry, residual symmetry, and symmetry-reduction-generated differentiation can be represented. They leave open the process that changes one relational configuration into another. This section addresses that gap at a deliberately provisional level. Its objective is to distinguish several candidate dynamical mechanisms that could support symmetry formation or symmetry reduction under different modeling assumptions. Perturbative, nonlinear, stochastic, critical-response, and historically cumulative mechanisms are treated as separate hypotheses whose applicability depends on the relational domain. The section does not propose a general dynamics of GR, and observing a reduction from an antecedent symmetry group to a residual subgroup does not by itself identify which mechanism produced the reduction.

9.1 Structural Transition and Unspecified Relational Dynamics

A structural comparison such as the one developed in Section 8 relates an antecedent frame to a later frame without specifying the law that connects them. Let $i \preceq j$ be two indices in the partially ordered historical set introduced in Section 6. A generic transition may be represented as in Equation 28.

$$\mathfrak{F}_j = \mathcal{D}_{j \leftarrow i}(\mathfrak{F}_i; \Theta_{ij}). \tag{28}$$

Equation 28 introduces $\mathcal{D}_{j \leftarrow i}$ as a placeholder for a relational dynamical process and Θ_{ij} as a collection of whatever endogenous, exogenous, historical, or stochastic factors an application requires. Neither symbol is assigned a universal mathematical type. The purpose of the representation is to keep the missing dynamical level explicit while allowing the structural formalism to remain agnostic about its eventual realization.

When both frames admit articulated relational symmetry, the corresponding symmetry structures can be extracted after the transition and compared. Under the residual-symmetry comparability assumption of Section 8, the later group can be represented by a residual subgroup $H \leq G^-$. The same structural outcome can nevertheless be compatible with multiple choices of $\mathcal{D}_{j \leftarrow i}$. Symmetry reduction therefore supplies an outcome constraint on a candidate dynamics rather than a unique inverse reconstruction of that dynamics.

This underdetermination is analytically important. A relational configuration can lose an exchange symmetry because an external perturbation distinguishes two positions, because a symmetric state becomes dynamically unstable, because stochastic fluctuations select one of several admissible branches, because a near-critical regime amplifies a small difference, or because historically ordered interactions accumulate into a persistent distinction. These mechanisms can also interact. The hypotheses below are therefore organized by additional structure and mechanism rather than presented as mutually exclusive alternatives.

9.2 Perturbative Symmetry Reduction

Perturbative reasoning supplies one comparatively simple candidate mechanism when an application admits a local state representation and a parameter measuring a small deviation from a reference dynamics. Let $z(s)$ denote a local coordinate representation of selected relational degrees of freedom over an evolution

parameter s. The parameter s is model-specific and need not represent universal clock time. A perturbative candidate dynamics is stated in Equation 29.

$$\frac{dz}{ds} = F_0(z;\mu) + \varepsilon V(z;\mu), \qquad |\varepsilon| \ll 1. \tag{29}$$

Equation 29 separates a reference vector field F_0 from a perturbing term V. Suppose a group G acts on the local state space and F_0 is G-equivariant over the regime of interest. If V preserves only a subgroup $H \leq G$, then the perturbed dynamics has an explicit structural bias toward H-compatible evolution.

Hypothesis 9.1 (Perturbative symmetry-reduction hypothesis)**.** For a relational regime that admits the local representation in Equation 29, a sufficiently small perturbation whose equivariance is reduced from G to H may generate or stabilize a later configuration whose retained symmetry is contained in H.

Hypothesis 9.1 is intentionally conditional. Smallness of ε does not guarantee small relational consequences, and reduced equivariance does not guarantee that every trajectory reaches an H-symmetric configuration. Stability, basin structure, nonlinear response, and the historical persistence of the differentiated positions remain additional questions. The hypothesis also covers only one form of perturbative breaking. In another case, the governing equations may retain the larger symmetry while a small initial displacement selects one among several symmetry-related branches. That second case is more naturally treated through nonlinear instability and bifurcation.

The analytical attraction of the perturbative hypothesis is its economy. It supplies a controlled way to ask how a small relationally selective term can alter an otherwise symmetric local model. Its limitation is equally important: the decomposition into F_0 and V, the meaning of the small parameter, and the local state coordinates are modeling choices. GR does not presently supply a general principle that guarantees such a decomposition for relational systems.

9.3 Nonlinear Instability and Bifurcation

A second mechanism allows the governing dynamics to retain a symmetry while the stable realized states cease to display it. Bifurcation theory studies qualitative changes in the invariant structures of dynamical systems as parameters vary, including changes in equilibria, periodic orbits, and their stability [5]. For the present paper, its value is conceptual: asymmetry can emerge because a symmetric relational state loses stability, without an explicitly asymmetric perturbing term in the dynamical law.

