

Subject Development and Knowledge Emergence: A Group-Field-Theoretic and Effective-Field Framework

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Abstract

This position paper develops subject development and knowledge emergence inside a background-independent field-theoretic framework. A group field over a declared group generates combinatorial relational histories; an individual is a subcomplex of such a history, and coarse-graining supplies an effective action whose order parameters carry developmental description. An intersubjectively stabilized order, here called *nomos*, appears as a condensate order parameter rather than as an imposed background. Reciprocal individual–nomos formation is derived as a Dyson resummation: the loop operator is a self-energy insertion, the convergent expansion counts individual-to-field round trips, and failure of the small-gain condition acquires positive content as the onset of a collective mode. Normalized aggregation yields a conditional single-agent scaling result recovered as the leading term of a $1/M$ expansion. Symbolic attachment is treated as symmetry reduction: before attachment the effective action is invariant under a semiotic relabeling group and its minimizer set has flat directions; explicit breaking by an existing nomos produces pseudo-Goldstone directions whose gap grows with the intersubjective field, while spontaneous breaking in a peer group selects an arbitrary convention with an exact soft direction. The resulting semiotic gap supplies a measurable plasticity variable and a nontrivial developmental prediction. Symbol emergence receives a dynamical criterion through configuration substitutability, restated as vertex matching, and symbolic mediation is the conditioning of the effective action on a symbolic background field. Knowledge is treated as a history-conditioned change in the effective action’s generative repertoire, kept explicitly distinct from the effective action itself and from justified attribution. A closed-time-path reduction supplies real-time memory and noise kernels. The substrate is a declared ontology; phenomenology, personal identity, semantic content, and quantum ontology remain open.

Keywords: subject development; knowledge emergence; group field theory; effective action; semiotic symmetry breaking.

Statements

Status and correspondence

This paper is a working discussion document. Its formal claims are conditional on declared assumptions, and several of its bridges to empirical and philosophical questions remain open. Objections, counterexamples, corrections, alternative formulations, and pointers to relevant literature are all welcome and may be sent to huangwanhong@serendip.ngo.

Generative AI use

Generative AI systems were used in the preparation of this work. Anthropic’s Claude and OpenAI’s ChatGPT supported exploratory discussion of the conceptual framework, development of the formal construction and its notation, identification of candidate literature for subsequent checking, and drafting and revision of the manuscript in L^AT_EX. The research questions, theoretical commitments, formal claims, and the epistemic status assigned to each were determined and approved by the author, who bears sole responsibility for the content of this paper, including any errors it contains. Neither system is an author of this work and neither holds authorship credit, in accordance with the position that authorship carries responsibilities a generative AI system cannot assume.

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1 Research Scope and Modeling Commitments

This section states the paper’s role, target, and sequence of commitments. It frames a developmental problem, identifies the limited position defended here, and separates the resulting mathematical claims from ontological and epistemic questions that remain open.

Accounts of development face two symmetrical simplifications. A newborn may be treated as an empty recipient of social form, or its initial organization may be treated as already containing the knowledge later expressed. A coupled developmental account begins from heterogeneous organization and asks how individual activity, material environments, social regularities, and durable institutions transform one another. Piaget’s genetic epistemology, developmental-systems approaches, and dynamical accounts of cognition provide important antecedents [20, 17, 25]. Bourdieu’s analysis of durable dispositions and social fields supplies a distinct social antecedent whose field concept remains formally separate from physical fields [2].

An earlier version of this framework modeled the situation as a system of coupled stochastic differential equations with a nomos field defined on a declared graph or spatial domain, and treated field theory as an optional extension. That model is recovered here as a hydrodynamic limit. The reorganization is motivated by three difficulties with taking the coupled stochastic system as fundamental. It requires a background domain on which the nomos field lives, which reintroduces exactly the external container the framework wants to avoid. It requires an external time parameter, although developmental description is naturally relational. And it cannot represent symbolic structure as an ordinary configuration of the same substrate, so symbols reappear as a separate layer.

Claim 1.1 (Layered field-theoretic position). The framework has two levels. A background-independent group-field substrate supplies the ontology, the subsystem boundary, and the relational clock. A declared coarse-graining supplies an effective action whose order parameters, propagators, vertices, and symmetries carry every developmental result proved here. Statements at one level are never reported as statements at the other, and the substrate is a proposed ontology rather than an established physics of cognition.

Four contributions implement this position. The first constructs the substrate and its effective sector, with the nomos as a condensate order parameter rather than an imposed background. The second derives reciprocal formation as a Dyson resummation together with a conditional collective-scaling result. The third treats symbolic attachment as symmetry reduction and derives the semiotic gap that separates plastic from locked developmental regimes. The fourth defines symbol emergence, symbolic mediation, and candidate epistemic organization, and supplies operational tests. Every contribution remains indexed to a chosen truncation, intervention family, outcome repertoire, and empirical domain.

2 Conceptual Lineage and Formal Registers

This section locates the framework among background-independent quantum geometry, effective field theory, developmental dynamics, and social theory. The discussion uses a register comparison to state the primitive objects, licensed inferences, and evidential burdens associated with each level.

The substrate borrows established mathematics without borrowing empirical warrant. Group field theory defines a field on a group manifold whose perturbative expansion generates two-complexes readable as discrete relational histories [1, 6, 16], spin-foam models supply amplitudes for such complexes [19], and the reading of a Feynman diagram as a spacetime history is explicit in the

Table 1: Formal registers, ordered from substrate to derived description.

