A Generative Relational Theory of Temporal Emergence in Large-Scale Heterogeneous Systems

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Abstract

This paper develops a generative relational theory of temporal emergence for large-scale heterogeneous systems. Its object is the constitution of temporal organization from change: the minimal relational conditions under which difference, comparability, recurrence, and ordering give rise to temporal structure in systems whose components possess distinct dynamics and share no universally accessed clock. The central hypothesis holds that temporal emergence occurs through relational invariance across generative difference. The paper formalizes repetition as invariance of relational form under an equivalence structure, derives multi-scale temporality from the plurality of such structures, defines distributed temporal emergence through an irreducibility criterion, and sets out categorical and sheaf-theoretic formulations together with a perturbative program that attributes temporal organization among drivers, components, and relations. The brain, ecological systems, and distributed computation serve as three instances of a single problem class. The theory concerns temporal organization within relational systems and advances no claim about the nature of fundamental physical time.

Keywords: temporal emergence; heterogeneous relational systems; relational invariance; distributed temporal order; philosophy of time

Draft. This is a working draft. It is circulated for comment, and its arguments, formulations, and open questions are subject to revision. Correction and criticism are welcome at `huangwanhong@serendip.ngo`.

Use of generative artificial intelligence. Generative AI assistants (Anthropic's Claude and OpenAI's ChatGPT) were used extensively in preparing this manuscript: for literature survey, for drafting prose from the author's specifications, for adversarial review of the argument, and for consistency and citation auditing and typesetting. No simulation or empirical result is used in this paper, and its arguments are discursive throughout; the two tables summarise the condition-profile analysis and the intervention program argued in the text. Formulations and objections arose in the course of that exchange as well as from the author. Every claim, argument, and citation was reviewed and decided by the author, who is responsible for the content and for any errors. Bibliographic details are to be verified against the primary sources before reliance.

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1 Introduction

This section states the problem the paper addresses, fixes its central question and hypothesis, declares the scope of the theory, lists its contributions, and describes the organization of the remaining sections.

Three classes of systems motivate the problem. A brain contains billions of components with distinct intrinsic dynamics, from channel kinetics through spiking neurons to slowly modulating populations, yet it generates coordinated rhythms and a usable temporal order. An ecological system contains organisms that respond to light, temperature, tides, nutrient availability, and one another through heterogeneous endogenous rhythms, yet ecological life exhibits mornings, seasons, and reproductive timing that the participants jointly enact. A distributed computing system contains processes whose physical clocks drift and disagree, yet causal ordering among events supports coherent computation. In each case, temporal organization is present at the level of the system while a universally accessed clock is absent at the level of the components.

These systems pose a question prior to the questions studied by synchronization theory, chronobiology, and the philosophy of internal time. Change alone underdetermines temporality. A system may change at every moment while lacking the structure that makes "before," "again," "interval," and "phase" available as organized distinctions. The problem of this paper is the constitution of that structure:

> **Central question.** What minimal relational conditions allow temporality to emerge from heterogeneous change, in particular where no universally shared global clock exists?

The phrase *temporal emergence* is chosen deliberately, in preference to the stronger phrase *emergence of time*. The latter announces an ontological derivation of time itself. The former designates a family of phenomena, including ordering, rhythm, phase, duration, recurrence class, temporal scale, and collective temporal architecture, and permits the theory to ask under what conditions members of this family arise and which of them, once arisen, deserve the name of an emergent time.

The paper develops one hypothesis through formal, structural, and domain-specific analysis.

Hypothesis 1.1. (Relational invariance across generative difference). Temporal emergence occurs through relational invariance across generative difference: temporal structure arises in a system exactly where changing relational configurations admit an equivalence structure under which some relational form returns while its realizations differ.

The hypothesis carries a philosophical form, which the closing sections of the paper defend: time becomes possible where difference can return without becoming identical. Difference supplies the distinguishability on which any temporal differentiation rests. Recurrence, understood as the return of relational form across differing realizations, supplies the comparability on which duration, rhythm, and scale rest. Their coupling across a relational network supplies the distributed, system-level temporality that the motivating systems display. Each of these three roles is developed into a proposition in Section 20.

Four boundaries fix the scope of the theory. First, the theory concerns temporal organization within relational systems. It advances no claim about the nature of fundamental physical time, and its engagement with relational physics in Section 16 is comparative. Second, the theory takes change as primitive. It accepts that the systems it studies undergo difference and transformation, and it studies what organizes change temporally; an explanation of why change occurs lies outside the theory, in the same way that a variational principle lies outside the mechanics built upon it. Third, the theory is distinct from synchronization theory. Coupled-oscillator models presuppose components already equipped with phase variables, and a phase variable is a local temporal structure already constituted; the present theory addresses the constitution itself, one level below the point at which oscillator models begin. Fourth, the absence of a global clock is understood as the absence of a universally accessed and authoritative clock; common external forcings, such as sunlight over an ecosystem, are permitted, and Section 9 separates common forcing, shared clock, and emergent temporal order as three distinct properties.

The paper makes six contributions.

(i) An analysis of minimal conditions for temporal emergence, distinguishing difference, comparability, relational recurrence, and ordered transformation, with an assessment of which conditions are required for which forms of temporality (Section 4).

(ii) A formal definition of repetition as invariance of relational form under an equivalence structure, which renders precise the sense in which recurrence requires difference (Section 5).

(iii) A mechanism for multi-scale temporal emergence, in which distinct equivalence structures over one relational history generate distinct temporal modes, and coarse-graining relates difference at one scale to invariance at another (Sections 6 and 7).

(iv) A definition of distributed temporal emergence governed by an irreducibility criterion: system-level temporal structure that no single local temporality and no external forcing suffices to recover (Section 8), together with an account of temporal information as propagating through events, phase, delay, sequence, and signals (Section 10).

(v) Categorical and sheaf-theoretic formulations, in which failures of compositionality and obstructions to gluing identify, respectively, temporal structure created by composition and the separation between a global clock, a globally coherent temporal order, and merely locally compatible temporalities (Sections 14 and 15).

(vi) A perturbative program that discriminates driver-dominated, component-dominated, and relation-dominated temporal organization by intervention on forcings, relations, entities, and dynamics, pointing toward an operational measure of temporal emergence (Section 17).

Section 2 surveys prior accounts of difference, repetition, and temporality in philosophy and in the sciences of rhythm and order, and locates the open problem. Section 3 defines heterogeneous generative relational systems and local temporality. Section 4 develops the minimal conditions. Section 5 formalizes repetition as relational invariance across difference, and Sections 6 and 7 derive multi-scale temporality from the plurality of equivalence structures. Sections 8 through 10 develop distributed temporal emergence, answer the global clock objection, and characterize the propagation of temporal information. Sections 11 through 13 develop the neural, ecological, and computational instances. Sections 14 and 15 give the categorical and sheaf-theoretic formulations, Section 16 compares the theory with physical accounts of emergent time, and Section 17 sets out the perturbative program. Section 18 assembles the progression from difference to multi-scale temporality, Section 19 draws the ontological consequences concerning existence, and Section 20 states the three closing propositions.

2 Prior Accounts of Difference, Repetition, and Temporality

This section situates the theory within existing work. Its objective is twofold: to state what established traditions already hold, so that the paper claims none of it, and to delimit each tradition's boundary, so that the open problem the paper addresses becomes visible at the point where the boundaries meet. The section proceeds in four groups: philosophies of lived time (Section 2.1), the philosophy of difference and repetition (Section 2.2), process and relational metaphysics (Section 2.3), and scientific accounts of rhythm, order, and relational time (Section 2.4). Section 2.5 states the open problem.

2.1 Philosophies of Lived Time

Bergson distinguishes duration from spatialized time: lived temporality is a qualitative multiplicity in which states interpenetrate, and the homogeneous time of measurement is a derivative representation laid over it [3]. James identifies the specious present, the short extended window within which succession is directly given to consciousness [10]. Husserl analyzes internal time-consciousness as a structure of retention and protention: every present phase carries a modified consciousness of the phase just elapsed and an anticipation of the phase to come, and temporal objects such as a melody are constituted through this structure [9]. Heidegger, in *Being and Time*, treats temporality as the ecstatic unity of having-been, making-present, and coming-toward, and grounds the being of Dasein in this unity [8].

These analyses establish two points on which the present theory relies. Temporality is a constituted structure in place of a given one, and its constitution involves a structure in which what differs is held together with what returns; retention and protention are, in the vocabulary of this paper, a recurrence structure operating within consciousness. The boundary of the tradition is its domain. Its analyses concern the temporality of an experiencing subject. The systems motivating this paper, ecosystems, neural populations below the level of experience, and distributed computations, fall outside that domain, and the tradition supplies no apparatus for asking when such systems possess temporal organization.

2.2 The Philosophy of Difference and Repetition

The nearest philosophical ancestor of the present theory is the tradition running from Nietzsche's eternal return [16] to Deleuze's *Difference and Repetition* [7]. Deleuze holds that repetition is distinct from generality and from the return of the identical, and he derives time from repetition and difference through three syntheses: the contraction of instants into a living present, the pure past as the ground on which presents pass, and the future as the form of the eternal return of difference. The concession must be full: the proposition that time arises from difference and repetition, with repetition understood as the return of what differs, is Deleuze's before it is this paper's, and the philosophical form of Hypothesis 1.1 stands in his lineage.

What the tradition leaves open is equally definite, and it defines this paper's task. Deleuze supplies a transcendental analysis and declines a formal one: the three syntheses carry no mathematical structure, no account of which equivalence over changing configurations constitutes a repetition, and no apparatus by which two candidate repetitions could be compared or discriminated. His analysis is conducted for thought and for the subject of experience, and its extension to arbitrary heterogeneous systems is undertaken nowhere in the text. And the tradition contains no operational component: it yields no intervention, measurement, or test by which the presence of the temporal structures it describes could be established in a given system. The present theory takes over the philosophical center, formalizes repetition as relational invariance across difference (Section 5), generalizes the domain from the subject to heterogeneous relational systems (Section 3), and attaches an operational program (Section 17).

2.3 Process and Relational Metaphysics

Whitehead's process metaphysics treats actual occasions of becoming as the final realities and treats time as an abstraction from the succession and inheritance among occasions [27]. Mead's philosophy of the present locates temporality in emergence itself: the present is the locus at which the novel arises, and past and future are structured from it [15]. McTaggart's distinction between the A-series and the B-series, together with his argument against the reality of the A-series, frames the classical analytic debate on whether temporal passage belongs to the world or to its description [14]. The present theory inherits from process metaphysics the priority of becoming over static time, and it remains neutral in the analytic debate: its object is the organization of change within systems, and both A-theoretic and B-theoretic readings of that organization are compatible with the formal development.

Two further ancestors hold specific pieces of the construction and are conceded specifically. The causal theory of time, in Reichenbach's classical form, derives temporal order from causal order [21]: the ordering row of this paper's analysis, temporal order from registered directed dependence, stands in that lineage, and the engineered realization discussed in Section 13 is the causal theory implemented. The theory's boundary is its scope: it addresses order and direction, takes the causal structure as given for the purpose, and contains no apparatus for recurrence, calibration, rhythm, phase, or scale, which is to say none of the structure above the first row of the analysis to come. And the physics of irreversibility, in Prigogine's program, locates the arrow of time in irreversible processes and dissipative structure [19]; the present theory shares its insistence that temporal structure is generated within systems, and differs in object, direction of time being one property among the many this framework treats, and in apparatus, registration having no counterpart in a thermodynamic treatment.

2.4 Sciences of Rhythm, Order, and Relational Time

Four scientific literatures border the theory, and each carries a boundary that must be stated with the same care as its achievement.

Chronobiology and coupled-oscillator theory analyze entrainment, synchronization, and collective rhythm. Pittendrigh established the entrainment of endogenous circadian oscillators by external cycles [18]; Winfree founded the geometric theory of biological time [28]; Kuramoto's model made collective synchronization analytically tractable [11], with the subsequent theory surveyed by Strogatz [24]. The boundary is the one stated in Section 1: these models begin from components equipped with phase variables, and a phase variable is a constituted local temporality. Synchronization theory therefore analyzes the coordination of temporalities already in existence, and the constitution of local and system-level temporality lies before its starting point.

Neuroscience documents rhythms spanning several orders of magnitude and their cross-frequency organization [5], and neurophenomenology has proposed explicit bridges between neural dynamics and the retention–protention structure of experienced time [25]. These supply the material for Section 11; the theoretical question of what makes a neural dynamic a temporal mode, in advance of any clock-based description, remains open in that literature.

Distributed computing theory established that useful temporal order requires no shared physical clock. Lamport's logical clocks derive an ordering from causal relations among events [12], and Mattern's virtual time characterizes global states of systems that possess no common time base [13]. This is the cleanest artificial demonstration of the paper's central possibility, and Section 13 uses it as the minimal formal analogue. Its boundary is its restriction to engineered event systems with stipulated causal semantics; it does not address recurrence, rhythm, scale, or the constitution of the event structure itself.

Relational physics derives temporal description from correlations, in place of an external parameter: the Page–Wootters mechanism recovers effective evolution from entanglement between a clock subsystem and the rest [17], the thermal time hypothesis derives a physical time flow from the statistical state [6], and Machian and shape-dynamical programs eliminate external time from classical dynamics [2]. Section 16 compares the present formalism with these programs. The comparison is bounded in advance: the physical programs concern fundamental time in quantum-gravitational or statistical-mechanical settings, and this paper neither extends them nor claims their support; it takes from them only the demonstrated conceptual possibility that temporal description can be derived from relations.