A minimal normal form uses a scalar order coordinate ϕ whose sign exchanges two symmetry-related differentiated states. The coordinate is application-specific and does not represent a universal GR order parameter. Equation 30 gives a symmetric pitchfork-type model.

$$\frac{d\phi}{ds} = \mu\phi - \beta\phi^3, \qquad \beta > 0. \tag{30}$$

Equation 30 is invariant under the exchange $\phi \mapsto -\phi$. For $\mu < 0$, the equilibrium $\phi = 0$ is locally stable. For $\mu > 0$, the symmetric equilibrium becomes unstable and the two nonzero equilibria $\phi = \pm\sqrt{\mu/\beta}$ are locally stable. This standard normal-form structure illustrates how symmetry can remain present in the governing equation while realized stable states select differentiated branches [5].

Hypothesis 9.2 (Bifurcation-mediated differentiation hypothesis)**.** A relational regime that admits a reduced nonlinear dynamics may develop persistent differentiation when a symmetric invariant state loses stability and symmetry-related differentiated branches become dynamically stable.

Hypothesis 9.2 makes a stronger modeling demand than the group-theoretic sections. An application must justify the reduced state variables, the control parameter, the stability concept, and the correspondence between the branches of the reduced model and relational positions in the underlying system. The pitchfork form in Equation 30 is therefore an illustrative local model rather than a claim that relational symmetry breaking generically has pitchfork geometry. Other bifurcations, discontinuous transitions, and nonlocal dynamical changes may be more appropriate in particular domains.

9.4 Stochastic Selection and Contingency

Relational histories can contain fluctuations, unmodeled interactions, and contingent events whose detailed realization is not represented deterministically. Stochastic-process formalisms provide established mathematical tools for representing noise-driven and diffusion-like dynamics [3]. In a reduced relational model, stochasticity can be introduced by augmenting the deterministic dynamics with a noise term, as in Equation 31.

$$d\phi_s = F(\phi_s; \mu)\, ds + \sigma(\phi_s; \mu)\, dW_s. \tag{31}$$

In Equation 31, W_s denotes a Wiener process within the candidate stochastic model and σ sets the local noise amplitude. This representation should be read as one possible mathematical idealization of contingent variation, not as an assertion that relational histories are literally Brownian.

Hypothesis 9.3 (Stochastic branch-selection hypothesis)**.** When a relational regime admits multiple dynamically accessible symmetry-related states, stochastic fluctuations may select among those states and thereby contribute to historically contingent relational differentiation even when the deterministic part of the reduced dynamics preserves the relevant symmetry.

The mechanism in Hypothesis 9.3 differs from explicit perturbative breaking. The stochastic realization can select a direction while the probability law or deterministic drift remains symmetric. Persistent asymmetry then requires additional structure that retains the selected branch, such as nonlinear stability, memory, hysteresis, institutional reproduction, or other history-sensitive relations. A transient fluctuation alone need not create a durable relational asymmetry.

The stochastic representation also distinguishes ontological claims from uncertainty models. A noise term can stand for genuine stochasticity, unresolved deterministic degrees of freedom, coarse-graining error, or deliberately aggregated historical contingencies. Choosing among these interpretations requires evidence beyond the formalism itself.

9.5 Susceptibility and Critical Regimes

The magnitude of a perturbation need not determine the magnitude of the resulting relational differentiation. Statistical mechanics and response theory provide a language in which a system's susceptibility measures the response of an order parameter to a conjugate perturbation, and large susceptibilities can occur near critical regimes [7]. GR can borrow this structure cautiously as a candidate representation of relational sensitivity.

Let h denote a weak symmetry-selective source in a reduced model and let $\langle\phi\rangle_h$ be the corresponding expected or coarse-grained order coordinate. A local susceptibility is defined in Equation 32.

$$\chi = \left.\frac{\partial\langle\phi\rangle_h}{\partial h}\right|_{h=0}. \tag{32}$$

Equation 32 supplies a model-specific response coefficient. A large $|\chi|$ indicates that a small symmetry-selective source can produce a comparatively large change in the chosen order coordinate. The definition alone does not establish that a relational system is critical, nor does it determine the source of a large response.

Hypothesis 9.4 (High-susceptibility amplification hypothesis)**.** A relational regime with high response susceptibility in a symmetry-relevant coordinate may amplify small perturbations or contingent differences into comparatively large relational differentiation, especially when the amplified state is subsequently stabilized by the system's dynamics or historical structure.

Hypothesis 9.4 links the perturbative and nonlinear pictures without identifying them. A perturbation supplies a source, susceptibility characterizes local response, and nonlinear dynamics can determine whether the response saturates, decays, or reorganizes the symmetry structure. Criticality is therefore retained as a possible dynamical condition rather than used as a general explanation for asymmetry.