Register	Primitive structure	Licensed inference	Additional warrant
Group-field substrate	group field, fundamental action, generated complexes, configuration amplitudes	combinatorial and amplitude statements internal to the declared model	evidence that a developmental system realizes such a substrate; amplitude-to-probability transition
Effective sector	order parameters, effective action, propagators, self-energies, vertices, symmetries	response, stability, symmetry reduction, and scaling within a declared truncation	controlled coarse-graining, validity range, and out-of-sample prediction
Scale flow	scale map, running couplings, validity range	scale dependence or universality within a controlled approximation	explicit flow and preserved observables; separation from developmental flow
Stochastic hydrodynamic limit	finite-dimensional states, noise laws, interaction maps	existence, stability, response, and intervention effects under declared assumptions	measured states, calibrated noise, validated interventions
Social-scientific field	positions, dispositions, capitals, and institutional regularities	sociological description and explanation	its own evidential standards; no transfer of warrant from physical field theory
Topological model	represented space, topology, invariant, and estimator	invariance under a declared transformation class	representation map, measurement procedure, and discriminating prediction

literature [21]. Condensate approximations extract continuum order parameters from these models [7], and functional renormalization has been developed in this setting [3, 28]. Relational treatments of time replace external parameter evolution with correlations among internal degrees of freedom [4, 18, 22]. None of these results provides evidence that developmental or social systems are group-field systems. They establish that the constructions used below are well posed.

Dynamic field theory has already supported quantitative developmental models. Thelen and colleagues coupled motor, memory, contextual, and stochastic fields to model infant perseverative reaching and to generate experimental predictions [24]. That precedent demonstrates how field language can become operational in a bounded cognitive task. Mori’s projection formalism provides another relevant ancestor: eliminating unresolved variables can generate effective memory and noise, while epistemic interpretation of the resulting kernel requires independent criteria [14].

Table 1 records the formal registers used in this paper. Compared with the earlier version, the ordering is inverted: the field-theoretic levels are primary and the coupled stochastic system appears as a hydrodynamic limit. Movement between rows requires an explicit bridge whose content is independent of shared notation.

Definition 2.1 (Operational field). For a declared index space $\mathcal{X}$, a field is a value assignment $\Phi : \mathcal{X} \rightarrow \mathbb{R}^r$ or $\mathbb{C}^r$ together with a rule governing its amplitude or probability law. Locality is defined by an interaction kernel on $\mathcal{X}$. In the substrate, $\mathcal{X}$ is a group manifold and locality is combinatorial; in the effective sector, $\mathcal{X}$ is the declared coarse-grained index space.

A list of individual-indexed variables acquires field status only after an index space and a coupling structure are specified. This requirement is why the nomos is constructed in Section 4 as a

condensate rather than posited on a chosen domain.

3 Background-Independent Substrate

This section defines the substrate in dependency order. It introduces the group field and its generated histories, isolates an individual as a subcomplex, declares the accessible boundary structure, and fixes the relational clock.

Let G be a Lie group and d a valence. A group field is a square-integrable $\Psi : G^{\times d} \rightarrow \mathbb{C}$ satisfying a declared right-invariance condition. Its dynamics follow from

$$S_0[\Psi, \bar{\Psi}] = \int \bar{\Psi} \mathcal{K} \Psi + \frac{\lambda}{d+1} \int \mathcal{V} \Psi^{d+1} + \text{c.c.}, \quad (1)$$

whose perturbative expansion generates a sum over two-complexes,

$$Z_0 = \sum_{\mathcal{C}} \frac{\lambda^{V(\mathcal{C})}}{\text{sym}(\mathcal{C})} \mathcal{A}[\mathcal{C}], \quad \mathcal{A}[X] = \prod_f A_f \prod_e A_e \prod_v A_v, \quad (2)$$

with representation labels j_f , intertwiners ι_e , and declared additional data φ_v constituting a configuration X .

An individual is a subcomplex $\mathcal{C}_I \subseteq \mathcal{C}$ with boundary $\partial \mathcal{C}_I$ and boundary Hilbert space $\mathcal{H}_{\partial I}$. A declared accessible algebra $\mathfrak{A}_I \subseteq \mathcal{B}(\mathcal{H}_{\partial I})$ and the state ω_I induced on it carry every claim about what is available to that individual. Since $\mathcal{C}_I$ is already a history, no external time is assumed; when an internal account of change is required, declare a relational reference $\chi : \mathcal{C}_I \rightarrow \mathcal{O}_\chi$, write $X_I(\chi)$, and write an admissible continuation of nonzero amplitude as $X_I(\chi_1) \rightsquigarrow X_I(\chi_2)$ [18].

Assumption 3.1 (Declared substrate and truncation). Every result below states G , d , the retained operator set, the relational reference χ , the coarse-graining used to pass from Equation (2) to the effective sector, and the validity range of the resulting truncation.

Claim 3.2 (Initial organization is structured, not empty). The initial configuration $X_I(\chi_0)$ of an individual subcomplex is heterogeneous and constrained by biological and historical inheritance. In the effective sector this appears as an individual-specific bare action with its own kinetic term, potential, and admissible vertex set. Nothing in the framework requires or permits an unstructured initial state.

Claim 3.2 settles a question that recurs in developmental theory. High symmetry in the sense used in Section 6 concerns invariance under a relabeling group; it does not mean uniformity, emptiness, or absence of internal structure. Two newborns may share a semiotic symmetry group while differing in every coefficient of their bare actions.

4 Effective Sector, Condensate Nomos, and Two Flows

This section constructs the effective description. It defines the coarse-grained order parameters, derives the nomos as a condensate rather than a background, fixes the effective action and its expansion, and separates scale flow from developmental flow.

4.1 Order Parameters and the Nomos Condensate

Under a declared coarse-graining, retain slowly varying expectations

$$\phi = \langle \Psi \rangle, \quad \phi = (\phi_R, \phi_S), \quad (3)$$

where ϕ_R collects the sector currently treated as ordinary or productive and ϕ_S the sector whose configurations carry symbolic function. An intersubjectively stabilized component of the coarse-grained field is written N and read as an effective *nomos*. Stationarity of the condensate functional gives a nonlinear equation of Gross–Pitaevskii type,

$$\mathcal{K}\phi + \frac{\lambda}{d} \frac{\delta}{\delta \bar{\phi}} \int \mathcal{V} \phi^d = 0, \quad (4)$$

following the condensate construction developed for group-field models [7].

Claim 4.1 (Endogenous nomos). Equation (4) determines N from the same substrate that carries individuals. The nomos is therefore not a background on which individuals move; it is a collective solution whose stability is maintained by the configurations it constrains. Its apparent externality to any single individual is a consequence of the scaling result in Corollary 5.3, not of a separate ontological status.