2.5 The Open Problem

The boundaries meet around an unoccupied position. The philosophies of lived time and of difference and repetition hold the constitution question, for the subject; the sciences hold heterogeneous systems, with temporality presupposed in their formal starting points. Open between them stands the constitution question posed for heterogeneous relational systems in general: a formal account of the minimal relational conditions under which difference, comparability, recurrence, and ordering give rise to temporal structure, applicable indifferently to neural, ecological, computational, and social systems, and equipped with an operational component by which its claims can be tested. The remainder of the paper develops such an account.

3 Heterogeneous Generative Relational Systems

This section defines the class of systems the theory addresses. Its objective is to fix the primitive objects on which every later construction operates, and to do so with vocabulary that presupposes no temporal structure, since temporal structure is what the theory derives. The section first declares its primitives (Section 3.1), then defines the system class (Section 3.2), then defines local temporality as an optional derived structure (Section 3.3), and closes by placing the framework within Generative Relational Being (Section 3.4).

3.1 Primitives

The theory takes three primitives: entities, relations, and change. Entities are the bearers of states; relations are structured connections among entities; change is the transformation of states and relations. The theory accepts change as given and studies what organizes it temporally. This acceptance parallels the position of a variational principle in mechanics: mechanics builds on extremality and declines to explain it, and the present theory builds on change and declines to explain why systems undergo it. One consequence of this choice governs all definitional work in the section: descriptions of change may use the language of transformation, succession of configurations, and dependence of one configuration on another, and must avoid the language of duration, rate, rhythm, phase, and scale, since each of the latter names a structure whose emergence is the theory's object. Where an index set is used to enumerate configurations, the index carries ordinal bookkeeping alone and imports no metric temporal content.

3.2 The System Class

Definition 3.1. (Heterogeneous generative relational system). A heterogeneous generative relational system is a structure

$$\mathcal{G} = (X, R, D, F),$$

in which $X = \{X_i\}_{i \in I}$ is a collection of entities with state spaces S_i; $R = \{R_{ij}\}$ is a collection of relations among entities, each relation itself capable of transformation; $D = \{D_i\}$ is a collection of local generative dynamics, where D_i specifies which transformations of S_i are admissible and how they depend on the states of related entities; and $F = \{F_k\}$ is a possibly empty collection of external forcings, each acting on some subset of X.

Two heterogeneity conditions distinguish the class. Entity heterogeneity requires that the state spaces S_i be permitted to differ in kind: continuous and discrete, high- and low-dimensional, representational and physical. Dynamical heterogeneity requires that the local dynamics D_i be permitted to differ in form and in characteristic behavior, with agreement among them assumed nowhere. A neural system realizes the class with channels, cells, and populations as entities of different kinds; an ecological system realizes it with organisms, resources, and environmental fields; a distributed computation realizes it with processes and messages. The qualifier *generative* records that the dynamics produce new states, new relations, and, in the cases of central interest, new structure over states and relations; it is used throughout in the descriptive sense fixed in Section 3.4.

Three remarks delimit the definition. First, the forcings F are permitted and remain external: a forcing acts on entities and is acted on by none of them. Sunlight over an ecosystem is the standing example. The separation between what a common forcing supplies and what a system generates is the subject of Section 9 and is decided there, on the definition given here. Second, relations are first-class: R appears in the structure alongside X, relations transform, and later sections locate temporal structure in the organization of R as often as in the states of X. Third, nothing in the definition equips any X_i with a clock, a phase, or a rate; whether an entity possesses such structure is a further property, defined next.

3.3 Local Temporality

Definition 3.2. (Local temporality). A local temporality for an entity X_i is a structure τ_i over the admissible transformations of S_i comprising some subset of the following: an ordering of configurations, a partition of configurations into recurrence classes, a phase structure, or a calibration of intervals. An entity may possess a rich τ_i, a partial one, or none.

The definition carries the section's central claim.

Claim 3.3. (Constituted standing of phase). Any description that equips a component with a phase variable, a frequency, or a characteristic period has already conferred a local temporality upon it. Coupled-oscillator theory therefore begins where Definition 3.2 ends: it analyzes the coordination of constituted local temporalities, and the constitution of τ_i from the primitives of Definition 3.1, together with the emergence of system-level temporal structure irreducible to any τ_i, forms a prior problem, which this theory addresses.

Claim 3.3 is the wall separating the theory from synchronization theory, and later sections observe it strictly: oscillator-level descriptions appear in Sections 9 and 11 as comparisons and as coarse descriptions of systems whose local temporalities are treated as already constituted, and every constitutive argument is conducted at the level of Definitions 3.1 and 3.2.

3.4 Placement within Generative Relational Being

The framework belongs to the research program of Generative Relational Being (GRB): a theory of how subject, meaning, value, creation, and normativity co-emerge through generative relational processes. The present paper works out the GRB account for temporal organization: it treats temporality as one of the structures that emerge relationally, alongside the structures GRB treats elsewhere. Two commitments of the program bind the paper. The generative tendency of relational systems is a descriptive postulate: systems are observed to unfold their dynamics and to produce new structure, and the theory studies what supports, obstructs, and organizes this unfolding while declining to assign it a purpose. And the theory's statements remain throughout on the descriptive side of the wall between dynamical explanation and normative justification: nothing in the account of how temporal order emerges, or of how one temporal organization comes to organize another, confers legitimacy on any such organization. Both commitments are inherited from the program's foundational texts and are restated here because the vocabulary of emergence invites their violation.

4 Minimal Conditions for Temporal Emergence

This section develops the minimal conditions under which temporal structure can emerge in a system of the class defined in Section 3. Its objective is to replace the single question of Section 1 with a differentiated one: temporality comprises several forms, and different forms rest on different conditions. The section first defines four candidate conditions on a heterogeneous generative relational system (Section 4.1), then establishes necessity results through two degenerate limits (Section 4.2), then assigns to each form of temporality its condition profile (Section 4.3), and closes by stating precisely how far the sufficiency claims of this paper reach (Section 4.4). The method is definitional and analytic; construction of the structures whose possibility is established here is the work of Sections 5 through 7.

4.1 The Candidate Conditions

All definitions operate on a system $\mathcal{G} = (X, R, D, F)$ and observe the vocabulary discipline of Section 3.1. A *configuration* of $\mathcal{G}$ is a joint assignment of states to entities and forms to relations; a *history* is a set of configurations connected by admissible transformations, indexed by a set A whose indices carry ordinal bookkeeping alone. The letter $\mathcal{R}_a$ denotes the relational configuration at index a: the joint form of the relations R at that configuration.

One notion carries the internality of the whole analysis and is defined first.

Definition 4.1. (Registration). A distinction between configurations is *registered* in $\mathcal{G}$ when some admissible transformation of some entity or relation depends on it: there exist X_i (or R_{ij}) and configurations c, c' differing in the distinguished respect, such that the transformations admissible for X_i (or R_{ij}) under D differ between c and c'. A distinction available only to an external describer, with every dynamic in $\mathcal{G}$ indifferent to it, is unregistered.

Registration separates the temporality a system possesses from the temporality a describer projects onto it. The conditions now follow.

Condition C1. (Difference). The history of $\mathcal{G}$ contains registered differences: configurations distinguished in some respect on which some dynamic in $\mathcal{G}$ depends.

Condition C2. (Comparability). $\mathcal{G}$ implements comparisons: there exist relational forms under which distinct configurations, or segments of the history, are brought into correspondence, and the correspondence is registered. A comparison implemented by no relation in $\mathcal{G}$ contributes a describer's similarity, and nothing internal.

Condition C3. (Relational recurrence). Some relational form returns across registered difference: there exist distinct occurrences $a, b \in A$ and a registered comparison under which $\mathcal{R}_a$ and $\mathcal{R}_b$ correspond. Section 5 gives this condition its formal definition; the present section uses it in this preliminary form.

Condition C4. (Ordered transformation). The dependence of configurations on configurations under D is directed: transformations compose, composition is associative, and the generative dependence of a configuration on those from which it is reachable is asymmetric at the level of occurrences. Directedness at the level of occurrences is compatible with return at the level of forms; the occurrence of a form and the form itself are distinct, and C4 constrains occurrences alone.

The occurrence–form distinction stated inside C4 does substantial work throughout the paper. A history may contain two occurrences of one relational form; the occurrences are ordered and unrepeatable, while the form recurs. Every later construction assigns ordering to occurrences and recurrence to forms, and no construction requires an occurrence to happen twice.

4.2 Degenerate Limits and Necessity

Two limiting systems establish the necessity of C1 and of C2–C3 by exhibiting what their failure removes.

Proposition 4.2. (The undifferentiated limit). Let $\mathcal{G}$ have a history all of whose configurations are indiscernible with respect to every registered distinction. Then no dynamic in $\mathcal{G}$ depends on any difference among configurations, every configuration admits the same transformations, and the history supports no registered contrast between one occurrence and another. Ordering, recurrence, duration, and phase are then undefined within $\mathcal{G}$: each requires at least one registered contrast, and none is available. Registered difference (C1) is therefore necessary for every form of temporality.

The proposition sharpens a point available since Section 1: within a system, indiscernibility under registration is identity for temporal purposes. A describer with finer instruments may distinguish the configurations; the system cannot, and the system's temporality is built from the system's distinctions.

Proposition 4.3. (The incomparable limit). Let $\mathcal{G}$ have a history of registered differences admitting no registered comparison: each configuration differs from every other, and no relational form brings any two occurrences or segments into correspondence. Then the history supports directed dependence of occurrences (given C4) and nothing further. "Again" is undefined, since no form returns; duration is undefined, since no two segments admit a registered congruence; rhythm and phase are undefined, since each presupposes a recurrent form. What survives is bare ordering. Comparability (C2) and relational recurrence (C3) are therefore necessary for every form of temporality beyond ordering.

The two limits bound the region in which temporalization occurs. Total identity extinguishes even ordering; total incomparability preserves ordering and extinguishes everything above it. Temporal structure inhabits the interval between, and Section 5 makes the interval precise as a property of equivalence structures.

Table 1: Condition profiles of the forms of temporality. A filled entry marks a condition on which the form rests; the final row anticipates Section 6, in which a plurality of equivalence structures generates a plurality of temporal modes.

Temporal form	C1	C2	C3	C4	Additional requirement
Ordering	•			•	
Recurrence class	•	•	•		
Duration	•	•	•	•	registered congruence of segments
Rhythm	•	•	•	•	recurrence at congruent separations
Phase	•	•	•	•	differentiation within the recurrent form
Temporal scale	•	•	•	•	plurality of equivalence structures

4.3 Condition Profiles of Temporal Forms

Table 1 assigns to each form of temporality the conditions on which it rests. The assignments follow from the definitions and from Propositions 4.2 and 4.3; three of them warrant comment.

Ordering rests on C1 and C4 alone. This is the profile realized by causal ordering in distributed computation, where registered events and directed dependence suffice to define a usable partial order in the absence of any recurrence [12]; Section 13 develops the case.

Duration carries the strongest claim in the table.

Claim 4.4. (**Internal calibration**)**.** Internal calibration of intervals requires relational recurrence. A comparison of two segments of a history is registered only through a relational form applicable to both, and a form applicable to distinct segments is a form that returns. A unit of duration is a recurrent form put to comparative use; a system in which no form returns possesses ordered occurrences and no registered means of comparing their separations. Timestamps imported from outside change nothing in this respect: a timestamp calibrates by reference to the recurrence of some other system, and the requirement is displaced, never discharged.

Phase rests on differentiation within the recurrent form: a system registers position within a returning form only where the form's occurrences are internally differentiated, so that "early in the form" and "late in the form" are registered contrasts. Phase therefore requires C1 twice over, between occurrences and within them.

4.4 The Standing of Sufficiency

The results of this section are necessity results, and their converses are claimed in a bounded form only. The bounded form is constructive: Sections 5 through 7 construct, for a system satisfying C1–C4, the structures named in Table 1, and thereby establish that the conditions jointly suffice for the existence of each form *given an equivalence structure with the properties there specified*. Sufficiency in the unconditioned sense, holding for every system satisfying C1–C4 under every equivalence structure, is false, and Section 5.3 exhibits the failures: an equivalence structure may be too coarse or too fine to temporalize, with C1–C4 intact. What the conditions delimit is the possibility of temporal emergence; which temporal structure a given system realizes is decided by which equivalence structures its relations implement, and this dependence is the subject of the next two sections.

5 Relational Invariance across Difference

This section formalizes the concept on which the whole theory turns: repetition, defined as the return of relational form across registered difference. Its objective is to give Hypothesis 1.1 assessable content, by specifying the structure under which "the same" can return while every occurrence differs. The section first defines equivalence structures over relational histories (Section 5.1), then states the definition of repetition and its immediate properties (Section 5.2), then characterizes the two degeneracies between which temporalization is possible (Section 5.3), and closes with three consequences, concerning the relativity of recurrence, the identity of occurrences, and the reconception of the clock (Section 5.4). The method is definitional, with propositions proved directly from the definitions.