9.6 Historical Accumulation and Path Dependence

GR places particular emphasis on historical trajectory, which suggests a mechanism not reducible to a single instantaneous perturbation. A relational asymmetry can be generated through the ordered composition of individually modest events whose consequences depend on the state produced by earlier events. Let $e_1, \ldots, e_n$ denote a historically ordered sequence of events, and let $\mathcal{D}_{e_k}$ denote the corresponding local transition map when it is defined. The composed historical transition is represented in Equation 33.

$$\mathcal{D}_\gamma = \mathcal{D}_{e_n} \circ \cdots \circ \mathcal{D}_{e_2} \circ \mathcal{D}_{e_1}, \qquad \gamma = (e_1, \ldots, e_n). \tag{33}$$

Equation 33 makes the ordered trajectory part of the dynamical description. If the local transitions fail to commute, changing the historical order can change the resulting configuration. The order sensitivity is stated in Equation 34 for two events whose alternative compositions are both defined.

$$\mathcal{D}_{e_2} \circ \mathcal{D}_{e_1} \neq \mathcal{D}_{e_1} \circ \mathcal{D}_{e_2}. \tag{34}$$

Equation 34 is a structural condition for path dependence, not a claim that every GR transition is noncommutative. It allows historical order to matter formally when an application supplies transitions with that property.

Hypothesis 9.5 (Historical accumulation hypothesis)**.** A persistent relational differentiation may arise through a history-dependent composition of locally limited transitions, even when no single event is sufficient to account for the final symmetry reduction on its own.

Hypothesis 9.5 is especially compatible with cases in which roles, expectations, access, dependencies, or capacities are repeatedly reinforced. The formalism does not require the increments to be monotone. An event can reverse, weaken, or redirect an earlier differentiation, and distinct event orders can lead to distinct residual symmetry structures. Historical accumulation therefore provides a mechanism family rather than a one-directional growth law.

9.7 Mechanism Identification and Scope

The candidate mechanisms developed above are summarized in Table 2. The table emphasizes the additional structure required by each hypothesis and the principal limitation that prevents it from functioning as a universal GR dynamics.

Table 2 also makes clear that these mechanisms can be combined. A near-critical nonlinear regime can have high susceptibility; a weak perturbation or stochastic fluctuation can select a branch; historical relations can then stabilize the selected configuration. Conversely, a historically accumulated asymmetry can alter the parameters of a later bifurcation or change which transformations remain meaningful. A composite model may therefore be necessary when the relational history spans several dynamical regimes.

The principal methodological constraint is identification. A later residual group H and an observed orbit split constrain candidate explanations, yet they do not determine whether the underlying transition was perturbative, bifurcational, stochastic, critical, cumulative, or mixed. Empirical applications require evidence capable of discriminating among these mechanisms, while theoretical applications must state which mechanism is assumed and what consequences follow from that assumption. The distinction between structural formalization and dynamical hypothesis should remain visible throughout.

The section therefore leaves the generic process $\mathcal{D}_{j \leftarrow i}$ in Equation 28 unresolved. This openness is deliberate. GR currently has stronger grounds for describing the relational structures before and after symmetry change than for asserting one universal law governing the transition between them. The next section uses the resulting asymmetry framework in a narrower illustrative domain: relational power. That application asks which differentiated relations become capable of systematically reshaping the feasible or weighted trajectories of other relational beings, without identifying asymmetry itself with power.

Table 2: Candidate dynamical hypotheses for symmetry formation or reduction.

Hypothesis family	**Additional modeling structure**	**Primary limitation**
Perturbative reduction	Local state coordinates, reference dynamics, small parameter, equivariance structure	Small perturbations need not produce small or persistent relational effects
Nonlinear bifurcation	Reduced nonlinear dynamics, stability notion, control parameter, order coordinate	Reduced normal forms are application-specific and local
Stochastic selection	Probability model or stochastic process, noise interpretation, accessible branches	Random selection does not by itself explain persistence
High susceptibility	Response coordinate, conjugate source, local response coefficient	Large response does not by itself establish criticality or symmetry reduction
Historical accumulation	Event ordering, composable transition maps, persistence/tracking conditions	The relevant event decomposition and path dependence are domain-specific

10 Relational Power within the Pre-Asymmetry Framework

This section uses relational power as a bounded application of the pre-asymmetry and asymmetry framework developed in the preceding sections. Its role is diagnostic rather than exhaustive: power makes clear why a theory of relational asymmetry requires more than a description of difference or orbit separation, while also showing why asymmetry alone does not yet constitute power. The section first identifies the additional trajectory-shaping condition that makes an asymmetry power-relevant, then introduces a minimal trajectory-profile representation for that condition, and finally locates symbolic and non-symbolic mediation within the distinction between relational and manifestation realities. A fuller account of power as a relational field, including operator-algebraic representations of its dynamical effects, is reserved for separate work.

10.1 Asymmetry and Power Relevance

Section 2 treated power as one domain in which directional relational differences can become especially consequential. Sections 7 and 8 then supplied a more precise account of interchangeability, orbit differentiation, and typed relational asymmetry. These results permit a sharper separation between the existence of an asymmetry and the conditions under which that asymmetry contributes to power.