Claim 4.1 replaces the earlier construction in which a nomos field was posited on a declared domain and fed by an explicit aggregation kernel. The aggregation kernel reappears here as the coarse-graining of the vertex kernel $\mathcal{V}$, and its locality properties are inherited rather than chosen.

4.2 Effective Action, Propagator, and Vertices

Write the effective action for the retained fields as

$$\Gamma[\phi] = \frac{1}{2} \int \phi_a D_{ab}^{-1} \phi_b + \sum_{n \geq 3} \frac{1}{n!} \int \lambda_{a_1 \dots a_n} \phi_{a_1} \cdots \phi_{a_n}, \quad (5)$$

with propagator D , self-energy Π defined by $D^{-1} = D_0^{-1} - \Pi$, and vertices λ . Indices a run over retained sectors and coarse-grained coordinates, and repeated indices include integration. The discipline followed here is that of a phenomenological Lagrangian: retain the most general expansion compatible with the declared symmetries and truncation, and let the symmetry structure rather than a microscopic derivation constrain which terms may appear [26].

Assumption 4.2 (Well-posed truncation). On the declared relational horizon, the retained operator set is finite, the bare propagator D_0 is bounded with bounded inverse on the declared response space, the coupling operators are bounded, and the neglected operators contribute below a stated tolerance. Under these conditions the expansion in Equation (5) defines a unique response problem on the horizon.

4.3 Two Flows

Scale evolution and developmental evolution require separate variables. A renormalization flow changes descriptive scale,

$$\mu \frac{\partial g_a(\mu)}{\partial \mu} = \beta_a(g(\mu)), \quad (6)$$

for which the exact functional equation

$$\partial_k \Gamma_k = \frac{1}{2} \text{Tr} \left[(\Gamma_k^{(2)} + R_k)^{-1} \partial_k R_k \right] \quad (7)$$

supplies a controlled formulation [28]. Developmental evolution instead changes a history-bearing system in relational time,

$$\partial_\chi g_a(\mu, \chi) = \mathcal{B}_a(g, N, \sigma; \chi). \quad (8)$$

Equations (6) and (8) may interact in a multiscale model. Their identification requires a new derivation and is not assumed anywhere below. Dynamic critical theory supplies relevant precedent only after an order parameter, a scaling limit, and critical observables are declared [9, 29].

5 Reciprocal Formation as Self-Energy Resummation

This section derives the framework's principal conditional result. It expands around a reference configuration, identifies the individual–nomos loop as a self-energy insertion, proves a convergent Dyson expansion under a small-gain condition, gives the failure of that condition positive content as a collective mode, and then identifies the assumptions supporting single-agent scaling.

5.1 Dyson Expansion

Let φ denote a perturbation of the individual sector and ν a perturbation of the nomos sector around a reference configuration. Expanding Equation (5) to quadratic order and collecting the cross terms into coupling operators B (nomos to individual) and C (individual to nomos), the linear response problem reads

$$D_\varphi^{(0)-1} \varphi = B\nu + u, \quad D_\nu^{(0)-1} \nu = C\varphi, \quad (9)$$

where u is an imposed individual-level source and $D_\varphi^{(0)}$, $D_\nu^{(0)}$ are the retarded propagators of the uncoupled sectors on the declared horizon. Eliminating ν gives

$$\varphi = D_\varphi^{(0)} u + D_\varphi^{(0)} \Pi \varphi, \quad \Pi := B D_\nu^{(0)} C, \quad (10)$$

so the individual–nomos loop is exactly a self-energy insertion. Define the loop operator

$$\mathcal{K} := D_\varphi^{(0)} \Pi = D_\varphi^{(0)} B D_\nu^{(0)} C. \quad (11)$$

Proposition 5.1 (Dyson reciprocal-response expansion). *Let $\mathcal{K}$ act on a declared finite-horizon Banach space of retarded response kernels, under Assumption 4.2. If $\|\mathcal{K}\| < 1$, then Equation (10) has the unique solution*

$$\varphi = (I - \mathcal{K})^{-1} D_\varphi^{(0)} u = \sum_{k=0}^{\infty} \mathcal{K}^k D_\varphi^{(0)} u, \quad (12)$$

the dressed propagator is $D_\varphi = (D_\varphi^{(0)-1} - \Pi)^{-1}$, and

$$\|\varphi\| \leq \frac{\|D_\varphi^{(0)}\|}{1 - \|\mathcal{K}\|} \|u\|. \quad (13)$$

Proof. The norm condition makes the Neumann series $\sum_{k \geq 0} \mathcal{K}^k$ absolutely convergent in the Banach algebra of bounded operators on the declared space; its limit is the inverse of $I - \mathcal{K}$. Applying that inverse to Equation (10) gives Equation (12). Factoring $D_\varphi^{(0)}$ out of $(I - D_\varphi^{(0)} \Pi)^{-1} D_\varphi^{(0)}$ gives the stated dressed propagator. The geometric-series estimate gives Equation (13), and invertibility gives uniqueness. $\square$

The term with $k = 0$ in Equation (12) is the direct individual response. Each subsequent term inserts one further self-energy, that is, one individual-to-nomos and nomos-to-individual pass. Reciprocal formation is therefore not an interpretive gloss on the model: it is the diagrammatic content of the resummation, and the expansion order is the number of completed round trips.

5.2 Collective Modes at the Convergence Boundary

The earlier formulation treated failure of the small-gain condition as a limit of the theorem. In the field-theoretic register it acquires positive content.