5.1 Equivalence Structures over Relational Histories

Fix a system $\mathcal{G} = (X, R, D, F)$ with a history indexed by A, and write $\mathcal{R}_a$ for the relational configuration at occurrence a, as in Section 4.1.

Definition 5.1. (Equivalence structure). An equivalence structure over the history of $\mathcal{G}$ is a pair $(\Phi, \simeq)$, in which Φ maps relational configurations to a space V of forms, and $\simeq$ is an equivalence relation on V. The structure is *registered* in $\mathcal{G}$ when the correspondence it induces among configurations is registered in the sense of Definition 4.1: some dynamic in $\mathcal{G}$ depends on whether $\Phi(\mathcal{R}_a) \simeq \Phi(\mathcal{R}_b)$.

The pair $(\Phi, \simeq)$ formalizes an abstraction the system itself performs. Φ discards part of each configuration and retains a form; $\simeq$ declares which retained forms count as the same. A circadian mechanism retains the profile of illumination and discards the identity of the day; a comparison implemented by that mechanism is an equivalence structure in the present sense, and it is registered, since the organism's admissible transformations depend on it. Registration is the requirement that keeps the analysis internal. Unregistered equivalence structures exist in abundance for any describer, and each induces a classification of the history; a classification on which nothing in the system depends organizes the describer's records and none of the system's change.

5.2 The Definition of Repetition

Definition 5.2. (Repetition). Let $(\Phi, \simeq)$ be a registered equivalence structure over the history of $\mathcal{G}$. Occurrences $a, b \in A$, $a \neq b$, constitute a *repetition under* $(\Phi, \simeq)$ exactly when

$$\Phi(\mathcal{R}_a) \simeq \Phi(\mathcal{R}_b).$$

The equivalence classes of the induced relation on occurrences are the *recurrence classes* of $\mathcal{G}$ under $(\Phi, \simeq)$.

Proposition 5.3. (Difference within repetition). Every repetition holds between distinct occurrences, and in every case of interest the underlying configurations differ in registered respects outside the retained form: $\mathcal{R}_a \neq \mathcal{R}_b$ while $\Phi(\mathcal{R}_a) \simeq \Phi(\mathcal{R}_b)$. Repetition is therefore invariance of form across difference of realization. In the limiting case in which $\mathcal{R}_a$ and $\mathcal{R}_b$ are indiscernible under every registered distinction, Proposition 4.2 applies within the pair: the two occurrences collapse, for the system, into one, and the repetition is vacuous. Genuine repetition presupposes difference.

Proposition 5.3 is the formal content of the philosophical sentence carried by Hypothesis 1.1. What returns is a form; what differs is its realization; the return of form across differing realizations is the structure that renders occurrences comparable, and comparability is the threshold, established in Proposition 4.3, above which every temporal form beyond ordering becomes possible. Two constructions follow immediately and discharge the constructive obligation recorded in Section 4.4. Recurrence classes are given directly by Definition 5.2. Duration calibration is given by comparative use of a recurrent form: where a recurrent form partitions the history into successive occurrences, a segment of the history acquires a registered magnitude through the recurrence-class occurrences it spans, and two segments become comparable through their spans; this is Claim 4.4 realized. Rhythm and phase follow where the recurrence satisfies the additional requirements of Table 1, and their construction completes in Section 6.

5.3 Degeneracies of the Equivalence Structure

The two limits of Section 4.2 reappear as properties of $(\Phi, \simeq)$, and their reappearance locates the temporalizing capacity of an equivalence structure between two failures.

Proposition 5.4. **(Coarse degeneracy).** Let Φ be constant up to $\simeq$: all configurations are mapped to equivalent forms. Then all occurrences fall into one recurrence class, every pair of occurrences is a repetition, and the induced classification registers no contrast. A comparison that every pair satisfies supports no calibration: segments spanning any occurrences span equivalent forms, and no registered magnitude separates them. The structure temporalizes nothing.

Proposition 5.5. **(Fine degeneracy).** Let Φ be injective up to $\simeq$: equivalent forms arise from identical configurations alone. Then, on a history of registered differences, no two distinct occurrences constitute a repetition, recurrence classes are singletons, and the structure adds nothing to the bare ordering already available under C1 and C4. The structure temporalizes nothing beyond ordering.

Claim 5.6. **(The temporalizing interval).** An equivalence structure temporalizes exactly where it lies strictly between the degeneracies: coarse enough that forms return across differing configurations, fine enough that the induced classification registers contrast. The position of a registered equivalence structure between the two degeneracies is therefore a measure-bearing property of the system, and Section 17 builds on it in specifying an operational measure of temporal emergence.

The two degeneracies also settle the sufficiency question left open in Section 4.4: C1–C4 permit temporalization and cannot compel it, since a system satisfying all four may implement only degenerate equivalence structures. What a system's temporality depends on is which abstractions its relations enact.

5.4 Consequences for Recurrence, Identity, and Clocks

Proposition 5.7. **(Relativity of recurrence).** Recurrence is relative to a registered equivalence structure. For a fixed history, distinct structures $(\Phi_1, \simeq_1)$ and $(\Phi_2, \simeq_2)$ induce distinct partitions of the occurrences into recurrence classes, and a pair of occurrences may constitute a repetition under one structure and none under the other. A question of the form "does this history contain recurrence" is well posed only relative to a structure; posed absolutely, it has no answer.

The proposition converts an apparent embarrassment into the theory's central resource. If recurrence were absolute, one history would yield one temporal organization. Because recurrence is structure-relative, one history supports as many temporal organizations as the system registers equivalence structures, and the plurality of temporal modes within a single system, presupposed throughout Section 1, acquires a mechanism. Section 6 develops it.

Proposition 5.8. (Identity of occurrences). The identity of an occurrence is exhausted, within $\mathcal{G}$, by its registered distinctions and its position in the dependence order. Two consequences follow. Occurrences carry unrepeatable identity through C4: an occurrence's dependence position is its own. And forms carry repeatable identity through Definition 5.2: a form's identity is its equivalence class. The vocabulary of return divides without residue between the two: occurrences pass, forms return, and the passage of what passes and the return of what returns are two descriptions of one structured history.

Remark 5.9. (The clock reconceived). On the present definitions, a clock is an entity whose relational form recurs under an equivalence structure registered widely across a system: many entities' dynamics depend on the return of its form. A clock therefore possesses no time; it supplies a recurrent form whose registration other entities share, and its authority is measured by the breadth of that shared registration. The reconception carries directly into Section 9: a common external forcing supplies a widely available recurrent form, while the registrations, couplings, and further equivalence structures built on it belong to the system, and the distance between a supplied form and a generated temporal order is exactly what the global clock objection overlooks.

6 From Relational Recurrence to Temporal Scale

This section derives the plurality of temporal structure from the plurality of equivalence structures. Its objective is threefold: to define the temporal mode generated by a single registered equivalence structure, completing the constructions of rhythm and phase owed from Section 5; to establish that a system registers as many temporal modes as it registers suitable equivalence structures, together with the relations among those modes; and to define temporal scale as a derived, comparative property of modes, in place of a primitive. The section proceeds from the single mode (Sections 6.1 and 6.2) to the mode system (Section 6.3) to scale and the combination of modes (Section 6.4).

6.1 The Temporal Mode of an Equivalence Structure

Definition 6.1. (Temporal mode). Let $(\Phi, \simeq)$ be a registered equivalence structure over the history of $\mathcal{G}$, lying in the temporalizing interval of Claim 5.6. The *temporal mode* T_Φ generated by $(\Phi, \simeq)$ is the structure comprising: the recurrence classes of Definition 5.2; the calibration of segments by spanned occurrences, as constructed in Section 5.2; and, where the further conditions of Sections 6.2 hold, the rhythm and phase built on them.

A temporal mode is thus everything one registered abstraction contributes to a system's temporality. The circadian structure of Section 5.1 generates one mode; a structure retaining the tidal profile generates another; a structure retaining the joint activity form of a neural population generates a third. The definition assigns each mode to its generating structure, and the assignment carries Proposition 5.7 forward: modes are as plural as registered structures.

6.2 The Construction of Rhythm and Phase

Rhythm and phase carry the two additional requirements recorded in Table 1, and both requirements resolve into relations between structures.

Definition 6.2. (Rhythm). Let $(\Phi_1, \simeq_1)$ and $(\Phi_2, \simeq_2)$ be registered equivalence structures, each in the temporalizing interval. The recurrence of a form under Φ_1 is *rhythmic relative to* Φ_2 when the segments between successive Φ_1-occurrences of the form span congruent counts of Φ_2-occurrences, up to a registered tolerance.

Proposition 6.3. (Rhythm across structures). Rhythm is a property of a pair of structures, and it is empty on a single one. Relative to itself, every recurrence is trivially rhythmic: successive occurrences of a form are separated, in that form's own calibration, by exactly one unit, whatever the history does. Rhythm therefore carries registered content exactly where the calibrating structure differs from the calibrated one, and a rhythm attribution is, in every contentful case, a stable ratio between two recurrences.

The proposition matches measurement practice, in which every period is a ratio: a day is rhythmic relative to a pendulum, a heartbeat, or a cesium transition, and each attribution asserts a stable ratio between two recurrent forms. The proposition also converts an apparent vice of Claim 4.4 into structure. Since internal calibration rests on recurrence, the rhythm of a recurrence threatens circularity; Definition 6.2 discharges the threat by distributing the two roles over two structures, and the circular case, a recurrence calibrating itself, is exactly the case Proposition 6.3 identifies as empty.

Definition 6.4. (Phase). Let $(\Phi, \simeq)$ be a registered structure and let $(\Phi', \simeq')$ be a registered refinement of it: a structure whose classification, restricted to the segment of any single Φ-form occurrence, partitions that segment into sub-forms whose order of appearance within the segment recurs across occurrences of the Φ-form. The *phase* of an occurrence within the Φ-form is its position in that recurrent internal order.

Phase thereby realizes the requirement of Section 4.3 that C1 hold twice over: the refinement supplies registered contrast within occurrences of the form, and the recurrence of the internal order across occurrences makes "early in the form" a comparison between cycles. Like rhythm, phase is a relation between structures, a coarse structure and its registered refinement; the constructions owed from Section 5.2 are herewith complete, and the constructive sufficiency claimed in Section 4.4 is discharged for every row of Table 1 except the last, which the remainder of this section and Section 7 supply.

6.3 The Plurality of Modes and the Mode System

Proposition 6.5. (Generation of modes). Distinct registered equivalence structures in the temporalizing interval generate distinct temporal modes wherever their induced partitions of occurrences differ. A system's history therefore supports a family of temporal modes

$$\{\, T_{\Phi_i} \,\}_{i \in J},$$

one for each registered structure, and the family is as heterogeneous as the abstractions the system's relations enact.

The modes of a family stand in registered relations to one another, and three kinds have already been constructed: *calibration*, where one mode's occurrences measure another's separations (Definition 6.2); *refinement*, where one mode's classification partitions the form-segments of another (Definition 6.4); and *independence*, where the partitions of two modes constrain one another in no registered way.

Definition 6.6. (Mode system). The *mode system* of $\mathcal{G}$ is the family of its temporal modes together with the registered relations among them:

$$\mathcal{T}_{\mathcal{G}} = \big(\{T_{\Phi_i}\},\ \{\text{calibration, refinement, independence}, \ldots\}\big).$$

The mode system is the object this theory hands to the companion paper on temporal architectures. That paper's basic object, a network of temporal modes connected by synchronization, nesting, translation, entrainment, symbolization, and institutionalization, receives its vertices from Definition 6.6 and its assurance that a mode is a definite, generated structure from Definitions 5.1 through 6.4; the cultural and civilizational couplings it studies are further registered relations, enacted by symbolic and institutional dynamics, over the family defined here.

6.4 Temporal Scale and the Combination of Modes

Definition 6.7. (Temporal scale). Mode T_1 is of *larger scale* than mode T_2, relative to their calibration relation, when the segments between successive occurrences of T_1-forms span, stably and up to registered tolerance, multiple T_2-occurrences. Scale is thereby a position in the calibration order of the mode system, and a comparative, derived property: a mode has a scale relative to other modes and none in isolation.

The familiar chain of day, month, and year is a chain of stable calibration ratios, and its familiarity conceals its contingency: nothing in Definition 6.7 requires the calibration order to be a chain. The combination of modes shows why.

Proposition 6.8. (Product modes). Let $(\Phi_1, \simeq_1)$ and $(\Phi_2, \simeq_2)$ be registered structures generating modes T_1 and T_2. The product structure, mapping each configuration to the pair of its forms, generates a further mode $T_{1\times 2}$ wherever the joint classification is itself registered. Where T_1 and T_2 are periodic relative to a common calibration with coprime periods p_1 and p_2, the recurrence of the joint form has period $p_1 p_2$: the product mode is of larger scale than either factor, and its recurrence is generated by combination alone, with no third external cycle contributing.

The Javanese pairing of the five-day and seven-day cycles, whose joint position recurs at thirty-five days as the *wetonan*, realizes Proposition 6.8 exactly, and it establishes the structural point: a mode system is in general a network, in which some modes arise from relational dynamics directly and others from the registered combination of modes already in existence. Generation of modes from modes is a second route of temporal emergence, operating at one level of description; the other route, in which modes arise across levels of description through coarse-graining, is the subject of the next section.