A relational asymmetry can remain locally descriptive. Two positions may differ in dependency, access, responsibility, or reachability without the difference giving either position a sustained capacity to reshape the other's future relational possibilities. Conversely, a power-relevant relation requires some mechanism through which the differentiated configuration affects what can subsequently occur. The relevant additional property is therefore generative and dynamical: the asymmetry must participate in changing the feasible, accessible, or effectively weighted trajectories available to one or more relational beings.

Definition 10.1 (Power-relevant relational asymmetry)**.** Let $\mathsf{A}_\alpha(a, b \mid \mathcal{C})$ denote a typed relational asymmetry in dimension α. The asymmetry is *power-relevant with respect to a trajectory dimension* β when the relational organization associated with that asymmetry participates in a persistent, directionally differentiated change in the future trajectory profile of at least one position in the configuration, according to an application-specific criterion for consequential change in dimension β.

Definition 10.1 is intentionally weaker than a definition of power. It identifies a threshold that a relational asymmetry should cross before a later theory treats it as part of a power structure. The definition

leaves open whether the relevant effect is generated by an actor's intentional transformation, an institutional arrangement, a historically stabilized dependency, a symbolic mediation, a material constraint, or a distributed configuration with no single directing subject. These alternatives matter because a relational theory of power should remain capable of representing both agent-centered and configuration-level effects.

The persistence requirement prevents a momentary perturbation with no continuing relational consequence from being treated automatically as power. Persistence need not imply permanence. It requires only that the effect survive or propagate far enough within the relevant historical horizon to alter subsequent relational possibilities in a way that matters for the application. The directional requirement likewise does not impose a scalar hierarchy. It indicates that the resulting transformation of future possibilities is relationally differentiated rather than exchange-neutral.

Claim 10.2 (Asymmetry is insufficient for power relevance)**.** A typed relational asymmetry can exist without satisfying Definition 10.1. In particular, orbit differentiation or a directional difference in a selected dimension need not produce a persistent consequential change in another position's future trajectory profile.

Claim 10.2 keeps the power application consistent with the earlier distinction between relational differentiation and relational asymmetry. A symmetry reduction can split an orbit without producing typed asymmetry, and typed asymmetry can in turn remain without substantial trajectory-shaping consequence. Power relevance therefore appears at a later analytical threshold.

10.2 Trajectory Profiles and Differential Shaping

The trajectory-shaping condition can be represented without committing the paper to a complete GR dynamics. Let $\Gamma_x(\mathcal{C}; h)$ denote the set of trajectories considered admissible for relational position x from configuration $\mathcal{C}$ over an application-specific historical horizon h. The term *admissible* can encode physical possibility, institutional permission, practical reachability, or another domain-specific feasibility condition. The set alone records which trajectories remain available; some applications also require a representation of their relative accessibility, propensity, or effective weight.

Let $\mathbb{W}_\beta$ denote a codomain appropriate to the chosen trajectory dimension β, and let $\omega^{\mathcal{C}}_{x,\beta}$ assign a weight to each admissible trajectory. The trajectory profile used in this section is defined in Equation 35.

$$\Theta_{x,\beta}(\mathcal{C}; h) := (\Gamma_x(\mathcal{C}; h),\, \omega^{\mathcal{C}}_{x,\beta}), \qquad \omega^{\mathcal{C}}_{x,\beta} : \Gamma_x(\mathcal{C}; h) \to \mathbb{W}_\beta. \tag{35}$$

Equation 35 is a structural representation rather than a probability model. The codomain $\mathbb{W}_\beta$ may carry probabilities in an empirically specified stochastic application, yet the present paper also permits ordinal accessibility, possibility weights, costs, or another application-appropriate structure. No probabilistic interpretation is assumed at this level.

Suppose a transformation τ is defined on $\mathcal{C}$ and produces a later configuration $\tau\mathcal{C}$. The comparison also requires a tracking relation for the position under study. Let x^τ denote the position in $\tau\mathcal{C}$ identified by the application as the continuation, image, or otherwise relevant counterpart of x. Let $\approx_{\beta,h}$ denote an application-specific equivalence relation on trajectory profiles that preserves the distinctions relevant to dimension β over horizon h. A transformation has a nontrivial trajectory-shaping effect on the tracked position when the condition in Equation 36 holds.

$$\Theta_{x^\tau,\beta}(\tau\mathcal{C}; h) \not\approx_{\beta,h} \Theta_{x,\beta}(\mathcal{C}; h). \tag{36}$$

Equation 36 allows two analytically distinct effects. A transformation can change the admissible set Γ_x, thereby opening or closing trajectories, or it can leave the admissible set unchanged while changing the weights assigned by $\omega^{\mathcal{C}}_{x,\beta}$. The second case is especially important for relations in which formal options remain available while their effective accessibility changes substantially.