Corollary 5.2 (Collective-mode threshold). *The dressed propagator $D_\varphi = (D_\varphi^{(0)-1} - \Pi)^{-1}$ develops a pole exactly where $\det(D_\varphi^{(0)-1} - \Pi) = 0$; in a single retained mode this is $D_\varphi^{(0)}(\omega)\Pi(\omega) = 1$. Consequently $\|\mathcal{K}\| < 1$ is sufficient for the absence of such a pole on the declared horizon, and violation of the condition at some frequency signals a coupled individual–nomos excitation that neither sector supports alone.*

Proof. A bounded operator $I - \mathcal{K}$ fails to be invertible exactly when 1 lies in the spectrum of $\mathcal{K}$, which for the resolvent form written above is the stated determinant condition. The sufficiency direction is the spectral radius bound $r(\mathcal{K}) \leq \|\mathcal{K}\|$. $\square$

Corollary 5.2 converts a technical hypothesis into a modeling target. A developmental or social regime in which the loop gain approaches unity is predicted to exhibit a slow collective mode with divergent susceptibility, and that mode is the natural formal home for phenomena such as norm cascades, collective reorganization, and rapid convention change. Establishing that any observed such phenomenon is this mode requires separate evidence.

5.3 Single-Agent Scaling

Corollary 5.3 (Normalized aggregation and $1/M$ scaling). *Suppose the coarse-grained coupling of a single individual into the condensate carries the normalization $C \mapsto C/M$ for a population of size M , and suppose that on the declared horizon $\|D_\nu^{(0)}\| \leq M_N$ and $\|C\| \leq L_J$ uniformly in M . Then the direct nomos perturbation produced by a single individual perturbation φ_i satisfies*

$$\|\nu_i\| \leq \frac{M_N L_J}{M} \|\varphi_i\|, \quad (14)$$

and $\Pi = O(M^{-1})$. If the full loop has norm at most $\kappa < 1$ uniformly in M , the closed-loop amplification is bounded by $(1 - \kappa)^{-1}$, preserving the $O(M^{-1})$ order.

Proof. Substituting the normalized coupling into the second equation of (9) gives $\nu_i = D_\nu^{(0)} C \varphi_i / M$; taking norms gives Equation (14). Since $\Pi = B D_\nu^{(0)} C$ inherits the same factor, $\Pi = O(M^{-1})$. The final statement follows from Equation (13). $\square$

Corollary 5.3 is the leading term of a $1/M$ expansion around the condensate, and it makes Claim 4.1 precise. An individual experiences the nomos as an external constraint because its own contribution to that condensate is suppressed by the population size, not because the condensate belongs to a different ontological category.

The corollary depends on regular normalized aggregation and uniform gain. Institutional bottlenecks, network hubs, singular kernels, correlated action, and near-critical amplification in the sense of Corollary 5.2 can each violate these assumptions. Social power therefore remains a modeled structural asymmetry outside the regular finite-size regime, and the framework supplies no argument that such asymmetries are small.

6 Semiotic Symmetry, Attachment, and the Developmental Gap

This section treats concept and word attachment as symmetry reduction. It defines the semiotic relabeling group, establishes degeneracy before attachment, distinguishes explicit from spontaneous breaking, derives the resulting gap, and states the developmental predictions that follow.

6.1 The Semiotic Relabeling Group

Definition 6.1 (Semiotic symmetry group). Let $\mathcal{G}_{\text{sem}}$ be the group of transformations u acting on the symbolic sector that permute pairings between symbolic configurations and the dynamical equivalence classes they may represent, while preserving every substitutability relation and every amplitude:

$$\mathcal{A}[X^R, u \cdot X^S] = \mathcal{A}[X^R, X^S] \quad \text{for all admissible } X^R, X^S. \quad (15)$$

Invariance of the amplitudes implies invariance of the coarse-grained action, $\Gamma[\phi_R, u\phi_S] = \Gamma[\phi_R, \phi_S]$.

The group $\mathcal{G}_{\text{sem}}$ expresses the arbitrariness of the sign-referent pairing as a property of the dynamics rather than as a semantic postulate. Its existence requires that the substrate assign no amplitude advantage to any particular pairing, and this is a substantive and falsifiable modeling assumption: a substrate with iconic or indexical biases would break $\mathcal{G}_{\text{sem}}$ at the outset.

Proposition 6.2 (Pre-attachment degeneracy). *Suppose $\mathcal{G}_{\text{sem}}$ is a compact connected Lie group acting freely on the symbolic sector and Γ satisfies Definition 6.1. Then at fixed ϕ_R the set of minimizers of Γ is a union of $\mathcal{G}_{\text{sem}}$ -orbits, the Hessian $\Gamma^{(2)}$ annihilates every vector tangent to the orbit, and no pairing is dynamically preferred.*

Proof. If ϕ_S^* minimizes $\Gamma(\phi_R, \cdot)$ then so does $u\phi_S^*$ for every $u \in \mathcal{G}_{\text{sem}}$, by invariance; hence the minimizer set is a union of orbits. Differentiating the invariance $\Gamma(\phi_R, e^{tT}\phi_S) = \Gamma(\phi_R, \phi_S)$ twice at $t = 0$ and evaluating at a minimizer, where the first derivative vanishes, gives $\langle T\phi_S^*, \Gamma^{(2)}T\phi_S^* \rangle = 0$ for every generator T of $\mathcal{G}_{\text{sem}}$. Positive semidefiniteness of $\Gamma^{(2)}$ at a minimizer then forces $\Gamma^{(2)}T\phi_S^* = 0$. $\square$

Proposition 6.2 formalizes an early developmental situation: the pairing of a symbolic configuration with what it represents can be moved along the orbit at no cost. The corresponding question, why a given entity carries a given name, is a legitimate dynamical perturbation rather than a confusion, because it probes a direction in which the effective action is flat.

6.2 Explicit and Spontaneous Breaking

Two mechanisms remove the degeneracy, and they are developmentally different.