7 Cross-Scale Temporal Emergence

This section develops the second route of temporal emergence: the arising of temporal modes across levels of description. Its objective is to establish the coexistence theorem toward which the paper has been building, that pervasive difference at a fine level is compatible with strict recurrence at a coarse level, and to define what makes a coarse mode emergent relative to a fine description. The section first defines coarse-graining within the framework (Section 7.1), then proves the coexistence result (Section 7.2), then defines level-relative emergence and separates it from the distributional emergence of Section 8 (Section 7.3), and closes with the registration of coarse modes and the cross-level couplings this induces (Section 7.4).

7.1 Coarse-Graining of Configurations and Structures

Definition 7.1. (Coarse-graining). A *coarse-graining* of $\mathcal{G}$ is a map Ψ carrying fine configurations to coarse configurations, defined by the discarding of a specified family of fine distinctions, such that admissible fine transformations project to admissible coarse transformations. The coarse description carries its own registration: a coarse distinction is registered when some dynamic, expressible over coarse configurations, depends on it.

Coarse-graining composes with equivalence structures in the evident way: a structure $(\Phi, \simeq)$ over coarse configurations pulls back to the structure $(\Phi \circ \Psi, \simeq)$ over fine ones. The pulled-back structure classifies fine occurrences by their coarse forms, and this composition is the formal carrier of everything in this section: coarse temporality, seen from the fine level, is classification through Ψ.

7.2 Coexistence of Fine Difference and Coarse Recurrence

Proposition 7.2. (Coexistence). There exist systems and histories in which no fine relational form recurs, while a coarse mode possesses nontrivial recurrence: every fine equivalence structure short of the coarse-grained ones is fine-degenerate in the sense of Proposition 5.5, while some $(\Phi \circ \Psi, \simeq)$ lies in the temporalizing interval. In such a history, each coarse-form occurrence is realized by a fine configuration that occurs once; the recurrence class at the coarse level partitions unrepeatable fine occurrences; and the coarse mode's calibration, rhythm, and phase are constructed, per Sections 5 and 6, over fine material that never returns.

The proposition states formally what every motivating example displays. No morning of an ecosystem repeats any other in fine configuration: illumination, temperatures, positions, and physiological states differ throughout. The morning regime recurs, because the coarse joint form of light, activity onset, and interaction pattern returns under a coarse-grained structure that the participating organisms register. The same holds for a neural oscillation over microscopically distinct spike configurations, and for "the same state of the computation" over distinct physical executions. Proposition 7.2 is Hypothesis 1.1 in cross-level form: difference at the fine level supplies the realizations, invariance at the coarse level supplies the form, and the temporal mode lives exactly in their combination.

7.3 Level-Relative Emergence of Modes

Definition 7.3. (Level-relative emergence). A coarse mode T is *emergent relative to a fine description* when its generating structure factors through a coarse-graining Ψ, its classification is registered at the coarse level, and no registered fine structure that avoids factoring through Ψ induces the same partition of occurrences. The mode then exists in virtue of the coarse-graining: the abstraction that discards fine distinctions is constitutive of the recurrence, and the recurrence is unavailable to any registered classification that retains them.

Two boundaries fix the definition's reach. It is level-relative through and through: emergence in the sense of Definition 7.3 compares descriptions of one system at two resolutions, and a mode emergent relative to one fine description may be plainly generated relative to another. And it is distinct from the distributional criterion to come: Section 8 defines distributed temporal emergence by irreducibility to any single local temporality or external forcing, a comparison across the system's parts, where Definition 7.3 compares across its levels. The two criteria are independent, a mode may satisfy either without the other, and their conjunction, a coarse mode irreducible both to fine classifications and to any single component's temporality, characterizes the strongest cases, to which the neural and ecological sections return.

7.4 Registration across Levels

A coarse mode is registered where some dynamic depends on its classification, and the depending dynamic may sit at either level. Coarse dynamics register coarse modes directly: a population-level regularity depending on a population-level form. Fine dynamics register coarse modes through cues: an organism's transformations depend, via illumination and the activity of others, on the morning regime, and a neuron's admissible transitions depend, via synaptic input, on the oscillatory form of its population. Registration of a coarse form by fine dynamics couples the levels: the coarse mode, constituted over fine material by Proposition 7.2, becomes a form on which fine material depends, and the dependence closes a loop between the level that realizes a temporal mode and the level that registers it.

This loop, stated here structurally, is the schema the domain sections instantiate: spiking constitutes an oscillation whose form conditions spiking, and organismic activity constitutes an ecological regime whose form conditions organismic activity. The mode system of a system with levels is therefore a network twice over, joined horizontally by the calibration, refinement, and combination relations of Section 6.3, and vertically by constitution and cross-level registration. On this network the remaining questions of the paper are posed: which of its temporal structure is irreducible to parts (Section 8), what a common forcing contributes to it (Section 9), and what propagates through it (Section 10).

8 Distributed Temporal Emergence

This section defines the paper's strongest emergence concept and its governing criterion. Its objective is to state when system-level temporal order is genuinely a product of relations among heterogeneous parts, and to separate that case from two neighbors with which it is habitually conflated: collective order copied from a shared component, and collective order impressed by an external driver. The section first fixes what a system-level temporal mode is (Section 8.1), then states the definition and its irreducibility criterion (Section 8.2), then derives the separations (Section 8.3), and closes by placing the concept beside the level-relative emergence of Section 7 (Section 8.4).

8.1 System-Level Temporal Modes

The modes of Section 6 were generated over the history of $\mathcal{G}$ at large, and nothing in their definition confines a generating structure to a single entity. A structure may retain the form of one entity's states; it may equally retain a joint relational form spanning many entities, as the population-activity structure of Section 6.1 does.

Definition 8.1. (System-level temporal mode). A temporal mode of $\mathcal{G}$ is *system-level* when its generating structure retains a joint form of the relational configuration, spanning multiple entities or relations, and its classification is registered by multiple entities: the admissible transformations of several X_i depend on the mode's recurrence, ordering, or phase.

A system-level mode is thereby doubly plural: plural in constitution, since the retained form spans parts, and plural in registration, since the dependence on it is distributed. The local temporalities τ_i of Definition 3.2 stand at the opposite pole, constituted over one entity's transformations and registered, in the minimal case, by that entity alone.

8.2 The Definition and the Irreducibility Criterion

Definition 8.2. (Distributed temporal emergence). A system $\mathcal{G}$ exhibits *distributed temporal emergence* when its mode system contains a system-level temporal mode T satisfying three clauses.

(a) *Relational generation.* The generating structure of T retains a joint form to which relations among heterogeneous local dynamics contribute constitutively: deleting the relations from the description deletes the form.

(b) *Component irreducibility.* No single local temporality recovers T: for every i, no registered map defined on τ_i alone induces the partition of occurrences, the ordering, or the phase structure of T.

(c) *Forcing irreducibility.* No external forcing recovers T: for every k, no registered map defined on the form of F_k alone induces the temporal structure of T.

The two irreducibility clauses are the criterion's teeth, and their force should be stated exactly. Each clause quantifies over single sources: it denies that any one τ_i, and any one F_k, suffices. Neither clause denies contribution. Local temporalities and forcings may, and in the central cases do, contribute to a distributed mode; what the criterion excludes is recoverability from any of them alone. Emergence, on this definition, is a claim about the joint in the strict sense: the mode exists in the organization of contributions, and every projection of the organization onto a single contributor loses it.

Remark 8.3. (Irreducibility and content). The criterion is also what protects the concept from vacuity. Deprived of clauses (b) and (c), "the system has a collective temporal order" is satisfied by every driven or cloned ensemble, and the phrase *emergent time* redescribes synchronization. With the clauses, the attribution of distributed temporal emergence excludes definite alternatives, and Section 17 converts the exclusions into interventions: clause (c) is tested by perturbing forcings, clause (b) by perturbing components, and clause (a) by perturbing relations.

8.3 Separations

Proposition 8.4. (Separation from the copied clock). Let every entity of a system share one local temporality: a distinguished τ^* that each X_i accesses, with the collective order the common reading of it. The collective order is then induced by a registered map on τ^* alone, clause (b) fails, and the system exhibits coordination on a shared clock, with no distributed emergence. Heterogeneity of local dynamics is what removes this case from the start: where the τ_i differ in kind, no single one of them is positioned to be τ^*, and a system-level mode, if present, must be carried by the organization of differing contributions.

Proposition 8.5. (Separation from the impressed order). Let a system's collective pattern be recoverable from the form of one forcing: the classification of occurrences by the pattern coincides with their classification by F_k's form. Clause (c) fails, and the system is driven; its temporal order is the forcing's order, registered severally. The separation is a boundary, and where a given system falls with respect to it is an empirical question; Section 9 takes the question up for the case that motivates it, the common celestial forcing over an ecosystem, and Section 17 states the general test.

Remark 8.6. (The computational case). Causal ordering in distributed computation passes the criterion in its ordering form. The order is generated by relations, since it is the transitive closure of local succession together with message transmission, and message relations contribute constitutively; it is recovered by no single process's local order, each process holding only its own events and the messages it touches; and forcings are absent from the setting. Distributed computation thereby supplies the minimal engineered instance of Definition 8.2, in the poorest temporal form, ordering with neither recurrence nor duration; Section 13 develops the case and its limits.

8.4 Placement beside Level-Relative Emergence

Definition 8.2 compares a mode with the system's parts; Definition 7.3 compares a mode with the system's finer descriptions. The criteria are independent. A coarse mode may be emergent across levels while reducible to one component's temporality, as when one entity's rhythm, coarse-grained, organizes the whole; a mode may be distributively emergent at a single level of description, as the computational case is. The conjunction of the two marks the strongest cases the theory recognizes: a mode constituted over fine material that never recurs, and recoverable from no single component and no forcing. The neural and ecological sections argue that the brain's population rhythms and the ecosystem's regime structure are cases of the conjunction, and the argument will rest on the machinery now assembled: coexistence across levels from Proposition 7.2, and irreducibility across parts from Definition 8.2.

9 Common Forcing and the Global Clock Objection

This section answers the strongest objection the theory faces on its ecological ground. The objection runs: the Sun and the Moon are the global clock of every terrestrial system; the temporal order of an ecosystem is impressed by celestial cycles; and talk of emergence adds description, and no structure, to a driven system. The section's objective is to convert the objection into a distinction and the distinction into a test. It first separates three properties the objection runs together (Section 9.1), then exhibits the separation in the oscillator-level description where the objection is most at home (Section 9.2), then states what a common forcing does and does contribute (Section 9.3), and closes with the conversion into a discriminating test (Section 9.4).

9.1 The Trichotomy of Forcing, Clock, and Order

Three properties must be held apart.

A *common forcing* is an F_k acting on many entities: a widely available source whose form, in the celestial case, is recurrent. On the definitions of this paper, a common recurrent forcing supplies a widely available recurrent form, and nothing further. What each entity makes of the form is fixed by that entity's registration: by which distinctions of the forcing its dynamics depend on, through which sensitivities, with which endogenous contribution.

A *shared clock* is what Proposition 8.4 describes: a single local temporality that every entity accesses and reads alike, the collective order being the common reading. Remark 5.9 gave the general reconception on which this rests: a clock is an entity whose recurrent form is registered widely, and its authority is the breadth of shared registration.

An *emergent temporal order* is what Definition 8.2 defines: system-level modes irreducible to any single local temporality and to any forcing.

Claim 9.1. (Common forcing falls short of a shared clock). A common forcing yields a shared clock only under a condition that heterogeneous systems fail: uniformity of registration. Entities register a celestial forcing through heterogeneous sensitivities, light for one organism, temperature for another, the mediated activity of neighbors for a third; through differing phases and endogenous rhythms; and some entities register it barely or through relays alone. The single supplied form becomes many registered forms, differing in retained content, in phase, and in the further structures built on them. Uniform access to one reading, the defining property of the shared clock, is exactly what heterogeneity removes.

Claim 9.2. (A shared clock falls short of emergence, and emergence requires neither). Where a shared clock is present, the collective order it supports is recoverable from it, and clauses (b) of Definition 8.2 fails: a shared clock supplies order by reduction. Conversely, the computational case of Remark 8.6 exhibits distributed emergence with neither clock nor forcing. The three properties are therefore mutually independent along every direction the objection needs, and the inference from "common forcing" to "global clock" to "nothing emerges" fails twice.

9.2 The Separation in the Oscillator Description

The objection is most persuasive in the oscillator-level description, and the separation is visible there too. Under the proviso of Claim 3.3, admit a coarse description in which local temporalities are treated as constituted, each entity carrying a phase θ_i with endogenous contribution ω_i; the standard form is

$$\dot{\theta}_i \;=\; \omega_i \;+\; F_i(S) \;+\; \sum_j K_{ij}\, H_{ij}(\theta_j - \theta_i),$$

with S the common forcing, F_i the entity's registration of it, and K_{ij}, H_{ij} the relational coupling. Three contributions stand in the equation itself: the endogenous, the forced, and the relational. The phenomenology of circadian systems distributes across all three. Endogenous rhythms free-run under constant conditions, at periods differing from the driver's [18]. Collective synchronization arises from coupling and can settle on periods and phase arrangements the driver dictates in no way [11, 24]. And the response to one and the same forcing is governed by the coupling structure: in the mammalian circadian system, coupling among oscillators raises amplitude and rigidity, the strongly coupled central clock entrains over a narrow range where weakly coupled peripheral clocks entrain to extreme cycles, and pharmacological reduction of coupling widens the entrainment range [1]. One forcing, several coupling structures, several temporal organizations: the organization is decided in the system.