The distinction also provides a precise sense in which power need not operate through prohibition. A relational configuration can preserve several formally admissible trajectories while making some of them costly, fragile, institutionally unsupported, normatively disfavored, or practically remote. A later GR theory

of power can therefore analyze both modification of reachability and deformation of relative trajectory weights without equating the two.

The present paper does not assign a unique source to τ. In an agent-centered application, τ may belong to the set of transformations available to a particular position. In an institutional application, the relevant transformation can be generated by rules or distributed organizational processes. In a historical application, the effective transformation can be a composition of events of the kind represented in Equation 33. This source-neutral representation keeps trajectory shaping compatible with the methodological pluralism established earlier.

Definition 10.3 (Trajectory-shaping relevance)**.** A relational structure is *trajectory-shaping for a tracked position x in dimension β over horizon h* when it supports or induces at least one admissible transformation τ together with an application-justified tracking relation $x \mapsto x^{\tau}$ satisfying Equation 36, and when the resulting difference meets the persistence criterion appropriate to the application.

Definition 10.3 supplies the dynamical component needed by Definition 10.1. It remains intentionally neutral about the algebra of the resulting transformations. A later treatment may represent repeated trajectory deformation through operators, fields, transition kernels, control structures, or other formalisms. Introducing one of those representations here would exceed the narrower objective of establishing the formal conditions preceding asymmetry and illustrating why some asymmetries become consequential for power.

10.3 Relational Mediation and Manifestation

The trajectory-profile formulation also helps separate the relational existence of a power-relevant structure from the way that structure is manifested. A relation can alter trajectory profiles without first being represented through an explicit title, command, rule statement, or symbolic classification. Repeated interaction, embodied response, spatial organization, habitual expectation, technical infrastructure, or historically stabilized dependency can already differentiate effective possibilities. Symbolic mediation can subsequently stabilize, amplify, contest, or reinterpret those differences, yet explicit symbolization is not required by Definition 10.1.

This distinction is consistent with GR's separation between relational reality and manifestation reality. At the relational level, the relevant question is whether a sufficiently persistent configuration differentially reshapes future possibilities. At the manifestation level, a modeled relational structure may become accessible through authority claims, prestige, legal status, ritual, institutional role, embodied deference, interface design, or other context-dependent forms. The framework does not assume a one-to-one correspondence between these levels: several manifestations may represent one relational structure within a model, while similar manifestations may be compatible with different relational structures. Manifestational similarity therefore provides insufficient grounds, by itself, for relational equivalence.

The separation also clarifies why symbolic orders can become dynamically important without being ontologically constitutive of power. A symbolic distinction can become part of the relational configuration once it acquires sufficient persistence and causal or generative efficacy. It can then alter admissible transformations, change trajectory weights, or reshape which relations become available to different positions. The symbol participates in the power structure through its relational effects, while the possibility of power-relevant differentiation remains broader than explicit symbolic mediation.

This account leaves room for feedback between relational and manifestation levels. A manifested hierarchy can stabilize a relational asymmetry; the stabilized asymmetry can then make the manifestation more credible, expected, or difficult to contest. Such feedback can enlarge the persistence of trajectory-shaping effects. The present section identifies this feedback as a possible mechanism of stabilization without supplying a general dynamics for it.

10.4 Scope of the Power Illustration

The power application clarifies the analytical sequence established by the paper. A relational configuration can first contain a difference, then a typed directional asymmetry, and then an asymmetry whose organization persistently reshapes future trajectories. Each transition adds a stronger condition. The later condition should therefore not be read backward into the earlier concepts. Relational difference does not carry an intrinsic power meaning, and relational asymmetry remains analytically useful in domains where no substantial power effect is present.

The application also keeps normative evaluation separate from structural description. A trajectory-shaping relation can enable coordination, expertise, care, collective action, or access to otherwise unavailable possibilities; it can also support exclusion, dependency, domination, or extraction. The existence of a power-relevant asymmetry therefore does not determine its legitimacy or value. Normative assessment requires additional criteria concerning formation, mediation, distribution of benefits and burdens, contestability, reversibility, responsibility, and the availability of alternative trajectories.

The present analysis stops before a full metaphysics or dynamics of power. It does not specify when multiple power-relevant asymmetries compose into a relational field, how such a field should be represented algebraically, or how repeated interventions deform trajectory weights through time. Those questions require a dedicated treatment. The contribution of the present section is narrower: it shows how the pre-asymmetry framework supports a layered account in which asymmetry can be distinguished from power relevance, while trajectory shaping supplies a formal bridge to later work on relational power dynamics.