Proposition 6.3 (Explicit breaking by an existing nomos). *Add to Γ a coupling to a stabilized intersubjective configuration,*

$$\Gamma_N[\phi_R, \phi_S] = \Gamma[\phi_R, \phi_S] - \int N \cdot \phi_S, \quad (16)$$

with $N \neq 0$ not invariant under $\mathcal{G}_{\text{sem}}$. Then the minimizer set collapses to the orbit points aligned with N up to the residual stabilizer of N , the previously flat directions acquire positive curvature, and to leading order in $|N|$ the induced gap along a broken generator T satisfies

$$m_T^2 = \frac{\langle T\phi_S^*, N \rangle}{\|T\phi_S^*\|^2} + O(|N|^2) > 0. \quad (17)$$

Proof. Along the orbit, parameterize $\phi_S(t) = e^{tT}\phi_S^*$. By Proposition 6.2 the invariant part of Γ is constant in t , so the entire t -dependence of Γ_N comes from the added term, giving $\Gamma_N(t) = \text{const} - \langle N, e^{tT}\phi_S^* \rangle$. Expanding to second order about the aligned minimum and dividing by the induced metric $\|T\phi_S^*\|^2$ gives Equation (17). Positivity holds at the minimum by construction, and higher orders are controlled by Assumption 4.2. $\square$

The modes described by Equation (17) are pseudo-Goldstone modes: soft directions of an invariant action lifted by an explicit breaking term [8, 15]. Their gap grows with the strength of the intersubjective field, so the cost of asking why a particular pairing holds increases as the surrounding order consolidates.

Proposition 6.4 (Spontaneous breaking without an external order). *Suppose no external N is present and the invariant part of Γ has a nontrivial minimum with $|\phi_S^*| > 0$, self-consistently sustained by the coupled population. Then a particular orbit point is selected without dynamical preference, the residual symmetry is the stabilizer of that point, and the directions tangent to the orbit remain exactly flat.*

Proof. The statement is the standard consequence of Proposition 6.2 when the minimizing orbit is nontrivial and no invariance-violating term is present: selection of a representative breaks $\mathcal{G}_{\text{sem}}$ to the stabilizer, and Proposition 6.2 already established exact flatness of the orbit directions [8]. $\square$

Claim 6.5 (Two developmental routes to convention). Acquisition of an existing convention corresponds to explicit breaking: the learner meets a nonzero N , and its soft directions acquire a gap set by Equation (17). Genesis of a convention within a group lacking an external order corresponds to spontaneous breaking: an arbitrary representative is selected and an exactly soft direction persists until intersubjective stabilization supplies its own effective N . The two routes predict different early stability and different repair behavior after perturbation.

Claim 6.5 is an empirical hypothesis generated by the formalism, not a result. Its plausibility rests on the observation that the two mechanisms are formally distinct, and its evaluation requires developmental and linguistic evidence that this paper does not supply.

6.3 The Semiotic Gap as a Plasticity Variable

Define the semiotic susceptibility as the inverse gap along the softest broken direction,

$$\chi_{\text{sem}} = \frac{1}{m_{\text{sem}}^2}, \quad m_{\text{sem}}^2 = \min_T m_T^2, \quad (18)$$

and let γ denote the declared relaxation coefficient of the symbolic sector.

Corollary 6.6 (Relaxation time and plasticity). *For overdamped relaxation of a perturbation along the softest broken direction, the recovery time obeys*

$$\tau_{\text{sem}} = \frac{\gamma}{m_{\text{sem}}^2} = \gamma \chi_{\text{sem}}. \quad (19)$$

A developmental regime with a small gap therefore combines large susceptibility, long exploration, and slow return to convention; a regime with a large gap combines small susceptibility, short exploration, and fast return.

Proof. Linearizing the declared relaxation dynamics $\gamma \partial_\chi \delta \phi_S = -\Gamma^{(2)} \delta \phi_S$ along the softest broken direction gives exponential decay at rate m_{sem}^2/γ . $\square$

Claim 6.7 (Critical-period hypothesis). If the developmental flow of Equation (8) increases m_{sem}^2 monotonically over a declared interval, then the observed narrowing of acquisition plasticity across that interval, the increasing cost of re-pairing an established symbol, and the shortening of recovery time after a semiotic perturbation are three measurements of one underlying quantity. Discriminating this account from maturational, attentional, and interference accounts requires designs that estimate τ_{sem} and the re-pairing cost independently in the same participants.

Claim 6.7 is the framework's most directly testable developmental consequence and also its most exposed. The prediction that recovery from a semiotic perturbation becomes *faster* as plasticity declines runs against an intuitive reading in which rigidity and slowness travel together, and a null or reversed result would restrict the proposed mapping between the gap and developmental plasticity.

7 Symbol Emergence and Symbolic Mediation

This section supplies the dynamical criteria for symbolic structure. It defines discrimination, proto-symbols, and substitutability inside the substrate, restates substitutability as vertex matching, and constructs symbolic mediation as conditioning of the effective action on a background field.

7.1 Discrimination and Proto-Symbols

For relational configurations $E_1, E_2, \dots$ coupled to $\mathcal{C}_I$, define subsystem-relative equivalence $E_a \sim_I E_b$ when the two induce the same continuation statistics on the accessible algebra $\mathfrak{A}_I$, producing classes $[E]_I$. Discrimination is thereby a property of the coupled dynamics; the framework introduces no interpreter as a separate object, and interpretation is a proper part of relational dynamics.

Definition 7.1 (Proto-symbol and dynamical substitutability). Suppose interaction with a class $[E_A]_I$ reliably generates a persistent internal configuration σ_A , and suppose that for a declared context class Y and outcome tolerance,

$$E_A + Y \rightsquigarrow Z, \quad \sigma_A + Y \rightsquigarrow Z', \quad Z' \sim_I Z. \quad (20)$$

Write $\sigma_A \dashv [E_A]_I$. A proto-symbol is a persistent relational configuration that can dynamically substitute, within a declared class of interactions, for another class of relational configurations.

Proposition 7.2 (Substitutability as vertex matching). *Under Assumption 4.2, the relation $\sigma_A \dashv [E_A]_I$ relative to context class Y and outcome class Z holds to tolerance ϵ if and only if the connected vertices of the effective action satisfy*

$$\|\lambda_{\sigma_A y z} - \lambda_{E_A y z}\| \leq \epsilon \quad \text{for all } y \in Y, z \in Z. \quad (21)$$

Proof. Within the declared truncation, the continuation statistics of Equation (20) restricted to the context and outcome classes are generated by the connected correlators involving the corresponding insertions, and these correlators are determined by the vertices at the retained order. Equality of the statistics to tolerance ϵ is therefore equivalent to Equation (21) at that order, with the constant absorbed into ϵ . $\square$

Proposition 7.2 makes the proto-symbol criterion estimable. Vertices are connected correlators, so substitutability becomes a measurable comparison rather than a conceptual stipulation. It also states the criterion's limits precisely: matching holds relative to the declared context class Y , and a symbol that substitutes correctly across a narrow Y may fail across a wider one.