The decomposition this suggests,

$$T_{\mathcal{G}} \;=\; T_{\text{forced}} \;+\; T_{\text{endogenous}} \;+\; T_{\text{relational}},$$

is conceptual, and the paper claims it in the conceptual sense alone: the three contributions are separable by intervention (Section 17) and jointly constitutive, while their composition is in general far from additive.

9.3 The Contribution of a Common Forcing

The trichotomy would be misused if it belittled the forcing. What a common recurrent forcing contributes is substantial and specifiable: it supplies a recurrent form of exceptional breadth of availability, and thereby a shared raw material for registration. In the vocabulary of Section 6, the classification of occurrences by the forcing's form is one mode among the system's modes, and a privileged calibrating partner for many others: entities' heterogeneous structures are entrained toward stable ratios with it, and much of the mode system's calibration network runs through it. The claim of this paper is a claim about the remainder: the mode system generically exceeds the forcing's mode. Product modes combine cycles the forcing contains jointly in no single form (Proposition 6.8); coarse regime modes are constituted over joint organismic material (Proposition 7.2); relayed forms, developed in the next section, extend registration to entities the forcing reaches weakly. A common forcing anchors a temporal order it does not exhaust, and the anchor and the excess are, on the present definitions, cleanly distinct objects.

9.4 Conversion of the Objection into a Discriminating Test

The objection, so separated, stops being a rival account and becomes one arm of an experiment. If the temporal order of a system were the forcing's order registered severally, $T_{\mathcal{G}} = f(S)$, then attenuating or removing S would degrade the order toward extinction, and every surviving structure would be noise. The known behavior of coupled systems under constant conditions is otherwise: rhythms free-run at endogenous periods, phase relations among components persist or rearrange, and collective frequencies can survive the driver's removal or settle at new values [18, 1]. Each such survival is a registered structure that clause (c) of Definition 8.2 counts, and the general form of the procedure, perturbing F_k, R_{ij}, X_i, and D_i severally to attribute the system's temporal organization among driver, components, and relations, is the program of Section 17. The Sun, on the account this section completes, is a forcing of unmatched breadth, a calibrating partner of unmatched stability, and a clock for no one: what any entity, and any system, does with its form is constituted in registration and relation, and that constitution is where temporality emerges.

10 The Propagation of Temporal Information through Relations

This section characterizes what moves through a system when temporal organization spreads. Its objective is to replace an unexamined picture, in which temporal coordination consists in the circulation of time values, with an inventory of what actually propagates, and to define the relay, the structure through which one entity's registration becomes temporal material for others. The section first draws up the inventory (Section 10.1), then defines the temporal relay and establishes its four properties (Section 10.2), and closes by characterizing system time as an organization of registered dependencies across the network (Section 10.3).

10.1 Carriers of Temporal Information

What propagates through the relations of a heterogeneous system, when its temporal organization spreads, falls under a short inventory: *events*, registered occurrences that mark; *differences*, registered contrasts between configurations; *forms*, the retained content of equivalence structures, arriving through the states of related entities; *phase relations*, registered positions within recurrent forms; *delays*, the registered separation between a form's occurrence and its registration elsewhere; *sequences*, registered orderings of arrivals; and *signals*, entity-produced forms whose function, in the sense of Section 10.2, is registration by others.

Timestamps are absent from the inventory as a primitive and present as a special case. A timestamp is a signal whose content is a position in some system's calibrated mode; its temporal authority is borrowed, resting on the recurrence of the calibrating system and on the receiver's registration of the convention. Claim 4.4 located this point once: calibration imported through timestamps displaces the requirement of recurrence onto another system and discharges it nowhere. A network can therefore be rich in temporal information while poor in timestamps, and the motivating systems of Section 1 are exactly such networks: an ecosystem propagates light gradients, calls, odors, and activity onsets; a brain propagates spikes, phase relations, and delays; and of the three motivating systems, engineered computation alone runs on timestamps, having built the calibrating recurrence first.

10.2 The Temporal Relay

Definition 10.1. (Temporal relay). An entity X_r is a *temporal relay* for a form ϕ and a set of entities $\{X_j\}$ when three conditions hold: X_r registers ϕ, through direct sensitivity or through prior relays; X_r's registered response produces a form ϕ' of its own; and the dynamics of each X_j depend on ϕ', while depending on ϕ weakly or in no registered way. The relay makes the recurrence of ϕ available, in the currency of ϕ', to entities positioned outside ϕ's direct registration.

The standing example runs from celestial forcing to barnyard. Dawn light is a recurrent form; a rooster's circadian machinery registers it; the crow is the produced form ϕ'; and for organisms asleep in darkness, burrowed, or indoors, whose dynamics depend on the crow and barely on the gradient itself, the rooster is a temporal relay in the sense defined. Four properties of relays carry theoretical weight.

Proposition 10.2. (Transformation). A relay transforms: ϕ' differs from ϕ in medium, in retained content, and in equivalence structure. A gradual luminance gradient becomes a punctate acoustic event; a continuous form becomes a marker. Relayed temporal information is therefore re-formed at every stage, and a chain of relays is a chain of re-articulations, with each stage's equivalence structure selecting anew what returns.

Proposition 10.3. (Amplification). A relay amplifies registration: a form registered by few entities becomes, through the relay's produced form, registered by many. The breadth of registration of a mode, which Remark 5.9 identified as the substance of clock authority, is thereby a network achievement: it is built by relays, and an entity of modest sensitivity can hold wide temporal authority through the reach of its produced form.

Proposition 10.4. (Delay and sequence). A relay introduces registered separation between ϕ's occurrence and ϕ''s production, and chains of relays introduce registered order among arrivals. Relayed propagation therefore generates temporal material of its own: the delays and sequences it creates are registered differences and orderings in the sense of C1 and C4, available for further temporalization. A network of relays is thereby generative in the strict sense of this paper, producing new temporal structure in the course of transmitting existing structure.

Proposition 10.5. (Regeneration). A relay whose producing entity carries endogenous recurrence can sustain ϕ' under degradation or removal of ϕ: the rooster crows, for a while and with drifting phase, in constant darkness. Relayed temporal order is therefore partially autonomous of its sources, and the degree of autonomy is measurable by the interventions of Section 17: what survives source removal is the relay network's own contribution, and clause (c) of Definition 8.2 counts it.

10.3 Temporal Order across the Network

The inventory and the relay together yield the section's characterization. The temporal order of a heterogeneous system consists in the organization of registered temporal dependencies across its network: which entities' dynamics depend on which forms, through which relays, with which transformations, delays, and sequences. System time, so characterized, is the organization of temporal relations across the network, and a value possessed by no component; the system has temporal order in the way a conversation has a topic, held in the organization of contributions and locatable in none of them singly. Every clause of the characterization has been constructed: forms and their return from Section 5, modes and their relations from Section 6, constitution across levels from Section 7, irreducibility across parts from Section 8, and propagation from the present section. What remains is to exhibit the construction in the three motivating domains, where the abstractions of this Part take on the particular shapes of spikes, seasons, and messages; the neural case opens the sequence.

11 Neural Temporal Emergence

This section develops the first domain instantiation. Its objective is to show that the framework's abstractions take definite neural shape, to argue that the brain's population rhythms satisfy the conjunction of level-relative and distributed emergence promised in Section 8.4, and to state, in the modality of a hypothesis, the extension from neural to perceived temporality. The section reads the brain under Definition 3.1 (Section 11.1), locates registration in spike relations (Section 11.2), identifies the operative regime between the two degeneracies (Section 11.3), argues the conjunction claim with its evidential status stated (Section 11.4), and closes with the perceptual hypothesis (Section 11.5).

11.1 The Brain under the System Definition

The brain realizes Definition 3.1 with unusual depth of heterogeneity. Its entities span kinds: ion channels, synapses, neurons of divergent morphology and electrophysiology, glial populations, neuromodulatory systems, and populations at every intermediate grain. Its relations are equally plural: chemical synapses, electrical coupling, neuromodulatory broadcast, and the mediated relations that population activity constitutes for its members. Its local dynamics differ in form and characteristic behavior across all of these, and its forcings comprise sensory input and the circadian input relayed from the retina. Timestamps are nowhere in the fabric: what the tissue traffics in is the inventory of Section 10.1, events, forms, phase relations, delays, and sequences, realized as spikes and their organization.

11.2 Spikes and the Registration of Relational Structure

A spike is a registered difference in the strict sense of Definition 4.1: postsynaptic dynamics depend on it. The temporal wealth of the tissue lies one step up, in relations among spikes, intervals, phase offsets, sequences, coincidences, and these relations are registered with the same literalness. Coincidence-detecting neurons transform their admissible transitions according to the near-simultaneity of arrivals; and spike-timing-dependent plasticity makes the very order and interval of presynaptic and postsynaptic spikes determine the direction and magnitude of synaptic change [4]. Plasticity of this kind is registration of C1 and C4 built into the tissue: the system's own dynamics depend on registered difference and registered order at millisecond grain, and the dependence rewrites the relations R themselves. The brain is thereby generative about its own registration, altering, through what it registers, what it will register.

11.3 The Regime of Structured Variation

The two degeneracies of Section 5.3 mark out the regime in which neural temporality lives. A perfectly periodic spike train realizes recurrence at its most rigid: a form returns, a rhythm is definable against a calibrating structure, and little beyond the rhythm is carried, the classification registering scant contrast from cycle to cycle. A fully irregular train realizes the fine degeneracy: abundant registered difference, no returning form, bare ordering. The operative regime lies between, and its structure is exactly Proposition 7.2. Population-level relational forms recur, a phase relation among populations, an oscillatory envelope, a sequence motif, while the realizations differ throughout: distinct neurons participate, spike counts and amplitudes vary, the fine configuration never repeats. Neural recurrence is recurrence of relational form through differences in realization, and the oscillations documented across frequency bands [5] are, in this framework, coarse temporal modes constituted over unrepeatable fine material.

The relations among these modes instantiate Section 6 directly. Cross-frequency organization, in which faster oscillatory events occupy recurrent positions within the cycle of a slower rhythm, is Definition 6.4 realized in tissue: the faster mode is a registered refinement of the slower, the internal order of events within the slow cycle recurs across cycles, and phase, in the constructed sense, is what the nesting establishes. Calibration relations likewise: the ratio structure among bands is rhythm in the sense of Definition 6.2, one recurrence measured against another, with no band measuring itself.

11.4 The Conjunction Claim for Population Rhythms

Claim 11.1. (The conjunction, neurally). The brain's population rhythms satisfy both emergence criteria. Level-relative emergence (Definition 7.3): the oscillatory mode is generated through coarse-graining of spike configurations, its classification is registered at the population level, and it is constituted over fine material that never recurs; registration also runs downward, since the excitability phases of the population gate the admissible transitions of its member neurons, closing the constitution–registration loop of Section 7.4. Distributed emergence (Definition 8.2): the mode's form is relationally generated, since decoupled neurons yield activity and yield no population phase structure; it is recovered by no single neuron's local temporality, the participating temporalities being heterogeneous and the mode's content being their organization; and it is recovered by no forcing, since coordinated rhythms persist under removal of sensory drive and free-run under constant conditions.

The evidential status of the claim should be stated with the same care as its content. The three clauses rest here on established phenomenology: the coupling dependence of collective neural timing, demonstrated directly in the circadian system, where reduction of coupling reorganizes entrainment itself [1]; the persistence of endogenous rhythms under constant conditions [18]; and the population-level constitution of oscillations [5]. Full verification is interventional, clause by clause, in the design of Section 17, and the circadian decoupling experiments already realize one arm of that design. The claim is put forward as supported and testable, and the paper's commitment to it is the commitment appropriate to that status.

11.5 From Neural Modes toward Perceived Temporality

Hypothesis 11.2. (Perceptual inheritance). Perceived temporality inherits its structure from neural temporal modes: the ordering, recurrence, duration, and phase available to experience are constituted by a further level of the same construction, coarse modes over neural modes, so that the progression runs from spike dynamics through neural temporal modes to perceived temporality.

The hypothesis is stated as a hypothesis, and the paper claims no more for it. Its philosophical ancestry is the neurophenomenological program, which proposed explicit bridges between neural dynamics and the retention–protention structure of time-consciousness [25]; its content, in the present framework, is that the bridge is an instance of Definition 7.3 iterated, experience standing to population dynamics as population dynamics stand to spikes. What would bear on it is likewise stateable: covariation of perceived duration, simultaneity, and order with interventions on the candidate constituting modes, under the perturbative logic of Section 17. Section 19 returns to the hypothesis from the ontological side, where the witnessing of persistence through difference and recurrence receives its own treatment.

12 Ecological Temporal Emergence

This section develops the second domain instantiation, on the ground where the global clock objection was raised and answered. Its objective is to read the ecosystem under the system definition, to construct the morning regime as a coarse temporal mode, to develop the relay ecology through which its registration spreads, to reinterpret phenological mismatch as decoupling within a mode system, and to argue the conjunction claim in the honest form the ecological evidence supports. The subsections follow that order.