11 Formal Scope and Research Interfaces

This section consolidates the formal scope of the paper and identifies the points at which the present framework remains deliberately open. Its role is to separate the structural claims developed in Sections 3–10 from broader ontological and dynamical questions that the paper does not settle. The discussion proceeds through four interfaces: the status of pre-asymmetry, the relation between partial and articulated symmetry, the explanatory limits of symmetry reduction, and the representational status of the mathematical tools used throughout. The objective is to make the paper's commitments sufficiently explicit that later GR work can extend the framework without treating provisional constructions as completed ontology or dynamics.

11.1 Structured Domain of Pre-Asymmetry

The analysis developed in this paper does not assign a single universal structure to what precedes relational asymmetry. Section 3 identified three admissible regimes because the antecedent conditions of asymmetry can differ across relational descriptions. An articulated symmetric regime permits symmetry reduction to become explanatory. A primordial-asymmetry regime begins from a relational difference whose earlier origin is left outside the selected domain. A pre-differentiated regime concerns a condition in which the operations required for symmetry judgment have not yet acquired a complete domain of applicability.

The three-regime classification is therefore a scope structure rather than an ontological ranking. It prevents one formal route from becoming a hidden metaphysical default. In particular, the existence of a mathematically tractable symmetry-breaking account for one class of relational differentiation does not establish that symmetry is the general antecedent of asymmetry. Likewise, the usefulness of partial operations in a pre-differentiated regime does not establish that partiality is an ultimate property of reality. Each regime identifies a conditional setting in which a distinct family of formal questions becomes meaningful.

This openness has a positive analytical consequence. A model can state which antecedent regime it assumes, what relational distinctions are already available within that regime, and what explanatory burden remains outside the model. A symmetry-breaking model, for example, can begin from an articulated group action and explain later orbit differentiation while explicitly leaving the formation of that group action to a different level of analysis. A model of primordial asymmetry can take a directional relation as given within its domain without converting that local starting point into a claim about ultimate beginnings. A

partial-symmetry model can study the growth or contraction of applicability domains without assigning a binary symmetry status where the relevant transformation is not yet defined.

The resulting research posture is one of conditional formal inquiry. Ontological suspension does not remove structure from the analysis; it relocates the burden of precision toward assumptions, domains, and transitions. The question becomes which relational structures are available under a stated regime, which transformations they support, and which later differentiations can be represented from them.

11.2 Definedness, Articulation, and Symmetry Formation

Sections 4–6 established a second boundary of the framework: symmetry judgment presupposes relational articulation. Identifiable positions, admissible transformations, criteria of equivalence, and domains of applicability must be available before a transformation can be evaluated for invariance. This requirement gives definedness an analytical role that precedes symmetry preservation.

The partial-symmetry formalism makes this boundary explicit. A transformation can be meaningful on one relational subdomain while remaining undefined elsewhere. Inverse-semigroup and restriction-categorical structures provide ways to organize such partial transformations and their domains. Their role in the present paper is representational: they make it possible to model a relational regime in which the availability of a transformation is itself part of the structure under study.

Articulated symmetry requires additional conditions. Section 6 treated historical articulation as distinct from the purely mathematical globalization of an already specified partial action. This distinction limits the interpretation of later group structure. When a configuration at a later historical stage admits a global action by a retained symmetry group, that fact characterizes the later regime. It does not by itself show that the same group structure was present, latent, or causally operative at earlier stages.

The paper therefore treats symmetry formation as an open generative problem. Applicability domains may expand, contract, or reorganize; local compositions may stabilize; equivalence criteria may become more coherent; and selected transformations may eventually become globally defined on a common relational domain. These developments can support articulated symmetry, yet the current formalism does not provide a general law governing their occurrence. The distinction between formal globalization and historical articulation should remain visible in future work, especially where mathematical completion procedures are used to represent historically evolving systems.

11.3 Symmetry Reduction and the Limits of Asymmetry Explanation

Once articulated symmetry is available, Sections 7 and 8 provide a more definite structural result. Relational positions that occupy the same orbit under a retained symmetry structure are interchangeable relative to the specified action and equivalence criterion. A later reduction of the relevant symmetry can refine those orbits and separate positions that were previously symmetry-related. Orbit splitting therefore supplies a formal representation of one route to relational differentiation.

The paper also preserves a further distinction between relational differentiation and typed relational asymmetry. Loss of interchangeability identifies a structural separation, while asymmetry requires a directional difference in a specified relational dimension. This distinction keeps the formalism from treating every broken symmetry as a hierarchy, inequality, dependency, or power relation. The type of asymmetry must be supplied by the domain-specific relational content that becomes directionally differentiated.

These results define the explanatory reach of symmetry reduction more clearly. Symmetry reduction can account for asymmetry when an antecedent symmetry is well-defined, historically comparable to the later regime, and connected to a typed directional consequence. The same construction does not explain primordial asymmetry whose antecedent conditions lie outside the model. It also does not directly apply to a pre-differentiated regime before the relevant transformations have acquired domains of applicability.