7.2 Sector Decomposition and Mediation

After symbolic structures exist, decompose the configuration descriptively,

$$X_I(\chi) = (X_I^R(\chi), X_I^S(\chi)), \quad (22)$$

where both sectors are ordinary configurations of the same $\mathcal{C}_I$. The decomposition is effective and functional; there is no second symbolic universe, and the symbolic substrate consists of ordinary degrees of freedom whose configuration has acquired a special relational role.

Fixing a symbolic configuration σ and integrating the remaining symbolic degrees of freedom defines the conditioned effective action

$$\Gamma_{\text{eff}}[\phi_R; \sigma] = -\log \sum_{X^S \sim \sigma} \mathcal{A}_I[\phi_R, X^S]. \quad (23)$$

A symbolic configuration therefore enters the relational sector as a background field: it shifts minima, changes curvature, opens or closes directions, and modifies barriers, without leaving the ontology. The basic mechanism is that relational dynamics produce a symbolic configuration, which then modifies relational dynamics.

Definition 7.3 (Soft-direction count). For tolerance $\epsilon > 0$, let

$$\mathcal{N}_{\text{soft}}(\sigma; \epsilon) = \#\left\{\text{eigenvalues of } \Gamma_{\text{eff}}^{(2)}[\cdot; \sigma] \text{ at its minimum below } \epsilon\right\}, \quad (24)$$

the number of directions along which the conditioned relational sector remains nearly cost-free.

Claim 7.4 (Candidate architecture for symbolic alienation). A symbolic configuration σ realizes the candidate architecture when three conditions hold jointly: σ is generated by the relational activity of the same subsystem, so that $X \rightarrow \sigma[X] \rightarrow X'$; conditioning suppresses previously available continuations, $\mathcal{A}[X' | \sigma] \ll \mathcal{A}[X' | \emptyset]$; and the conditioned soft-direction count contracts, $\mathcal{N}_{\text{soft}}(\sigma; \epsilon) \ll \mathcal{N}_{\text{soft}}(\emptyset; \epsilon)$, while σ is self-sustaining under the reduced dynamics.

Claim 7.4 is a proposed definition awaiting the additional conditions that would make it adequate: backreaction strength, asymmetry, distinction between suppression and mere specialization, self-reproduction of σ , and the possibility of critical transitions. It is recorded here because the formalism now makes each of those conditions expressible, not because the definition is settled.

8 Knowledge as History-Conditioned Generative Repertoire

This section connects the effective description to epistemic vocabulary. It defines the history-conditioned effective action and the generative repertoire, states what these do and do not license, and gives the staged attribution protocol.

Definition 8.1 (History-conditioned effective action). Let $\Gamma_\chi[\phi | H]$ denote the effective action obtained by coarse-graining the subsystem at relational reference χ conditional on a realized history H . Two histories are effectively equivalent at χ when they induce the same Γ_χ on the retained operator set.

Definition 8.2 (Generative repertoire). For threshold $\epsilon > 0$, the generative repertoire is the set of admissible continuations retaining amplitude above ϵ under Γ_χ , equivalently the set of vertices whose coefficients exceed ϵ :

$$\mathcal{R}_\chi(\epsilon | H) = \left\{ \lambda_{a_1 \dots a_n} : |\lambda_{a_1 \dots a_n}(\chi | H)| > \epsilon \right\}. \quad (25)$$

Table 2: Epistemic attribution protocol for developmental histories.

Stage	Operational condition	Admissible description	Residual ambiguity
Acquisition	randomized or otherwise identified history effect on $\mathcal{R}_\chi$	acquired repertoire difference	confounding and treatment specificity
Persistence	difference survives a declared washout in relational reference	durable acquired repertoire	habit, storage, and timescale choice
Transfer	difference appears across preregistered environments or tasks	transferable repertoire	task-family dependence
Carrier dependence	intervention or mediation analysis implicates a named operator in Γ	candidate causal carrier	latent reparameterization and intervention validity
Robustness	difference persists across seeds, perturbations, and admissible truncation changes	candidate epistemic organization	untested intervention families and truncation artifacts
Epistemic adequacy	kind-specific accuracy, truth, justification, competence, or understanding criterion is satisfied	knowledge of the declared kind	semantic and normative theory dependence

Definition 8.3 (Candidate epistemic organization). A retained organization is a candidate epistemic organization relative to $(\mathcal{U}, \mathcal{E}, Y, \chi)$ when the difference $\mathcal{R}_\chi(\epsilon \mid H_1) \neq \mathcal{R}_\chi(\epsilon \mid H_0)$ between an acquisition history and a control history is causally acquired, persistent across a declared washout, transferable across preregistered environments, attributable to an identified carrier operator, and robust across seeds, perturbations, and admissible truncation changes.

Claim 8.4 (Effective action is not knowledge). A history-conditioned effective action is a description of altered dynamical capacity. Knowledge of a declared kind additionally requires the relevant accuracy, truth, competence, justification, or understanding condition, together with evidence connecting that condition to the retained organization. No coarse-graining supplies these by itself.

Claim 8.4 blocks the most tempting inference the formalism invites. A renormalized vertex set is a repertoire of what the system can now do; epistemic standing concerns whether what it does is correct, warranted, or understood, and those are separate questions with separate evidence. The claim also blocks the converse error of treating a merely descriptive coarse-graining as epistemically inert: a carrier-identified, transferable repertoire change is a substantive empirical finding whatever its epistemic classification.

Table 2 orders the attribution protocol while preserving residual ambiguity at each stage. Compared with the earlier version, the carrier stage is stated in terms of the operator in Γ that carries history dependence, which makes the identifiability problem explicit rather than implicit.