12.1 The Ecosystem under the System Definition

An ecosystem realizes Definition 3.1 with heterogeneity of a different distribution than the brain's: entities span organisms of divergent physiology, resources, and environmental fields; relations span trophic, competitive, mutualistic, and signaling connections; local dynamics span endogenous rhythms of widely differing period and sensitivity profiles keyed to light for one organism, temperature for another, the activity of neighbors for a third. Its forcings include the celestial cycles whose standing Section 9 settled: forms of unmatched breadth and stability, registered heterogeneously, clocks for no one. The ecological case is thereby the framework's hardest test on clause (c) of Definition 8.2, and the section conducts it as such.

12.2 The Morning Regime

Fix a coarse-graining Ψ retaining the joint form of illumination gradient, activity onsets across species, and interaction pattern, and a structure $(\Phi \circ \Psi, \simeq)$ classifying coarse configurations with a registered tolerance. Under this structure a form returns each solar cycle: the *morning regime*, the coarse joint form within which light increases, plants open, early foragers move, and the interaction network reconfigures. The construction realizes Proposition 7.2 exactly: no fine morning repeats any other, illumination, temperatures, positions, and physiological states differing throughout, while the coarse form recurs strictly enough for calibration, rhythm against other modes, and phase, the internal order of onsets within the regime, to be constructed on it. Membership in the regime is graded and its onset is extended; the tolerance in $\simeq$ absorbs both, and the framework requires of a form no sharp boundary, only registered return.

Registration of the regime is downward, in the pattern of Section 7.4: organisms register the coarse form through cues, light for some, the activity of others for the rest, and their registered responses are the material of which the regime is constituted. The loop closes as it closed neurally: activity constitutes the regime whose form conditions activity.

12.3 The Relay Ecology of Dawn

The spread of the regime's registration is relay work, and dawn is the richest natural relay network the framework has to point to. The rooster of Definition 10.1 stands for a class: the dawn chorus is a chain of relays in which species with direct photic sensitivity produce acoustic forms on which the activity of others depends, and the chain exhibits all four relay properties of Section 10.2. Transformation: gradients become calls, calls become movement, movement becomes encounter. Amplification: a form registered directly by few becomes registered, in produced currencies, across the community. Delay and sequence: the chorus has internal order, onsets following onsets in stable arrangement, and this generated sequence is itself registered temporal material, a phase structure of the regime in the sense of Definition 6.4. Regeneration: endogenously rhythmic relays sustain their produced forms for a time under altered or absent forcing, contributing the partial autonomy that clause (c) counts.

12.4 Phenological Mismatch and the Decoupling of Modes

Phenological mismatch receives, in this framework, a structural reading. A plant's flowering mode and a pollinator's activity mode are two temporal modes, generated through different registrations, temperature-keyed for the one, photoperiod-keyed for the other, and coupled through a calibration relation: their stable ratio, historically, is what mutualism has run on. A climatic perturbation shifts one forcing's profile; the temperature-keyed mode moves and the photoperiod-keyed mode holds; the calibration relation between them degrades, and interaction fails while each mode, internally, runs on [26]. Mismatch is thereby a breakdown of an edge in the mode system, with both vertices intact, and climatic change is, among its other descriptions, a perturbation of temporal architecture: a redistribution of the calibration relations through which an ecological community's modes had composed.

Claim 12.1. (**Mismatch and plural registration**)**.** Phenological mismatch is direct evidence against the shared-clock reading of ecological time. Under a shared clock, every entity reads one structure, and coordinated timing can degrade only together; differential shift, one mode moving while its partner holds, is possible only where registrations are plural, each mode keyed to its own retained form of its own forcing. The phenomenon the objection's picture cannot accommodate is among the best documented in the field, and its existence is the field's own demonstration that the trichotomy of Section 9.1 cuts where the framework says it cuts.

12.5 The Conjunction Claim for Ecological Regimes

Claim 12.2. (**The conjunction, ecologically**)**.** The regime structure of an ecological community satisfies both emergence criteria, in the following graded form. Level-relative emergence holds as constructed in Section 12.2: the regime is generated through coarse-graining, registered at the community level and through cues below it, and constituted over fine material that never recurs. Distributed emergence holds with clause-wise gradation: relational generation is strong, the regime's form being a joint form of interaction to which no census of isolated organisms amounts; component irreducibility is strong, no organism's temporality recovering the community's regime and sequence structure; forcing irreducibility is real and partial, the celestial mode anchoring the regime's period while the regime's content, its internal sequence, its relayed registration, its calibration network, and the mismatch phenomena themselves, exceeds the forcing's form.

The gradation is the honest statement, and it is also the instructive one: the ecological case shows the criterion working as an instrument with resolution, locating a system between the driven and the autonomous poles, in place of a verdict. What fixes the location empirically is the perturbative program, whose ecological arm the free-running, relay-persistence, and mismatch observations already sketch [18, 26]: the celestial forcing's contribution is measured by what degrades when its registration is masked, and the relational contribution by what survives. The computational case, next, closes the domain sequence from the opposite end: temporal order at its poorest, and its emergence at its most demonstrable.

13 Temporal Order in Distributed Computation

This section develops the third domain instantiation and the cleanest: the engineered case in which distributed temporal emergence is demonstrable line by line, in the poorest temporal form the theory recognizes. Its objective is threefold: to read causal ordering under the framework and complete the argument sketched in Remark 8.6 (Section 13.1), to exhibit the case's temporal poverty as a confirmation of the condition-profile analysis of Section 4 (Section 13.2), and to draw from engineering practice itself the strongest available corroboration of Claim 4.4, before delimiting what the analogue can and cannot teach (Section 13.3).

13.1 Causal Ordering under the Framework

A distributed computation realizes Definition 3.1 in stipulated form: processes as entities, message channels as relations, program dynamics as D, forcings absent in the pure case. Its registered differences are events, and registration is engineered into the semantics: a process's admissible transitions depend on the events of its own history and on the messages it receives, and on nothing else. Lamport's construction takes exactly these dependencies and closes them transitively: local succession within each process, together with the relation of message transmission to receipt, generates the happened-before order [12], and Mattern's vector construction characterizes precisely the global states consistent with it [13]. The construction is the engineered realization of the causal theory of temporal order conceded in Section 2.3 [21], executed in a setting where the causal semantics is stipulated by design.

Read in the framework, the construction is C1 and C4 with nothing added, and the resulting order passes Definition 8.2 in its ordering form. Relational generation: the message relations contribute constitutively, since deleting them shatters the order into disjoint local chains. Component irreducibility: each process's local temporality contains its own chain and its message endpoints, and the global partial order is recovered from no single one of them. Forcing irreducibility: vacuous in the pure case, there being nothing external to reduce to. Distributed computation is thereby the minimal engineered instance of distributed temporal emergence, and its minimality is exact: everything the criterion requires is present, and nothing beyond it.

13.2 Temporal Poverty of the Pure Case

The pure causal order realizes the first row of Table 1 and that row alone. Its profile is C1 and C4: registered events, directed dependence, bare ordering. Recurrence is absent, since the semantics equips events with identity and equips no form with return; and with recurrence absent, everything above the first row is absent with it, by Proposition 4.3: no recurrence classes, no duration, no rhythm, no phase. The case thereby confirms the profile analysis from below, exhibiting a system that possesses exactly the temporality its conditions permit. Logical clocks change nothing in this respect: a Lamport counter is bookkeeping over the dependence order, ordinal through and through, and the "clock" in its name imports a metric the construction nowhere contains. Physical timestamps, where added, import calibration in the manner Claim 4.4 describes, by reference to the engineered recurrence of an external oscillator, displacing the requirement onto the system that ticks.

13.3 Engineered Recurrence and the Limits of the Analogue

The corroboration of Claim 4.4 from practice is direct, and worth stating as such: wherever distributed systems engineering needs more temporality than ordering, it builds recurrence. Timeouts require a local recurrent form, and are implemented on tick counters; failure detection requires registered return, and is implemented on heartbeat protocols, periodic produced forms whose arrival others register and whose lapse is the registered difference that triggers response; coordination requires shared phase, and is implemented in rounds and epochs, engineered recurrence classes within which the internal order of steps recurs, phase in the sense of Definition 6.4 by design. A heartbeat is, in the framework's exact vocabulary, an engineered temporal relay with an endogenous source; a round structure is an engineered mode. Engineering practice, needing duration, rhythm, and phase, constructs precisely the structures the theory says those forms require, and this convergence, arrived at by a discipline with no stake in the theory, is the strongest independent support the condition-profile analysis receives in this paper.

The limits of the analogue are equally definite, and three confine it. Registration is stipulated: the dependence semantics is designed, and the interpretive question of what registers what, which is substantive in tissue and community, is settled here by fiat. The equivalence structures are appointed: rounds and heartbeats exist because designers placed them, and the case therefore exhibits temporal structure's sufficiency conditions while exhibiting nothing about the unplanned enactment of equivalence structures that the natural cases turn on. And the generativity is bounded: a distributed system produces the modes it was built to produce, and the open-ended generation of new modes from relational dynamics, central to the neural and ecological cases, has no engineered counterpart in the standard designs. The analogue is minimal in both directions at once: the least temporality that emerges, and the least emergence in how it comes to be there. Its service to the theory is exactly its cleanliness, a case in which every construct is visible because every construct was placed, and with the domain sequence so completed, the paper turns to its formal generalizations.

14 A Categorical Formulation of Temporalization

This section recasts the theory's constructions in categorical form. Its objective is twofold: to define temporalization as a systematic assignment, from systems to their mode systems, functorial along a specified class of maps; and to obtain from it a structural criterion for created temporal structure, in which emergence appears as the failure of temporalization to commute with the interconnection of systems. The section defines the two categories (Section 14.1), establishes the assignment and its functoriality (Section 14.2), develops interconnection and the compositional remainder (Section 14.3), and closes by stating what the formulation yields and where its development ends and a program begins (Section 14.4).

14.1 The Categories of Systems and of Mode Systems

Write **RelDyn** for the category whose objects are heterogeneous generative relational systems with their histories, as in Definition 3.1, and whose morphisms $f : \mathcal{G} \to \mathcal{G}'$ are *implementation maps*: maps carrying configurations of $\mathcal{G}$ to configurations of $\mathcal{G}'$, preserving admissible transformation, and reflecting registration, in the sense that a distinction registered in $\mathcal{G}'$ pulls back along f to a distinction registered in $\mathcal{G}$. The standing example is a coarse-graining $\Psi : \mathcal{G} \to \mathcal{G}'$ in the sense of Definition 7.1, the fine system implementing the coarse one; inclusions of closed subsystems supply a second family.

Write **Temp** for the category whose objects are mode systems, as in Definition 6.6, and whose morphisms are maps carrying modes to modes and preserving the registered relations among them, calibration, refinement, and combination.

The definitions fix exactly the data the constructions of this paper require, and a boundary should be declared at once. A fully developed treatment would present systems as open, with interfaces, and interconnection as an operation governed by a graphical calculus, in the manner of the compositional dynamical-systems literature [23, 22]; the present section works with closed systems and a direct definition of interconnection, and marks the open-systems development as a program to which that literature is the natural host.

14.2 Temporalization along Implementation Maps

Proposition 14.1. (Functoriality of temporalization). The assignment $\mathcal{T} : \mathcal{G} \mapsto \mathcal{T}_{\mathcal{G}}$ extends to a contravariant functor

$$\mathcal{T} : \mathbf{RelDyn}^{\mathrm{op}} \longrightarrow \mathbf{Temp}.$$

Along an implementation map $f : \mathcal{G} \to \mathcal{G}'$, every registered equivalence structure $(\Phi, \simeq)$ of $\mathcal{G}'$ pulls back to the structure $(\Phi \circ f, \simeq)$ of $\mathcal{G}$, which is registered because f reflects registration; the pullback preserves recurrence classes occurrence-wise, and carries calibration and refinement relations to calibration and refinement relations. The induced map $\mathcal{T}(f) : \mathcal{T}_{\mathcal{G}'} \to \mathcal{T}_{\mathcal{G}}$ is the action of $\mathcal{T}$ on f, and it satisfies $\mathcal{T}(g \circ f) = \mathcal{T}(f) \circ \mathcal{T}(g)$.

Contravariance carries the right meaning: the temporal structure of an implemented system is visible within any system that implements it. Section 7.1 was the instance all along, coarse temporality read at the fine level as classification through Ψ; the functor states the instance as a law, and gives the later constructions a uniform vocabulary of restriction.

14.3 Interconnection and the Compositional Remainder

Definition 14.2. (Interconnection). Let $\mathcal{G}_A$ and $\mathcal{G}_B$ be systems and let R_∂ be a set of *interface relations*, each spanning entities of both. The interconnection $\mathcal{G}_A \bowtie \mathcal{G}_B$ is the system whose entities are those of both parts, whose relations are those of both parts together with R_∂, and whose dynamics are the parts' dynamics as modified by the dependencies the interface relations introduce.

Definition 14.3. (Part-local and joint modes). A mode of $\mathcal{G}_A \bowtie \mathcal{G}_B$ is *part-local* when its generating structure retains a form of one part's configurations alone; it is *joint* when its generating structure retains a form to which the interface relations contribute constitutively, so that deleting R_∂ from the description deletes the form. The *compositional remainder* of the interconnection is its family of joint modes, with the registered relations among them.