Section 9 adds another limitation. A structural observation of residual symmetry does not identify the dynamics that produced it. Perturbative reduction, nonlinear bifurcation, stochastic branch selection, high susceptibility, and historically ordered accumulation can generate similar structural outcomes under different assumptions. The present paper therefore treats mechanism identification as a separate research

problem. Future empirical or domain-specific models will need to specify which dynamical family is appropriate, what evidence distinguishes it from alternatives, and which parameters can be interpreted rather than merely fitted.

11.4 Formal Pluralism and Representational Status

The mathematical languages used in this paper occupy different structural roles. Partial operations represent limited domains of applicability. Inverse semigroups organize partial reversible transformations. Restriction categories internalize information about where partial maps are defined. Groupoids may represent local reversible relations among differentiated objects. Groups and group actions become appropriate when symmetry is sufficiently articulated to support global composition and orbit analysis. Dynamical-system tools enter only when the research question concerns the evolution of these structures rather than their static organization.

This division of labor expresses a form of formal pluralism under relational constraint. The formalisms are not interchangeable, because they preserve different structures and answer different questions. Their plurality also does not imply that the choice among them is arbitrary. A mathematical language is adequate to the extent that it preserves the relational distinctions relevant to the local problem, states its assumptions clearly, and exposes the information it omits.

The representational status of these tools is especially important for GR. A group action can model relational interchangeability without showing that relational reality is fundamentally group-theoretic. A restriction category can encode partial definedness without establishing partiality as an ultimate ontology. A bifurcation normal form can illustrate a class of symmetry-breaking dynamics without demonstrating that a historical social process literally instantiates the selected differential equation. The paper consequently treats structural formalization, dynamical hypothesis, and ontological claim as distinct epistemic categories.

The same discipline applies to the power illustration in Section 10. The trajectory-profile construction identifies an additional condition under which typed asymmetry becomes power-relevant: a persistent, directionally differentiated reshaping of future relational possibilities. It remains a minimal representation. A later theory may use fields, operators, stochastic kernels, control structures, or other mathematical languages to model such trajectory shaping. Selection among those options requires further assumptions about the relational dynamics and cannot be settled by the asymmetry formalism alone.

11.5 Research Interfaces for Further Formal Development

The present formalism leaves several interfaces intentionally open. The first concerns the mathematical representation of relational configuration itself. The paper has used configurations, positions, transformation domains, and equivalence criteria at a level sufficient for the current symmetry analysis. A more complete GR formalism may need higher-order relations, typed diagrams, event structures, causal partial orders, or other constructions when the internal organization of a configuration becomes the primary research object.

The second interface concerns the formation of applicability domains. Section 6 permits domain development along a historical index, yet the mechanisms through which a previously undefined relational operation becomes available remain underdetermined. The question includes conceptual differentiation, stabilization of relational identities, institutional articulation, repeated interaction, and other processes whose formal treatment may differ across domains.

The third interface concerns the transition from structural differentiation to dynamics. The hypothesis family in Section 9 provides candidate mechanisms, while a general GR dynamics remains open. Later work will need to determine whether trajectory evolution is best represented through deterministic flows, stochastic processes, operator algebras, event-composition structures, or a composite formalism. The answer may vary across relational regimes.

The fourth interface concerns manifestation. The present paper concentrates primarily on relational reality, with manifestation entering only where power-relevant structures become symbolically, institutionally, habitually, or bodily expressed. A fuller account will need to formalize how relational asymmetry can

be amplified, attenuated, hidden, or transformed at the manifestation level, while preserving the distinction between manifestation equivalence and relational interchangeability.

These research interfaces define the paper's completion boundary. The current contribution is a preliminary structure for making pre-asymmetry, partial symmetry, articulated symmetry, symmetry reduction, and typed relational asymmetry formally distinguishable within GR. The framework remains deliberately revisable at the points where stronger ontological or dynamical assumptions would be required. That boundary prepares the concluding section to state the contribution without converting an exploratory formalism into a completed theory.

12 Conclusion

This paper has developed a preliminary formal account of pre-asymmetry within Generative Relational Theory. Its central objective has been to clarify the conditions under which relational asymmetry can become formally meaningful without presupposing a single ontological origin for asymmetry. The analysis therefore began from GR's commitments to relationality, historicity, generativity, and ontological suspension, and treated symmetry as a structure whose applicability itself requires explanation.

The first contribution is a three-regime account of pre-asymmetry. An antecedent-symmetry regime permits later asymmetry to be studied through symmetry reduction. A primordial-asymmetry regime allows a directional relational structure to be treated as antecedent relative to the selected domain without converting that local starting point into an ultimate ontological claim. A pre-differentiated regime identifies a condition in which the transformations and comparison structures required for symmetry judgment have not yet acquired a complete domain of applicability. These regimes are conditional analytical possibilities. Their purpose is to keep the formalism from turning one mathematically convenient origin story into a universal metaphysics.