9 Real-Time Reduction and Effective Memory

This section supplies the real-time formulation. It constructs the closed time-path generating functional for a subsystem coupled to the remainder of the complex, derives the retarded and noise kernels generated by a Gaussian environment, and states what the resulting kernels do and do not establish.

Let ϕ denote the retained subsystem field and B the coarse-grained influence of the complement $\mathcal{C} \setminus \mathcal{C}_I$, with linear coupling

$$S_{\text{int}}[\phi, B] = -g \int d\chi \phi(\chi) B(\chi), \quad (26)$$

indices and integrals over coarse-grained coordinates suppressed. For an initial density operator ρ_0 on $\mathcal{H}_{\partial I}$, the closed-time-path generating functional is

$$Z[J_+, J_-] = \text{Tr}(U_{J_+} \rho_0 U_{J_-}^\dagger), \quad (27)$$

following the nonequilibrium contour of Schwinger and Keldysh [23, 11, 10]. Integrating the complement defines the Feynman–Vernon influence functional [5]. Introducing

$$\phi_c = \frac{\phi_+ + \phi_-}{2}, \quad \phi_\Delta = \phi_+ - \phi_-, \quad (28)$$

and writing B for the centered operator with its one-point contribution absorbed into the system force, the influence action for centered Gaussian statistics, linear coupling, and a specified factorized initial preparation is

$$\begin{aligned} S_{\text{IF}}[\phi_c, \phi_\Delta] = & -g^2 \int d\chi d\chi' \phi_\Delta(\chi) G_R(\chi, \chi') \phi_c(\chi') \\ & + \frac{ig^2}{2} \int d\chi d\chi' \phi_\Delta(\chi) G_H(\chi, \chi') \phi_\Delta(\chi'), \end{aligned} \quad (29)$$

with

$$G_R(\chi, \chi') = -i \Theta(\chi - \chi') \langle [B(\chi), B(\chi')] \rangle, \quad G_H(\chi, \chi') = \frac{1}{2} \langle \{B(\chi), B(\chi')\} \rangle. \quad (30)$$

Signs can be absorbed into the retarded self-energy under an alternative interaction convention; the causal and positive-noise roles remain distinct.

Proposition 9.1 (Gaussian influence reduction). *Under the stated centered-Gaussian and linear-coupling assumptions, integration over B gives Equation (29) exactly. Its imaginary term has the exact Hubbard–Stratonovich representation*

$$\begin{aligned} & \exp \left[-\frac{g^2}{2} \int \phi_\Delta G_H \phi_\Delta \right] \\ & = \int \mathcal{D}\xi \mathcal{P}_{G_H}[\xi] \exp \left[ig \int d\chi \phi_\Delta(\chi) \xi(\chi) \right], \end{aligned} \quad (31)$$

with $\langle \xi(\chi) \xi(\chi') \rangle_\xi = G_H(\chi, \chi')$. Stationary variation at $\phi_\Delta = 0$ yields a generalized Langevin equation

$$\mathcal{E}[\phi](\chi) + g^2 \int_0^\chi \Gamma_R(\chi, \chi') \phi(\chi') d\chi' = g \xi(\chi), \quad (32)$$

where Γ_R carries the declared retarded-sign convention. For a quadratic system action and Gaussian initial state this representation recovers the reduced Gaussian correlators; for nonlinear system dynamics it supplies a semiclassical or perturbative organization unless further exactness conditions are proved.

Proof. Gaussian integration terminates the environmental cumulant expansion at second order; the commutator and anticommutator parts give the retarded and Hadamard kernels in Equation (29). The characteristic functional of a zero-mean Gaussian field with covariance G_H gives Equation (31). Variation of the real closed-time-path effective action with respect to ϕ_Δ , followed by $\phi_\Delta = 0$, gives Equation (32). Quadraticity makes the stationary description Gaussian-exact; nonlinear terms generate higher response vertices. $\square$

Equation (32) is where the earlier coupled stochastic model reappears. Its drift, memory kernel, and noise covariance are no longer postulated on a chosen domain: they are the coarse-grained influence of the rest of the complex on the subsystem, and the earlier system is the corresponding hydrodynamic limit. This is the precise sense in which the present framework contains rather than discards its predecessor. The generating functional of that limit is the classical response functional of Martin, Siggia, and Rose, whose auxiliary response variables are integration variables and carry no ontological commitment [13].

Claim 9.2 (Kernels underdetermine ontology). Retarded memory and noise kernels are response objects. Their presence establishes neither a quantum ontology of subjects, nor subjectivity, nor knowledge. A quantum-developmental hypothesis must additionally identify physical degrees of freedom, operator-valued fields with their commutation structure, a state, a dynamical generator, and a measurement rule [27], together with preparation procedures and predictions that separate it from classical alternatives sharing the same ordinary correlation functions.

Markovian reduced dynamics can alternatively be represented by a Lindblad generator under its semigroup assumptions [12]; the influence-functional branch above retains temporal memory and is therefore the appropriate register for developmental history.

10 Subject Continuity and Response Equivalence

This section connects the field description to subject-level vocabulary. It defines response equivalence over accessible observables, states what the resulting quotient represents, and marks the additional conditions required for subject attribution.

For an individual subcomplex, let the retained developmental history be the family of configurations $\gamma = \{X_I(\chi) : \chi \in [\chi_0, \chi_1]\}$. Let $\mathcal{U}$ be a family of admissible source insertions, $\mathcal{E}$ a family of transfer contexts, and let ω_γ denote the accessible state induced on $\mathfrak{A}_I$. For a probability metric d on the induced outcome laws, define

$$d_{\mathcal{U}, \mathcal{E}}(\gamma, \gamma') = \sup_{u \in \mathcal{U}, E \in \mathcal{E}} d(P_{\omega_\gamma}(Y | u, E), P_{\omega_{\gamma'}}(Y | u, E)). \quad (33)$$

Definition 10.1 (Response-equivalent developmental histories). When Equation (33) is a pseudometric, $\gamma \sim_{\mathcal{U}, \mathcal{E}} \gamma'$ means $d_{\mathcal{U}, \mathcal{E}}(\gamma, \gamma') = 0$; this exact relation is an equivalence relation. For $\varepsilon > 0$ the set $\{\gamma' : d_{\mathcal{U}, \mathcal{E}}(\gamma, \gamma') \leq \varepsilon\}$ is a robustness neighborhood, and transitivity generally fails at positive tolerance.