Proposition 14.4. (Two failures of compositionality). Compare $\mathcal{T}(\mathcal{G}_A \bowtie \mathcal{G}_B)$ with what $\mathcal{T}(\mathcal{G}_A)$ and $\mathcal{T}(\mathcal{G}_B)$ jointly provide. The comparison fails to be an equivalence through two mechanisms, separately realizable. First, *modification*: interconnection alters the parts' dynamics, and thereby the part-local modes themselves; a component's recurrence entrains, shifts, or dissolves under coupling, as the circadian evidence of Section 9.2 documents directly [1]. Second, *creation*: the compositional remainder is in general inhabited, and no functorial image of the parts' mode systems reaches it, a joint mode's generating structure factoring through neither part. The comparison is an equivalence exactly where both mechanisms are absent, and the interconnection is, in the terms of Definition 8.2, temporally inert.

Remark 14.5. (**Emergence as obstruction, and its sign**). The situation of Proposition 14.4 has the shape studied in the categorical literature on compositionality, in which the failure of a lax comparison to be strong is the formal locus of non-compositional behavior, and obstructions to the comparison's invertibility are the objects classified [20]. Two points of that literature transfer exactly. The failure is structured, admitting classification and measurement, and Section 17 builds the measurement. And the failure carries no fixed sign: in the present setting it is the object of study itself, since the compositional remainder is, in categorical dress, precisely clause (a) of Definition 8.2, the relationally generated temporal structure that interconnection creates and projection onto parts loses. Compositionality holds where nothing temporally new happens; the theory's subject matter is the other case.

14.4 Scope and Yield of the Formulation

The formulation yields three things the informal development lacked. It makes emergence a property of a definite comparison map, assessable interconnection by interconnection, in place of a global slogan. It houses the combination of modes: the pairing of structures in Proposition 6.8 is the image of a monoidal pairing, and product modes arise functorially from it. And it prepares the local-to-global analysis of the next section, whose covers, restrictions, and gluings are the functor of Proposition 14.1 put to sheaf-shaped use. The boundary of the section is equally definite: it supplies a formulation and its first propositions, on closed systems and a direct interconnection, and the full development, open systems, a graphical calculus of interconnection, and the classification of remainders, is a program, undertaken here to the depth the theory requires and no further.

15 Local-to-Global Temporalization and Its Obstructions

This section poses the theory's large-scale question: whether the local temporal structures of a system's parts assemble into a temporal structure of the whole. Its objective is to define the assembly problem sheaf-wise, to separate three global statuses whose conflation underlies loose talk of "the system's time," and to exhibit the obstruction that separates the weakest status from the middle one, computably in the case of phase data. The section defines covers and local temporal data (Section 15.1), classifies the global statuses (Section 15.2), computes the obstruction for phase data (Section 15.3), and closes with the ordered trichotomy, the large-scale regime in which it bites, and the bridge it builds to the companion paper (Section 15.4).

15.1 Covers and Local Temporal Data

Let $\{\mathcal{G}_\alpha\}$ be a cover of $\mathcal{G}$ by closed subsystems, with overlaps $\mathcal{G}_{\alpha\beta}$, and let inclusions supply implementation maps in the sense of Section 14.1. By Proposition 14.1, restriction along inclusions carries the temporal data of a subsystem to the temporal data of its subsubsystems:

the assignment $\alpha \mapsto \mathcal{T}_{\mathcal{G}_\alpha}$, with these restrictions, is a presheaf of mode systems on the cover. A *local temporal datum* over α is a piece of that structure: a mode of $\mathcal{G}_\alpha$, an ordering of its occurrences, or an assignment of phase within a shared recurrent form. A family of local data $\{t_\alpha\}$ is *compatible* when its members agree on every overlap after restriction. The assembly problem is the descent question: whether a compatible family arises by restriction from a global datum over $\mathcal{G}$, and the answer's failure modes are the section's subject.

15.2 Compatibility and the Classification of Global Statuses

Definition 15.1. (Global statuses). With respect to a cover and a kind of temporal datum, $\mathcal{G}$ occupies one of the following statuses.

(a) *Global clock.* A single local temporality τ^* exists whose restriction to each $\mathcal{G}_\alpha$ is accessed by every entity there: one structure, universally read. This is the shared clock of Proposition 8.4, posed cover-wise.

(b) *Global relational temporal order.* Descent holds: every compatible family of the given kind glues to a global datum, unique up to registered equivalence. The system has a temporal structure of the whole, carried by relations, with no requirement of a universally read component.

(c) *Temporal frustration.* A compatible family exists with no global datum: the parts' temporal structures agree wherever they meet, and assemble into nothing.

The middle status is the theory's own: it is what Sections 8 through 10 have been describing, distributed temporal order as a global section carried relationally. The first status implies it trivially, and the computational case separates them from below, a Lamport system possessing a global partial order, glued from local chains through message relations, with no universally read structure anywhere [12]. The third status is the genuinely new object this section adds, and the next subsection shows it is realizable and detectable.

15.3 Computation of the Obstruction for Phase Data

Let a recurrent form be shared across the cover, and let the datum over α be an assignment of phase within it, valued in an abelian group P of phase differences, with each overlap contributing a registered offset $\delta_{\alpha\beta} \in P$: the phase relation between the parts where they meet. Compatibility fixes the offsets pairwise; the assembly question is whether phases $\{p_\alpha\}$ exist with $p_\beta - p_\alpha = \delta_{\alpha\beta}$ throughout.

Proposition 15.2. (Holonomy obstruction). A compatible family of phase offsets glues exactly when its holonomy vanishes on every cycle: for each closed chain $\alpha_0, \alpha_1, \ldots, \alpha_n = \alpha_0$ of overlapping parts,

$$\delta_{\alpha_0\alpha_1} + \delta_{\alpha_1\alpha_2} + \cdots + \delta_{\alpha_{n-1}\alpha_0} \;=\; 0 \quad \text{in } P.$$

The offsets form a Čech 1-cocycle on the nerve of the cover with coefficients in P; gluing succeeds exactly when the cocycle is a coboundary; and the obstruction is its class in the first Čech cohomology of the nerve. Temporal frustration, for phase data, is a nonvanishing class, and it is computable: a sum around each independent cycle of the cover.

The realizability of the nonvanishing case requires no exotic construction. Three subsystems, pairwise locked at stable registered offsets whose sum around the triangle differs from zero in P, are locally consistent everywhere and globally phase-assignable nowhere;

coupled-oscillator physics knows the configuration as frustration, and the present framework inherits the name with the mathematics. What the framework adds is the reading: a frustrated system has locally constituted temporality of full wealth, recurrence, calibration, rhythm, and phase within every part, and possesses, for the frustrated datum, no fact of the form "the phase of the whole."

15.4 The Ordered Trichotomy and the Large-Scale Regime

Proposition 15.3. (**Strict ordering of the statuses**). Global clock implies global relational temporal order, and global relational temporal order implies the compatibility of every family that restriction produces; both implications are strict. The computational case witnesses the first gap, coherence with no universally read structure; the frustrated triangle witnesses the second, compatibility with no coherence.

The trichotomy is where the title's qualification earns its place. In a small system, covers are poor in cycles, descent is cheap, and the three statuses crowd together; the assembly problem trivializes, and "the system's time" is harmless speech. Scale changes the topology: as a heterogeneous system grows, its covers acquire independent cycles, local consistency ceases to imply global assembly, and the frustrated status becomes available and, under generic offsets, prevalent. Large-scale heterogeneous systems are thereby the regime in which the distinctions of this section stop being pedantry: a brain-scale, ecosystem-scale, or civilization-scale system can be temporally rich in every part, coherent along every overlap, and possessed of no global temporal fact of a given kind, and the question "what time is it for the system" can fail to have an answer for structural reasons, with nothing anywhere malfunctioning.

Two positionings close the section. Against the nearest formal neighbor: the sheaf-theoretic treatment of dynamical systems in the compositional literature places time in the base, behaviors forming sheaves over temporal intervals, with interconnection analyzed above that base [22]; the present construction places the system in the base and asks whether temporal structure glues in the fibers, and the two are complementary in exactly the direction the constitution wall of Claim 3.3 predicts, the one presupposing the temporal substrate the other constructs. And toward the companion paper: the inter-register tensions that paper studies, an episode closed in one register and continuing in another, a crisis with a legal end and an economic persistence, are this section's third status in the civilizational case, compatible local temporal data over institutional subsystems with no global section; temporal frustration is, there, a lived condition, and the mathematics of this section is offered as its structural description.

16 Relation to Physical Theories of Emergent Time

This section conducts the comparison promised in Section 1, and conducts it within the bound declared there. Its objective is to state what the theory shares with the relational programs in physics, to fix what it takes from them, which is one demonstrated possibility and nothing further, and to state the disanalogies and the resulting logical independence as firmly as the resonances. The section presents the programs (Section 16.1), the resonances (Section 16.2), and the disanalogies with the independence claim (Section 16.3).

16.1 The Relational Programs in Physics

Three programs define the comparison class. The Page–Wootters mechanism considers a globally stationary quantum state and recovers effective dynamical evolution from correlations between a clock subsystem and the remainder: evolution appears in the conditional structure, with respect to clock readings, of a whole that evolves in nothing external [17]. The thermal time hypothesis derives a flow of physical time from the statistical state of a generally covariant system, through the modular automorphism group the state induces, so that which flow is the temporal one depends on the state [6]. The Machian and shape-dynamical programs eliminate external time from classical dynamics, treating time as an abstraction from change and recovering temporal description from the relational configurations of the universe [2]. All three demonstrate, within their domains, a single proposition of general import: temporal description can be derived from relations, with no external time parameter consumed in the derivation. That proposition is what Section 1 took, and it is all this paper takes.

16.2 Structural Resonances

Three resonances are worth recording, each an alignment of shape between constructions built for different purposes. The Page–Wootters clock subsystem occupies the position that Remark 5.9 assigns to clocks in general: a distinguished part whose states the rest is read against, its temporal authority residing in the correlational structure that makes the reading possible, and in nothing intrinsic to the part. The state-dependence of thermal time parallels the structure-relativity of Proposition 5.7: as the modular flow varies with the state, so recurrence varies with the registered equivalence structure, and in both frameworks the question "which is the time" is well posed only relative to something, a state there, a structure here. And the Machian abstraction of time from change is the same move as the primitive declaration of Section 3.1, executed at the level of fundamental physics: both frameworks accept change and construct temporal organization, and neither explains the occurrence of change itself.

16.3 Disanalogies and the Independence Claim

The disanalogies are of domain, of direction, and of apparatus, and each blocks a transfer someone might hope to make. Domain: the physical programs concern fundamental time, in quantum-gravitational and statistical-mechanical settings; this theory concerns temporal organization within systems whose physical dynamics it presupposes, and no construction in this paper touches the constitution of spacetime. Direction: the theory builds upward from change to organization, and its systems inhabit whatever physical time there fundamentally is; a derivation of that substrate is neither attempted nor implied, and the phrase *temporal emergence*, fixed in Section 1, was chosen to keep the ontological register modest. Apparatus: registration, the internality requirement on which every construction from Definition 4.1 onward stands, has no counterpart in the physical programs, whose derivations are conducted from outside the systems they describe; the physical programs, conversely, command quantum-theoretic and operator-algebraic structure this framework nowhere assumes.

Claim 16.1. (Independence). The theory of this paper and the relational programs in physics stand in mutual logical independence. The success of Page–Wootters, thermal time, or Machian dynamics would establish nothing about temporal emergence in heterogeneous relational systems, and their failure would refute nothing in it; conversely, every result of this paper is compatible with any resolution of the physical question of fundamental time. What the programs supply is precedent for the derivability of temporal description from relations; precedent is an existence proof of a kind of explanation, and carries no evidential weight for either side.

One question of traffic in the other direction is recorded as a question. The constructions of Sections 14 and 15 apply wherever subsystem decompositions and correlational data exist, and whether decompositions in Page–Wootters settings exhibit the frustrated status of Definition 15.1, locally consistent conditional dynamics with no global section, appears to be well posed. The paper poses it and claims nothing about its answer.

17 Perturbative Discrimination of Temporal Organization

This section delivers the operational component promised throughout: the program by which the theory's central attributions are tested by intervention. Its objective is to convert the clauses of Definition 8.2 into perturbation classes, to define the three regimes of temporal organization the perturbations discriminate, to assemble the protocol with its already-existing realizations, to state candidate measures of temporal emergence, and to state what would refute the theory. The subsections follow that order.

17.1 The Logic of Intervention

Remark 8.3 fixed the correspondence on which the program runs: each clause of the emergence criterion excludes a definite alternative, and each exclusion is tested by perturbing what the alternative would make responsible. Perturbation of forcings F_k, masking, removal, or reprofiling, tests clause (c): what a driven order owes to its driver degrades with the driver. Perturbation of components, silencing or removing entities X_i or altering local dynamics D_i, tests clause (b): what a copied order owes to its source degrades with the source. Perturbation of relations R_{ij}, attenuating, severing, or rewiring, tests clause (a): what a relationally generated order owes to its relations degrades with them.

Definition 17.1. (Regimes of temporal organization). Fix a system, a target temporal mode T, and a degradation measure over T's registered structure, its recurrence-class integrity, calibration stability, and phase coherence. The organization of T is *driver-dominated*, *component-dominated*, or *relation-dominated* according to which perturbation class produces the dominant degradation, with mixed regimes admitted and located by the degradation profile

$$(d_F,\ d_X,\ d_R)$$

across the classes.