The second contribution concerns the formal status of symmetry itself. The paper has treated definedness as analytically prior to invariance. A symmetry judgment requires identifiable relational positions, admissible transformations, a retained criterion of relational equivalence, and a domain on which the relevant transformation is defined. Partial operations therefore provide more than a technical convenience. They allow the formalism to represent relational conditions in which a transformation is meaningful only on part of the available configuration space. Partial group actions, inverse semigroups, and restriction categories then provide complementary languages for organizing such partially defined symmetry structures. Their use remains representational and problem-relative; none is presented as the ontology of relational reality.

The third contribution is the distinction between symmetry formation and symmetry breaking. A later articulated group action does not by itself establish that the same group structure was already present at an earlier historical stage. The paper consequently separated the historical formation of applicability domains, comparison structures, and stable compositions from the mathematical globalization of an already specified partial action. This distinction preserves the generative and historical commitments of GR by allowing the structures required for symmetry to emerge, reorganize, or disappear across relational trajectories.

Once articulated symmetry is available, ordinary group-theoretic tools provide a more definite account of relational interchangeability. Positions in the same orbit are symmetry-related relative to the retained action and equivalence structure. A later reduction of the relevant symmetry can refine the orbit structure and separate positions that were previously interchangeable. The resulting orbit differentiation supplies one formal route to relational differentiation. The paper further distinguished this structural differentiation from typed relational asymmetry: a broken symmetry becomes an asymmetry in the stronger GR sense only when the differentiated positions acquire directionally different consequences, capacities, constraints, dependencies, access conditions, or other specified relational properties.

The dynamical account remains intentionally open. Perturbative symmetry reduction, nonlinear bifurcation, stochastic branch selection, high-susceptibility amplification, and historically ordered accumulation were formulated as candidate mechanisms with different assumptions and explanatory ranges. An observed reduction of symmetry does not identify which mechanism produced it. The present framework therefore

separates structural description from mechanism identification and treats a general dynamics of symmetry formation and breaking as an open research problem.

Relational power illustrated the practical significance of these distinctions. Typed asymmetry alone is insufficient for power relevance. A stronger condition concerns the persistent and directionally differentiated shaping of relational trajectories. This application showed how the pre-asymmetry framework can support later analysis of power while preserving distinctions among difference, asymmetry, trajectory shaping, manifestation, and normative evaluation. A full theory of relational power fields and operator-algebraic dynamics lies beyond the scope of the present paper.

The broader methodological commitment can be stated succinctly. GR does not foreclose ontological possibilities in advance. While suspending final ontological commitment, it can conditionally, formally, and generatively analyze the different structures that may be possible. Ontological suspension therefore leaves room for substantive formal inquiry into the domains in which predicates become applicable, the transformations they support, and the historical processes through which relational structures become articulated.

The framework remains preliminary in several respects. The internal mathematics of relational configurations requires further development; the formation of applicability domains lacks a general dynamics; the relation between relational and manifestation realities requires additional formal treatment; and the empirical or domain-specific conditions under which particular dynamical hypotheses become adequate remain to be established. These limitations delineate the next research interfaces. The present paper provides a structured starting point: pre-asymmetry can be studied as a conditional formal domain in which definedness, partial symmetry, articulated symmetry, symmetry reduction, and typed relational differentiation occupy distinct analytical roles.

References

[1] J. R. B. Cockett and Stephen Lack. “Restriction Categories I: Categories of Partial Maps”. In: *Theoretical Computer Science* 270.1–2 (2002), pp. 223–259. DOI: 10.1016/S0304-3975(00)00382-0.

[2] Ruy Exel. “Partial Actions of Groups and Actions of Inverse Semigroups”. In: *Proceedings of the American Mathematical Society* 126.12 (1998), pp. 3481–3494. DOI: 10.1090/S0002-9939-98-04575-4.

[3] Crispin Gardiner. *Stochastic Methods: A Handbook for the Natural and Social Sciences*. 4th ed. Vol. 13. Springer Series in Synergetics. Berlin, Heidelberg: Springer, 2009. ISBN: 978-3-540-70712-7.

[4] Johannes Kellendonk and Mark V. Lawson. “Partial Actions of Groups”. In: *International Journal of Algebra and Computation* 14.1 (2004), pp. 87–114. DOI: 10.1142/S0218196704001657.

[5] Yuri A. Kuznetsov. *Elements of Applied Bifurcation Theory*. 4th ed. Cham: Springer, 2023. ISBN: 978-3-031-22007-4. DOI: 10.1007/978-3-031-22007-4.

[6] Mark V. Lawson. *Inverse Semigroups: The Theory of Partial Symmetries*. Singapore: World Scientific, 1998. ISBN: 9789810233167. DOI: 10.1142/3645.

[7] James P. Sethna. *Statistical Mechanics: Entropy, Order Parameters, and Complexity*. 2nd ed. Oxford University Press, 2021. ISBN: 9780198865247. DOI: 10.1093/oso/9780198865247.001.0001.