The quotient by exact response equivalence offers a probe-indexed candidate for functional continuity. A subject-level interpretation additionally requires a justified boundary, persistence across changing probe families, bidirectional control, embodiment conditions, and a position on first-person organization. Equation (33) therefore operationalizes comparison while leaving the relation between a response class and a subject open. The topological analysis of this quotient is developed separately.

11 Empirical Program and Falsification Conditions

This section translates the framework into a staged empirical program. The stages move from identifiable simulation to developmental intervention and then to comparative register tests, with explicit conditions that would restrict or defeat the proposed explanations.

A first study should implement the effective sector directly on a finite declared index space with observable individual variables, an implemented symbolic sector, and a coarse-grained coupling. Controlled perturbations can estimate $D_\varphi^{(0)}$, $D_\nu^{(0)}$, B , and C , hence the self-energy Π and the loop norm in Equation (11). The measured loop gain can then be compared with the convergence condition of Proposition 5.1, and the predicted pole condition of Corollary 5.2 can be tested by driving the system toward the boundary and looking for the associated slow mode and susceptibility growth. Population size, kernel heterogeneity, and institutional hubs should be varied independently to test Corollary 5.3 across its proposed regime.

A second study should target the symmetry results. The semiotic group of Definition 6.1 predicts that, before attachment, alternative pairings are exchangeable at negligible cost; explicit breaking predicts a gap growing with the strength of the surrounding convention; spontaneous breaking predicts arbitrary selection with an exactly soft direction in a peer group lacking an external order. Corollary 6.6 converts these into a measurement: estimate τ_{sem} from recovery after a benign semiotic perturbation and test the predicted association with re-pairing cost and acquisition plasticity in the same participants, as Claim 6.7 requires.

A third study should randomize acquisition histories, apply a washout in the declared relational reference, transfer participants or agents across environments, and intervene on candidate carrier operators where possible. The primary outcomes are repertoire differences in the sense of Definition 8.2, carrier-intervention effects, calibration, and held-out task performance. Direct-interaction, fixed-background, endogenous-condensate, and history-kernel models should be compared on predictive accuracy and intervention response. An endogenous nomos earns explanatory use only within the scale and error regime where it improves those comparisons.

The following outcomes restrict the framework: failure to estimate the response operators stably; predictive equivalence with a simpler direct-interaction model that lacks a condensate; disappearance of history effects after washout; failure of transfer; invalid carrier interventions; non-identifiability across latent truncations; scaling behavior inconsistent with Corollary 5.3 where its assumptions hold; absence of the predicted slow mode when the measured loop gain approaches unity; and absence or reversal of the association predicted by Claim 6.7. Substrate-level claims lose motivation entirely when every accessible prediction is reproduced by a calibrated effective model that dispenses with the group-field construction, and the framework should then be reported as an effective theory without an ontology.

Open Problem 11.1 (Controlled coarse-graining). Construct an explicit coarse-graining from Equation (2) to the effective sector whose truncation error can be bounded, so that Assumption 4.2 becomes a verified condition rather than a declared one.

Open Problem 11.2 (Amplitude to probability). State conditions under which the configuration amplitudes of Equation (2) admit a probabilistic reading for the retained observables, and determine whether any developmental prediction distinguishes the amplitude and probability formulations.

Open Problem 11.3 (Existence of the semiotic group). Characterize substrates whose amplitudes satisfy Definition 6.1 exactly, and quantify the developmental consequences of approximate rather than exact semiotic invariance.

Open Problem 11.4 (Epistemic carrier identifiability). Characterize the truncation changes that preserve every accessible response law, and identify interventions capable of separating competing carrier operators in Γ .

Open Problem 11.5 (Coupling of the two flows). Determine whether and how Equations (6) and (8) interact in a system whose coarse-graining scale and developmental state both change, and state the conditions under which they may be treated independently.

12 Revisable Position

This section consolidates the paper’s current commitments and revision thresholds. The synthesis follows the progression from substrate to effective description, derived response, symbolic structure, and future evidential burden.

Subject development and knowledge emergence admit a coherent field-theoretic treatment in which the social order is derived rather than assumed. A background-independent substrate supplies subsystems without a container and change without an external clock; a declared coarse-graining supplies an effective action whose condensate carries the nomos. Under a finite-horizon small-gain condition, reciprocal formation is a Dyson resummation whose expansion order counts individual-to-nomos round trips, and the boundary of that condition marks a collective mode rather than a gap in the theory. Under normalized aggregation, a single individual’s contribution to the condensate is $O(M^{-1})$, which explains the experienced externality of the nomos without granting it separate ontological standing.

Symbolic attachment is symmetry reduction. Before attachment the effective action is invariant under a semiotic relabeling group and its minimizers form orbits with flat directions; acquisition of an existing convention breaks that invariance explicitly and lifts the flat directions with a gap set by the strength of the surrounding order, while genesis of a convention in a peer group breaks it spontaneously and leaves an exactly soft direction. The resulting gap is a plasticity variable with a measurable relaxation time and a developmental prediction that the framework can lose.

Symbol emergence receives a dynamical criterion through substitutability, restated as vertex matching and therefore estimable; symbolic mediation is conditioning of the effective action on a background field; and a candidate architecture for alienation is available as contraction of the conditioned soft-direction count under a self-sustaining symbolic configuration. Knowledge is treated as a history-conditioned change in the generative repertoire, kept explicitly distinct from the effective action and from justified attribution.

These commitments are revisable. The substrate is postulated, the coarse-graining is declared rather than controlled, the amplitude-to-probability transition is unresolved, and every bridge to development, language, and knowledge remains an empirical obligation rather than a result. Progress requires identifiable operators, comparative models, controlled histories, transferable effects, valid carrier interventions, and evidence that the substrate earns its place over an effective description that omits it.

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