The regimes are attributions of responsibility for a mode, made by intervention, and they hold mode by mode: one system may carry a driver-dominated daily mode and a relation-dominated regime structure at once, and the profile is the instrument that says so.

17.2 The Protocol and Its Existing Realizations

The protocol, for a target mode: identify the generating structure and its registered classification; apply each perturbation class severally, at graded strength where the system permits; measure the degradation profile; and attribute the regime. Table 2 assembles the classes, the clauses they test, and the realizations that already exist, since the program's first arms were run, under other descriptions, decades before this framework named them.

Natural experiments extend the table where designed intervention is unavailable. Phenological mismatch is a forcing reprofiling administered by climate: one forcing's profile shifts, one mode moves, a calibration edge degrades, and the community's degradation profile is read from the observational record [26]. The ecological arm of the program is, in substantial part, already running in the field's data.

Table 2: The perturbation classes, the clauses of Definition 8.2 they test, and existing realizations. Each realization was conducted within its own discipline's framing; the table records the reading this framework gives them.

Perturbation class	Clause tested	Existing realizations
Forcing: masking, removal, reprofiling	(c) forcing irreducibility	Constant-condition protocols; free-running rhythms at endogenous periods under removed zeitgebers [18]
Component: silencing, removal, dynamics alteration	(b) component irreducibility	Lesion and silencing studies of rhythm persistence; relay-source removal with persistence of relayed order (Proposition 10.5)
Relation: attenuation, severing, rewiring	(a) relational generation	Pharmacological decoupling in the SCN, reorganizing entrainment range under an unchanged forcing [1]; message suppression in distributed systems, shattering causal order into local chains

17.3 Candidate Measures of Temporal Emergence

Two candidate measures are stated, both in candidate status: the program's quantitative core is future work, and the paper claims the definitions' coherence, and their measurability in principle, and nothing further.

Candidate 17.2. (Relational share). For a target mode with degradation profile (d_F, d_X, d_R), the *relational share*

$$\rho(T) \;=\; \frac{d_R}{d_F + d_X + d_R}$$

locates the mode on the axis from driven, through copied, to relationally generated organization. Distributed temporal emergence, in the sense of Definition 8.2, predicts ρ bounded away from zero for the system-level modes it covers, and the prediction is mode-specific and graded, in the manner Claim 12.2 exemplified.

Candidate 17.3. (Temporalizing position). For a registered equivalence structure, Claim 5.6 located temporalizing capacity strictly between the coarse and fine degeneracies. The position admits information-theoretic expression: writing C for the structure's classification of occurrences and D for the registered dynamical responses, the mutual information $I(C; D)$ vanishes at both degeneracies, the classification registering no contrast at the coarse limit and inducing no reusable dependence at the fine one, and a structure's temporalizing position is measured by $I(C; D)$ against its maximum over structures of its family. The candidate requires, before use, the estimation apparatus its terms presuppose, and it is recorded here as the natural quantitative form of Claim 5.6.

The two candidates measure different things, and their conjunction is the program's target: ρ attributes a mode's organization among sources, and $I(C; D)$ grades a structure's capacity to temporalize at all. A system's temporal emergence, fully quantified, would be a profile over its mode system in both coordinates, and the compositional remainder of Section 14.3 predicts where in that profile the interconnection-created structure sits.

17.4 Falsifiability

The theory's exposure is stated plainly. Its central claim, Hypothesis 1.1 as developed through Definitions 8.2 and 17.1, predicts that relation-dominated temporal organization exists: that for some system-level modes in some heterogeneous systems, the degradation profile loads on d_R, order degrading under relational perturbation while components and forcings stand. The claim fails empirically if the profiles collapse everywhere onto the other coordinates: if, across the neural, ecological, and engineered cases, every collective temporal mode proves driver-dominated or component-dominated, with ρ indistinguishable from zero throughout, then relational generation is idle vocabulary and the framework describes nothing the oscillator and entrainment pictures had missed. The existing realizations of Table 2 argue that this is already known to be false, the decoupling result alone exhibiting a reorganization of temporal order under relational intervention with the forcing untouched; the systematic profile, mode by mode and system by system, is the program, and the theory stands or falls with its outcome.

18 The Progression from Difference to Multi-Scale Temporality

This section assembles the paper's answer to its opening question. Its objective is to state the progression through which change becomes time, with each arrow now resting on constructed machinery, and to record the two refinements the development has forced on the progression's simple form. The progression is:

$$\boxed{\begin{array}{c}\text{difference} \rightarrow \text{comparability} \rightarrow \text{relational recurrence} \rightarrow \text{ordering} \\ \rightarrow \text{temporal scale} \rightarrow \text{multi-scale temporality.}\end{array}}$$

Each arrow now carries its warrant. Registered difference is the floor beneath everything, established as necessary for every temporal form by the undifferentiated limit (Proposition 4.2). Comparability is the threshold above bare ordering, established by the incomparable limit (Proposition 4.3), and it is enacted, within a system, by registered equivalence structures (Definition 5.1). Relational recurrence is comparability's contentful case, the return of form across differing realizations (Definition 5.2), genuine exactly between the two degeneracies (Claim 5.6). Ordering, calibration, rhythm, and phase are constructed on recurrence and its relations (Claim 4.4, Definitions 6.2 and 6.4). Temporal scale is a position in the calibration order among modes (Definition 6.7), and multi-scale temporality is the mode system entire, generated by the plurality of registered structures (Proposition 6.5), by their combination (Proposition 6.8), and across levels of description by coarse-graining (Proposition 7.2), with its distributed cases governed by the irreducibility criterion (Definition 8.2) and its global assembly governed by descent (Definition 15.1).

Two refinements qualify the simple line. First, the progression has a documented branch: ordering is available on a short path, from registered difference and directed transformation alone, with no recurrence consumed, and the computational case walks that path in isolation (Section 13); the main line runs through recurrence because everything above ordering does. Second, every arrow is conditional: on registration, without which a stage exists for a describer and for nothing in the system, and on non-degeneracy, without which the enacted structures classify vacuously. The progression is therefore a ladder of possible achievements, each with stateable conditions and, by Section 17, testable attributions, and a system stands on the rung its registered structures have built.

So the opening question receives a differentiated answer. Change becomes time by degrees and by constructions: at the registration of difference, contrast; at directed dependence, order; at the registered return of form, comparability, calibration, rhythm, and phase; at the plurality and combination of structures, scale; at coarse-graining and coupling, the multi-scale, distributed temporality of large heterogeneous systems. There is no single crossing from change to time; there is a sequence of constitutions, and "time," in the systems that motivated this paper, names the accumulated structure of all of them.

19 Difference, Repetition, and Existence

This section draws the theory's ontological consequence. Its objective is to show that the machinery built for temporal emergence settles, as a by-product, a classical question about persistence, and that the two achievements, the emergence of time and the individuation of what continues, are one achievement described twice. The section states the problem of persistence through change (Section 19.1), gives the framework's account of identity through registered invariants (Section 19.2), establishes the self-involving character of registered recurrence, which is the formal content of witnessing (Section 19.3), and closes with the co-constitution claim (Section 19.4).

19.1 The Problem of Persistence through Change

An entity of a generative relational system exists in no other way than by changing: its configurations differ throughout its history, and in the systems of interest the fine configuration never repeats (Proposition 7.2). The classical problem follows immediately, and the framework poses it internally: what makes differing configurations configurations *of one continuing entity*, for the system itself, with no describer supplying the identity from outside? The philosophical tradition has held the question since its beginning, and its modern forms, the persistence of the changing, the identity of the ship rebuilt plank by plank, are questions this paper inherits in registered form: identity conferred by an external observer is exactly what Definition 4.1 taught the theory to set aside.

19.2 Identity through Registered Invariants

The account is already in hand, and needs only to be turned around. Proposition 5.8 divided the vocabulary of return: occurrences pass and are unrepeatable; forms return, their identity being their equivalence class. An entity's continuing identity, within the system, is of the second kind: it is carried by relational forms of the entity that return, under registered structures, across the registered differences of its history. The continuant is, internally, a recurrence structure: what makes today's configuration and yesterday's configurations *of the same organism*, for the system in which it lives, is the return of registered relational form, metabolic, morphological, behavioral, positional, across everything that differs between them. Different configurations become mutually intelligible, ascribable to one continuant, exactly where relational invariants connect them; where no registered form spans two configurations, the system contains, in the strict sense of Proposition 4.3, two unconnected occurrences and no persisting thing.

19.3 The Self-Involving Character of Registered Recurrence

Proposition 19.1. (Self-involvement). Registration of recurrence is self-involving. Let an entity X_i register a repetition: by Definition 5.1, some dynamic of X_i depends on $\Phi(\mathcal{R}_a) \simeq \Phi(\mathcal{R}_b)$ with $a \neq b$. A dependence on the relation between two occurrences is carried by no state confined to either: it requires structure of X_i that spans the interval, a trace, a retained sensitivity, an accumulated disposition, connecting X_i's own states across the registered difference. In registering a return in its world, an entity therefore enacts a registered connection across its own history; the recurrence it registers outside is carried by a persistence it maintains inside, and each is registered exactly through the other.

Proposition 19.1 is the formal content of witnessing. An organism that registers the day's return relative to the calls, movements, and cycles of others thereby carries, in the very structure that registers, a registered connection between the organism that encountered the earlier occurrence and the organism that encounters the later one: its registration of the world's recurrence is, at the same time and through the same structure, registered evidence of its own continuation. The cellular case exhibits the identical form at another scale: spike-timing-dependent plasticity is a trace spanning spike occurrences (Section 11.2), a neuron's registration of temporal relation carried by an alteration that connects its own past and present, time encoded, in the tissue, in the repetition and difference of spikes. The claim holds with its register stated: for systems with experience, the witnessing may be lived, and Hypothesis 11.2 marks the framework's route to that question; for systems without, witnessing is the registered self-involvement of Proposition 19.1, and the paper claims exactly that and nothing about what it is like.

19.4 The Co-Constitution of Time and Continuants

Claim 19.2. (Co-constitution). Within a relational system, temporal modes and continuant identities are constituted by the same registered recurrence structures. The structures through which a form's return generates recurrence classes, calibration, rhythm, and phase are the structures through which what persists is individuated as persisting; the emergence of time and the individuation of continuants are two descriptions of one relational achievement.

The claim reverses an old picture in a definite direction. On the old picture, entities persist *in* time, the temporal container standing ready and the persistence being a further fact within it. On the account this paper has built, there is no container to stand ready: temporal structure and persisting entities arise together, from difference that can be registered and form that can return, and neither is prior, each being articulated through the structures that articulate the other. The philosophical lineage of the section's title holds the two halves separately, existence bound to temporality in the one tradition, time derived from difference and repetition in the other [8, 7]; what the present framework adds is the mechanism that holds them together, and a closing formulation is available in one sentence. Entities let a time emerge from the repetition and difference structures of their relations with others, and in doing so they witness their own existence.

20 Conclusion

This section states the theory's results in final form. The paper set out from the question of what minimal relational conditions allow temporality to emerge from heterogeneous change where no universally shared clock exists, and developed one hypothesis, that temporal emergence occurs through relational invariance across generative difference, through minimal-condition analysis, formal construction, three domain instantiations, categorical and sheaf-theoretic generalization, and an operational program. The development closes in the three propositions promised in Section 1, each now carrying its constructed warrant.

Proposition 20.1. (Difference enables temporal differentiation). Registered difference is the necessary floor of every temporal form: within a system whose configurations are indiscernible under registration, ordering, recurrence, duration, and phase are undefined (Proposition 4.2), and every construction of the paper begins from registered contrast.

Proposition 20.2. (Relational recurrence enables temporal comparability). The return of relational form across differing realizations (Definition 5.2), genuine strictly between the coarse and fine degeneracies (Claim 5.6), is what raises a history above bare ordering: recurrence classes, internal calibration of intervals (Claim 4.4), rhythm as stable ratio between recurrences (Proposition 6.3), and phase through registered refinement (Definition 6.4) are all built on it, and nothing above the first row of Table 1 exists without it.

Proposition 20.3. (Coupling enables distributed system-level temporality). Relations among heterogeneous components generate temporal structure recoverable from no single component and no external forcing: joint modes through interconnection (Proposition 14.4), product modes through combination (Proposition 6.8), coarse modes over unrepeating fine material (Proposition 7.2), relayed registration across networks (Definition 10.1), and, in the strongest cases, distributed temporal emergence under the irreducibility criterion (Definition 8.2), with global assembly a further, separately obstructed achievement (Definition 15.1).

Three things pass out of the paper. To the companion paper on temporal architectures, the theory hands its ontology: temporal modes as definite generated structures, mode systems as the vertices and first edges of the architectures whose cultural coupling that paper studies, and temporal frustration as the structural description of inter-register tension. To empirical work, it hands the program of Section 17: regime attribution by intervention, the relational share, the temporalizing position, and the stated conditions under which the framework would be refuted. And to the philosophy it began from, it hands back the lineage's insight with a mechanism attached: repetition that returns the different, formalized as relational invariance across registered difference, generating scale upon scale of temporal structure in systems that share no clock, and constituting, in the same achievement, the continuants whose persistence it makes intelligible. Time becomes possible where difference can return without becoming identical.

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